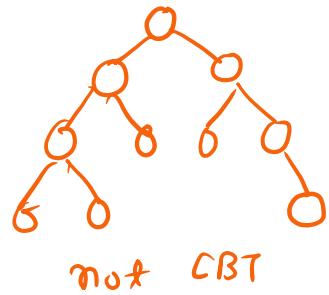
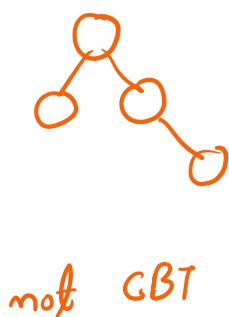
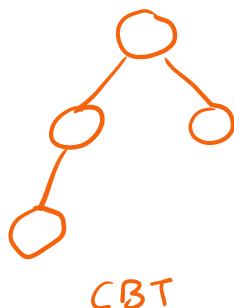
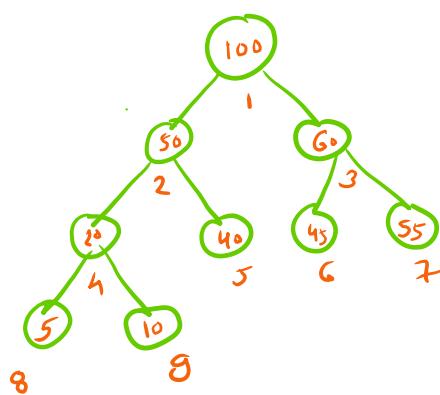


→ Heap → D.S.  
 ↳ complete Binary Tree.  
 ↳ that holds a Heap property  
 CBT  
 ↳ all levels are completely filled (except last one)  
 (leaf → right build)



Creation in array form.  
 but visualization is tree form.

Max-Heap → Parent are both child che  
 min-Heap → Parent are both child che  
 max  
 min.



using Arrays

1 sc start  
 parent = i  
 left child index =  $2*i$   
 Right child index =  $2*i+1$   
 child → parent  
 index =  $i/2$

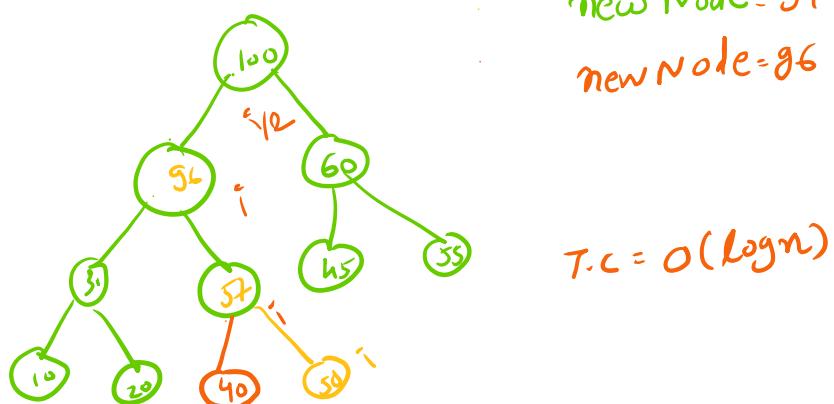
0 sc start  
 i  
 left =  $2*i+1$   
 right =  $2*i+2$



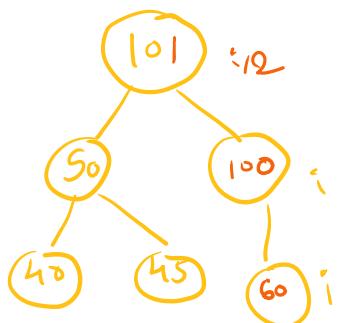
using Arrays

-1	100	50	60	20	40	15	55	5	10
0	1	2	3	4	5	6	7	8	9

insertion

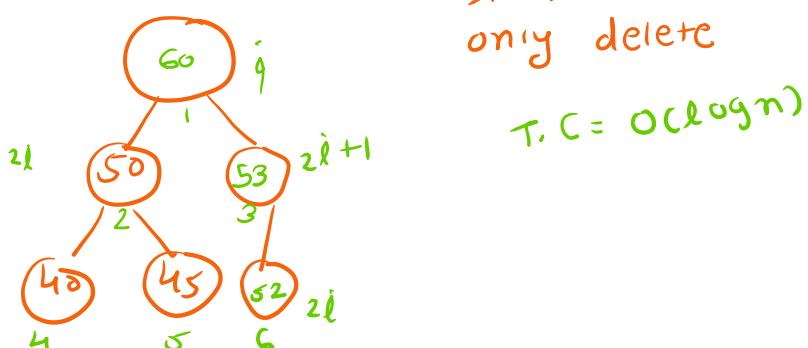


- (1) Add node at the end of array
- (2) swap place push letar suo



-1, 100, 50, 60, 40, 45

Deletion

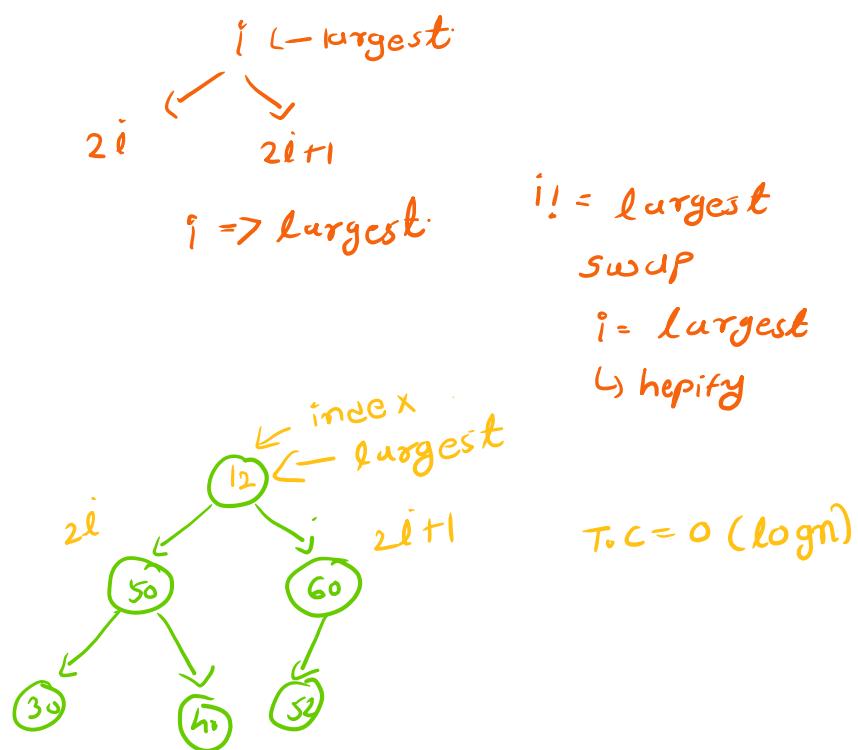




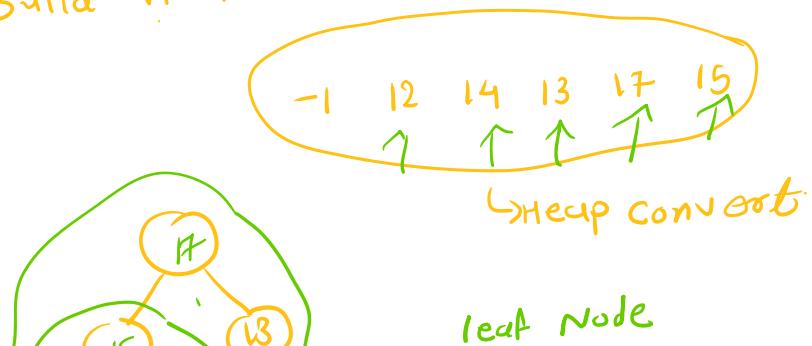
- ① Replace last value with root node.
- ② Root node ko right (correct) place par replace

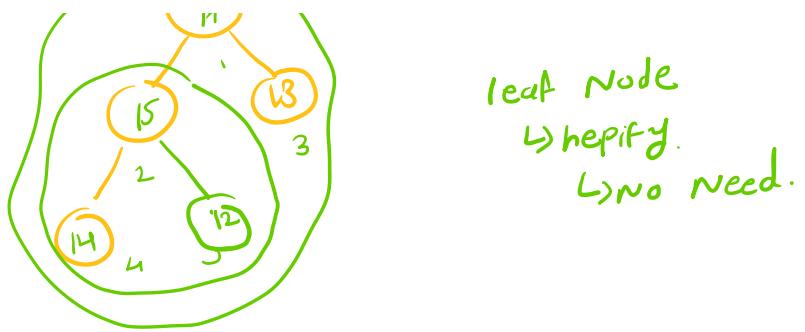
heapify :- Node ko sahi place par  
 ↑ place karna  
 any error use convert to Heap  
 this function using

Heapify      Heapify(492, size, index)



Build Heap





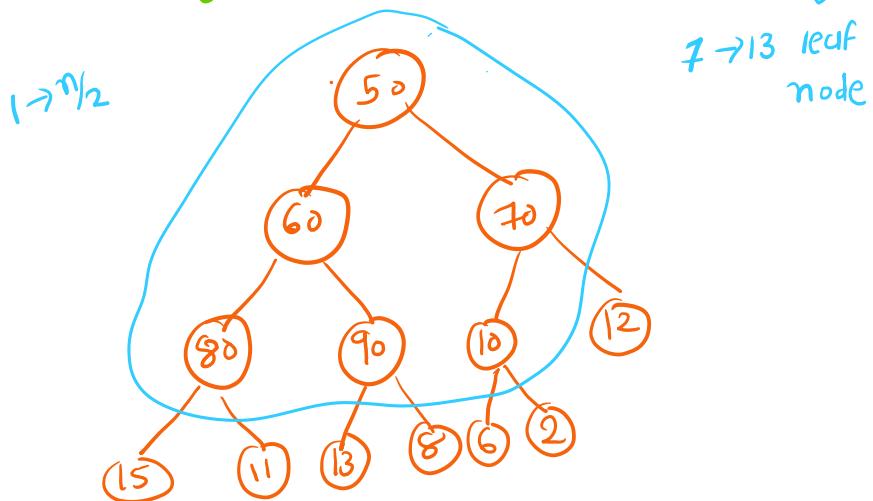
$\left(\frac{n}{2} + 1\right) \rightarrow n \rightarrow$  all node use  
 ↳ leaf node.

$$\left(\frac{5}{2} + 1\right) \rightarrow 5$$

$$3 \rightarrow 5$$

$\frac{n}{2} \rightarrow > 0$  heapify

no need to  
 heapify ↳  
 $7 \rightarrow 13$  leaf  
 node



T.C =  $O(n)$

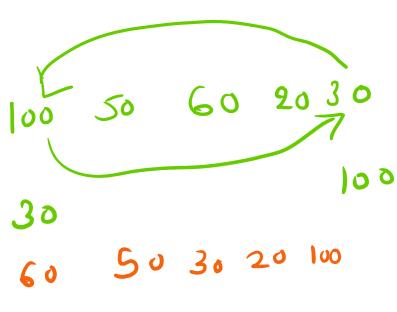
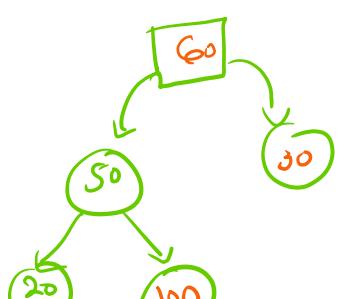
buildheap

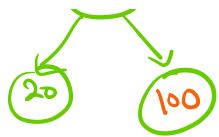
for ( $\frac{n}{2} \rightarrow > 0$ )

    2 heapify()

    3

Heap Sort T.C =  $O(n \log n)$





- ① First element / last element  
Scrap
- ② root node ko sahi place par replace  
karna (heapify)

## Priority-queue

↳ Heap behaviour

→  $k^{\text{th}}$  smallest no.  $\xrightarrow{\text{size.}}$   
 i/p arr →  $\boxed{3 \mid 6 \mid 9 \mid 12 \mid 11 \mid 4 \mid 2 \mid 8}$

#1 arr sort

sort(arr, arr+1)  
 ↳  $k^{\text{th}}$  ele → arr[k-1]

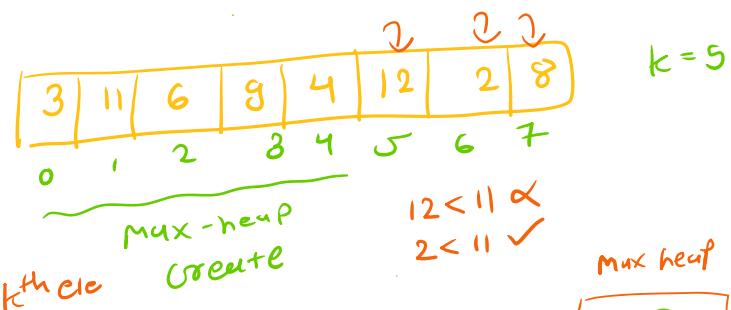
$$\text{T.C} = n \log n$$

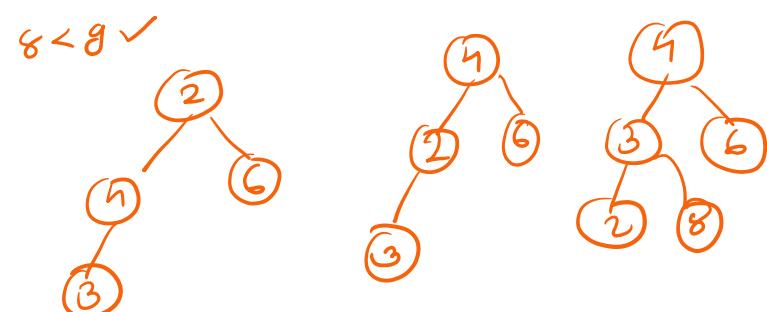
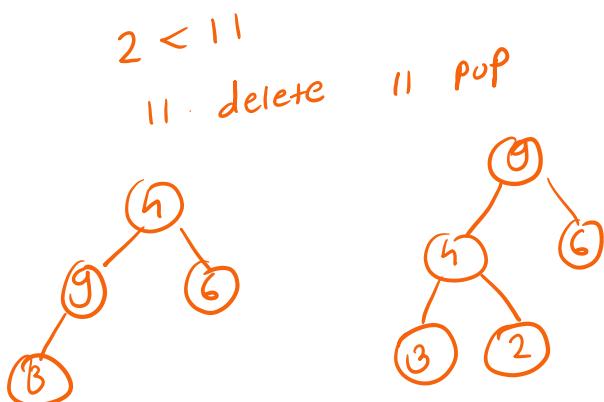
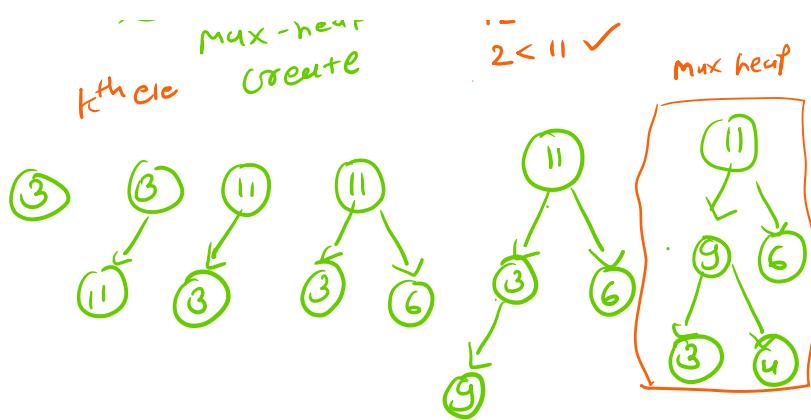
#2 Min-Heap

$n \cdot \text{size} \rightarrow \text{heap create}$   $S.C = O(n)$   
 $(n-k)$  time pop  
 $1^{\text{st}}$  pop →  $1^{\text{st}}$  small ele  $T.C = O(n)$   
 $2^{\text{nd}}$  pop →  $2^{\text{nd}}$  small ele  
 $\vdots$   
 $k^{\text{th}}$  pop →  $k^{\text{th}}$  small ele.

#3 Max-Heap

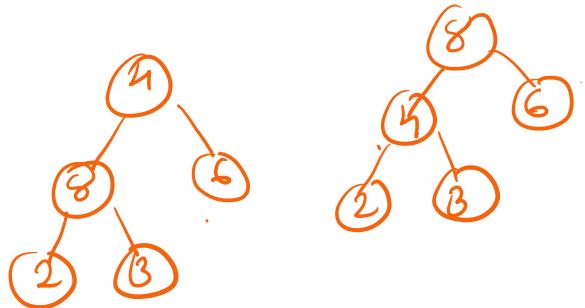
first k ele → Max heap  
 $\uparrow$   
 max heap  
 ↳ k-size  
 ↳ k small element  
 ↳ top →  $k^{\text{th}}$  small element  
 insert →  
 $\text{new element} < \text{heap.top()}$   
 $\text{heap.pop()}$   
 $\text{heap.insert(new element)}$





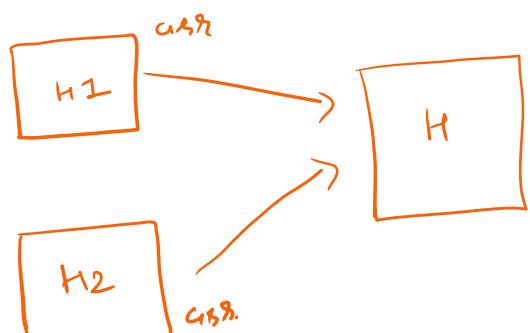
algo :-

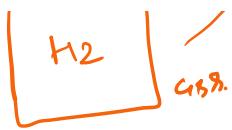
- ① kth element Create a max heap



- ② another element compare the heap top element  
 if (element < heap.top())  
 {  
 delete heap.top();  
 insert(element)  
 }

→ Merge 2 max-heap





H2 heap  
 T.C  $O(n \log n)$       H2 insert in H1 }  
 H2      }

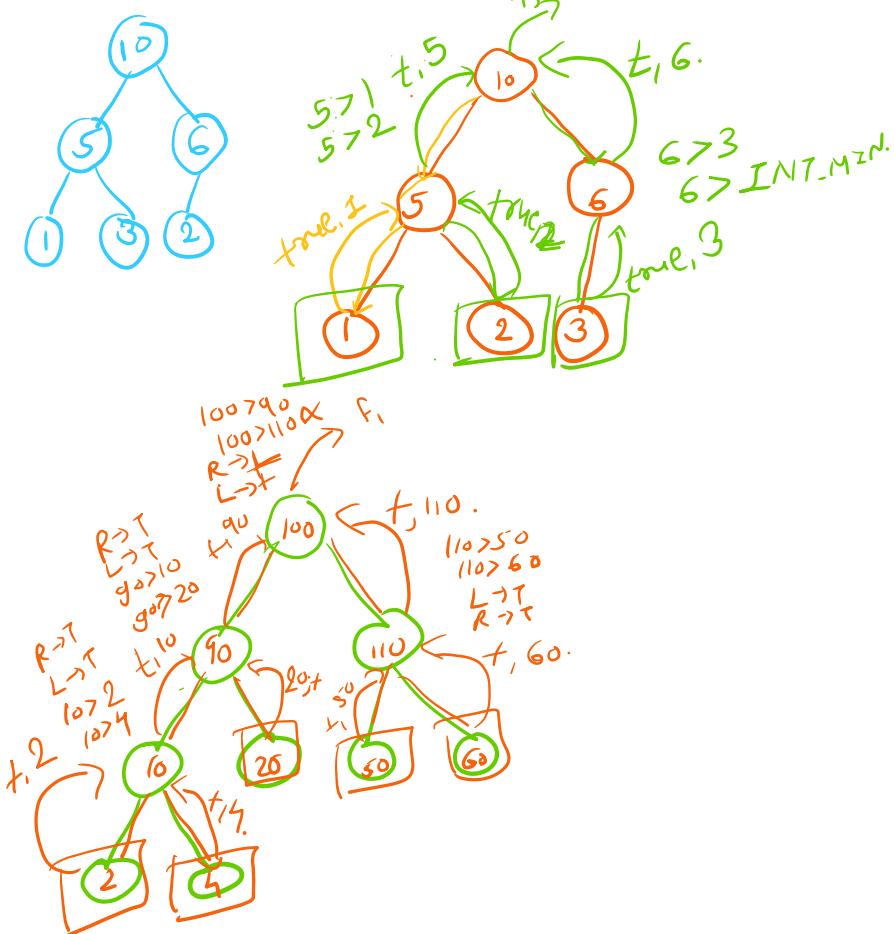
- ① ans  $\rightarrow$  Merge(H1, H2)  $\rightarrow O(m+n)$   
 Build heap  $O(n)$

T.C  $O(m+n)$

- ② H2 ✓  
 H2 each element insert H1  
 T.C  $O(n \log n)$

Complete.  
 → Binary Tree

↳ Heap or not  
 max



↳ ↳

```

for C
{
    if B.C
        if(L.N)
            return { true, root->data };
        if( == null)
            return { true, INT.MIN };
    left Ans -> Rec
    Right Ans -> Rec
  
```

(g)

CBT

→ BST

convert

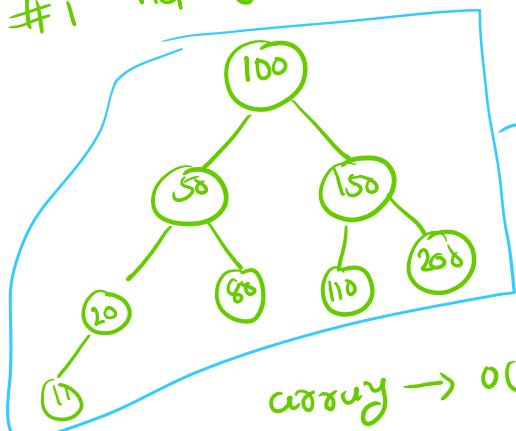
Max  
Heap

↳ CBT

↳ Max heap

Property.

#1 heapify



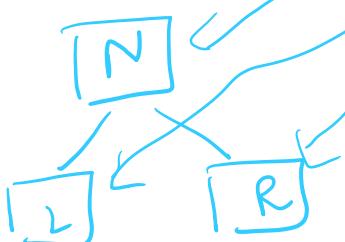
structure  
same  
only

value  
change  
Heap

LRN ← post order insert

inorder  
store

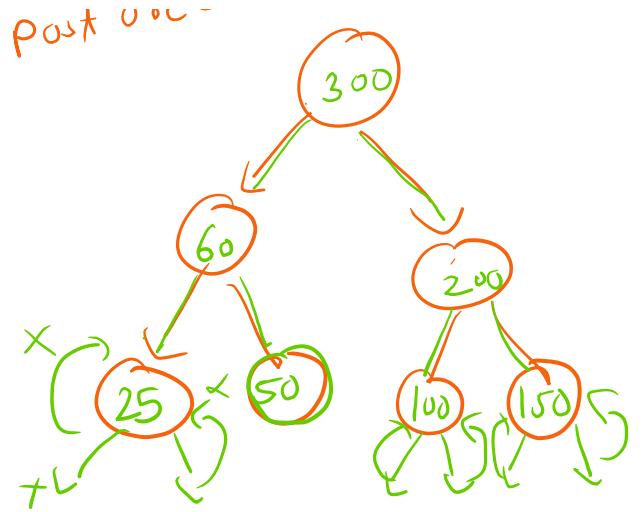
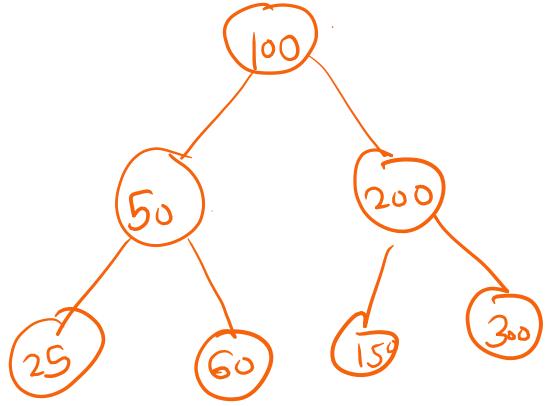
L-N-R



N > L  
N > R

post order

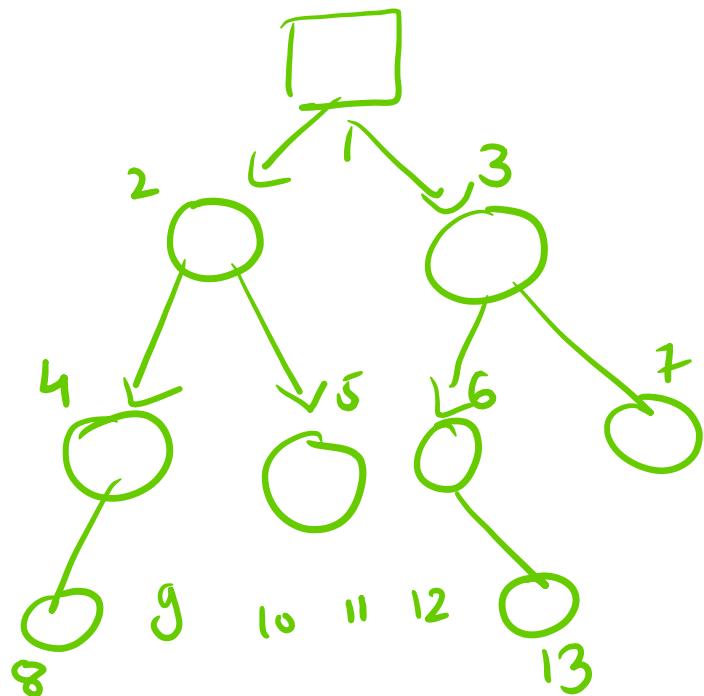
300



inorder

25 50 60 100 150 200 300

Check tree → is CBT or not

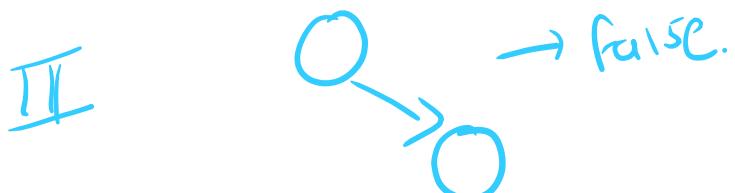
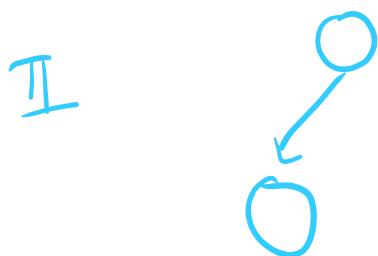
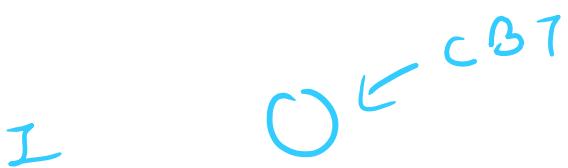


# 1 level order

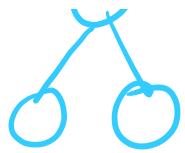
# 2 Total Node = 9

$13 > 9$

↳ not CBT.

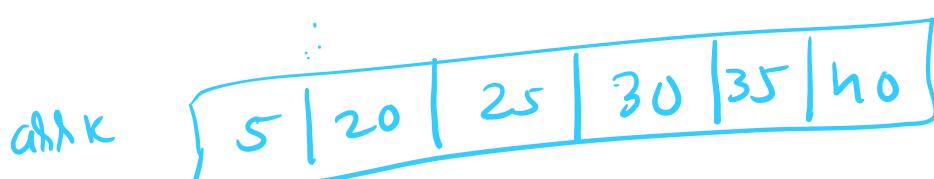
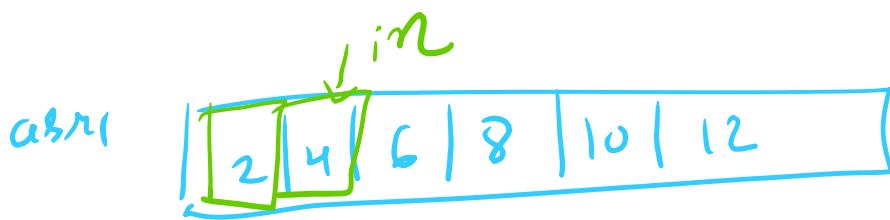


N



\*\*\*

## Merge k-Sorted array.



# Move singular arr. T.C = nk log(nk)  
sort

# ans-vector

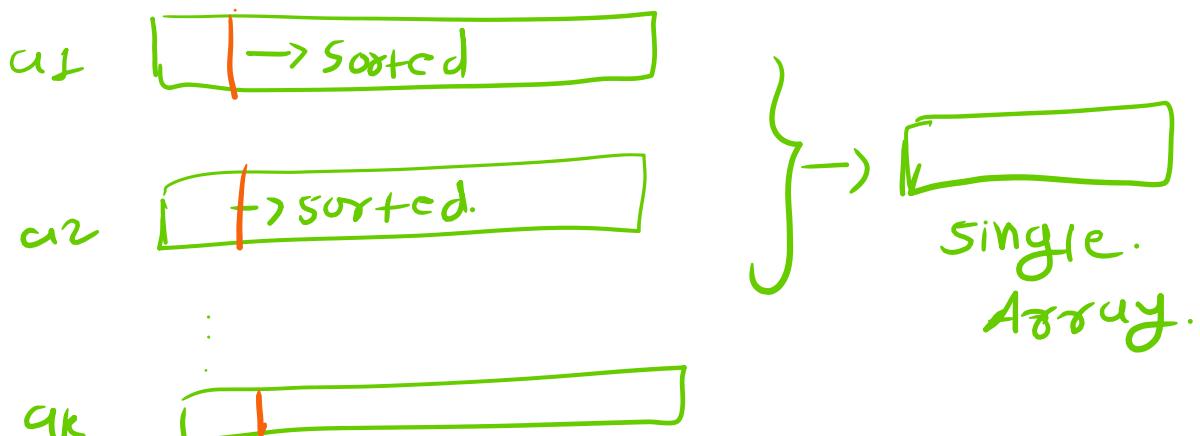


Min-Heap-create  $\rightarrow$  all elems in first element

& top ye elements insert them into. then that goes next

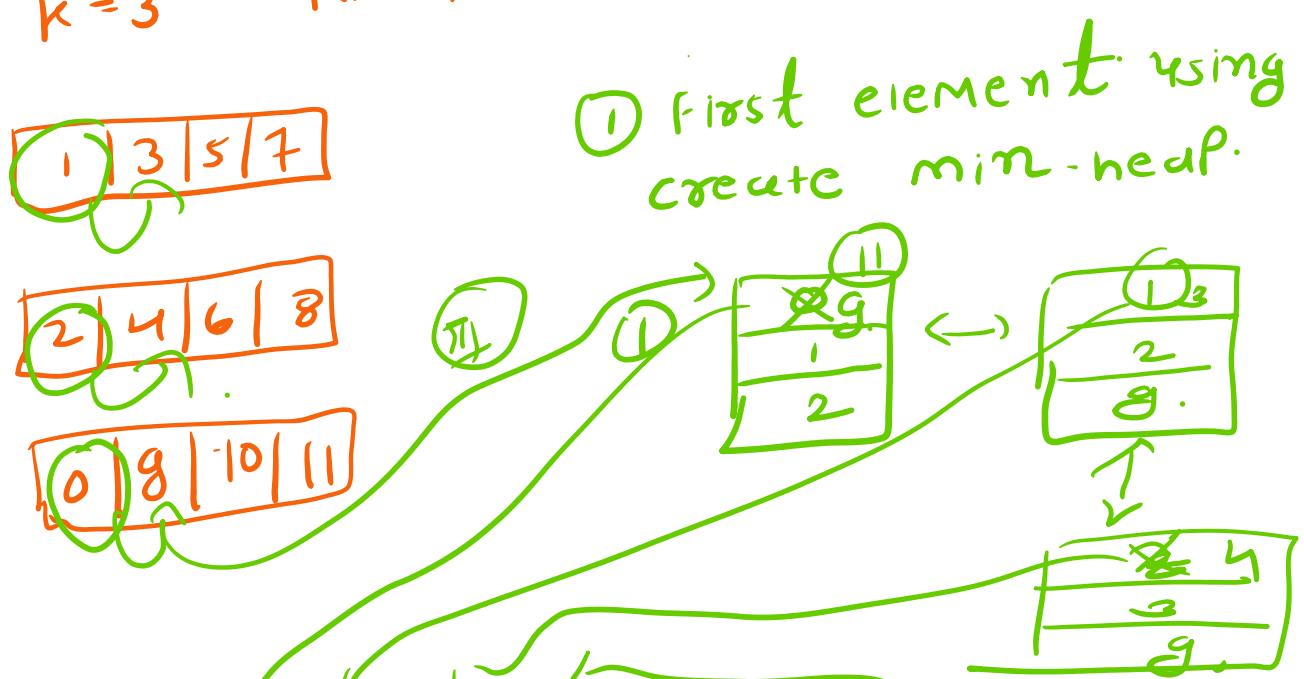
To implement delete, then that goes next  
element insert.

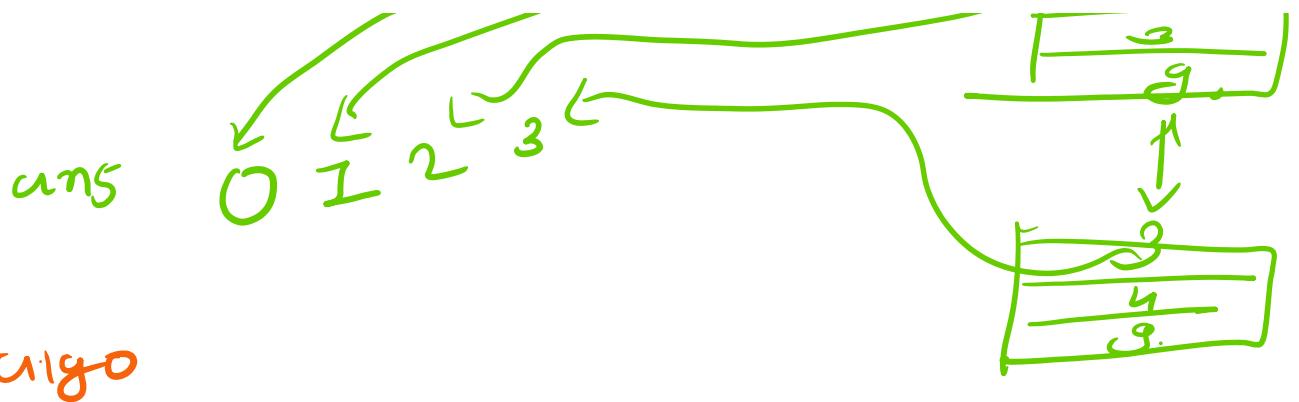
## Merge K Sorted Array.



$k$ -size min-heap  $\rightarrow$  min number.  
 min number - push - single array.  
 min number next number insert in  
 heap

$$k=3 \quad n=4$$





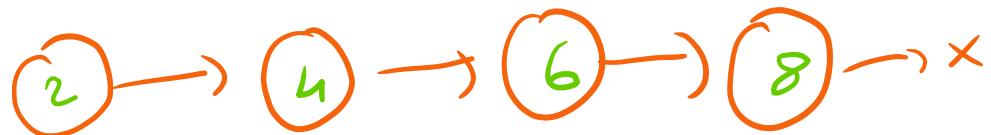
algo

- ① init array first element create min-heap
- ② min-heap top ele push ans array. and delete the. top ele.
- ③ push element next element push in heap.

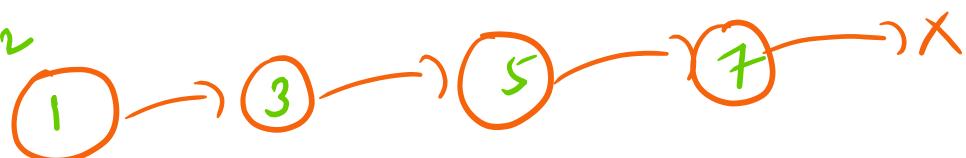
$$T.C = O(nk \log k)$$

28 → Merge k sorted linked list

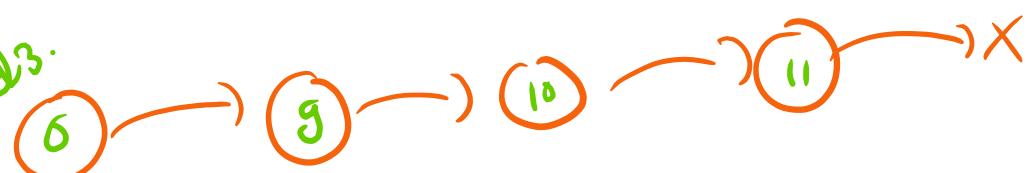
head 1



head 2

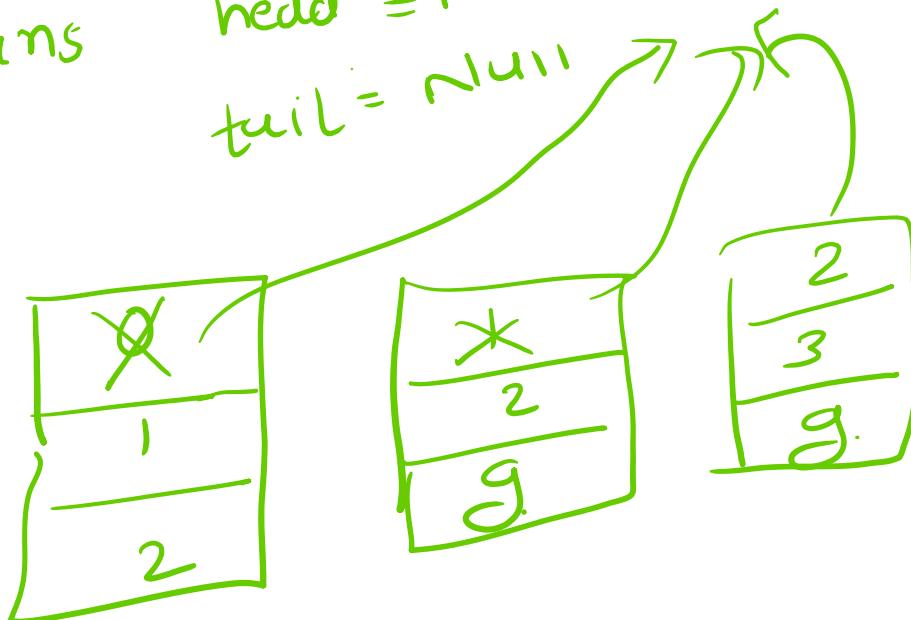


head 3



ans

head = null  
tail = null

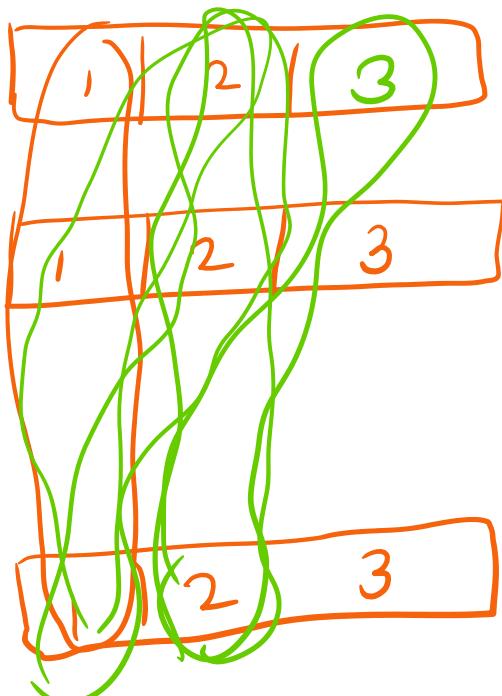
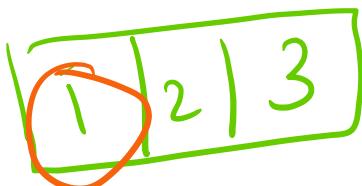
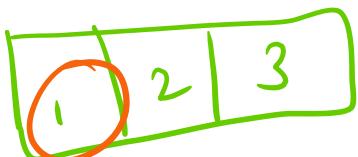


632 ~~imp~~

→ Smallest range in k lists



Max - Min  
↳ small



$$\text{Min} = 1 - 1$$

$$\text{Max} = 2 = 1$$

$$\text{Min} = 1$$

$$\text{Max} = 2 = 1$$

$$\text{Min} = 2 = 0$$

$$\text{Max} = 2$$

$$\text{Min} = 2 = 1$$

$$\min = -\infty$$
$$\max = 3 \approx 1$$

if any list  
finished then break guy

smallest range.

↳ min - max.

min ↑      min Heap used  
Max ↓

4	10	15	24	26
---	----	----	----	----

0	9	12	20
---	---	----	----

5	18	22	30
---	----	----	----

$0 \rightarrow 5$

$4 \rightarrow g$

$5 \rightarrow 16$

$g \rightarrow 18$

$10 \rightarrow 18$

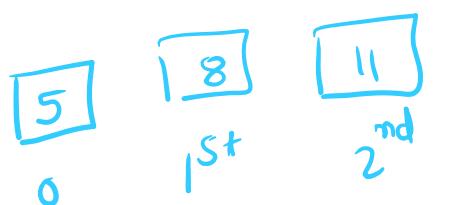
$12 \rightarrow 18$

$15 \rightarrow 20$

$18 \rightarrow 24$

$20 \rightarrow 24$

1962 Remove stones to minimize the total



$\text{K} \rightarrow \text{operation}$

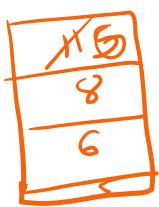
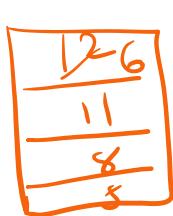
$\text{total} - \text{Floor}(\text{total}/2)$

Min stones after K options

Max heap

$\hookrightarrow K$  operation

$K=3$

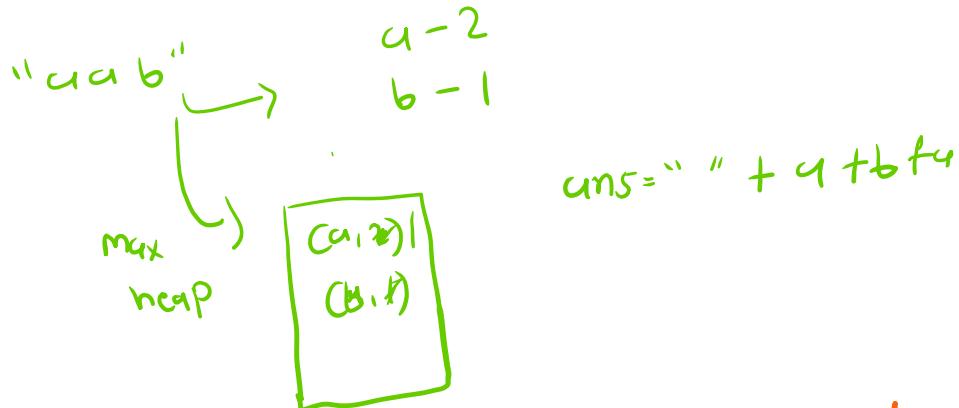


STL: `heifly()`

767

## ReOrganize String

Min cost to cut groups.

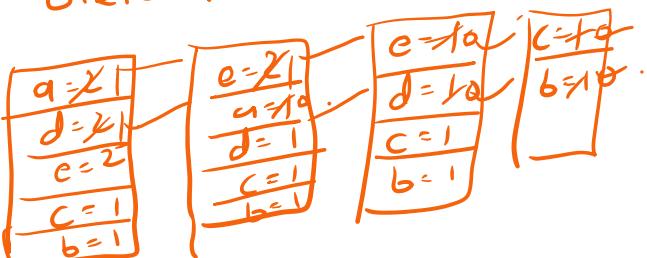


aaabcc dd ee

$a=3$   
 $b=1$   
 $c=2$   
 $d=2$   
 $e=2$

a=3	2
c=2	1
d=2	1
e=2	1
b=1	1

two element together  
 use Nikalna



$ans = " " + a1 + c1 + d1 + e1 + a1 + b1 + d1 + c1 + b1 + e1$

## 1405 Longest Happy String

$a=0$        $b=8$        $c=11$

$c=11 \neq 8321$   
 $b=8 \neq 520$

$4^{n+5} = cc + bb + cc + bb + cc + bb + cc + bb + cc$   
 $cc + bb + cc + bb + cc + bb + cc + bb + cc + bb + cc$

## Medium in a Stream

data stream

2, 3, 5, 1, 4, 9, 8, ...  
 ↓    ↓    |  
 Med Med. Med.

Medium

2 3 5 10 11 12 13 odd  
 ↑  
 Median.

2 3 4 5 6 7 even  
 ↑  
 avg.

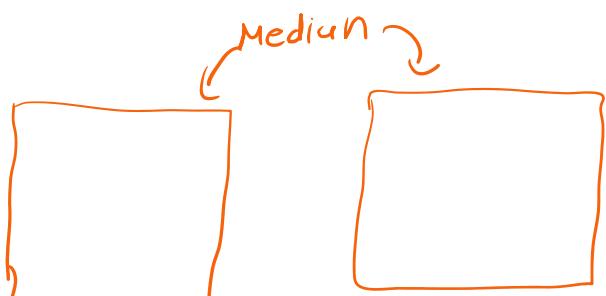
2, 20, 11, 4, 8, 25, ...

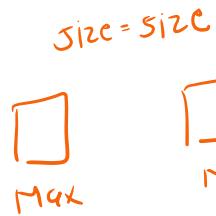
X #1 Vector → push → sort → Middle de  
 $O(n^2 \log n)$  ↴ Medium

X #2 BST → no-BST create → log n.  
 $O(n^2)$  mid. ← inorder.

#3 Heap  
 ↴ Max-Heap  
 ↴  $\frac{n}{2}$  pop. X

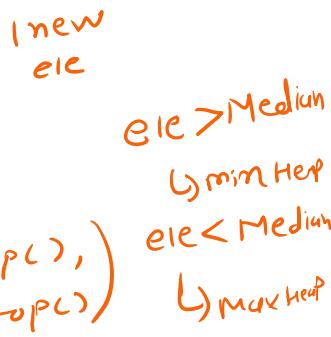
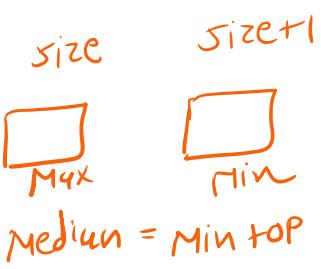
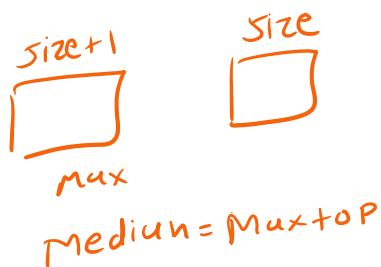
Min + Max heap      pop K





$$\text{size} = \text{size}_c$$

$$\text{Medium} = \text{avg}(\text{Max.top}(), \text{Min.top}())$$



$$\text{Medium} = \text{Min.top}$$

$$\text{Medium} = \text{avg}$$

one element  
pop on min and  
that element  
push in MaxHeap

ele < Medium.

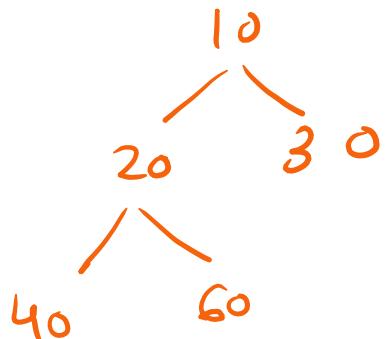
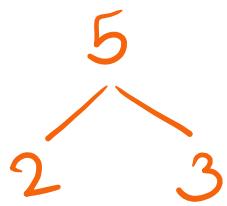


ele  
ele > Medi.  
↳ right.

ele < Medi.  
↳ left.

pop element of Maxheap  
pop and that element  
push on Minheap

gfg check if Binary Tree is Heap

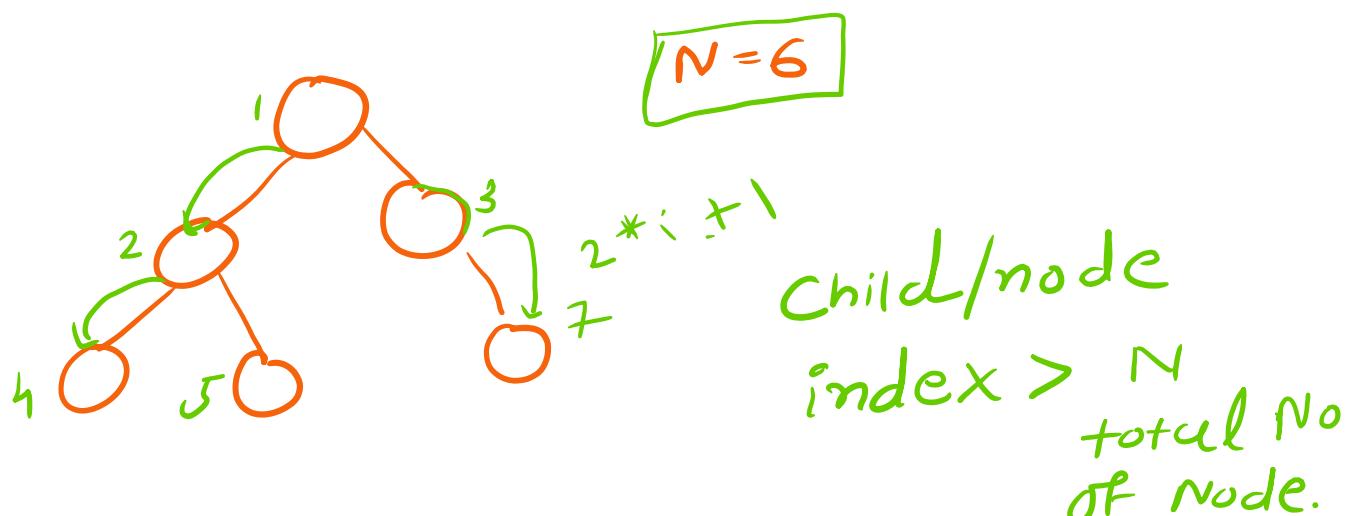


① is complete BT

② does it satisfy max heap order

How to check CBT?

- total no. of nodes

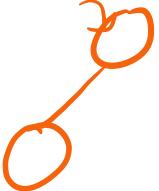


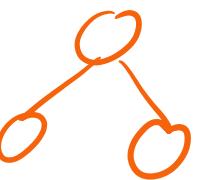
$$7 > 6$$

for every no. check Max order.  
parent value  $>$  children

rule - ✓  
parent value > children  
value.

case 1: leaf node ✓

case 2:  left child only  
parent > child ✓

case 3:  both child available  
parent > LL  
> RL

gfg merge two Binary Max Heap

a → 10, 5, 6, 2

b → 12, 7, 9

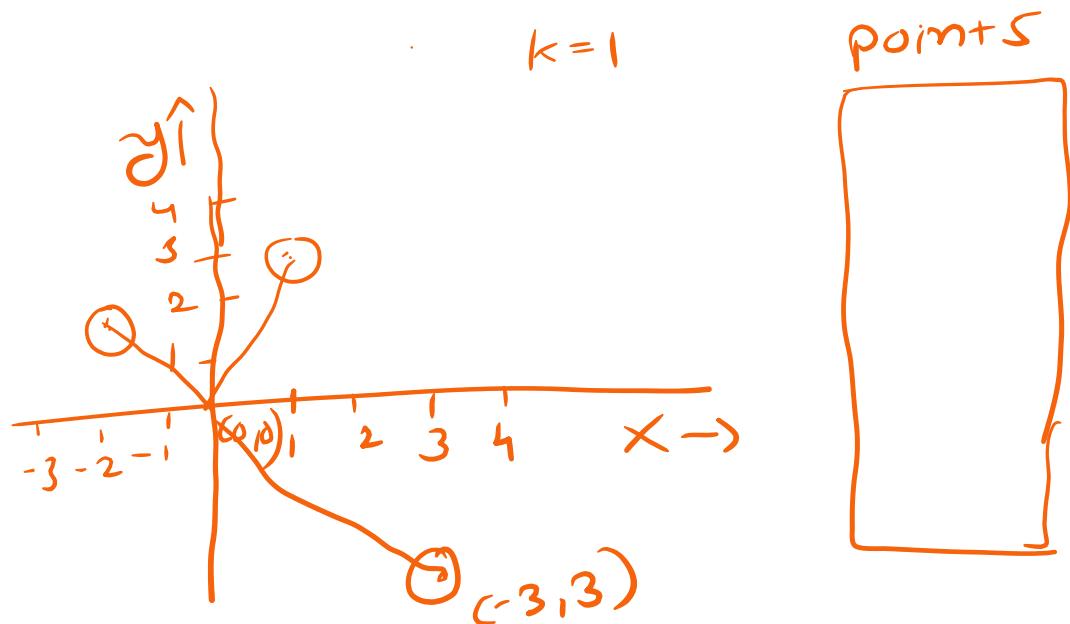
① Merge two heap vector

10, 5, 6, 2, 12, 7, 9

② heapify function convert into heap

③ C → heap

# g73 k-closest points to origin



$$\Rightarrow \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}$$

$$\sqrt{x_i^2 + y_i^2}$$

$$d^2 = x_i^2 + y_i^2$$

$$d = 2 \quad 2 < 3$$

$$d = 3 \quad \uparrow$$

more closer

① 1, 3

$$d^2 = 1^2 + 3^2 = 10$$

$$d = 4$$

$$d = 8$$

② -2, 2

$$1^2 + 2^2 = 5$$

$$k=1$$

$$d^2 = h + h = 8 \quad k=1$$

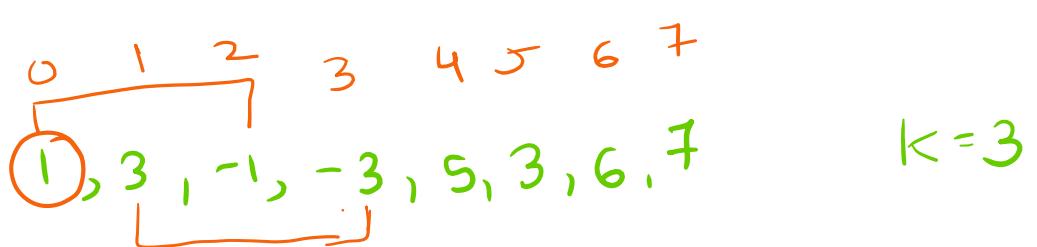
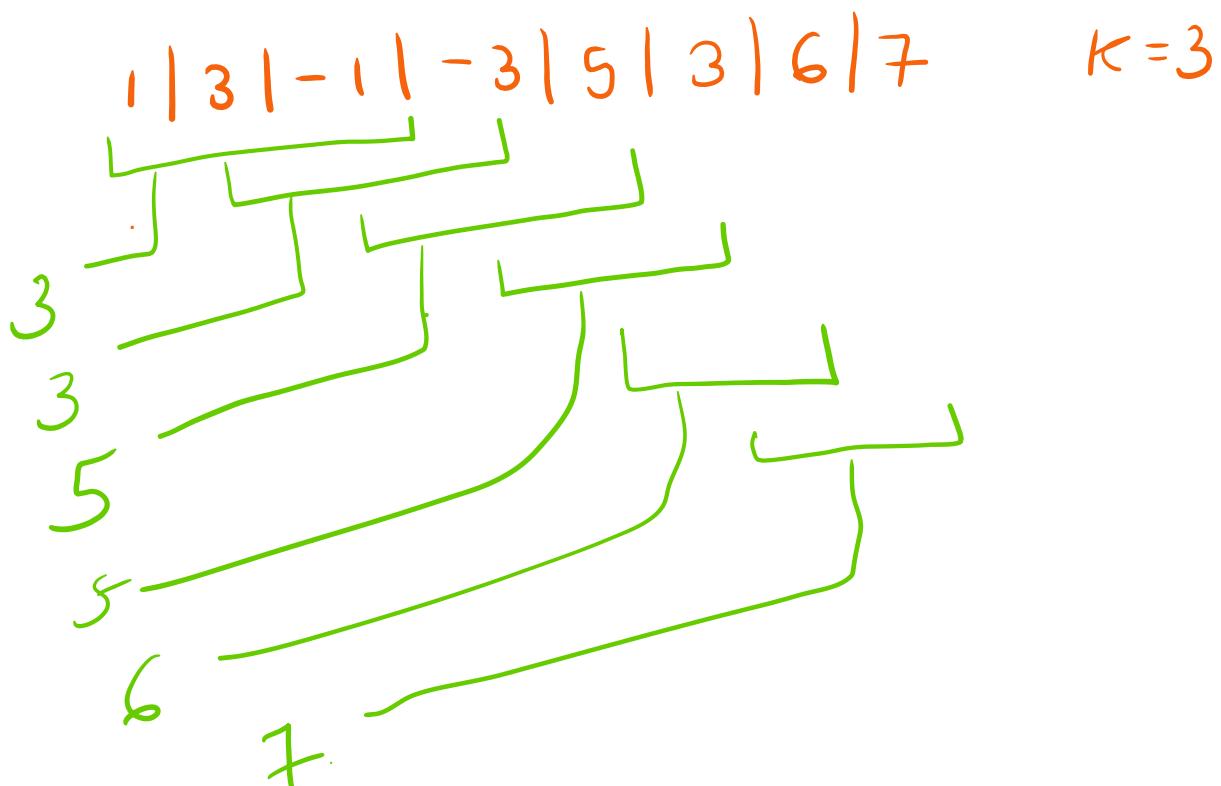
ans :- (-2, 2)

⑤ -3, 3

$$d^2 = g + g = 18 \quad k=2$$

ans :- (-2, 2)  
(1, 3)

## 239 Sliding window max

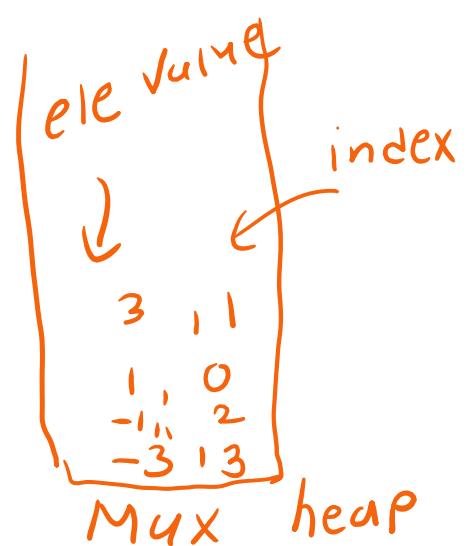


loop = 0  $\rightarrow$   $K-1$   
Next window  
 $i = K$

lets remove old  
window max

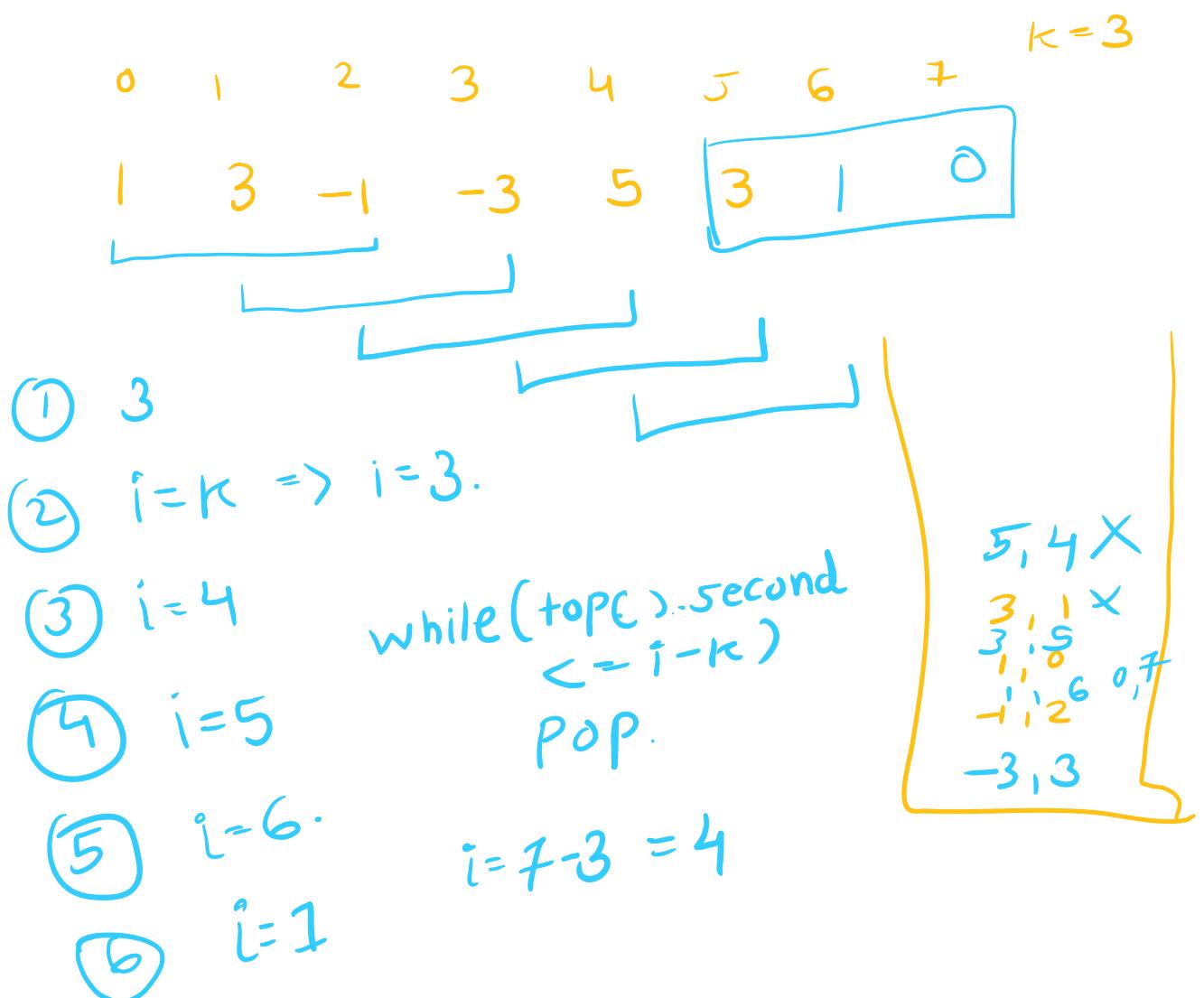
while (`top().second`  
 $\leq i-K$ ) {

  ^ ^



$\dots \leftarrow \leftarrow \leftarrow \leftarrow \leftarrow \leftarrow \dots$   
 $\uparrow \quad \uparrow$   
 $3 - 3 = 0$   
 $\}$  PQ-POP( $\triangleright$ )

ans  $\rightarrow 3$



ans  $\rightarrow 3 3 5 5 5$

→ for every window push into heap  
& check if the max top(), belongs  
to previous window  $\rightarrow$  pop()

1878 Get Biggest Three Rhombus Sums In A Grid.

- ① full grid  $\rightarrow$  cell  $[i,j] \Rightarrow$  needs to be considered
- ② Make Rhombus using each cell

## 2163 Minimum Difference in Sums After Removal of Elements

7 9 5 8 1 3

$$N = 6$$

$$3 \times n = 6$$

$$n = 2$$

7 9 ~~X~~ ~~X~~ 1 3



$$2 \times n = 4$$

7 9    1 3

$$n$$

$$n$$

Sum first = 16

Sum second = 4

$$16 - 4 = 12$$

7 ~~5~~ 8 ~~3~~ 3

7 5    8 3

$$12$$

$$11$$

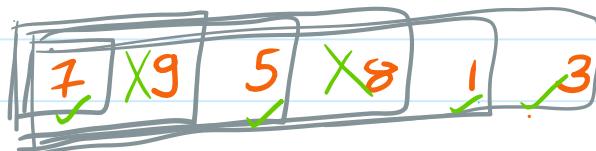
$$12 - 11 = 1$$

(M1)

Brute force  $\rightarrow$  Try and remove all  $n$  subsequences

①  $S_F - S_S \rightarrow \text{Minimize}$

$\downarrow$        $\downarrow$   
 Mini      Max  
 possible      possible



$$3 \times n = 6$$

$$n = 2$$

Mini      -1      16      12      12      6      4  
 Possible

$\downarrow$       17      17      13      11      4      -1

Max  
possible

Prefix  
 $\downarrow$

sum of first  $n$  ans  
min-element  
from left.

$\downarrow$   
Suffix

sum of first  $n$   
max. element from  
right.

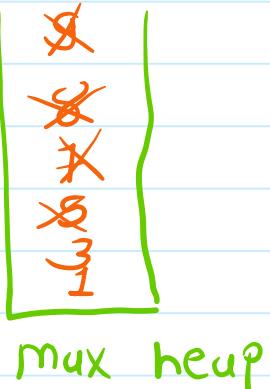
7 9 5 8 1 3  
0 1 2 3 4 5

[-1, 16, 12, 12, 6, 4]

$i=0$ , sum = 7  
 $i=1$ , sum =  $7+9=16$

minimum possible  
sum of n  
element

if (heap.size == n)  
{  
    prefix[i] = sum;  
}



$i=2$  sum =  $16+5=21$        $21-9=12$   
if (heap.size > n)  
{  
    heap.top element ko pop kusana.  
    sum -= heap.top();  
    pop();  
}

$i=3$  sum =  $12+8=20$   
 $20-8=12$

$i=4$  sum =  $12+1=13$   
 $13-7=6$

$i=5$  sum =  $6+3=9$   
 $9-5=4$

## 871 Minimum Number of Refueling stops

