Probability and Probability Distribution

Unit No 5: Discrete & Continuous Probability Distribution

Syllabus

Unit 5- Discrete & Continuous Probability Distribution

-**Discrete Probability Distribution**

Binomial Distribution

Finding Mean and variance of Binomial Distribution

Poisson Distribution

Finding Mean and variance of Poisson Distribution

-**Continuous Probability Distribution**

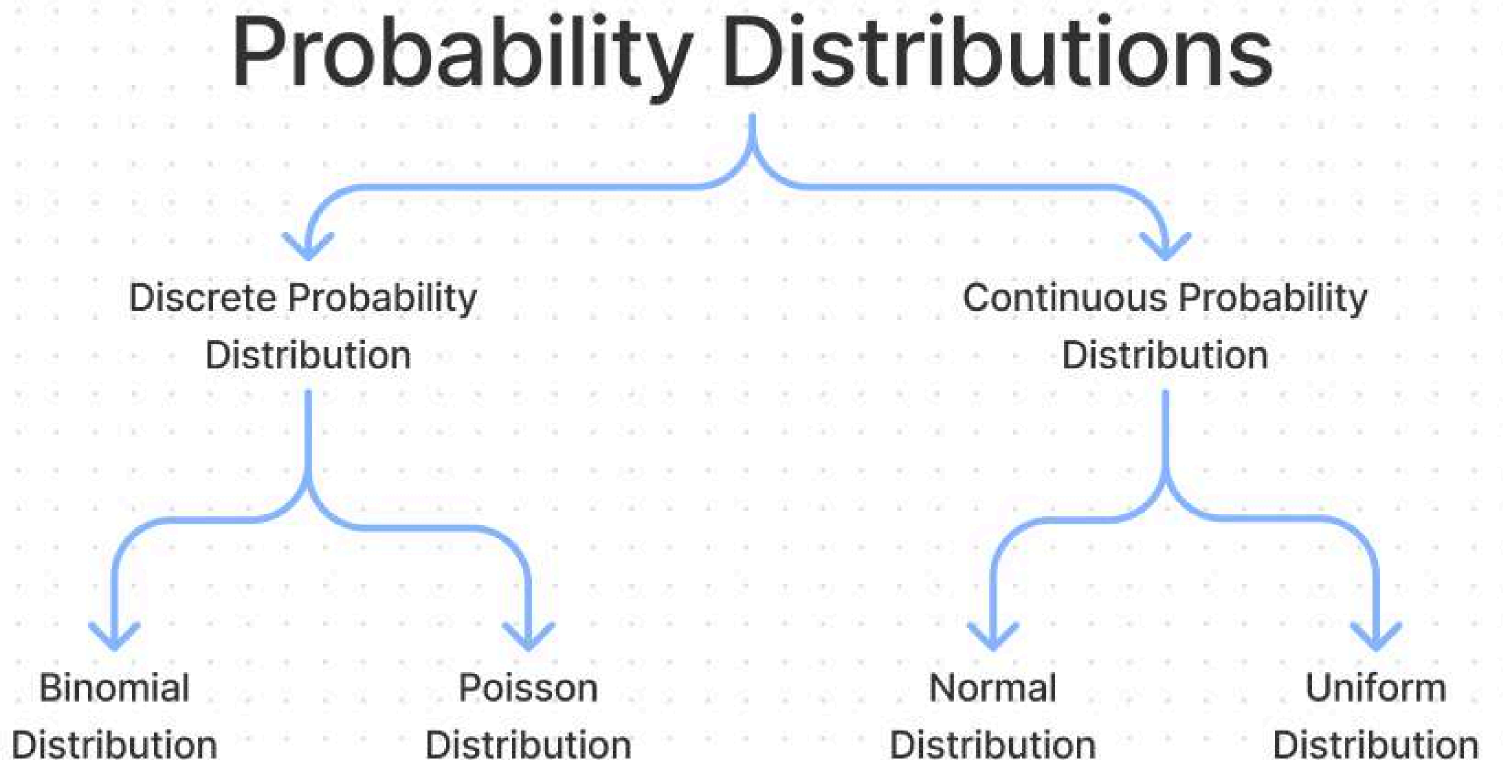
Uniform Distribution

Finding Mean and variance of uniform Distribution

Normal Distribution

Finding Mean and variance of normal Distribution

Types of Probability Distribution



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Types of Probability Distribution continue..................

A **probability distribution** describes how the values of a random variable are distributed and how likely each value is to occur.

A random variable can be:

Discrete → takes countable values

Continuous → takes values over an interval

Accordingly, probability distributions are broadly classified into:

Discrete Probability Distributions

Continuous Probability Distributions

Types of Probability Distribution continue……………..

A **probability distribution** is a mathematical description that specifies how probabilities are assigned to the possible values of a random variable. It provides a complete picture of the uncertainty associated with a random experiment.

Probability distributions help in:

- Understanding randomness
- Making predictions
- Performing statistical inference
- Modeling real-world phenomena

Based on the nature of the random variable, probability distributions are broadly classified into:

Discrete Probability Distributions

Continuous Probability Distributions

1. Discrete Probability Distributions

A **discrete probability distribution** is associated with a **discrete random variable**, which can take only a **finite or countably infinite number of values**.

Characteristics of Discrete Distributions

Probabilities are defined using a **Probability Mass Function (PMF)**

The sum of probabilities of all possible values is **1**

Each possible value has a positive probability

Mathematically,

$\sum P(X=x) = 1$

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1. Discrete Probability Distributions

a. Binomial Distribution

Definition:

The binomial distribution models the number of successes in a fixed number of independent Bernoulli trials.

Theoretical Assumptions:

Fixed number of trials

Each trial has two outcomes

Probability of success remains constant

Trials are independent

Significance:

Widely used in quality control and testing

Models repeated experiments with fixed probability

1. Discrete Probability Distributions

b. Poisson Distribution

Definition:

The Poisson distribution models the number of times an event occurs in a **given interval of time or space**, when events occur randomly and independently.

Theoretical Conditions:

Events occur independently

Average rate of occurrence is constant

Two events cannot occur simultaneously

Importance:

Useful when events are rare

Often used as an approximation to binomial distribution for large samples

2. Continuous Probability Distributions

A **continuous probability distribution** is associated with a **continuous random variable**, which can take **any value within a given interval**.

Characteristics of Continuous Distributions

- Defined using a **Probability Density Function (PDF)**
- Probability at any exact point is zero
- Probabilities are calculated over intervals
- Total area under the curve is equal to 1

2. Continuous Probability Distributions

a. Continuous Uniform Distribution

Definition:

In a continuous uniform distribution, all values within a specified interval have equal likelihood.

Theoretical Features:

PDF is constant over the interval

Represents complete randomness over a range

Used as a baseline distribution in simulations

Types of Probability Distribution continue..................

2. Continuous Probability Distributions

Normal Distribution

Definition:

The normal distribution, also known as the Gaussian distribution, is a **symmetrical, bell-shaped distribution** characterized by its mean and standard deviation.

Theoretical Properties:

- Mean = Median = Mode
- Symmetric about the mean
- Defined by two parameters (μ and σ)
- Follows the empirical rule (68–95–99.7)

Importance:

- Most widely used distribution in statistics
- Central to the Central Limit Theorem
- Approximates many natural and social phenomena

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Conclusion

Probability distributions are fundamental tools in statistics that describe how random variables behave. Understanding both **discrete and continuous distributions** enables accurate modeling of real-world problems and supports data-driven decision-making across disciplines.

-**Discrete Probability Distribution**

Binomial Distribution

Finding Mean and variance of Binomial Distribution

1. What is a Binomial Distribution?

A **Binomial Distribution** is a discrete probability distribution that describes the number of **successes** in a fixed number of **independent trials**, where each trial has only **two outcomes**:

Success (with probability **p**)

Failure (with probability **q = 1 – p**)

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Conditions of a Binomial Distribution

A random variable **X** follows a Binomial Distribution if:

1. The number of trials **n** is fixed.
2. Each trial has only two outcomes: success or failure.
3. The probability of success **p** remains constant for each trial.
4. The trials are independent (result of one trial does not affect others).

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Binomial Probability Mass Function (PMF)

The probability of getting exactly **x successes** out of **n trials** is:

$$P(X = x) = \binom{n}{x} p^x q^{n-x}$$

where

$$q = 1 - p$$

P(X=x)

This represents the **probability** that the random variable X(number of successes) is exactly **x** or P(x) is the probability of the Binomial random variable X

n-x = number of failure in n trials

nCx = Binomial Coefficient

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Mean (Expected Value) of Binomial Distribution

Formula

$$\boxed{E(X) = np}$$

Explanation

- Out of **n trials**, each trial contributes a success with probability **p**.
- So the expected number of successes is simply **n multiplied by p**.

Variance of Binomial Distribution

Formula

$$\boxed{\mathrm{Var}(X) = npq}$$

where

$$q = 1 - p$$

Explanation

- Variance measures how spread out the distribution is.
- It increases when:
  - the number of trials increases (**n** ↑), or
  - success and failure probabilities are closer to 0.5 (maximum uncertainty).

Standard Deviation

$$\boxed{\sigma = \sqrt{npq}}$$

Summary Table

Concept	Formula
Mean (Expectation)	$E(X) = np$
Variance	$Var(X) = np(1-p)$
Standard Deviation	$\sigma = \sqrt{np(1-p)}$

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Example 1

It is believed that 50% students use internet for academic purposes. In a sample of 4 students, calculate the probability.

a) None of the students use internet

b) Only one student uses internet

c) Two students use internet

d) Three students use internet

e) All the four students use internet

SOLUTION: Given: $p = 0.5; n = 4$

(a) $P(X = 0) = {}^4C_0 (0.5)^0 (0.5)^4 = 0.0625$

(b) $P(X = 1) = {}^4C_1 (0.5)^1 (0.5)^3 = 0.25$

(c) $P(X = 2) = {}^4C_2 (0.5)^2 (0.5)^2 = 0.375$

(d) $P(X = 3) = {}^4C_3 (0.5)^3 (0.5)^1 = 0.25$

(e) $P(X = 4) = {}^4C_4 (0.5)^4 (0.5)^0 = 0.0625$

Example 2

It is believed that 20% of the employees in an office are usually late. If 10 employees report for duty on a given day, what is the probability that

a) Exactly 3 employees are late

b) At most 3 employees are late

Solution (Binomial, $n = 10,\ p = 0.20$)

Let X = number of employees late. $X \sim \text{Binomial}(n = 10, p = 0.2)$ so

$$P(X = k) = \binom{10}{k}(0.2)^k(0.8)^{10-k}.$$

I'll show the intermediate probabilities used.

Component probabilities:

- $P(X = 0) = \binom{10}{0}0.2^0\,0.8^{10} = 0.8^{10} = 0.1073741824$
- $P(X = 1) = \binom{10}{1}0.2^1\,0.8^9 = 10 \cdot 0.2 \cdot 0.8^9 = 0.2684354560$
- $P(X = 2) = \binom{10}{2}0.2^2\,0.8^8 = 45 \cdot 0.04 \cdot 0.8^8 = 0.3019898880$
- $P(X = 3) = \binom{10}{3}0.2^3\,0.8^7 = 120 \cdot 0.008 \cdot 0.8^7 = 0.2013265920$

Exactly 3 employees are late → P(X = 3)

You plug **x = 3** into the formula:

$$P(X = 3) = \binom{10}{3}(0.2)^3(0.8)^7$$

Calculate:

- $\binom{10}{3} = 120$
- $(0.2)^3 = 0.008$
- $(0.8)^7 = 0.2097152$

Multiply:

$$P(X = 3) = 120 \times 0.008 \times 0.2097152 = 0.2013$$

At most 3 employees are late → P(X ≤ 3)

This includes:

$$P(0) + P(1) + P(2) + P(3)$$

We already know $P(3) = 0.20$.

We add:

- $P(0) = 0.11$
- $P(1) = 0.27$
- $P(2) = 0.30$
- $P(3) = 0.20$

Add them:

$$0.11 + 0.27 + 0.30 + 0.20 = 0.88$$

Example 3

An Insurance salesman sells policies to 5 men of identical age and good health . According to the actuarial tables, the probability that a man of this particular age will be alive 30 years hence is 2/3 . Find the probability that 30 years hence.

a)At least 3 men will be alive.

Example 4

Metro rail is concerned about the safe maintenance of its property and found that a vandal will be caught is 0.3 What is the probability that exactly 3 vandals will be arrested in the next five cases of vandalism

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Solution

Use the binomial distribution.

Let $n = 5, p = 0.3$ (probability a vandal is caught), and $k = 3$. The binomial probability is

$$P(X = 3) = \binom{5}{3} p^3 (1 - p)^{5-3}.$$

Compute step-by-step:

$$\binom{5}{3} = 10, \qquad p^3 = 0.3^3 = 0.027, \qquad (1 - p)^2 = 0.7^2 = 0.49.$$

$$P(X = 3) = 10 \times 0.027 \times 0.49 = 10 \times 0.01323 = 0.1323.$$

So the probability exactly 3 vandals are arrested in the next 5 cases is **0.1323** (or **13.23%**).

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Example 5

A software company installs a new high-security biometric system for employee entry.
Based on past data, the probability that the biometric scanner correctly authenticates an employee on the **first attempt** is **0.78**.

On a particular day, **12 employees** arrive at the same time at the entrance gate.

Assume each authentication attempt is independent.

Find:

1. The probability that **exactly 9 employees** are authenticated on the first attempt.

2. At least 10 authenticated on first attempt

3 The **expected number** of successful authentications.

4. The **variance** of the number of successful authentications.

5. SD

1. Exactly 9 authenticated on first attempt

$$P(X=9)=\binom{12}{9}(0.78)^9(0.22)^3\approx\mathbf{0.25035}.$$

2) Probability at least 10 authenticated on first attempt

$$P(X\geq 10)=P(X=10)+P(X=11)+P(X=12).$$

Calculating those (using the binomial formula) and summing gives

$$P(X\geq 10)\approx 0.4886432,$$

i.e. ≈ **48.86%**.

(So there's about a 48.9% chance that 10 or more of the 12 succeed on first try.)

3. Expected number of successful authentications

$$E[X]=np=12\times 0.78=\mathbf{9.36}.$$

4. Variance of the number of successful authentications

$$\mathrm{Var}(X)=np(1-p)=12\times 0.78\times 0.22=\mathbf{2.0592}.$$

Example 6

A large IT service company upgrades its login system to an AI-based face-recognition access control at all building entrances.
From system testing, the probability that the AI correctly verifies an employee on the first attempt is 0.85.

During peak hours, 15 employees attempt to enter the office through the system.

Assume each authentication attempt is independent.

Find:

1. The probability that exactly 12 employees are authenticated on the first attempt.1
2. The probability that at least 13 employees are authenticated on the first attempt.
3. The expected number of successful authentications.
4. The variance of the number of successful authentications.
5. The standard deviation.

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1) Probability exactly 12 succeed

$$P(X = 12) = \binom{15}{12}(0.85)^{12}(0.15)^{3}.$$

Compute pieces:

- $\binom{15}{12} = \binom{15}{3} = 455.$
- $(0.85)^{12} \approx 0.1422417571.$
- $(0.15)^{3} = 0.003375.$

So

$$P(X = 12) = 455 \times 0.1422417571 \times 0.003375 \approx 0.218430.$$

Answer: $P(X = 12) \approx 0.21843$ (≈ **21.84%**).

2) Probability at least 13 succeed

$$P(X \geq 13) = P(X = 13) + P(X = 14) + P(X = 15).$$

Calculate each term:

- $\binom{15}{13} = 105, \quad (0.85)^{13} \approx 0.1209054936, \quad (0.15)^2 = 0.0225$
  $P(X = 13) = 105 \times 0.1209054936 \times 0.0225 \approx 0.285639.$
- $\binom{15}{14} = 15, \quad (0.85)^{14} \approx 0.1027696695, \quad (0.15)^1 = 0.15$
  $P(X = 14) = 15 \times 0.1027696695 \times 0.15 \approx 0.231232.$
- $\binom{15}{15} = 1, \quad (0.85)^{15} \approx 0.0873542191, \quad (0.15)^0 = 1$
  $P(X = 15) = 1 \times 0.0873542191 \times 1 \approx 0.087354.$

Sum:

$$P(X \geq 13) \approx 0.285639 + 0.231232 + 0.087354 \approx 0.604225.$$

3) Expected number

For binomial, $E[X] = np$:

$$E[X] = 15 \times 0.85 = 12.75.$$

4) Variance

$\mathrm{Var}(X) = np(1 - p)$:

$$\mathrm{Var}(X) = 15 \times 0.85 \times 0.15 = 1.9125.$$

5) Standard deviation

$\sigma = \sqrt{\mathrm{Var}(X)} = \sqrt{1.9125} \approx 1.382932.$

-**Discrete Probability Distribution**

Poisson Distribution

Finding Mean and variance of Poisson Distribution

Poisson Distribution Formula

Poisson distribution is characterized by a single parameter, lambda (λ), which represents the average rate of occurrence of the events. The probability mass function of the Poisson distribution is given by:

$$P(X = r) = \frac{e^{-\lambda}\lambda^{r}}{r!}$$

Where,

- **P(X = r)** is the Probability of observing k Events
- **e** is the Base of the Natural Logarithm (approximately 2.71828)
- **λ** is the Average Rate of Occurrence of Events
- **r** is the Number of Events that Occur

Example 1

A Manufacturer who produces medicine bottles finds that 0.1% of the bottles are defective. The bottles are packed in boxes containing 500 bottles. A drug manufacturer buys 100 boxes from the producer of bottles. Using poisson distribution, find how many boxes will contain

1) No defectives
2) At least two defectives

(Given $e^{-0.5} = 0.6065$)

$\lambda = 500\times0.001 = 0.5$ (defects per box). Use Poisson(λ=0.5).

1. No defectives

   $P(0) = e^{-0.5} = 0.6065$ (given).

   Expected number of boxes (out of 100) = $100 \times 0.6065 =$ **60.65 ≈ 61 boxes.**

2. At least two defectives

   $P(1) = e^{-0.5}\cdot 0.5 = 0.6065\times0.5 = 0.30325$.

   $P(\geq 2) = 1 - P(0) - P(1) = 1 - 0.6065 - 0.30325 = 0.09025$.

   Expected number of boxes = $100 \times 0.09025 =$ **9.025 ≈ 9 boxes.**

So about 61 boxes with no defectives and about 9 boxes with at least two defectives (expected values).

Example 2

A server receives crash alerts at an average rate of 1.2 alerts per day. The admin analyses data for 90 days.

Find: The number of days with no crash alerts. The number of days with at least two alerts.

($e^{-1.2}$=0.3010(approx))

Solution

1 Probability of no alerts: P(0)

$$P(0) = e^{-1.2} = 0.3010$$

Expected number of days:

$$90 \times 0.3010 = 27.09 \approx \mathbf{27\ days}$$

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2 Probability of at least two alerts: P(X ≥ 2)

$$P(X \geq 2) = 1 - P(0) - P(1)$$

Step A: Find P(1)

$$P(1) = e^{-1.2} \times 1.2$$

$$P(1) = 0.3010 \times 1.2 = 0.3612$$

Step B: Now calculate P(X ≥2)

$$P(X \geq 2) = 1 - 0.3010 - 0.3612$$

$$P(X \geq 2) = 1 - 0.6622 = 0.3378$$

Expected number of days:

$$90 \times 0.3378 = 30.40 \approx \mathbf{30\ days}$$

Example 3

Assume that the number of network errors experienced in day on a local area network is distributed as a Poisson random variable. The average number of network errors experienced in a day is 2.4 . What is the probability that is any given day:

1. Exactly one network error will result?
2. Two or more network errors will result?

(Given : e−2.4≈0.0907)

Solution

✅ 1) Probability of *exactly one* network error

$$P(1) = e^{-2.4}\frac{(2.4)^1}{1!}$$

Substitute values:

$$P(1) = 0.0907 \times 2.4$$

$$P(1) = 0.21768$$

$$\boxed{P(1) \approx 0.218}$$

✅ 2) Probability of *two or more* network errors

$$P(X \geq 2) = 1 - P(0) - P(1)$$

First find P(0):

$$P(0) = e^{-2.4} = 0.0907$$

We already found:

$$P(1) = 0.21768$$

Now calculate:

$$P(X \geq 2) = 1 - 0.0907 - 0.21768$$

$$P(X \geq 2) = 1 - 0.30838$$

$$\boxed{P(X \geq 2) \approx 0.6916}$$

Example 4

An antivirus software generates **3.2 alerts** per day on average.
Alerts follow a Poisson distribution.

Find the probability that on any given day:

1)Exactly one alert occurs.

2)Three or more alerts occur.
(Given: $e^{-3.2} \approx 0.0408$)

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1) Probability of exactly one alert

$$P(1) = e^{-3.2}\frac{(3.2)^1}{1!}$$

Substitute values:

$$P(1) = 0.0408 \times 3.2$$

$$P(1) = 0.13056$$

$$\boxed{P(1) \approx 0.131}$$

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$$P(0)+P(1)+P(2)=0.0408+0.13056+0.2089$$

$$=0.38026$$

Finally:

$$P(X \geq 3)=1-0.38026$$

$$\boxed{P(X \geq 3) \approx 0.620}$$

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Example 5

A cybersecurity monitoring system records the number of suspicious login attempts detected per hour. Past data shows that the system flags an average of 4.2 suspicious login attempts per hour.

Questions:

1. What is the mean and variance of the number of suspicious login attempts per hour?
2. What is the probability that in a given hour, exactly 3 suspicious attempts will occur?
3. What is the probability that two or more suspicious attempts will occur in an hour?
4. What is the probability that no suspicious attempt will be recorded?

(Given : $e^{-4.2} \approx 0.0149$)

Solution

1. Mean and Variance

For a Poisson distribution with parameter λ:

$$\text{Mean} = \lambda, \quad \text{Variance} = \lambda$$

Here:

$$\text{Mean} = 4.2, \quad \text{Variance} = 4.2$$

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2. Probability of exactly 3 suspicious attempts

The Poisson probability formula:

$$P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

For $k = 3$ and $\lambda = 4.2$:

$$P(X = 3) = \frac{4.2^3 \cdot e^{-4.2}}{3!}$$

Step by step:

- $4.2^3 = 4.2 \cdot 4.2 \cdot 4.2 = 74.088$
- $3! = 6$
- $e^{-4.2} \approx 0.0149$

$$P(X = 3) = \frac{74.088 \cdot 0.0149}{6} = \frac{1.1037}{6} \approx 0.184$$

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3. Probability that two or more suspicious attempts occur

$$P(X \geq 2) = 1 - P(X < 2) = 1 - P(X = 0) - P(X = 1)$$

Step 1: Compute $P(X = 0)$ **and** $P(X = 1)$

$$P(X = 0) = \frac{4.2^0 \cdot e^{-4.2}}{0!} = e^{-4.2} \approx 0.0149$$

$$P(X = 1) = \frac{4.2^1 \cdot e^{-4.2}}{1!} = 4.2 \cdot 0.0149 \approx 0.0626$$

Step 2: Compute $P(X \geq 2)$

$$P(X \geq 2) = 1 - 0.0149 - 0.0626 = 1 - 0.0775 = 0.9225$$

So, the probability is approximately **0.923**.

4. Probability that no suspicious attempt occurs

$$P(X = 0) = e^{-4.2} \approx 0.0149$$

✅ Summary

Event	Probability
Mean	4.2
Variance	4.2
Exactly 3 attempts	0.184
Two or more attempts	0.923
No attempt	0.0149

-**Continuous Probability Distribution**

Normal Distribution

Finding Mean and variance of Normal Distribution

A **Normal Distribution** is a continuous probability distribution that is **symmetrical**, **bell-shaped**, and centered around a **mean** (average).
It is the most important distribution in statistics because many natural and real-life phenomena follow it.

Key Characteristics of Normal Distribution

a) **Bell-shaped Curve**

The curve is highest at the mean.

It decreases symmetrically on both sides.

b) Symmetry

Mean = Median = Mode

The left and right sides are mirror images.

c) Total Area = 1

The entire area under the curve represents total probability = **100%**.

d) Defined by Two Parameters

Mean (μ) – determines the center of the distribution.

Standard Deviation (σ) – determines the spread or width.

Example 1

In a programming exam, a student who scored 50 marks has a z-score of −1.2, and another student who scored 90 marks has a z-score of +1.8.

Find:

The mean of the exam scores.

The standard deviation of the exam scores.

Solution

The formula for z-score is:

$$z = \frac{X - \mu}{\sigma}$$

Where:

- X = score
- μ = mean
- σ = standard deviation

Set up two equations using the given z-scores and scores, then solve for μ and σ.

Solution continue.....

Given

- Score $X_1 = 50$, z-score $z_1 = -1.2$
- Score $X_2 = 90$, z-score $z_2 = +1.8$

Formula for z-score:

$$z = \frac{X - \mu}{\sigma}$$

So we form two equations:

Equation 1

$$-1.2 = \frac{50 - \mu}{\sigma}$$

Equation 2

$$1.8 = \frac{90 - \mu}{\sigma}$$

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Solution continue.....

Step 1: Eliminate σ by multiplying

Multiply both equations by σ:

1. $-1.2\sigma = 50 - \mu$
2. $1.8\sigma = 90 - \mu$

Subtract Eq. 1 from Eq. 2:

$$1.8\sigma - (-1.2\sigma) = (90 - \mu) - (50 - \mu)$$

$$3\sigma = 40$$

$$\sigma = \frac{40}{3} \approx 13.33$$

Solution continue.....

Step 2: Substitute σ back into one equation to find μ

Use:

$$-1.2 = \frac{50 - \mu}{13.33}$$

$$50 - \mu = -1.2 \times 13.33 \approx -16$$

$$\mu = 50 + 16 = 66$$

✅ Final Answers

Quantity	Value
Mean (μ)	66 marks
Standard deviation (σ)	13.33 marks (approx.)

Example 2

In a Java test, a student scoring 55 has a z-score of –1.2,
and another student scoring 95 has a z-score of +1.3. Find the mean and standard deviation of the test scores.

Solution

Given

- Score $X_1 = 55$, z-score $z_1 = -1.2$
- Score $X_2 = 95$, z-score $z_2 = +1.3$

Formula for z-score:

$$z = \frac{X - \mu}{\sigma}$$

So we form two equations:

Equation 1

$$-1.2 = \frac{55 - \mu}{\sigma}$$

Equation 2

$$1.3 = \frac{95 - \mu}{\sigma}$$

Step 1: Multiply both equations by σ

1. $-1.2\sigma = 55 - \mu$
2. $1.3\sigma = 95 - \mu$

Subtract Eq. 1 from Eq. 2:

$$1.3\sigma + 1.2\sigma = 95 - \mu - (55 - \mu)$$

$$2.5\sigma = 40$$

$$\sigma = \frac{40}{2.5} = 16$$

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Step 2: Substitute σ = 16 into one equation to find μ

Use:

$$-1.2\sigma = 55 - \mu$$

$$-1.2(16) = 55 - \mu$$

$$-19.2 = 55 - \mu$$

$$\mu = 55 + 19.2 = 74.2$$

✅ Final Answers

Quantity	Value
Mean (μ)	74.2 marks
Standard deviation (σ)	16 marks

Example 3

In a Cloud Computing course, assignment scores follow a normal distribution. A student who scored **30** has a **z-score of –1.8**, and another who scored **75** has a **z-score of +1.0**.

Determine the mean score and standard deviation.

Solution

✅ Given

- $\text{Score}_1 = 30$, $z_1 = -1.8$
- $\text{Score}_2 = 75$, $z_2 = +1.0$

Formula:

$$z = \frac{X - \mu}{\sigma}$$

So we write two equations:

(1)

$$30 = \mu - 1.8\sigma$$

(2)

$$75 = \mu + 1\sigma$$

Solution

Subtract (1) from (2):

$$75 - 30 = (\mu + \sigma) - (\mu - 1.8\sigma)$$

$$45 = 2.8\sigma$$

$$\sigma = \frac{45}{2.8} = 16.07 \approx 16.1$$

✅ Step 2: Substitute σ into one equation

Use equation (2):

$$75 = \mu + 16.07$$

$$\mu = 75 - 16.07$$

$$\mu = 58.93 \approx 58.9$$

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Example 4

In a Cloud Computing course, assignment marks follow a normal distribution with a mean of 60 and a standard deviation of 12. If a student's performance corresponds to a z-score of +1.5, what is the student's assignment score?

Solution

where

- $z = 1.5$
- $\mu = 60$
- $\sigma = 12$

Rearrange to find the score x:

$$x = \mu + z\sigma$$

Substitute the values:

$$x = 60 + (1.5)(12)$$

$$x = 60 + 18$$

$$\boxed{x = 78}$$

Example 5

Machine Learning mid-term scores have a **mean of 55** and a **standard deviation of 11**.
A student's performance places them at a **z-score of –1.2**.
Find the actual marks obtained by the student.

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Solution

Given:

- Mean $\mu = 55$
- Standard deviation $\sigma = 11$
- $z = -1.2$

Substitute:

$$x = 55 + (-1.2)(11)$$

$$x = 55 - 13.2$$

$$x = 41.8$$

✅ **The student's actual mid-term score is 41.8 marks (≈ 42 marks).**

Example 6

In a Data Analytics lab test:

A student scored **72**,

The mean of the test is **60**,

And the z-score for this student is **+1.5**.

Find the:

a) **Standard deviation**
b) **Variance**

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Example 7

$$1.5 = \frac{72 - 60}{\sigma}$$

$$1.5 = \frac{12}{\sigma}$$

Cross multiply:

$$\sigma = \frac{12}{1.5}$$

$$\sigma = 8$$

✅ **Standard deviation = 8**

Step 2: Find Variance

$$\sigma^2 = 8^2 = 64$$

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Example 7

Machine Learning Project Evaluation

In Machine Learning project scores:

A student got **64 marks**

Mean = **52**

z-score = **+2.0**

Calculate:

a) **Standard deviation**
b) **Variance**

Solution

Step 1: Find Standard Deviation

$$2.0 = \frac{64 - 52}{\sigma}$$

$$2.0 = \frac{12}{\sigma}$$

Cross-multiply:

$$\sigma = \frac{12}{2.0}$$

$$\sigma = 6$$

✅ Standard deviation = 6

Step 2: Find Variance

$$\sigma^2 = 6^2 = 36$$

Syllabus

-Continuous Probability Distribution

Uniform Distribution

-Finding Mean and variance of uniform Distribution

Continuous Uniform Distribution

Definition:

A uniform distribution is a probability distribution in which all possible values of a random variable are equally likely(Every value has the same chance of occurring).

Theoretical Features:

- PDF is constant over the interval
- Represents complete randomness over a range
- Used as a baseline distribution in simulations

Why need to Study Uniform Distribution

- Used in computer simulations
- Helps in performance analysis
- Basis for random number generation
- Important in probability theory and statistics

Real-Life Examples

- File upload time between 12 and 20 seconds
- CPU processing time between 8 and 18 milliseconds
- Online quiz completion time between 30 and 50 minutes
- Waiting time for a system response

Formulas for Continuous Uniform Distribution

- Mean:

$$\mu = \frac{a+b}{2}$$

- Variance:

$$\sigma^2 = \frac{(b-a)^2}{12}$$

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Example 1

The marks in an online quiz are uniformly distributed between 20 and 60.

Find

- the mean of the distribution.
- the variance
- standard deviation

Solution

Let the marks be uniformly distributed on the interval

$$a = 20, \quad b = 60$$

For a **Uniform(a, b)** distribution:

Mean

$$\mu = \frac{a+b}{2}$$

$$\mu = \frac{20+60}{2} = \frac{80}{2} = 40$$

Solution

Variance

$$\sigma^2 = \frac{(b-a)^2}{12}$$

$$\sigma^2 = \frac{(60-20)^2}{12}$$

$$= \frac{40^2}{12}$$

$$= \frac{1600}{12}$$

$$= 133.33$$

Standard Deviation

$$\sigma = \sqrt{133.33} \approx 11.55$$

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Example 2

Upload time for a file is uniformly distributed from **12 to 20 seconds**.

Find the **mean upload time**.

Find the **variance**.

Solution

1 Mean Upload Time

$$\mu = \frac{a+b}{2}$$

$$\mu = \frac{12+20}{2} = \frac{32}{2} = 16$$

✅ Mean upload time = 16 seconds

2 Variance

$$\sigma^2 = \frac{(b-a)^2}{12}$$

$$\sigma^2 = \frac{(20-12)^2}{12}$$

$$= \frac{8^2}{12}$$

$$= \frac{64}{12}$$

$$= 5.33$$

✅ Variance ≈ 5.33

Example 3

The processing time of a CPU task is uniformly distributed between **8 and 18 milliseconds**.

Find:

Mean processing time

Variance

Solution

1. Mean Processing Time

$$\mu = \frac{8+18}{2} = \frac{26}{2} = 13 \text{ ms}$$

2. Variance

$$\sigma^2 = \frac{(18-8)^2}{12} = \frac{10^2}{12} = \frac{100}{12} = 8.33 \text{ ms}^2 \text{ (approx)}$$

Example 4

The time taken by students to complete an online quiz is uniformly distributed between **30 and 50 minutes**.

Find:

Mean completion time

Variance

Solution

1. Mean Completion Time

$$\mu = \frac{30 + 50}{2} = \frac{80}{2} = 40 \text{ minutes}$$

2. Variance

$$\sigma^2 = \frac{(50 - 30)^2}{12} = \frac{20^2}{12} = \frac{400}{12} = 33.33 \text{ minutes}^2 \text{ (approx)}$$

Thank You

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