

Probability and Probability Distribution

Unit No 4: Random Variables and Mathematical Expectation

Faculty Name :Dr. Vidhya Kshirsagar

Syllabus

Unit 4- Random variables and Mathematical Expectation

- ❑ Random Variable (Discrete and continuous),
- ❑ Probability Distribution of a Random Variable
- ❑ Probability Mass Function
- ❑ Cumulative Distribution Function
- ❑ Mathematical Expectation of Probability Distribution, Theorems
- ❑ Calculation of Mean and Variance using Mathematical Expectation



Random Variable (Discrete and continuous)

What is a Random Variable?

A Random Variable (RV) is a function that assigns a numerical value to each outcome of a random experiment. It helps in quantitatively analyzing uncertainty. Random variables are of two types — Discrete (countable values) and Continuous (uncountable values).

Important Notation

Random Variable is usually denoted as:

X, Y, Z

Individual values are:

x_1, x_2, x_3, \dots



Why the name “Random Variable”?

✓ Why the name “Random”?

Because which value will occur is **not known in advance** → it depends on chance.

✓ Why the name “Variable”?

Because its value **changes** depending on the outcome of the experiment.

Example: toss 3 coins

Sometimes you get 0 heads, sometimes 1, sometimes 2, sometimes 3.

So value **varies** → that's why it's called variable.



Why do we need Random Variables?

❑ To analyze uncertain events mathematically:

A random variable (often denoted as X) assigns a numerical value to each possible outcome of a random phenomenon. This conversion allows the application of mathematical tools like calculus, algebra, and analysis to events that are inherently unpredictable.



❑ **To calculate probabilities, mean, and variance:** Once outcomes are represented numerically, the *probability distribution* of the random variable describes the likelihood of each value occurring. This distribution is used to calculate key metrics:

Probabilities: Determining the chance that an event (e.g., a stock price reaching a certain level) falls within a specific range .

Mean (Expected Value): Representing the long-run average outcome of the variable. It is a weighted average of all possible values

Variance: Measuring the spread or dispersion of the possible outcomes around the mean. It quantifies the level of risk or volatility associated with an uncertain event



Why do we need Random Variables?

❑ To model real-world business, finance, and data problems:

Random variables are essential for creating models in various fields:

Finance: Modeling stock price movements, interest rate fluctuations, and portfolio risk .

Business: Forecasting demand for a product, analyzing insurance claims, and assessing project completion times .

Data Science: Characterizing data distributions, performing hypothesis testing, and building predictive models .



Why do we need Random Variables?

□ Random variables help convert uncertainty into numbers for decision-making:

By providing quantifiable risk and expected value metrics, random variables allow decision-makers to evaluate different strategies objectively. For example, a business can compare two investment projects based on their expected returns (mean) and associated risks (variance), leading to more informed choices



Random Variable vs. Outcome

| Outcome | Random Variable Value |
|---------|-----------------------------|
| HH | 2 heads $\rightarrow X = 2$ |
| HT | 1 head $\rightarrow X = 1$ |
| TT | 0 heads $\rightarrow X = 0$ |

Outcomes are **events**,
Random variable is **number assigned to results**.



✨ Types of Random Variables

| Type | Meaning | Examples |
|----------------------------|--|--|
| Discrete Random Variable | Takes countable values (finite or infinite) | No. of heads in coin toss, No. of calls per hour |
| Continuous Random Variable | Takes uncountably infinite values | Height, weight, time, temperature, rainfall |

Examples

1 — Coin Toss

Toss a coin 2 times

Let X = number of Heads

Possible values of $X \rightarrow \{0, 1, 2\}$

2 — Rolling a Dice

Let X = number appearing on dice

$X = \{1, 2, 3, 4, 5, 6\}$

3 — Continuous RV (Height)

Let X = height of students

Values like 160.5 cm, 161.7 cm \rightarrow infinite possible values



Discrete Random Variable (DRV)

Definition

A random variable that can take **specific separate values** (countable).

Examples

| Experiment | Random Variable X | Possible Values |
|--------------------|----------------------|------------------|
| Toss 2 coins | Number of heads | 0, 1, 2 |
| Roll a die | Number shown | 1, 2, 3, 4, 5, 6 |
| Call center | No. of calls in 1 hr | 0, 1, 2, 3, ... |
| Bank loan defaults | No. of defaults | 0, 1, 2, ... |

Simple example

Toss a coin 3 times.

Let **X = Number of heads**

Possible outcomes = HHH, HHT, HTH, HTT, THH, THT, TTH, TTT

X can be:

$X = \{0, 1, 2, 3\}$




Continuous Random Variable (CRV)

Important

You **cannot count** values of a continuous random variable, you can only **measure** them.

A random variable that can take values from a **range or interval** (uncountably infinite possibilities).

Examples

| Real situation | Random Variable X |  |
|---------------------------|------------------------|---|
| Time to complete a task | 0 to ∞ minutes | |
| Height of students | Continuous measurement | |
| Amount of rain | Continuous | |
| Amount of petrol consumed | Continuous | |
| Stock prices | Continuous | |

Example of a Continuous Random Variable

Some examples of Continuous Random Variable are:

The height of an adult male or female.

The weight of an object.

The time is taken to complete a task.

The temperature of a room.

The speed of a vehicle on a highway.



Difference Between Discrete Random Variable And Continuous Random Variable

| | Discrete Random Variable | Continuous Random Variable |
|---|---|--|
| Definition | Takes on a finite or countably infinite set of possible values. | Takes on any value within a range or interval i.e., can be uncountably infinite as well. |
| Probability Distribution | Described by a probability mass function (PMF), which gives the probability of each possible value. | Described by a probability density function (PDF), which gives the probability density at each possible value. |
| Example | Number of heads in three coin tosses. | Height of a person selected at random. |
| Probability of a single value | Non-zero probability at each possible value. | Zero probability at each possible value. |
| Cumulative Distribution Function | Describes the probability of getting a value less than or equal to a particular value. | Describes the probability of getting a value less than or equal to a particular value. |
| Mean and Variance | Mean and variance can be calculated directly from the PMF. | Mean and variance can be calculated using the PDF and integration. |
| Probability of an Interval | The probability of an interval is the sum of the probabilities of each value in the interval. | The probability of an interval is the area under the PDF over the interval. |



Probability Distribution of a Random Variable



1. Tossing Two/three Coins

Let X = number of heads obtained.

2. A card is drawn at random from a deck of 52 cards.

Let $X=1$ if an ace appears, and $X=0$ if not.

Example

3. A dealer in ready-made shirts has studied his record and notices that for the past 310 working days in the year demand for the shirts has varies as follows:

| Demand('000 units) | 5 | 6 | 7 | 8 | 9 | 10 |
|--------------------|----|----|----|-----|----|----|
| Number of days | 20 | 60 | 80 | 120 | 20 | 10 |

Probability Mass function

1. Meaning

A Probability Mass Function (PMF) gives the probability that a discrete random variable takes on a particular value.

In other words, it tells us how the total probability (1) is distributed among all possible values of a discrete random variable.



Probability Mass function

In probability theory, we often perform experiments where the outcomes are uncertain — for example, tossing a coin, rolling a die, or drawing a card from a deck.

Each possible outcome of such an experiment can be represented numerically using a random variable.

When this random variable takes only a finite or countable number of values, it is called a discrete random variable.

For such variables, we use a Probability Mass Function (PMF) to describe the probability distribution.



Probability Mass Function vs Probability Density Function

| Probability Mass Function | Probability Density Function |
|---|---|
| The PMF is the probability that a discrete random variable takes at an exact value. | The PDF is the probability that a continuous random variable takes at a specified interval. |
| The PMF deals with the discrete random variables. | The PDF deals with the continuous random variables. |
| PMF is evaluated at specific point. | PDF is evaluated at specified interval |
| $f(x) = P(X = x)$ | $P(x) = F'(x)$ where, $F(x)$ is CDF |



2. Definition

- Let X be a **discrete random variable** that can take values $x_1, x_2, x_3, \dots, x_n$.

Then the **probability mass function (PMF)** of X is written as:

$$P(X = x_i) = p(x_i)$$

where:

- $p(x_i) \geq 0$ for all i
- The total probability is 1

$$\sum_i p(x_i) = 1$$



Important Points

- PMF is used only for discrete random variables (not for continuous).
- PMF gives probability for exact values of X (e.g., $P(X=2)$).
- It helps create the probability distribution table of X .



Mathematical Expectation or expected value

Meaning

Expected value of a random variable is the sum of the products obtained by multiplying the various values that the variable can take by their corresponding probabilities and is usually denoted by $E(X)$



E(X) for a Discrete Random Variable

The **Expected Value** of a discrete random variable gives the **average or mean value** of the random variable over a large number of repetitions of an experiment. It represents the **long-term average outcome**.

◆ Definition

If a random variable X takes possible values $x_1, x_2, x_3, \dots, x_n$ with corresponding probabilities $p(x_1), p(x_2), p(x_3), \dots, p(x_n)$, then the **expected value** of X is given by:

$$E(X) = \sum_{i=1}^n x_i p(x_i)$$

That is,

$$E(X) = x_1 p_1 + x_2 p_2 + x_3 p_3 + \dots + x_n p_n$$

Example 1

A die is thrown at random

- a) What is the expectation of the number on it?
- b) What is the variance of the expectation?



The random variable X can take the values

1, 2, 3, 4, 5, or 6.

Each value has an equal probability of $\frac{1}{6}$.

So, the probability distribution of X is:

| | | | | | | |
|--------|---------------|---------------|---------------|---------------|---------------|---------------|
| X | 1 | 2 | 3 | 4 | 5 | 6 |
| $P(X)$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ |



(a) Expectation of X

$$E(X) = \sum xP(x)$$

$$E(X) = 1\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 3\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right) + 6\left(\frac{1}{6}\right)$$

$$E(X) = \frac{1+2+3+4+5+6}{6} = \frac{21}{6} = 3.5$$

Hence, the Expectation (Mean) = 3.5



(b) Variance of X

Variance is given by:

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

We first calculate $E(X^2)$:

$$E(X^2) = \sum x^2 P(x)$$

$$E(X^2) = 1^2 \left(\frac{1}{6}\right) + 2^2 \left(\frac{1}{6}\right) + 3^2 \left(\frac{1}{6}\right) + 4^2 \left(\frac{1}{6}\right) + 5^2 \left(\frac{1}{6}\right) + 6^2 \left(\frac{1}{6}\right)$$

$$E(X^2) = \frac{1 + 4 + 9 + 16 + 25 + 36}{6} = \frac{91}{6}$$

Now,

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

$$\text{Var}(X) = \frac{91}{6} - \left(\frac{21}{6}\right)^2$$

$$\text{Var}(X) = \frac{91}{6} - \frac{441}{36} = \frac{35}{12}$$

So, **Variance** = 35/12

And the **Standard Deviation** = $\sqrt{35/12}$.



Final Answers:

- $E(X) = 3.5$
- $\text{Var}(X) = 35/12 = 2.9167$
- $\text{SD}(X) = \sqrt{35/12} \approx 1.7078$



Example 2

If three coins are tossed
find standard Deviation of
the expectation of the
number of heads.



| X (Number of Heads) | Possible Outcomes | Frequency | Probability P(X) |
|---------------------|-------------------|-----------|------------------|
| 0 | TTT | 1 | 1/8 |
| 1 | HTT, THT, TTH | 3 | 3/8 |
| 2 | HHT, HTH, THH | 3 | 3/8 |
| 3 | HHH | 1 | 1/8 |

Find the Expectation (Mean)

$$E(X) = \sum xP(x)$$

$$E(X) = 0(1/8) + 1(3/8) + 2(3/8) + 3(1/8)$$

$$E(X) = (0 + 3 + 6 + 3)/8 = 12/8 = 1.5$$

$$E(X) = 1.5$$



: Find $E(X^2)$

$$E(X^2) = \sum x^2 P(x)$$

$$E(X^2) = 0^2(1/8) + 1^2(3/8) + 2^2(3/8) + 3^2(1/8)$$

$$E(X^2) = (0 + 3 + 12 + 9)/8 = 24/8 = 3$$

: Find Variance

$$Var(X) = E(X^2) - [E(X)]^2$$

$$Var(X) = 3 - (1.5)^2 = 3 - 2.25 = 0.75$$



Find Standard Deviation

$$SD(X) = \sqrt{Var(X)} = \sqrt{0.75} = 0.866$$

✓ Final Answers:

- Expectation (Mean) = 1.5
- Variance = 0.75
- Standard Deviation = 0.866 (approx.)



Example 3

If a player plays a game of chance where he can win Rs. 5000 with Probability 0.6, win Rs. 2,500 with probability 0.3 and lose Rs. 15,000 with probability 0.1. What is his expected gain in one play of the game.



Solution

A player can have three outcomes:

| Outcome | Value (Rs.) | Probability |
|---------|-------------|-------------|
| Win | 5000 | 0.6 |
| Win | 2500 | 0.3 |
| Lose | -15000 | 0.1 |



Expected Gain ($E(X)$)

$$E(X) = \sum (x \cdot p)$$

Compute each term:

1. $5000 \times 0.6 = 3000$

2. $2500 \times 0.3 = 750$

3. $-15000 \times 0.1 = -1500$

Now add them:

$$E(X) = 3000 + 750 - 1500$$

$$E(X) = 2250$$



Example 4

A company rents cloud servers. For each hour:

Saves ₹8,000 due to autoscaling with probability 0.5

Saves ₹3,000 with probability 0.3

Loses ₹10,000 due to downtime with probability 0.2

Find expected hourly cost/benefit.



Solution

Outcomes: +₹8,000 ($p=0.5$), +₹3,000 ($p=0.3$), -₹10,000 ($p=0.2$)

- $8000 \times 0.5 = 4000$
- $3000 \times 0.3 = 900$
- $-10000 \times 0.2 = -2000$

$$E = 4000 + 900 - 2000 = ₹2,900 \text{ (expected hourly benefit)}$$



Example 5

A company faces cyber-attack attempts daily:

No loss (attack blocked): probability 0.7

Minor data breach costing ₹20,000: probability 0.2

Major ransomware loss of ₹2,00,000: probability 0.1

Find expected daily loss.



Solution

Outcomes: ₹0 ($p=0.7$), -₹20,000 ($p=0.2$), -₹200,000 ($p=0.1$)

- $0 \times 0.7 = 0$
- $-20000 \times 0.2 = -4000$
- $-200000 \times 0.1 = -20000$

$$E = 0 - 4000 - 20000 = -₹24,000 \text{ (expected daily loss of ₹24,000)}$$



Example 6

A system sends 3 data packets. The probability distribution of successfully delivered packets (X) is:

X = number of packets
delivered

0

1

2

3

$P(X)$

0.1

0.2

0.4

0.3

Find:

1. Expectation $E(X)$
2. Variance $V(X)$
3. Standard deviation $SD(X)$

Solution

PMF:

| | | | | |
|---|-----|-----|-----|-----|
| x | 0 | 1 | 2 | 3 |
| p | 0.1 | 0.2 | 0.4 | 0.3 |

$$1. E(X) = \sum xP(x) = 0 \cdot 0.1 + 1 \cdot 0.2 + 2 \cdot 0.4 + 3 \cdot 0.3 = 0 + 0.2 + 0.8 + 0.9 = 1.9.$$

$$2. E(X^2) = 0^2 \cdot 0.1 + 1^2 \cdot 0.2 + 2^2 \cdot 0.4 + 3^2 \cdot 0.3 = 0 + 0.2 + 1.6 + 2.7 = 4.5.$$

$$3. \text{Var}(X) = E(X^2) - [E(X)]^2 = 4.5 - (1.9)^2 = 4.5 - 3.61 = 0.89.$$

$$4. \text{SD}(X) = \sqrt{0.89} \approx 0.943.$$



Example 7

A dealer in television sets estimates from his past experience the probabilities of his selling television sets in a day is given below. Find the expected number of sales in a day.

| Number of TV Sold in a day | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|----------------------------|------|------|------|------|------|------|------|
| Probability | 0.02 | 0.10 | 0.21 | 0.32 | 0.20 | 0.09 | 0.06 |

Solution

We observe that the number of television sets sold in a day is a random variable which can assume the values 0, 1, 2, 3, 4, 5, 6 with the respective probabilities given in the table.

$$\begin{aligned}\text{Now the expectation of } x &= E(X) = \sum x_i p_i \\ &= x_1 p_1 + x_2 p_2 + x_3 p_3 + x_4 p_4 + x_5 p_5 + x_6 p_6 \\ &= (0)(0.02) + (1)(0.010) + 2(0.21) + (3)(0.32) + 4(0.20) \\ &\quad + (5)(0.09) + (6)(0.06)\end{aligned}$$

$$E(X) = 3.09$$

The expected number of sales per day is 3



Example 8

There are three alternative proposals before a businessman to start a new project:

Proposal X: Profit of Rs. 4 lakhs with a probability of 0.7 or a loss of Rs. 60,000 with a probability of 0.3.

Proposal Y: Profit of Rs. 8 lakhs with a probability of 0.3 or a loss of Rs. 1.50 lakhs with a probability of 0.7.

Proposal Z: Profit of Rs. 3 lakhs with a probability of 0.6 or a loss of Rs. 40,000 with a probability of 0.4. If he wants to maximize the profits and minimize the loss, which proposal he should prefer.

Solution

Convert amounts to lakhs (1 lakh = Rs.100,000) to keep calculations simple:

- Proposal X: profit = 4.00 lakhs ($p = 0.7$), loss = 0.60 lakhs ($p = 0.3$)
- Proposal Y: profit = 8.00 lakhs ($p = 0.3$), loss = 1.50 lakhs ($p = 0.7$)
- Proposal Z: profit = 3.00 lakhs ($p = 0.6$), loss = 0.40 lakhs ($p = 0.4$)

Compute expected value $E = (\text{probability of profit} \times \text{profit}) + (\text{probability of loss} \times (-\text{loss}))$.

Proposal X

$$E_X = 0.7 \times 4.00 + 0.3 \times (-0.60)$$

$$\text{Stepwise: } 0.7 \times 4.00 = 2.80$$

$$0.3 \times 0.60 = 0.18 \text{ so loss term} = -0.18$$

$$E_X = 2.80 - 0.18 = 2.62 \text{ lakhs} = \text{Rs. } 2,62,000$$



Proposal Y

$$E_Y = 0.3 \times 8.00 + 0.7 \times (-1.50)$$

$$\text{Stepwise: } 0.3 \times 8.00 = 2.40$$

$$0.7 \times 1.50 = 1.05 \text{ so loss term} = -1.05$$

$$E_Y = 2.40 - 1.05 = 1.35 \text{ lakhs} = \text{Rs. 1,35,000}$$

Proposal Z

$$E_Z = 0.6 \times 3.00 + 0.4 \times (-0.40)$$

$$\text{Stepwise: } 0.6 \times 3.00 = 1.80$$

$$0.4 \times 0.40 = 0.16 \text{ so loss term} = -0.16$$

$$E_Z = 1.80 - 0.16 = 1.64 \text{ lakhs} = \text{Rs. 1,64,000}$$

Conclusion

Expected values:

- $E_X = 2.62$ lakhs (Rs.262,000)
- $E_Z = 1.64$ lakhs (Rs.164,000)
- $E_Y = 1.35$ lakhs (Rs.135,000)

To maximise expected profit and minimise expected loss, the businessman should prefer **Proposal X** (highest expected value).



Example 9

A company is choosing one of three server upgrade proposals:

Proposal P:

Savings of ₹10 lakhs with probability 0.45

Extra cost of ₹5 lakhs with probability 0.55

Proposal Q:

Savings of ₹6 lakhs with probability 0.75

Extra cost of ₹2 lakhs with probability 0.25

Proposal R:

Savings of ₹12 lakhs with probability 0.3

Extra cost of ₹7 lakhs with probability 0.7

Which proposal minimizes risk while maximizing expected savings?



Solution

Proposal P

- Savings: $+10 \text{ lakhs} \times 0.45 = +4.5 \text{ lakhs}$
- Loss: $-5 \text{ lakhs} \times 0.55 = -2.75 \text{ lakhs}$

$$\text{EMV(P)} = 4.5 - 2.75 = +1.75 \text{ lakhs}$$

Proposal Q

- Savings: $+6 \text{ lakhs} \times 0.75 = +4.5 \text{ lakhs}$
- Loss: $-2 \text{ lakhs} \times 0.25 = -0.5 \text{ lakhs}$

$$\text{EMV(Q)} = 4.5 - 0.5 = +4 \text{ lakhs}$$

Proposal R

- Savings: $+12 \text{ lakhs} \times 0.3 = +3.6 \text{ lakhs}$
- Loss: $-7 \text{ lakhs} \times 0.7 = -4.9 \text{ lakhs}$

$$\text{EMV(R)} = 3.6 - 4.9 = -1.3 \text{ lakhs (overall loss)}$$



✓ Proposal Q has:

- Highest expected savings: +4 lakhs
- Lowest probability of loss: 0.25
- Smallest possible loss: 2 lakhs

**Therefore:

👉 Proposal Q is the best choice.**

It maximizes expected savings and minimizes risk, so the company should choose Proposal Q.

Cumulative Distribution Function



What is a Cumulative Distribution Function?

A cumulative distribution function (CDF) describes the probabilities of a random variable having values less than or equal to x . It is a cumulative function because it sums the total likelihood up to that point. Its output always ranges between 0 and 1.

CDFs have the following definition:

$$\text{CDF}(x) = P(X \leq x)$$

Where X is the random variable, and x is a specific value. The CDF gives us the probability that the random variable X is less than or equal to x . These functions are non-decreasing. As x increases, the likelihood can either increase or stay constant, but it can't decrease.



What is a Cumulative Distribution Function?

The **Cumulative Distribution Function (CDF)**, of a real-valued random variable X , evaluated at x , is the probability function that X will take a value less than or equal to x . It is used to describe the probability distribution of random variables in a table. And with the help of these data, we can easily create a CDF plot in an excel sheet.

In other words, CDF finds the cumulative probability for the given value. To determine the probability of a random variable, it is used and also to compare the probability between values under certain conditions. For discrete distribution functions, CDF gives the probability values till what we specify and for continuous distribution functions, it gives the area under the probability density function up to the given value specified.



Example 1

A random variable X has the following PMF:

| | | | | | |
|----------|------|------|------|------|------|
| x | 0 | 1 | 2 | 3 | 4 |
| $P(X=x)$ | 0.10 | 0.20 | 0.30 | 0.25 | 0.15 |

(a) Find the CDF $F(x) = P(X \leq x)$.

(b) Find $P(1 < X \leq 3)$.

(c) Find $P(X = 4)$.

CDF Table

| x | $F(x) = P(X \leq x)$ |
|---------|----------------------|
| $x < 0$ | 0 |
| 0 | 0.10 |
| 1 | $0.10 + 0.20 = 0.30$ |
| 2 | $0.30 + 0.30 = 0.60$ |
| 3 | $0.60 + 0.25 = 0.85$ |
| 4 | $0.85 + 0.15 = 1.00$ |
| $x > 4$ | 1 |



So the CDF:

$$F(x) = \begin{cases} 0, & x < 0 \\ 0.10, & 0 \leq x < 1 \\ 0.30, & 1 \leq x < 2 \\ 0.60, & 2 \leq x < 3 \\ 0.85, & 3 \leq x < 4 \\ 1.00, & x \geq 4 \end{cases}$$



(b) Find $P(1 < X \leq 3)$

Values between 1 and 3 (not including 1, include 3):

- $X = 2$
- $X = 3$

$$P(1 < X \leq 3) = P(X = 2) + P(X = 3)$$

$$= 0.30 + 0.25 = \boxed{0.55}$$



(c) Find $P(X = 4)$

Directly from PMF:

$$P(X = 4) = \boxed{0.15}$$

Final Answers

(a) CDF values: 0, 0.10, 0.30, 0.60, 0.85, 1

(b) $P(1 < X \leq 3) = \boxed{0.55}$

(c) $P(X = 4) = \boxed{0.15}$



Example 2

A random variable X has the PMF:

| x | 0 | 1 | 2 | 3 |
|----------|------|------|------|------|
| $P(X=x)$ | 0.15 | 0.25 | 0.40 | 0.20 |

(a) Find the CDF $F(x)$

(b) Find $P(1 < X \leq 3)$

(c) Find $P(X=0)$



solution

(a) Find the CDF $F(x)$

Add probabilities cumulatively:

- $F(0) = 0.15$
- $F(1) = 0.15 + 0.25 = 0.40$
- $F(2) = 0.40 + 0.40 = 0.80$
- $F(3) = 0.80 + 0.20 = 1.00$

(b) Find $P(1 < X \leq 3)$

Values: $X = 2, 3$

$$P = 0.40 + 0.20 = 0.60$$

(c) Find $P(X = 0)$

From table: 0.15



Example 3

A random variable X has CDF:

| x | $F(x)$ |
|-----|--------|
| 0 | 0.10 |
| 1 | 0.30 |
| 2 | 0.55 |
| 3 | 0.85 |
| 4 | 1.00 |

- (a) Find $P(X=2)$
- (b) Find $P(1 < X \leq 3)$
- (c) Find $P(X=4)$



Solution

(a) Find $P(X = 2)$

$$P(X = 2) = F(2) - F(1) = 0.55 - 0.30 = 0.25$$

(b) Find $P(1 < X \leq 3)$

$$P = F(3) - F(1) = 0.85 - 0.30 = 0.55$$

(c) Find $P(X = 4)$

$$P(X = 4) = 1.00 - 0.85 = 0.15$$



Example 4

Given PMF:

| | | | | |
|----------|------|------|------|------|
| x | -1 | 0 | 2 | 4 |
| $P(X=x)$ | 0.10 | 0.20 | 0.50 | 0.20 |

Find $P(0 < X \leq 4)$

Solution

Find $P(0 < X \leq 4)$

Values: 2, 4

$$P = 0.50 + 0.20 = 0.70$$



Example 5

Given PMF:

| | | | | | |
|----------|------|------|------|------|------|
| x | 1 | 3 | 5 | 6 | 8 |
| $P(X=x)$ | 0.10 | 0.15 | 0.20 | 0.30 | 0.25 |

(a) Find

CDF.

(b) Find $P(3 < X \leq 8)$

(c) Find $P(2 < X < 6)$



Solution

(a) Find CDF.

- $F(1) = 0.10$
- $F(3) = 0.10 + 0.15 = 0.25$
- $F(5) = 0.25 + 0.20 = 0.45$
- $F(6) = 0.45 + 0.30 = 0.75$
- $F(8) = 0.75 + 0.25 = 1.00$

(b) Find $P(3 < X \leq 8)$

Values: 5, 6, 8

$$P = 0.20 + 0.30 + 0.25 = 0.75$$

(c) Find $P(2 < X < 6)$

Values: 3, 5

$$P = 0.15 + 0.20 = 0.35$$



Thank You



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