Probability and Probability Distribution

Unit No 3: Probability

Syllabus

Unit 3- Probability

- ❑Trail, Events, Sample spaces,
- ❑Probability axioms
- ❑Independent and Dependent Events
- ❑Conditional probability and its applications.
- ❑Bayes's Theorem and its applications.

Probability

Probability is the mathematical measurement of the likelihood of an event occurring, expressed as a value between 0 (impossible) and 1 (certain). It's calculated as the ratio of favorable outcomes to the total possible outcomes within a sample space. Key concepts include the sample space (all possible outcomes) and an event (a subset of the sample space). For example, the probability of getting a head when flipping a coin is 1/2, with the sample space being {Head, Tail} and the event being {Head}.

Probability

The Probability of a given event is an expression of likelihood or chance of occurrence of an event.

A probability is a number which ranges from 0 (zero) to 1 (one) –zero for an event which cannot occur and 1 for an event certain to occur.

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Applications of Probability

Probability is used in many areas of life, including:

- **Weather Forecasting:** A 30% chance of rain indicates the likelihood of that event occurring.
- **Statistics and Research:** Analyzing data and making predictions about future events.
- **Everyday Decisions:** Helping people make choices when faced with uncertainty, such as deciding whether to take an umbrella.

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Applications of Probability in Computer Science

1. Artificial Intelligence (AI) & Machine Learning (ML)

Bayesian Networks: Model relationships between uncertain events.

Naive Bayes Classifier: Used in spam detection, sentiment analysis.

Probabilistic Models: Hidden Markov Models (HMM) for speech recognition, NLP, and gesture recognition.

Recommendation Systems: Netflix, Amazon, YouTube use probability to predict user preferences.

2. Science & Big Data

Predictive Analytics: Probability predicts trends and outcomes from large datasets.

Uncertainty Handling: Probabilistic models handle missing or noisy data.

Statistical Inference: Helps in drawing conclusions from samples about populations.

Applications of Probability in Computer Science

3. Computer Networks

Reliability Analysis: Probability calculates the likelihood of network failure.

Traffic Modeling: Helps predict congestion and optimize data routing.

Error Detection & Correction: Probabilistic models improve data transmission reliability.

4. Computer Security & Cryptography

Random Key Generation: Keys are generated using probabilistic models to ensure unpredictability.

Probabilistic Encryption Algorithms: Ensure security by using randomness.

Risk Assessment: Probability helps quantify chances of attacks or breaches.

Applications of Probability in Computer Science

5. Computer Vision & Image Processing

Object Detection: Probability helps in identifying objects in uncertain or noisy images.

Pattern Recognition: Probabilistic models classify images, handwriting, or facial features.

6. Natural Language Processing (NLP)

Word Prediction & Autocomplete: Probability predicts the next word in a sentence.

Speech Recognition: Probabilistic models handle ambiguous sounds and words.

Text Classification: Spam filtering, sentiment analysis use probability-based algorithms.

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Applications of Probability in Computer Science

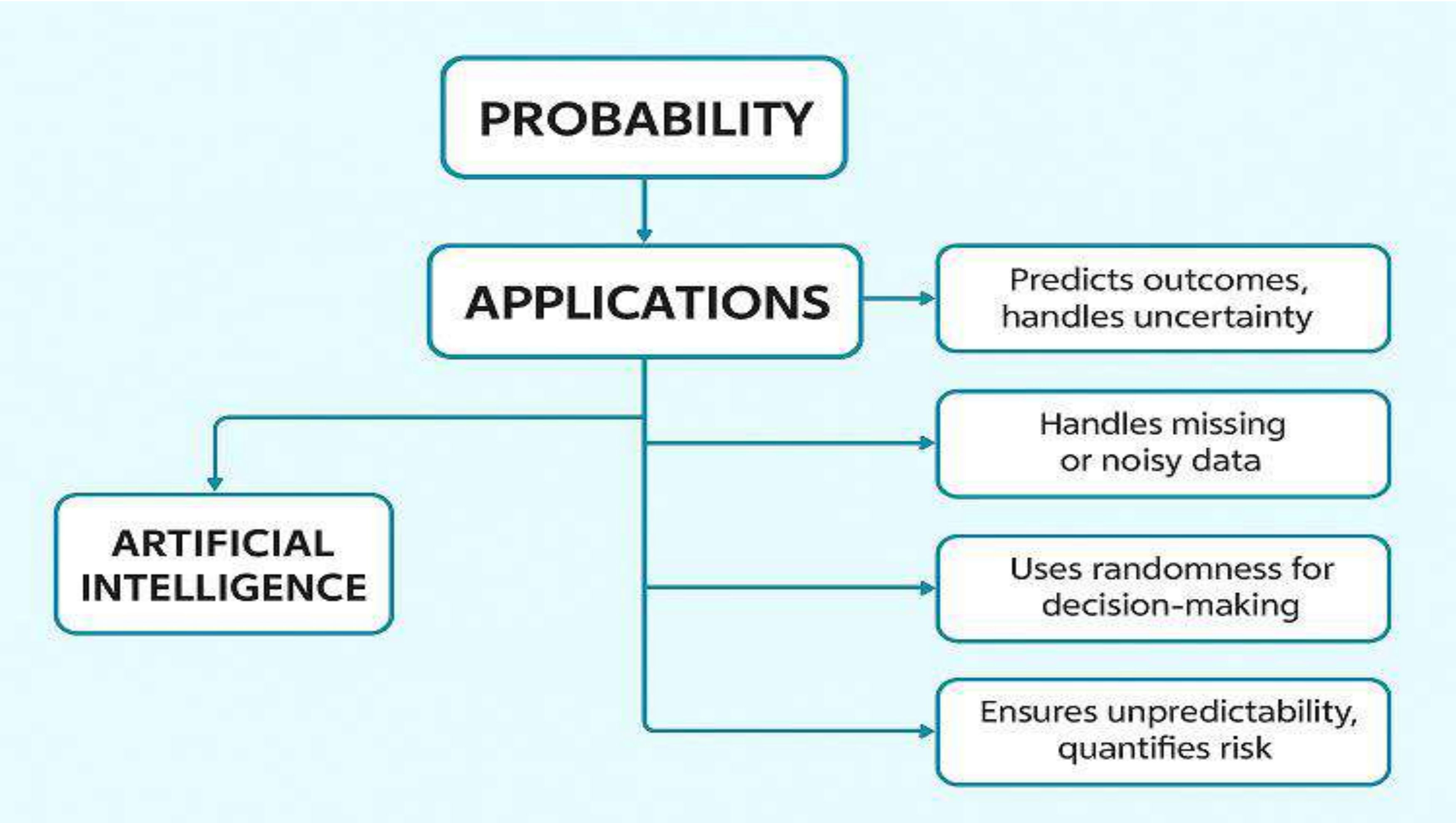
Gaming & Simulations

Game AI: Probability models opponent behavior, random events, or strategy optimization.

Monte Carlo Simulations: Probabilistic simulations to predict outcomes in complex systems.

Probability is **everywhere in computer science** — wherever there's uncertainty, randomness, or prediction, probability is used to make systems smarter, efficient, and reliable.

Applications of Probability in Computer Science



Computer Science Field	Example / Application	How Probability is Used
Artificial Intelligence & Machine Learning	Spam filters, recommendation systems, Hidden Markov Models	Predicts outcomes, handles uncertainty, builds probabilistic models
Data Science & Big Data	Predictive analytics, trend analysis	Handles missing/noisy data, estimates probabilities from samples
Algorithms	Randomized algorithms, Monte Carlo simulations	Uses randomness for decision-making and performance analysis
Computer Networks	Traffic modeling, network reliability	Predicts congestion, calculates failure probability, optimizes routing
Computer Security & Cryptography	Random key generation, probabilistic encryption	Ensures unpredictability, quantifies risk of attacks
Computer Vision & Image Processing	Object detection, pattern recognition	Classifies images under uncertainty, detects patterns probabilistically
Natural Language Processing (NLP)	Speech recognition, text classification	Predicts words, classifies text, handles ambiguous input
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Trial, Events, Sample spaces

Random Experiment(Trial): An experiment with multiple possible outcomes that cannot be predicted with certainty.

Example: Tossing a coin, rolling a die.

Outcome: A single result of a random experiment.

Sample Space (S): The set of all possible outcomes of a random experiment.

Tossing one coin → S={H,T}
Rolling one die → S={1,2,3,4,5,6}

Event (E): A collection of favorable outcomes from the sample space; it's a subset of the sample space.

Example: Rolling an even number → E={2,4,6}

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Key Concepts

- **The Scale:** Probability is on a scale from 0 to 1.
  - **0 (Impossible):** An event that cannot happen.
  - **1 (Certain):** An event that is guaranteed to happen.
  - **0.5 (50%):** An even chance for an event to occur or not occur.
- **Favorable Outcomes:** The specific results you are interested in.
- **Total Outcomes:** All possible results of an experiment.

How to Calculate Probability

To find the probability of an event, you use the following formula:

Probability = (Number of favorable outcomes) / (Total number of possible outcomes)

Examples

- **Coin Flip:**

The probability of getting tails when flipping a coin is 1/2 (or 0.5 or 50%) because there is one way to get tails, and two total possible outcomes (heads and tails).

- **Rolling a Die:**

The probability of rolling a 4 on a six-sided die is 1/6 because there is one favorable outcome (rolling a 4) and six total possible outcomes (1, 2, 3, 4, 5, 6).

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Example

a. If two fair coins are flipped, What is the probability of getting at least one head?

b. If three coins are flipped what is the probability of getting at least two tails.

c. If three coins are flipped, what is the probability of getting exactly one tail

Solution

a. What is the probability of getting atleast one head?

S = {HH,HT, TH, TT} - 4
A = { HH,HT, TH } - 3

P= ¾ = 0.75 = 75%

b. If three coins are flipped what is the probability of getting atleast two tails.

S={HHH,HHT,HTH,THH,HTT,THT,TTH,TTT} = 8

A = {HTT,THT,TTH,TTT} = 4

P= 4/8= ½ = 0.5 = 50 %

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c. If three coins are flipped, what is the probability of getting exactly one tail

S={HHH,HHT,HTH,THH,HTT,THT,TTH,TTT}

A= {HHT,HTH,THH}

P = 3/8 = 0.375

Example 2 :

2. A six sided die is tossed

a.What is probability of getting a 2?

b.What is the probability of getting a 3 or 5

c.What is the probability of getting a number that is at most 4

d.What is the probability of getting a number that is greater than 3

e.What is the probability of getting a number that is less than or equal to 5?

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Solution

We are tossing a **fair six-sided die**.
The **sample space** is:
S={1,2,3,4,5,6}
So, total outcomes n(S)=6

a. What is probability of getting a 2?

Probability of getting a 2

There is only **one outcome** that gives 2: {2}

P(2)=Number of favorable outcomes/Total outcomes=1/6= 0.167 = 16.7%

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Solution

B . What is the probability of getting a 3 or 5

Probability of getting a 3 or 5

Favorable outcomes: {3,5} → 2 outcomes

P(3 or 5)=2/6=1/3P= 0.333=33.3%

c. What is the probability of getting a number that is at most 4

Probability of getting a number that is at most 4

"At most 4" means numbers ≤ 4 → {1,2,3,4} → 4 outcomes

P(at most 4)=4/6=2/3 = 0.667 = 66.7 %

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Solution

d. What is the probability of getting a number that is greater than 3

Probability of getting a number greater than 3

Numbers > 3 → {4,5,6} → 3 outcomes

P(greater than 3)=3/6=1/2=0.5=50%

e. What is the probability of getting a number that is less than or equal to 5?

Probability of getting a number less than or equal to 5

Numbers ≤ 5 → {1,2,3,4,5} → 5 outcomes

P(≤ 5)=5/6=0.833=83.3%

Example

A teacher randomly picks one student from a class of 20 students (12 girls and 8 boys) what is the probability that the chosen student is girl?

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Solution

Total students in class = 20 (12 girls + 8 boys)

Favourable cases (girl chosen) = 12

Sample space size = 20

P(Girl)=Number of girls/Total students=12/20

The probability that the chosen student is a **girl** is **3/5 or 0.6**.

Example :

One card is drawn from a well-shuffled pack of 52 cards. Find the probability of getting:

a) a red card

b) a king

c) neither a king nor a queen.

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Solution

Sample Space: $n(S)=52$

a)Red cards = 26

$P(E)=26/52=1/2$

b) Kings = 4

$P(E)=4/52=1/13$

c) Neither King nor Queen = 52 – (4 + 4) = 44

$P(E)=44/52=11/13$

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Example

From a bag containing 10 black and 20 white balls, a ball is drawn at random. What is the probability that it is black?

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Axioms of Probability (Kolmogorov's Axioms)

Axioms of Probability

The axioms of probability, also known as Kolmogorov's axioms, are three fundamental rules that define a valid probability function:

❑non-negativity ($P(E) \geq 0$ for any event E)

❑normalization ($P(S) = 1$ for the entire sample space S)

❑, and additivity ($P(A \cup B) = P(A) + P(B)$ for mutually exclusive events A and B).

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Axioms of Probability

Axiom 1: Non-Negativity

Probability of an event is never negative. meaning it can't be less than zero.

For any event A,

$P(A) \geq 0$

Axioms of Probability

Axiom 2: Normalization

The probability that **something in the sample space happens is 1** (certainty).

For the sample space S,

$P(S)=1$

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Axioms of Probability

Axiom 3: Additivity

Probability of either A or B happening is the sum of their probabilities (when they don't overlap).

If A and B are mutually exclusive events (cannot occur together)or disjoint events, then

$P(A \cup B)=P(A)+P(B)$

Ex. Toss two coins

Sample space = {HH, HT, TH, TT}   A = {HH} B= {HT}

P(A U B) means Probability of A or B = P(A) + P(B)= ¼+¼ = 1/2

Formula for probability of union of 3 disjoint sets:

If A,B,C are disjoint, then

$P(A \cup B \cup C)=P(A)+P(B)+P(C)$

Because for disjoint events, the probability of intersection is **0**, so the Inclusion–Exclusion formula simplifies.

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If A and B are not disjoint sets

Additional law of probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Example : Rolling a Dice = SS {1, 2, 3, 4, 5, 6}

A= Dice show up even numbers = {2, 4, 6}

B = Dice show up value >3 = {4, 5, 6}

$P(A) = 3/6$ $P(B) = 3/6$ **$P(A \cap B) = 2/6$**

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 3/6 + 3/6 - 2/6 = 4/6 = 2/3$

Formula for probability of union of 3 sets

$P(A \cup B \cup C)=P(A)+P(B)+P(C)-P(A \cap B)-P(B \cap C)-P(C \cap A)+P(A \cap B \cap C)$

Example : Rolling a Dice = SS {1, 2, 3, 4, 5, 6}

First event is A = {1,2,5}

Second event is B= {2,5,6}

Third event is C= {3,5,6}

P(A) = 3/6 P(B) = 3/6 P(C) = 3/6

P(A∩B)= 2/6 P(B∩C)= 2/6 P(A∩C)= 1/6 P(A∩B∩C) = 1/6

$P(A \cup B \cup C)=P(A)+P(B)+P(C)-P(A \cap B)-P(B \cap C)-P(A \cap C)+P(A \cap B \cap C)$

Summary

From these axioms, we derive useful properties:

1. $0 \leq P(A) \leq 1$

2. $P(\emptyset)=0$ (probability of impossible event is 0)

3. For any event A,

$P(A^c)=1-P(A)$

(where A^c is the complement of A)

$P(A) + P(A^c) = 1$

Example : Rolling a Die

- Event A: Getting an **even number** = $\{2, 4, 6\}$

$$P(A) = \frac{3}{6} = \frac{1}{2}$$

- Complement A^c: Getting an **odd number** = $\{1, 3, 5\}$

$$P(A^c) = \frac{3}{6} = \frac{1}{2}$$

$$P(A) + P(A^c) = \frac{1}{2} + \frac{1}{2} = 1$$

Axioms of Probability Continue......

These axioms provide the mathematical foundation for calculating and understanding probabilities. These axioms allow for the development of various probability properties and theorems, making the axiomatic approach a cornerstone of modern probability theory.

It has wide applications in fields ranging from mathematics and statistics to physics, engineering, and artificial intelligence, helping to model and predict outcomes in uncertain situations.

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These axioms are crucial for several reasons:

- They ensure that probability is a well-defined and consistent mathematical concept.
- They serve as a foundation for deriving other important rules and theorems of probability.
- They are used in various fields, including statistics, artificial intelligence, and real-world analysis to model and understand uncertain phenomena.

Sample Problems on Axiomatic Approach to Probability

Example 1 :A class is choosing their class sports captain through a random draw. The class has 30% students who like cricket, 50% students who like football, and 20% students who like basketball. Calculate the probability that the chosen captain will be a cricket lover.

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Solution – Example 1

Problem Restatement

30% of students like **Cricket**

50% of students like **Football**

20% of students like **Basketball**

A captain is chosen at random. Find P(Cricket lover) = ?

Step 1: Define Sample Space (Axiom 2 – Normalization)

Let

C = Event that a student likes cricket

F = Event that a student likes football

B = Event that a student likes basketball

The sample space is:

$S=\{C,F,B\}$

By **Axiom 2 (Normalization):**

$P(S)=1$ So, $P(C)+P(F)+P(B)=1$ $0.30+0.50+0.20=1$

Condition satisfied.

Example 2 : Find out the probability of getting an even number when a die is tossed.

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Solution : Example 2

We know that possible outcomes when a die is tossed are,

{1, 2, 3, 4, 5 and 6}

We want to calculate the probability for getting an even number. Even number are {2,4,6}

Number of favorable outcomes = 3

Total Number of outcomes = 6.

So, the probability of getting an even number, P(Even)

= Number of favourable Outcomes/Total number of possible outcomes

P(Even) =3/6

⇒ P(Even) = 1/2

Independent and Dependent Events

Independent Events:

Two or more events are said to be independent when the outcome of one does not affect, and is not affected by the other.

For example : if a coin is tossed twice the result of the second throw would in no way be affected by the result of the first throw. the event of tossing two coins simultaneously the outcome of one coin does not affect the outcome of another coin then they are independent events.

Similarly the results obtained by throwing a dice are independent of the results obtained by drawing an ace from a pack of cards.

Examples of independent events:

- Getting a paycheck when you own a car.
- When you own a book and look for a café.
- Buying a coffee and then buying a pencil.

Dependent Events

Dependent events are those events that are affected by the outcomes of events that had already occurred previously. i.e. Two or more events that depend on one another are known as dependent events. If one event is by chance changed, then another is likely to differ. Thus, If whether one event occurs does affect the probability that the other event will occur, then the two events are said to be dependent.

Hence Dependent events are events that are affected by the occurrence of other events.

Example :

A bag has 3 red and 2 blue balls. Two balls are drawn **one after the other without replacement**. Find the probability that **both are red**.

Step 1: First draw

Total balls = 5

Probability first ball is red = $P(A)=3/5$

Step 2: Second draw (depends on first)

One red is removed → remaining balls = 2 red + 2 blue = 4

Probability second ball is red given first was red: $P(B|A)=2/4=1/2$

Step 3: Combined probability

$P(A\cap B)=P(A)\cdot P(B|A)=3/5\cdot 1/2=3/10$

✅ Notice: If the first ball **wasn't removed**, the probability of the second being red **would have stayed 3/5**, and the events would have been independent.

Examples of dependent events:

- ❑ Going to jail after you have robbed a bank.
- ❑ Buying a flight ticket and boarding your flight.
- ❑ Not paying telephone bills when you did not make any calls.

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There are two important theorems of Probability

1The Addition theorem

2The Multiplication theorem

Addition Theorem

The addition theorem states that if two events A and B are mutually exclusive the probability of the occurrence of either A or B is the sum of the individual probability of A and B.

Symbolically :

$P(A \text{ or } B) = P(A) + P(B)$

$P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

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Example 1:

One card is drawn from a standard pack of 52. What is the probability that it is either a king or a queen?

Solution 1 :

There are 4 Kings and 4 Queens in a pack of 52 cards

The probability that the card drawn is a king = 4/52

The probability that the card drawn is a queen = 4/52

Since the events are mutually exclusive, the probability that the card drawn is either a king or a queen

$4/52 + 4/52 = 8/52 = 2/13$

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Example 2

The Managing committee of Vaishali Welfare Association formed a sub-committee of 5 persons to look into electricity problem. Profiles of the 5 persons are

1. Male age 40
2. Male age 43
3. Female age 38
4. Female age 27
5. Male age 65

If a chairperson has to be selected from this, what is the probability that he would be either female or over 30 years.

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Solution 2 :

So, total sample space size = 5

We need the probability that the selected chairperson is **either female OR over 30 years old**.

This is a union of two events:

A person is **female**

B: person is **over 30 years old**

We want:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Females (A): Female (38), Female (27) → **2** persons

Over 30 (B): Male (40), Male (43), Female (38), Male (65) → **4** persons

Both Female AND Over 30 (A ∩ B): Female (38) → **1** person

P (Female or over 30) = P(female) + P(over 30) – P(Female and over 30)

= 2/5 + 4/5 – 1/5 = 5/5 = 1

Example 3 :

In a cloud database system:

Probability that a query is **slow** due to heavy load = 0.30

Probability that a query is **slow** due to indexing issue = 0.25

Probability that it is slow due to **both load and indexing issue** = 0.10

Find the probability that the query is slow due to **either heavy load or indexing issue**.

SOLUTION

$$P(\text{Load or Index})$$
$$= P(\text{Load})+P(\text{Index})-P(\text{Load} \cap \text{Index})$$
$$=0.30+0.25-0.10=0.45$$

So, 45% chance of query being slow.

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Example 4 :

Probability of a system facing a **malware attack** = 0.20

Probability of a **phishing attack** = 0.25

Probability of facing **both attacks together** = 0.05

Find the probability of facing **either malware or phishing attack**.

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Solution

$$P(\text{Malware or Phishing})$$
$$=0.20+0.25-0.05$$
$$=0.40$$

So, 40% chance of facing at least one type of attack.

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Example 5

A cloud service can go down due to three reasons:

Network failure (N): $P(N)=0.15$

Server overload (S): $P(S)=0.10$

Software bug (B): $P(B)=0.12$

The probabilities of joint failures are:

$P(N \cap S)=0.05$

$P(N \cap B)=0.04$

$P(S \cap B)=0.03$

$P(N \cap S \cap B)=0.02$

Find:

The probability that the service is down due to **at least one of the three causes**

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Solution 5

Use the **general addition theorem for 3 events**:

$$P(N \cup S \cup B) = P(N) + P(S) + P(B) - P(N \cap S) - P(N \cap B) - P(S \cap B) + P(N \cap S \cap B)$$

Answer

$$P(N \cup S \cup B) = 0.27$$

Multiplication Theorem

This theorem states that if two events A and B are independent, the probability that they both will occur is equal to the product of their individual probability.

If the events are independent, the formula is:

P(A and B) = P(A) * P(B)

If the events are dependent, the formula is:

P(A and B) = P(A) * P(B|A)

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In probability:

A∪B (read as *A or B*) means **union** → at least one of the events occurs.

A∩B (read as *A and B*) means **intersection** → both events occur together.

Example 1

A secure login requires passing **3 independent checks**:

Password correct: 0.90

OTP correct: 0.85

Biometric correct: 0.95

Find the probability that a user **passes all three checks**.

Solution

Events: A,B,C independent.

$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$

$= 0.9 \times 0.85 \times 0.95$

$= 0.72675$

Answer: $P(\text{all checks passed}) = 0.7268$

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Example 2

Two cards are selected without replacing the first card from the deck. Find the probability of selecting a king and then selecting a queen.

Solution

Total events = 52

Since the first card is not replaced, the events are dependent.

Probability of selecting a king = P(K) = 4/52

Probability of getting a queen = P(Q) = 4/51 (one card drawn first has not been replaced)

P(a king & a then queen) = $P(K).P(Q|K)$

$=4/52 * 4/51 = 16/2652 = 1/166.$

Therefore, the probability of selecting a king and then selecting a queen is 1/166.

Example 3

A man wants to marry a girl having qualities: white complexion –the probability of getting such a girl is one in twenty; handsome dowry – the probability of getting this is one in fifty; westernized manners and etiquettes-the probability here is one in hundred. Find out the probability of his getting married to such a girl when the possession of these three attributes is independent.

Solution

Probability of a girl with white complexion
1/20 = 0.05

Probability of a girl with handsome dowry
=1/50 = 0.02

Probability of a girl with westernized manners
=1/100

Since the events are independent, the probability of simultaneous occurrence of all these qualities

=1/20 *1/50*1/100= 0.05*0.02*0.01= 0.00001

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Example 4

The probability of a student passing in **Maths** is 0.7 and in **English** is 0.8.
If the events are independent, find the probability that the student passes in **both subjects**.(Ans. : 0.56)

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Example 5

Two independent machines A and B function correctly with probabilities 0.60and 0.50 respectively.

Find the probability that **both machines fail**.

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Solution

Failure probabilities:

$P(A^c) = 1 - 0.6 = 0.4$,

$P(B^c) = 1 - 0.5 = 0.5$

Independent $\Rightarrow P(\text{both fail}) = 0.4 \times 0.5 = 0.20$

Example 6

Three independent components X, Y and Z work with probabilities 3/4, 2/5, 3/4 respectively.
Find the probability that **X and Y both work**.

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Solution

Independent $\Rightarrow$ $P(X \cap Y) = 3/4 \times 2/5 = 6/20 = 3/10 = 0.3$

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Example 7

Three independent checks with success probabilities $0.90,\ 0.80,\ 0.50$. Find probability **none succeed**.

Solution:

P(none succeed)= all fail = product of complements:

$=(1-0.90)(1-0.80)(1-0.50)$

$= 0.10 \times 0.20 \times 0.50$

Solution:

$=0.0100$

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Practice Example 1

The probability that A can solve a problem in statistics is ½ . That B can solve is 1/3 and C can solve it is ¼ . If all of them try independently, then find the probability that the problem will be solved.

Solution

By Addition Theorem

For three events A, B, and C:

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

Let events A,B,C be "A solves", "B solves", "C solves" with

$P(A)=1/2$, $P(B)=1/3$, $P(C)=1/4$,

and they are independent, so pairwise and triple intersections are products.

$P(A \cap B) = P(A) * P(B) = ½ * 1/3 = 1/6$

$P(A \cap C) = P(A) * P(C) = ½ * 1/4 = 1/8$

$P(B \cap C) = P(B) * P(C) = 1/3 * ¼ = 1/12$

$(A \cap B \cap C) = P(A) * P(B) * P(C) = ½ * 1/3 * 1/4 = 1/24$

$½+1/3+1/4-1/6-1/8-1/12+1/24 = (12+8+6-4-3-2+1)/24 = 18/24=3/4=0.75$

$$\boxed{P(\text{problem solved}) = \frac{3}{4} = 75\%}$$

Practice Example 2

A salesman is known to sell a product in 3 out of 5 attempts while another salesman in 2 out of 5 attempts. Find the probability that

1. No sale will take place when they both try to sell the product
2. Either of them will be succeed in selling the product

Given:

- Salesman A sells a product in 3 out of 5 attempts → $P(A \text{ sells}) = \frac{3}{5}$

  So, probability that A does **not sell**:

$$P(A') = 1 - \frac{3}{5} = \frac{2}{5}$$

- Salesman B sells a product in 2 out of 5 attempts → $P(B \text{ sells}) = \frac{2}{5}$

  So, probability that B does **not sell**:

$$P(B') = 1 - \frac{2}{5} = \frac{3}{5}$$

We assume that their attempts are **independent**, meaning what one does does not affect the other.

1 Probability that no sale will take place when both try

"No sale" means **neither A nor B sells**, i.e., both fail.

$$P(\text{No Sale}) = P(A' \cap B') = P(A') \cdot P(B') \quad (\text{because independent})$$

$$P(\text{No Sale}) = \frac{2}{5} \cdot \frac{3}{5} = \frac{6}{25}$$

✅ So, the probability that no sale takes place is **6/25.**

Solution:

2. P(Either of them will succeed in selling the product =

$= P(A) + P(B) – P(A \cap B)$

$= 3/5 + 2/5 - 6/25$

$= 19/25 = 0.76$

Answer:

- Probability of no sale = **6/25**
- Probability that either succeeds = **19/25**

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Practice Example 3

A class consists of 100 students; 25 them are girls and 75 boys . 80 of them are rich and 20 are poor. 40 of them have brown eyes and 60 have black eyes. What is the probability of selecting a brown eyed rich girl.

Solution

Let,

A be the event that the selected student is a girl

B be the event that the selected student is rich

C be the event that the selected student is brown eyed.

Then, it is given that

$$P(A) = 25/100 = 1/4$$

$$P(B) = 80/100 = 4/5$$

$$P(C) = 40/100 = 4/10 = 2/5$$

Now, the probability of a student being brown-eyed rich girl is given by $P(ABC)$.

Assuming that the sex, richness and colour of eyes are independent of each other, we can us the Multiplication Theorem of Probability, and therefore,

$$P(ABC) = P(A)\,P(B)\,P(C) = (1/4) \times (4/5) \times (2/5)$$

$$= 2/25$$

Practice Example 4

A candidate is selected for interviews for 3 posts. For the first post, there are 3 candidates, for the second, 4 and for the third post there are 2 candidates. What is the probability that the candidate is selected for at least one post.

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Solution

Let

A represent the event that a candidate is selected for the first post $P(A) = 1/3$

B represent the event that a candidate is selected for the second post$P(B)= 1/4$

C represent the event that a candidate is selected for the third post $P(C) = ½$

Therefore the probability that a candidate is selected for at least one post is given by the theorem of total probability as

$$P(A \cup B \cup C)=P(A)+P(B)+P(C)-P(A \cap B)-P(A \cap C)-P(B \cap C)+P(A \cap B \cap C)$$

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Solution Continue....

- $P(A \cap B) = \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$
- $P(A \cap C) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$
- $P(B \cap C) = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$
- $P(A \cap B \cap C) = \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{24}$

Plug into the formula:

$$P(A + B + C) = \frac{1}{3} + \frac{1}{4} + \frac{1}{2} - \frac{1}{12} - \frac{1}{6} - \frac{1}{8} + \frac{1}{24}$$

$$P(A + B + C) = \boxed{\frac{3}{4}}$$

Practice Example 5

A system works if at least one of its 3 independent backup batteries functions. Each battery has a 90% chance of working. What is the probability the system works?

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Solution

There are **3 independent backup batteries**.

Each battery has a **90% chance of working**.

The system works if **at least one battery functions**.

We are to find the **probability that the system works**.

Probability that a single battery fails

Each battery has a 90% chance of working ⇒ 10% chance of failing.

So,

$P(\text{Battery fails}) = 1 - 0.9 = 0.1$

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Solution Continue......

Probability that all three batteries fail

Since the batteries fail **independently**, we multiply their failure probabilities:

$P(\text{All three fail}) = 0.1 \times 0.1 \times 0.1 = 0.001$

Probability that system works

$P(\text{System works}) = 1 - P(\text{All fail}) = 1 - 0.001 = 0.999 = 99.9\ \%$

Result :

So, there's a **99.9% chance** the system works with 3 independent 90%-reliable backup batteries.

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Practice example 6

A student goes to the college store. For writing he can either buy a ball pen or a pencil or an ink pen with equal probability. Suppose he chooses only one item, what is the probability that the student buys either a pencil or an ink pen?

Solution

Total possible outcomes = 3 (Ball pen, Pencil, Ink pen).

Favorable outcomes = 2 (Pencil, Ink pen).

Probability formula:

P(E)=Number of favorable outcomes/Total outcomes

P(E) = 2/3

Practice example 7

In a MCA class E:

Probability that projector stops working = 0.3 (Event A)

Probability that Wi-Fi goes down = 0.4 (Event B)

Probability that teacher gives surprise Test = 0.2 (Event C)

👉 Find the probability that **at least one disaster** (A, B, or C) happens in the lecture.

Answer : Probability = 0.664

Practice example 6

Following table shows the probability distribution of salary of a fresh MCA student at a college campus placement.

Salary in Rs. Lakhs per annum	Less than 4	4-5	5-6	6-7	Above 7
Probability	0.2	0.3	0.25	0.18	0.07

What is the Probability that the MCA student will be placed in the salary Bracket of

1. 4 to 6 Lakhs
2. 4 to 7 Lakhs

Solution

1. P(A+B) = P(A) + P(B)=0.30+0.25=0.55

1. P(A+B+C+) = P(A) + P(B) + P(C) = 0.30+0.25+0.18= 0.73

Practice example 7

Indian Oil limited assesses that the probability of successful strike in the Bombay high sea is 0.6 while in Assam the probability is only 0.4. If it drills two bore holes, one in the Bombay high and other in Assam, simultaneously, what is the probability of striking oil in both the cases?

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Solution

Let A= Success in Bomaby High Sea

B= Success in Assam

$P(A \cap B) = P(A) * P(B)$

$= 0.6*0.4$

$=0.24$

Practice example 8

The probability that an MCA student gets a job in Company A is 0.70 and in Company B is 0.50.

1. What is the probability that the student gets selected in both companies?
2. What is the probability that the student gets selected in at least one company?

Solution

Both ⇒ Multiplication Theorem:

$P(A \cap B) = 0.7 \times 0.5 = 0.35$

At least one ⇒ Addition Theorem:

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.7 + 0.5 - 0.35 = 0.85$

Practice example 9

In a company, there are two independent projects, Project A and Project B. The probability that Project A is completed on time is $P(A) = \frac{3}{4}$, and the probability that Project B is completed on time is $P(B) = \frac{2}{3}$.

1. What is the probability that **at least one project is completed on time**?
2. What is the probability that **exactly one project is completed on time**?
3. If a bonus is given **only if both projects are completed on time**, what is the probability that the bonus **will not be given**?

Solution

Step 1: Probability that **at least one project is completed on time**

Using the **Addition Theorem:**

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Since A and B are independent:

$$P(A \text{ and } B) = P(A) \cdot P(B) = \frac{3}{4} \cdot \frac{2}{3} = \frac{1}{2}$$

So,

$$P(A \text{ or } B) = \frac{3}{4} + \frac{2}{3} - \frac{1}{2}$$

=11/12

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Practice example 9

There are two senior executives in the finance department of an IT company. Both of them are eligible for promotion . The probability that A may be promoted is ½, the Probability that A or B may be promoted is ¾ and the probability that B may not be promoted is 5/8. Calculate the probability that

1. A and B may be promoted
2. Neither A nor B may be Promoted
3. B but not A is promoted

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Solution

If you see **AND** → Check whether the events are **independent**.

If yes → Multiply.

If no → Use **addition theorem** to find the overlap.

If you see **OR** → Always apply the **addition theorem**.

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SOLUTION: Given $P(A) = 1/2$; $P(A \cup B) = 3/4$; $P(B^c) = 5/8$

(i) $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 1/2 + 3/8 - 3/4 = 1/8$

(ii) $P(A^c \cap B^c) = 1 - P(A \cup B) = 1 - 3/4 = 1/4$

(iii) $P(A^c \cap B) = P(B) - P(A \cap B) = 3/8 - 1/8 = 1/4$

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Conditional probability and its applications.

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What is Conditional Probability?

Conditional probability is the probability of an event occurring **given that another event has already occurred.**

Formula:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad \text{where } P(B) > 0$$

Here:

- $P(A|B)$ = Probability of A happening given that B has already happened.
- $P(A \cap B)$ = Probability of both A and B happening.
- $P(B)$ = Probability of B happening.

The **conditional probability** of **A given B** is:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \quad \text{provided } P(B) > 0$$

This means: *the probability that event A occurs, under the condition that event B has already occurred.*

Similarly, the **conditional probability** of **B given A** is:

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}, \quad \text{provided } P(A) > 0$$

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Complement of an Event

The complement of event A is denoted by A' (or sometimes A^c).

- A' means **A does not happen.**
- Also, $P(A') = 1 - P(A)$.

Conditional Probability of Complement

If we want $P(A' \mid B)$, then:

$$P(A' \mid B) = \frac{P(A' \cap B)}{P(B)}$$

But since $B = (A \cap B) \cup (A' \cap B)$, we can rewrite:

$$P(A' \mid B) = \frac{P(B) - P(A \cap B)}{P(B)}$$

$$P(A' \mid B) = 1 - \frac{P(A \cap B)}{P(B)}$$

$$P(A' \mid B) = 1 - P(A \mid B)$$

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Applications of Conditional Probability

- **Everyday Life**

Weather: Probability of rain given that it is cloudy.
Traffic: Probability of reaching office late given heavy traffic.

- **Sports**

Example: The probability of a batsman scoring a century given that he has survived the first 10 overs.
Coaches use these stats for strategy.

- **Finance & Insurance**

In stock markets, probability of stock price rise given good earnings report.
In insurance, probability of claim given car accident.

- **Computer Networks**

Packet delivery: If a packet is sent, the probability that it reaches successfully given no congestion is a conditional probability problem.
Example:
P(Successful Transmission | No Congestion)

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Applications of Conditional Probability

❖ **Natural Language Processing (NLP)**

Predicting words in speech recognition or text prediction:

P(Next Word | Previous Words)

❖ **Cyber security**

Intrusion detection systems (IDS):

Probability of attack **given** unusual network traffic.

Example:

P(Malware|Suspicious download detected)

❖ **Information Retrieval (Search Engines)**

Ranking web pages:

P(Document Relevant | Query)

Google and other search engines estimate the relevance of documents using probabilistic models.

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Applications of Conditional Probability

❖ **Recommendation Systems**

Netflix, Amazon, YouTube recommend items using:

P(User likes Movie | User's past choices)

❖ **Robotics**

Decision-making under uncertainty:

A robot navigating a maze calculates

P(Correct path | Sensor input)

Helps robots act correctly despite noisy sensor data.

❖ **Fault Tolerant Systems**

Probability of system failure given a specific component failure.

Example: In cloud computing,

P(System Down | Server Crash)

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Applications of Conditional Probability

❖ **Algorithms & Data Science**

Markov Chains (used in Google PageRank, speech recognition, etc.) depend on conditional probability:

P(Next State | Current State)

Conditional probability is the backbone of intelligent decision-making in computer science—from algorithms and AI to networking and cybersecurity.

Example 1

A bag contains 5 white and 3 black balls. Two balls are drawn at random one after the other without replacement. Find the probability that both balls drawn are black.

Solution. Probability of drawing a black ball in the first attempt is

$$P(A) = \frac{3}{5+3} = \frac{3}{8}$$

Probability of drawing the second black ball given that the first ball drawn is black

$$P(B/A) = \frac{2}{5+2} = \frac{2}{7}$$

$\therefore$ The probability that both balls drawn are balck is given by

$$P(AB) = P(A) \times P(B/A) = \frac{3}{8} \times \frac{2}{7} = \frac{3}{28}.$$

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Example 2

Find the probability of drawing a queen, a king and knave in that order from a pack of cards in three consecutive draws, the cards drawn not being replaced.

Solution. The probability of drawing a queen $= \frac{4}{52}$.

The probability of drawing a king after a queen has been drawn $= \frac{4}{51}$

The probability of drawing a knave given that a queen and king have been drawn $= \frac{4}{50}$

Since they are dependent events, the required probability of the compound event is :

$$\frac{4}{52} \times \frac{4}{51} \times \frac{4}{50} = \frac{64}{1,32,600} = 0.00048.$$

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Apply conditional probability multiplication rule

$$P(Q1 \cap K2 \cap J3) = P(Q1) \times P(K2|Q1) \times P(J3|Q1 \cap K2)$$

$$= 4/52 * 4/51 * 4/50 = 0.00048$$

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Example 3

A network log shows that 12% of packets were **both** on a congested link and delivered successfully. Overall, 30% of packets traveled on congested links. What is the probability a packet is delivered successfully **given** it was on a congested link?

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Solution

Given:

P(success ∩ congested)=0.12,

P(congested)=0.30.

$$P(\text{success} \mid \text{congested}) = \frac{P(\text{success} \cap \text{congested})}{P(\text{congested})} = \frac{0.12}{0.30} = 0.40.$$

Answer: 40%.

Example 4

Among the workers in a factory only 30% receive a bonus. Among those receiving the bonus only 20% are skilled. What is the probability of a randomly selected worker who is skilled and receiving bonus.

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Solution

A = {The event of receiving bonus}

B = {The event of considering skilled workers}

Given: $P(A) = 30 / 100 = 0.3$

$P(B/A) = 20 / 100 = 0.2$

$P(B/A) = P(B \cap A) / P(A)$

To find the probability of the event $A \cap B$

(i.e.,) $P(A \cap B) = P(A)\, P(B/A)$

$= (0.3)\,(0.2) = 0.06$

$P(B/A) \rightarrow$ The conditional probability of event B when the event

A has already happened.

Example 5

Out of 100 MCA students:

60 know Python,

50 know Java,

30 know both.

If a student is chosen at random and is known to know **Python**, find the probability that the student also knows **Java**.

Solution

Solution:

$$P(Java \mid Python) = \frac{P(Java \cap Python)}{P(Python)} = \frac{30/100}{60/100} = \frac{30}{60} = \frac{1}{2}$$

Example 6:

In a class, 30% of the students failed in Physics, 25% failed in Mathematics and 15% failed in both Physics and Mathematics. If a student is selected at random failed in Mathematics, find the probability that he failed in Physics also.

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Solution:

Let A be the event "failed in Physics" and B be the event "failed in Mathematics". We want to find $P\left(\frac{A}{B}\right)$.

It is given that P(A) = $\frac{300}{100}$ and P(B) = $\frac{25}{100}$

Also P(A∩B)= $\frac{15}{100}$

Therefore $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} \frac{\left(\frac{15}{100}\right)}{\left(\frac{25}{100}\right)} = \frac{15}{25} = \frac{3}{5}$

Example 7

Example : The table below shows the occurrence of diabetes in 100 people. Let D and N be the events where a randomly selected person "has diabetes" and "not overweight". Then find P(D | N).

	Diabetes (D)	No Diabetes (D')
Not overweight (N)	5	45
Overweight (N')	17	33

Solution:

From the given table, $P(N) = (5+45) / 100 = 50/100$.

$P(D \cap N) = 5/100$.

By the conditional probability formula,

$P(D \mid N) = P(D \cap N) / P(N)$

$= (5/100) / (50/100)$

$= 5/50$

$= 1/10$

Answer: $P(D \mid N) = 1/10$.

Example 8

A manufacturer of aeroplane parts knowns that the probability is 0.8 that an order will be ready for shipment on time, and it is 0.7 that an order will be ready for shipment and will be delivered on time. What is the probability that such an order will be delivered on time given that it was also ready for shipment on time?

Solution

Solution: Let 'A' be an event that an order is ready for shipment on time.

Let 'D' be an event that an order is delivered on time.

Given that $P(A) = 0.8$ and $P(A \cap D) = 0.7$

To find $P(D/A)$

$P(D/A) = P(A \cap D) / P(A) = 0.7 / 0.8 = 7/8 = 0.875$.

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Example 9

Two manufacturing plants produce similar parts. Plant I produces 1,000 parts, 100 of which are defective. Plant II produces 2,000 parts, 150 of which are defective. A part is selected at random and found to be defective. What is the probability that it come from plant I.

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Solution :

A - → the part selected come from plant I (A)

B -→ the part selected is defective.

A ∩ B - → the part selected is defective and came from plant I

A/B - → The conditional probability of event A when the event B has already happened.

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{100}{3000} = \frac{1}{30}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{250}{3000} = \frac{1}{12}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\left(\frac{1}{30}\right)}{\left(\frac{1}{12}\right)} = \left(\frac{1}{30}\right)\left(\frac{12}{1}\right) = \frac{2}{5} = 0.4$$

Example 10

Delhi traffic police found that 55 percent of all road accidents in Delhi are caused by trucks, 70 percent occur at night and 48 percent are caused by trucks at night

1. Given that an accident has occurred at night what is the probability that it was caused by a truck
2. Given that a truck has caused the accident what is the probability it has occurred at night?

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Solution

SOLUTION: $P(T) = 0.55$ accident caused by truck; $P(N) = 0.70$ accident has occurred at night; $P(T \cap N) = 0.48$

$$P(T|N) = \frac{P(T \cap N)}{P(N)} = \frac{0.48}{0.70} = \frac{24}{35}$$

$$P(N|T) = \frac{P(T \cap N)}{P(T)} = \frac{0.48}{0.55} = \frac{48}{55}$$

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Example 11

EXAMPLE 36. There are 40 girls and 20 boys in a class. 8 students of this class of which 3 were girls formed a Violin Club. (*i*) Given that a student of this class does not play violin, what is the probability that the student is a boy? (*ii*) Given that the student is girl, what is the probability she plays violin?

Solution

SOLUTION: The above information can be expressed in a tabular form.

	Girls = G	*Boys = B*	*Total*
Member of Violin club = M	3 $(G \cap M)$	5 $(B \cap M)$	8
Not member = N	37 $(G \cap N)$	15 $(B \cap N)$	52
Total	40	20	60

$$(i) P(B \mid N) = \frac{P(B \cap N)}{P(N)} = \frac{15/60}{52/60} = 15/52$$

$$(ii) P(M \mid G) = \frac{P(G \cap M)}{P(G)} = \frac{3/60}{40/60} = 3/40$$

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Example 12

EXAMPLE 37. The HR manager of Quick Software Solutions, Bengaluru, collected the following data relating to a sample of two hundred employees.

Division of work	Geographic origin of employees				
	South	North	West	East	Total
Software Development	40	36	28	14	118
Finance	18	12	10	12	52
HR	10	8	4	8	30
Total	68	56	42	34	200

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Example continue.......

If an employee is selected at random from the above employees

(a) What is the probability that the employee works in the Software Development Department?

(b) What is the probability that the employee belongs to Eastern states?

(c) What is the probability that the employee belongs to Western India or works for the Finance Department?

(d) What is the probability that the employee is a North Indian working in the HR Department?

(e) Given an employee is working in the Software Development Department what is the probability that the employee is a north Indian?

(f) What is the probability that the employee is neither a south Indian nor working in the Finance Department?

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SOLUTION:

Division of work	Geographic origin of employees				
	South (A)	North (B)	West (C)	East (D)	Total
Software Development (E)	0.20	0.18	0.14	0.07	$P(E) = 0.59$
Finance (F)	0.09	0.06	0.05	0.06	$P(F) = 0.26$
HR (G)	0.05	0.04	0.02	0.04	$P(G) = 0.15$
Total	$P(A) = 0.34$	$P(B) = 0.28$	$P(C) = 0.21$	$P(D) = 0.17$	1.00

(a) $P(E) = 0.59$

(b) $P(D) = 0.17$

(c) $P(C \cup F) = P(C) + P(F) - P(C \cap F) = 0.21 + 0.26 - 0.05 = 0.42$

(d) $P(B \cap G) = 0.04$

(e) $P(B \mid E) = \dfrac{P(B \cap E)}{P(E)} = \dfrac{0.18}{0.59} = \dfrac{18}{59}$

(f) $P(\bar{A} \cap F) = 1 - P(A \cup F) = 1 - [P(A) + P(F) - P(A \cap F)] = 1 - [0.34 + 0.26 - 0.09] = 0.49$

Example 13

A web API logged **1200** requests.

300 requests came from **mobile** devices.

Of those 300 mobile requests, **225** completed **successfully**.

Overall, **900** of the 1200 requests were successful.

Question: **What is the probability that a request came from a mobile device, given that it was successful?**

We use the definition:

$$P(\text{mobile} \mid \text{success}) = \frac{P(\text{mobile} \cap \text{success})}{P(\text{success})}.$$

Compute the probabilities from counts (total = 1200):

1. $P(\text{mobile} \cap \text{success}) = \dfrac{\text{mobile \& success count}}{\text{total}} = \dfrac{225}{1200}.$

   Simplify: divide numerator & denominator by 75 → $225/1200 = 3/16 = 0.1875.$

2. $P(\text{success}) = \dfrac{\text{success count}}{\text{total}} = \dfrac{900}{1200} = \dfrac{3}{4} = 0.75.$

Now divide:

$$P(\text{mobile} \mid \text{success}) = \frac{0.1875}{0.75}.$$

Answer

$$\boxed{P(\text{mobile} \mid \text{success}) = 0.25 = 25\%}$$

—

Interpretation / takeaway: Among successful requests, **25%** came from mobile devices.

Example 14

At a business school, the probability that a student takes all the courses in one semester and does a job is 0.045. The probability that the student takes all the courses in the semester is 0.75. What is the probability that the student does a job, given that he/she has taken all the courses in that semester?

Solution

The probability of taking all the courses in a semester and doing a job = $P(A \cap B) = 0.045$

The probability that a student takes all the courses in the semester = $P(A) = 0.75$

The probability of student doing a job, given that he/she is taking all the course = $P(B|A) = ?$

We will use the following formula to calculate the probability:

$P(B|A) = \frac{0.045}{0.75}$

$= 0.06$

Hence, the probability that a student is doing a job, given that he/she has taken all the courses is 0.06 or 6%.

Example 15

In a system, Process A is running 40% of the observed time $P(A)=0.4$. Process B holds a resource 25% of the time $P(B)=0.25$. Observations show A and B hold/run concurrently 10% of the time. Find the probability that A is running given B holds the resource

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Solution:

$P(A|B)=P(A\cap B)/P(B)=0.10\ /.25$

$0.10 \div 0.25 = (10/100) \div (25/100) = 10/25 =$ divide by $5 \rightarrow 2/5 = 0.4$

✅ **Answer:** P(A|B)=0.4P(A\mid B)=0.4P(A|B)=0.4 (40%)

Decision: When the resource is held, A runs only 40% of time → resource scheduling can reduce contention by focusing on other processes.

High-latency event (L) occurs in 12% of I/O operations. Heavy I/O (H) occurs in 20% of operations. Both happen together in 6%: $P(L \cap H) = 0.06$. Find the probability that an I/O is high-latency **given** it is heavy: $P(L \mid H)$.

Solution:

$$P(L \mid H) = \frac{P(L \cap H)}{P(H)} = \frac{0.06}{0.20}.$$

Arithmetic: 0.06 ÷ 0.20 = (6/100) ÷ (20/100) = 6/20 = divide by 2 → 3/10 = 0.3

✅ **Answer:** $P(L \mid H) = 0.3$ (30%)

Decision: 30% of heavy I/O results in high latency → optimize heavy I/O path or schedule heavy jobs off-peak.

Example 16

The Probability that a MCA intern will remain with a company is 0.60. The Probability that an employee earns more than Rs. 50,000/- per year is 0.50. the probability that an employee is a MCA intern who remained with the company or who earns more than Rs. 50000/- per year is 0.70. What is the probability that an employee earns more than Rs. 50,000 per year, given that he is a MCA intern who stayed with the company?

Given:

Let

- A: Employee is an MCA intern who remained with the company
- B: Employee earns more than ₹50,000 per year

We are told:

$$P(A) = 0.60$$

$$P(B) = 0.50$$

$$P(A \cup B) = 0.70$$

We need to find:

$$P(B|A)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Substitute values:

$$0.70 = 0.60 + 0.50 - P(A \cap B)$$

$$P(A \cap B) = 1.10 - 0.70 = 0.40$$

Step 2: Use the conditional probability formula

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Substitute values:

$$P(B|A) = \frac{0.40}{0.60} = 0.6667$$

Interpretation:

There is a **66.7% probability** that an MCA intern who stayed with the company earns more than ₹50,000 per year.

In a locality out of 5000 people residing, 1200 are above 30 years of age and 3,000 are females. Out of the 1200 who are above 30, 200 are females. Suppose after a person is chosen you are told that the person is female. What is the probability that she is above 30 years of age?

Given:

- Total people = 5000

Step 1: Write down known probabilities

$$P(\text{Above } 30) = \frac{1200}{5000}$$

$$P(\text{Female}) = \frac{3000}{5000}$$

$$P(\text{Above 30 and Female}) = \frac{200}{5000}$$

Step 1: Write down known probabilities

$$P(\text{Above } 30) = \frac{1200}{5000}$$

$$P(\text{Female}) = \frac{3000}{5000}$$

$$P(\text{Above 30 and Female}) = \frac{200}{5000}$$

Step 2: Use the conditional probability formula

$$P(\text{Above 30} \mid \text{Female}) = \frac{P(\text{Above 30 and Female})}{P(\text{Female})}$$

Substitute values:

$$P(\text{Above 30} \mid \text{Female}) = \frac{\frac{200}{5000}}{\frac{3000}{5000}} = \frac{200}{3000}$$

Step 3: Simplify

$$P(\text{Above 30} \mid \text{Female}) = \frac{2}{30} = \frac{1}{15} \approx 0.0667$$

Refer for additional examples for practice

https://www.analyzemath.com/probabilities/conditional-probabilities.html

Bayes Theorem and its applications.

Introduction

Bayes’s Theorem is a fundamental concept in probability theory used to determine the probability of an event based on prior knowledge of related events.
It provides a way to **update probabilities** when new evidence or information is available.

Statement of Bayes's Theorem

If $A_1, A_2, A_3, \ldots, A_n$ are **mutually exclusive and exhaustive events**, and B is any event related to them, then

$$P(A_i|B) = \frac{P(A_i)\,P(B|A_i)}{\sum_{j=1}^{n} P(A_j)\,P(B|A_j)}$$

Where:

- $P(A_i)$: Prior probability of event A_i
- $P(B|A_i)$: Conditional probability of B given A_i
- $P(A_i|B)$: Posterior probability of A_i given that B has occurred

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Bayes Theorem Formula

For any two events A and B, **Bayes's** formula for the Bayes theorem is given by:

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

Formula for the Bayes theorem

Where,

- **P(A)** and **P(B)** are the probabilities of events A and B; also, P(B) is never equal to zero.
- **P(A|B)** is the probability of event A when event B happens,
- **P(B|A)** is the probability of event B when A happens.

Applications of Bayes’s Theorem in Computer Science

Bayes’s Theorem plays a crucial role in **decision-making under uncertainty**, which is a core challenge in computer science. It helps computers and intelligent systems update their predictions or beliefs when new data becomes available.

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1. Machine Learning and Artificial Intelligence

(a) Naïve Bayes Classifier

One of the most popular algorithms in **supervised learning**.

Based on Bayes's Theorem with the assumption that features are independent.

Used for:

- **Email spam detection** – classifies emails as *spam* or *not spam*.
- **Sentiment analysis** – identifies positive or negative reviews.
- **Document classification** – sorts news articles, blogs, or web content.

Example:

If a message contains the words "win", "lottery", and "free", Bayes's theorem helps calculate the probability that the message is spam based on how often those words appear in known spam emails.

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2. Medical Diagnosis Systems

Used in **AI-based healthcare** applications to estimate the probability of a disease given certain symptoms or test results.

Helps reduce false positives and false negatives by updating probabilities as new patient data becomes available.

Example:
If a test for COVID-19 is 95% accurate and 2% of people are infected, Bayes's theorem determines the actual probability that a person with a positive result truly has the disease.

3. Data Mining and Predictive Analytics

Helps discover patterns and make predictions based on past data.

Used to estimate the likelihood of future events, such as:

- Customer purchasing behavior
- Loan default prediction
- Fraud detection

Example:

In banking, Bayes's theorem can calculate the probability that a transaction is fraudulent based on transaction amount, time, and location.

4. Network Security and Intrusion Detection

Used to determine the probability of a **cyberattack** given certain network behaviors (like unusual traffic patterns).

Helps security systems update threat levels dynamically.

Example:
If a system observes multiple failed login attempts, Bayes's theorem helps compute the probability that these attempts are part of a brute-force attack.

5. Information Retrieval and Search Engines

Used in **ranking and relevance estimation**.

When a user searches for "AI in education," Bayes's theorem helps estimate which documents are most likely relevant to the query.

Example:

Search engines apply Bayesian models to calculate the probability that a document is relevant given the words in the user's query.

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6. Robotics and Autonomous Systems

Robots and self-driving cars use Bayes's Theorem to **update their understanding of the environment** as they receive new sensor data.

It helps in **localization**, **mapping**, and **path planning**.

Example:
If a robot detects an obstacle using sensors, Bayes's theorem helps determine the probability that it is actually a wall, human, or object, considering previous observations.

7. Natural Language Processing (NLP)

Bayes's theorem supports models that interpret human language.

Used for **speech recognition**, **language translation**, and **text prediction**.

Example:

In a predictive keyboard, Bayes's theorem estimates the probability of the next word based on previous words typed.

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Conclusion

Bayes's Theorem is the mathematical foundation for **probabilistic reasoning** in computer science. It allows systems to make intelligent decisions using evidence and prior knowledge — forming the basis of many modern technologies in AI, ML, and data analytics.

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Example 1

A diagnostic test for a computer virus gives the following data:

Probability that a computer has the virus =0.05

Probability that the test is positive given that virus is present =0.92

Probability that the test is positive overall (from lab data) = 0.10

Find the probability that a computer actually has the virus given that the test is positive.

Solution

$$P(V|T) = \frac{P(T|V)\,P(V)}{P(T)}$$

Here:

- $P(T|V) = 0.92$
- $P(V) = 0.05$
- $P(T) = 0.10$

$$P(V|T) = \frac{0.92 \times 0.05}{0.10}$$

$$P(V|T) = 0.46 = 46\%$$

Example 2

In a hardware company, 4% of the chips are defective. The testing machine identifies defective chips correctly 90% of the time. It is known that 10% of all chips test positive.

Find the probability that a chip is actually defective when it tests positive.

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Solution

$$P(\text{Defective} \mid \text{Test} +) = \frac{P(\text{Test} + \mid \text{Defective}) \times P(\text{Defective})}{P(\text{Test} +)}$$

$$= \frac{0.90 \times 0.04}{0.10} = 0.36$$

Hence, there is a **36% chance** that a chip is truly defective when the test shows it as defective.

Example 3

In a programming course, 70% of the students pass the exam. Among those who pass, 80% study regularly. From overall data, 65% of students study regularly.

Find the probability that a student passed, given that they study regularly.

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Solution

$$P(\text{Pass} \mid \text{Study}) = \frac{P(\text{Study} \mid \text{Pass}) \times P(\text{Pass})}{P(\text{Study})}$$

$$= \frac{0.80 \times 0.70}{0.65} = 0.8615$$

Therefore, the probability that a student passed given they study regularly is **86%.**

Example 4

A factory has two machines:

Machine	Probability of choosing	Defective rate
A	0.6	0.03
B	0.4	0.05

Total probability a product is defective = **0.038** (given).

Find: $P(\mathrm{A}|\mathrm{Defective})$ — Probability product came from Machine A if defective.

Solution

$$P(A|D) = \frac{P(D|A)P(A)}{P(D)} = \frac{0.03 \times 0.6}{0.038}$$

$$= \frac{0.018}{0.038} \approx 0.4737$$

Example 5

Two jars:

Jar X: 4 Red, 1 Blue (so $P(R|X)=0.8$

Jar Y: 1 Red, 4 Blue (so $P(R|Y)=0.2$

Both jars are chosen with equal chance.
A red ball is picked.
Find probability ball came from **Jar X**.

$P(R)=P(R|X)P(X)+P(R|Y)P(Y)$
$P(R)=(0.8)(0.5)+(0.2)(0.5)=0.4+0.1=0.5$

Solution :

Given

- $P(X) = 0.5$
- $P(Y) = 0.5$
- $P(R|X) = 0.8$
- $P(R|Y) = 0.2$
- $P(R) = 0.5$ *(given)*

Solution

$$P(X|R) = \frac{P(X)P(R|X)}{P(R)} = \frac{0.5 \times 0.8}{0.5} = \frac{0.4}{0.5} = 0.8$$

✅ **Answer:** $0.8 = 80\%$

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Law of Total Probability and Bayes Theorem

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Law of total Probability

Concept

The **Law of Total Probability** helps us find the probability of an event by **considering all possible ways** that event can happen.

If $B_1, B_2, \ldots, B_n$ form a **partition of the sample space** (mutually exclusive & exhaustive events), then for any event A:

$$P(A) = P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + \ldots + P(B_n)P(A|B_n)$$

Where:

- $P(B_i)$ = Probability of the i^{th} case
- $P(A|B_i)$ = Probability of event A happening given B_i

Example

A company manufactures computer circuit boards at three different plants: Plant X, Plant Y, and Plant Z. The daily production volumes and defective rates are as follows:

Plant X: Produces 2,000 boards per day, with a 3% defective rate.

Plant Y: Produces 3,000 boards per day, with a 4% defective rate.

Plant Z: Produces 5,000 boards per day, with a 2% defective rate.

Question: If a randomly selected circuit board is found to be defective, what is the probability that it was produced by Plant Y?

Solution

Let X, Y, and Z represent the events that the board was produced by the respective plants, and D represent the event that the board is defective. The total production is 10,000 boards per day.

Given probabilities:

- $P(X) = 2000/10000 = 0.20$
- $P(Y) = 3000/10000 = 0.30$
- $P(Z) = 5000/10000 = 0.50$
- $P(D|X) = 0.03$
- $P(D|Y) = 0.04$
- $P(D|Z) = 0.02$

We want to find $P(Y|D)$, the probability the board is from Plant Y given it's defective.

Using Bayes' theorem:

$$P(Y|D) = \frac{P(D|Y)P(Y)}{P(D)}$$

First, find the total probability of a board being defective, $P(D)$:

$$P(D) = P(D|X)P(X) + P(D|Y)P(Y) + P(D|Z)P(Z)$$

$$P(D) = (0.03)(0.20) + (0.04)(0.30) + (0.02)(0.50) = 0.006 + 0.012 + 0.010 = 0.028$$

Now, calculate $P(Y|D)$:

$$P(Y|D) = \frac{(0.04)(0.30)}{0.028} = \frac{0.012}{0.028} \approx 0.4286$$

Answer: The probability that a defective board came from Plant Y is approximately 42.86%.

Example

A factory produces three types of products: P_1, P_2, and P_3. The production of P_1, P_2, and P_3 is 30%, 20%, and 50% of the total production, respectively. The probability that a product is defective is 1% for P_1, 2% for P_2, and 3% for P_3. If a product is selected at random, what is the probability that it is defective?

Answer :

$P(D)=0.003+0.004+0.015 = P(D)=0.022$

Example

A company recruits from 2 institutes:

Institute	% Selected	Probability of Good Performance
X	70%	90%
Y	30%	60%

Find probability a randomly selected employee performs well.

Solution

Given

$P(\text{from } X)=0.70$, $P(\text{good}|X)=0.90$

$P(\text{from } Y)=0.30$, $P(\text{good}|Y)=0.60$

Write the total probability formula

$P(G)=P(\text{from } X)\cdot P(G|\text{from } X) + P(\text{from } Y)\cdot P(G|\text{from } Y)$

$P(G)=0.70\times0.90+0.30\times0.60$

For institute X: $0.70\times0.90=0.63$

63% of all hires are from X **and** perform well.

For institute Y: $0.30\times0.60=0.18$

Interpretation: 18% of all hires are from Y **and** perform well.

$P(G)=0.63+0.18=0.81$

So there is an **81% chance** that a randomly selected employee (from the company's hires) will perform well.

In words: most hires come from X (70%) and X has a high success rate (90%), so overall performance is high.

In 2013 there will be three candidates for the post of professor X, Y & Z whose chances of getting the promotion are in the proportions 4 : 2 : 3 respectively. The probability that X if selected would introduce new subjects of today's requirement in the department is 0.35. The probability of Y & Z doing the same are respectively 0.52 & 0.80. What is the probability that there will be new subjects in the department in the year 2014.

Given:

- Candidates: **X, Y, Z**
- Chances of promotion are in the ratio **4 : 2 : 3**.

So,

$$P(X) = \frac{4}{4+2+3} = \frac{4}{9}, \quad P(Y) = \frac{2}{9}, \quad P(Z) = \frac{3}{9} = \frac{1}{3}.$$

Given probabilities of introducing new subjects:

$$P(\text{New subject}|X) = 0.35, \quad P(\text{New subject}|Y) = 0.52, \quad P(\text{New subject}|Z) = 0.80.$$

Required:

The total probability that **there will be new subjects introduced** in 2014:

$$P(\text{New subject}) = P(X)P(\text{New subject}|X) + P(Y)P(\text{New subject}|Y) + P(Z)P(\text{New subject}|Z)$$

Substitute values:

$$P(\text{New subject}) = \frac{4}{9}(0.35) + \frac{2}{9}(0.52) + \frac{3}{9}(0.80)$$

Now calculate step by step:

$$\frac{4}{9} \times 0.35 = \frac{1.4}{9} = 0.1556$$

$$\frac{2}{9} \times 0.52 = \frac{1.04}{9} = 0.1156$$

$$\frac{3}{9} \times 0.80 = \frac{2.4}{9} = 0.2667$$

Now add them:

$$0.1556 + 0.1156 + 0.2667 = 0.5379$$

✅ **Final Answer:**

$$\boxed{P(\text{New subjects in 2014}) = 0.538 \text{ (approximately)}}$$

Interpretation:

There is a **53.8% chance** that new subjects will be introduced in the department in 2014.

A software company receives applications from two groups:

Source	% of Applications	Probability Candidate is Skilled
Engineering Graduates (E)	70%	80%
Non-Engineering Graduates (N)	30%	40%

A candidate is selected for interview and **performs highly in the technical test.**

The company wants to decide whether to prioritize **Engineering candidates** in further rounds.

If a candidate performs well, what is the probability that they are from Engineering background?

Step 1: Find Total Probability of Skilled Candidate

$$P(Skill) = P(E)P(Skill|E) + P(N)P(Skill|N)$$

$$= 0.7(0.8) + 0.3(0.4)$$

$$= 0.56 + 0.12 = 0.68$$

$P(Skill) = 0.68$

Step 2: Apply Bayes' Theorem

$$P(E|Skill) = \frac{P(E) \cdot P(Skill|E)}{P(Skill)}$$

$$= \frac{0.7 \cdot 0.8}{0.68}$$

$$= \frac{0.56}{0.68}$$

$$= 0.8235 \approx 82.35\%$$

If a candidate appears skilled, the probability they are from Engineering is **~82.35%**

Example

An insurance company insured 2,000 scooter drivers, 4,000 car drivers and 6,000 truck drivers. The probability of their accident is 0.1, 0.3 and 0.2 respectively. One of the insured persons met with an accident. What is the probability that he is a scooter driver?

Solution

Let B_1, B_2, B_3 be the respective events that a scooter driver, a car driver and a truck driver is insured.

$$P(B_1) = \frac{2}{12} = \frac{1}{6}, \quad P(B_2) = \frac{4}{12} = \frac{1}{3}, \quad P(B_3) = \frac{6}{12} = \frac{1}{2}$$

Let A be the event that an accident occurs.

$$P(A|B_1) = 0.1, \quad P(A|B_2) = 0.3, \quad P(A|B_3) = 0.2$$

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Required Probability

$$P(B_1|A) = \frac{P(B_1) \cdot P(A|B_1)}{P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + P(B_3)P(A|B_3)}$$

$$= \frac{\frac{1}{6} \times 0.1}{\frac{1}{6} \times 0.1 + \frac{1}{3} \times 0.3 + \frac{1}{2} \times 0.2}$$

$$= \frac{\frac{1}{60}}{\frac{1}{60} + \frac{1}{10} + \frac{1}{10}} = \frac{\frac{1}{60}}{\frac{1}{60} + \frac{6}{60} + \frac{6}{60}}$$

$$= \frac{\frac{1}{60}}{\frac{13}{60}} = \frac{1}{13}$$

Thus, the probability that the insured person who met with an accident is a scooter driver is

Example

A disease affects 1 in 1,000 people.
A test for the disease is **99% accurate** (meaning it gives a positive result for 99% of sick people and a false positive for 1% of healthy people).
If a person tests positive, what is the probability that they actually have the disease?

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Given data

Prevalence of disease:
$P(D)=1/1000=0.001$

Probability of a positive test if the person has the disease (true positive):
$P(+|D)=0.99$

Probability of a positive test if the person does **not** have the disease (false positive):
$P(+|\text{no } D)=0.01$

We need to find:

$P(D|+)=?$

(the probability the person actually has the disease given they tested positive)

Given:

- $P(D) = 0.001$
- $P(+|D) = 0.99$
- $P(+|\bar{D}) = 0.01$

By Bayes' Theorem:

$$P(D|+) = \frac{P(D)P(+|D)}{P(D)P(+|D) + P(\bar{D})P(+|\bar{D})}$$

$$P(D|+) = \frac{0.001 \times 0.99}{0.001 \times 0.99 + 0.999 \times 0.01} = 0.09$$

✅ Only **9%** chance that the person actually has the disease!

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A company develops three types of software projects:

Project Type	% of Total Projects	Probability of Failure
Web Apps	30%	0.05
Mobile Apps	50%	0.08
AI Systems	20%	0.15

One project from the company has **failed.**

What is the probability that it was an **AI project**?

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Let

B_1= Web project

B_2= Mobile project

B_3= AI project

A= failure

$$P(B_1) = 0.3, \quad P(B_2) = 0.5, \quad P(B_3) = 0.2$$

$$P(A|B_1) = 0.05, \quad P(A|B_2) = 0.08, \quad P(A|B_3) = 0.15$$

$$P(B_3|A) = \frac{P(B_3)P(A|B_3)}{P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + P(B_3)P(A|B_3)}$$

$$= \frac{0.2 \times 0.15}{0.3 \times 0.05 + 0.5 \times 0.08 + 0.2 \times 0.15} = \frac{0.03}{0.015 + 0.04 + 0.03}$$

$$= \frac{0.03}{0.085} = 0.353 \approx 35.3\%$$

Example

An email contains the word "offer" 80% of the time in spam messages but only 10% of the time in legitimate emails. If 30% of all emails are spam, what is the probability that a new email containing the word "offer" is spam?

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Given data

P(S)=0.30→ Probability an email is **spam**

P(L)=0.70 → Probability an email is **legitimate (not spam)**

P("offer"|S)=0.80 Probability the word *"offer"* appears **given spam**

P("offer"|L)=0.10 → Probability the word *"offer"* appears **given legitimate email**

We need to find:

P(S|"offer")

(the probability that an email **is spam given** it contains the word *"offer"*).

- **Solution:**
  - Let S be the event that an email is spam.
  - Let O be the event that the email contains the word "offer".
  - We are given: $P(S) = 0.30$, $P(O|S) = 0.80$, and $P(O|\text{not } S) = 0.10$.
  - We need to find $P(S|O)$. Using Bayes' theorem:
  - $P(S|O) = \dfrac{P(O|S)P(S)}{P(O|S)P(S) + P(O|\text{not } S)P(\text{not } S)}$
  - Since $P(\text{not } S) = 1 - P(S) = 1 - 0.30 = 0.70$, we have:
  - $P(S|O) = \dfrac{(0.80)(0.30)}{(0.80)(0.30) + (0.10)(0.70)} = \dfrac{0.24}{0.24 + 0.07} = \dfrac{0.24}{0.31} \approx 0.7742$
  - The probability that the email is spam, given it contains the word "offer," is approximately 0.7742.

Example

A disease affects 2% of a population. The test for it is **95% accurate** (positive if diseased) and **5% false positive** (positive even if not diseased).

Solution:

Given:

$P(\text{disease}) = 0.02$

$P(\text{no disease}) = 0.98$

$P(\text{positive} / \text{disease}) = 0.95$

$P(\text{positive} / \text{no disease}) = 0.05$

Find $P(\text{disease} / \text{positive})$

$P(\text{positive}) = 0.95(0.02) + 0.05(0.98) = 0.019 + 0.049 = 0.068$

$$P(\text{disease} / \text{positive}) = 0.95(0.02) / 0.068$$
$$= 0.019 / 0.068$$
$$\approx 0.279$$

Thank You

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