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Probability and Probability Distribution
Unit No 2: Principle of Inclusion and Exclusion

Principle of Inclusion and Exclusion:

- Principle of Inclusion and Exclusion theorem and applications.
- Derangement theorem and its applications
- Non negative integer value solution
- Multinomial Theorem and application

Principle of Inclusion and Exclusion theorem and applications.

Principle of Inclusion and Exclusion theorem

What is the Principle of Inclusion and Exclusion?

The **Principle of Inclusion and Exclusion (PIE)** is a counting technique in combinatorics used to calculate the **number of elements in the union** of multiple sets, especially when there is **overlap between the sets.**

Let A and B be subsets of a finite universal set U. Then

(a) $|A \cup B| = |A| + |B| - |A \cap B|$

(b) $|A \cap B| \leq \min\{|A|, |B|\}$, the minimum of $|A|$ and $|B|$

(c) $|A \setminus B| = |A| - |A \cap B| \geq |A| - |B|$

(d) $|A^c| = |U| - |A|$

(e) $|A \oplus B| = |A \cup B| - |A \cap B| = |A| + |B| - 2|A \cap B| = |A \setminus B| + |B \setminus A|$

(f) $|A \times B| = |A| \times |B|$

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Principle of Inclusion and Exclusion theorem

(a) **$|A \cup B| = |A| + |B| - |A \cap B|$**

Meaning: The number of elements in the union = sum of the sizes minus the overlap.

Why subtract? Because elements in the intersection get counted twice when we add.

Example (live):

In a class, 20 students know **Java** (A), and 15 students know **Python** (B).

10 students know **both Java and Python** ($A \cap B$).

$|A \cup B| = 20 + 15 - 10 = 25$

👉 25 students know **Java or Python or both**.

(b) $|A \cap B| \leq \min(|A|, |B|)$

Meaning: The size of the intersection cannot exceed the size of the smaller set.

Example:

If 12 people like **Tea** and 8 people like **Coffee**, at most 8 people can like both.

So $|A \cap B| \leq 8$.

(c) $|A \setminus B| = |A| - |A \cap B||$

So, the symbol “\” is read as **“set minus”** or **“set difference”**.

Meaning: Elements in A but not in B.

Example:

In a company, 40 employees know **C++**, 25 know **Java**, and 15 know **both**.

Employees who know **C++ but not Java**:

$|A \setminus B| = 40 - 15 = 25$

Continue.....

1. With two sets

$|A \setminus B| = |A| - |A \cap B|$

A \setminus B| = |A| - |A∩B|

This is simply "all of A, but remove those that also belong to B.“

2. With three sets

If you want **elements in A but not in B or C**, that means:

$A \setminus (B \cup C)$

So:

$|A \setminus (B \cup C)| = |A| - |A \cap (B \cup C)|$

Now expand $A \cap (B \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

By inclusion–exclusion:

$|A \cap (B \cup C)| = |A \cap B| + |A \cap C| - |A \cap B \cap C|$

Final formula

$|A \setminus (B \cup C)| = |A| - (|A \cap B| + |A \cap C| - |A \cap B \cap C|)$

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(d) $|A^c| = |U| - |A|$

Meaning: The complement is everything in the universe except the set.

Example:

If the universal set is a class of 60 students, and 40 passed in **Maths** (A), then students who **did not pass Maths**:

$|A^c| = 60 - 40 = 20$

- **(e) Symmetric Difference:** $\oplus$ is read as **symmetric difference**
- $|A \oplus B| = |A \cup B| - |A \cap B| = |A| + |B| - 2|A \cap B|$
- **Meaning:** Elements in either A or B but not both.
- **Example:**

Example:
$A=\{1,2,3,4\}$
$B=\{3,4,5,6\}$
Then,
$A \oplus B=\{1,2,5,6\}$.

In a survey: 18 people like **Cricket**, 12 like **Football**, and 6 like **both**. People who like **only one of the two sports**:

- $|A \oplus B|=18+12-2(6)=18$

(f) $|A \times B| = |A| \times |B|$

Meaning: Cartesian product (pairs from A and B).

Example:
4 choices of **starters** (A),
3 choices of **drinks** (B).

Total meal combinations =

$|A \times B| = 4 \times 3 = 12$

Principle of Inclusion and Exclusion

The principle of inclusion-exclusion is a mathematical counting technique used to find the total number of elements in the union of multiple sets by summing the individual counts of each set, subtracting the counts of their pairwise intersections, adding back the counts of their triple intersections, and so on. This alternating process ensures that overlapping elements are not overcounted, providing an exact count of all unique elements.

Formula

Formula for Two Sets

For two sets, A and B:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$|A \cup B|$ is the number of elements in A or B or both.

$|A|$ is the number of elements in set A.

$|B|$ is the number of elements in set B.

$|A \cap B|$ is the number of elements in both A and B.

Formula for Three Sets

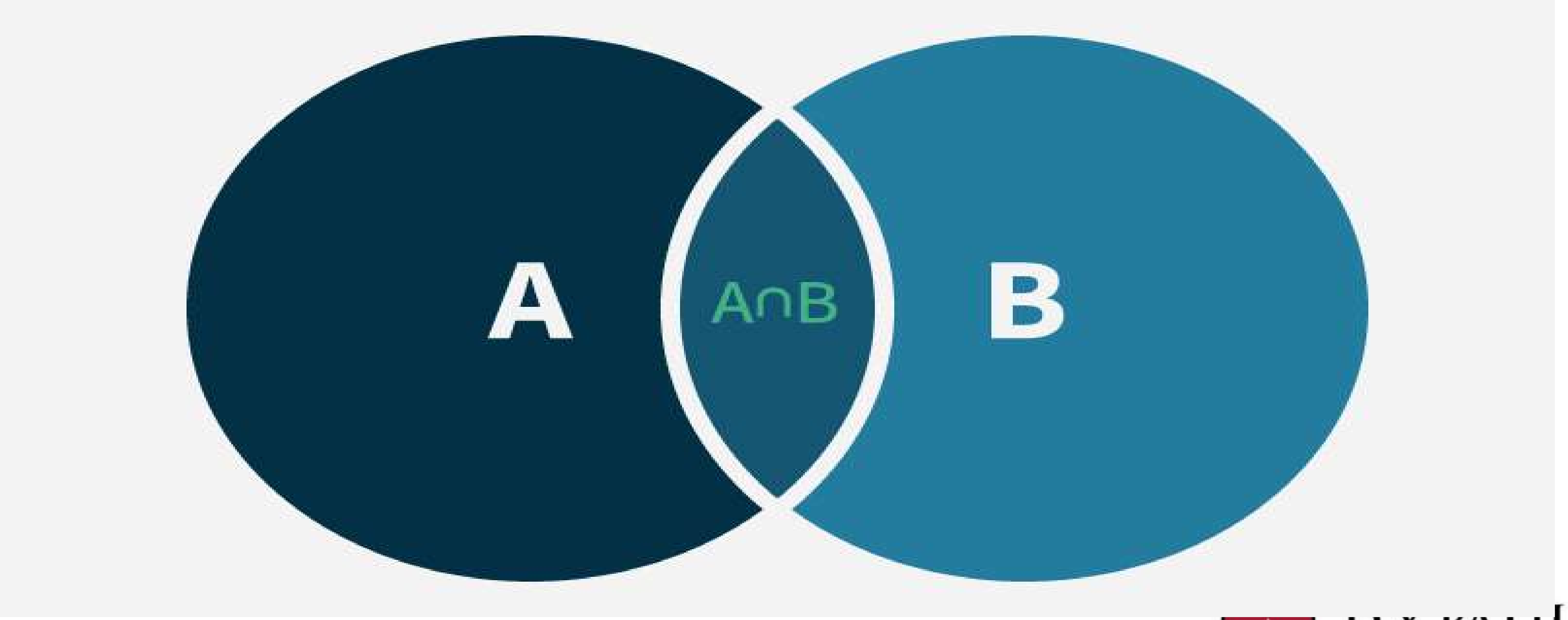
For three sets, A, B, and C:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

Principle of Inclusion and Exclusion for two and three sets

✅ For Two Sets A and B:

$$|A \cup B| = |A| + |B| - |A \cap B|$$



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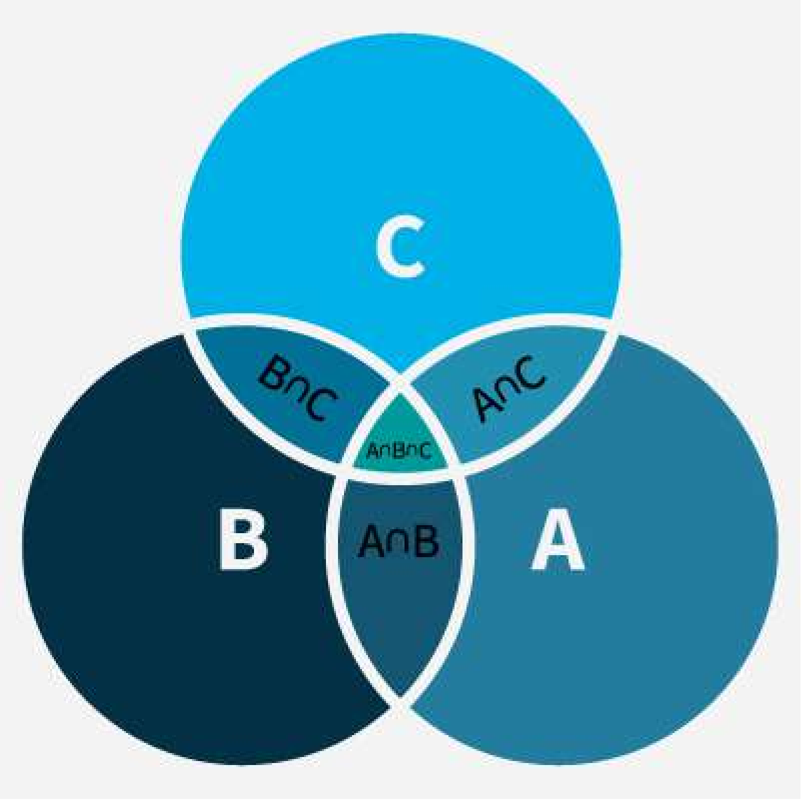
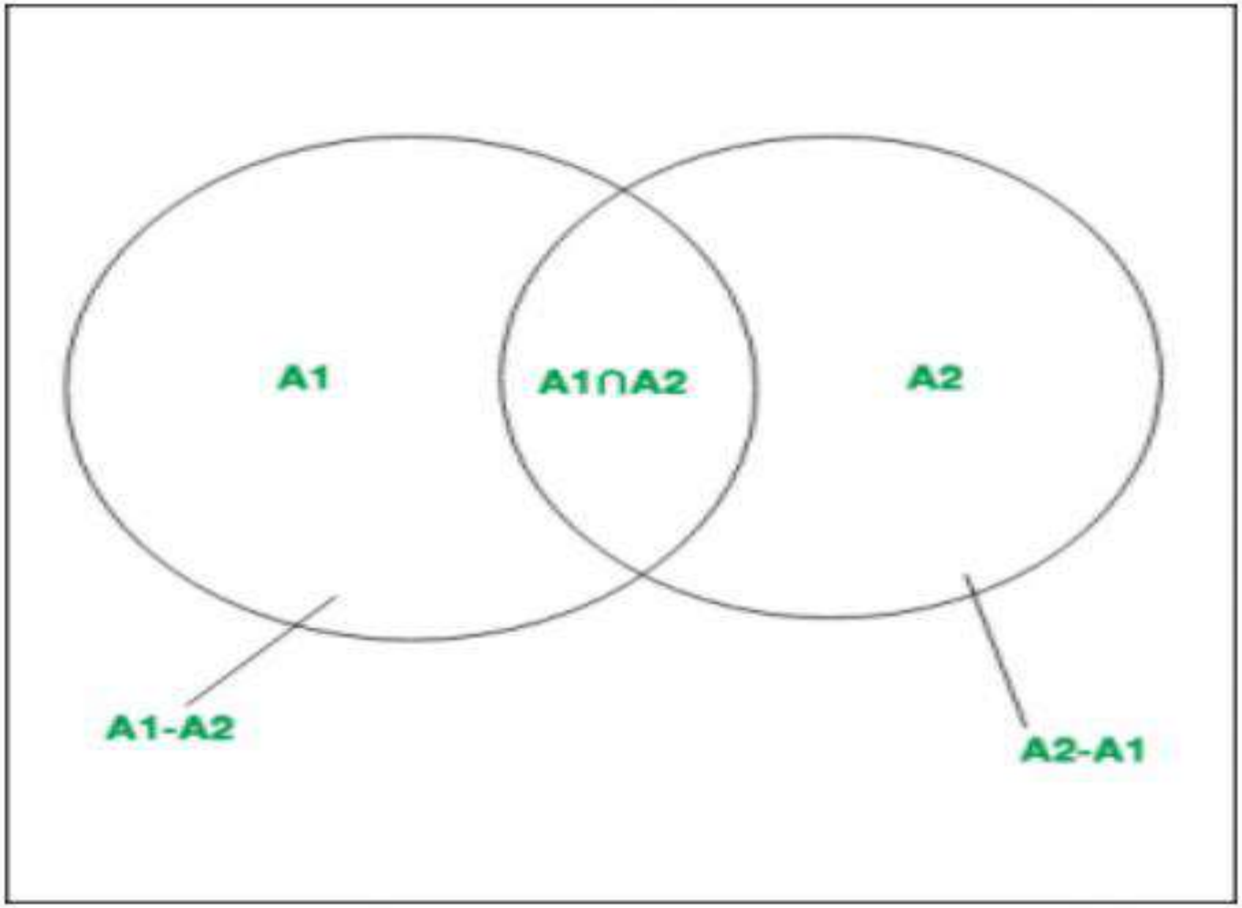
✅ For Three Sets A, B, and C:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$




A1
A1∩A2
A2
A1-A2
A2-A1

Example

Imagine you want to count how many people in a group like either cats or dogs.

Include: Count everyone who likes cats and everyone who likes dogs.

Exclude: Subtract the people who like both cats and dogs (as they were counted twice).

Result: The result is the accurate total number of people who like at least one of the two pets.

Example

1. Counting Numbers with Certain Properties

Example: Between 1 and 100, how many numbers are divisible by 2 or 3?

$|A|$ = numbers divisible by 2 = $\lfloor 100/2 \rfloor = 50$

$|B|$ = numbers divisible by 3 = $\lfloor 100/3 \rfloor = 33$

$|A \cap B|$ = numbers divisible by 6 = $\lfloor 100/6 \rfloor = 16$

$|A \cup B| = 50 + 33 - 16 = 67$

So, **67 numbers** between 1 and 100 are divisible by 2 or 3.

Example

If in a class:

20 know Java,

15 know Python,

10 know both,

Then students knowing at least one language =

$20+15-10=25$

Example

In a university with 200 students:

- ❑120 students attended a **Cloud Computing** seminar.
- ❑80 attended a **DevOps** seminar.
- ❑40 attended **both**.

How many students **did not attend either seminar**?

Solution:

Solution:

First, use the union formula:

$|A \cup B| = 120 + 80 - 40 = 160$

Now, total students = 200

Students who attended neither $= 200 - 160 = 40$

Example

Example : A company has 30 employees, 10 are assigned to Task A, 15 to Task B, and 5 are assigned to both. How many employees are assigned to at least one task?

Solution : **Solution:**

As we know, $(A \cup B) = (A) + (B) - (A \cap B)$

Given:

$|A| = 10$ Task A

$|B| = 15$ Task B

$|A \cap B| = 5$ Both tasks

Substitute the Values, We get

$(A \cup B) = 10 + 15 - 5 = 20$

Thus, 20 employees are assigned to at least one task.

Example

Example: In a survey of 200 people:
120 likes pizza.
100 like burgers.
80 like tacos.
60 likes both pizza and burgers.
40 likes both burgers and tacos.
30 likes both pizza and tacos.
20 likes all three: pizza, burgers, and tacos.
How many people like at least one of these three foods?
Solution:
As we know, $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$
Given:
$|A| = 120$ Pizza lovers
$|B| = 100$ Burger lovers
$|C| = 80$ Taco lovers
$|A \cap B| = 60$
$|A \cap C| = 30$
$|B \cap C| = 40$
$|A \cap B \cap C| = 20$
Substituting in the formula, we get,
$|A \cup B \cup C| = 120 + 100 + 80 - 60 - 30 - 40 + 20 = 190$
Thus, 190 people like at least one of the three foods.

Example: In a university of 300 students:
150 students take Math.
120 students take Physics.
100 students take Chemistry.
80 students take both Math and Physics.
60 students take both Physics and Chemistry.
50 students take both Math and Chemistry.
30 students take all three subjects.
How many students are taking at least one subject?

Solution:

As we know, $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

Given:

$|A| = 150$ (Math),
$|B| = 120$ (Physics),
$|C| = 100$ (Chemistry),
$|A \cap B| = 80$
$|A \cap C| = 60$
$|B \cap C| = 50$
$|A \cap B \cap C| = 30$

Substituting values, we get
$|A \cup B \cup C| = 150 + 120 + 100 - 80 - 60 - 50 + 30 = 210$
Thus, 210 students are taking at least one subject.

Practice Questions

Question 1: In a group of 200 people:
120 people like chocolate.
90 people like vanilla.
50 people like both chocolate and vanilla.
How many people like either chocolate or vanilla?(Ans : 160)

Question 2: In a class of 100 students:
70 students play football.
60 students play basketball.
50 students play cricket.
30 students play both football and basketball.
25 students play both basketball and cricket.
20 students play both football and cricket.
15 students play all three sports.
How many students play at least one of these three sports?
(Ans: 120)

Practice Questions

Question 3: Out of 150 attendees at an event:

80 attended Workshop A.

70 attended Workshop B.

40 attended both Workshop A and Workshop B.

How many people attended at least one workshop?(Ans:110)

Question 4: In a school of 300 students:

180 students are enrolled in Math.

150 students are enrolled in Science.

120 students are enrolled in English.

90 students are enrolled in both Math and Science.

80 students are enrolled in both Science and English.

70 students are enrolled in both Math and English.

50 students are enrolled in all three subjects.

How many students are enrolled in at least one subject?

(Ans:260)

Practice Questions

Question 5 : A total of 1232 students have taken a course in Spanish, 879 have taken a course in French, and 114 have taken a course in Russian. Further, 103 have taken courses in both Spanish and French, 23 have taken courses in both Spanish and Russian, and 14 have taken courses in both French and Russian. If 2092 students have taken a course in at least one of Spanish French and Russian, how many students have taken a course in all 3 languages.

(Ans:7)

Solution

$|A|=1232$ (students in Spanish)
$|B|=879$ (students in French)
$|C|=114$ (students in Russian)
$|A\cap B|=103$
$|A\cap C|=23$
$|B\cap C|=14$
Let $x=|A\cap B\cap C$
By the **principle of inclusion–exclusion**:
$|A\cup B\cup C|=|A|+|B|+|C|-(|A\cap B|+|A\cap C|+|B\cap C|)+|A\cap B\cap C|$

$2092=1232+879+114-(103+23+14)+x$
$2092=2225-140+x$
$2092 = 2085 + x$
$x = 7$
So, **7 students** took **all three courses** ($A\cap B\cap C$)

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Practice Questions

Question 6 : In a group of 200 people, each of whom is at least accountant or management consultant or sales manager, it was found that 80 are accountants, 110 are management consultants and 130 are sales managers, 25are accountants as well as sales managers, 70 are management consultants as well as sales managers, 10 are accountants, management consultants as well as sales managers. Find the number of those people who are accountants as well as management consultants but not sales managers.

Solution

Given

Total =200 (everyone is in at least one of the three groups)

$|A|=80$ (Accountants)

$|B|=110$ (Management consultants)

$|C|=130$ (Sales managers)

$|A \cap C|=25$ $|B \cap C|=70$

$|A \cap B \cap C|=10$

PIE for three sets:

$|A \cup B \cup C|=|A|+|B|+|C|-(|A \cap B|+|A \cap C|+|B \cap C|)+|A \cap B \cap C|$

$200=80+110+130-(|A \cap B|+25+70)+10$

$|A \cap B|=235-200=35.$

Solution continue.....

Now the requested count (accountants AND management consultants but NOT sales managers)

So 35 people are accountants **and** consultants (maybe some are also sales managers).

We know 10 people are in **all three** groups:

$|A \cap B \cap C| = 10$

So:

Only accountants + consultants (not sales) $= |A \cap B| - |A \cap B \cap C|$

$= 35 - 10$

$= 25$

Final Answer = 25 people

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Question 7 : In a pollution study of 1,500 Indian rivers, the following data were reported: 520 were polluted by sulphur compounds, 335 were polluted by phosphates, 425 were polluted by crude oil, 100 were polluted by both crude oil and sulphur compounds, 180 were polluted by sulphur compounds and phosphates, 150 were polluted by both phosphates and crude oil and 28 were polluted by sulphur compounds, phosphates and crude oil. How many of the rivers were polluted by at least one of the three impurities? How many rivers were not polluted by exactly one of the three impurities? How many of the rivers were not polluted?

(a) Number of rivers polluted by at least One of the three impurities(Ans: 878)

((b)Number of rivers polluted by exactly sulphur compounds (Ans: 268)

(c) Number of rivers polluted by exactly phosphate (ans: 33)

(d) Number of rivers polluted by exactly crude oil (ans:203)

Thus, the number of rivers which were polluted by exactly one of the three impurities =268 + 33 + 203 = 504

and so the number of rivers which were not polluted at all = 1500 – 878 = 622.

Solution : Q7

Let

A = rivers polluted by sulphur compounds = 520

B = rivers polluted by phosphates = 335

C = rivers polluted by crude oil = 425

$A \cap B = 180$, $A \cap C = 100$, $B \cap C = 150$, $A \cap B \cap C = 28$

Total = 1500

Elements in A but not in B or C

We want $|A \setminus (B \cup C)|$

Formula:

$$|A \setminus (B \cup C)| = |A| - |A \cap (B \cup C)| = |A| - (|A \cap B| + |A \cap C| - |A \cap B \cap C|)$$

Plugging numbers:

$$|A \setminus (B \cup C)| = 520 - (180 + 100 - 28) = 520 - 280 + 28 = 268.$$

So **268 rivers are in A only** (sulphur only).

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Solution : Q7 continue...

Similarly for B only

$|B \setminus (A \cup C)| = |B| - (|A \cap B| + |B \cap C| - |A \cap B \cap C|) =$

$335 - (180 + 150 - 28) = 335 - 330 + 28 = 33$

Similarly for C only

$|C \setminus (A \cup B)| = |C| - (|A \cap C| + |B \cap C| - |A \cap B \cap C|) = 425 - (100 + 150 - 28) =$

$425 - 250 + 28 = 203$

So **B only = 33**, **C only = 203**.

Solution : Q7 continue...

Exactly two (A∩B only, etc.)

To get the elements in A∩BA\cap BA∩B but **not** CCC:

$|(A\cap B)\setminus C|=|A\cap B|-|A\cap B\cap C|=180-28=152$

Similarly:

$|(A\cap C)\setminus B|=100-28=72$

$|(B\cap C)\setminus A|=150-28=122$

Solution : Q7 continue...

Results

At least one impurity (PIE): 878

Exactly one = 268+33+203=504

Exactly two = 152+72+122=346

All three = 28

Check: 504+346+28=878

Not polluted at all = 1500−878=622

Practice Question No. 8

Of a group of 20 persons, 10 are interested in music, 7 are interested in photography, and 4 like swimming; further more 4 are interested in both music and photography, 3 are interested in both music and swimming, 2 are interested in both photography and swimming and one is interested in music, photography and swimming. How many are interested in photography but not in music and swimming?

Solution :

Let A=Music, B=Photography, C=Swimming.
Given:

$|A|=10, |B|=7, |C|=4,$

$|A \cap B|=4, \ |A \cap C|=3, \quad |B \cap C|=2, \ |A \cap B \cap C|=1.$

$(B \setminus A \cup C) = |B| - (|B \cap (A \cup C)|)$

$= |B| - (|B \cap A| + |B \cap C| - |A \cap B \cap C|)$

$= 7 - (4+2-1)$

$= 7-4-2+1$

$= 2$

Practice Question : Homework

Question 9: 30 personal computers (PCs) owned by faculty members in a certain university department. 20 run windows, eight have 21 inch monitors, 25 have CD-ROM, drives, 20 have at least two of these features, and six have all three.

a)How many PCs have at least one of these features.

b)How many have none of these features

c)How many have exactly one feature.

Derangement theorem and its applications

Suppose there are 4 guests who give their hats to a cloakroom attendant. At the end, the attendant returns the hats randomly. What's the chance that *no guest gets back their own hat*?"

Derangement

The word derangement in simple words means any change in the existing order of things. Mathematically derangement refers to the permutation consisting of elements of a set in which the elements don't exist in their respective usual positions.

We consider a simple example to understand this concept. Suppose that a teacher has given assignments to 4 groups. Now, obviously a group cannot grade or assess his own assignment and so the teacher wants the groups to grade each other's assignment. Our first concern is that in how many ways the teacher can provide the assignments to groups 1, 2, 3 and 4 the so that none of the group gets its own. In this case, the total possible number of permutations is 4! = 24. But, out of these there are only 9 derangements. The possible derangements are listed below:

2143	2341	2413
3142	3412	3421
4123	4312	4321

Derangement Theorem

1. Introduction

A **derangement** is a permutation of objects in which **no element appears in its original position**. In simpler terms, it's a complete "scrambling" of a sequence.

Example: Suppose 3 letters **A, B, C** are to be placed in 3 envelopes **1, 2, 3**. A derangement means **A is not in 1, B is not in 2, C is not in 3**.

For the set {1, 2, 3}, the permutation (2, 3, 1) is a derangement because no number is in its original spot (1 is not first, 2 is not second, 3 is not third). The permutation (2, 1, 3) is *not* a derangement because the number 3 is in its correct position.

2. Formula for Number of Derangements (!n)

The number of derangements of **n** objects is denoted by **!n** (read as subfactorial of n).

$$!n = n!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots + (-1)^n \frac{1}{n!}\right)$$

- The Derangement theorm is part of combinatorics and discrete mathematics – which are foundation for computer science.
- students often need to solve **arrangement, permutation, and probability problems** in algorithms, data science, and AI.
- Derangements model situations where **no element maps to itself**, useful in designing secure key exchanges.
- **Parallel Processing & Scheduling**: Task assignment problems where tasks must not be assigned back to their original processor resemble derangement problems.
- students are trained to think algorithmically.
- Derangement problems develop **stepwise problem-solving skills**, especially in using **inclusion–exclusion principle**

Case Study 1: Password Shuffling for Security

Suppose a system randomly shuffles passwords for maintenance or encryption.

To prevent **predictable outcomes**, no user should get their original password back (otherwise it becomes insecure).

This is exactly a **derangement problem** → number of ways passwords can be reassigned with **no one getting their own**.

Example 1 — Small case by formula: $n = 3$

Use the inclusion–exclusion formula:

$$!n = n! \sum_{i=0}^{n} \frac{(-1)^i}{i!}$$

For $n = 3$:

$$!3 = 3! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!}\right) = 6\left(\frac{6}{6} - \frac{6}{6} + \frac{3}{6} - \frac{1}{6}\right) = 6\left(\frac{2}{6}\right) = 2.$$

So there are **2 derangements** of 3 items. (You can list them to check: for items A,B,C the derangements are B C A and C A B.)

Example

Supposing 4 letters are placed in 4 different envelopes. In how many ways can they be taken out from their original envelopes and distributed among the 4 different envelopes so that no letter remains in its original envelope?

Solution: This is clearly a case of derangement. Using the formula for the number of derangements that are possible out of 4 letters in 4 envelopes, we get the number of ways as:

$4!(1 - 1 + 1/2! - 1/3! + 1/4!)$

$= 24(1 - 1 + 1/2 - 1/6 + 1/24)$

$= 9.$

At a party, **four friends** (A, B, C, and D) check in their **hats** at the entrance. When they leave, the attendant returns the hats **randomly** without keeping track of whose hat belongs to whom.

1. In how many ways can the hats be returned such that **no friend receives their own hat?**

1. Total possible ways

All permutations of 4 hats → $4! = 24$.

2. Number of derangements D_4 (no one gets own hat)

Method A — Inclusion–Exclusion formula

$$D_n = n! \sum_{k=0}^{n} \frac{(-1)^k}{k!}.$$

For $n = 4$:

$$D_4 = 4! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right).$$

Compute inside:

$$1 - 1 + \tfrac{1}{2} - \tfrac{1}{6} + \tfrac{1}{24} = 0 + \tfrac{12-4+1}{24} = \tfrac{9}{24}.$$

So

$$D_4 = 24 \cdot \tfrac{9}{24} = 9.$$

Example : At the cloakroom 8 distinct coats are returned randomly to their owners. How many ways can the coats be returned so that nobody gets their own coat.(Ans: 14833)

Non negative integer value solution

Combination

Meaning of Combination

Combination is the selection of objects from a given set where the **order of selection does not matter**.

Unlike permutations (arrangements), in combinations **(A, B, C) = (B, C, A) = (C, A, B)** → all are considered the same.

👉 Think of **forming a committee** vs. **arranging chairs**.
Committees → Combination (order doesn't matter).
Arrangements → Permutation (order matters).

Permutation → *arranging books on a shelf.*

Combination → *selecting books to carry in your bag.*

👉 In permutation, order matters. In combination, it doesn't.

Difference between Permutation and Combination

1. Definition

Permutation → Arrangement of objects in a **specific order**.
Order **matters**.

Combination → Selection of objects without considering order.
Order **does not matter**.

2. Formula

- Permutation (selecting r objects from n):

$$P(n, r) = \frac{n!}{(n-r)!}$$

- Combination (choosing r objects from n):

$$C(n, r) = \binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

3. In **Permutation**,
(A,B)and (B,A) are considered **different**.

In **Combination**,
(A,B) and (B,A) are considered the **same**.

Permutation → Position matters (arrangement).

Combination → Committee matters (selection).

Compare with Permutation

Permutation → *arranging books on a shelf.*

Combination → *selecting books to carry in your bag.*

👉 In permutation, order matters. In combination, it doesn't.

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Combination formula- No repetition

$$\frac{n!}{r!(n-r)!} = \binom{n}{r}$$

where ***n*** is the number of things to choose from,
and we choose ***r*** of them,
no repetition,
order doesn't matter.

It is often called "n choose r" (such as "16 choose 3")

And is also known as the Binomial Coefficient.

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Notation

All these notations mean "n choose r":

$$C(n,r) = {}^{n}C_{r} = {}_{n}C_{r} = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Just remember the formula:

$$\frac{n!}{r!(n-r)!}$$

Combination formula- Repetition allowed

$$^{n}C_{r}\ \textbf{(with repetition)} = \binom{n+r-1}{r}$$

where ***n*** is the number of things to choose from,
and we choose ***r*** of them
repetition allowed,
order doesn't matter.

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Example 1:

From a group of 10 students, a committee of 3 students is to be formed. In how many ways can this be done?

Solution:

$$^{n}C_{r} = \binom{10}{3} = \frac{10!}{3!(10-3)!}$$

$$= \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120$$

👉 So, the committee can be formed in **120 ways**.

Example 2:

How many ways can 5 cards be chosen from a pack of 52 cards?

Solution:

$$\binom{52}{5} = \frac{52!}{5!(52-5)!}$$

$$= \frac{52 \times 51 \times 50 \times 49 \times 48}{5 \times 4 \times 3 \times 2 \times 1}$$

$$= 2,598,960$$

👉 There are **2,598,960 ways** to choose 5 cards.

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Example 3:

In how many ways can a team of 11 players be selected from 15 players?

Solution:

$$\binom{15}{11} = \binom{15}{4} \quad (\text{since } \binom{n}{r} = \binom{n}{n-r})$$

$$= \frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1}$$

$$= 1365$$

👉 The team can be selected in **1365 ways.**

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A company has 12 software projects. Out of these, 5 projects are to be assigned to a team of interns. In how many different ways can the projects be assigned if order doesn't matter?

Solution:

$$\binom{12}{5} = \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1}$$

$$= 792$$

👉 792 ways.

Combination formula- Repetition allowed

$$^nC_r \text{ (with repetition)} = \binom{n+r-1}{r}$$

where ***n*** is the number of things to choose from,
and we choose ***r*** of them
repetition allowed,
order doesn't matter.

Combination when repetition is allowed

A password consists of 3 digits chosen from {0,1,2,3,4} where digits may repeat. How many different passwords are possible if order **does not matter**?

Solution:

This is choosing 3 digits from 5 with repetition allowed.

$$\binom{5+3-1}{3} = \binom{7}{3} = 35$$

👉 35 passwords

In how many ways can 6 ice creams be chosen from 10 flavours (repetition allowed)?

Solution:

$$\binom{10+6-1}{6} = \binom{15}{6}$$

$$= \frac{15 \times 14 \times 13 \times 12 \times 11 \times 10}{6 \times 5 \times 4 \times 3 \times 2 \times 1} = 5005$$

👉 5005 ways

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A person wants to buy 4 fruits from a shop that has 3 types of fruits (apple, mango, banana). Repetition allowed. How many ways?

Solution:

$$\binom{3+4-1}{4} = \binom{6}{4} = 15$$

👉 15 ways

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Thank You

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