Unit-1 : Counting Principles.

1) Fundamental Counting principle

• Addition Principle :-

• Formula :- $m + n$

• eg :- Student wants to ~~by~~ buy a drink & can choose from 5 drinks or 4 sodas. Total no. of ways to choose one drink is

$$5 + 4 = 9 \text{ ways.}$$

• Multiplication Principle :-

• Formula : $n_1 \times n_2 \ldots\ldots n_k$

• eg :- Person is choosing an outfit consist of one shirt and one pair of pants. if they have 10 shirts and 7 pants, the total no. of different outfit possible is

$$10 \times 7 = 70 \text{ outfits.}$$

2. Linear Permutation :-

• Permutation of n distinct objects:-

$$\boxed{\text{• Formula : } n!}$$

• eg:- Calculating the number of ways to arrange the letters in the word "FLOWERS". Since there are 7 distinct letters, the total arrangements are:

$\rightarrow 7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 5,040.$

• Permutations of n objects taken r at a time:

$$\boxed{\text{• Formula: } P(n, r) = \frac{n!}{(n-r)!}}$$

• eg:- in a race with 10 sprinters, how many ways can the gold, silver and bronze medals be awarded? This is a permutation of 10 object taken 3 at a time:

$$P(10,3) = \frac{10!}{(10-3)!} = \frac{10!}{7!}$$

$$= \frac{10 \times 9 \times 8 \times \cancel{7!}}{\cancel{7!}} = \underline{\underline{720}}$$

EXPERIMENT : No. Page No. Date

3. Circular Permutation :-

- **Clockwise and anticlockwise arrangements are different:**

  - Formula: $(n-1)!$

  - eg:- How many ways can 5 friends be seated around a campfire? Since the relative position matters, no. of arrangements is

$$(5-1)! = 4! = \underline{\underline{24}}$$

- **Clockwise and Anticlockwise arrangements are identical:-**

  - Formula: $\frac{(n-1)!}{2}$

  - eg:- A jeweler is making a bracelet using 8 different colored gemstone. Because the bracelet can be flipped over, the clockwise & anticlockwise orders are the same. No. of ways to arrange a stone is:

$$\frac{(n-1)!}{2} = \frac{7!}{2} = \underline{\underline{2520}}$$

4) Permutation with Repetitions

- **Objects of the same kind :-**

> • Formula :- $\dfrac{n!}{n_1 \times n_2 \cdots \times n_k}$

- eg:- Find the number of distinct ways to arrange the letters in the word "MISSISSIPPI".

  - $(n) = 11$

    $I = 4, \; S = 4, \; P = 2$

$$\frac{11!}{4! \times 4! \times 2!} = \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times \cancel{4!}}{4! \times \cancel{4!} \times 2!}$$

$$= \frac{11 \times 10 \times 9 \times \overset{4}{\cancel{8}} \times 7 \times \overset{3}{\cancel{6}} \times 5}{\cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{2}}$$

$$= 11 \times 10 \times 9 \times 7 \times 5$$

$$= 110 \times 9 \times 35$$

$$= 990 \times 35 = \underline{\underline{34650}}$$

EXPERIMENT : No. Page No. Date

Unit 2 - Principle of Inclusion & Exclusion

1.) Principle of Inclusion & Exclusion :- (PIE) :-

- **Formula for 2 sets (A & B) :-**

→ $\boxed{|A \cup B| = |A| + |B| - |A \cap B|}$

eg:- in a class, 20 students knows Java (A) and 15 knows Python (B). if 10 knows both ($A \cap B$), the no. of students who know atleast one is

$20 + 15 - 10 = \underline{25}$

- **Formula for 3 sets (A, B & C)**

→ $\boxed{A \cup B \cup C = |A| + |B| + |C| - (|A \cap B|) + |(A \cap C)| + |(B \cap C)| + |A \cap B \cap C|}$

eg:- in a survery of 200 people, 120 like pizza (A), 100 like burger (B), and 80 like tacos (C), intersections are $A \cap B = 60$, $A \cap C = 30$, $B \cap C = 40$ and all three $(A \cap B \cap C) = 20$

$120 + 100 + 80 - (60 + 30 + 40) + 20$

$= \underline{190}$

2) Set Difference or Set Minus

- **Formula for 2 sets.**

$$\boxed{|A \setminus B| = |A| - |A \cap B|}$$

→ eg:- if 40 employees know C (A) and 15 of them also know Java $(A \cap B)$, those who know C++ but not Java.

$$|A \setminus B| = 40 - 15 = 25$$

- **Formula for 3 sets:-**

$$\boxed{|A \setminus (B \cap C)| = |A| - (|A \cap B| + |A \cap C| - |A \cap B \cap C|}$$

→ in a study of 1500 river, 520 were polluted by sulphur (A), 335 by phosphate (B), and 425 by oil (C). To find rivers polluted only by sulphur:

$$520 - (180 + 100 - 28) = \underline{\underline{268}}.$$

3) Other set difference :-

- **Symmetric difference $\oplus$:-**

$$\boxed{|A \oplus B| = |A \cap B| - |A \cup B| = |A| + |B| - 2|A \cap B|}$$

→ eg:- if 18 like cricket, 12 like football, and 6 like both, those who like only one spot:

$$18 + 12 - 2(6) = \underline{\underline{18}}$$

EXPERIMENT : No. Page No. Date

- **Complement (A^c):**

→ $\boxed{|A^c| = |U| - |A|}$

→ eg:- in a class of 60 students (U), if 40 classes Maths (A), then (20) did not pass (60 - 40)

- **Cartesian Product:**

→ $\boxed{|A \times B| = |A| \times |B|}$

→ eg:- 4 starter choice (A) and 3 drink choices (B) result in $4 \times 3 = 12$

4) **Dearrangement :-**

→ $$\boxed{!n = n!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots + (-1)^n \frac{1}{n!}\right)}$$

→ eg:- if 8 distinct coats are returned randomly, there are 14,833 ways for nobody to get their own coat back.

5) Non-negative integer value solution

- **Permutation:-**

$$\rightarrow \boxed{P(n,r) = \frac{n!}{(n-r)!}}$$

- **Combination:-**

$$\rightarrow \boxed{C(n,r) = \binom{n}{r} = \frac{n!}{r!\,(n-r)!}}$$

eg:- from a group of 10 students, a committee of 3 students is to be formed. In how many way can this be done?

$$^{n}C_{r} = \binom{10}{3} = \frac{10!}{3!\,(10-3)!}$$

$$= \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = \underline{120}$$

- **$^{n}C_{r}$ (with repetition):-**

$$\rightarrow \binom{n+r-1}{r}$$

→ eg:-

in how many ways can 6 icecream be choses from 10 flavours

$$\binom{10+6-1}{6} = \binom{15}{6}$$

$$= \frac{\overset{5}{\cancel{15}} \times \overset{7}{\cancel{14}} \times 13 \times \overset{\overset{1}{\cancel{2}}}{\cancel{12}} \times 11 \times \cancel{10}\,\cancel{8}}{\cancel{6} \times \cancel{5} \times \underset{\cancel{2}}{\cancel{4}} \times \cancel{3} \times \cancel{2} \times 1}$$

$$= 5 \times 7 \times 13 \times 11 = \underline{5005}$$

EXPERIMENT : No. Page No. Date

Unit 3 - Probability

1) How to calculate Probability :-

- **Probability**

→ $\boxed{\dfrac{\text{(No. of favourable outcomes)}}{\text{(Total no. of possible outcomes)}}}$

→ eg :- what is the probability of getting atleast one head?

→ $S = \{HH, HT, TT, TH\}$

$A = \{HH, HT, TH\}$

$P = A/S = 3/4 = 0.75 = 75\%$

→ eg :- if 3 coins are flipped, what is the probability of getting exactly 1 tail.

→ $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$A = \{HHT, HTH, THH\}$

$P = A/S = 3/8 = 0.375$

2) Axioms of Probability :-

- **Non - Negativity :-**

→ Probability of an event is never negative, meaning it can't be less than zero

→ for any event A,

$$P(A) \geq 0$$

- **Normalization:**
  - → probability that something in the sample space happen is 1
  - → for the sample space S,

$$\boxed{P(S) = 1}$$

- **Additivity:-**
  - → if A and B are mutually exclusive events.
  - → $\boxed{P(A \cup B) = P(A) + P(B)}$
  - → eg:- Toss 2 coins.

    sample space = {HH, HT, TT, TH}

    $A = \{HH\}$ $B = \{HT\}$

    $P(A \cup B) = P(A) + P(B) = 1/4 + 1/4 = 1/2$

- if A, B, C are disjoint, then;
  - Formula for probability of union of 3 disjoint sets.
  - → $\boxed{P(A \cup B \cup C) = P(A) + P(B) + P(C)}$

- if A & B are not disjoint sets:-
  - → $\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$

- formula for probability of union of 3 sets
  - → $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

EXPERIMENT: No. Page No. Date

→ eg:-

Rolling a Dice = $SS = \{1, 2, 3, 4, 5, 6\}$

1st event $A = \{1, 2, 5\}$

2nd event $B = \{2, 5, 6\}$

3rd event $C = \{3, 5, 6\}$

$P(A) = 3/6$ $P(B) = 3/6$ $P(C) = 3$

$P(A \cap B) = 2/6$ $P(B \cap C) = 2/6$ $P(A \cap C) = 1/6$

$P(A \cap B \cap C) = 1/6$

→ For these axioms, we derive useful properties:

(i) $0 \leq P(A) \leq 1$

(ii) $P(\emptyset) = 0$

(iii) $\boxed{P(A^c) = 1 - P(A)}$

↓

$\boxed{P(A) + (A^c) = 1}$

3) Addition theorem :-

→ $\boxed{P(A \text{ or } B) = P(A) + P(B)}$ (2 events)

→ $\boxed{P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C)}$

→ $\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$ (2 events)

→ eg:- 4 king and 4 queen in pack of 52 cards

king = 4/52

queen = 4/52

Probability that it is either a king or queen = 4/52 + 4/52

= 8/52 = 2/13

Teacher's Sign. :

→ eg:- Probability that query is slow due to heavy load = 0.30.
Probability that query is slow due to indexing issue = 0.25
Probability that query is slow due to both load & indexing issue = 0.10

$$P(A) + P(B) - P(A \cap B) = 0.30 + 0.25 - 0.10$$
$$= \underline{0.45}$$

- for 3 events:-

→ $$\boxed{P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)}$$

4) Multiplication Theorem:-

- events are idependent:-

→ $$\boxed{P(A \text{ and } B) = P(A) * P(B)}$$

- events are dependent:-

→ $$\boxed{P(A \text{ and } B) = P(A) * P(B|A)}$$

- union:-

→ $\boxed{(A \cup B) \rightarrow \text{Read as } A \text{ or } B}$

- intersection:-

→ $\boxed{(A \cap B) \rightarrow \text{Read as } A \text{ and } B}$

→ eg:- 3 independent checks.
password correct : 0.90
OTP correct : 0.85
Biometric correct : 0.95

$$P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B) \cdot P(C)$$
$$= \underline{0.72}$$

- Neither A or B, when both try :-

→ $\boxed{P(\text{NO SALE}) \to P(A' \cap B') = P(A') \cdot P(B')}$

5) Conditional Probability :-

→ $\boxed{P(A|B) = \dfrac{P(A \cap B)}{P(B)} \text{, where } P(B) > 0}$

- conditional probability of A given B :

✓ → $\boxed{P(A|B) = \dfrac{P(A \cap B)}{P(B)} \text{, provided } P(B) > 0}$

- conditional probability of B given A :-

→ $\boxed{P(B|A) = \dfrac{P(A \cap B)}{P(A)} \text{, provided } P(A) > 0}$

- Complement of an Event :-

→ $\boxed{P(A') = 1 - P(A)}$

- if $A_1, A_2, A_3 \dots A_n$ are mutually exclusive and exhaustive events:-

$$\rightarrow \boxed{P(A_i|B) = \frac{P(A_i)\cdot P(B|A_i)}{\sum_{j=1}^{n} P(A_j)\cdot P(B|A_j)}}$$

- Baye's Theorem Formula :-

$$\rightarrow \boxed{P(A|B) = \frac{P(B|A)\cdot P(A)}{P(B)}}$$

6) Law of Total Probability :-

$$\rightarrow \boxed{P(A) = P(B_1)\cdot P(A|B_1) + P(B_2)\cdot P(A|B_2) + \dots + P(B_n)\cdot P(A|B_n)}$$

EXPERIMENT : No.

Page No
Date

Unit 4:
Random variable & Mathemathical Expectation.

- Discrete Random variable :-

  - expected value :-

$$\rightarrow \boxed{E(x) = \sum_{i=1}^{n} x_i\, p(x_i)}$$

i.e

$$E(x) = x_i p_i + x_2 p_2 + x_3 p_3 + \dots + x_n p_n$$

- variance :-

$$\rightarrow \boxed{var(x) = E(X^2) - [E(x)]^2}$$

- Standard deviation :-

$$\rightarrow \boxed{\sqrt{var(x)}}$$

Teacher's Sign. :

EXPERIMENT : No. Page No Date

Unit 5:-

Discrete and Continues Probability Distribution

- Discrete probability distribution :-

  - Binomial distribution :-

→ 'x' success out of 'n' trials

$$P(X = x) = \binom{n}{x} p^x q^{n-x}$$

→ Mean:

$$E(X) = np$$

→ Variance :

$$var(X) = npq$$

→ Failure :

$$q = 1 - p$$

→ Standard Deviation:

$$\sigma = \sqrt{npq}$$

- **Poisson Distribution:-**

→ PMF of PD is:

$$P(X=r) = \frac{e^{-\lambda}\lambda^{r}}{r!}$$

→ Mean = λ, variance = λ

2) Continuous Probability Distribution:-

- **Normal distribution:-**

→ • z-score is

$$z = \frac{x-\mu}{\sigma}$$

where x = score
μ = mean
σ = standard deviation

- **Uniform distribution:-**

→ • Mean:- $\mu = \frac{a+b}{2}$

→ • variance:- $\sigma^2 = \frac{(b-a)^2}{12}$

→ • Standard Deviation:- $\sigma = \sqrt{\sigma^2}$