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Data Representation & Arithmetic Algorithms.

Topics:

1. Unsigned Binary Arithmetic
2. Sign-Magnitude Numbers
3. Binary Subtraction Using 1's & 2's complement
4. Addition, Subtraction
5. Unsigned Multiplication
6. Signed Multiplication (Booth's Algorithm)
7. Restoring Division
8. Non-Restoring Division.

Unsigned Binary Arithmetic

In some applications, all the data is either positive or negative.

Then we can just forget about the + and − signs, and concentrate only on the magnitude of the data.

For example, the smallest 8 bit binary number is 00000000, i.e all zeros, and the largest 8 bit binary number is 11111111.

Hence the complete range of unsigned 8 bit binary numbers extends from

$(00)_H$ to $(FF)_H$ or from

$(00)_{10}$ to $(255)_{10}$

All the numbers being added or subtracted must be in the range 0 to 255.

The answer obtained should also be in the same range i.e 0 to 255.

For magnitudes greater than 255, we have to use 16-bit arithmetic

Overflow ??

If the addition or multiplication of two 8-bit numbers results in generation of a number, greater than $(255)_{10}$, then it is said that overflow has taken place.

Binary Arithmetic

Binary Addition:

	A		B	Addition
Case 1	0	+	0	0
Case 2	0	+	1	1
Case 3	1	+	0	1
Case 4	1	+	1	10

10 → Binary 2.

{0 with a carry 1}

For example, Add 011 & 101

$$
\begin{array}{cccccc}
 & 1 & 1 & 1 & & \\
A & 0 & 1 & 1 & = & 3 \\
B & 1 & 0 & 1 & = & 5 \\
\hline
1 & 0 & 0 & 0 & = & 8
\end{array}
$$

Binary Subtraction

	A		B	Subtraction	Borrow
Case 1	0	−	0	0	0
Case 2	1	−	0	1	0
Case 3	1	−	1	0	0
Case 4	0	−	1	1	1

For example,

$A = (11011)_2$

$B = (10110)_2$, obtain $A-B$

$$
\begin{array}{r}
11011 \\
10110 \\
\hline
00101
\end{array}
\qquad
\begin{array}{l}
= 27 \\
= 22 \\
= 5
\end{array}
$$

Practice

Convert $(38)_{10}$ & $(29)_{10}$ to binary & perform binary subtraction.

Unsigned Binary Multiplication

The procedure used for binary multiplication is exactly same as that for the decimal multiplication.

In fact binary multiplication is simpler than decimal multiplication because only 0's and 1's are involved.

$$0 \times 0 = 0$$
$$0 \times 1 = 0$$
$$1 \times 0 = 0$$
$$1 \times 1 = 1$$

Example: Perform 101.11 × 111.01

Sol.

```
A         10111
B         11101
     ______________
          10111
        000000x
       10111xx
      10111xxx
     10111xxxx
   ______________
     1010011011
```

The binary point is placed after 4 positions from LSB

1010 01.1011

Practice

Perform $(11001)_2 \times (101)_2$

Unsigned Binary Division

The division of binary numbers take place in a similar way as that of decimal numbers.

It is called as the long Division method.

Q. Perform $(11110)_2 \div (110)_2$

```
110 ) 11110 ( 101 → Quotient
      110
      -----
       110
      -110
      -----
         0 → Remainder
```

$\therefore (11110)_2 \div (110)_2 = (101)_2$

Practice

1. Perform the following operation without converting to any other base.

$$(10101011)_2 \div (101)_2$$

2. Perform $(11010)_2 \div (101)_2$

3. Perform $(11001)_2 \div (101)_2$

Sign-Magnitude Numbers

If the data has positive as well as negative numbers, then the signed binary numbers should be used for their representation.

The + sign is represented by a 0.

The − sign is represented by a 1.

The MSB of a binary number is used to represent the sign and the remaining bits are used for representing the magnitude.

Complements :

Complements are used in the digital computers in order to simplify the subtraction operation and for logical manipulations.

Types of Complements :

For each radix - r system, there are two types of complements :

1. The radix complement.
2. The diminished radix complement.

The radix complement is referred to as the r's complement & the diminished radix complement is referred to as (r-1)'s complement

1's Complement

The 1's and 2's complement of a binary number are important because we can use them for representation of negative numbers.

The 1's complement of a number is found by inverting all the bits in that number.

eg. 1010 $\Rightarrow$ 0101

2's Complement

The 2's complement of a binary number is obtained by adding 1 to the LSB of 1's complement of that number.

The method of 2's complement arithmetic is commonly used in computers to handle negative numbers.

☆ <u>Obtain 2's complement of $(10110010)_2$</u>

Given Number	:	10110010
1's Complement	:	01001101
Add 1 to 1's Complement	:	+ 1
		01001110

Hence, the 2's complement of $(10110010)_2$ is $(01001110)_2$

Practice

Q. Find the 1's & 2's complement of $(57)_{10}$

Binary Subtraction Using 1's & 2's complements.

The direct binary subtraction becomes complicated as the number size increases.

Therefore we can represent the subtraction of $\underline{A - B}$ in the form of addition as :- $A + (-B)$

We can represent number B in its 1's or 2's complement form and use addition instead of subtraction to get the result.

Subtraction using 1's Complement

Step 1: Convert the number to be subtracted $(B)_2$ to its 1's complement.

Step 2: Add $(A)_2$ and 1's complement of $(B)_2$ using the rules of binary addition.

Step 3: If final carry is 1, then add it to the result of addition obtained in step 2 to get the final result of $(A)_2 - (B)_2$.

Note, if the final carry is 1, then the subtraction is positive and in its true form.

Step 4: If the final carry produced in step 2 is 0, then the result obtained in step 2 is negative, and in the 1's complement form. So convert it into the true form by complementing all the bits.

Q.1. Subtract $(32)_{10}$ from $(85)_{10}$ using 1's complement binary arithmetic.

Sol. Convert both the numbers to binary:

$(85)_2 = 1010101$

$(32)_2 = 0100000$

Step 1 Obtain 1's complement of $(32)_2$

$\Rightarrow 1011111$

Step 2 Add $(85)_2$ and 1's complement of $(32)_2$

$$
\begin{array}{r}
1010101 \\
+\ 1011111 \\
\hline
\boxed{1}\,0110100
\end{array}
$$

Final Carry

Step 3: Add the final carry to the result obtained in step 2.

$$
\begin{array}{r}
0110100 \\
+ \quad\quad\quad 1 \\
\hline
0110101 \\
\hline
\end{array}
$$

Answer in True Form

i.e $(53)_{10}$

Q.2. Perform the following

$(1011001)_2 - (1101010)_2$

Q.3. $(1111)_2 - (0110)_2$

Binary Subtraction Using 2's Complement

Steps:

1. Add $(A)_2$ to the 2's complement of $(B)_2$.

2. If the carry is generated, then the result is positive & in its true form.

3. If the carry is not produced, then the result is negative & in it's 2's complement form.

Carry always needs to be discarded in the subtraction using 2's complement.

Q.1. Perform $(9)_{10} - (5)_{10}$ using 2's complement method.

<u>Step 1.</u> Obtain 2's complement of $(5)_{10}$.

$(5)_{10} \rightarrow 0101 \rightarrow 1010 \rightarrow 1011$

1's Complement

2's Complement

<u>Step 2:</u> Add $(9)_{10}$ to the 2's complement of $(5)_{10}$

$$
\begin{array}{r}
1001 \\
1011 \\
\hline
\cancel{\boxed{1}}\,0100
\end{array}
$$

Discard Carry

Final Carry indicates that the answer is positive & in it's true form.

Q.2 Perform subtraction using 2's Complement for given numbers

$$(1101)_2 - (1001)_2$$

Step 1 2's complement of B

$1001 \longrightarrow 0110 \longrightarrow 0111$

Step 2: Addition :

$$\begin{array}{r} 1101 \\ 0111 \\ \hline \boxed{1}\,\underbrace{0100}_{\text{Answer}} \end{array}$$

Final Carry

Answer

As final carry is 1, the answer is positive and in it's true form.

$\therefore (1101)_2 - (1001)_2 = (0100)_2$

Practice ①

Perform binary subtraction using 2's complement for $(62)_{10}$ & $(99)_{10}$

Practice ②

Subtract using 1's & 2's complement method $(73)_{10} - (49)_{10}$

Practice ③

Subtract using 1's & 2's complement method

$(15)_{10} - (21)_{10}$

Practice 4

a) $(11)_{10} - (22)_{10}$ using 2's complement

b) $(33)_{10} - (44)_{10}$ using 1's complement

c) $(56)_{10} - (76)_{10}$ using both methods

d) $(52)_{10} - (65)_{10}$ using 2's complement

e) $(10)_{10} - (7)_{10}$ using 2's complement