ASSIGNMENT - 1

Q1. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

$$A^{-1} = \frac{1}{-2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

$$(AB)A^{-1} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$$

$$= \begin{bmatrix} -38+33 & 19-11 \\ -86+75 & 43-25 \end{bmatrix} = \begin{bmatrix} -5 & 8 \\ -11 & 18 \end{bmatrix}$$

$$Tr(ABA^{-1}) = Tr(B)$$

$$\Rightarrow Tr\begin{bmatrix} -5 & 8 \\ -11 & 18 \end{bmatrix} = Tr\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$\Rightarrow -5+18 = 5+8$$

$$\Rightarrow 13 = 13$$

Hence proved

Q2. $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$; $A^2 = 0$ $\qquad B = \begin{bmatrix} 0 & 2 & 1 & 6 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$; $B^4 = 0$

$C = \begin{bmatrix} 5 & -3 & 2 \\ 15 & -9 & 6 \\ 10 & -6 & 4 \end{bmatrix}$; $C^2 = 0$ $\qquad D = \begin{bmatrix} 5 & 5 & 5 \\ 6 & 6 & 6 \\ -11 & -11 & -11 \end{bmatrix}$; $D^2 = 0$

Q3. (a) $u = [2, -5, 6]$ and $v = [2, 8, -1]$

$u \cdot v = 2\times2 - 5\times8 - 6\times1 = 4 - 40 - 6 = -42$

(b) $u = [6, 6, -9, 6, -4]$ and $v = [-3, 5, -1, 0, 5]$

$u v = -6\times3 + 6\times5 + 9\times1 + 6\times0 - 4\times5$

$= -18 + 30 + 9 + 0 - 20 = 1$

Q4 $u = [3, 1, 6]$ and $v = [-2, 1, 4]$

$|u| = \sqrt{3^2+1^2+6^2} = \sqrt{46}$

$|v| = \sqrt{(-2)^2+1^2+4^2} = \sqrt{21}$

$u \cdot v = -3\times2 + 1\times1 + 6\times4 = -6+1+24 = 19$

$\cos\theta = \frac{u\cdot v}{|u||v|} = \frac{19}{\sqrt{46\times21}} \approx \frac{19}{31}$

(a) $\theta = \cos^{-1}\left(\frac{19}{31}\right) = 0.911$ rad

(b) $proj(u,v) = \frac{u\cdot v}{|v|^2} \cdot v = \frac{19}{(\sqrt{21})^2}[-2,1,4] = \left[\frac{-38}{21}, \frac{19}{21}, \frac{76}{21}\right]$

Q5. (a) $u = \vec{PQ} = [6-1, 1-(-2), -5-4] = [5, 3, -9]$

(b) $u = \vec{PQ} = [1-2, 5-3, -3-(-6), 6-5] = [-1, 2, 3, 1]$

Q6 $u = 3i - 4j + 2k$, $v = 2i + 5j - 3k$, $w = 4i + 7j + 2k$

(a)
$$u\times v = \begin{vmatrix} i & j & k \\ 3 & -4 & 2 \\ 2 & 5 & -3 \end{vmatrix} = i(12-10) - j(-9-4) + k(15+8) = 2i + 13j + 23k$$

$$u\times w = \begin{vmatrix} i & j & k \\ 3 & -4 & 2 \\ 4 & 7 & 2 \end{vmatrix} = i(-8-14) - j(6-8) + k(21+16) = -22i + 2j + 37k$$

$$v\times w = \begin{vmatrix} i & j & k \\ 2 & 5 & -3 \\ 4 & 7 & 2 \end{vmatrix} = i(10+21) - j(4+12) + k(14-20) = 31i - 16j - 6k$$

b) Volume $= u \cdot (v \times w)$

$$v \times w = \begin{vmatrix} i & j & k \\ 2 & 5 & -3 \\ 4 & 7 & 2 \end{vmatrix} = 31i - 16j - 6k$$

$u = 3i - 4j + 2k$

$u \cdot (v \times w) = 93 + 64 - 12 = 145$ cube units

Q7

$$\begin{aligned} x + 2y + 3z &= 1 \\ -x \quad + 2z &= 2 \\ -2y + z &= -2 \end{aligned} \Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$$

$D = 1(4) - 2(-1) + 3(2) = 12$

$$D_x = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 0 & 2 \\ -2 & -2 & 1 \end{vmatrix} = 1(4) - 2(2+4) + 3(-4) = -20$$

$$D_y = \begin{vmatrix} 1 & 1 & 3 \\ -1 & 2 & 2 \\ 0 & -2 & 1 \end{vmatrix} = 1(2+4) - 1(-1) + 3(2) = 13$$

$$D_z = \begin{vmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 0 & -2 & -2 \end{vmatrix} = 1(4) - 2(2) + 1(2) = 2$$

$x = \frac{D_x}{D} = \frac{-20}{12} = \frac{-5}{3}$; $y = \frac{D_y}{D} = \frac{13}{12}$; $z = \frac{D_z}{D} = \frac{2}{12} = \frac{1}{6}$

Q8

$$D(A) = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} = 1\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & 2 \\ 2 & 1 \end{vmatrix} + 2\begin{vmatrix} 2 & 1 \\ 2 & 2 \end{vmatrix}$$

$= 1(1-4) - 2(2-4) + 2(4-2)$

$= 1(-3) - 2(-2) + 2(2)$

$= -3 + 4 + 4$

$= 5$

Q9

(a) $a\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} + b\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix} + c\begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 12 \end{bmatrix}$

(b) Augmented matrix $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ -2 & 0 & 2 & -2 \\ 3 & 5 & 4 & 12 \end{array}\right]$

Applying these operations-

$R_2 \to \frac{R_2}{2} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ -1 & 0 & 1 & -1 \\ 3 & 5 & 4 & 12 \end{array}\right]$ $R_2 \to R_1 + R_2 \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 1 \\ 3 & 5 & 4 & 12 \end{array}\right]$

$R_3 \to R_3 - 3R_1 \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 2 & 4 & 6 \end{array}\right]$ $R_3 \to \frac{R_3}{2} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{array}\right]$

$R_3 \to R_3 - R_1 \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$ $R_2 \to R_2 - R_3 \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right]$

$R_1 \to R_1 - R_2 \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right]$ Now in echelon form $\Rightarrow$ One unique solution where $a = 3, b = -1, c = 2$

$\Rightarrow 3v_1 - v_2 + 2v_3 = b$

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Q10. $A = \begin{bmatrix} 0 & 3 & 5 \\ 3 & 5 & 0 \\ 5 & 0 & 10 \end{bmatrix}$ $|A| = -3(30) + 5(-25)$

$= -90 - 125$

$= -215 \neq 0$

$\Rightarrow \text{Rank}(A) = 3$

$B = \begin{bmatrix} 6 & -4 & 0 \\ -4 & 0 & 2 \\ 0 & 2 & 6 \end{bmatrix}$ $|B| = 6(-4) + 4(-24)$

$= -24 - 96$

$= -120 \neq 0$

$\Rightarrow \text{Rank}(B) = 3$