Quantitative Aptitude

Absolute Value

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Instructor:

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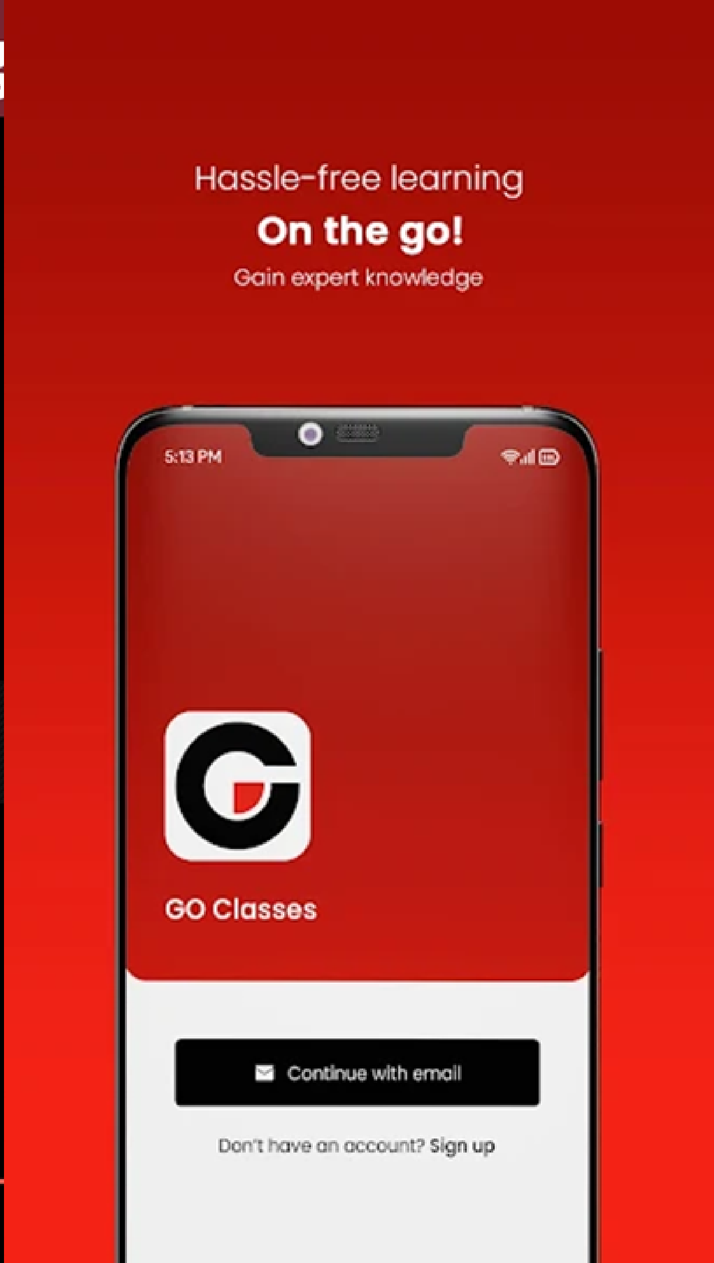

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Absolute Value

Other Names:

Absolute Value

Modulus

Magnitude

Numerical Value

Same

Absolute Value Definition 1:

(Simple Definition)

Absolute Value of a Real Number:

Absolute Value of a Real Number x, denoted by $|x|$, is the non-negative value of x without regard to its sign.

$x = -4$

Absolute value of $x = |x| = 4$

Modulus of $x = |x| = 4$

Magnitude of $x = |x| = 4$

Numerical value of $x = |x| = 4$

$x = 4$

Absolute value of $x = |x| = 4$

Modulus of x

Magnitude of x

Numerical value of x

$|-2.5|$ = Absolute Value of $-2.5 = 2.5$

$|\pi| = \pi$

$|-3| = 3$

$|-\pi| = \pi$

$|3| = 3$

$|0| = 0$

Q:
True/False??

Absolute Value of any real number x is always positive.

Q:

True/~~False~~??

$|0| = 0$

Absolute Value of any real number x is always positive.

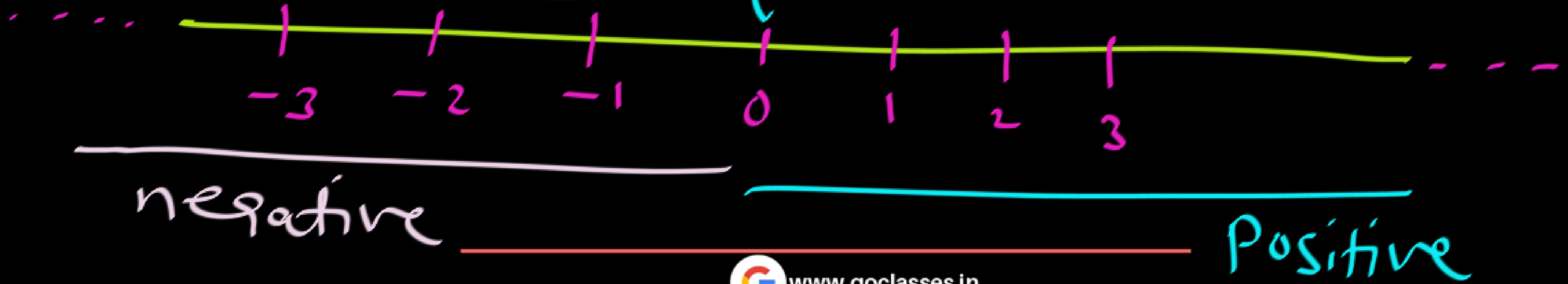
false

Counter: $x = 0$

0 : Neither positive Nor Negation

Real line :

0 : Neither Positive Nor Negative



Positive Real numbers ; $x > 0$

Negative Real no. : $x < 0$

0.1 ✓

0.0001 ✓

0.15 ✓

1 ✓

2 ✓

2.3 ✓

Q:
True/False??

Absolute Value of any non-zero real number x is always positive.

Q:
True/False??

$|0| = 0$

Absolute Value of any non-zero real number x is always positive.

if $x \neq 0$ $|x| > 0$

If $x \neq 0$

$x = 2 \quad |x| = 2 > 0$

$= -2 \quad |x| = 2 > 0$

$= 2.5 \quad |x| = 2.5$

$= -2.5 \quad |x| = 2.5$

Q:
True/False??

Absolute Value of any negative real number x is always positive.

Q:

True/False??

Absolute Value of any negative real number x is always positive.

Absolute Value Definition 2:

(As a piecewise function)

x : Real no.

If $x < 0$ then $|x| = -x > 0$

$\boxed{x = -2.5}$ $|x| = -x = -(-2.5) = 2.5$

$x = -10$ $|x| = -x = -(-10) = 10$

If $x \geq 0$ then $|x| = x$

$x = 10$ then $|x| = 10 = x$

x : Real no.

$$|x| = \left\{ \begin{array}{ll} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{array} \right\}$$

Piecewise function

Real numbers [edit]

For any real number x, the **absolute value** or **modulus** of x is denoted by $|x|$, with a vertical bar on each side of the quantity, and is defined as[8]

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0. \end{cases}$$

The absolute value of x is thus always either a positive number or zero, but never negative. When x itself is negative ($x < 0$), then its absolute value is necessarily positive ($|x| = -x > 0$).

Absolute Value Definition 3:

(Geometric Interpretation)

Real line:

$\boxed{\text{Distance} \geq 0}$

Distance b/w 0 & -3 : 3 = |-3|

" " 0 & 3 : 3 = |3|

$|x|$ = Distance of x from 0.

$|-10.2|$ = Absolute Value of -10.2

= Distance of -10.2 from 0

= 10.2

Absolute Value

The distance between a number x and 0 on the number line is called the **absolute value** of x, written as $|x|$. Therefore, $|3| = 3$ and $|-3| = 3$ because each of the numbers 3 and −3 is a distance of 3 from 0. Note that if x is positive, then $|x| = x$; if x is negative, then $|x| = -x$; and lastly, $|0| = 0$. It follows that the absolute value of any nonzero number is positive. Here are three examples.

Example 1.5.1: $|\sqrt{5}| = \sqrt{5}$

Example 1.5.2: $|-23| = -(-23) = 23$

Example 1.5.3: $|-10.2| = 10.2$

Absolute Value:

In mathematics, the **absolute value** or **modulus** of a real number x, denoted $|x|$, is the non-negative value of x without regard to its sign. Namely, $|x| = x$ if x is a positive number, and $|x| = -x$ if x is negative (in which case negating x makes $-x$ positive), and $|0| = 0$. For example, the absolute value of 3 is 3, and the absolute value of −3 is also 3. The absolute value of a number may be thought of as its distance from zero.

Absolute Value:

Standard Properties

Basic Properties

a : Real number

1. $|a| >= 0$
2. $|a| = 0$ Iff $a = 0$.
3. $||a|| = |a|$
4. $|-a| = |a|$

a : Real no.

① $|a| \geq 0$

$|a|$: Always Non-Neg.

② $a = 0 \longrightarrow |a| = 0$

$|a| = 0 \longrightarrow a = 0$

$a = 0$ iff $|a| = 0$

iff <u>means</u> if and only if

$\boxed{P \text{ iff } Q}$ <u>means</u> $P \longleftrightarrow Q$

$a = 0 \longleftrightarrow |a| = 0$

$\boxed{|a| = 5 \text{ iff } a = 5}$

True / ~~false~~ ?

~~false~~

$a = 5 \longrightarrow |a| = 5$

$a = 5 \;\not\longleftarrow\; |a| = 5$

$|a| = 5 \longrightarrow a = 5 \text{ or } -5$

3

$$||a|| = |a|$$

$a = -4$ $\quad ||a|| = |4| = 4 = |a|$

$$||a|| = |a| \checkmark$$

(4) for any Real a

$\boxed{0 = -0}$

$|a| = |-a|$ ✓

$|4| = |-4|$

$|0| = |-0| = |0| = 0$

0

Basic Properties:

For any real number x,

$|x| = \max\{x, -x\}$ and $\pm x \leq |x|$

From your definition of the absolute value, establish first $|x| = \max\{x, -x\}$ and $\pm x \leq |x|$.

Real number x

$-0 = 0$

1. $\boxed{|x| = \max\{x, -x\}}$

$|4| = \max\{4, -4\} = 4$

$|-2| = \max\{-2, 2\} = 2$

$|0| = \max\{0, -0\} = 0$

2 $\boxed{x \leq |x|}$

$x = -4.4$ $\quad -4.4 \leq 4.4$ ✓

$x = 10.2$ $\quad 10.2 \leq 10.2$ ✓

$x = 0$ $\quad 0 \leq 0$ ✓

$x \leq |x|$; $-x \leq |x|$

$x = 4$	$4 \leq 4$ ✓	$-4 \leq 4$ ✓
$x = 0$	$0 \leq 0$ ✓	$0 \leq 0$ ✓
$x = -2$	$-2 \leq 2$ ✓	$2 \leq 2$ ✓

Useful:

1. $a \le |a|$; $-a \le |a|$

2. $|x| \ge 0$

Basic Properties:

If $|x| = t$ then $x = t$ OR $x = -t$.

$$|t| = 5 \longrightarrow t = ?$$

$$t = 5 \text{ or } -5$$

$$|x| = t \longrightarrow x = t \text{ or } -t$$

$$t \geq 0$$

$|x| = 6 \longrightarrow x = 6$ OR -6

$|x| = 0 \longrightarrow x = 0$

$|x| = -6$

Nonsense

Never happen

Guideline:

Generally, the best way to solve questions involving $|x|$ is to create two cases:

Case 1: If $x >= 0$

Case 2: If $x < 0$

$|x|$

$$\begin{cases} \underline{\text{Case 1:}} \text{ if } x \geq 0 \text{ then } ---- \\ \underline{\text{Case 2:}} \text{ if } x < 0 \text{ then } ---- \end{cases}$$

$$|x| = x \quad \text{if } x \geq 0$$

$$|x| = -x \quad \text{if } x < 0$$

Q :

$2 \leq |x| \leq 3$ then $x = ?$

Q:

$2 \le |x| \le 3$ then $x = ?$

method 1:

Case 1

If $x \ge 0$ then

$|x| = x$

$\boxed{2 \le x \le 3}$

Case 2

if $x < 0$ then

$|x| = -x$

$2 \le -x \le 3$

Case 1:

if $x \geq 0$

$$\boxed{2 \leq x \leq 3}$$

OR

Case 2:

if $x < 0$

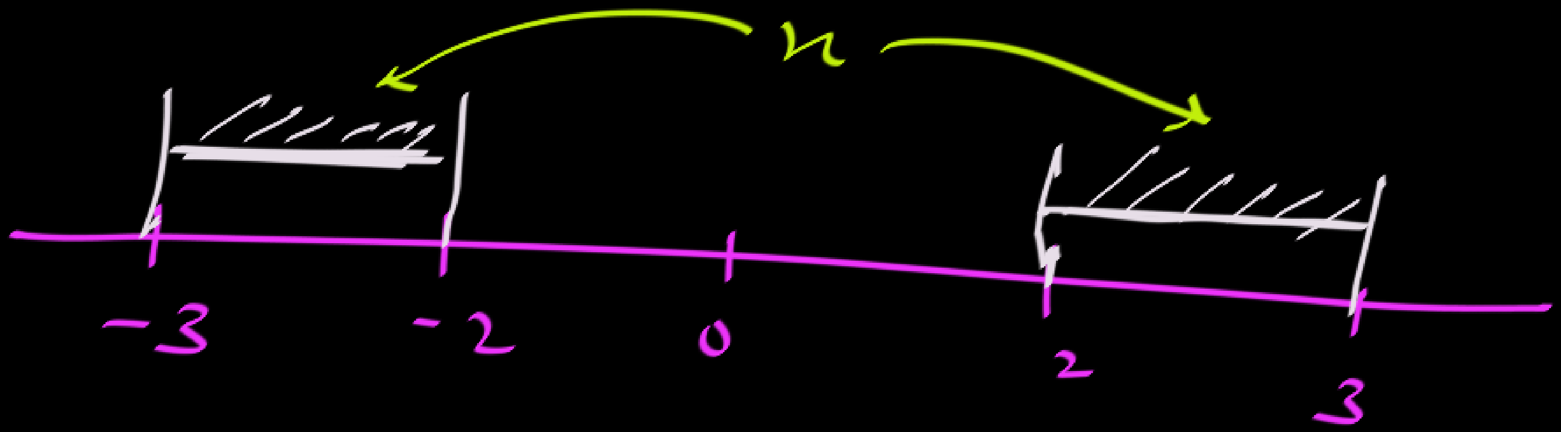
$$2 \leq -x \leq 3$$

$$-2 \geq x \geq -3$$

$$\boxed{-3 \leq x \leq -2}$$

final Ans:

$\boxed{2 \leq x \leq 3}$ or $\boxed{-3 \leq x \leq -2}$



Note:

inequality:

if $x < a$	if $x > a$
$-x > -a$	$-x < -a$
Eg: $5 < 7$	Eg: $10 > 5$
$-5 > -7$	$-10 < -5$

Note:

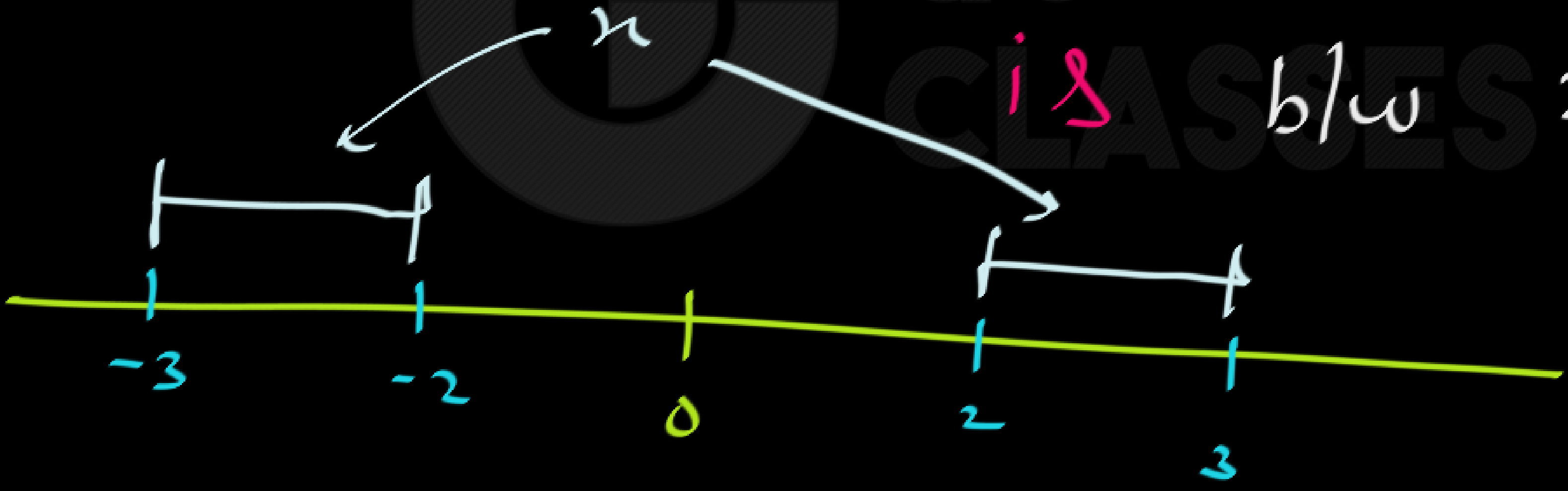
if $x \geq a$	if $x \leq a$
$-x \leq -a$	$-x \geq -a$

Eg: $5 \geq 5$

$-5 \leq -5$

Q: $\boxed{2 \le |x| \le 3}$ then $x = ?$

Method 2: Distance of x from 0 is b/w 2 & 3.



Q :

$|x| \leq 5$ then $x = ?$

Q:

$|x| \leq 5$ then $x = ?$

method 1:

Case 1:

if $x \geq 0$

$|x| = x$

$x \leq 5$

Case 2:

if $x < 0$

$|x| = -x$

$-x \leq 5$; $x \geq -5$

Q:

$|x| \leq 5$ then $x = ?$

Method 1: Case 1:

$\boxed{\text{if } x \geq 0}$

$|x| = x$

$x \leq 5$ → $x = -10$ possible??

Q:

$|x| \leq 5$ then $x = ?$

$x = 2$ possible??

No

why?? $\Rightarrow$ Case 2: $x < 0$

Case 2:

$\boxed{\text{if } x < 0}$

$|x| = -x$

$-x \leq 5$; $x \geq -5$

Case 1: $x \geq 0$

$x \leq 5$

Case 2: $x < 0$

$x \geq -5$

Case 1:

$0 \leq x \leq 5$

Case 2:

$-5 \leq x < 0$

Case 1:

$0 \leq x \leq 5$

Case 2:

$-5 \leq x < 0$

$$\boxed{-5 \leq x \leq 5}$$ ✓

Q:

$|x| \le 5$ then $x = ?$

$x \le 5$ OR $-x \le 5$

$x \ge -5$

$\boxed{-5 \le x \le 5}$

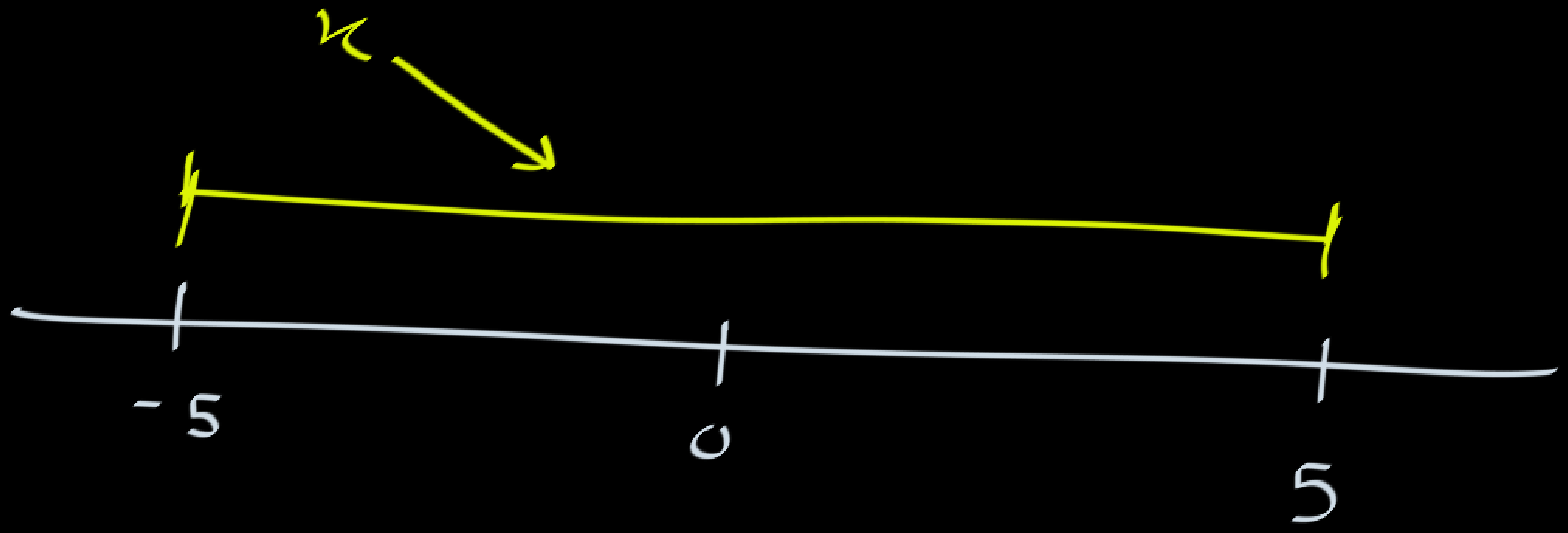
Q:

$|x| \le 5$ then $x = ?$

Method 2 : Geometric interpretation :

$\boxed{|x| \le 5}$ means Distance of x from 0 is ≤ 5.

x lives within 5 km from 0's House.



x
-5
0
5

Q:
Prove that

Two other useful properties concerning inequalities are:

$$|a| \leq b \iff -b \leq a \leq b$$

$$|a| \geq b \iff a \leq -b \text{ or } a \geq b$$

Q:
Prove that

Two other useful properties concerning inequalities are:

$$|a| \leq b \iff -b \leq a \leq b$$
$$|a| \geq b \iff a \leq -b \text{ or } a \geq b$$

using Geometric interpretation

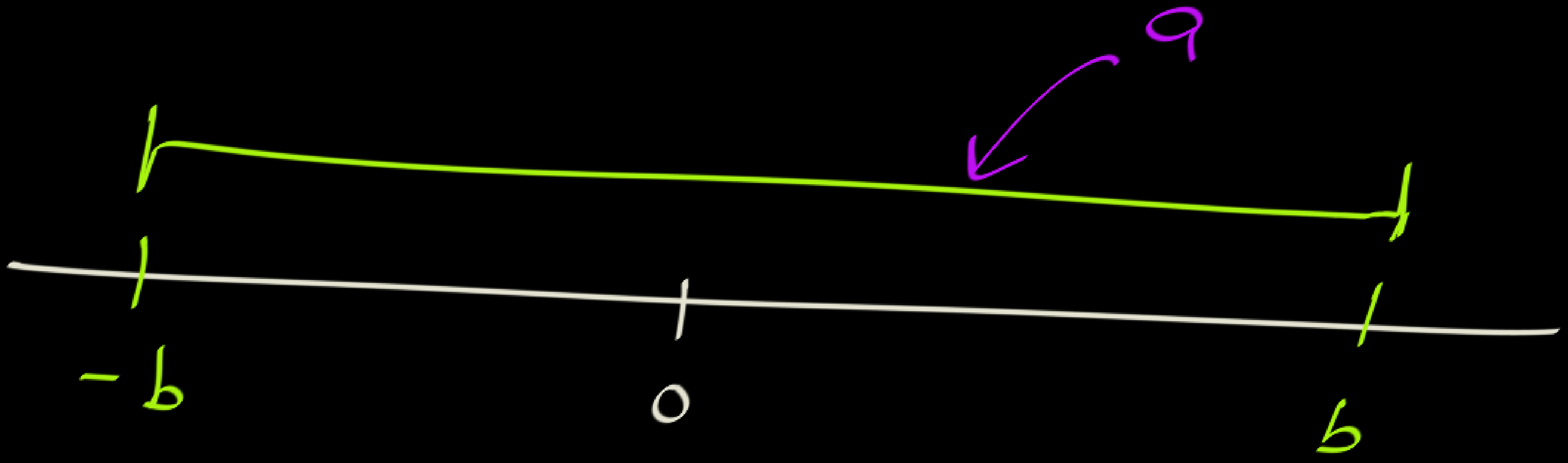
if $|a| \leq b$ then a ?

Geometric interpretation:

$|a| \leq b$

Distance of a from 0 $\leq b$

a lives within b kms from 0's House.



$$\boxed{-b \le a \le b}$$

if $|a| \le b$

method 2 :

Case 1 :

if $a \ge 0$

then $|a| = a$

$a \le b$

Case 2 :

if $a < 0$

$|a| = -a$

$-a \le b$; $a \ge -b$

if $|a| \le b$

Case 1:

$\boxed{0 \le a \le b}$

Case 2

$\boxed{-b \le a < 0}$

$\boxed{-b \le a \le b}$

Conclusion:

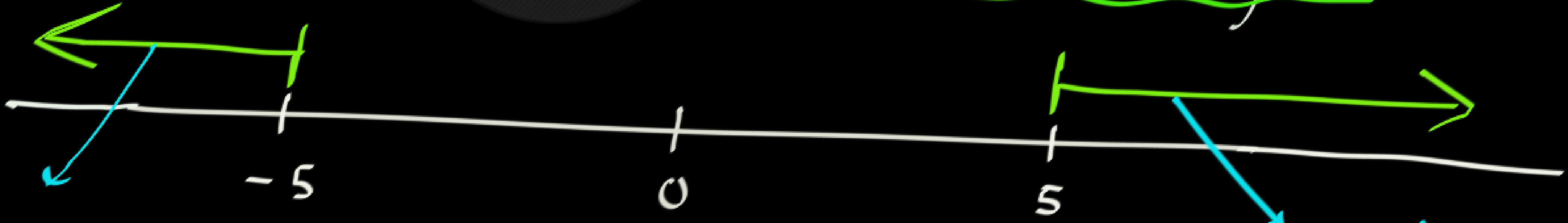
if $|x| \le t$ then $\boxed{-t \le x \le t}$

Q: $|x| \geq 5$ then $x = ?$

Q: $|x| \geq 5$ then $x = ?$

Distance of x from $0 \geq 5$

x lives at least 5 km away from 0.



$x \leq -5$ $x \geq 5$

So ;

$x \geq 5$ OR $x \leq -5$

if $|x| \geq 5$ then $x = ?$

Method 2:

Case 1:

if $x \geq 0$

$|x| = x$

$x \geq 5$

Case 2

if $x < 0$

$|x| = -x$

$-x \geq 5$; $x \leq -5$

$|x| \geq 5$

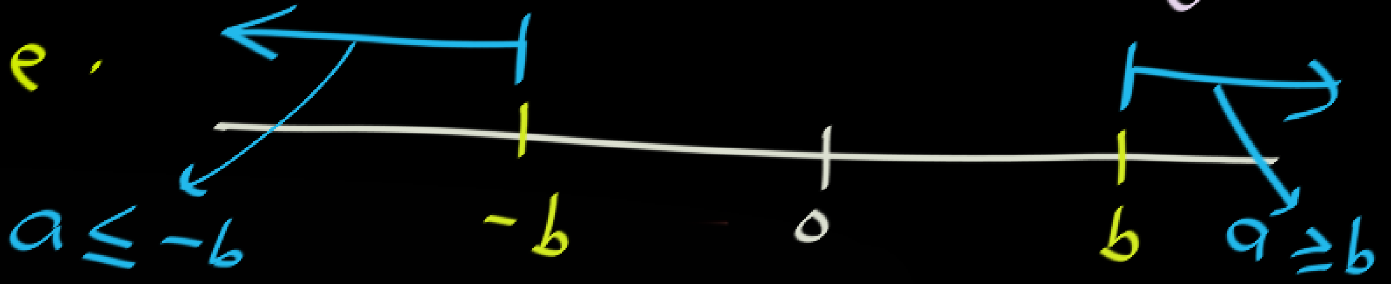
$x \geq 5$ OR $x \leq -5$

Q: $|a| \geq b$ then a?

Q: $|a| \geq b$ then a?

Distance of a from 0 $\geq b$

So a lives at least b km away from 0's House.



So;

$a \geq b$ OR $a \leq -b$

Conclusion:

① $|a| \le b$ then $-b \le a \le b$

② $|a| \ge b$ then $a \ge b$ OR $a \le -b$

Conclusion:

(1) $|a| < b$ then $-b < a < b$

(2) $|a| > b$ then $a > b$ OR $a < -b$

Conclusion:

(1) $n \le |a| \le m$ then

Conclusion:

① $n \le |a| \le m$ then

$n \le a \le m$ or

$-m \le a \le -n$

Q:

Given that: $|x - 3| <= 9$

What is x ??

Q:

$-a \leq m$

inequality

equality

Equality

inequality

=

<

>

$t = ml$

$-t = -ml$

equality

$a < b$

$-a > -b$

inequality

$-a \leq m$

$a \geq -m$

$5 \leq 5$ ✓

$-5 \geq -5$ ✓

Absolute Value:

GATE Questions

9 General Aptitude: Quantitative Aptitude (376)

9.1 Absolute Value (6) top

9.1.1 Absolute Value: GATE CSE 2014 Set 2 | Question: GA-8 top

If x is real and $|x^2 - 2x + 3| = 11$, then possible values of $|-x^3 + x^2 - x|$ include

A. $2, 4$ B. $2, 14$ C. $4, 52$ D. $14, 52$

gatecse-2014-set2 quantitative-aptitude normal absolute-value

9 General Aptitude: Quantitative Aptitude (376)

9.1 Absolute Value (6) top

9.1.1 Absolute Value: GATE CSE 2014 Set 2 | Question: GA-8 top

If x is real and $\boxed{|x^2 - 2x + 3| = 11}$, then possible values of $|-x^3 + x^2 - x|$ include

A. $2, 4$ B. $2, 14$ C. $4, 52$ D. $14, 52$

gatecse-2014-set2 quantitative-aptitude normal absolute-value

two Cases

Case 1 :

$x = \boxed{4, -2}$

Case 2 : No Real x

9 General Aptitude: Quantitative Aptitude (376)

9.1 Absolute Value (6) top

9.1.1 Absolute Value: GATE CSE 2014 Set 2 | Question: GA-8 top

If x is real and $|x^2 - 2x + 3| = 11$, then possible values of $|-x^3 + x^2 - x|$ include

A. $2, 4$ B. $2, 14$ C. $4, 52$ D. $14, 52$

gatecse-2014-set2 quantitative-aptitude normal absolute-value

$x = -2 \longrightarrow |+8+4+2| = 14$

$x = 4, -2$

$x = 4 \longrightarrow |-64+16-4| = 52$

$|y| = t$ then $y = t$ $\underline{\underline{\text{OR}}}$ $-t$

$$|x^2 - 2x + 3| = 11$$

x : Real.

then

$$\left.\begin{array}{l}\boxed{x^2 - 2x + 3 = 11} \\ \text{OR} \\ \boxed{x^2 - 2x + 3 = -11}\end{array}\right\}$$

$$x^2 - 2x + 3 = 11$$

x : Real.

$$ax^2 + bx + c = 0$$

$$x^2 - 2x - 8 = 0$$

$a = 1$; $b = -2$; $c = -8$

$$x = \frac{2 \pm \sqrt{4 + 32}}{2}$$

$$\frac{2+6}{2} = 4$$

$$\frac{2-6}{2} = -2$$

$$\boxed{x^2 - 2x + 3 = 11}$$

$$\boxed{x = 4, -2}$$

x : Real

Quadratic Equation :

$$\boxed{ax^2 + bx + c = 0}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\boxed{x^2 - 2x + 3 = -11}$$

x : Real.

$$\Rightarrow x^2 - 2x + 14 = 0$$

$$x = \frac{2 \pm \sqrt{4 - 14 \times 4}}{2}$$

$\sqrt{-\text{value}}$

NOT Real

NOT Real

$$|x^2 - 2x + 3| = 11$$

Given

x : Real.

$$\boxed{x^2 - 2x + 3 = 11}$$

$x = 4, -2$

Real

$$x^2 - 2x + 3 = -11$$

$x =$ NoT Real

$$\left|\underbrace{x^2 - 2x + 3}_{m}\right| = 11$$

$|m| = 11$

$m = -11$ → No Real x

$m = 11$ → $x = 4, -2$

9.1.2 Absolute Value: GATE CSE 2017 Set 1 | Question: GA-8 top

The expression $\frac{(x+y)-|x-y|}{2}$ is equal to :

A. The maximum of x and y
B. The minimum of x and y
C. 1
D. None of the above

gatecse-2017-set1 general-aptitude quantitative-aptitude maxima-minima absolute-value

9.1.2 Absolute Value: GATE CSE 2017 Set 1 | Question: GA-8 top

The expression $\frac{(x+y)-|x-y|}{2}$ is equal to :

A. The maximum of x and y
B. The minimum of x and y
C. 1
D. None of the above

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Method 1: During those 3 Hours: $\frac{(x+y) - |x-y|}{2}$

$x = 2$, $y = 3$

$\frac{5 - |-1|}{2}$

$= 2$

$x = 3$, $y = 2$

$\frac{5 - |1|}{2}$

$= 2$

$x = -2$, $y = -2$

$\frac{-4 - 0}{2} = -2$

$$\boxed{\begin{array}{ll} |m| = m & \text{if } m \geq 0 \\ |m| = -m & \text{if } m < 0 \end{array}}$$

method:

$$E = \frac{x + y - |\overset{m}{\boxed{x-y}}|}{2}$$

Case 1: $\boxed{x - y \geq 0}$ $\rightarrow x \geq y$

$$|x-y| = x - y$$

$$E = \frac{x+y-(x-y)}{2} = y$$

$= \min\{x,y\}$

Case 2: $\boxed{x - y < 0}$ $\rightarrow x < y$

$$|x-y| = -(x-y) = y - x$$

$$E = \frac{x+y-(y-x)}{2} = x$$

$\min\{x,y\}$

method :

$$\frac{x+y-|x-y|}{2} = \min\{x, y\}$$

9.1.3 Absolute Value: GATE2011 AG: GA-7 top

Given that $f(y) = \frac{|y|}{y}$, and q is non-zero real number, the value of $|f(q) - f(-q)|$ is

A. 0

B. -1

C. 1

D. 2

general-aptitude quantitative-aptitude gate2011-ag absolute-value

9.1.3 Absolute Value: GATE2011 AG: GA-7 top

Given that $\boxed{f(y) = \frac{|y|}{y}}$, and q is non-zero real number, the value of $|f(q) - f(-q)|$ is

A. 0
B. -1
C. 1
D. 2 ✓

general-aptitude quantitative-aptitude gate2011-ag absolute-value

$q \neq 0$;

$f(y) = \frac{|y|}{y}$

if $q > 0$ then

$f(q) = 1$

if $q < 0$ then

$f(q) = -1$

$f(5) = 1$

$f(-5) = -1$

$f(0) = \frac{0}{0}$

We don't care.

Undefined

$|f(q) - f(-q)|$

$q \neq 0$

Case 1: $q > 0$

$f(q) = 1$; $f(-q) = -1$

$|1 - (-1)| = 2$ ✓

Case 2: $q < 0$

$f(q) = -1$; $f(-q) = 1$

$|-1 - (1)| = 2$ ✓

$$f(y) = \frac{|y|}{y}$$

$$f(q) = \frac{|q|}{q}$$

if $q > 0$

Ex: $q = 2$

$$f(q) = \frac{|2|}{2} = \frac{2}{2} = 1$$

9.1.4 Absolute Value: GATE2013 AE: GA-8 top

If $|-2X+9|=3$ then the possible value of $|-X|-X^2$ would be:

A. 30 B. -30 C. -42 D. 42

gate2013-ae quantitative-aptitude absolute-value

9.1.4 Absolute Value: GATE2013 AE: GA-8 top

If $|-2X+9|=3$ then the possible value of $|-X|-X^2$ would be:

A. 30 B. -30 C. -42 D. 42

gate2013-ae quantitative-aptitude absolute-value

$|a| = b$ then $a = b$ or $a = -b$

$$\left|\underbrace{\boxed{-2x+9}}_{}\right| = 3$$

a

Case 1:

$$-2x+9=3$$

$$x=3$$

Case 2:

$$-2x+9=-3$$

$$x=6$$

$x = 3$ or 6.

E: $|-x| - x^2$

if $x=3$; $E = 3-9$

$= -6$ ✓

if $x=6$; $E = 6-36$

$= -30$ ✓

9.1.5 Absolute Value: GATE2013 CE: GA-7 top

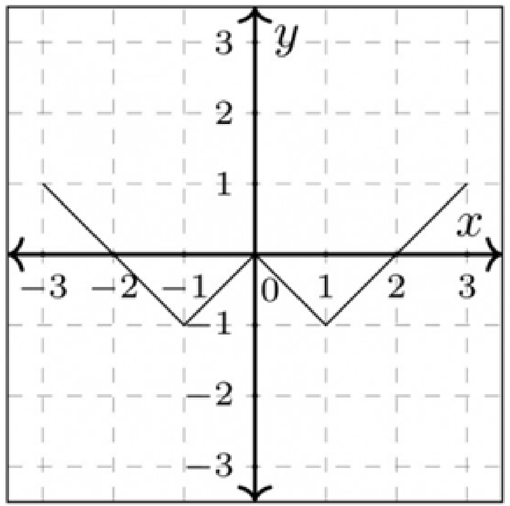
If $|4X - 7| = 5$ then the values of $2\,|X| - |-X|$ is:

A. $2, \left(\frac{1}{3}\right)$ B. $\left(\frac{1}{2}\right), 3$ C. $\left(\frac{3}{2}\right), 9$ D. $\left(\frac{2}{3}\right), 9$

gate2013-ce quantitative-aptitude absolute-value

9.1.6 Absolute Value: GATE2018 ME-1: GA-9 top

Which of the following functions describe the graph shown in the below figure?



A. $y = || x | + 1 | - 2$
B. $y = || x | - 1 | - 1$
C. $y = || x | + 1 | - 1$
D. $y = || x - 1 | - 1|$

2. Which of the following numbers is farthest from the number 1 on the number line?

(A) -10

(B) -5

(C) 0

(D) 5

(E) 10

Source: GRE

Explanation

Circling each of the answer choices in a sketch of the number line (Figure 4) shows that of the given numbers, −10 is the greatest distance from 1.

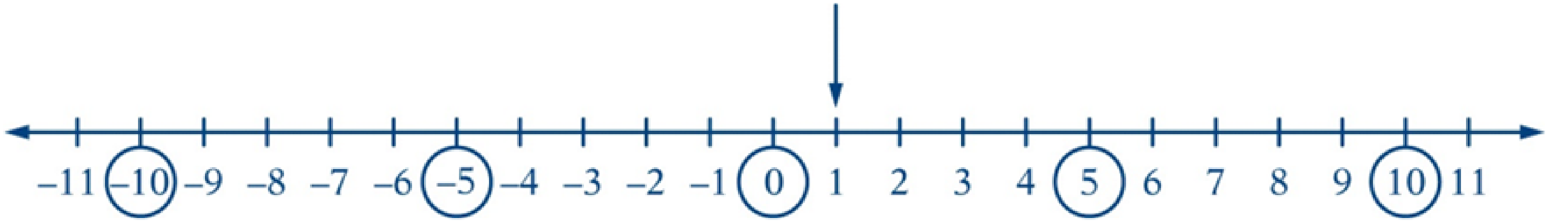


Figure 4

Another way to answer the question is to remember that the distance between two numbers on the number line is equal to the absolute value of the difference of the two numbers. For example, the distance between −10 and 1 is $|-10-1| = 11$, and the distance between 10 and 1 is $|10-1| = |9| = 9$.

The correct answer is Choice A, −10.

7.

X is the set of all integers n that satisfy the inequality $2 \le |n| \le 5$.

<u>Quantity A</u>

The absolute value of the greatest integer in X

<u>Quantity B</u>

The absolute value of the least integer in X

(A) (B) (C) (D)

(A) **Quantity A is greater.**

(B) **Quantity B is greater.**

(C) **The two quantities are equal.**

(D) **The relationship cannot be determined from the information given.**

Source: GRE

Explanation

When comparing these quantities, it is important to remember that a nonzero number and its negative have the same absolute value. For example, $|-2| = |2| = 2$. Keeping this in mind, you can see that the positive integers 2, 3, 4, and 5 and the negative integers −2, −3, −4, and −5 all satisfy the inequalities $2 \le |n| \le 5$, and that these are the only such integers. Thus, the set X consists of the integers −5, −4, −3, −2, 2, 3, 4, and 5. The greatest of these integers is 5, and its absolute value is 5. The least of these integers is −5, and its absolute value is also 5. Therefore, Quantity A is equal to Quantity B. The correct answer is **Choice C**.

Absolute Value:

Some More

Standard Properties

Q:

Prove that

The absolute value of the product of two real numbers is the product of their absolute values.

To make the proof correct, consider the four exhaustive cases

1. $a \geq 0, b \geq 0$
2. $a \geq 0, b < 0$
3. $a < 0, b \geq 0$
4. $a < 0, b < 0$

separately, and verify that the equation $|ab| = |a||b|$ holds in each case.

For all $a, b \in \mathbb{R}$, $|a| \cdot |b| = |ab|$.

$$|ab| = \begin{cases} ab & \text{if } ab \geq 0 \\ -ab & \text{if } ab < 0 \end{cases}$$

Case 1. $a \geq 0$, $b \geq 0$

$$|a| \cdot |b| = a \cdot b = |ab|$$

Case 2. $a < 0$, $b < 0$

$$|a| \cdot |b| = (-a)(-b) = ab = |ab|$$

Case 3. $a < 0$, $b \geq 0$

$$|a| \cdot |b| = (-a)(b) = -ab = |ab|$$

$\square$

Q:

Prove that:

The absolute value of the sum of two real numbers does not exceed the sum of the absolute values of these real numbers.

For all $a, b \in \mathbb{R}$, $|a+b| \le |a| + |b|$.

$a \le |a|$ $\quad b \le |b|$

$-a \le |a|$ $\quad -b \le |b|$

$$|a+b| = \begin{cases} a+b & a+b \ge 0 \\ -(a+b) & a+b < 0 \end{cases}$$

$a+b \le |a| + |b|$

$-(a+b) = -a + (-b) \le |a| + (-b) \le |a| + |b|$

$$|a+b| \le |a| + |b|.$$

$\square$

Q:

For any reals a,b
Provide a geometric interpretation
for $|a-b|$.

Note:

The absolute value of the difference of two real numbers is the distance between them.

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