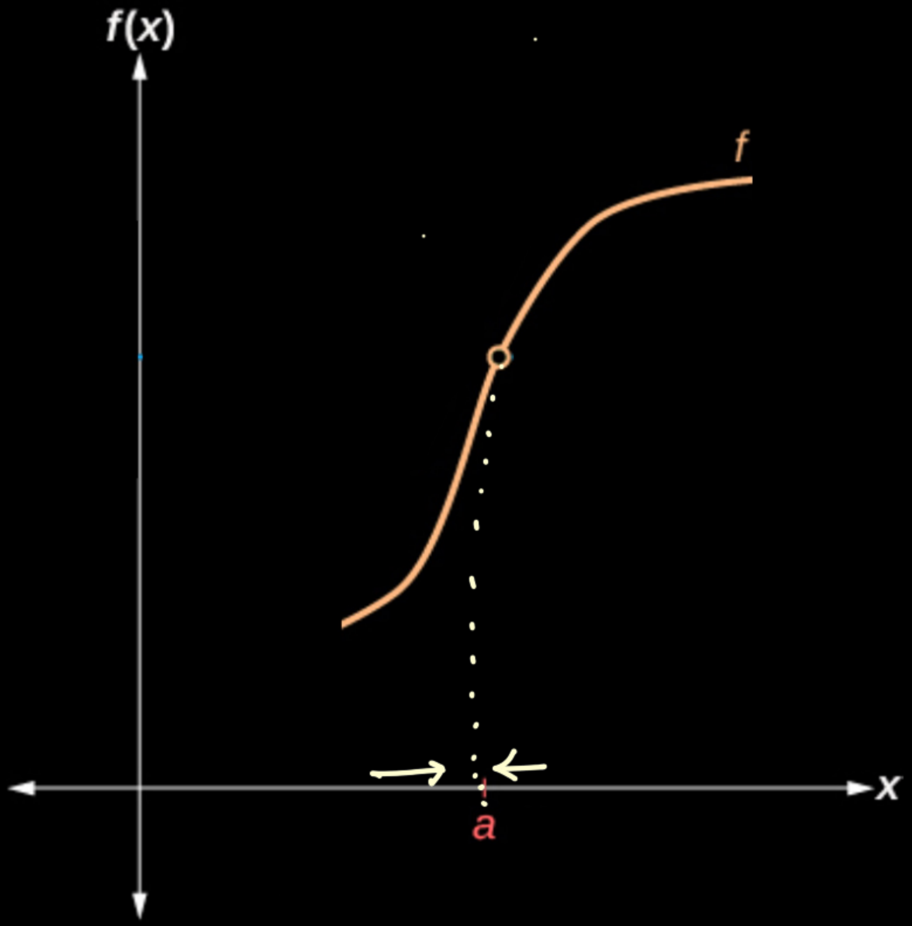
Limit



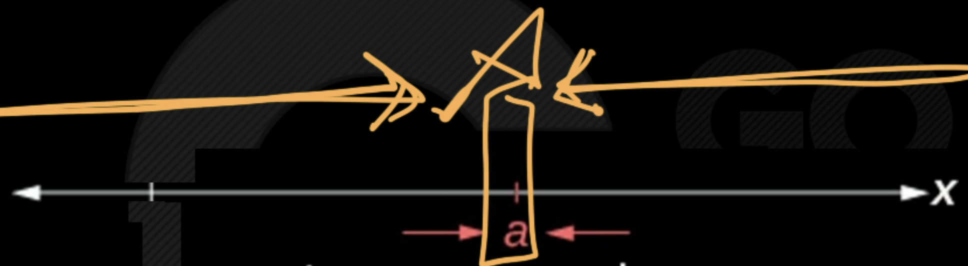
$$\lim_{x \to a} f(x)$$

are we interested in $f(a)$?

$\Rightarrow$ no.

$$\boxed{x \to a \quad \neq \quad x = a}$$

$x \to a \neq x = a$



Input x approaches a

$x \to 2$

$1.9, 1.99, 1.999$

$2.001 \quad 2.000001$

2

$x \to 2$ $\neq$ $x = 2$

$x = 1.99999$

$x = 2.0001$

$x = 2.00000000000001$

$$f(x) = \frac{(x-2)(x-3)}{x-2}$$

can't cancel

$$f(x) = \frac{(x-2)(x-3)}{x-2}$$

$\boxed{\text{at } x=2}$

$\frac{\cancel{0}}{\cancel{0}}$ (-1)

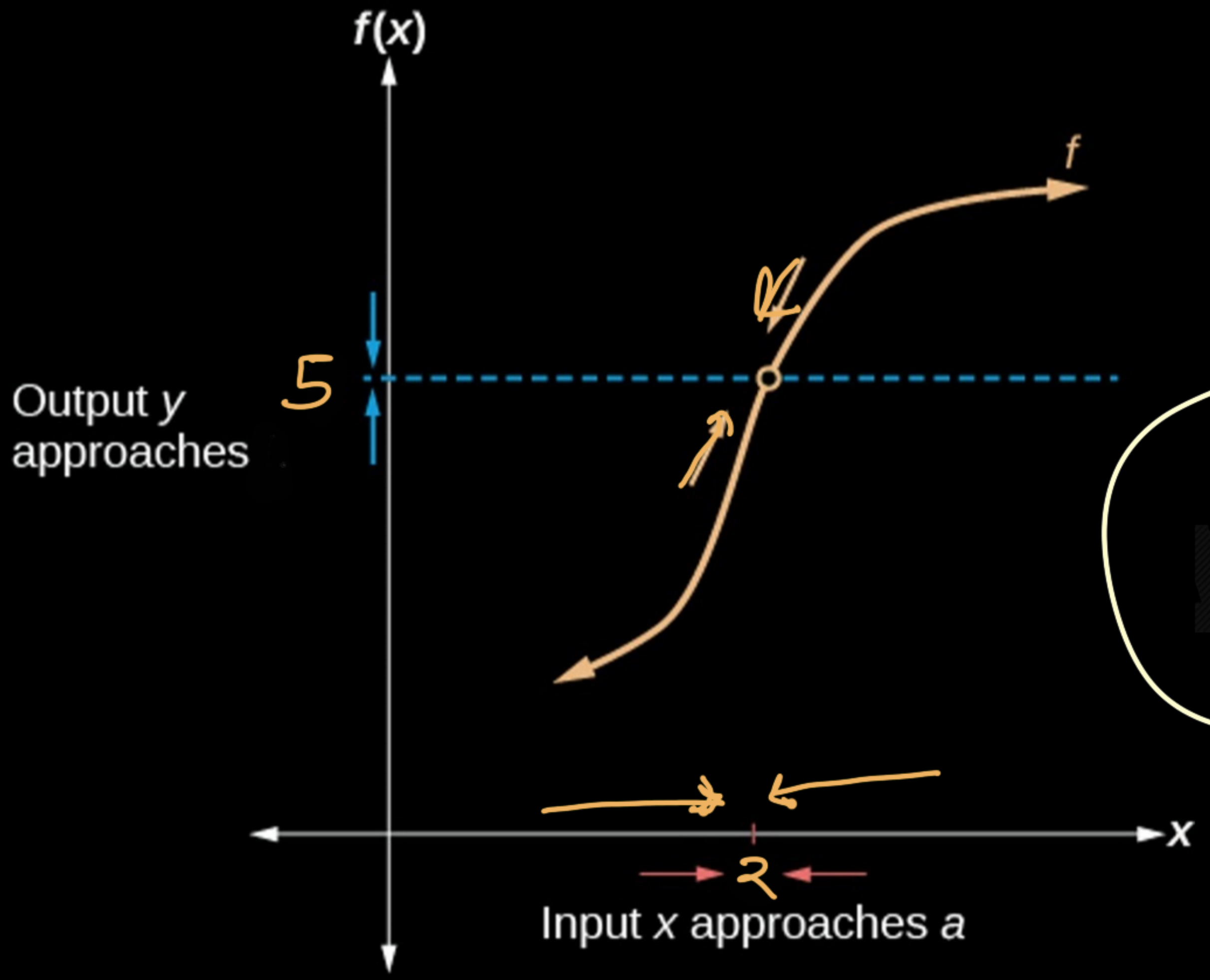
$$\lim_{x \to 2} \frac{\cancel{(x-2)}(x-3)}{\cancel{(x-2)}}$$

valid

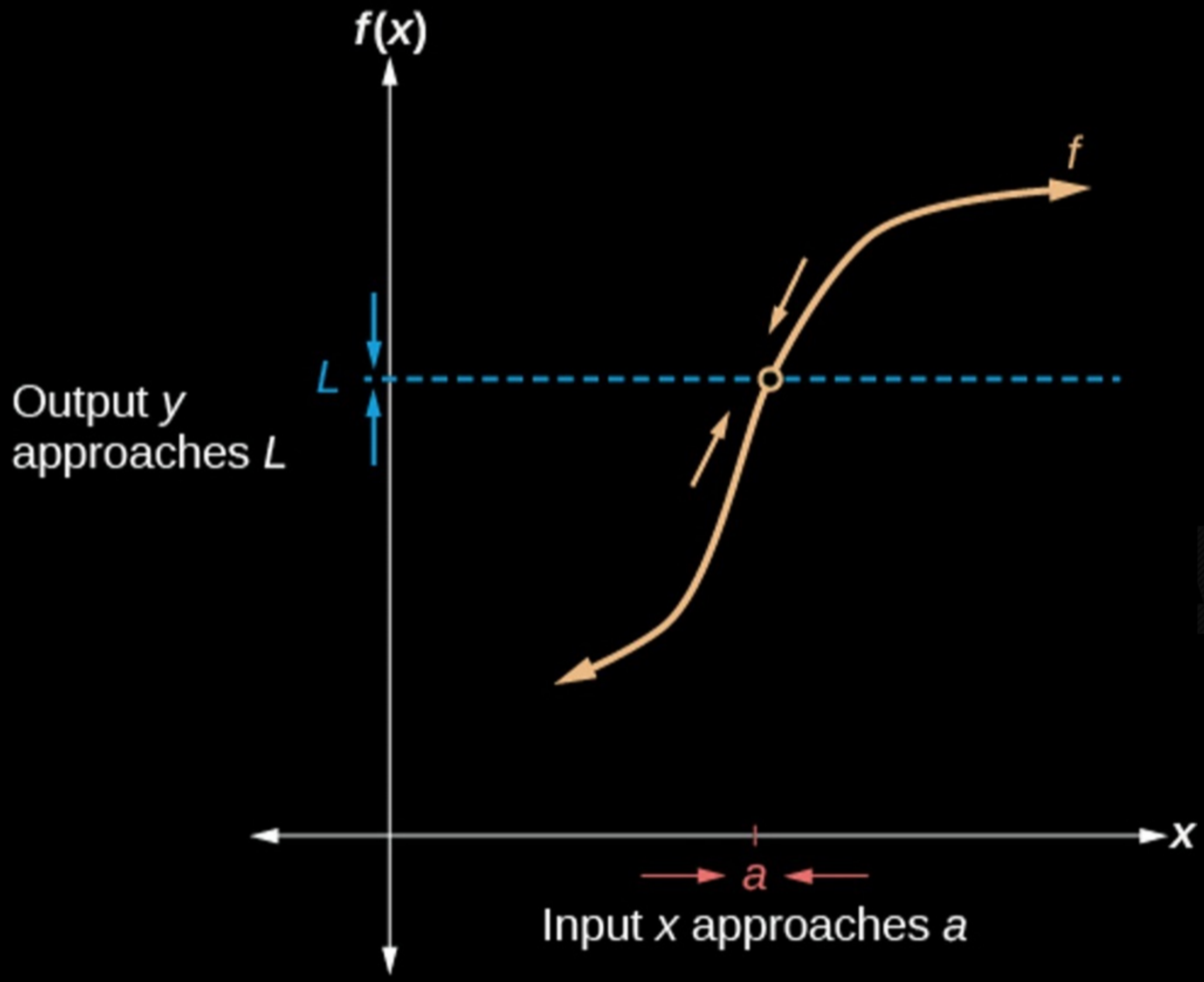
$\frac{\cancel{2}}{\cancel{2}}$ $\frac{\cancel{3}}{\cancel{3}}$

2.0000000 . . . 1

1.9999999999



$$\lim_{x \to 2} f(x) = 5$$


f(x)
f
L
Output y
approaches L
x
a
Input x approaches a

$$\lim_{x \to a} f(x)$$

$\hookrightarrow$ we can approach to "a" either from left or right

$$\lim_{h \to 0^+} f(a-h) \qquad \lim_{h \to 0^+} f(a+h)$$

LHL RHL

LHL :

$$\lim_{x \to a^-} f(x) = \lim_{h \to 0^+} f(a-h)$$

RHL :

$$\lim_{x \to a^+} f(x) = \lim_{h \to 0^+} f(a+h)$$

$\lim_{x \to a} f(x) \rightarrow$ LHL

$\lim_{x \to a} f(x) \rightarrow$ RHL

limit exist if

$\boxed{LHL = RHL}$

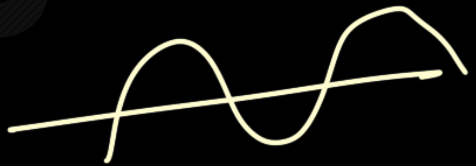
and finite

in what case limit does not exist?

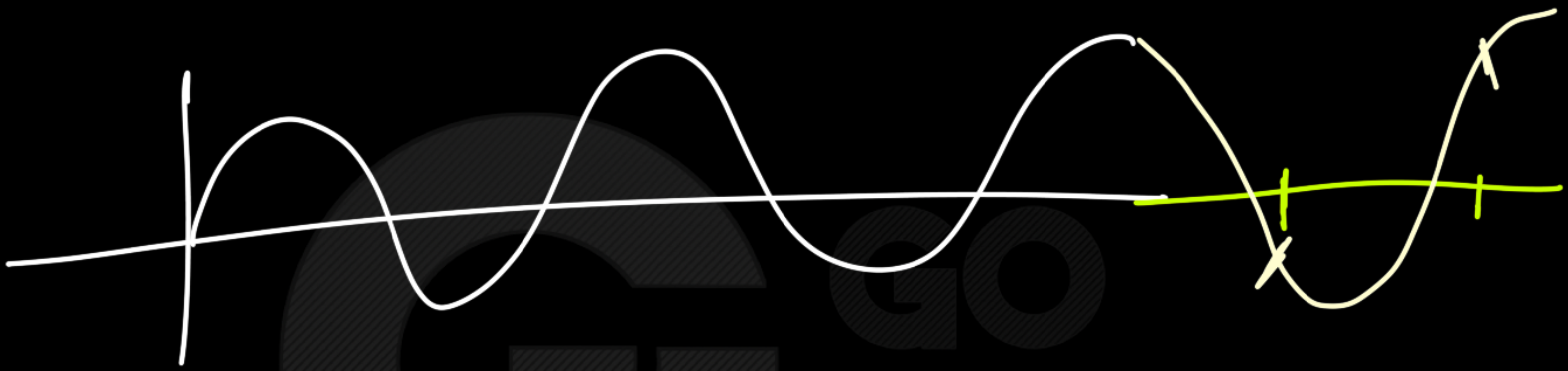
1) LHL & RHL both exists but

$LHL \neq RHL$

2) LHL or RHL is ∞



3) function value fluctuate $\lim_{x \to \infty} \sin x$



$x \to a \implies f(x) \to L$

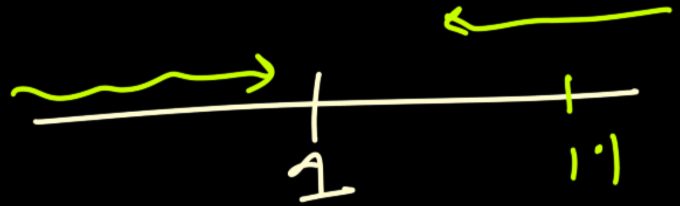
$\equiv \lim_{x \to a} f(x) = L$

$$f(x) = 2x + 3$$

$$\lim_{x \to 1} f(x) = 5$$

x	$2x+3$
0.9	4.8
0.99	4.98
0.999	4.998
1	
1.001	5.002
1.01	5.02
1.1	5.2

Not interested



$f(x) = 2x + 3$

$\lim_{x \to 1} f(x)$

x	$2x+3$
0.9	4.8
0.99	4.98
0.999	4.998
1	5
1.001	5.002
1.01	5.02
1.1	5.2

$$\underbrace{\lim_{x \to -3} \frac{x^2 + x - 6}{x + 3}}_{\text{not interested}} = -5$$

-3.01 -3.001 -3.0001

-3 -2.9999 -2.999 -2.99

x	-3.01	-3.001	-3.0001	-3	-2.9999	-2.999	-2.99
$\dfrac{x^2 + x - 6}{x + 3}$	-5.01	-5.001	-5.0001	?	-4.9999	-4.999	-4.99

Find the limit.

$$\lim_{x \to -3} \frac{x^2 + x - 6}{x + 3}$$

Solution:

Begin by factoring the numerator and dividing out any common factors.

$$\lim_{x \to -3} \frac{x^2 + x - 6}{x + 3} = \lim_{x \to -3} \frac{(x - 2)(x + 3)}{x + 3}$$ Factor numerator.

$$= \lim_{x \to -3} \frac{(x-2)\cancel{(x+3)}}{\cancel{x+3}}$$ Divide out common factor.

$$= \lim_{x \to -3} (x-2)$$ Simplify.

$$= -3 - 2$$ Direct substitution

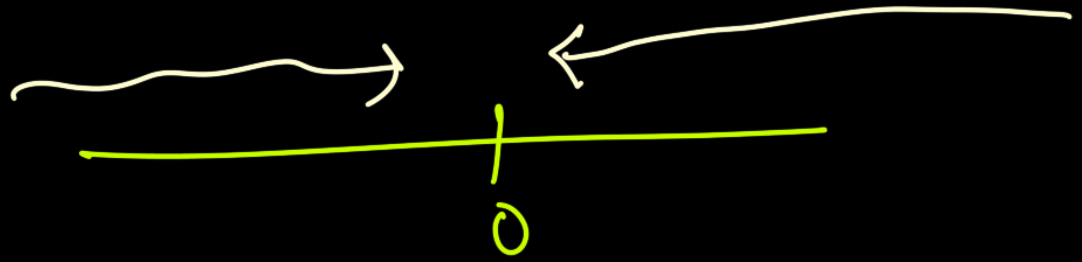
$$= -5$$ Simplify.

Example

Find $\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{(x+1)^2 - 1}{x}$. $= 2$

at $x = 0$
we are not
interested

x	-0.01	-0.001	-0.0001	0.0001	0.001	0.01
$f(x)$	1.99	1.999	1.9999	2.0001	2.001	2.01



Example

Find $\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{(x+1)^2 - 1}{x}$.

Solution

The idea is to evaluate $f(x)$ for various values of x which are close to 0 e.g. $x = 0.01, 0.001, -0.01, -0.001$ and so on to guess the limit. Consider the following table:

x	-0.01	-0.001	-0.0001	0.0001	0.001	0.01
$f(x)$	1.99	1.999	1.9999	2.0001	2.001	2.01

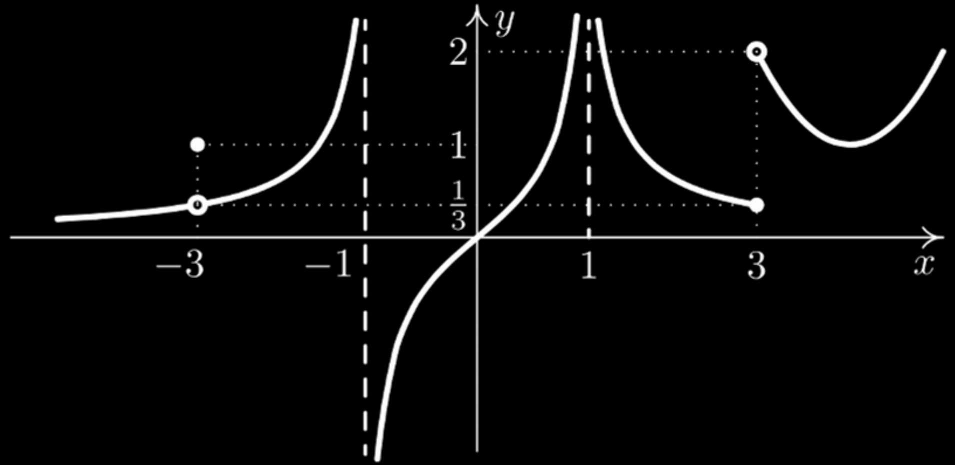
Notice that $f(x)$ tends to 2 as x tends to 0. Therefore,

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{(x+1)^2 - 1}{x} = 2$$

Question

Example

Let the function f be given by the graph:

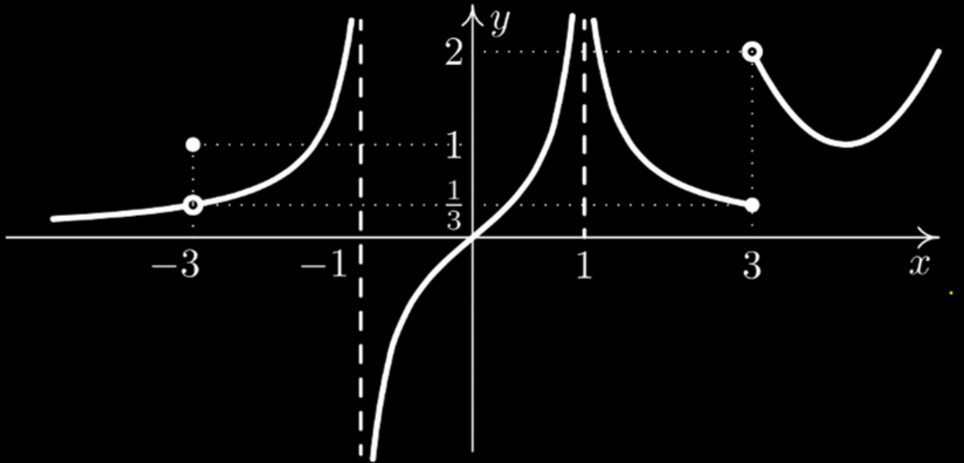


- $\lim_{x\to-\infty} f(x)$
- $\lim_{x\to-3} f(x)$
- $\lim_{x\to-1} f(x)$
- $\lim_{x\to1} f(x)$
- $\lim_{x\to3} f(x)$
- $\lim_{x\to\infty} f(x)$

Question

Example

Let the function f be given by the graph:



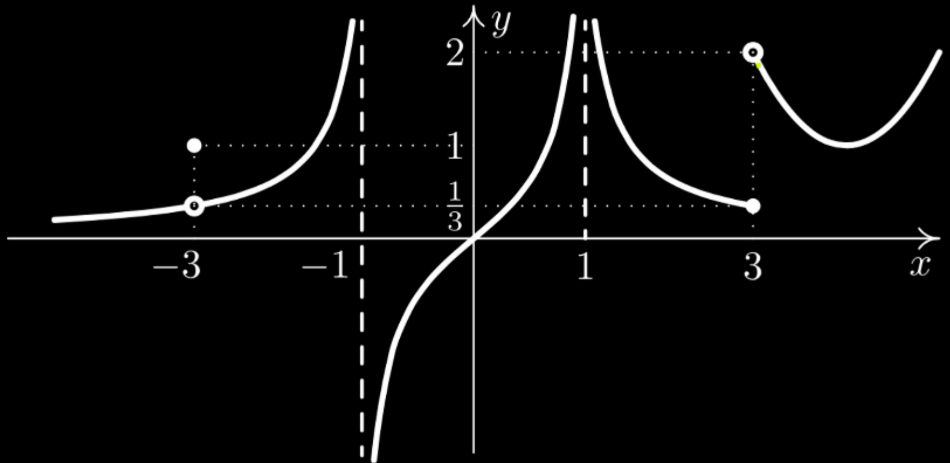
- $\lim_{x\to-\infty} f(x)$ $= 0$
- $\lim_{x\to-3} f(x)$ $= 1/3$
- $\lim_{x\to-1} f(x)$

DNE

- $\lim_{x\to1} f(x)$
- $\lim_{x\to3} f(x)$
- $\lim_{x\to\infty} f(x)$

Example

Let the function f be given by the graph:



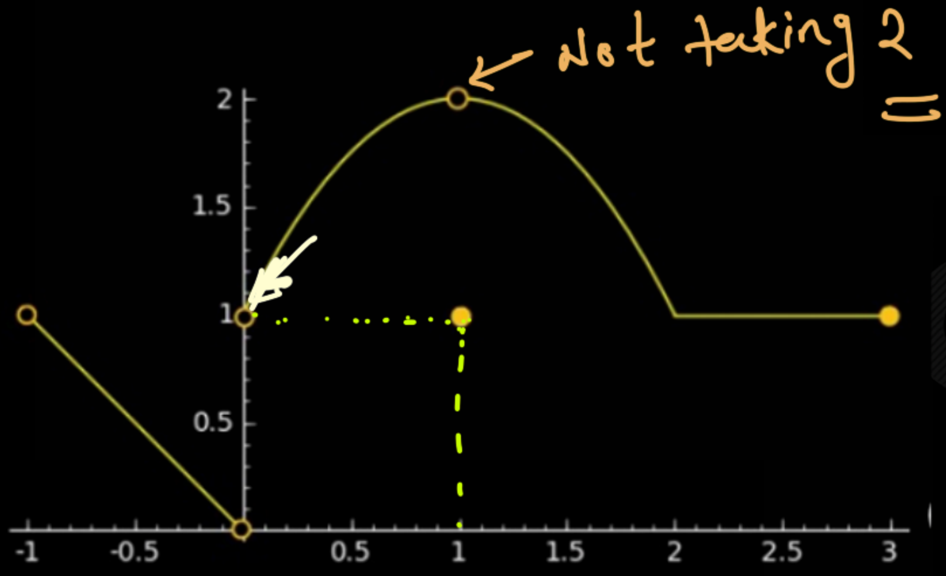
- $\lim_{x\to-\infty} f(x) = 0$
- $\lim_{x\to-3} f(x) = \frac{1}{3}$
- $\lim_{x\to-1} f(x)$ does not exist, since $\lim_{x\to-1^-} f(x) = \infty$, $\lim_{x\to-1^+} f(x) = -\infty$
- $\lim_{x\to1} f(x) = \infty$

DNE

- $\lim_{x\to3} f(x)$ does not exist, since $\lim_{x\to3^-} f(x) = \frac{1}{3}$, $\lim_{x\to3^+} f(x) = 2$
- $\lim_{x\to\infty} f(x) = \infty$

Question

(1) (5 pts) Find the following values from the graph of $f(x)$. If it does not exist, write DNE. You don't need to write the reason.



(a) $\lim_{x\to -1^+} f(x) = 1$

(b) $\lim_{x\to 0^-} f(x) = 0$

(c) $\lim_{x\to 0^+} f(x) = 1$

(d) $\lim_{x\to 0} f(x) \Rightarrow$ DNE

(e) $f(0)$ $\longrightarrow$ Not defined

(f) $\lim_{x\to 1^-} f(x) \longrightarrow 2$

(g) $\lim_{x\to 1^+} f(x) \longrightarrow 2$

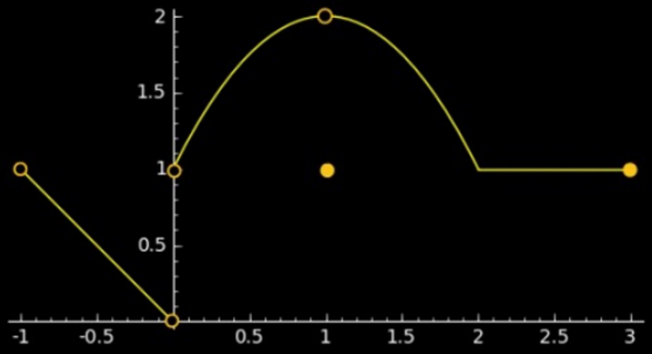
(h) $\lim_{x\to 1} f(x) \longrightarrow 2$

(i) $f(1) \longrightarrow 1$

(j) $\lim_{x\to 3^-} f(x) \longrightarrow 1$

Solution

(1) (5 pts) Find the following values from the graph of $f(x)$. If it does not exist, write DNE. You don't need to write the reason.



(a) $\lim_{x\to -1^+} f(x) = 1$

(b) $\lim_{x\to 0^-} f(x) = 0$

(c) $\lim_{x\to 0^+} f(x) = 1$

(d) $\lim_{x\to 0} f(x) = \text{DNE}$

(e) $f(0) = \text{DNE}$

(f) $\lim_{x\to 1^-} f(x) = 2$

(g) $\lim_{x\to 1^+} f(x) = 2$

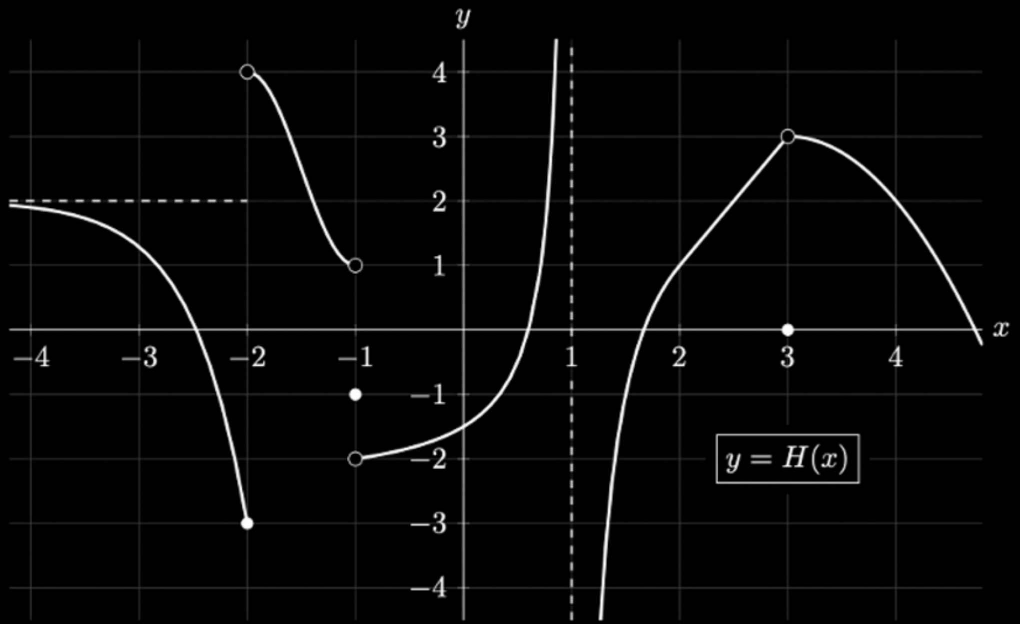
(h) $\lim_{x\to 1} f(x) = 2$

(i) $f(1) = 1$

(j) $\lim_{x\to 3^-} f(x) = 1$

Question

5. [11 points] Below is a portion of the graph of the function $H(x)$, which is defined for all $x < 1$ and all $x > 1$. Note that $H(x)$ is linear for $2 \le x < 3$, and that $H(x)$ has a horizontal asymptote at $y = 2$ and a vertical asymptote at $x = 1$.



DNE

a. [1 point] $\lim_{x\to -2} H(x)$

LHL

$\lim_{x\to -2^-} H(x) = -3$

RHL

$\lim_{x\to -2^+} H(x) = 4$

b. [1 point] $\lim_{x\to 3} H(x)$

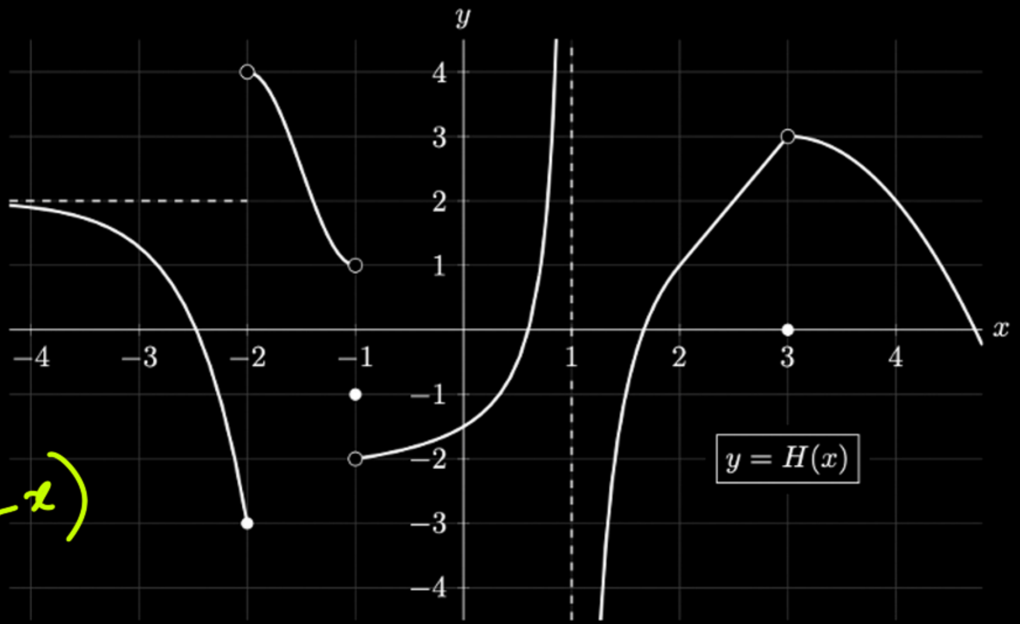
c. [1 point] $\lim_{x\to -\infty} H(x)$

d. [2 points] $\lim_{x\to 4^+} H(3-x)$

https://dhsp.math.lsa.umich.edu/exams/115/f21s1.pdf

Question

5. [11 points] Below is a portion of the graph of the function $H(x)$, which is defined for all $x < 1$ and all $x > 1$. Note that $H(x)$ is linear for $2 \le x < 3$, and that $H(x)$ has a horizontal asymptote at $y = 2$ and a vertical asymptote at $x = 1$.



$\lim_{x \to 4^-} H(3-x)$

a. [1 point] $\lim_{x \to -2} H(x)$

DNE

LHL

$\lim_{x \to -2^-} H(x) = -3$

RHL

$\lim_{x \to -2^+} H(x) = 4$

b. [1 point] $\lim_{x \to 3} H(x)$

c. [1 point] $\lim_{x \to -\infty} H(x)$

d. [2 points] $\lim_{x \to 4^+} H(3-x)$

-1^-

2

3

https://dhsp.math.lsa.umich.edu/exams/115/f21s1.pdf

$x \to 4^{+} \quad \Rightarrow \quad 3-x \longrightarrow -1^{-}$

$3-5 = -2$

$3-6 = -3$

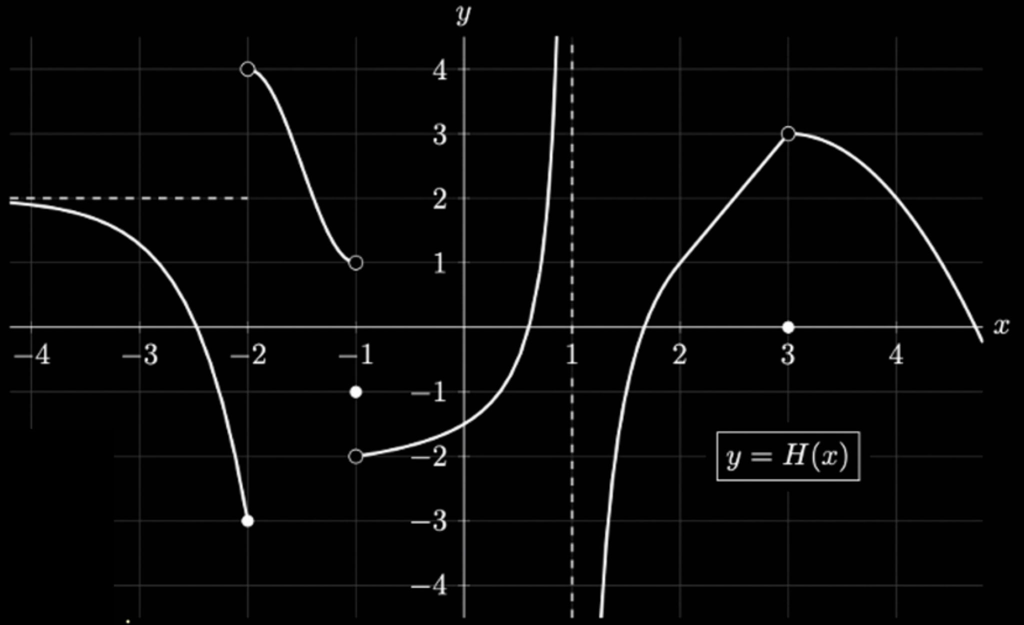
$x \to 4^{-} \quad \Rightarrow \quad 3-x \longrightarrow -1^{+}$

$3-3 = 0$

$3-2 = 1$

$x \to 4^- \implies 3 - x \to -1^+$

$3 - 3 = 0$



$$\underbrace{\lim_{x \to 4^-} H(3-x)}_{\to -2}$$

$x \to 4^-$ $\implies$ $3 - x \to -1^+$

$x = 3$

$x = 3.9$

$x = 3.99$

$x = 3.9999$

$3 - x = 0$

$3 - 3.9 = -0.9$

$3 - 3.99 = -0.99$

$3 - 3.9999 = -0.9999$

Use this graph to find the numerical value of each of the following limits. If a limit does not exist, including if it diverges to $\pm\infty$, write DNE. You do not need to show work.

a. [1 point] $\lim_{x \to -2} H(x)$

Answer: ____________________

b. [1 point] $\lim_{x \to 3} H(x)$

Answer: ____________________

c. [1 point] $\lim_{x \to -\infty} H(x)$

Answer: ____________________

d. [2 points] $\lim_{x \to 4^+} H(3 - x)$

Answer: ____________________

e. [2 points] $\lim_{h \to 0} \dfrac{H(2.5 + h) - H(2.5)}{h}$

Answer: ____________________

Solution

Use this graph to find the numerical value of each of the following limits. If a limit does not exist, including if it diverges to $\pm\infty$, write DNE. You do not need to show work.

a. [1 point] $\lim_{x \to -2} H(x)$

Answer: DNE

b. [1 point] $\lim_{x \to 3} H(x)$

Answer: 3

c. [1 point] $\lim_{x \to -\infty} H(x)$

Answer: 2

d. [2 points] $\lim_{x \to 4^+} H(3-x)$

Answer: 1

e. [2 points] $\lim_{h \to 0} \frac{H(2.5+h) - H(2.5)}{h}$

Answer: 2

↳ skiping for now (related to diffrentiation)

Indeterminate forms in limit

1. $0 \cdot \infty = 0$
2. $\frac{0}{0}$
3. $\frac{\infty}{\infty}$
4. $\infty - \infty$
5. 0^0
6. ∞^0
7. 1^∞

In determinate

something which we can't determine easily

Indeterminate forms in limit

In determinate

something which we can't determine easily

1. $0 \cdot \infty$
2. $\frac{0}{0}$
3. $\frac{\infty}{\infty}$
4. $\infty - \infty$
5. 0^0
6. ∞^0
7. $\boxed{1}^\infty$

this is Not exact zero. it is either 0^+ or 0^-

this is also not exact 1

Product of two terms

$x \cdot y$

$3\uparrow \quad 5\uparrow \longrightarrow 15\uparrow$

$2.9999 \quad 4.9999 \approx 14.999$

$3.000001 \quad 5.000001 \approx 15.000$

Product of two terms

$x \cdot y$

$3\uparrow \quad 5\uparrow \longrightarrow 15\uparrow$

$2.9999 \quad 4.9999 \approx 14.999$

$3.000001 \quad 5.000001 \approx 15.0001$

$x \cdot y$

$3\uparrow \cdot 0\uparrow \Rightarrow 0\uparrow$

$3\uparrow \cdot 0^{+} \Rightarrow 0^{+}$

$3\uparrow \cdot 0^{-} \Rightarrow 0^{-}$

$x \cdot y$

$\infty\uparrow \quad 5\uparrow \quad = \quad \infty\uparrow$ ✓

$x \cdot y$

$\underbrace{0\uparrow \cdot \infty\uparrow} \Rightarrow 0$

x	$\cdot$	y		
1		100	$\Rightarrow$	100
10^{-3}		1000	$\Rightarrow$	1
10^{-6}		10^4	$\Rightarrow$	10^{-2}
10^{-9}		10^5	$\Rightarrow$	10^{-4}

x y ✓

$0 \uparrow$ $\infty \uparrow$

1 $100 \Rightarrow 100$

0.1 $10^6 \Rightarrow 10^5$ ∞

10^{-2} $10^{10} \Rightarrow 10^8$

$x \cdot y$

$0^{\uparrow} \cdot \infty^{\uparrow}$

pulling towards zero

pulling towards infinity

$\underbrace{c \cdot \infty}$

this is NOT indeterminant form

$$\lim_{x \to 0} \underbrace{x}_{\downarrow \atop 0} \cdot \underbrace{\frac{1}{x^2}}_{\downarrow \atop \infty} \Rightarrow \infty$$

$$\lim_{x \to 0} x \cdot \frac{1}{x} \Rightarrow 1$$

$$\lim_{x \to 0} \underbrace{x^2}_{\downarrow \atop 0} \cdot \underbrace{\frac{1}{x}}_{\downarrow \atop \infty} \Rightarrow 0$$

x $\quad$ y

1 $\quad$ 100 $\quad \Rightarrow$ 100

0.1 $\quad$ 1000 $\quad \Rightarrow$ 100

10^{-2} $\quad$ 10^{4} $\quad \Rightarrow$ 100

10^{-3} $\quad$ 10^{5} $\quad \Rightarrow$ 100

Indeterminate forms in limit

1. $0 \cdot \infty = 0$
2. $\frac{0}{0}$
3. $\frac{\infty}{\infty}$
4. $\infty - \infty$
5. 0^0
6. ∞^0
7. 1^∞

$c - \infty$

$0 \cdot 0$

$\infty \cdot \infty$

$-\infty \cdot \infty$

$-\infty$

$\frac{0\uparrow}{0} \Rightarrow$ undefined

$\underbrace{0}$

exact zero

$\frac{0}{0\uparrow}$ ← exact zero

$\Rightarrow 0$

$\frac{c}{0\uparrow} \Rightarrow \infty\uparrow$

Division of two terms

$$\frac{0\uparrow}{c} \Rightarrow 0$$

$$\frac{\boxed{0\uparrow}}{\infty\uparrow} \Rightarrow 0\uparrow$$

$$\frac{0\uparrow}{0\uparrow}$$

$$\frac{8\uparrow}{5\uparrow} \Rightarrow \checkmark$$

$$\frac{\infty\uparrow}{0\uparrow} \Rightarrow \infty\uparrow$$

$$\lim_{x \to 0} \frac{x^2}{x} \Rightarrow 0$$

$$\lim_{x \to 0} \frac{x}{x} \Rightarrow 1$$

$$\lim_{x \to 0} \frac{x}{x^2} \Rightarrow \infty$$

$$\frac{0\uparrow}{c\uparrow} \Rightarrow 0$$

$$\boxed{\frac{0\uparrow}{0\uparrow}} \Rightarrow \text{indeterminate}$$

$$\Downarrow$$

it will depend which 0 is dominating

$$\frac{c\uparrow}{0\uparrow} \Rightarrow \infty$$

$$\frac{\boxed{\infty}}{\boxed{\infty}} \Rightarrow \text{in determinate}$$

$$\frac{c}{\boxed{\infty}} \Rightarrow 0$$

$$\frac{\boxed{\infty}}{c} \Rightarrow \infty$$

$$\frac{c}{\infty} \qquad \frac{0}{c} \qquad \frac{\infty}{c} \qquad \frac{0}{0} \qquad \frac{\infty}{\infty}$$

$$\lim_{x \to 0} \frac{(1/x)}{(1/x^2)} \Rightarrow 0$$

$$\lim_{x \to 0} \frac{(1/x^2)}{(1/x^2)} = \underline{1}$$

$$\lim_{x \to 0} \frac{(1/x^2)}{(1/x)} \Rightarrow \infty$$

Difference of two terms

$x - y$

$3\uparrow - 2\uparrow \Rightarrow 1\uparrow$

$\infty + \infty \Rightarrow \infty$

$\infty - \infty$

$\frac{1}{x} \left(\frac{1}{x} + 2\right)$

$$\lim_{x \to 0} \frac{1}{x} - \left(\frac{1}{x} + 2\right) \Rightarrow -2$$

$$\lim_{x \to 0} \frac{1}{x} - \frac{1}{x} \Rightarrow 0$$

$$\lim_{x \to 0} \frac{1}{x^2} - \frac{1}{x}$$

$$\frac{1 - x}{x^2} \Rightarrow \infty$$

Power of two numbers

5. 0^0

6. ∞^0

7. 1^∞

x^y

$0^{+\infty} \Rightarrow 0$

$\infty^\infty \Rightarrow \infty$

$(1/5)^\infty \Rightarrow 0$

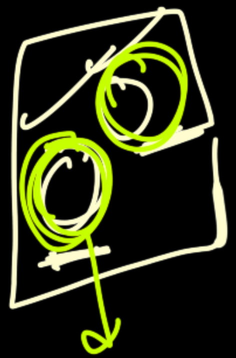
$c^{0} = 1$

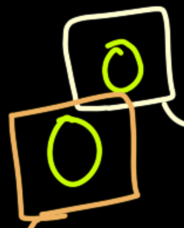
$5^\infty \Rightarrow \infty$

$0.2 \times 0.2 = 0.04$

$0.2 \times 0.2 \times 0.2 = 0.008$

$0.2 \times .02 \times .0.2 \times 0.2 = 0.0016$





this says final answer should be 1

this says final answer should be 0.

final answer should be 1

final answer should be ∞

inderminate Vs undefined

↓

we need to analyse

final answer could be anything

↓

Simply undefined

divide by zero

1^{∞}

$\boxed{1^{\infty} = 1 \quad \text{exact } 1}$

Indeterminate Form	Example
$0 \cdot \infty$	$\lim_{x \to 0} x \cdot \frac{1}{x}$
$\frac{0}{0}$	$\lim_{x \to 0} \frac{\sin x}{x}$
$\frac{\infty}{\infty}$	$\lim_{x \to \infty} \frac{x^2}{x^3}$
$\infty - \infty$	$\lim_{x \to \infty} (x - x^2)$
0^0	$\lim_{x \to 0^+} x^x$
∞^0	$\lim_{x \to \infty} x^{\frac{1}{x}}$
1^∞	$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$

Question

1. <u>QUOTIENTS</u>: So far we have primarily looked at <u>INDETERMINATE FORMS</u> that are quotients: $\frac{0}{0}, \frac{\pm\infty}{\pm\infty}$. Hence, the following quotients are <u>DETERMINATE FORMS</u>, ***meaning you know the limit***. Indicate the following limits, for a constant $c \neq 0$ [Note: You may indicate $\pm\infty$]:

(a). $\frac{c}{0} \to$ **(b).** $\frac{0}{c} \to$ [redacted] **(c).** $\frac{\pm\infty}{c} \to$ **(d).** $\frac{c}{\pm\infty} \to$

https://crawford.elmhurst.edu/classes/m152/2016F/wk6.8_IndetForms.pdf

Question

1. <u>QUOTIENTS</u>: So far we have primarily looked at <u>INDETERMINATE FORMS</u> that are quotients: $\frac{0}{0}, \frac{\pm\infty}{\pm\infty}$. Hence, the following quotients are <u>DETERMINATE FORMS</u>, ***meaning you know the limit***. Indicate the following limits, for a constant $c \neq 0$ [Note: You may indicate $\pm\infty$]:

(a). $\frac{c}{0} \to$ ∞ **(b).** $\frac{0}{c} \to$ 0 **(c).** $\frac{\pm\infty}{c} \to$ $\pm\infty$ **(d).** $\frac{c}{\pm\infty} \to$ 0

https://crawford.elmhurst.edu/classes/m152/2016F/wk6.8_IndetForms.pdf

1. Quotients: So far we have primarily looked at Indeterminate Forms that are quotients: $\frac{0}{0}, \frac{\pm\infty}{\pm\infty}$. Hence, the following quotients are Determinate Forms, ***meaning you know the limit.*** Indicate the following limits, for a constant $c \neq 0$ [Note: You may indicate $\pm\infty$]:

(a). $\frac{c}{0} \to \pm\infty$ **(b).** $\frac{0}{c} \to 0$ **(c).** $\frac{\pm\infty}{c} \to \pm\infty$ **(d).** $\frac{c}{\pm\infty} \to 0$

Question

2. <u>PRODUCTS</u>. Only one of the following products gives an indeterminate form. Cirle this one and then state the value of the limit for the other four forms.

$$0 \cdot \pm\infty \qquad c \cdot 0 \qquad c \cdot \pm\infty \qquad \infty \cdot \infty \qquad -\infty \cdot \infty$$

Question

2. PRODUCTS. Only one of the following products gives an indeterminate form. Cirle this one and then state the value of the limit for the other four forms.

$\underbrace{0 \cdot \infty}$ $\Downarrow$ $0, \infty$

$\underbrace{c \cdot 0}$ $\Downarrow$ 0

$\underbrace{c \cdot \infty}$ $\Downarrow$ ∞

$\underbrace{\infty \cdot \infty}$ $\Downarrow$ ∞

$\underbrace{-\infty \cdot \infty}$ $\Downarrow$ $-\infty$

2. <u>Products</u>. Only one of the following products gives an indeterminate form. Cirle this one and then state the value of the limit for the other four forms.

$0 \cdot \pm\infty$ IND $c \cdot 0 = 0$ $c \cdot \pm\infty = \pm\infty$ $\infty \cdot \infty = \infty$ $-\infty \cdot \infty = -\infty$

0 ∞ could be some other constant

$\lim_{x \to 0} x \cdot \frac{1}{x} = 1$

Question

3. SUMS AND DIFFERENCES. Only one of the following gives and indeterminate form. Circle this one and then state the value of the limit for the other two forms.

$$-\infty - \infty \qquad\qquad \infty - \infty \qquad\qquad \infty + \infty$$

Question

3. Sums and Differences. Only one of the following gives and indeterminate form. Circle this one and then state the value of the limit for the other two forms.

$$-\infty - \infty \qquad \boxed{\infty - \infty} \qquad \infty + \infty$$

$\Downarrow$

$-\infty$

$\Downarrow$

∞

3. <u>Sums and Differences</u>. Only one of the following gives and indeterminate form. Circle this one and then state the value of the limit for the other two forms.

$$-\infty - \infty = -\infty \qquad \infty - \infty \ \text{IND} \qquad \infty + \infty = \infty$$

Question

4. <u>Powers</u>. Fill in the following blanks. The following two powers are <u>Determinate Forms</u>:

$$0^{\infty} \qquad\qquad 0^{-\infty}$$

(a). Zero multiplied by itself over and over is still __________ . So in the limit 0^{∞}, we get ________ since it doesn't matter whether you approach zero from above or below, it still goes to zero.

(b). $0^{-\infty} = \dfrac{1}{0^{\infty}} = \dfrac{1}{0} \rightarrow$

Question

4. <u>POWERS</u>. Fill in the following blanks. The following two powers are <u>DETERMINATE FORMS</u>:

$$0^{\infty} \Rightarrow 0 \qquad\qquad 0^{-\infty} \Rightarrow \infty$$

(a). Zero multiplied by itself over and over is still __zero__ . So in the limit 0^{∞}, we get ______ since it doesn't matter whether you approach zero from above or below, it still goes to zero.

(b). $0^{-\infty} = \dfrac{1}{0^{\infty}} = \dfrac{1}{0} \to$

$$\frac{1}{0^{\infty}}$$

4. <u>Powers</u>. Fill in the following blanks. The following two powers are <u>Determinate Forms</u>:

$$0^{\infty} \qquad\qquad 0^{-\infty}$$

(a). Zero multiplied by itself over and over is still ___zero___ . So in the limit 0^{∞}, we get ___0___ since it doesn't matter whether you approach zero from above or below, it still goes to zero.

(b). $0^{-\infty} = \dfrac{1}{0^{\infty}} = \dfrac{1}{0} \to \pm\infty$

Question

H.W.:

The following three powers are <u>INDETERMINATE FORMS</u>:

0^0 ∞^0 1^∞

(a). Zero raised to any number should still be ________ . But any number raised to the ________ should be one. So 0^0 is INDETERMINATE because "reason" gives two plausible answers.

(b). Infinity (or a really big number) raised to a power should still be ________ , but any number raised to the zero should be ________ . So ∞^0 is INDETERMINATE because "reason" gives two plausible answers.

(c). 1.00001 multiplied by itself over and over will get larger. But 0.99999 multiplied by itself over and over will get ________ . So the limit 1^∞ is also an ________ form.

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0^0 ∞^0 1^∞

(a). Zero raised to any number should still be ___zero___ . But any number raised to the ___zero___ should be one. So 0^0 is INDETERMINATE because "reason" gives two plausible answers.

(b). Infinity (or a really big number) raised to a power should still be ___a really big number___ , but any number raised to the zero should be ___one___ . So ∞^0 is INDETERMINATE because "reason" gives two plausible answers.

(c). 1.00001 multiplied by itself over and over will get larger. But 0.99999 multiplied by itself over and over will get ___smaller___ . So the limit 1^∞ is also an ___INDETERMINATE___ form.

We will cover -

- Methods for solving $\frac{0}{0}$
  - Factorization
  - Rationalization
- Use of standard limits
- Use of Substitution
- Solving $\frac{\infty}{\infty}$
- Solving $\infty - \infty$
- Solving 1^{∞}
- L hospital rule

$$\lim_{x \to 3} x+3 = 6$$