Maxima and minima

Basic Properties and Formulas

If $f(x)$ and $g(x)$ are differentiable functions (the derivative exists), c and n are any real numbers,

1. $(cf)' = c f'(x)$
2. $(f \pm g)' = f'(x) \pm g'(x)$
3. $(fg)' = f'g + fg'$ – **Product Rule**
4. $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$ – **Quotient Rule**
5. $\frac{d}{dx}(c) = 0$
6. $\frac{d}{dx}(x^n) = n x^{n-1}$ – **Power Rule**
7. $\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$
   This is the **Chain Rule**

Common Derivatives

$\frac{d}{dx}(x) = 1$

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{\sin x}{\cos x}$

$\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(a^x) = a^x \ln(a)$

$\ln \to \log_e$

$\frac{d}{dx}(\mathbf{e}^x) = \mathbf{e}^x$

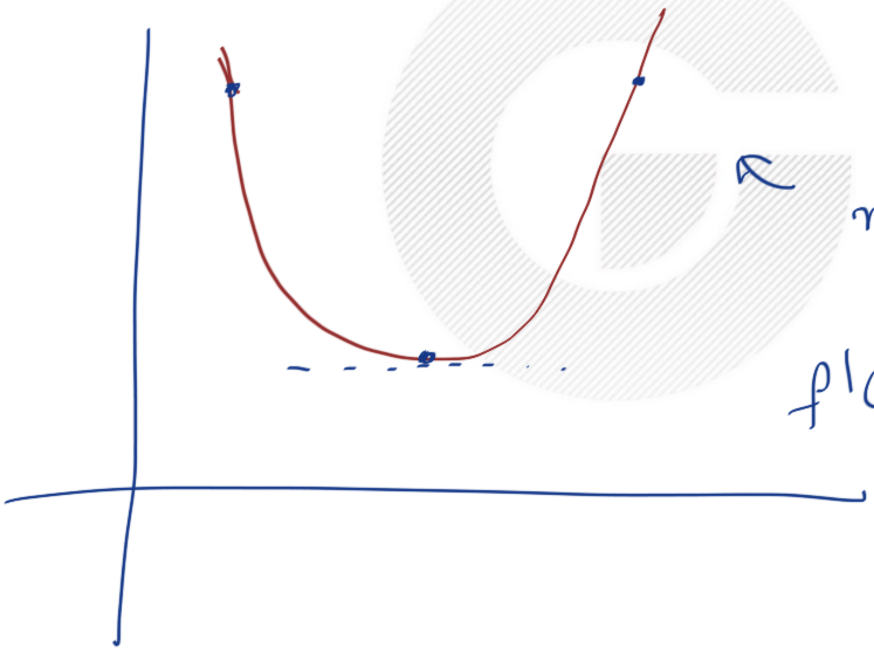
$\frac{d}{dx}(\ln(x)) = \frac{1}{x}, \quad x > 0$

$\frac{d}{dx}(\log_a(x)) = \frac{1}{x \ln a}, \quad x > 0$

$\underline{\sin x^2} \to \cos x^2 \times 2x$

$\log_a x = \frac{\log_e x}{\log_e a}$

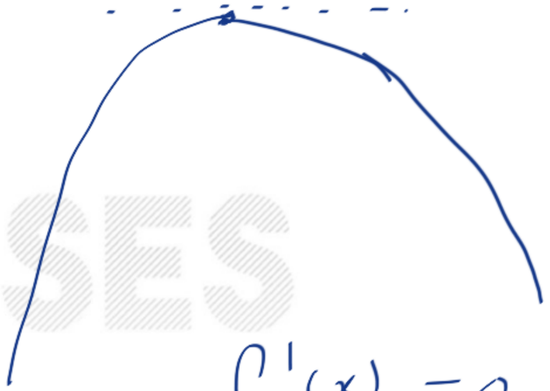
Maxima and minima



convex

minima

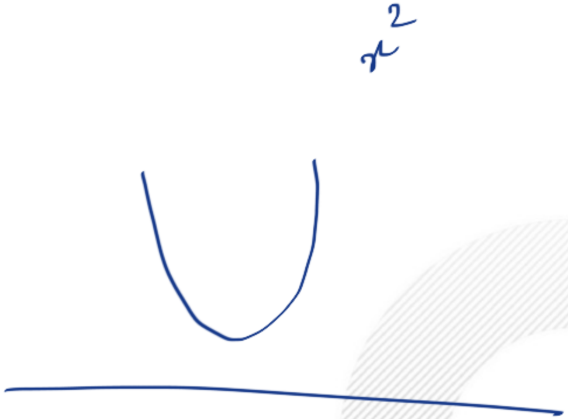
$f'(x) = 0$

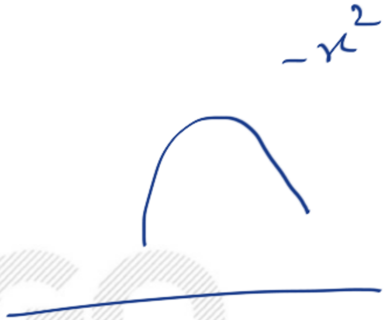


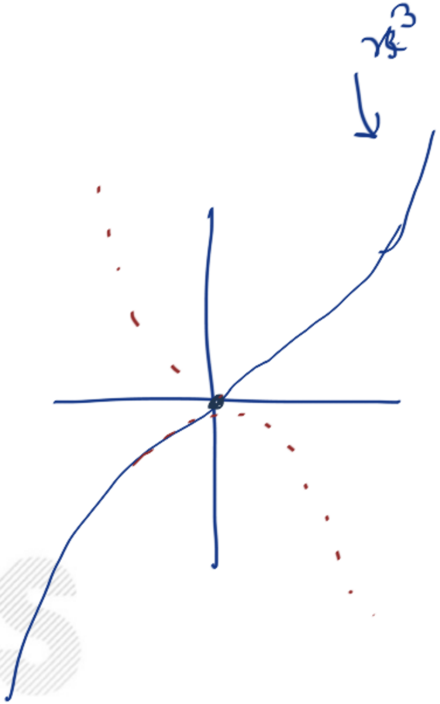
Concave

maxima

$f'(x) = 0$







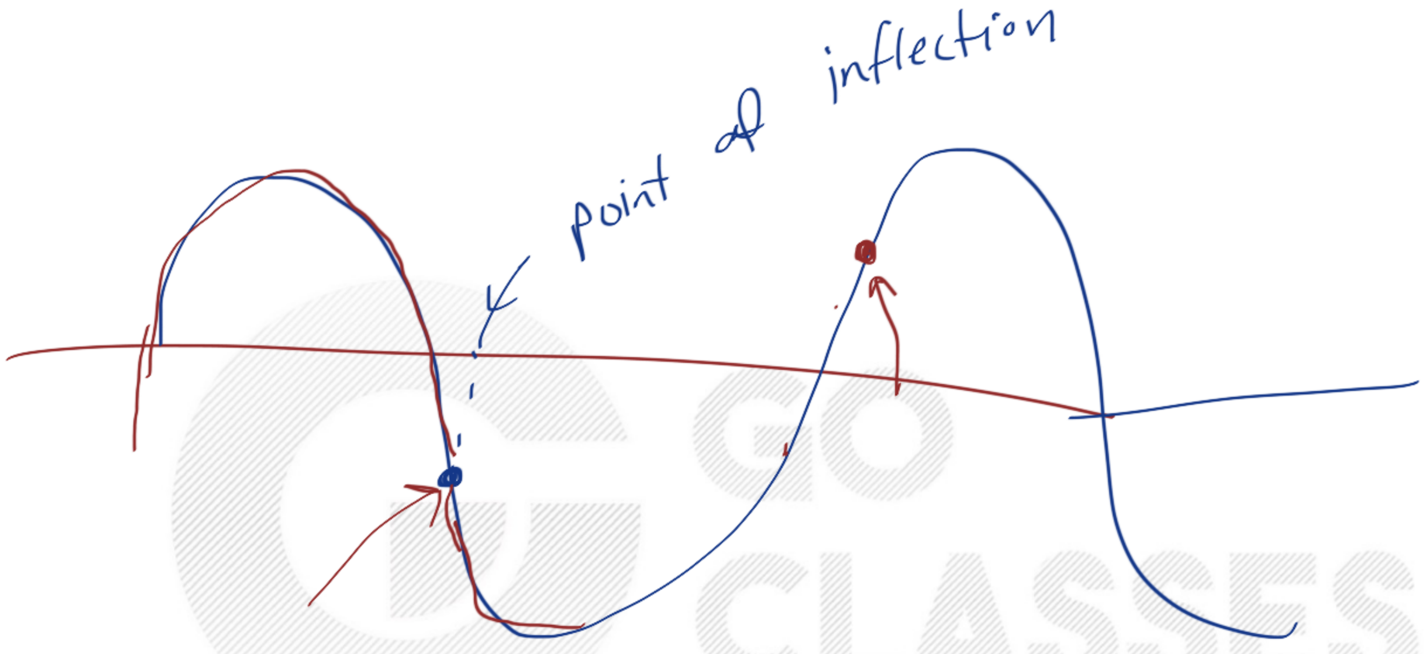
$f'(x) = 0$

$\hookrightarrow x = \_\_, \_\_, \_\_$

point of inflection —

$f'(x) = x^2 = 0$

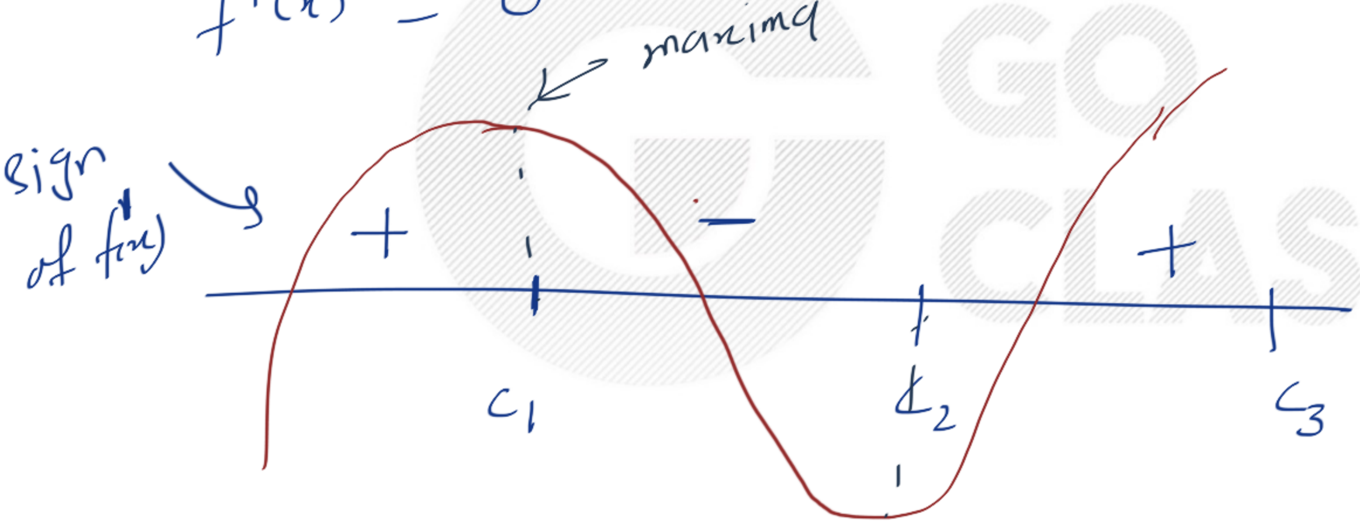
$x = 0$



$\cap \rightleftarrows \cup$ $\quad$ $\underline{f'(x) = 0}$

first derivative test

$f'(x) = 0 \leftarrow x = c_1, c_2, c_3$ } $\leftarrow$ critical points or extremum

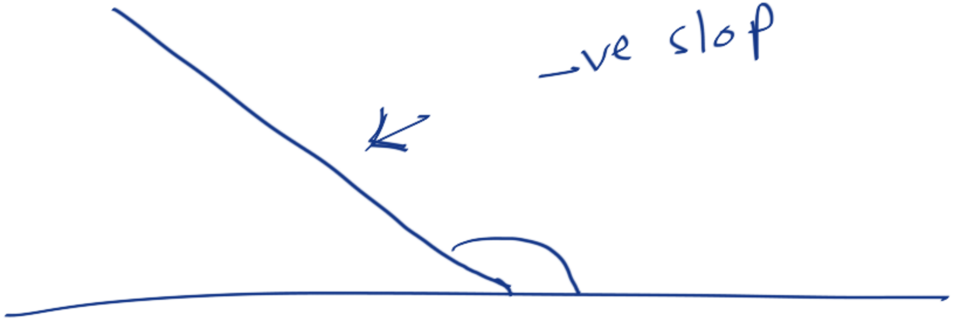


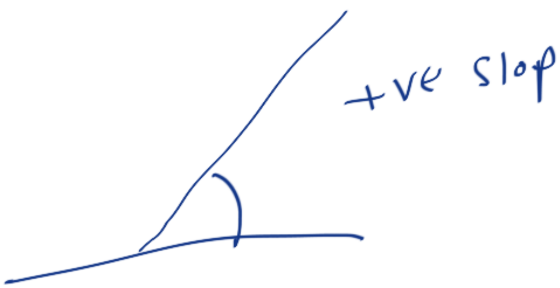
$c_1, c_4 \to$ local maxima

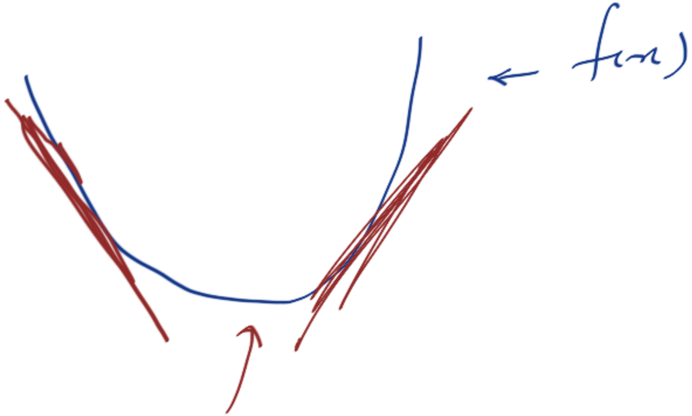
minim

$f(c_1)$ $f(c_4)$

local minima and local maxima







$f'(x) \to -ve$

$\equiv f^n$ is decreasing

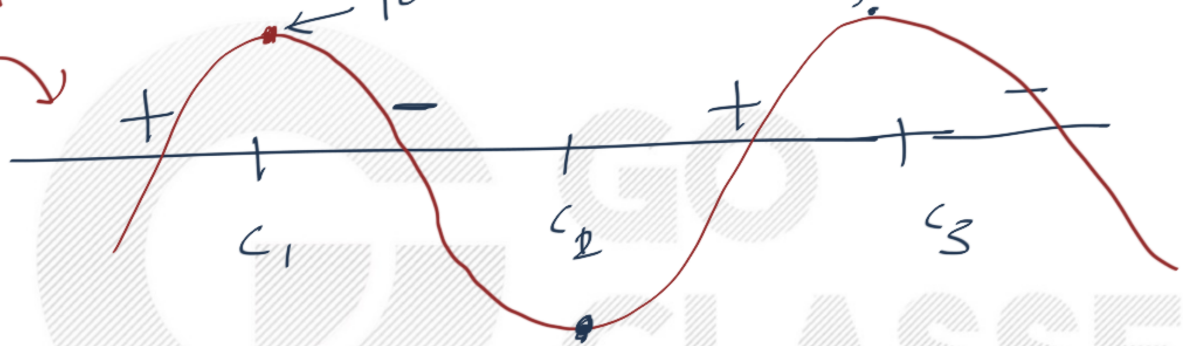
$f'(x) \to +ve$ $\quad f^n$ is increasing

① $f'(x) = 0 \leftarrow$ critical points

first derivative

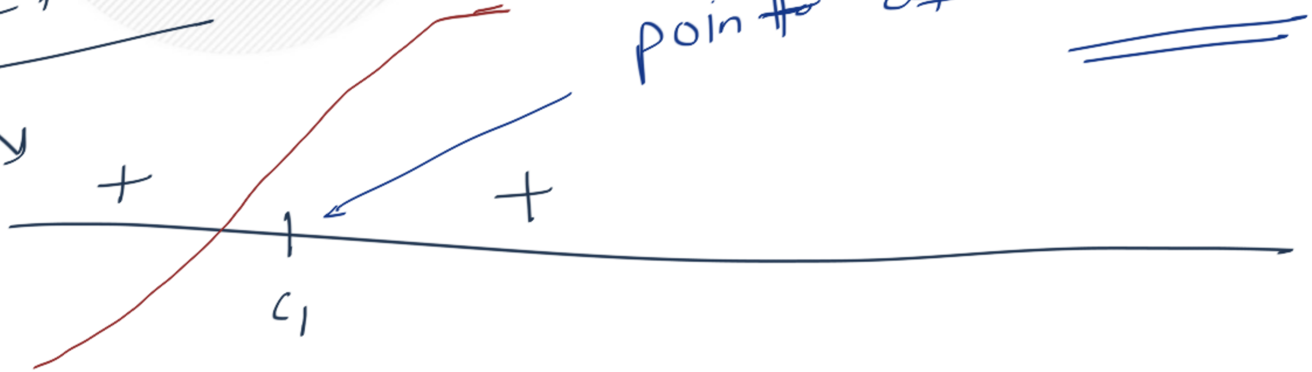
② Sign of $f'(x)$

local maxima



Sign of $f'(x)$

point of inflection



$f(x) = x^3 - 3x + 3$

$f'(x) = 3x^2 - 3 = 0$

$x^2 - 1 = 0$

$x = \pm 1$

$f(1) = 1 - \cancel{3} + \cancel{3} = 1$

min

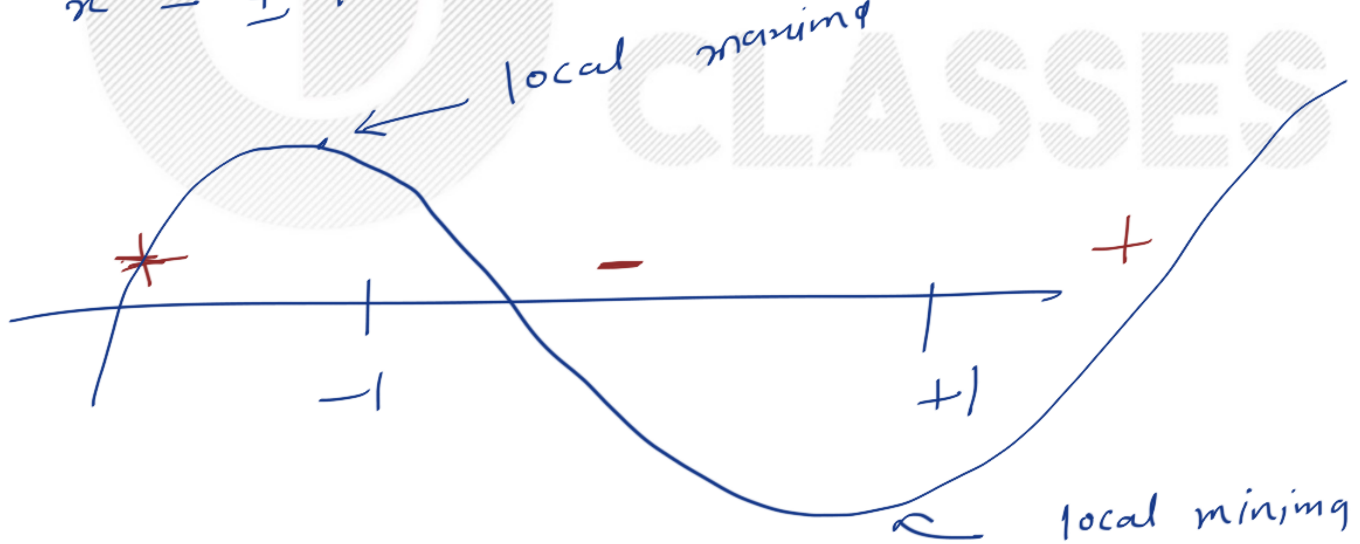
$3(x^2 - 1)$

0

$f(-1) = -1 + \cancel{3} + \cancel{3}$

$= 5$

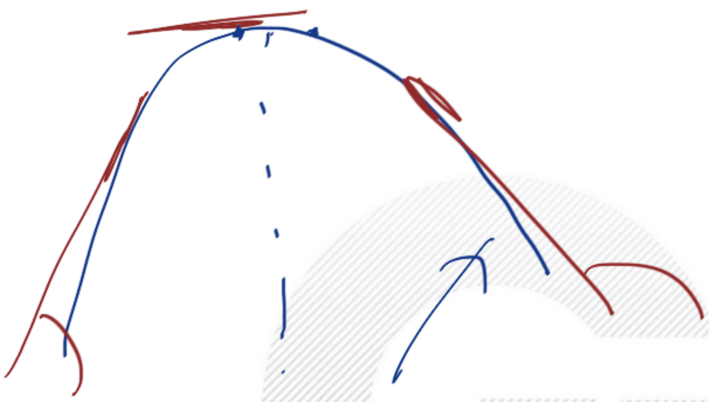
max



$f(x) \longrightarrow f'(x)$ $\qquad\qquad$ $f''(x)$

$f(x)$ is increasing $\Leftrightarrow$ $+ve$

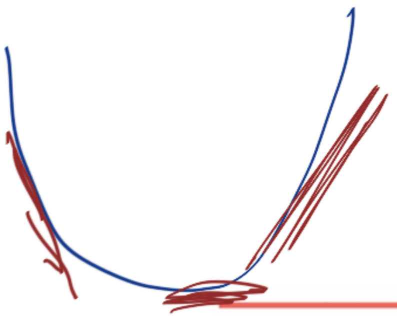
$f(x)$ is decreasing $\Leftrightarrow$ $f'(x)$ $-ve$



$f(x)$

$f'(x)$ is decreasing $\Rightarrow$ $f''(x)$

$f''(x) < 0$



$f'(x)$ is increasing $\Rightarrow$ $f''(x) > 0$

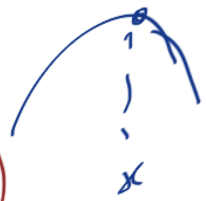
we can't say $\rightarrow$ $f''(x) = 0$

$$f(x+h) = f(x) + \cancel{h \cdot f'(x)}^{\,0} + \frac{h^2}{2} f''(x) + \underbrace{\frac{h^3}{\ldots}\cdots}_{0}$$

$$h \to 0$$

$$\boxed{f(x+h) < f(x)}$$

$$f(x) + \frac{h^2}{2} f''(x) < f(x) \Rightarrow f''(x) < 0$$



2^{nd} derivative test

① $f'(x) = 0$ $\qquad x = c_1, c_2, c_3$

② $f''(x) > 0$ $\qquad$ minima

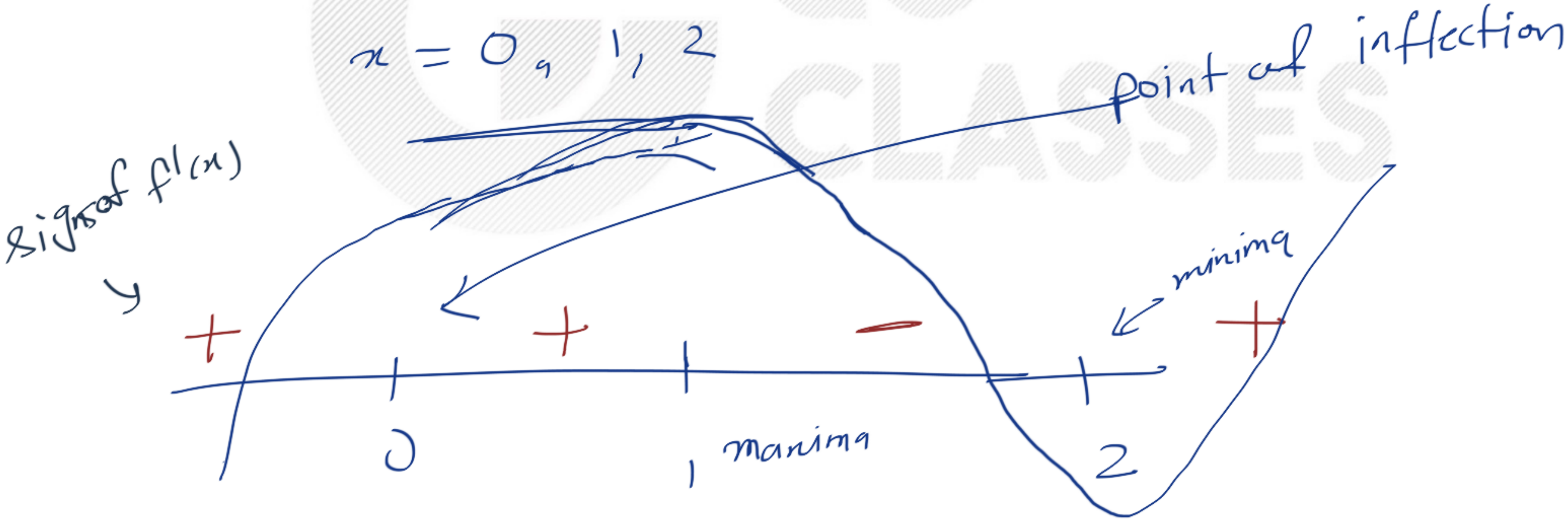
$f''(x) < 0$ $\qquad$ maxima

$f''(x) = 0$ $\qquad$ we can't say

$$f(x) = 12x^5 - 45x^4 + 40x^3 + 40$$

$$f'(x) = 60x^2(x-1)(x-2) = 0$$

$$x = 0, 1, 2$$



$f'(x) = 60x^2(x-1)(x-2)$

$(x^3 - x^2)(x-2) = x^4 - x^3 - 2x^3 + 2x^2$

$x^4 - 3x^3 + 2x^2$

$f''(x) = 60(4x^3 - 9x^2 + 4x)$

$4 - 9 + 4$

$f''(0) = 0 \rightarrow$ can't say

$f''(1) < 0 \rightarrow$ maxima

$f''(2) > 0 \rightarrow$ minima