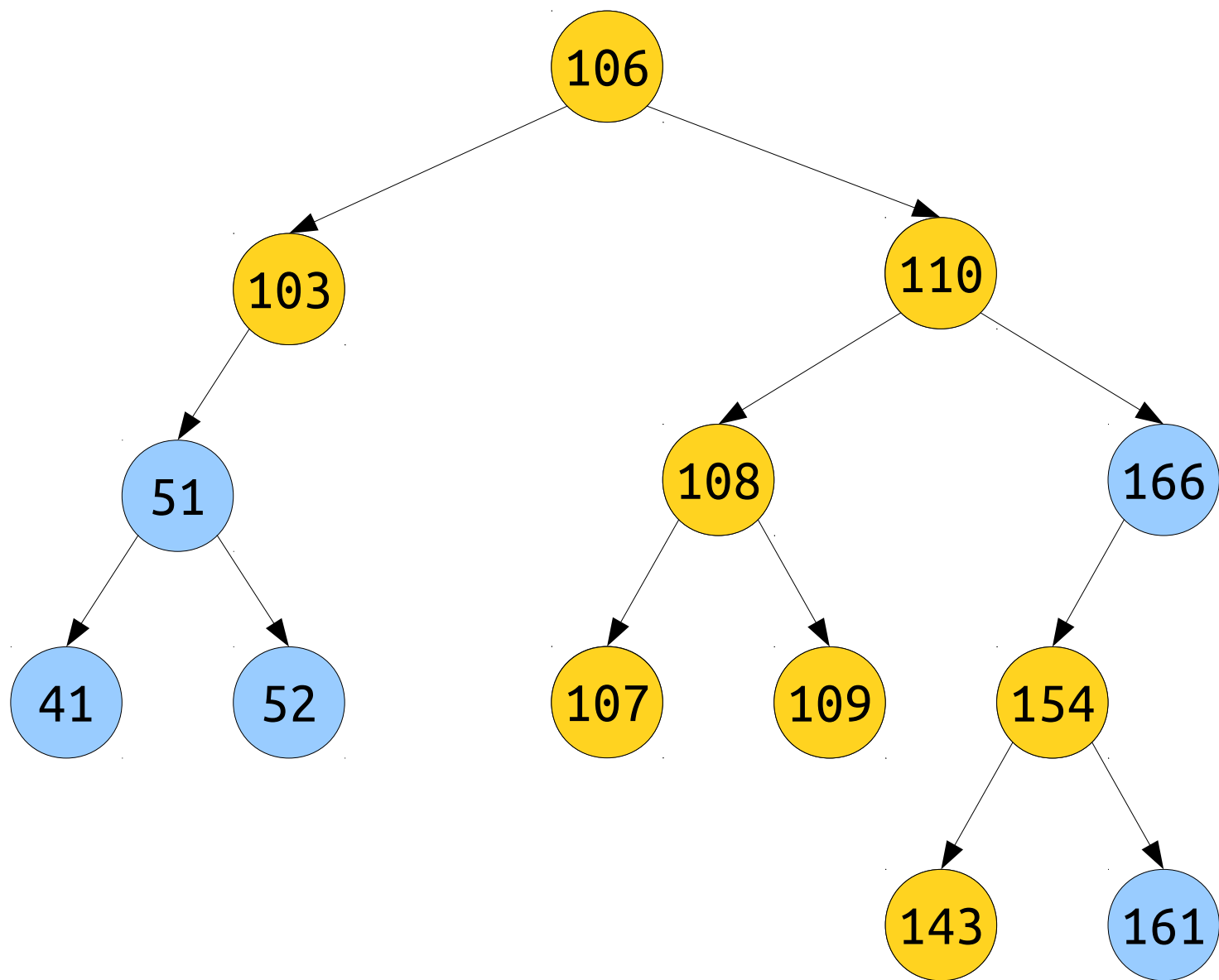
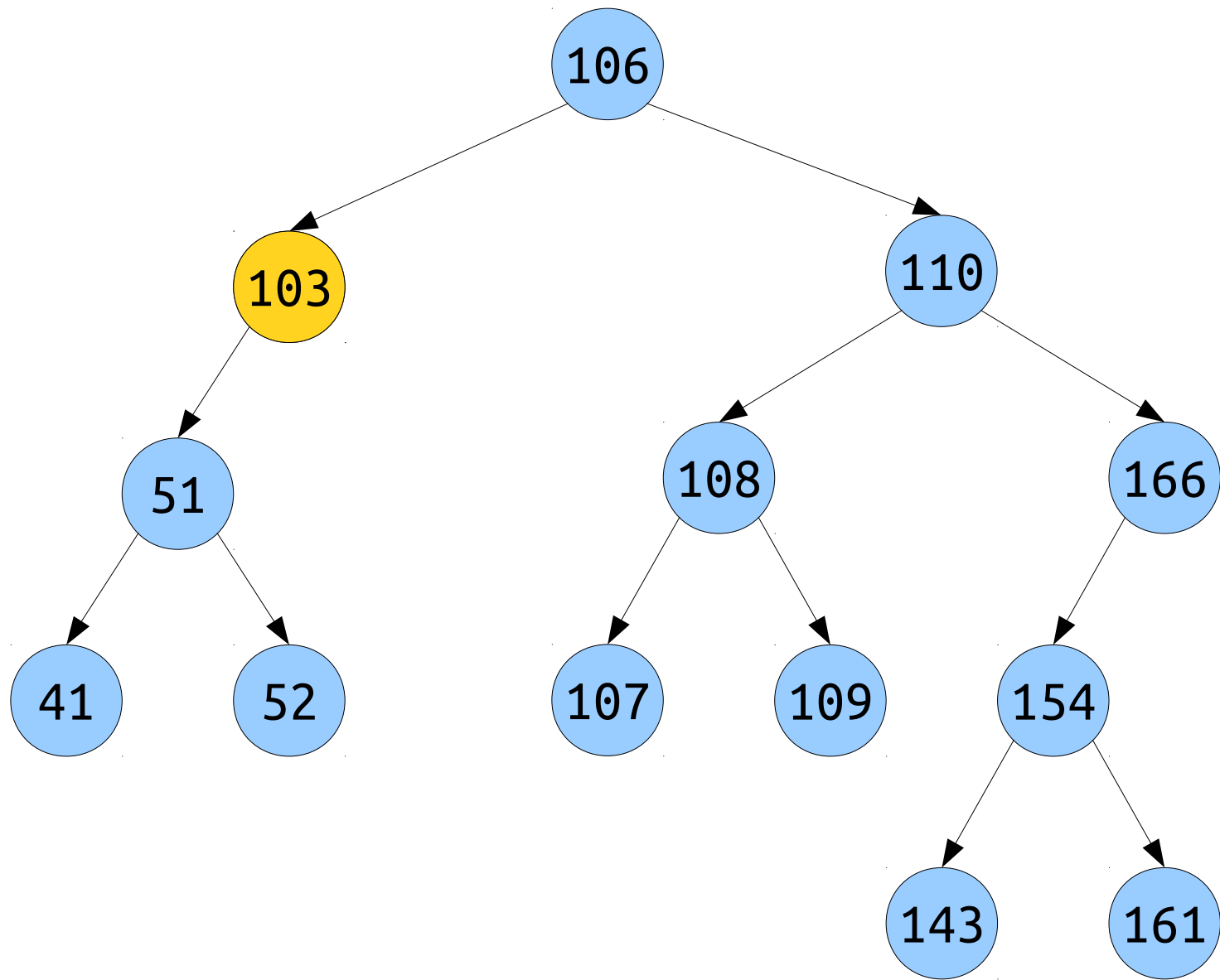


# Range Searches

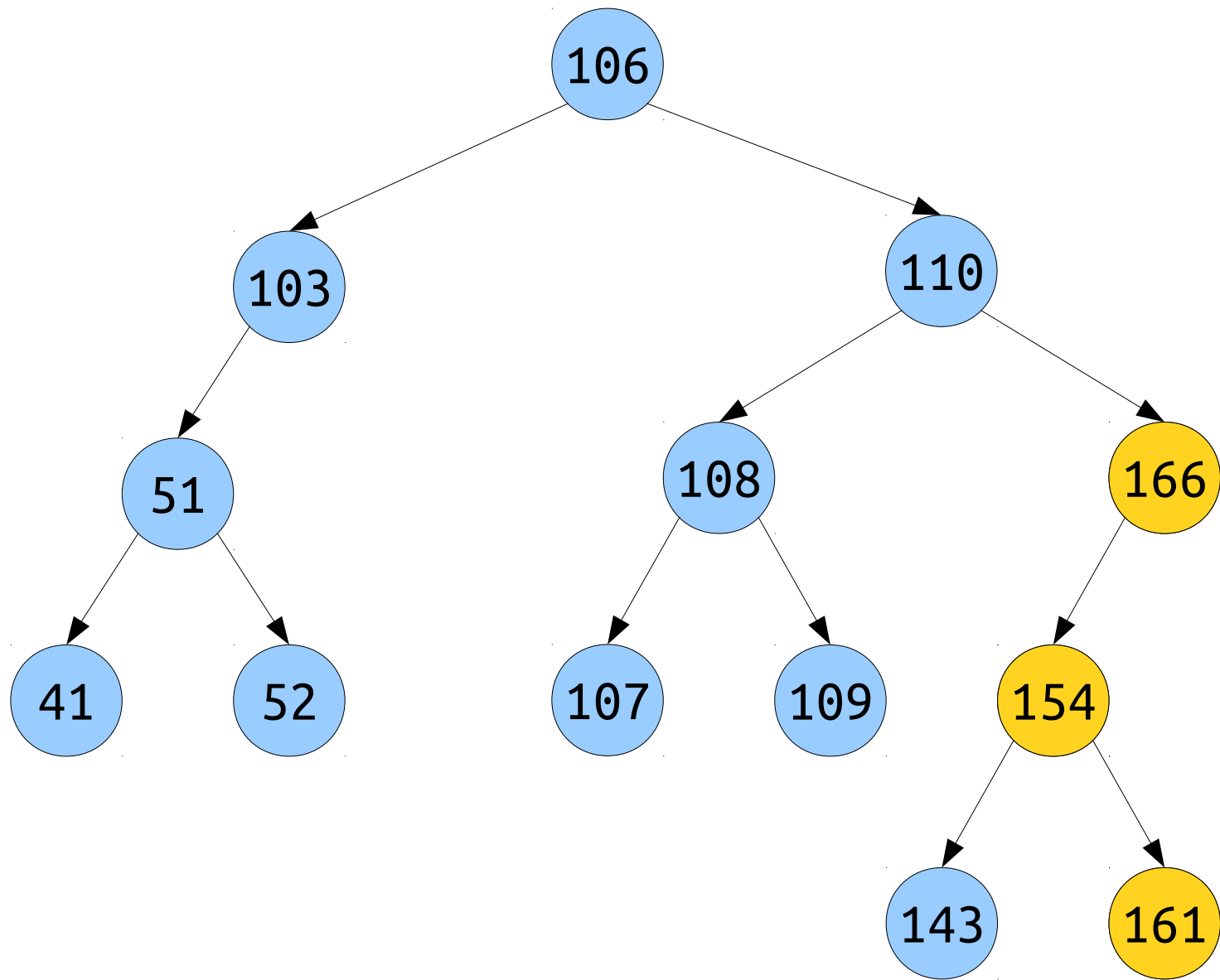




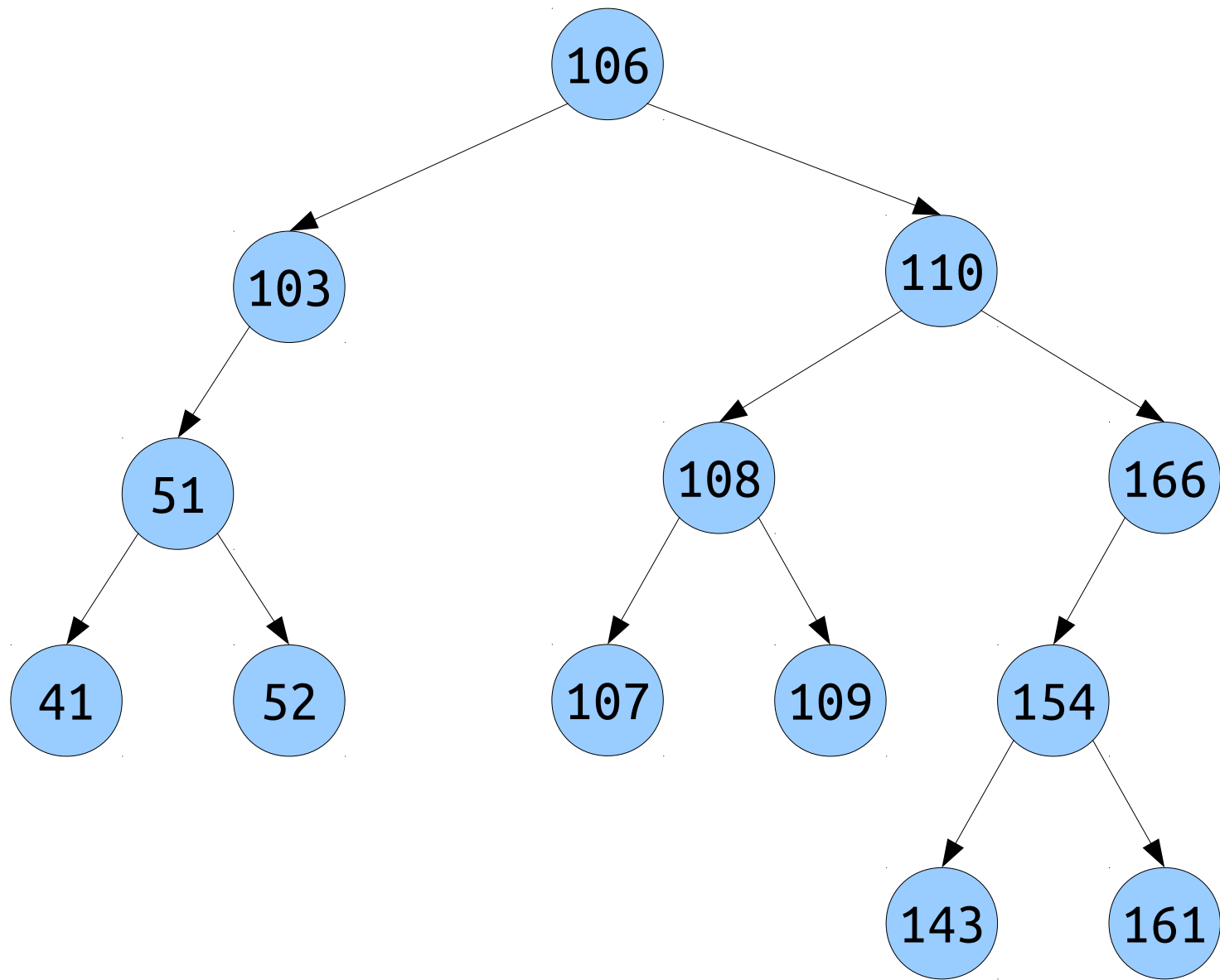
Find all elements in this tree in the range **[103, 154]**.



Find all elements in this tree in the range **[99, 105]**.



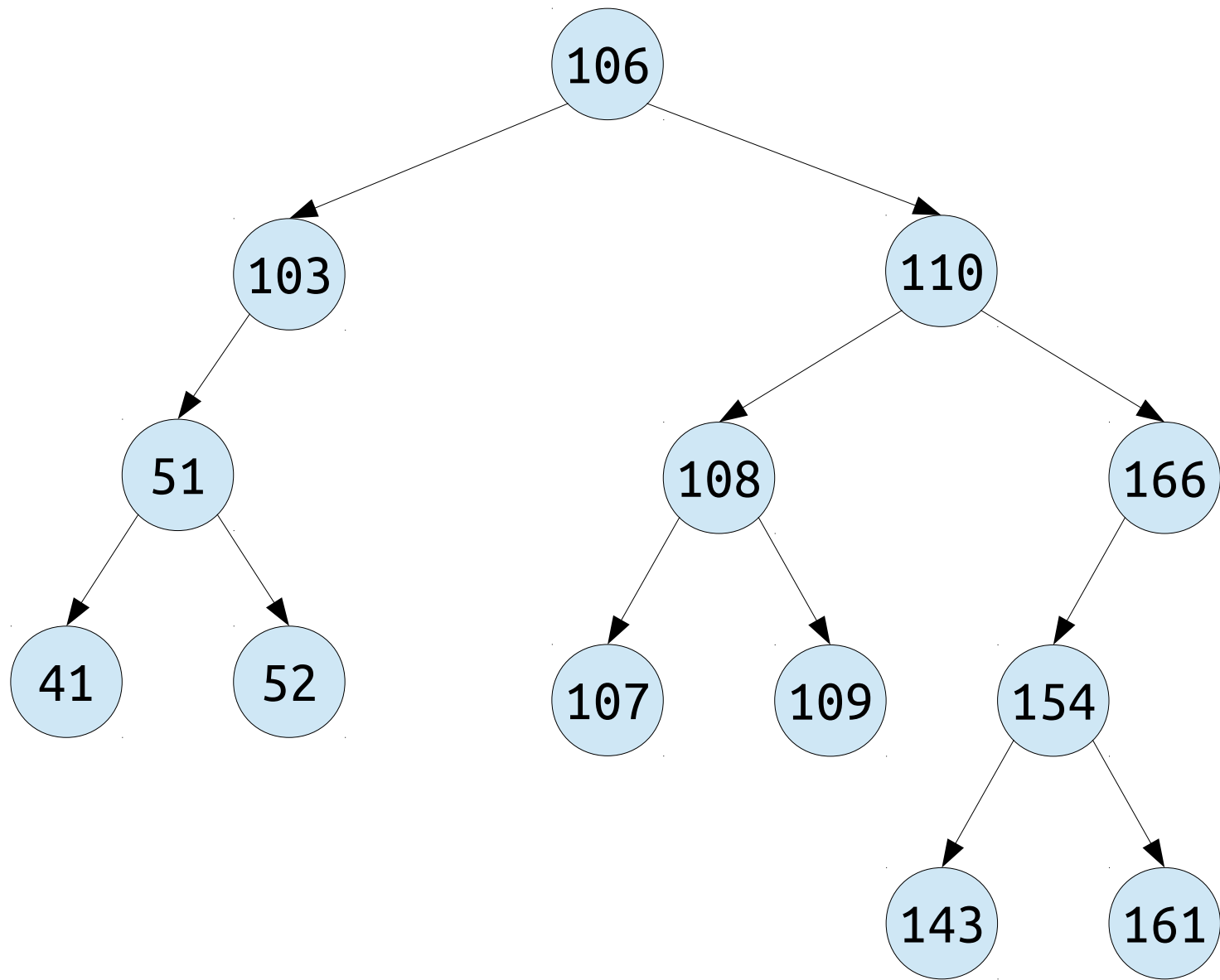
Find all elements in this tree in the range **[150, 170]**.



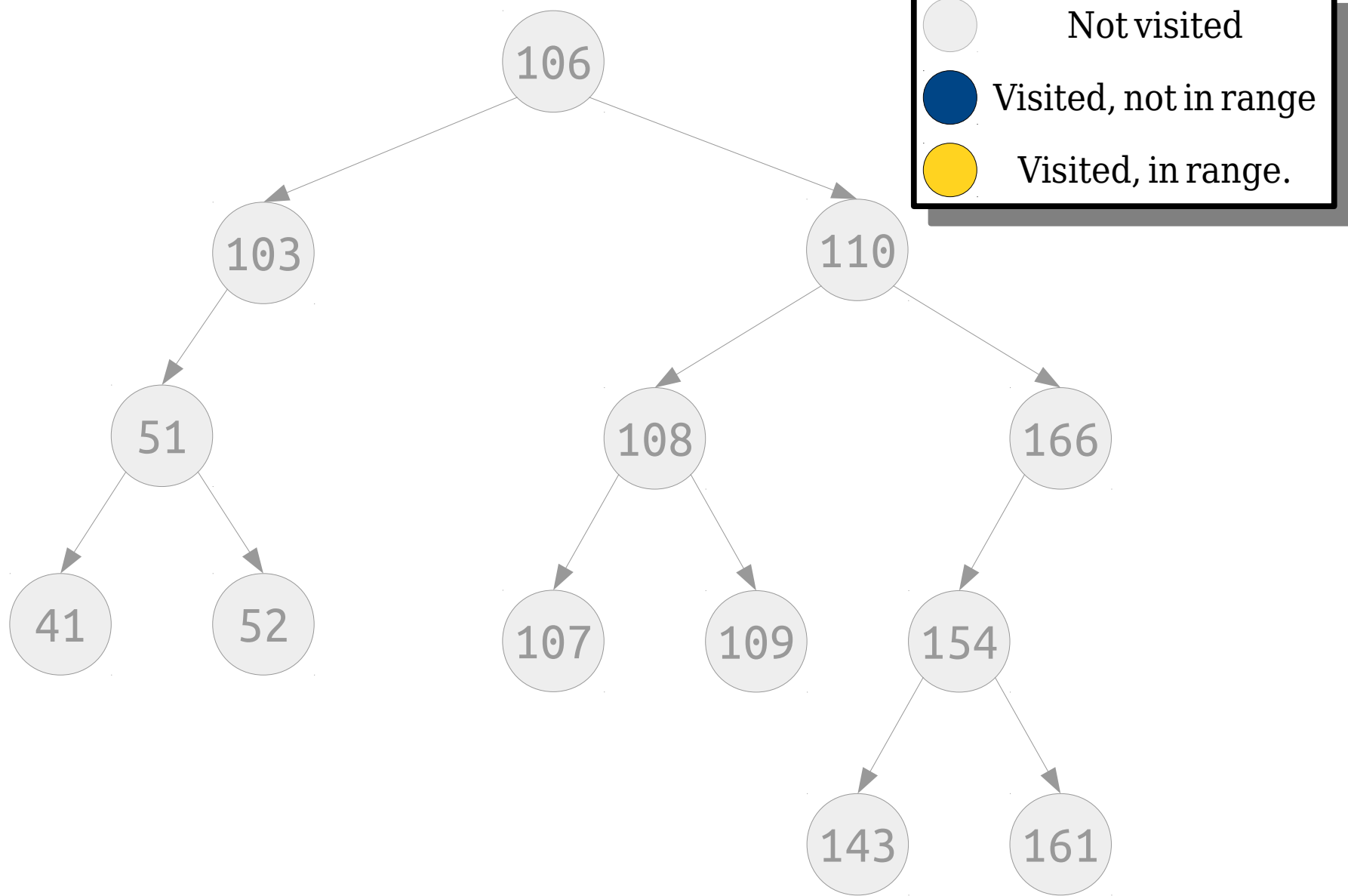
Find all elements in this tree in the range **[137, 138]**.

# Range Searches

- We can use BSTs to do *range searches*, in which we find all values in the BST within some range.
- For example:
  - If the values in the BST are dates, we can find all events that occurred within some time window.
  - If the values in the BST are number of diagnostic scans ordered, we can find all doctors who order a disproportionate number of scans.

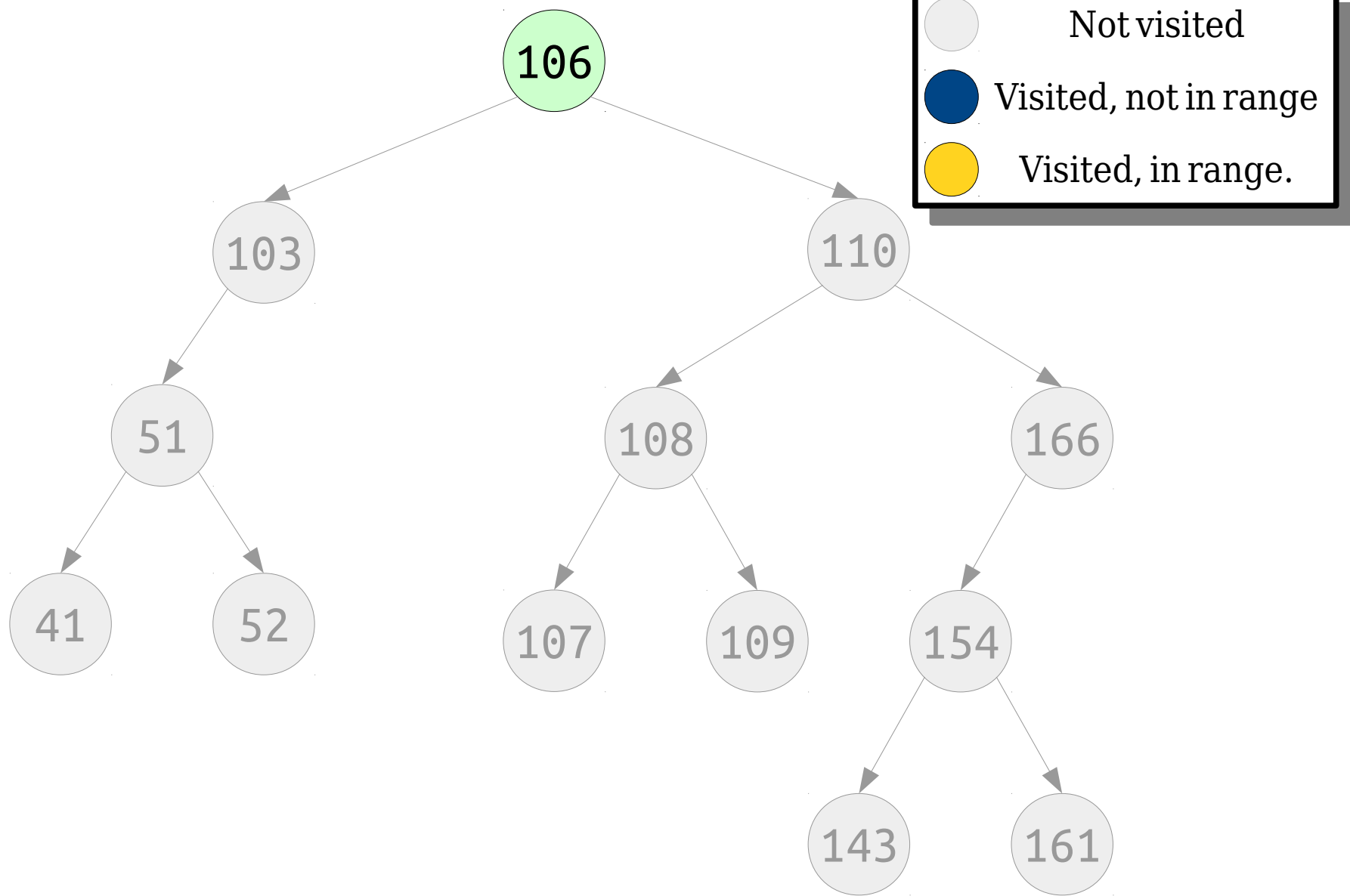


Find all elements in this tree in the range **[109, 163]**.

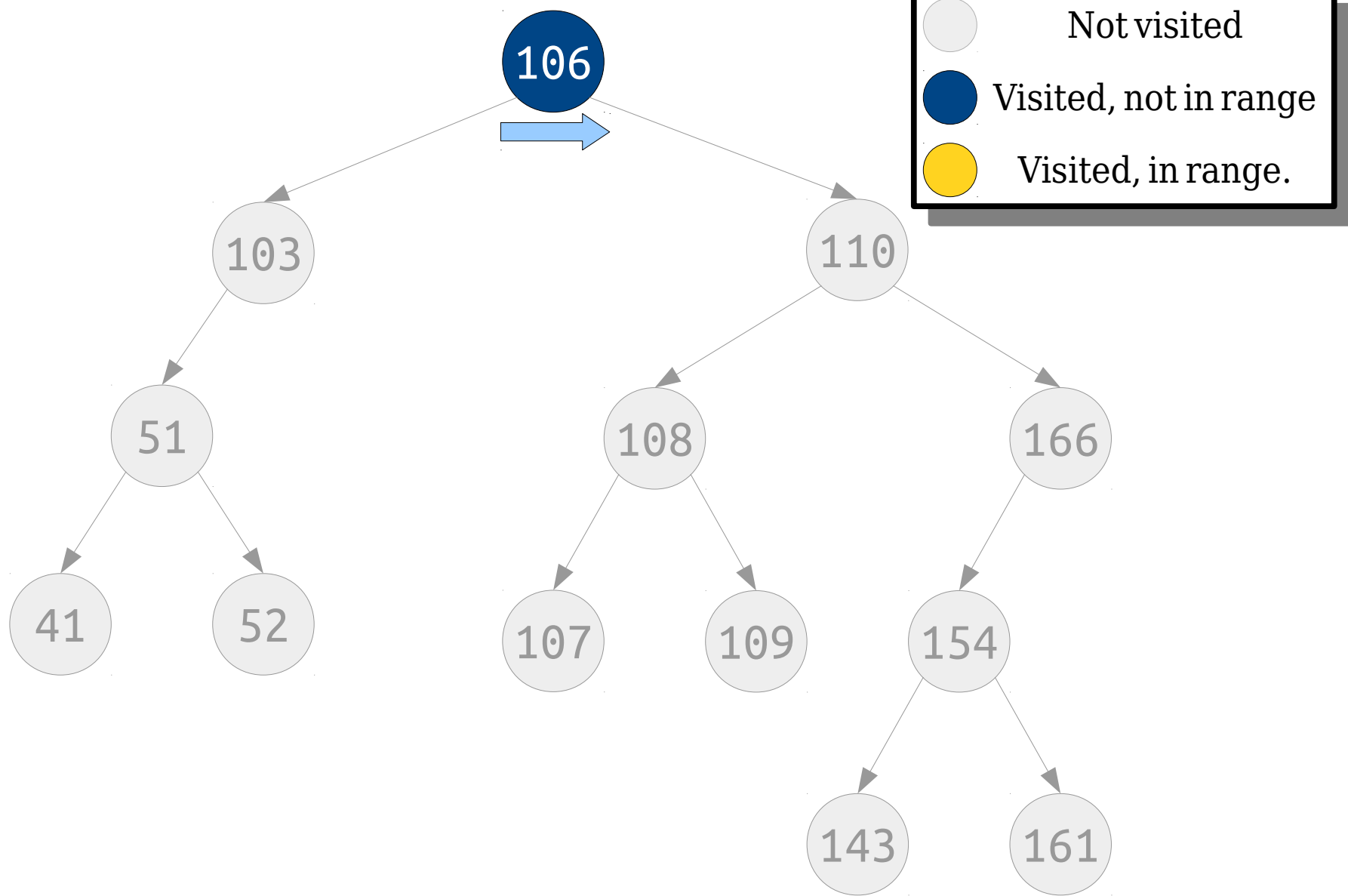


Find all elements in this tree in the range **[109, 163]**.

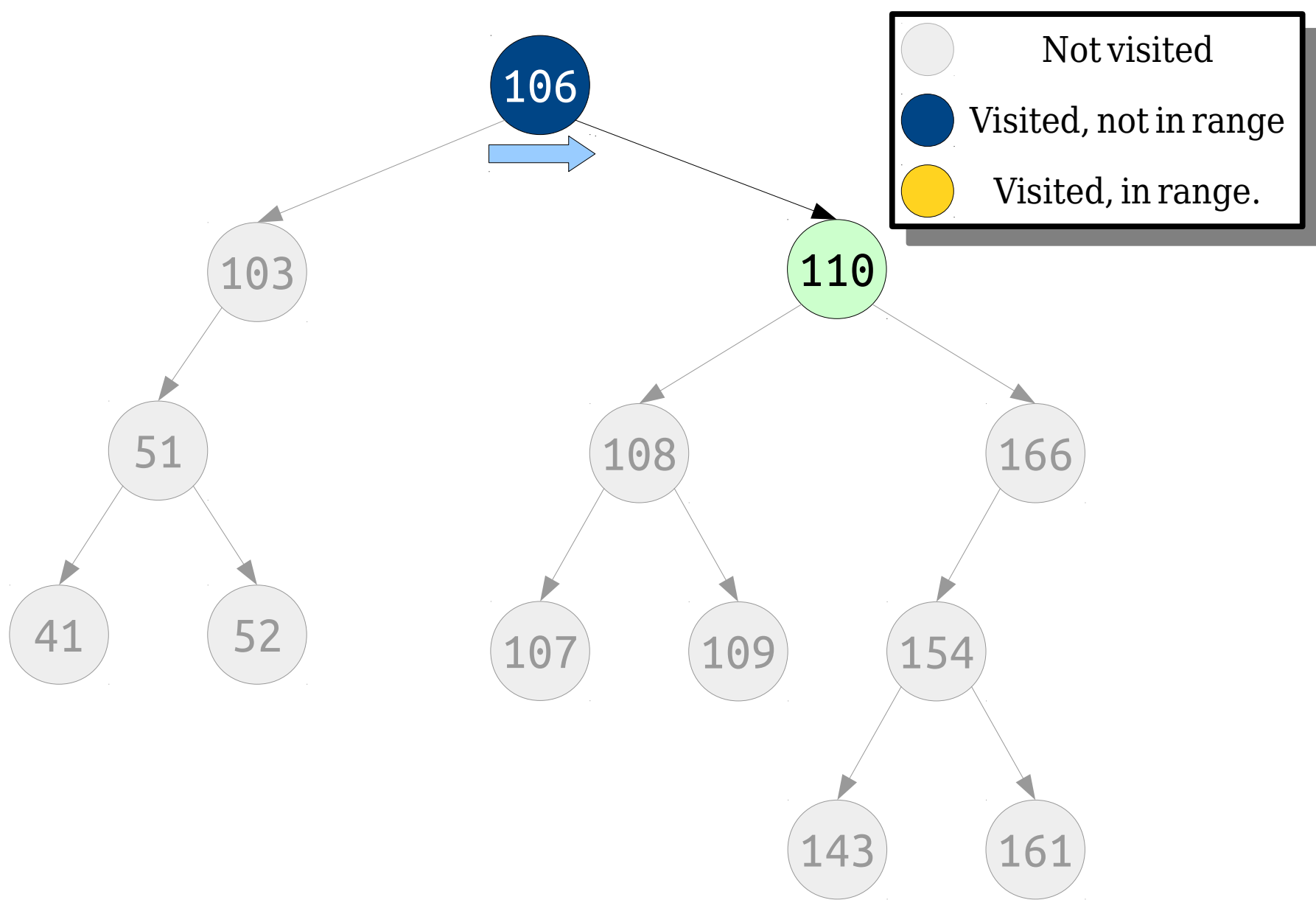




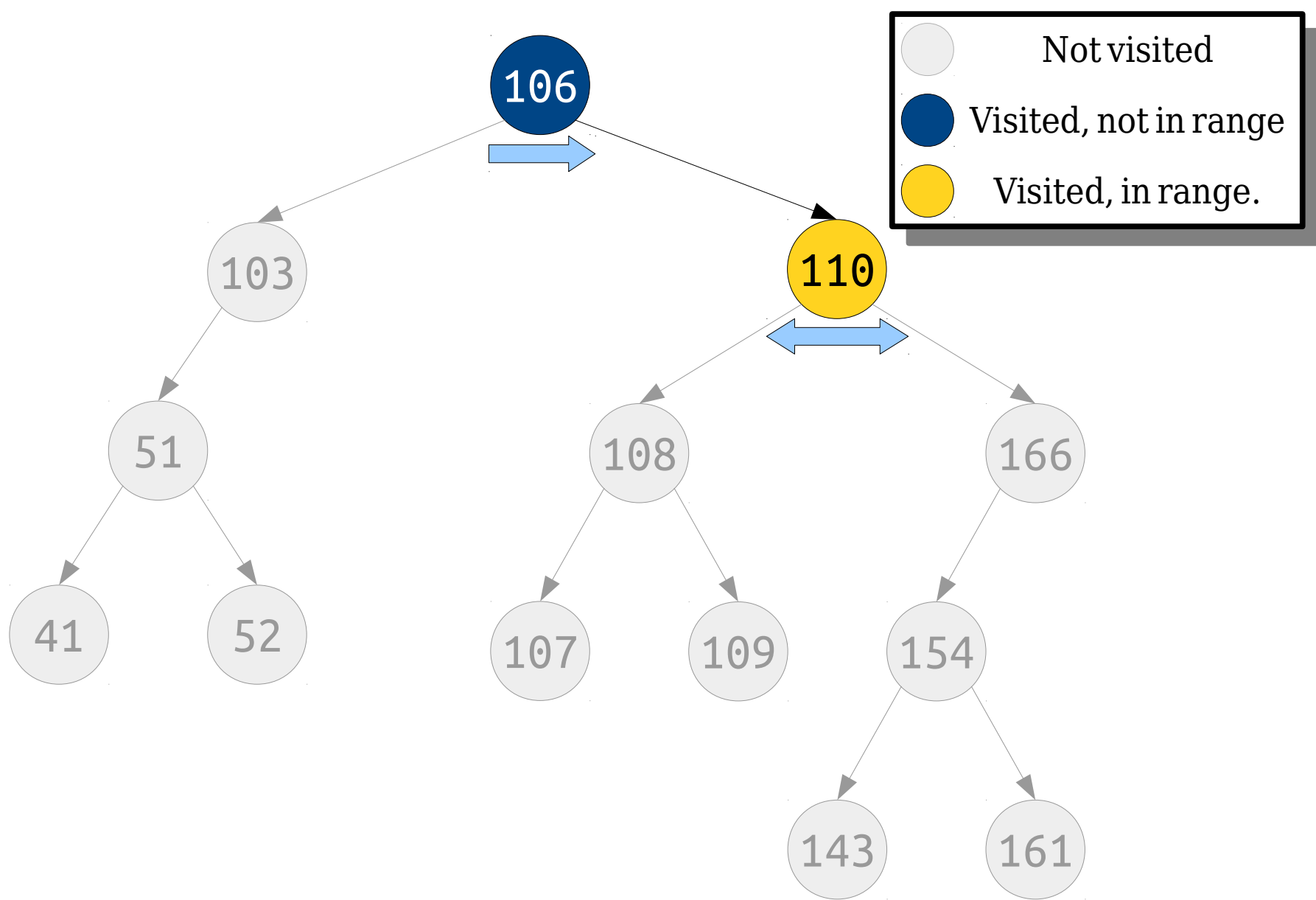
Find all elements in this tree in the range **[109, 163]**.



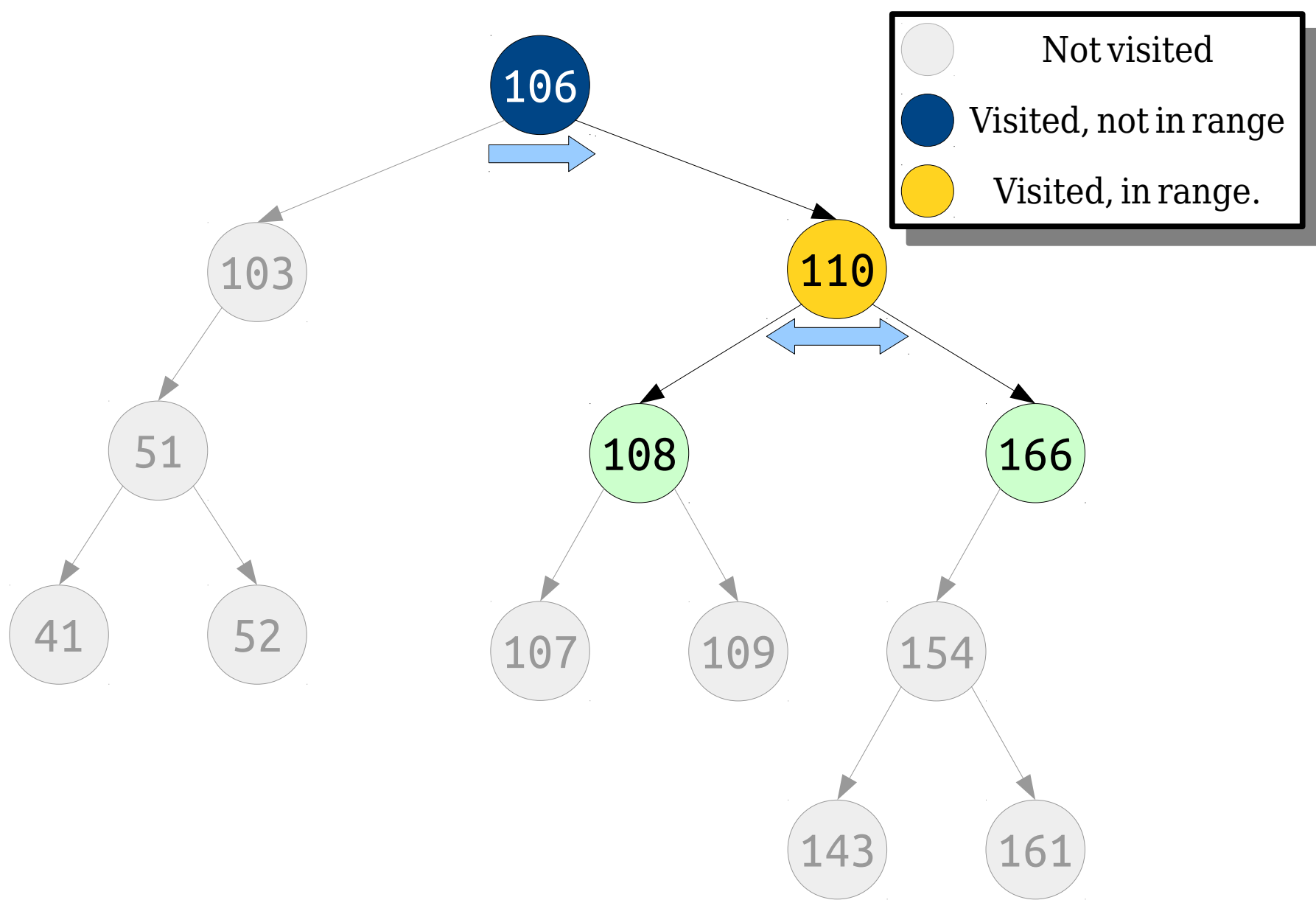
Find all elements in this tree in the range **[109, 163]**.



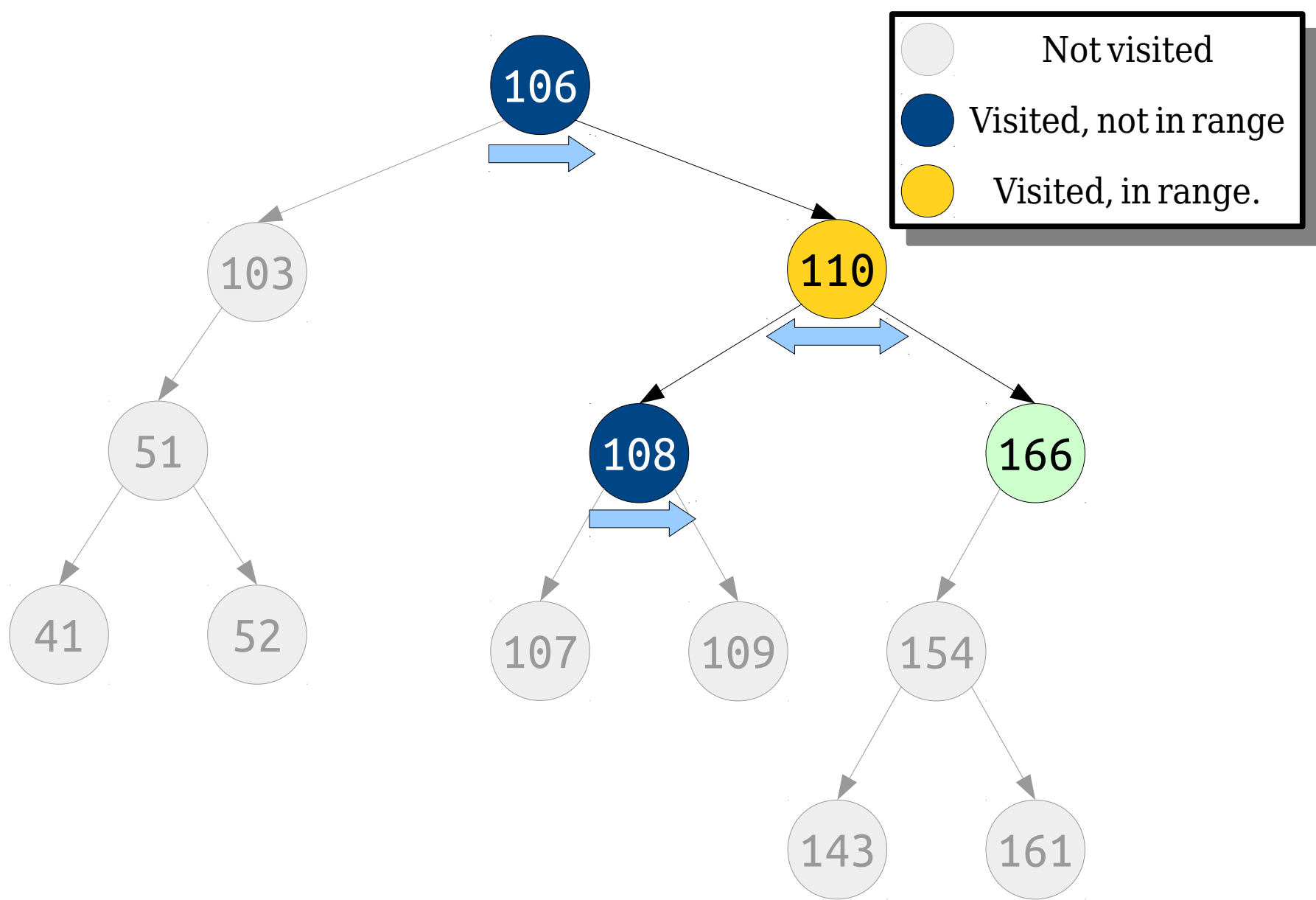
Find all elements in this tree in the range **[109, 163]**.



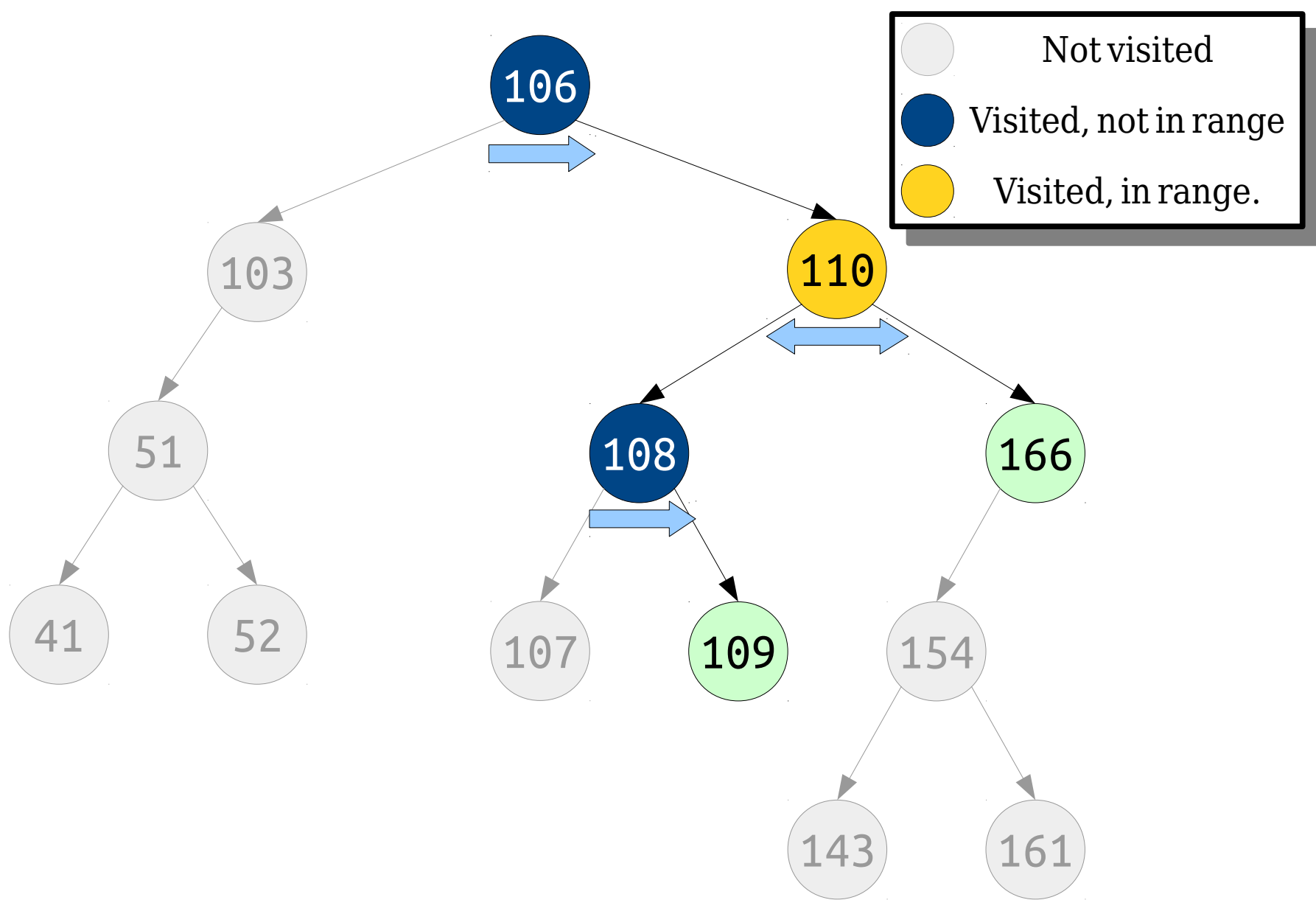
Find all elements in this tree in the range **[109, 163]**.



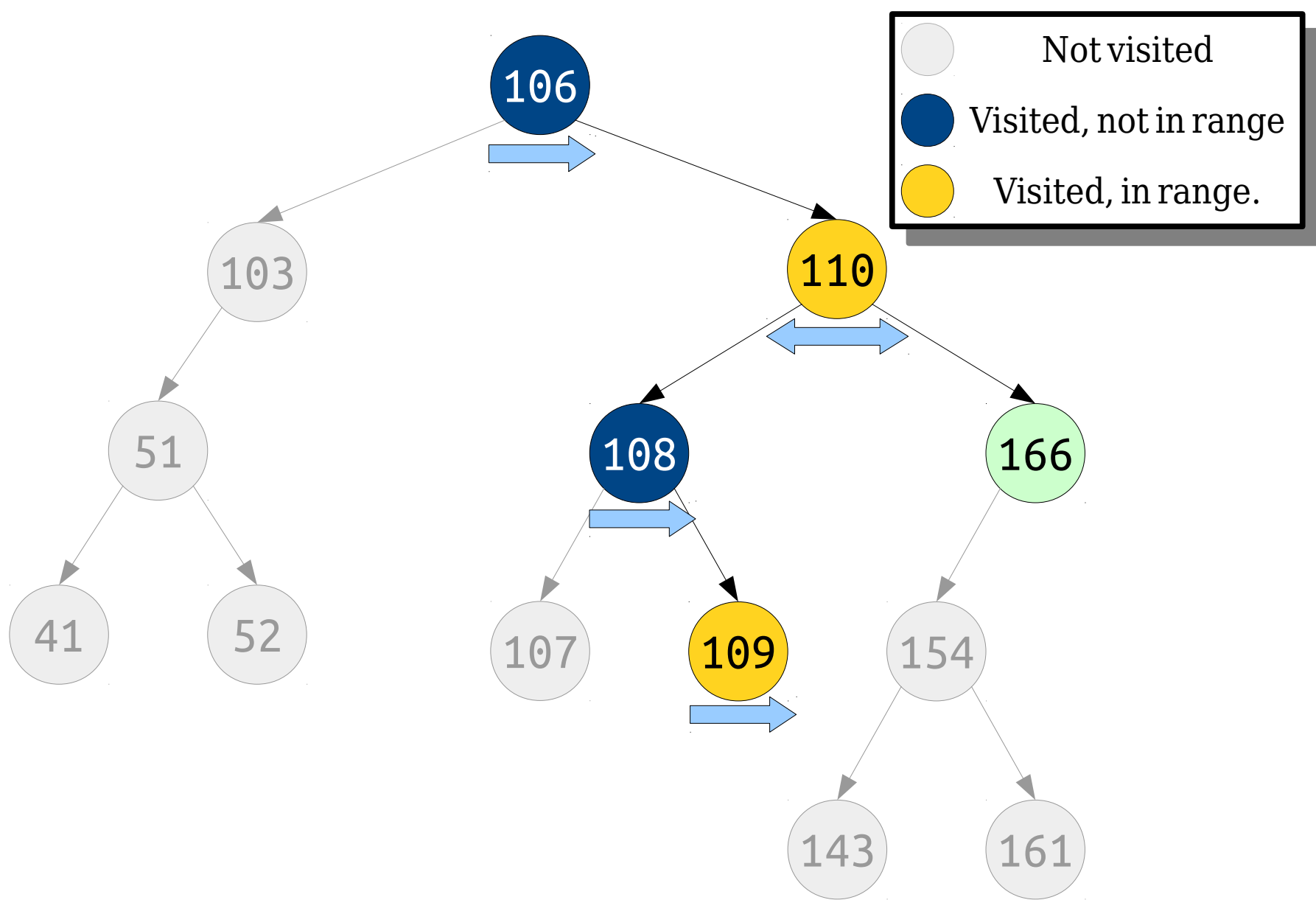
Find all elements in this tree in the range **[109, 163]**.



Find all elements in this tree in the range **[109, 163]**.

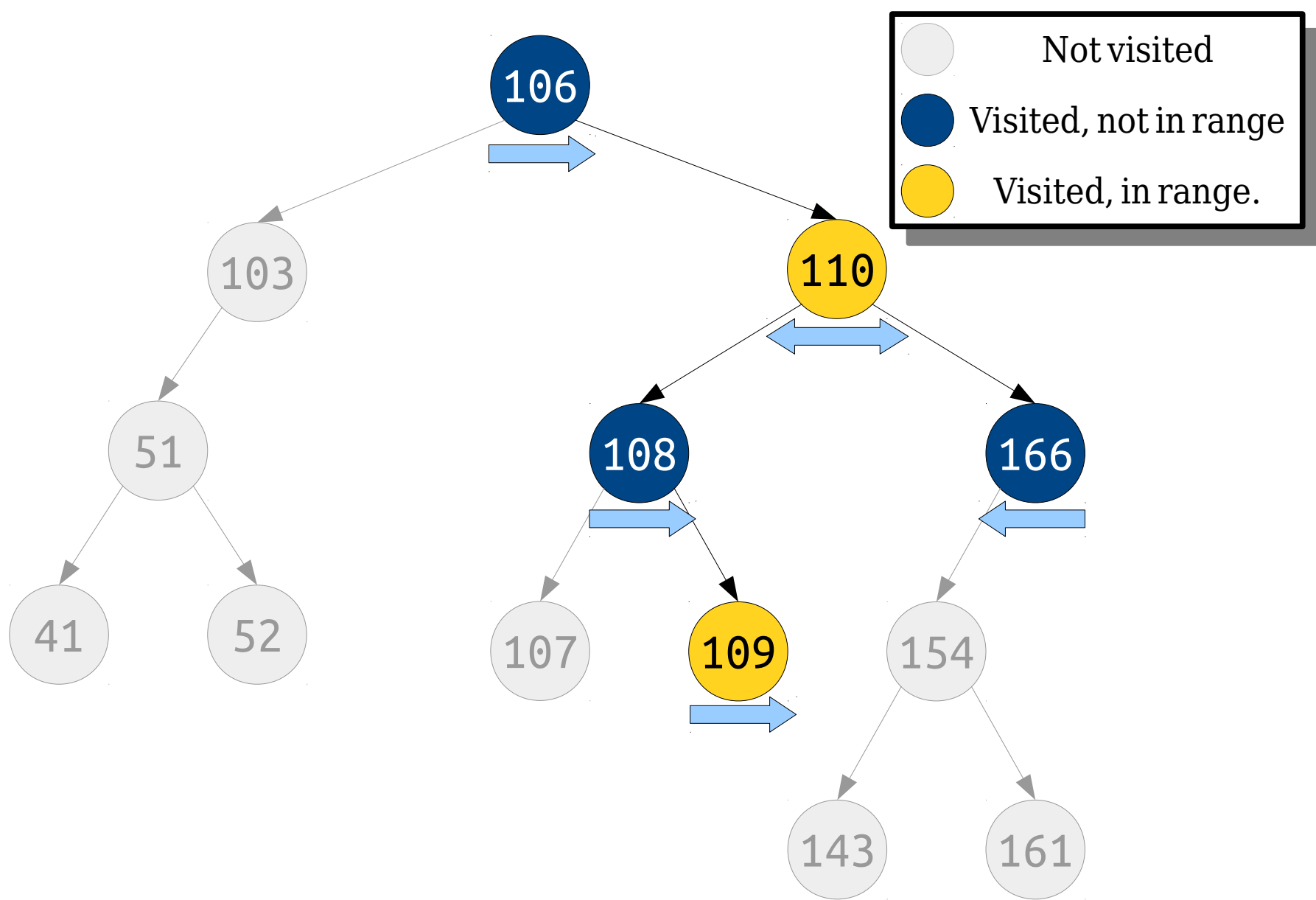


Find all elements in this tree in the range **[109, 163]**.

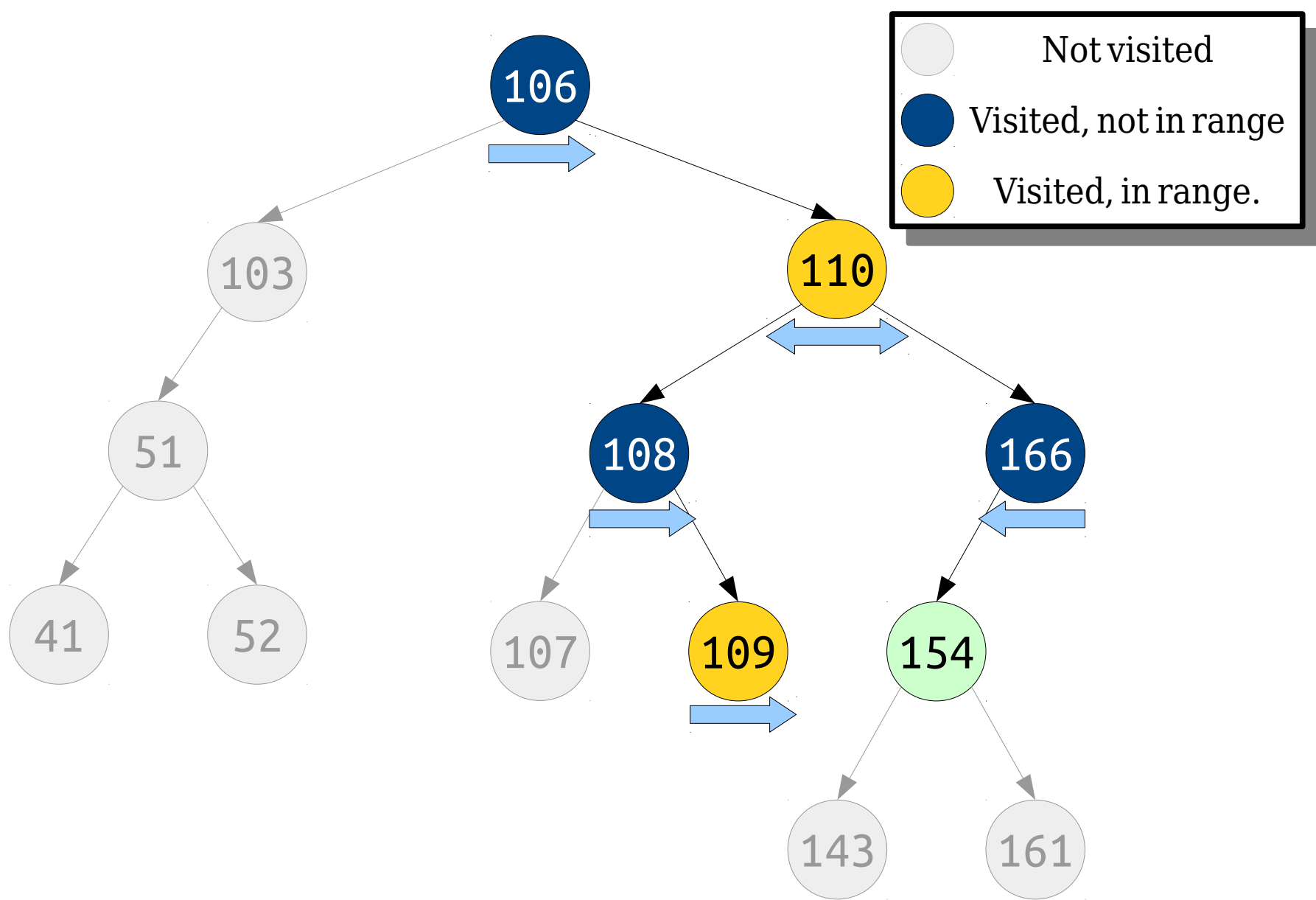


Find all elements in this tree in the range **[109, 163]**.

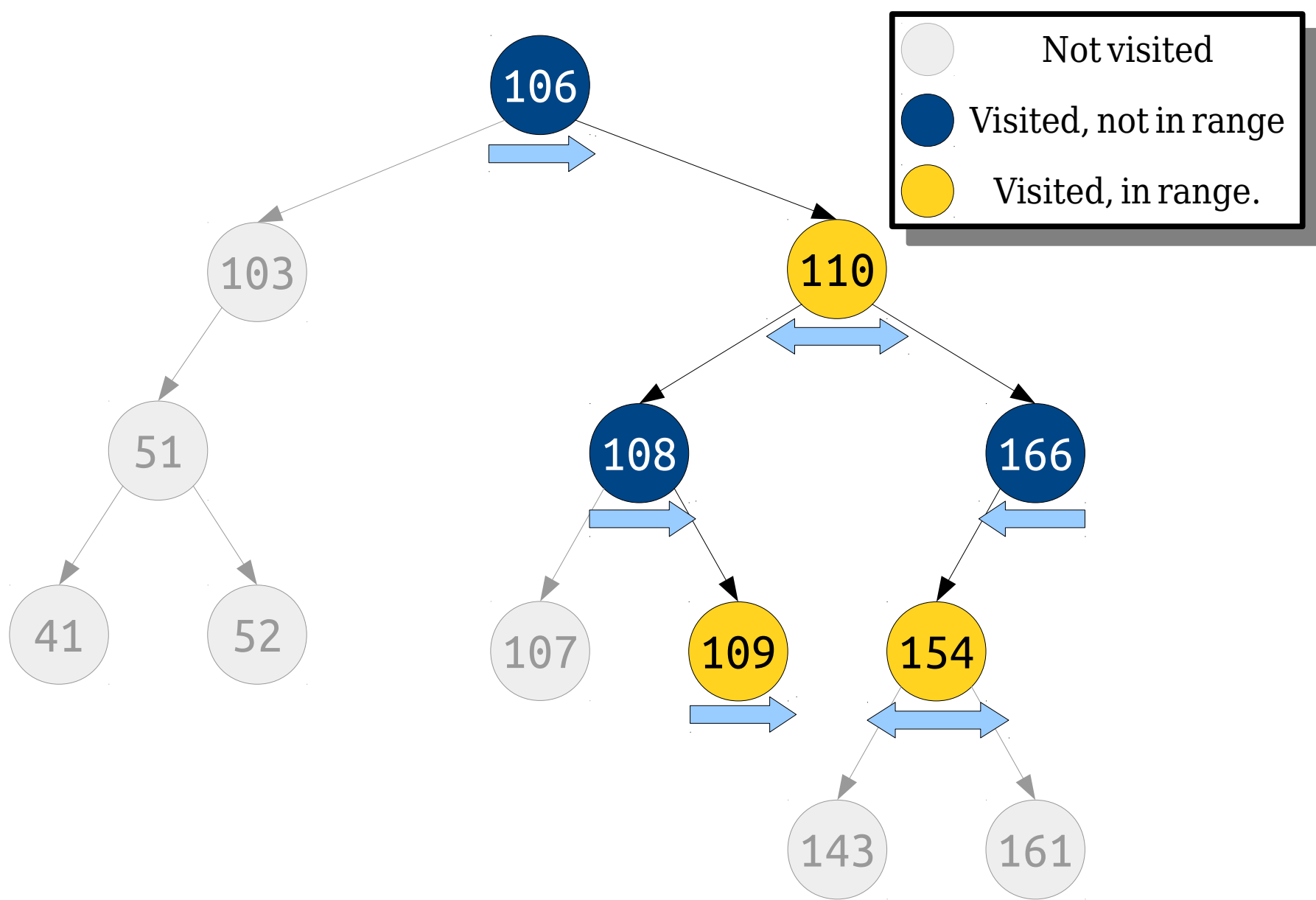




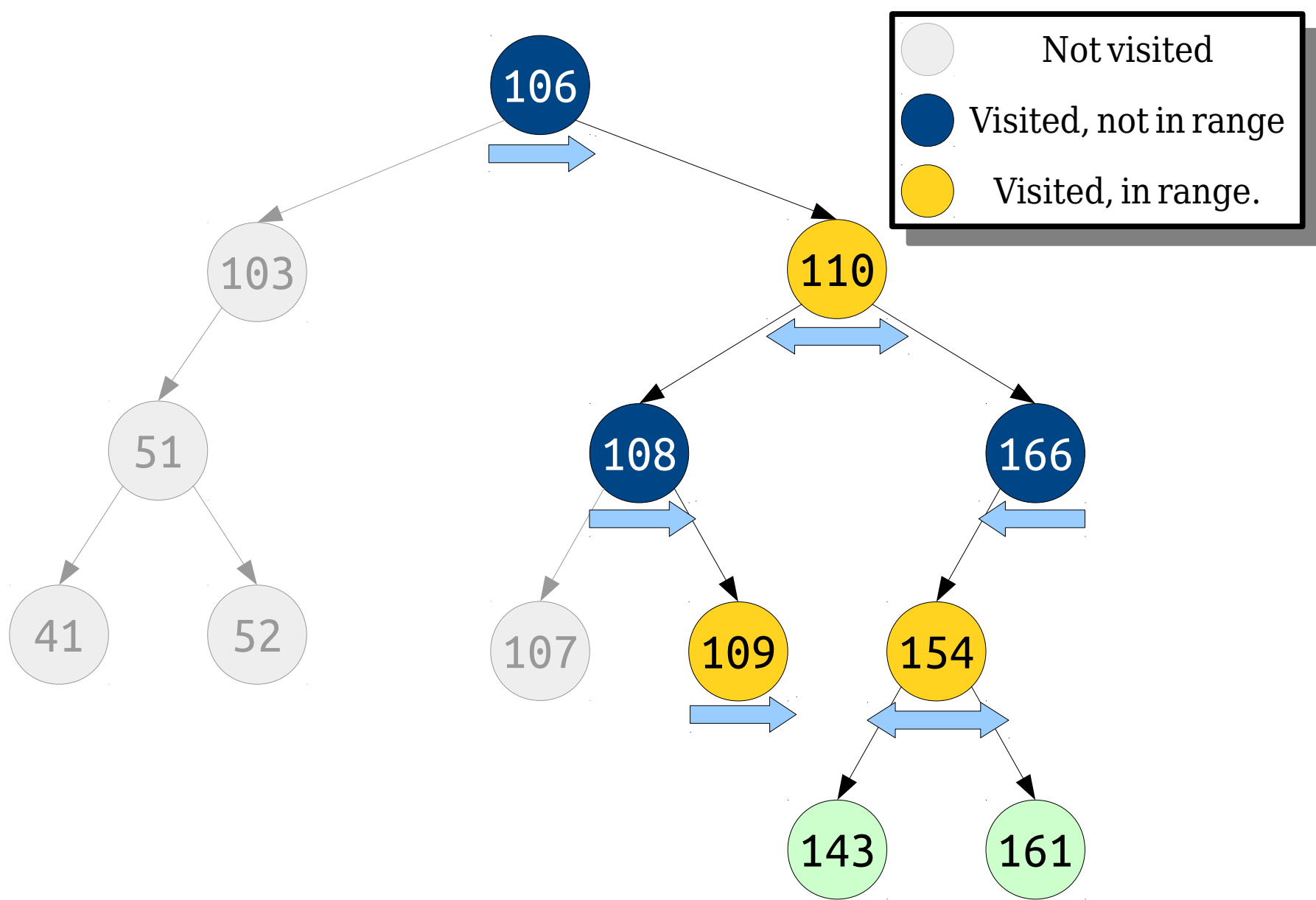
Find all elements in this tree in the range **[109, 163]**.



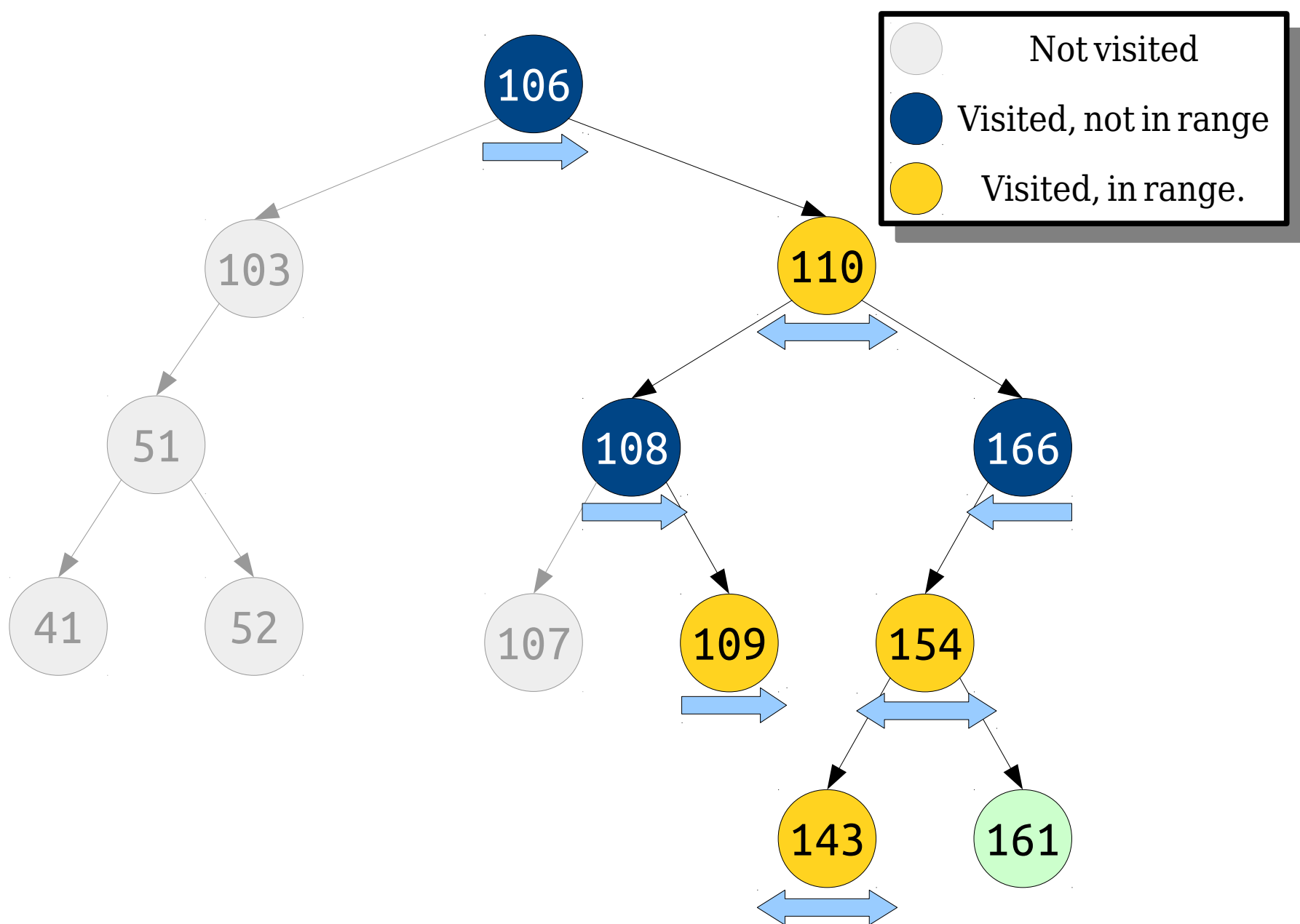
Find all elements in this tree in the range **[109, 163]**.



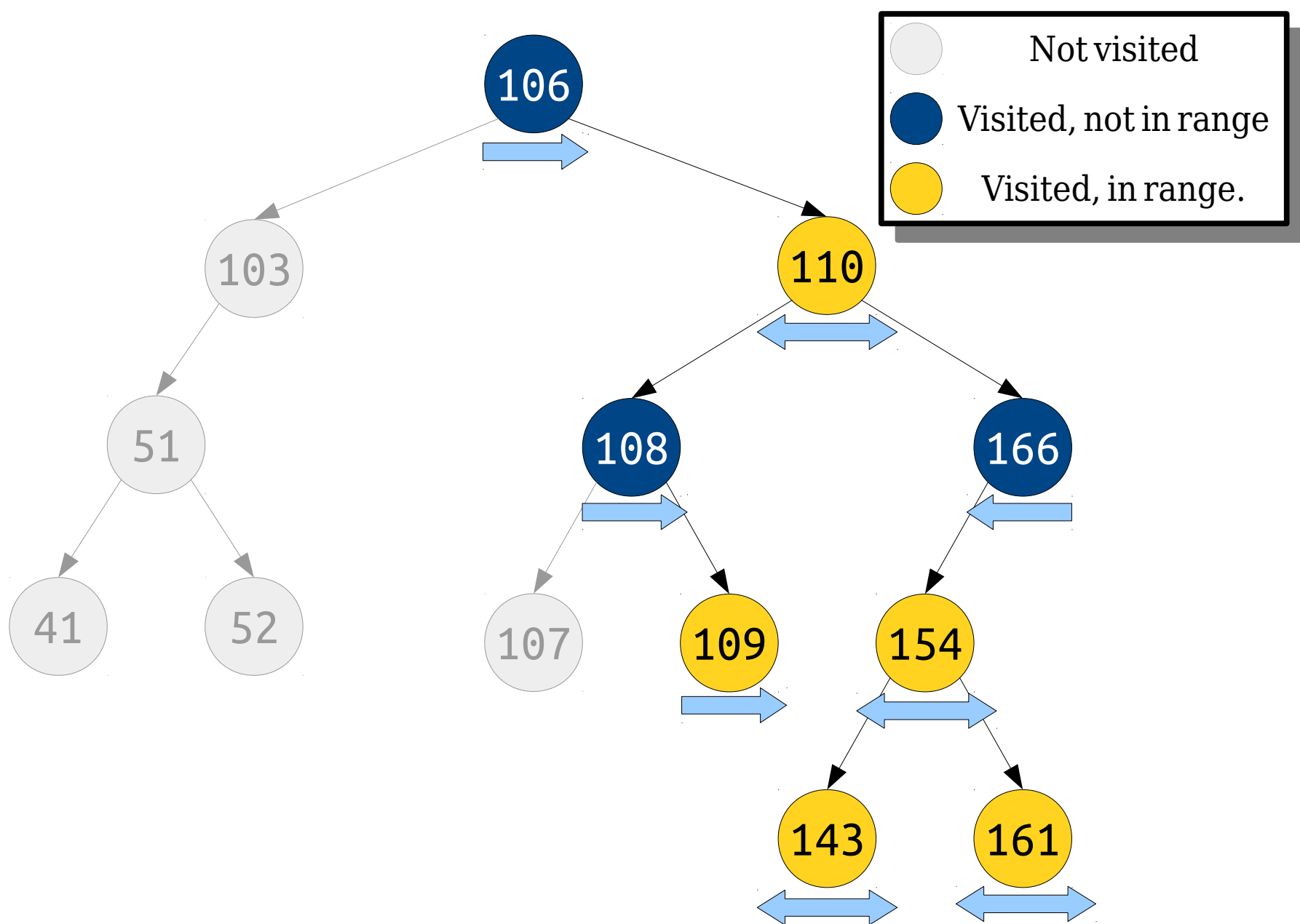
Find all elements in this tree in the range **[109, 163]**.



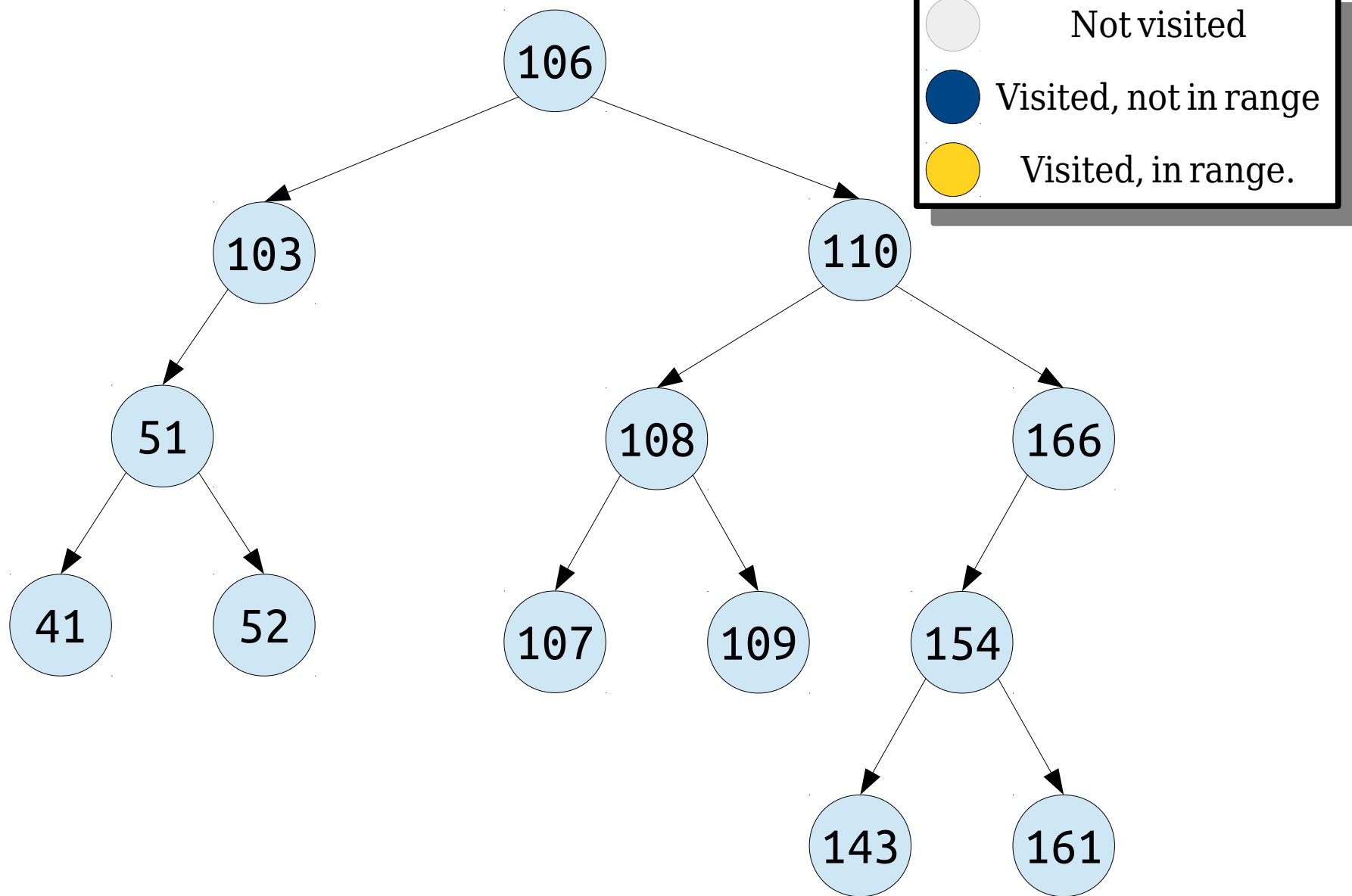
Find all elements in this tree in the range **[109, 163]**.



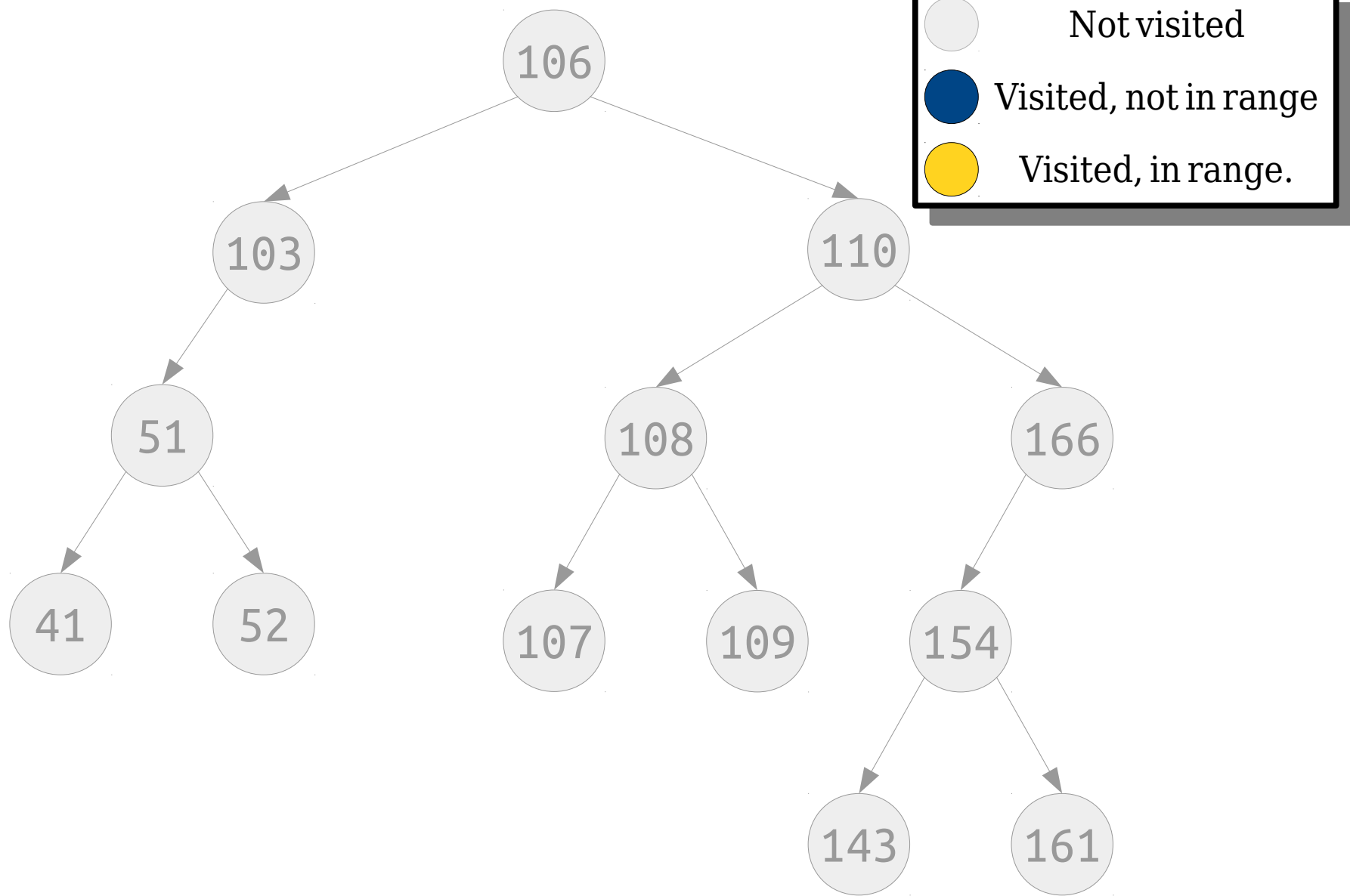
Find all elements in this tree in the range **[109, 163]**.



Find all elements in this tree in the range **[109, 163]**.

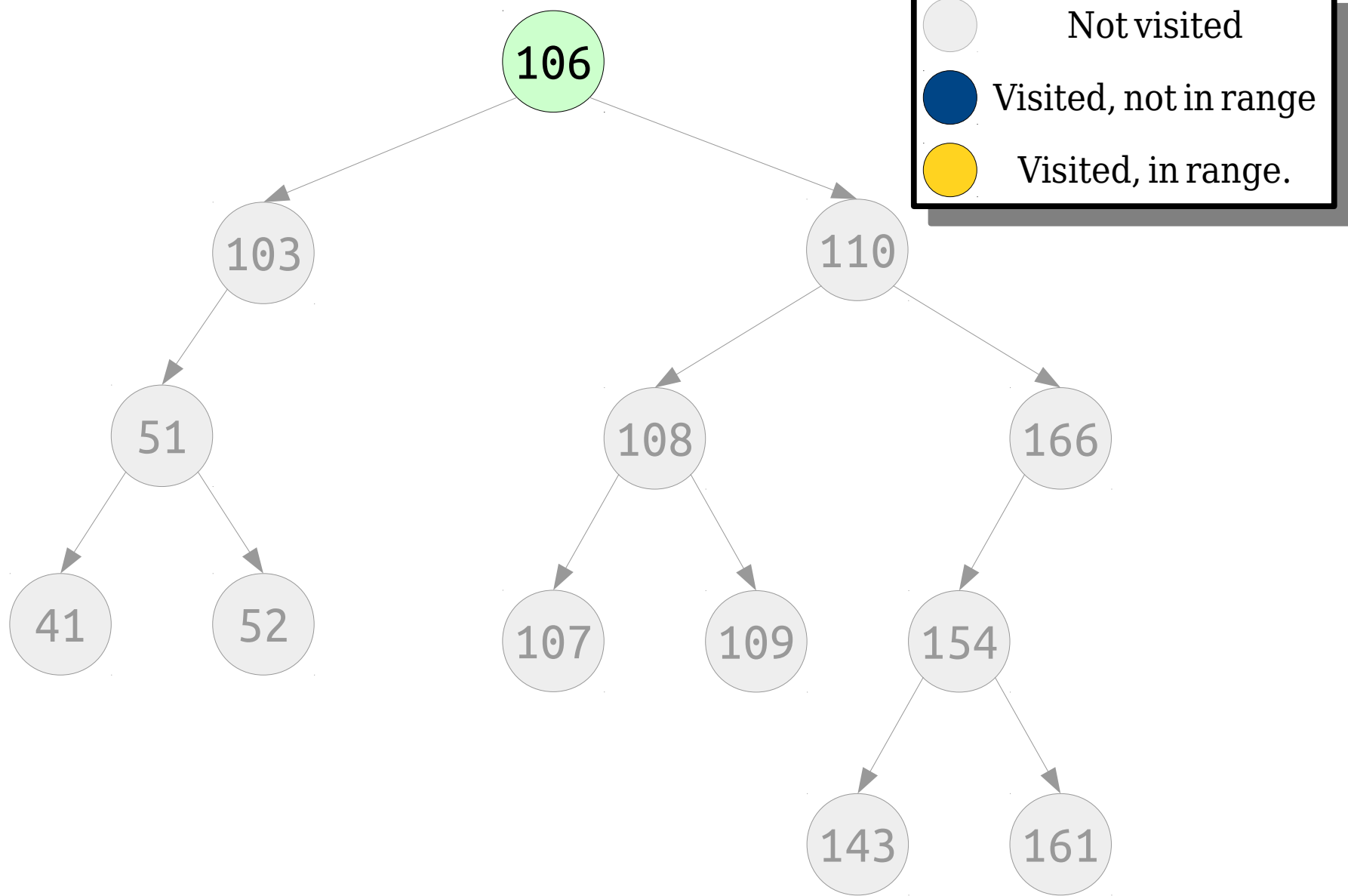


Find all elements in this tree in the range **[124, 155]**.

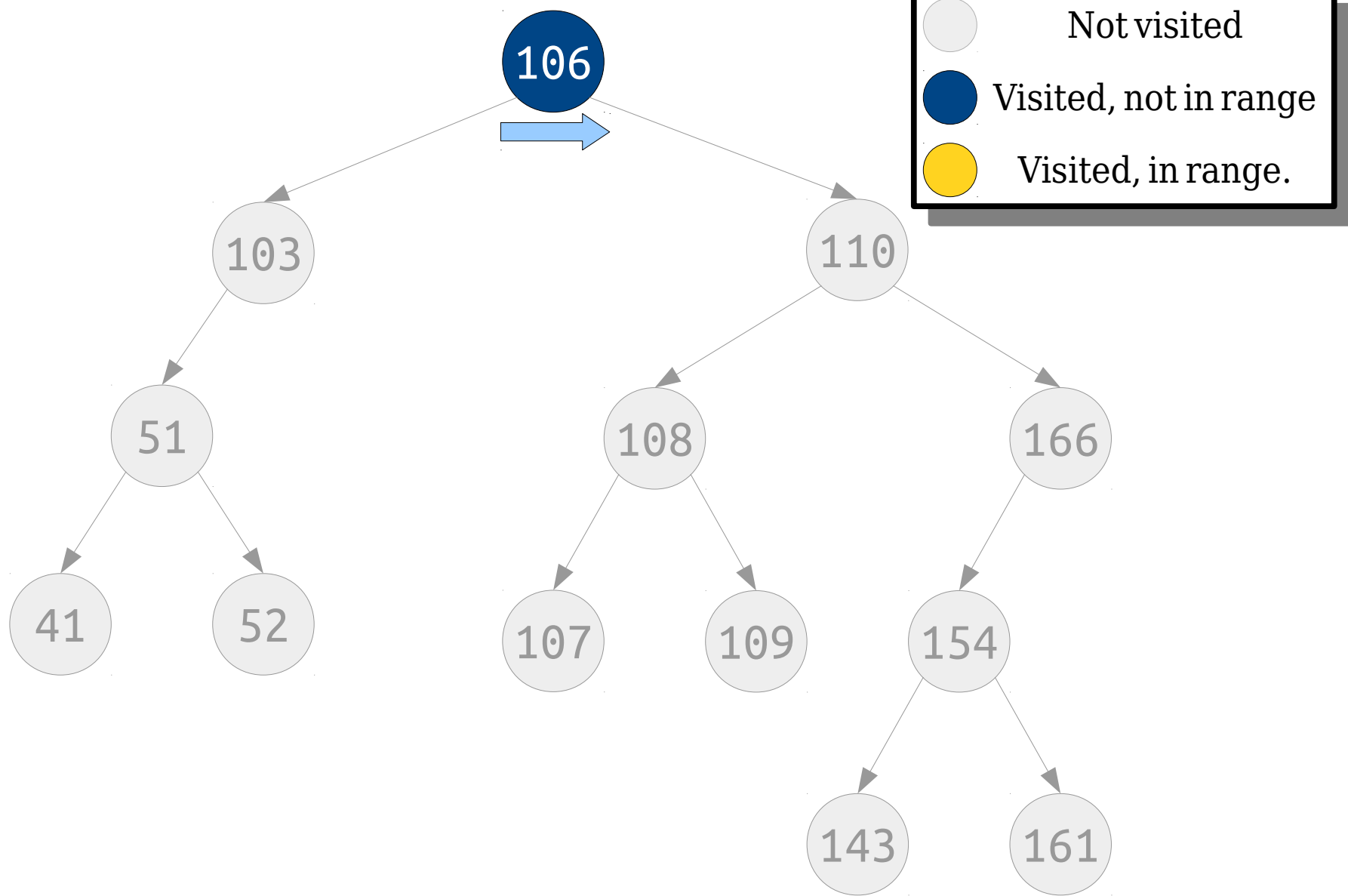


Find all elements in this tree in the range **[124, 155]**.

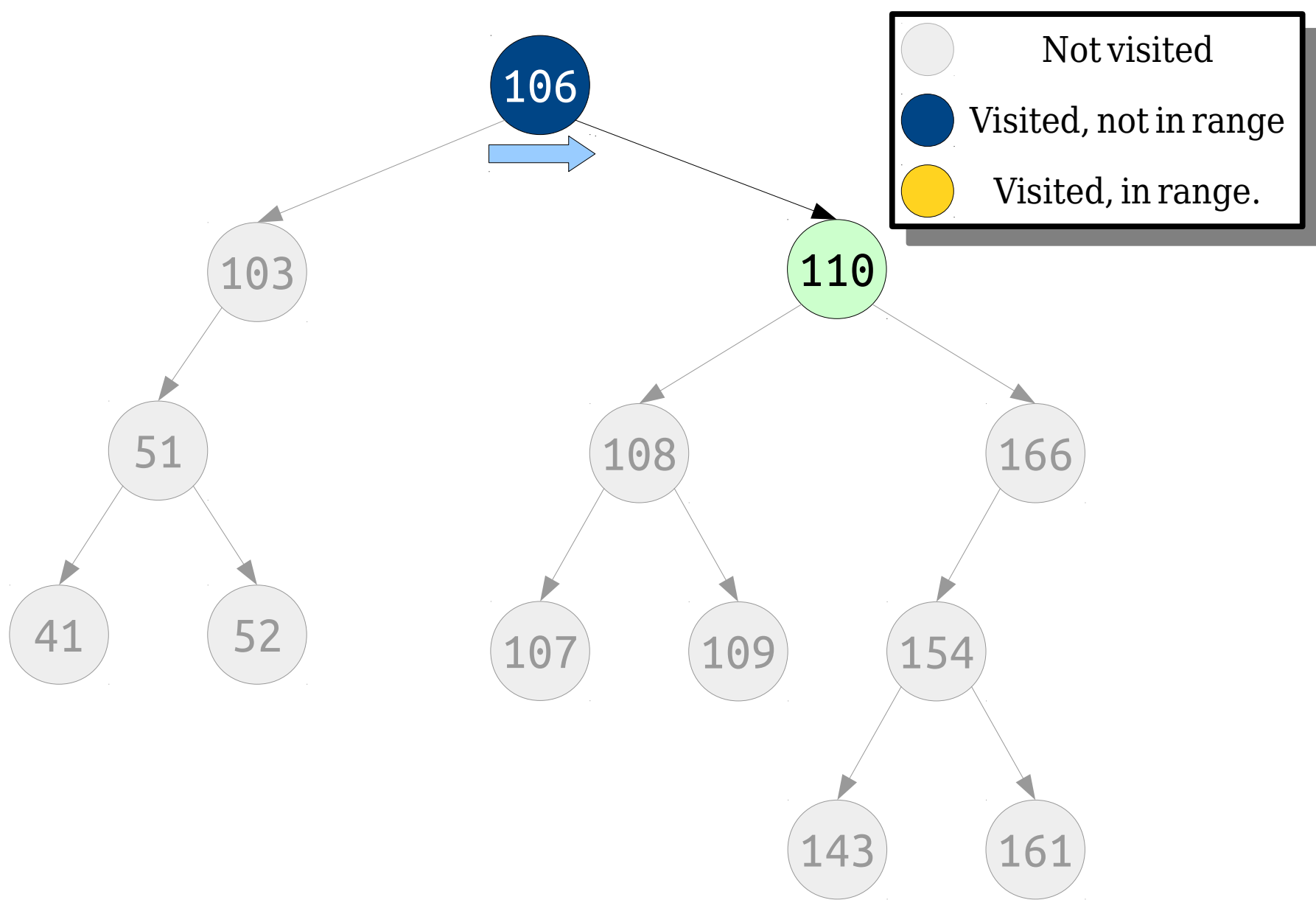




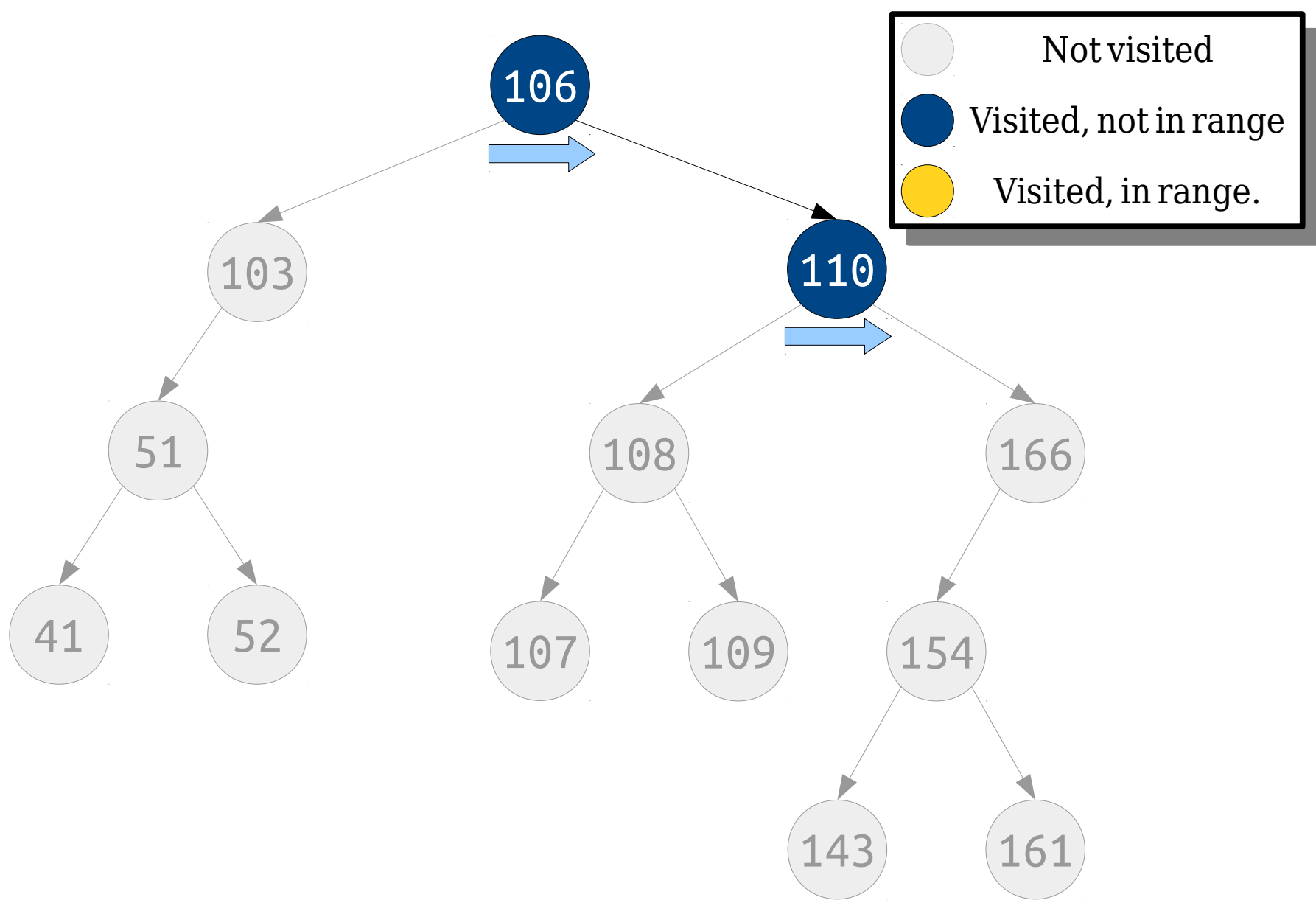
Find all elements in this tree in the range **[124, 155]**.



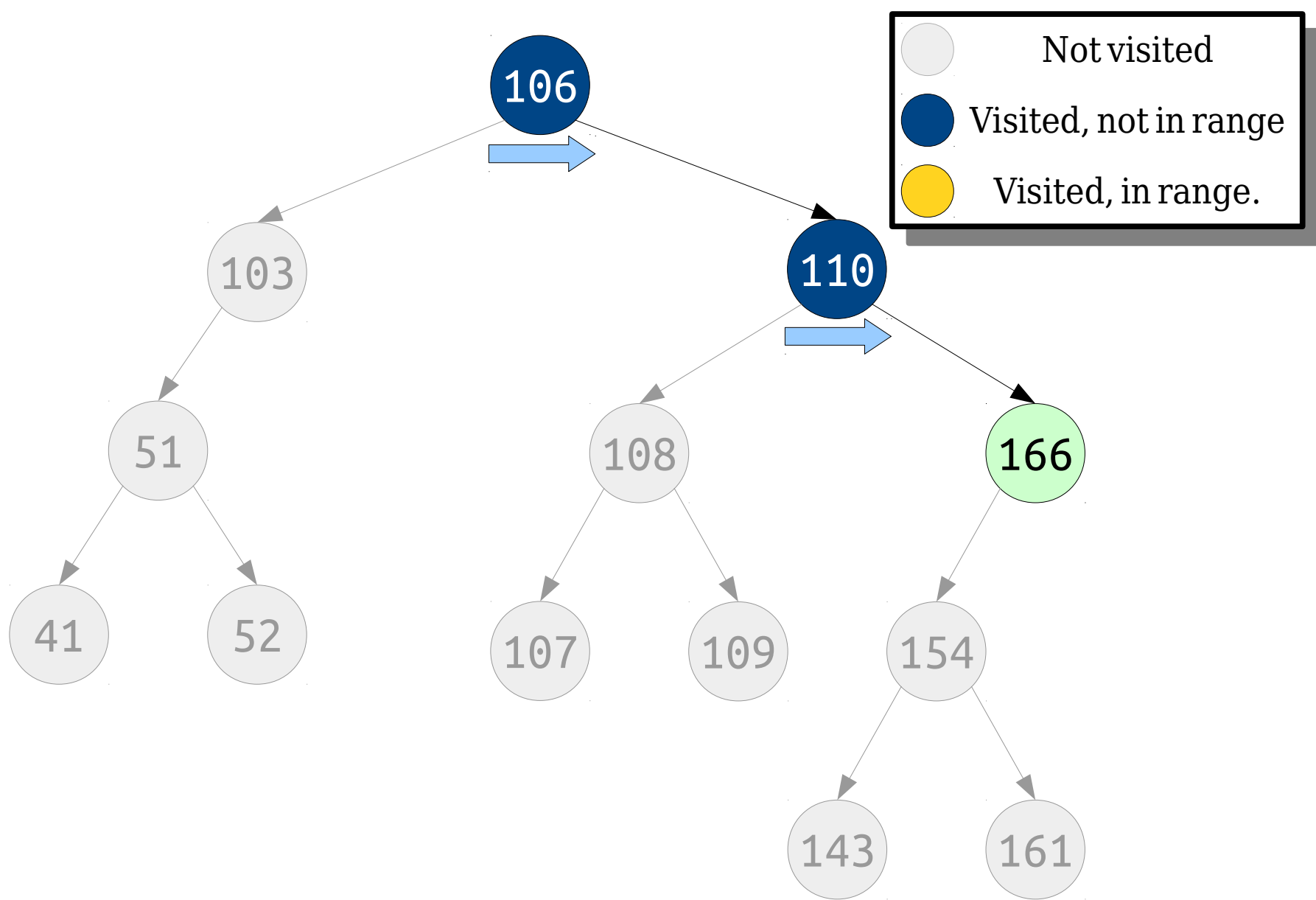
Find all elements in this tree in the range **[124, 155]**.



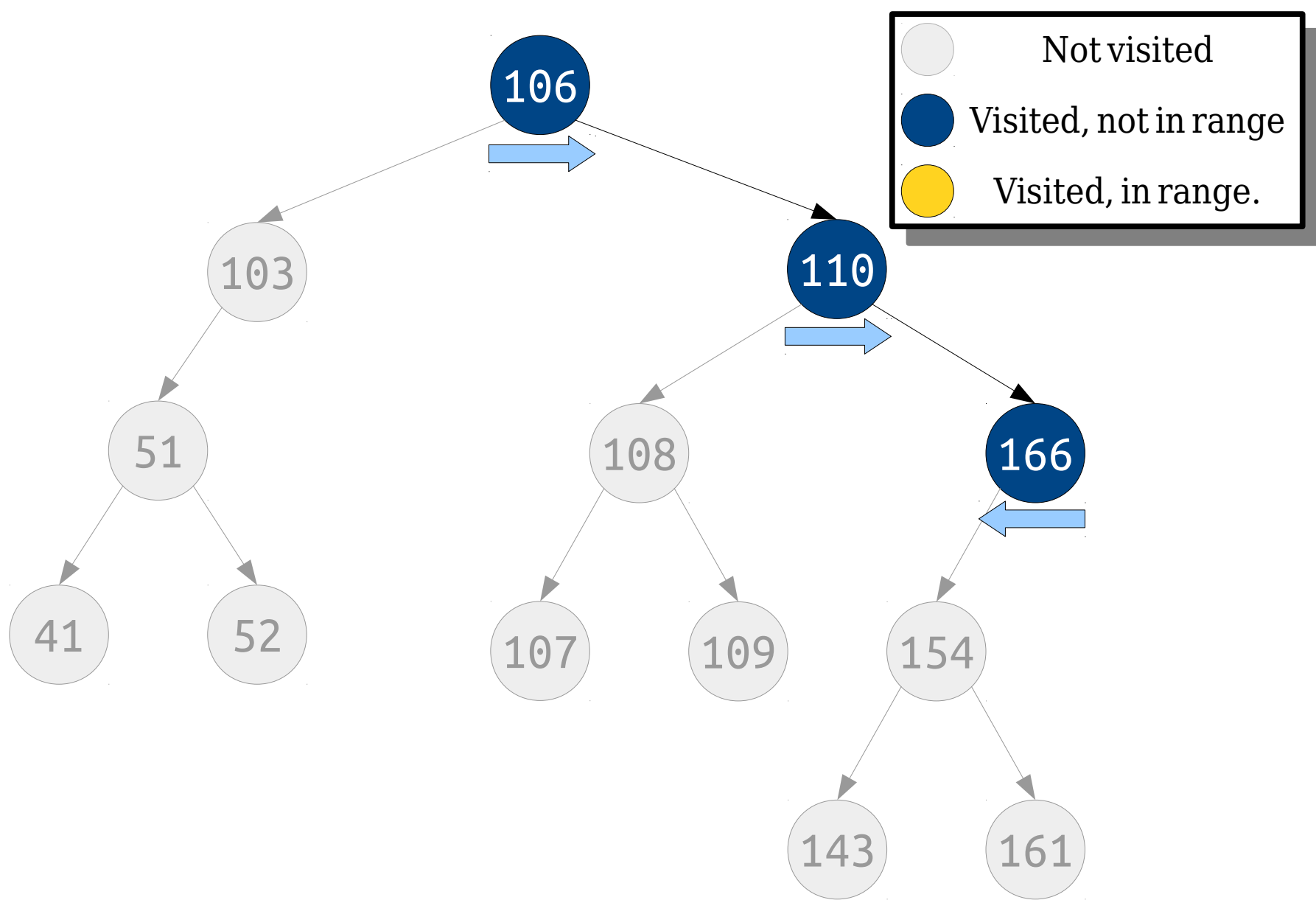
Find all elements in this tree in the range **[124, 155]**.



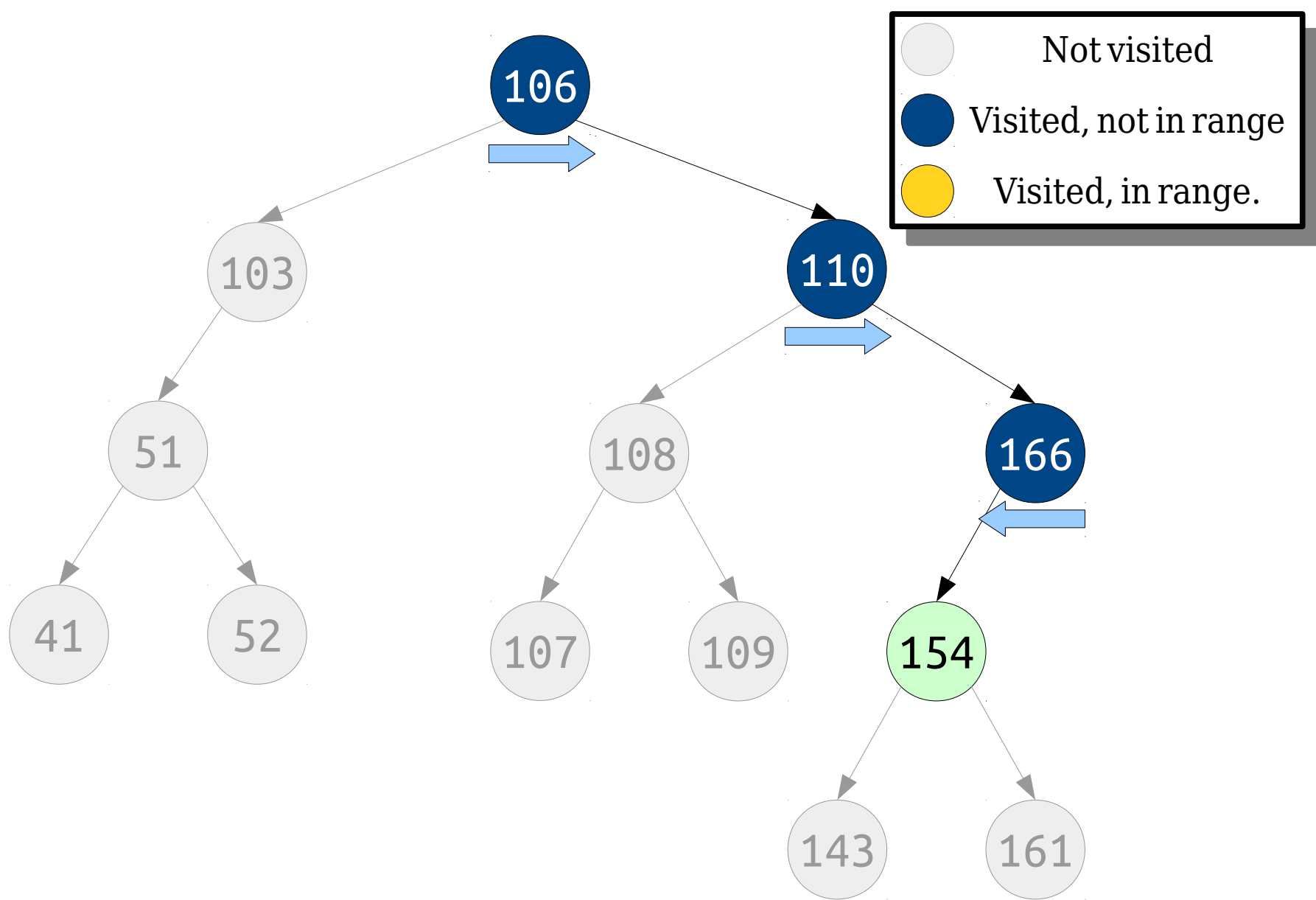
Find all elements in this tree in the range **[124, 155]**.



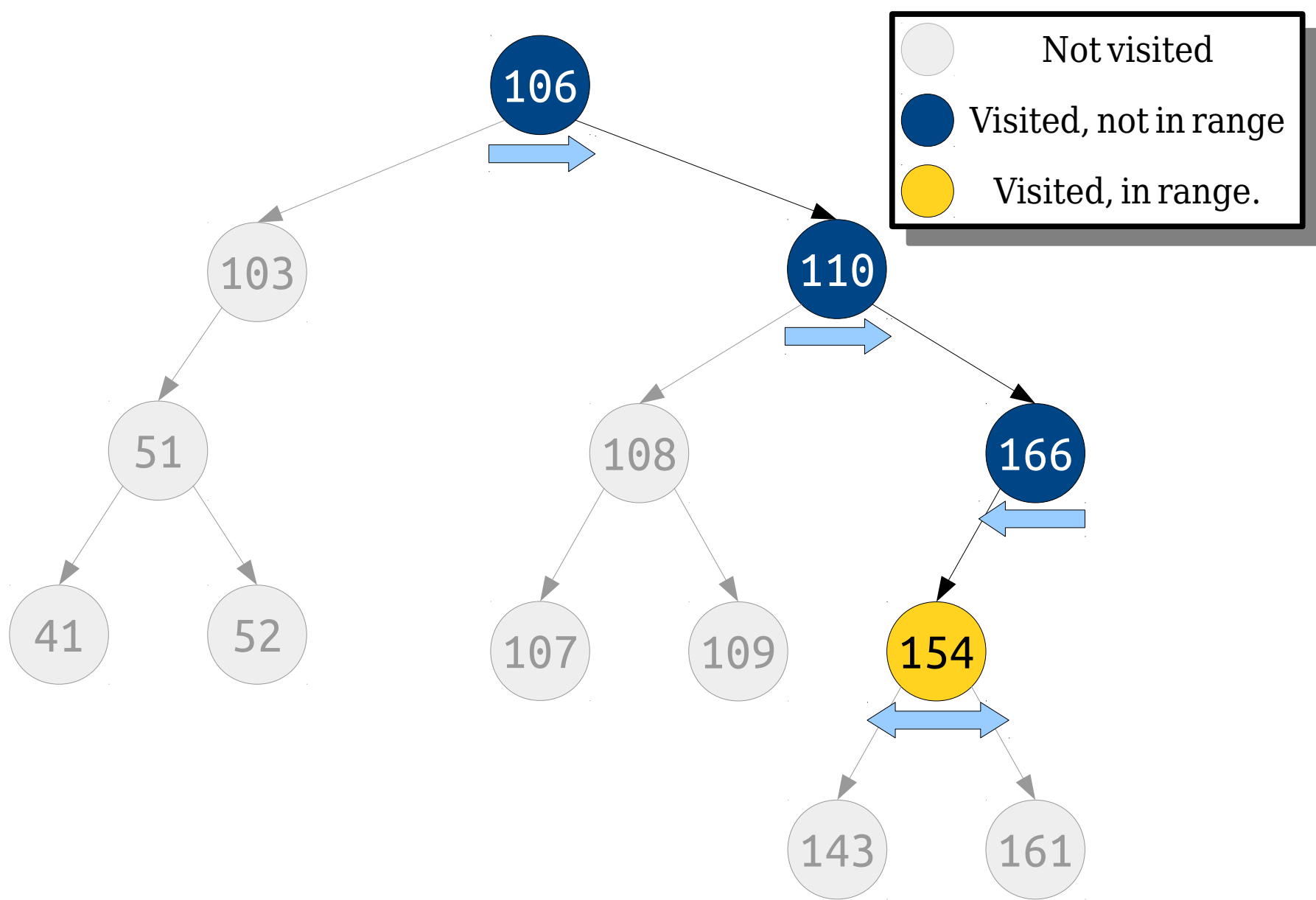
Find all elements in this tree in the range **[124, 155]**.



Find all elements in this tree in the range **[124, 155]**.

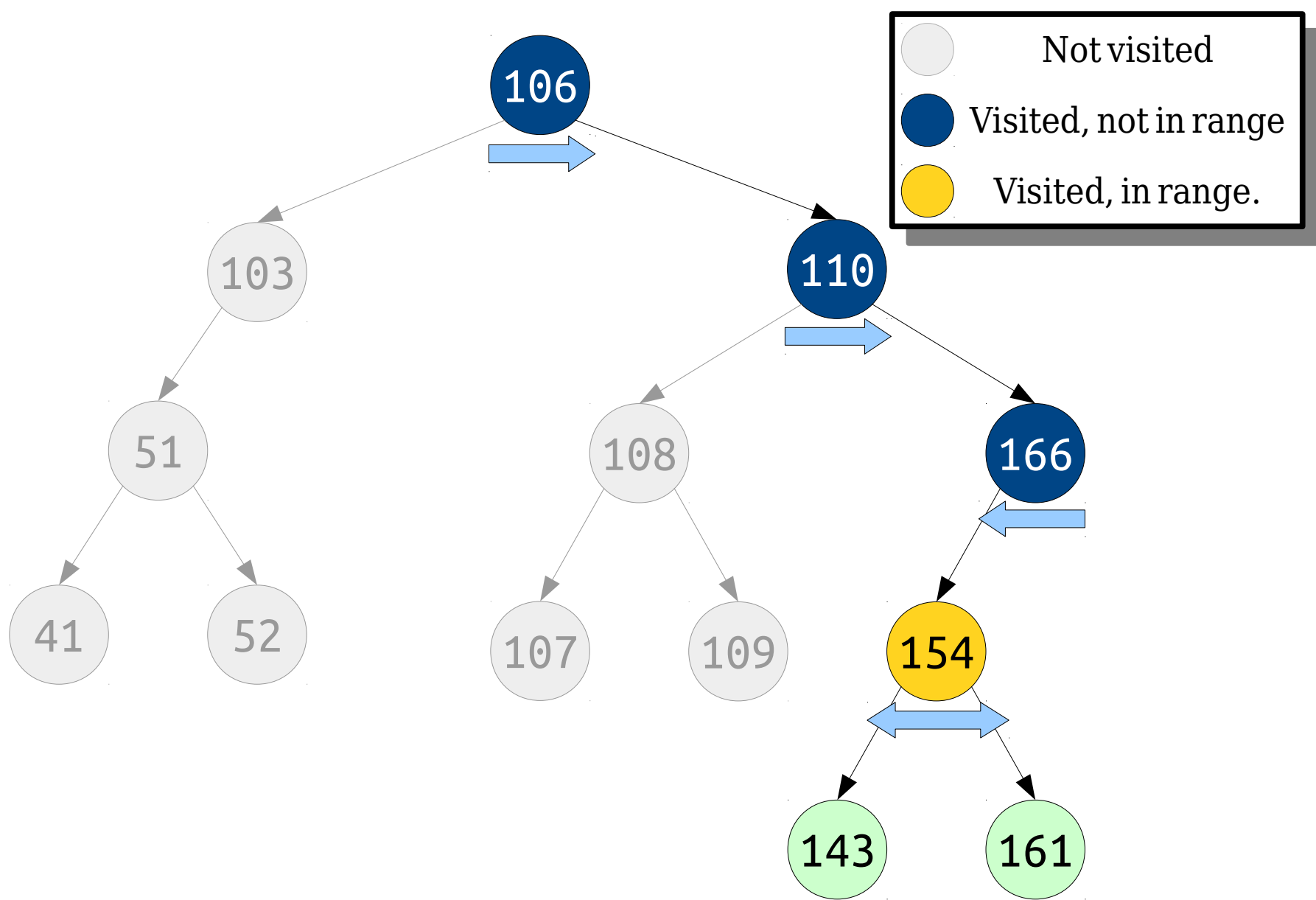


Find all elements in this tree in the range **[124, 155]**.

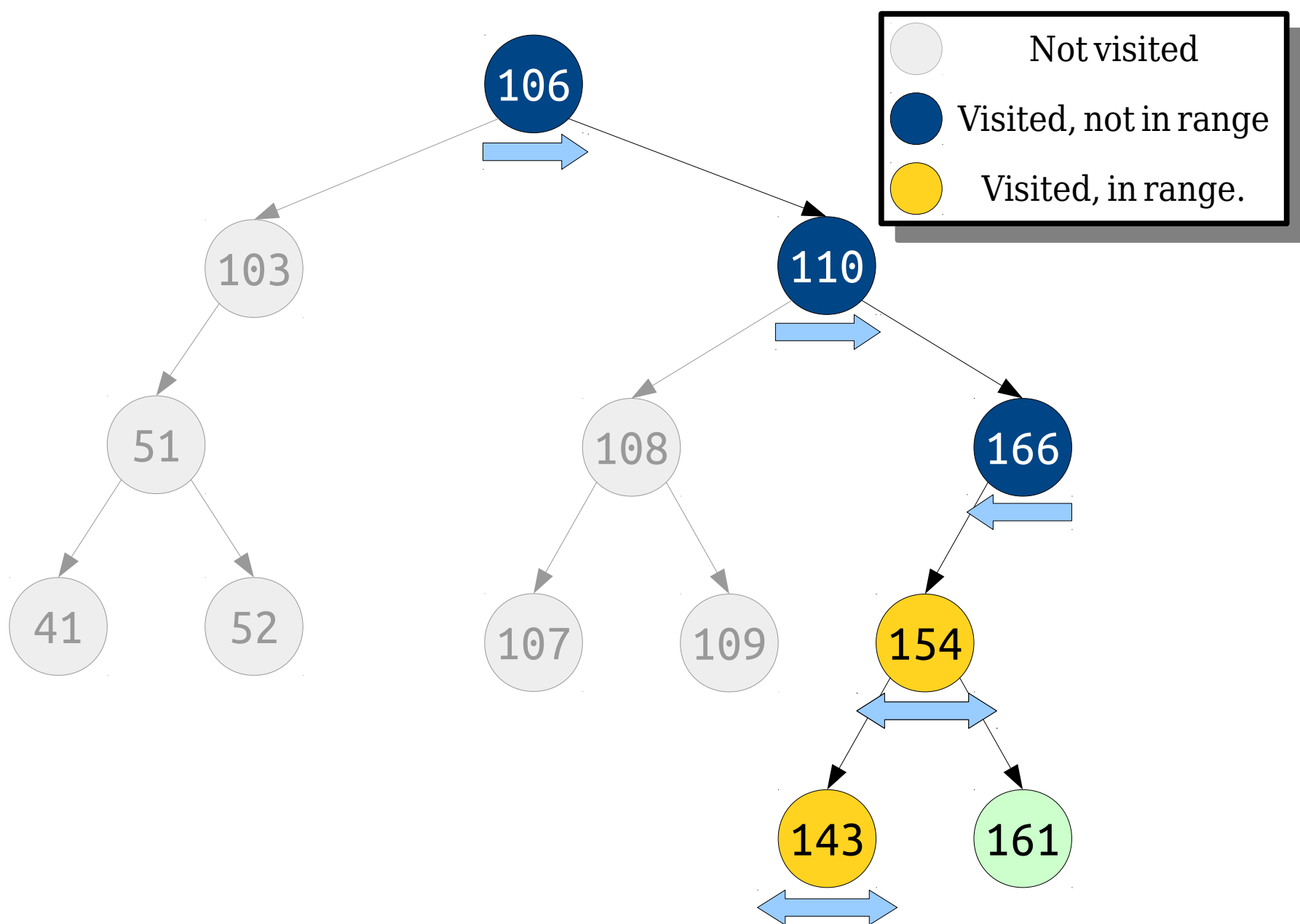


Find all elements in this tree in the range **[124, 155]**.

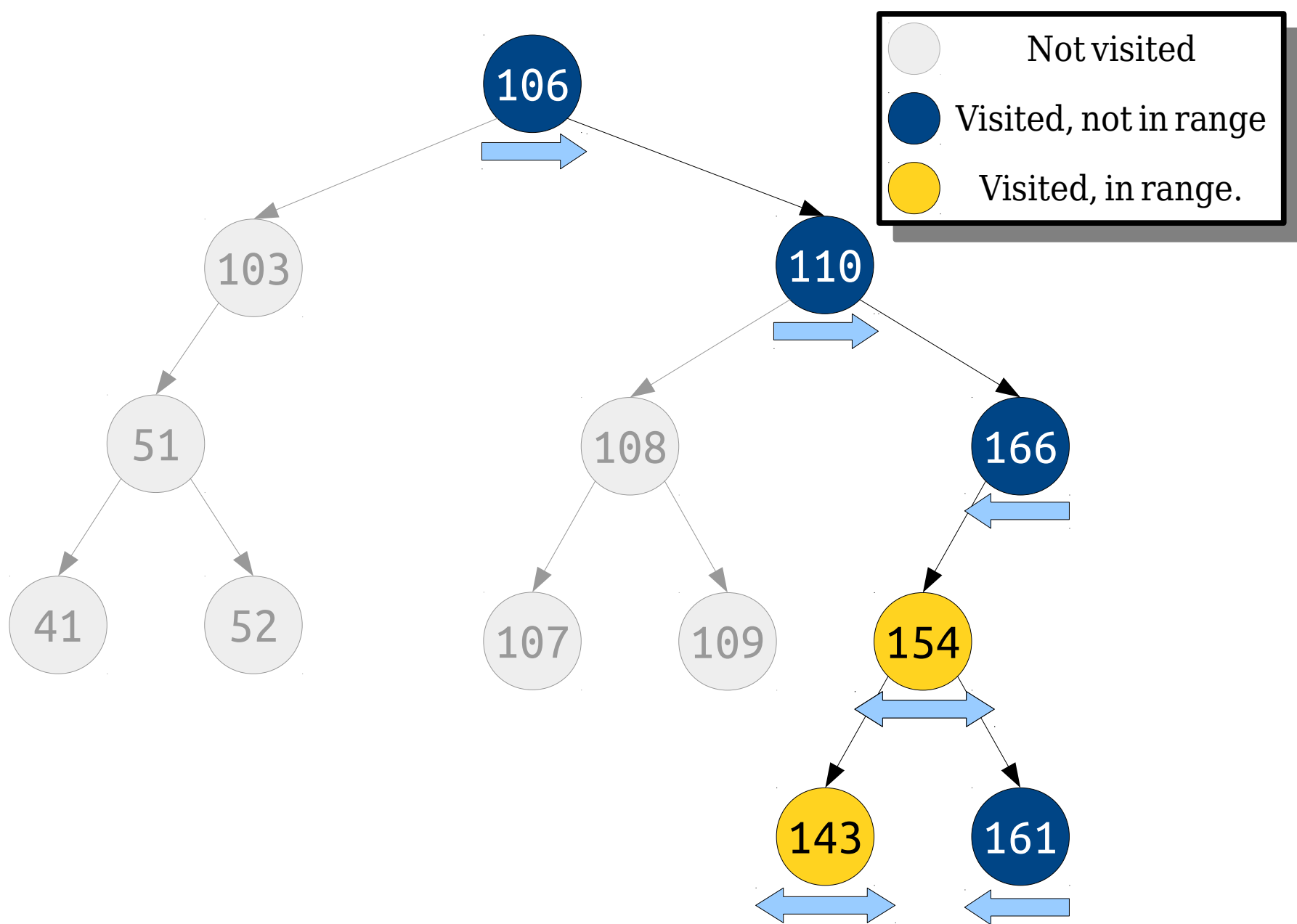




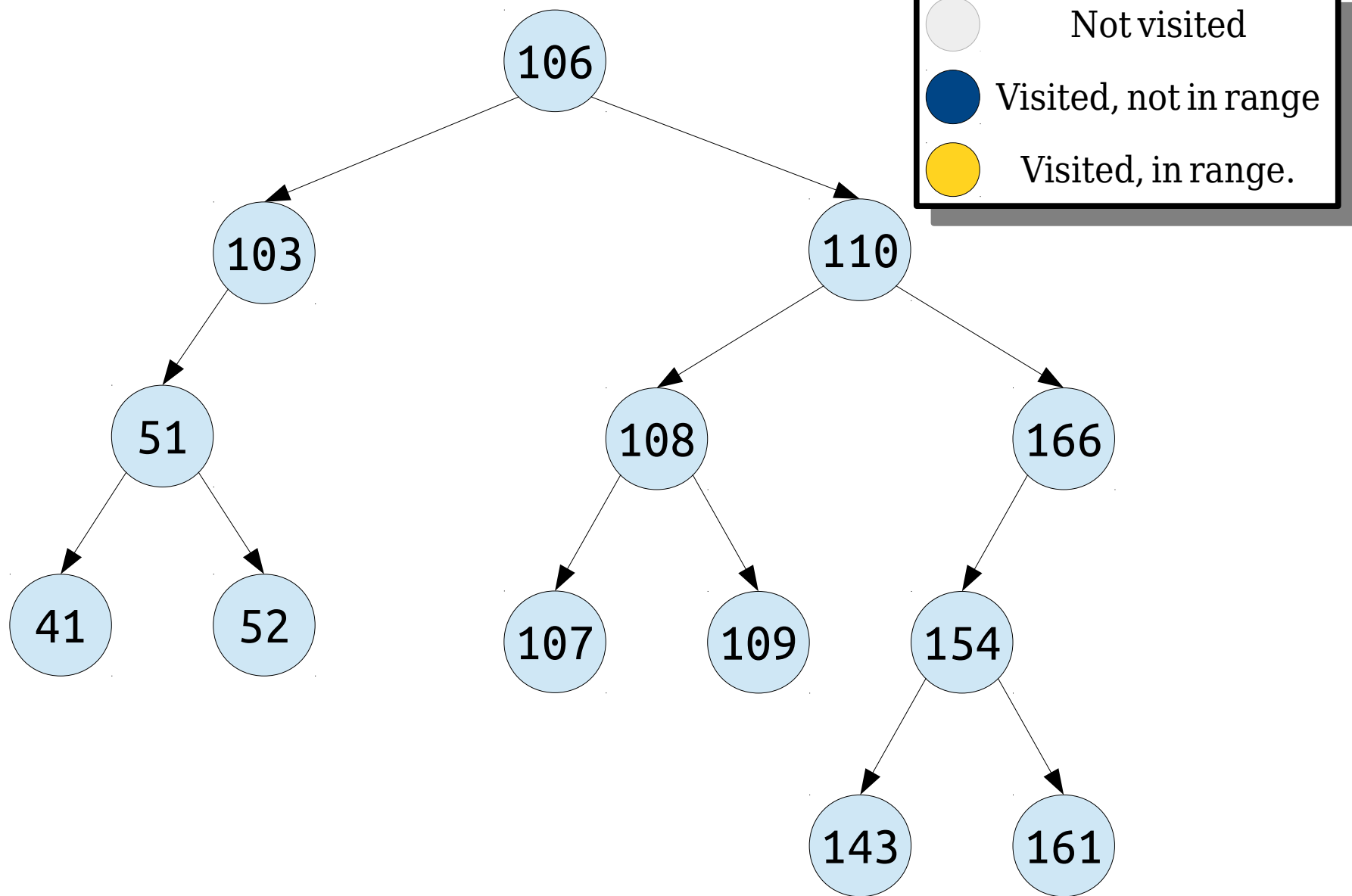
Find all elements in this tree in the range **[124, 155]**.



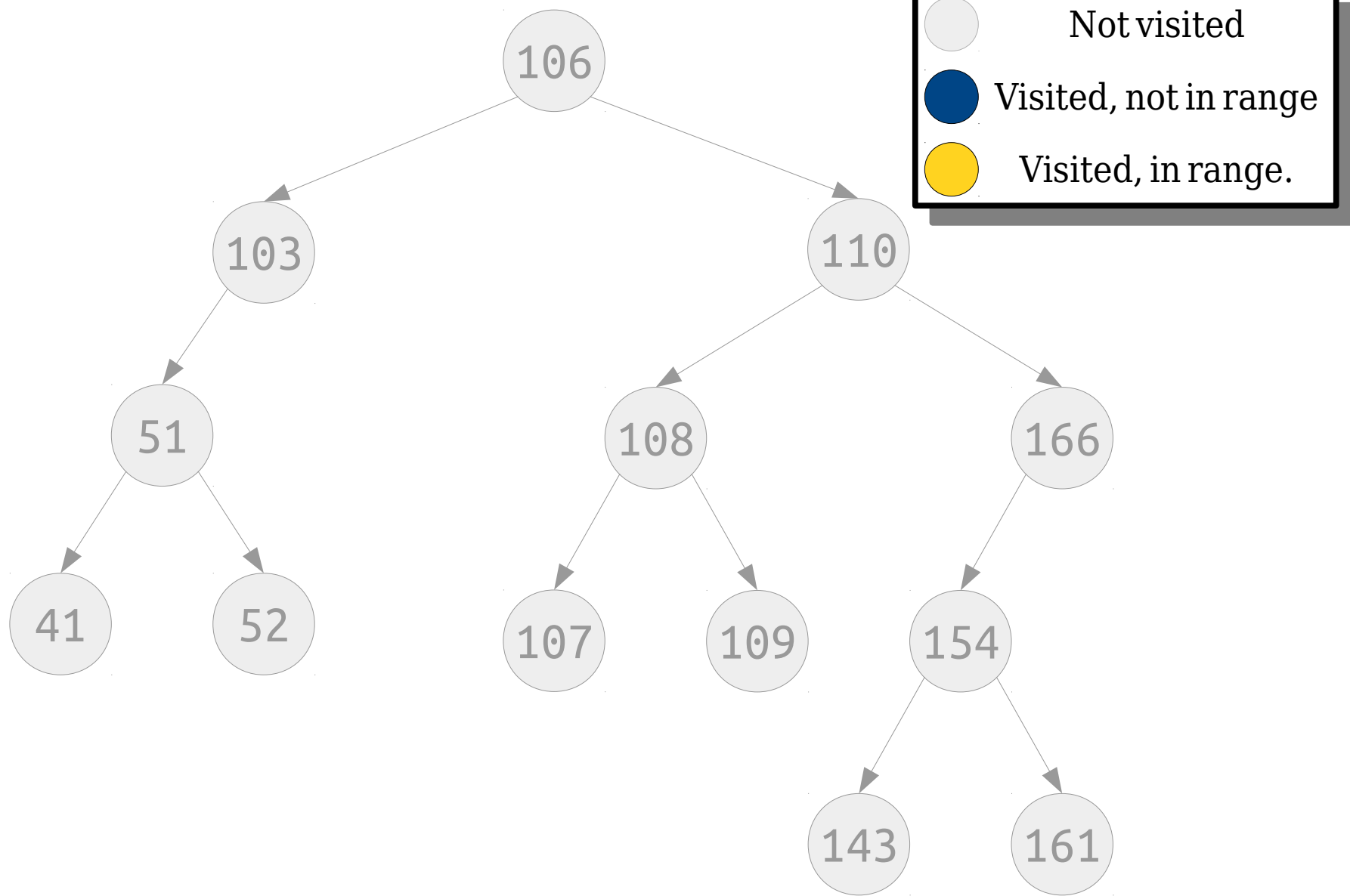
Find all elements in this tree in the range **[124, 155]**.



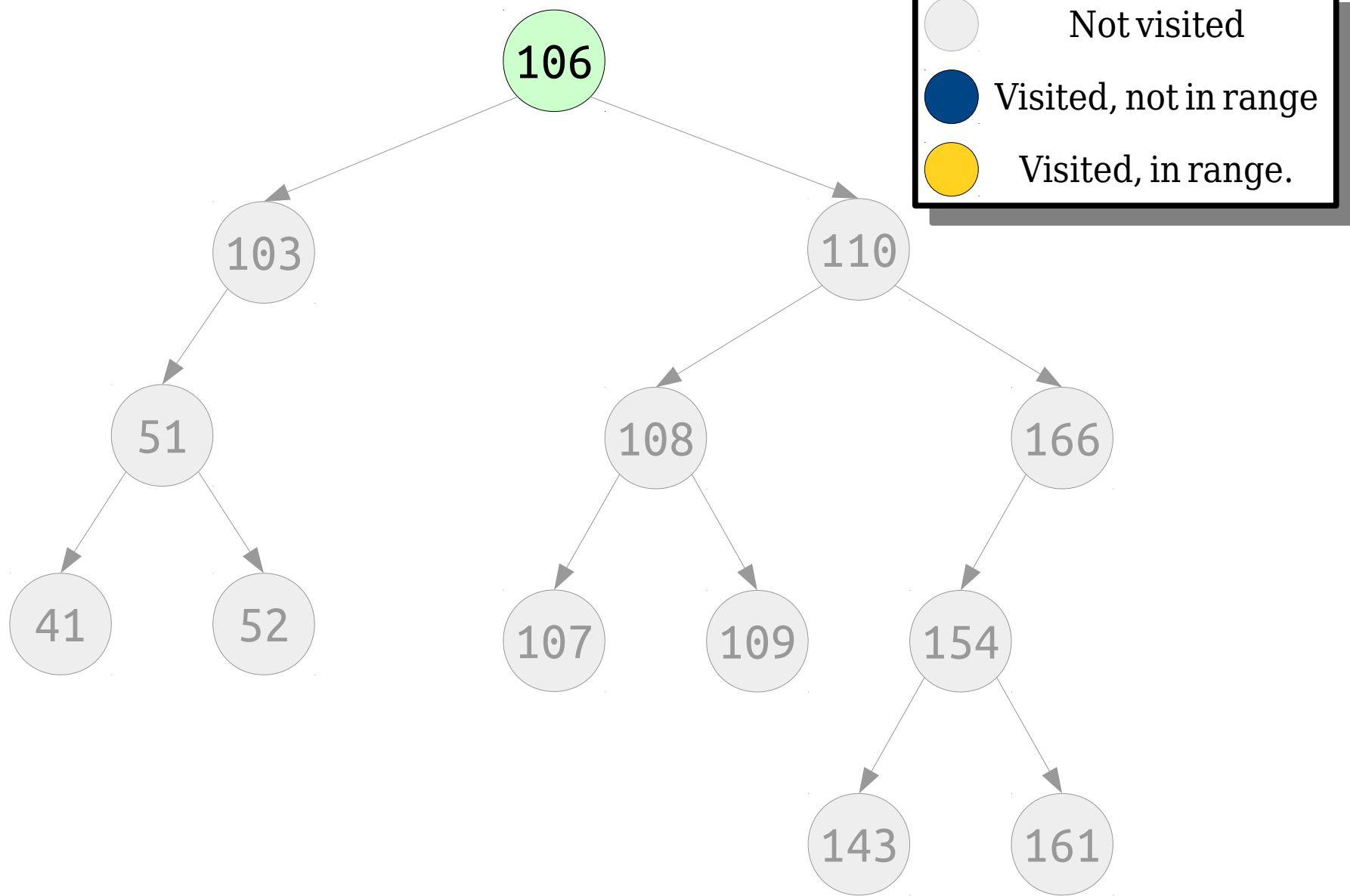
Find all elements in this tree in the range **[124, 155]**.



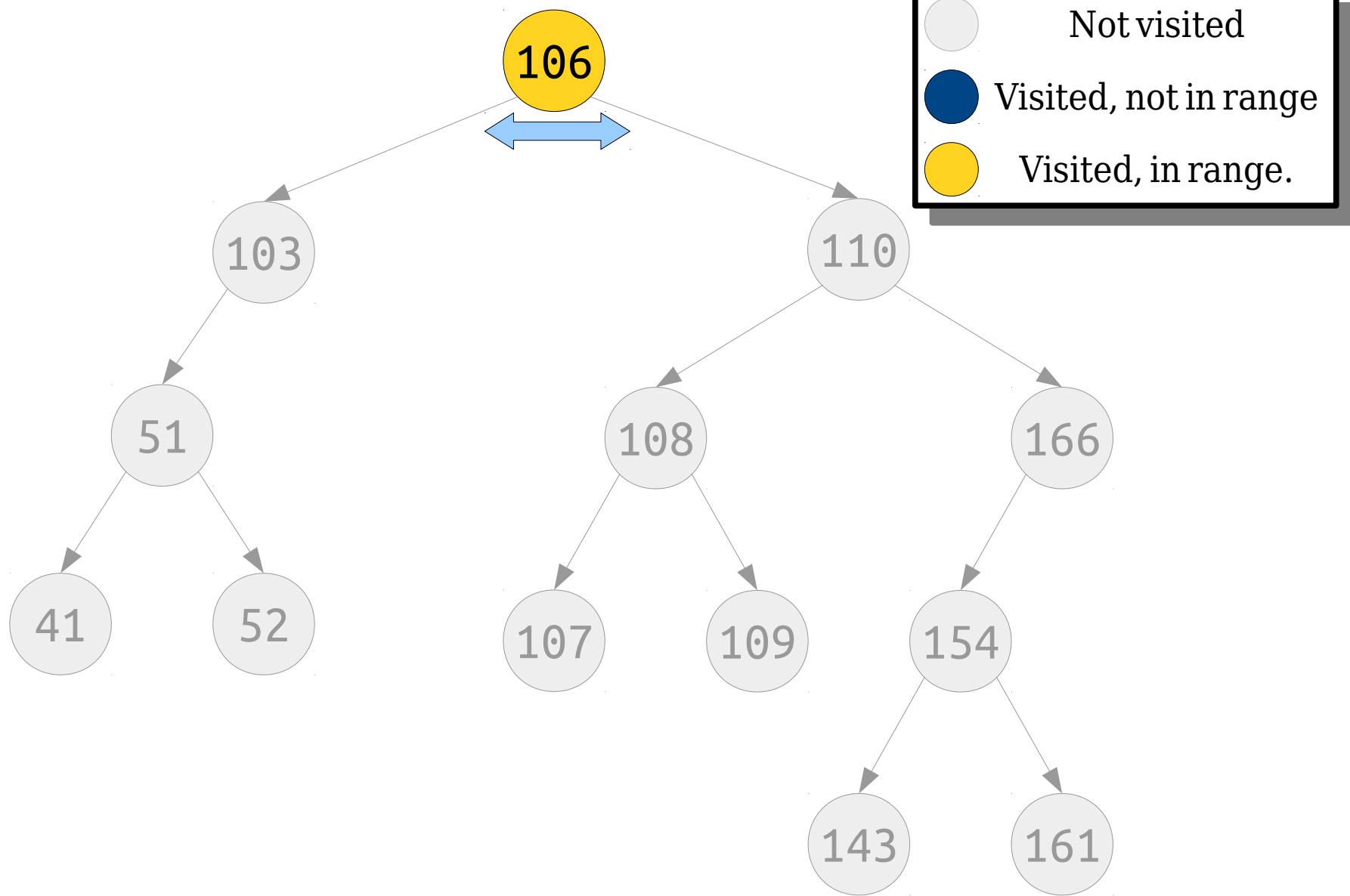
Find all elements in this tree in the range **[42, 165]**.



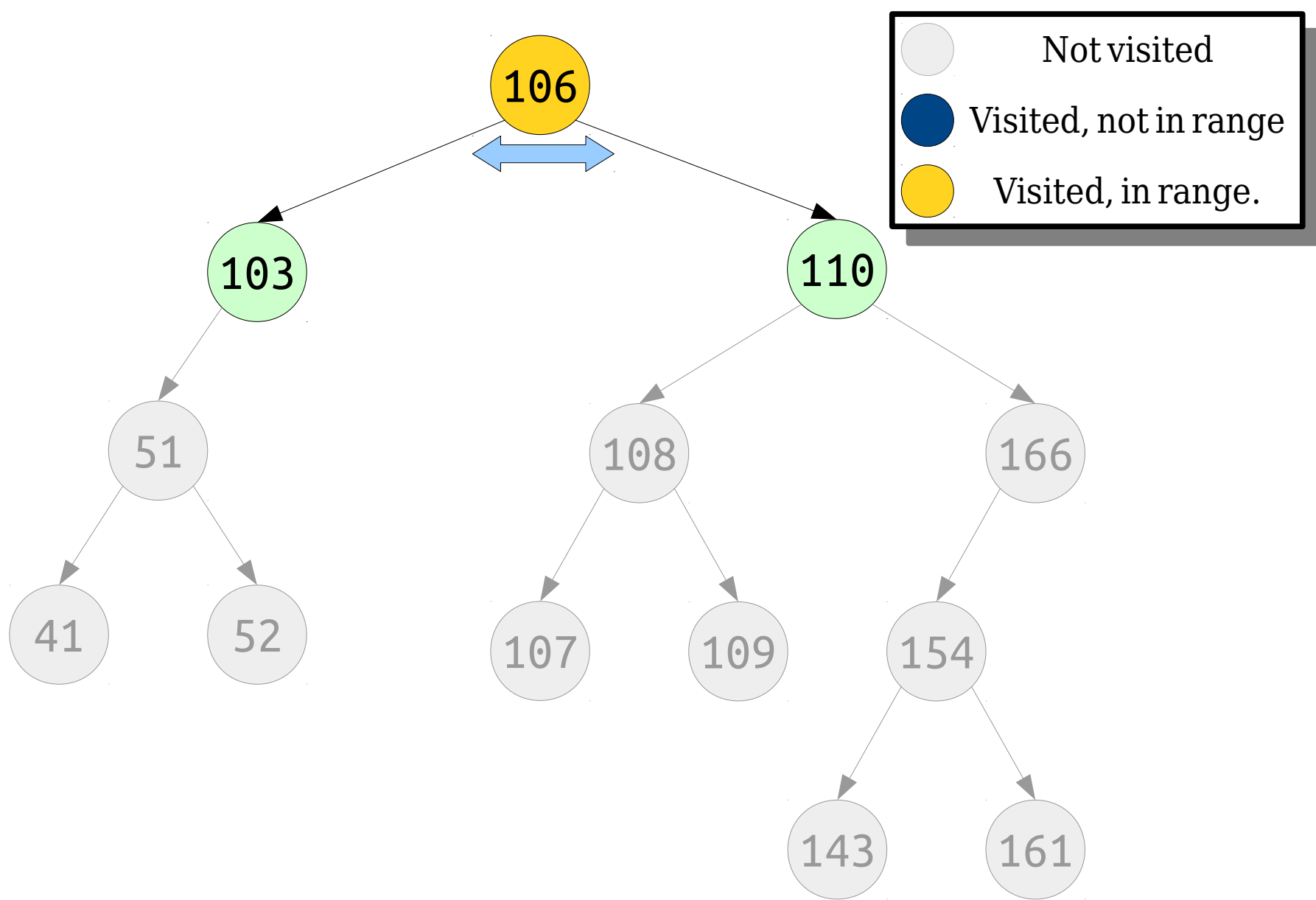
Find all elements in this tree in the range **[42, 165]**.



Find all elements in this tree in the range **[42, 165]**.

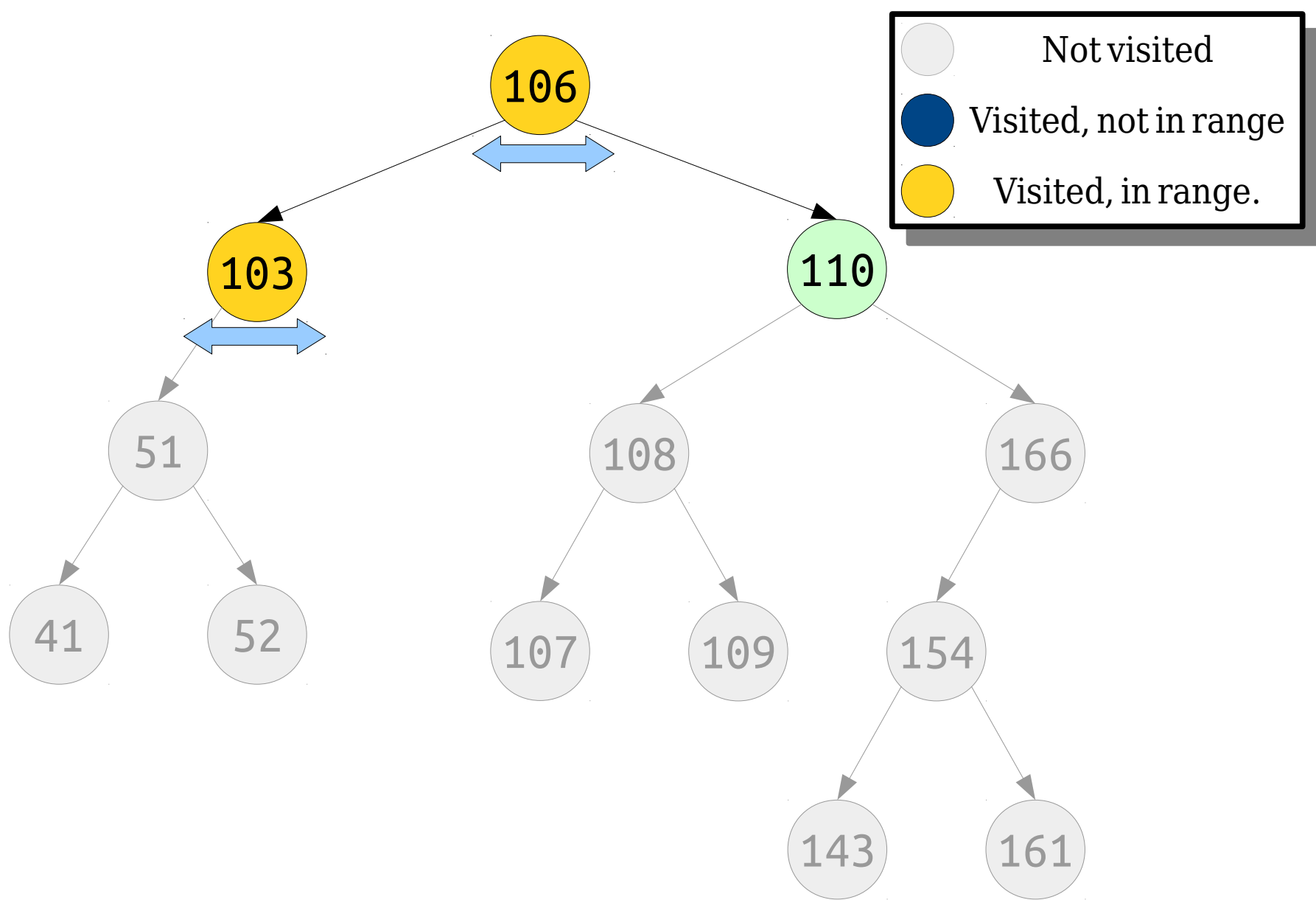


Find all elements in this tree in the range **[42, 165]**.

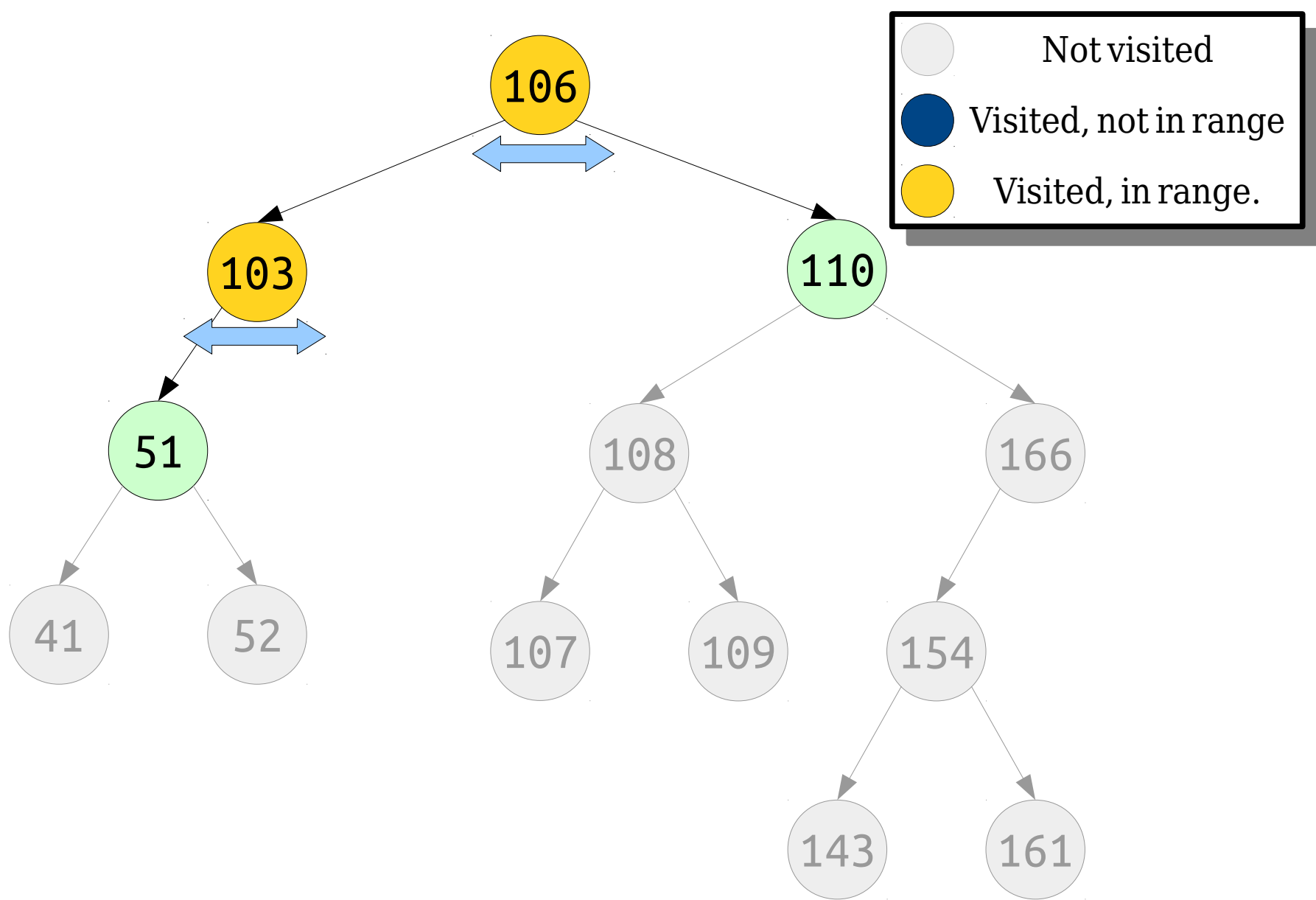


Find all elements in this tree in the range **[42, 165]**.

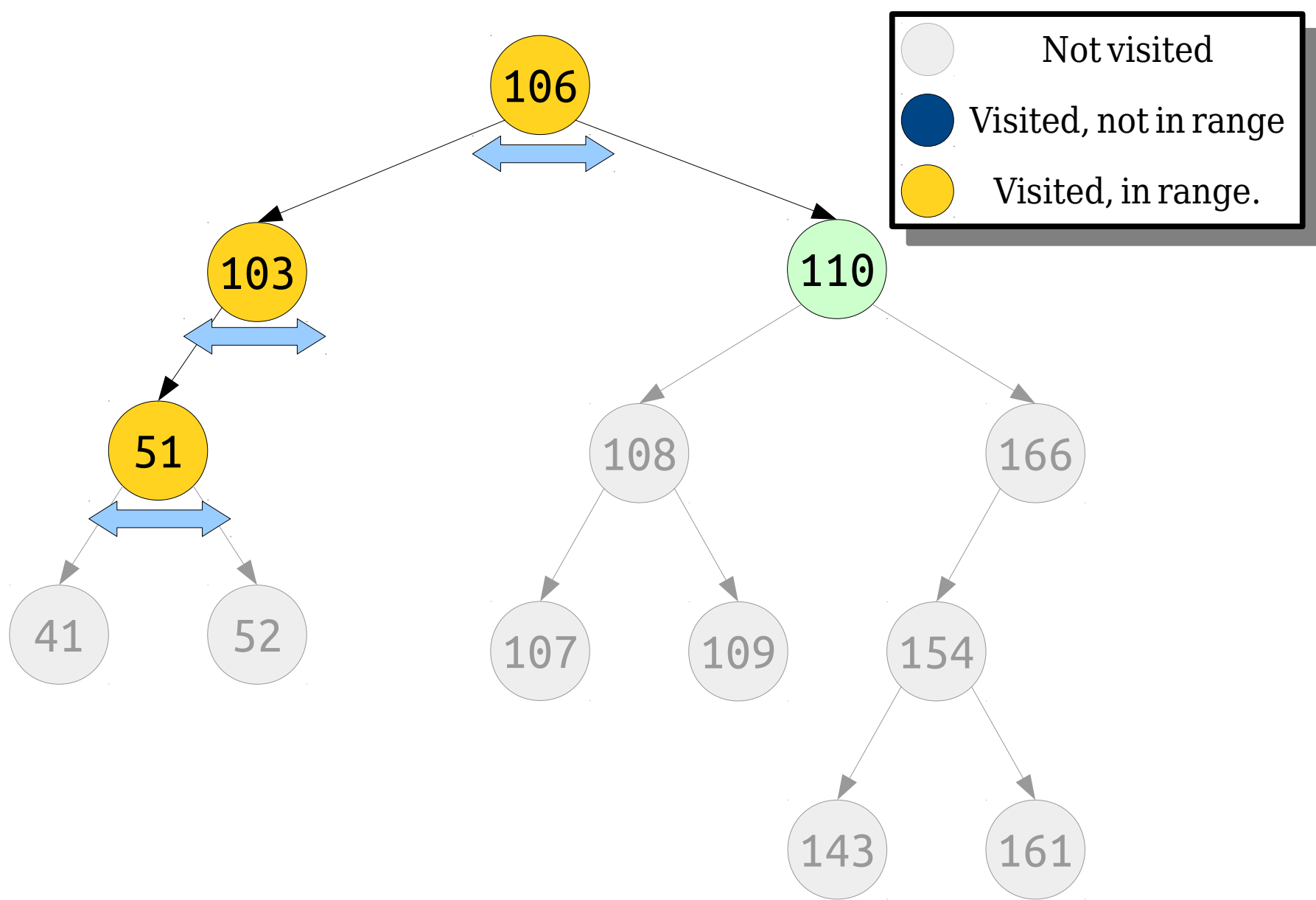




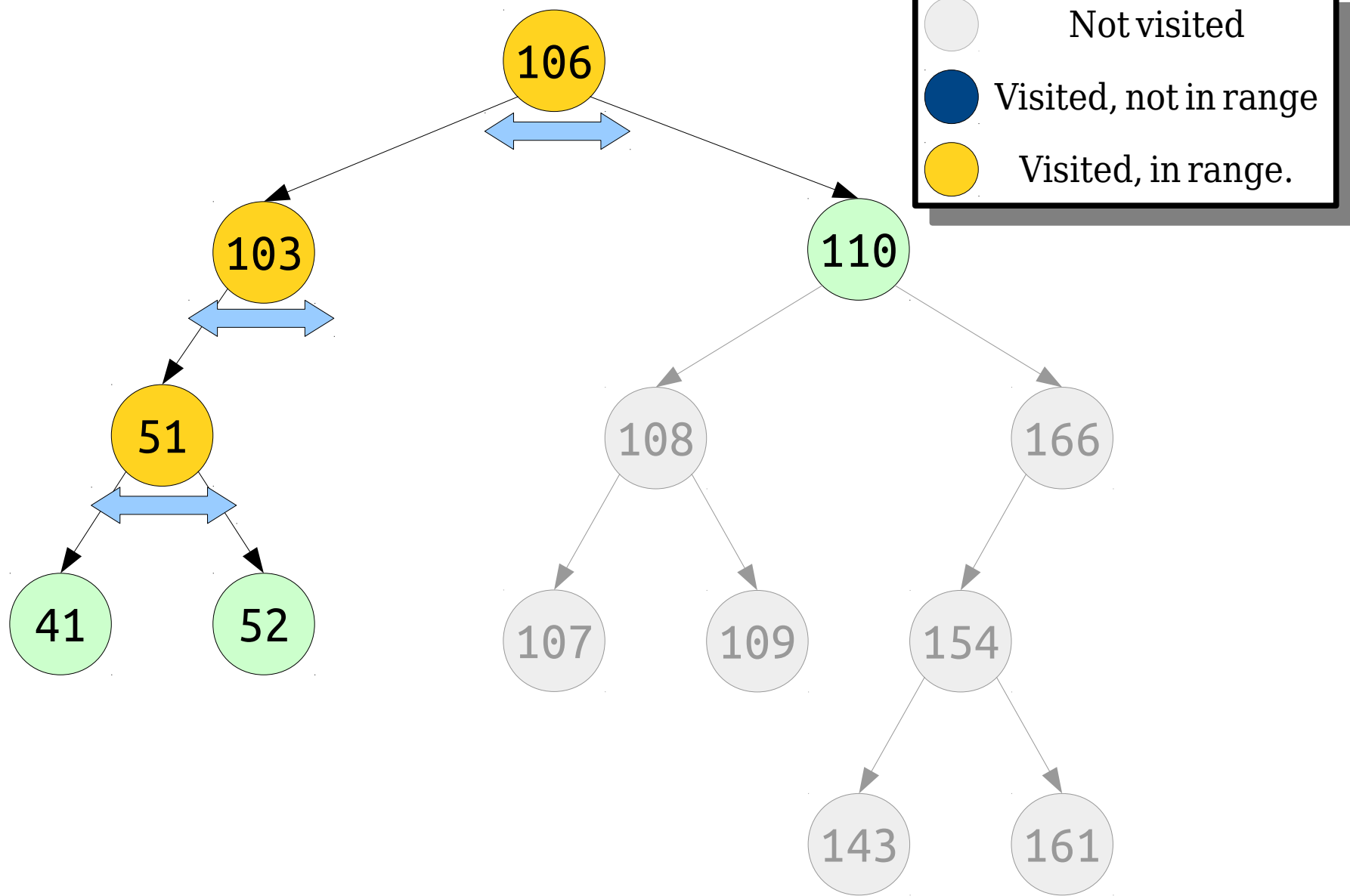
Find all elements in this tree in the range **[42, 165]**.



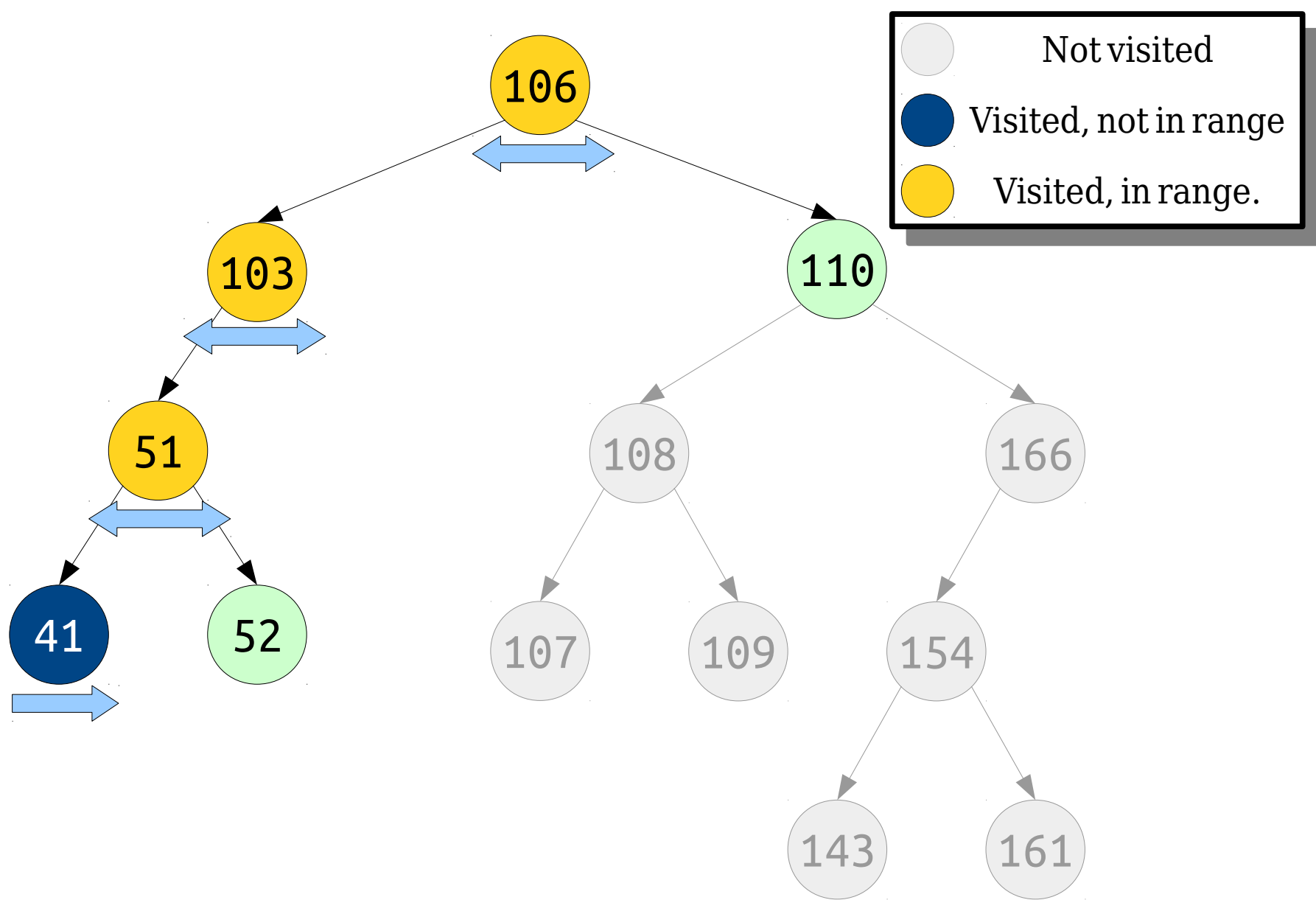
Find all elements in this tree in the range **[42, 165]**.



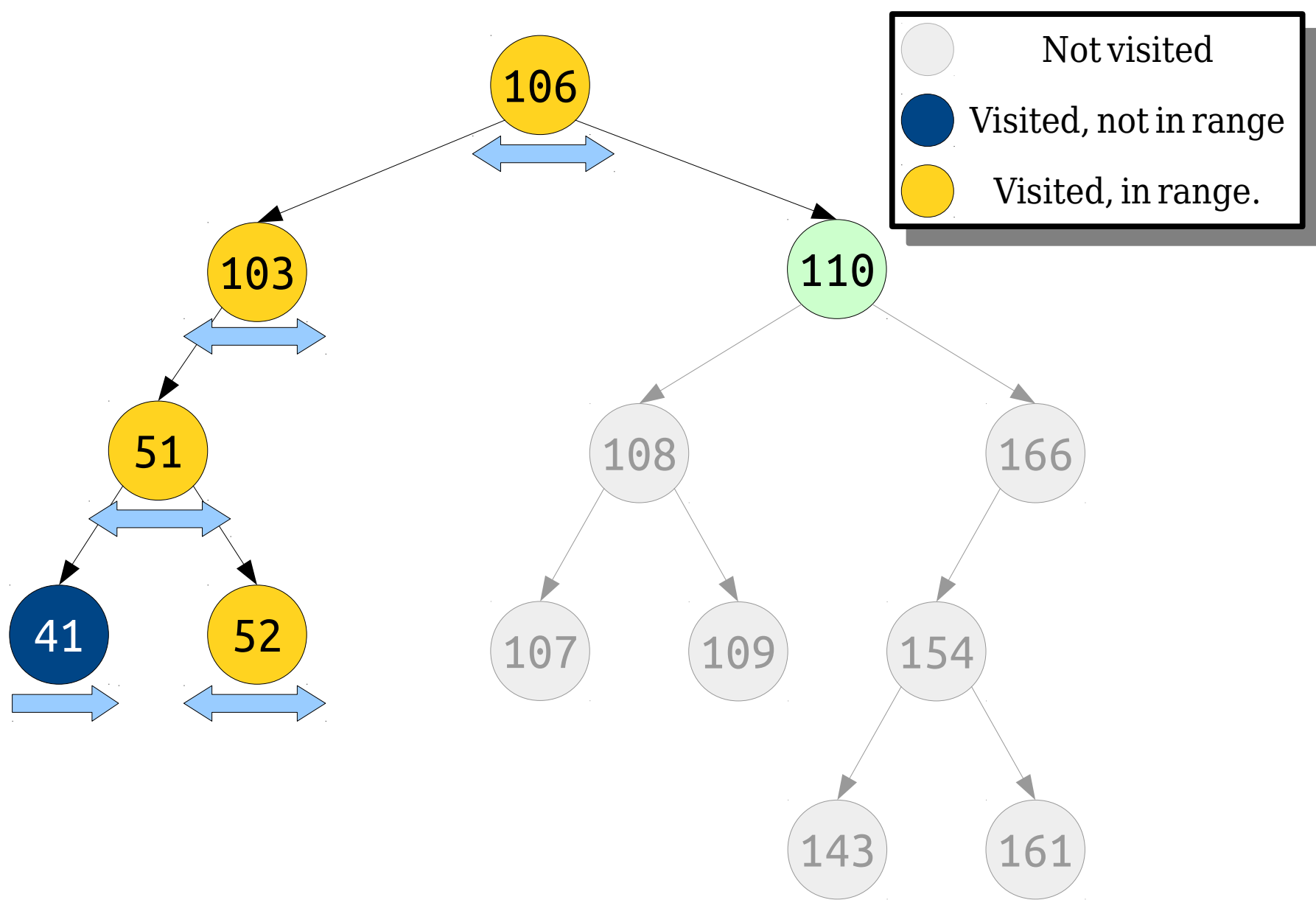
Find all elements in this tree in the range **[42, 165]**.



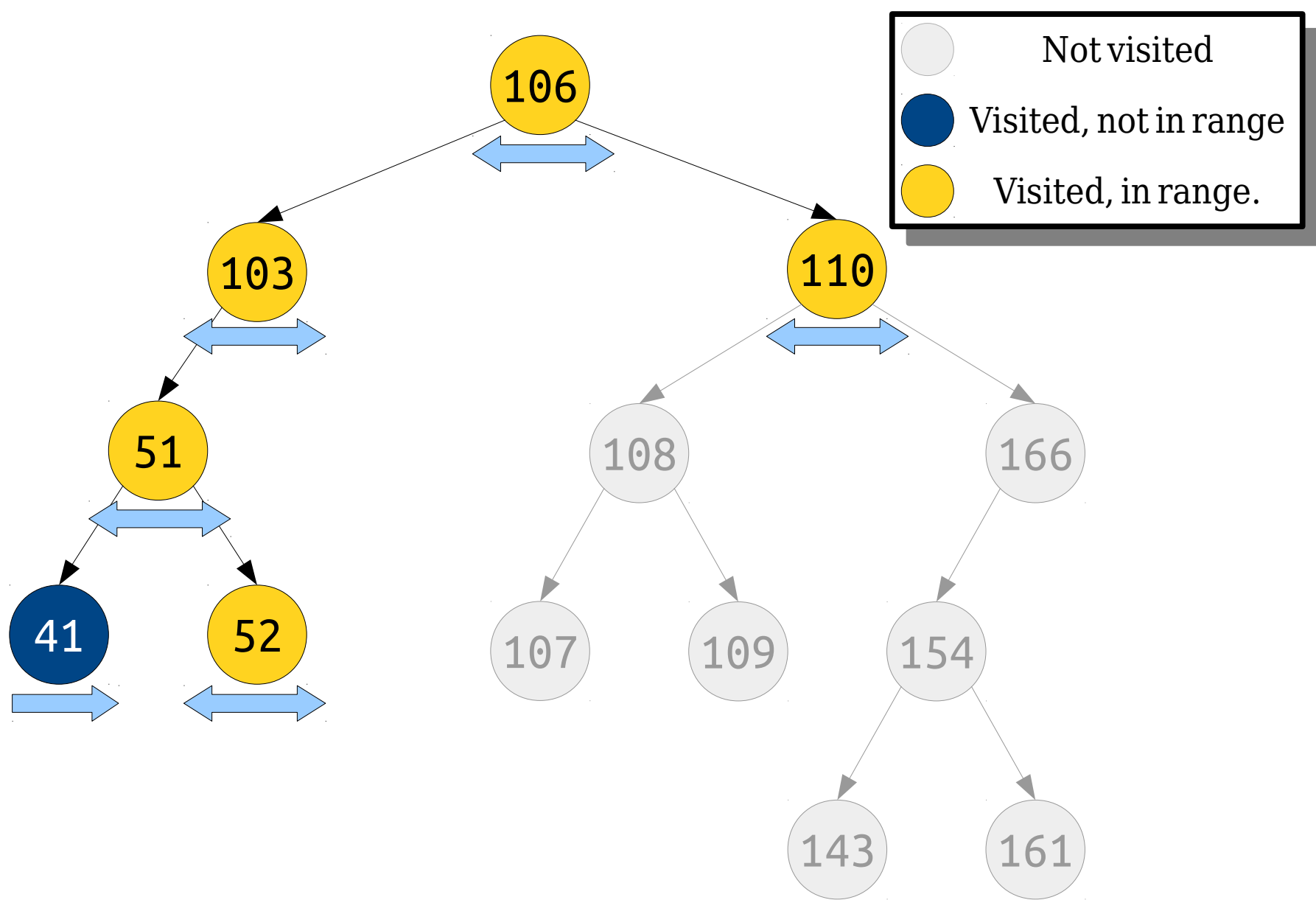
Find all elements in this tree in the range **[42, 165]**.



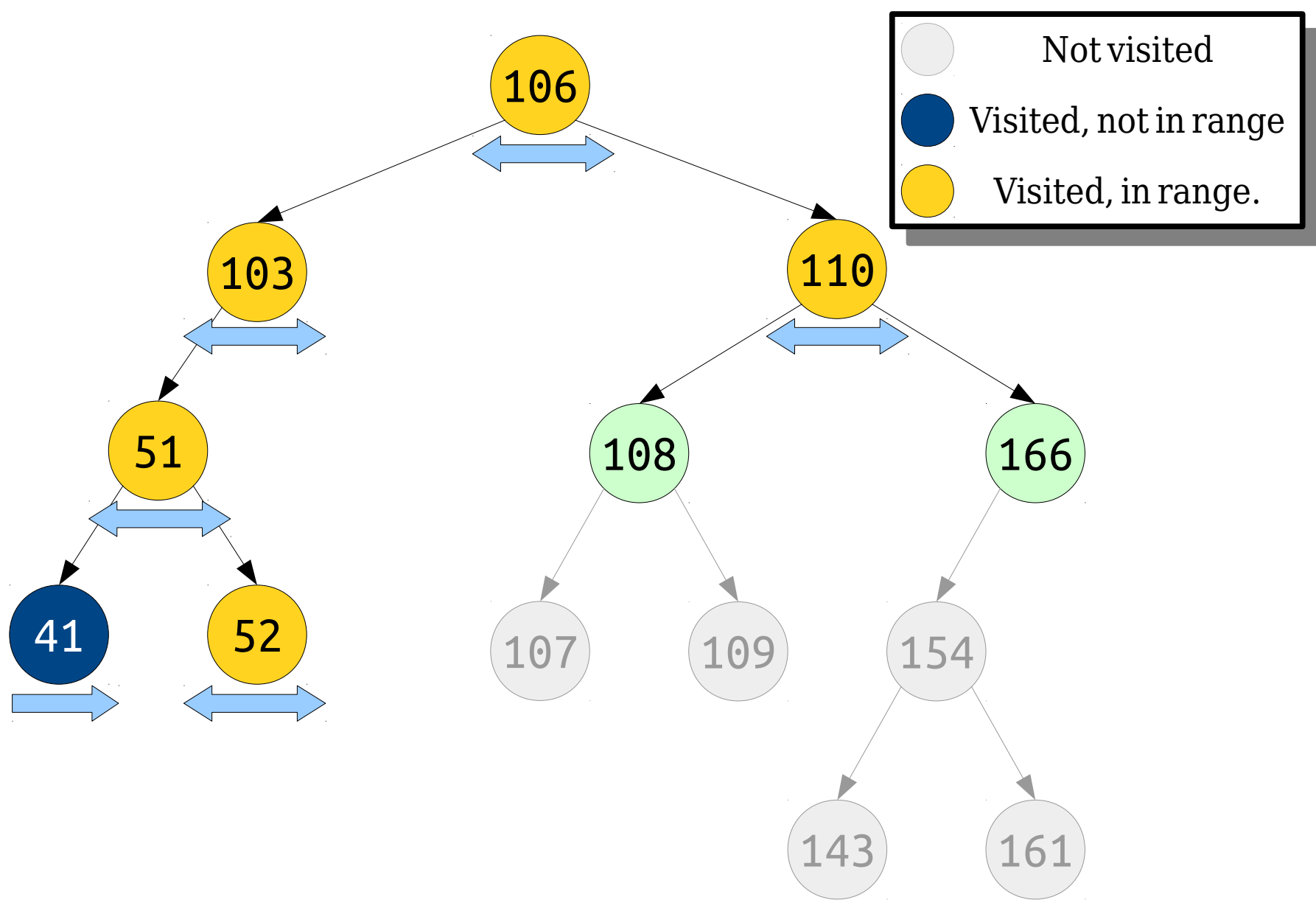
Find all elements in this tree in the range **[42, 165]**.



Find all elements in this tree in the range **[42, 165]**.

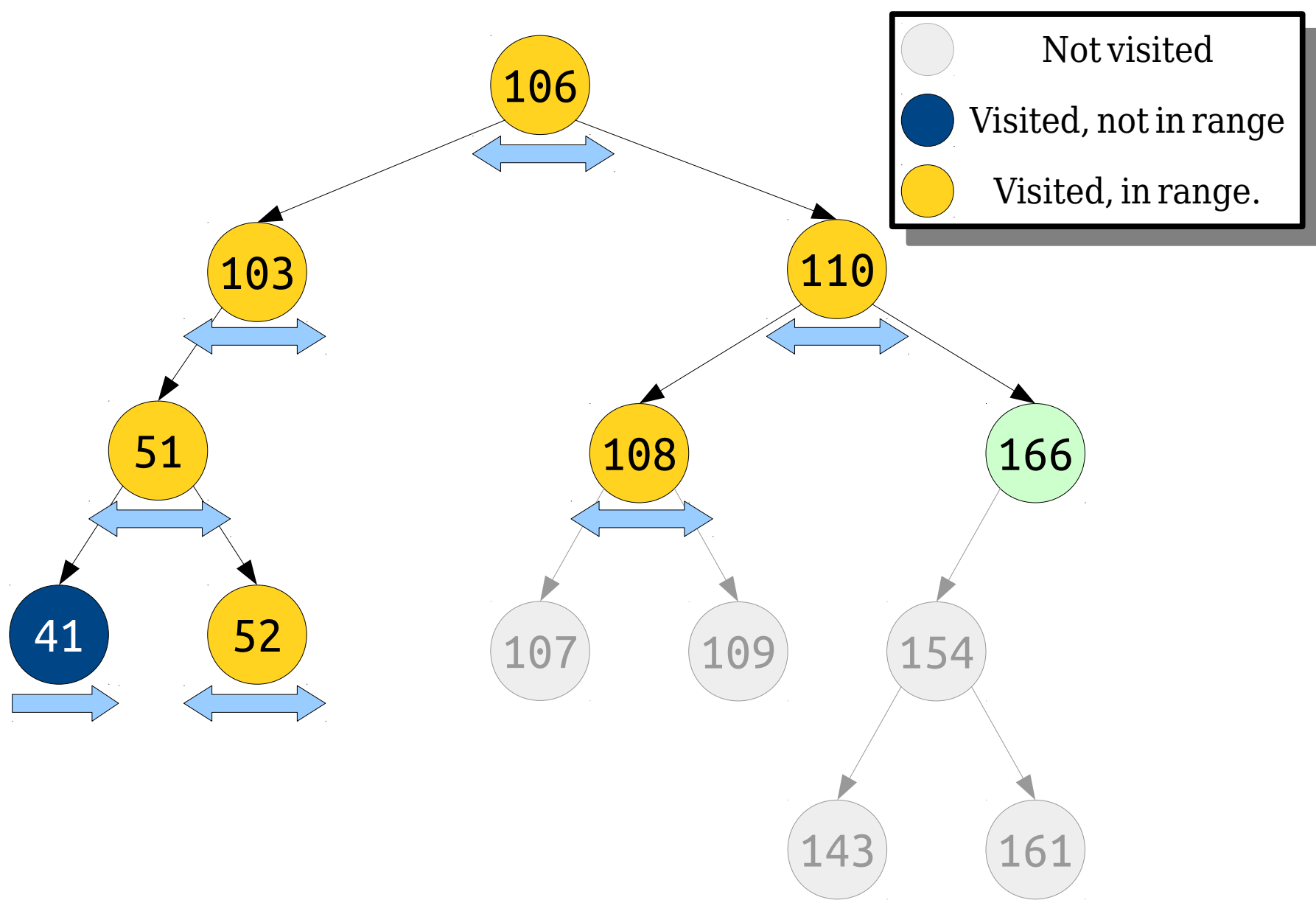


Find all elements in this tree in the range **[42, 165]**.

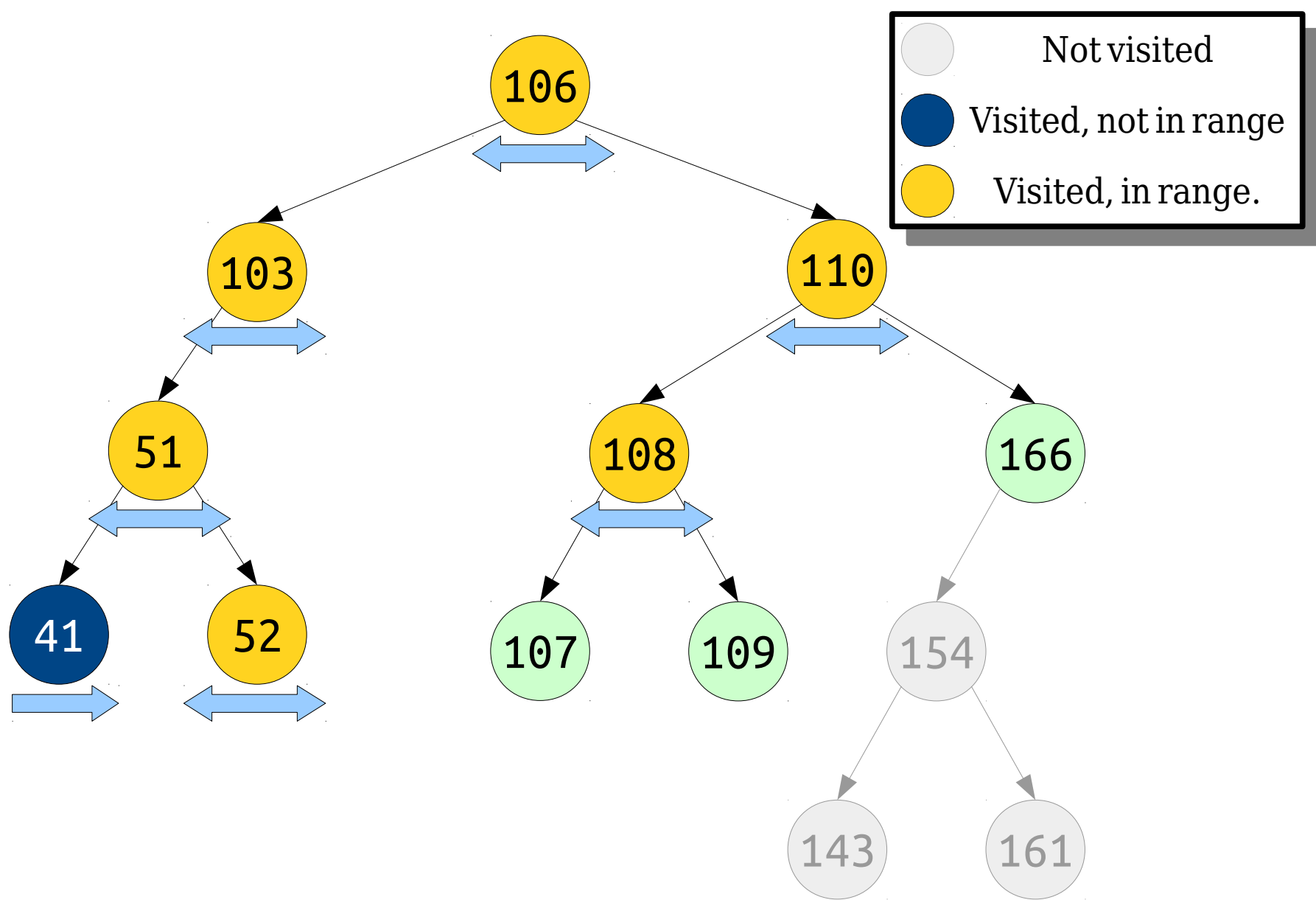


Find all elements in this tree in the range **[42, 165]**.

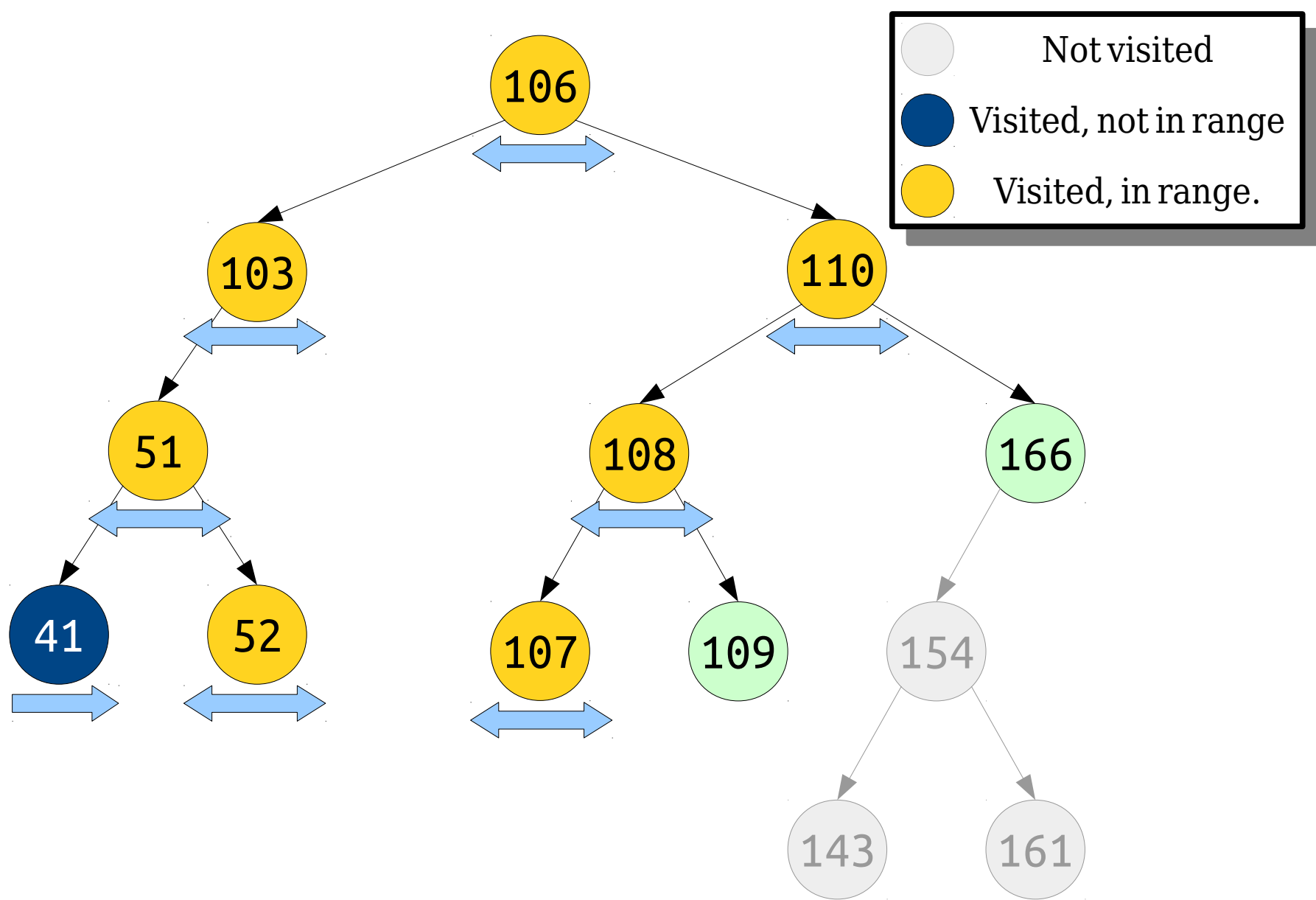




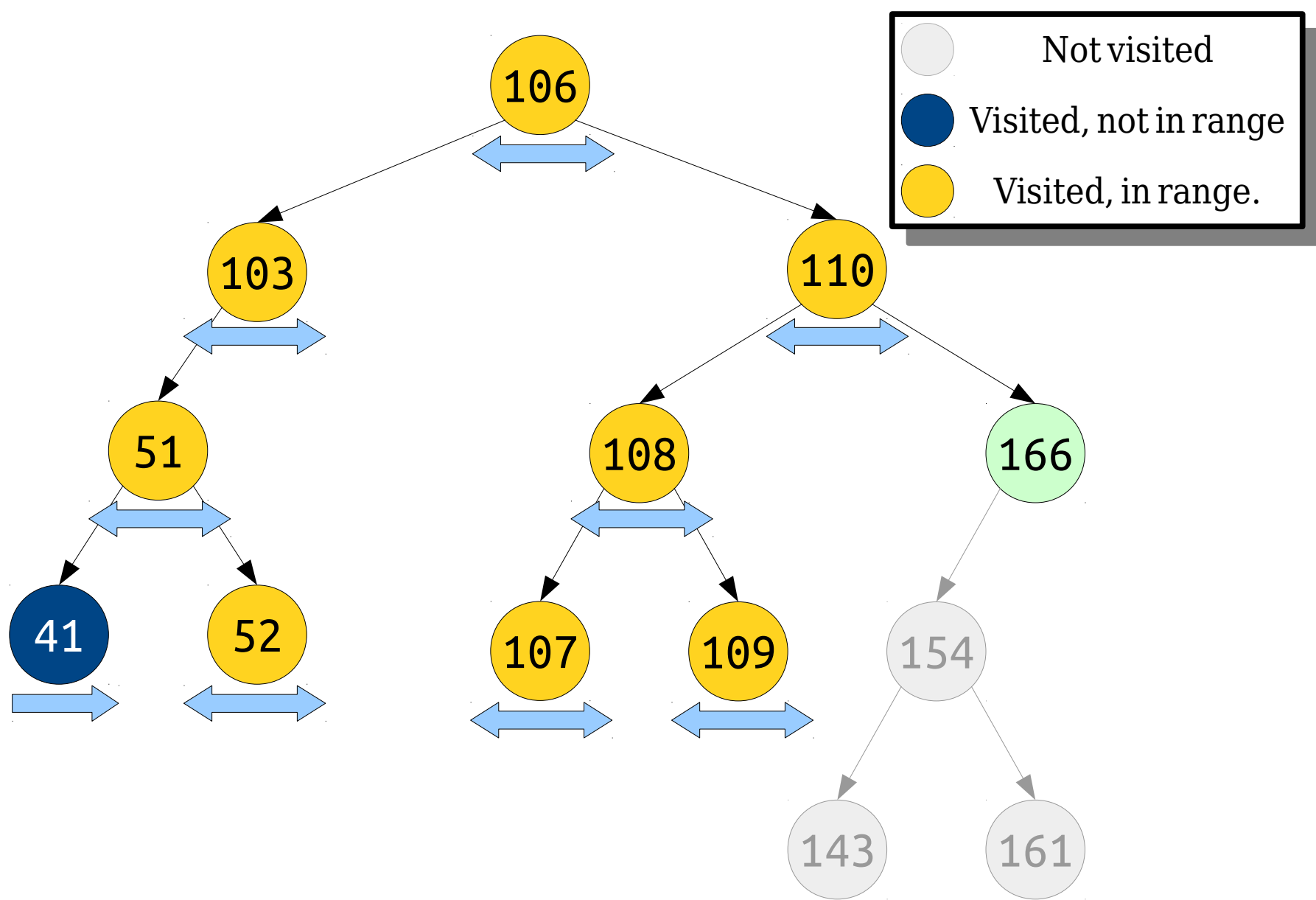
Find all elements in this tree in the range **[42, 165]**.



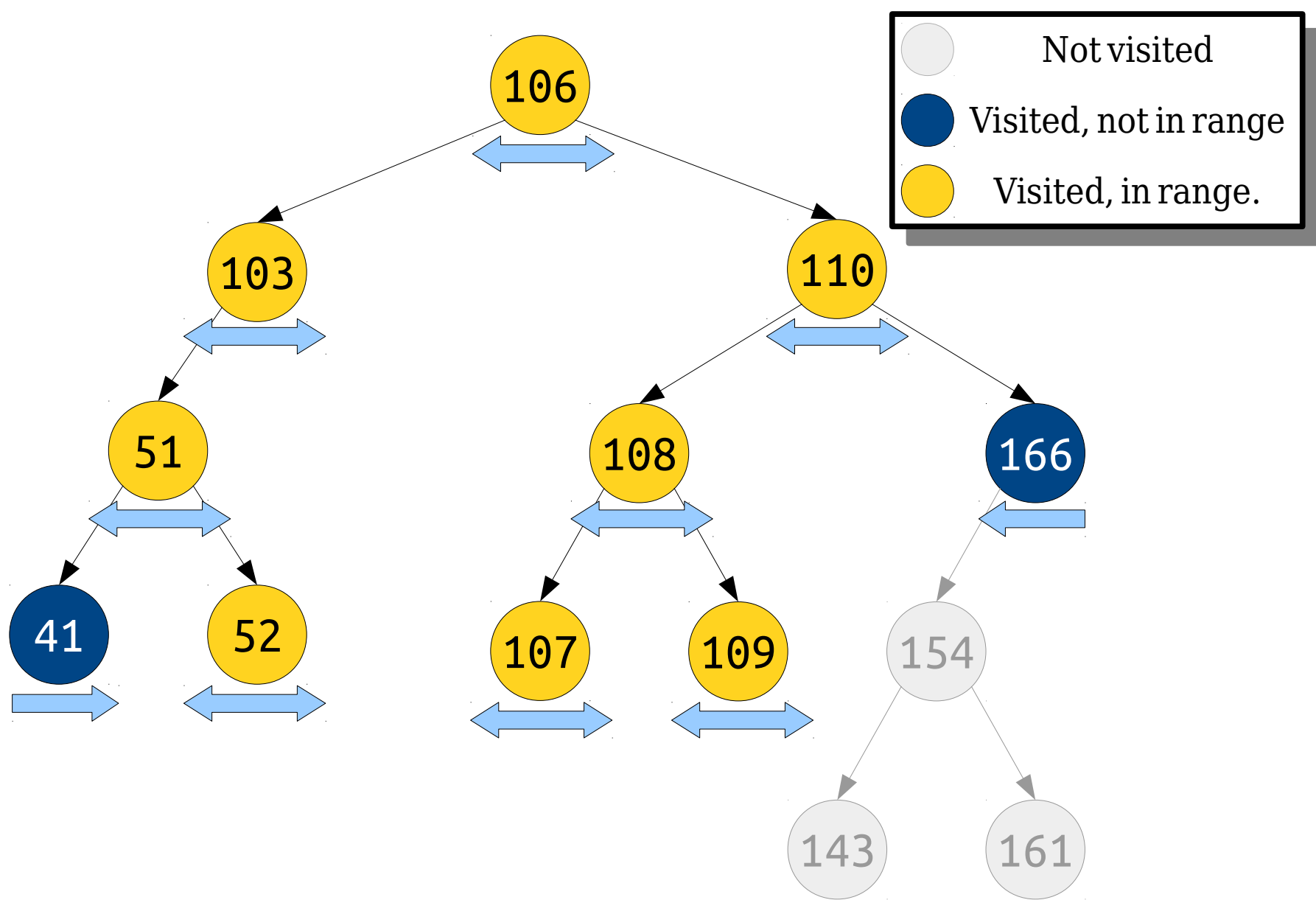
Find all elements in this tree in the range **[42, 165]**.



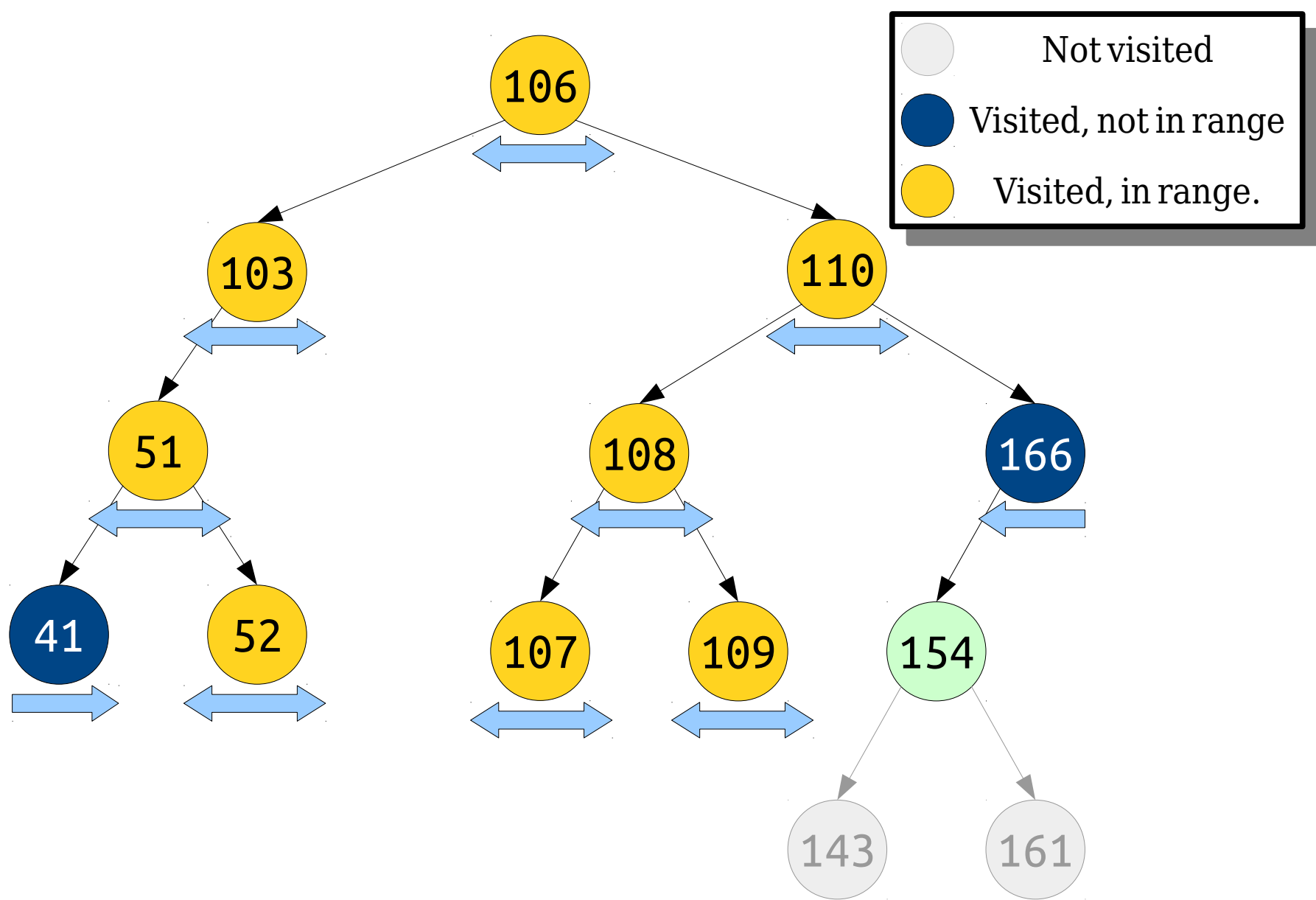
Find all elements in this tree in the range **[42, 165]**.



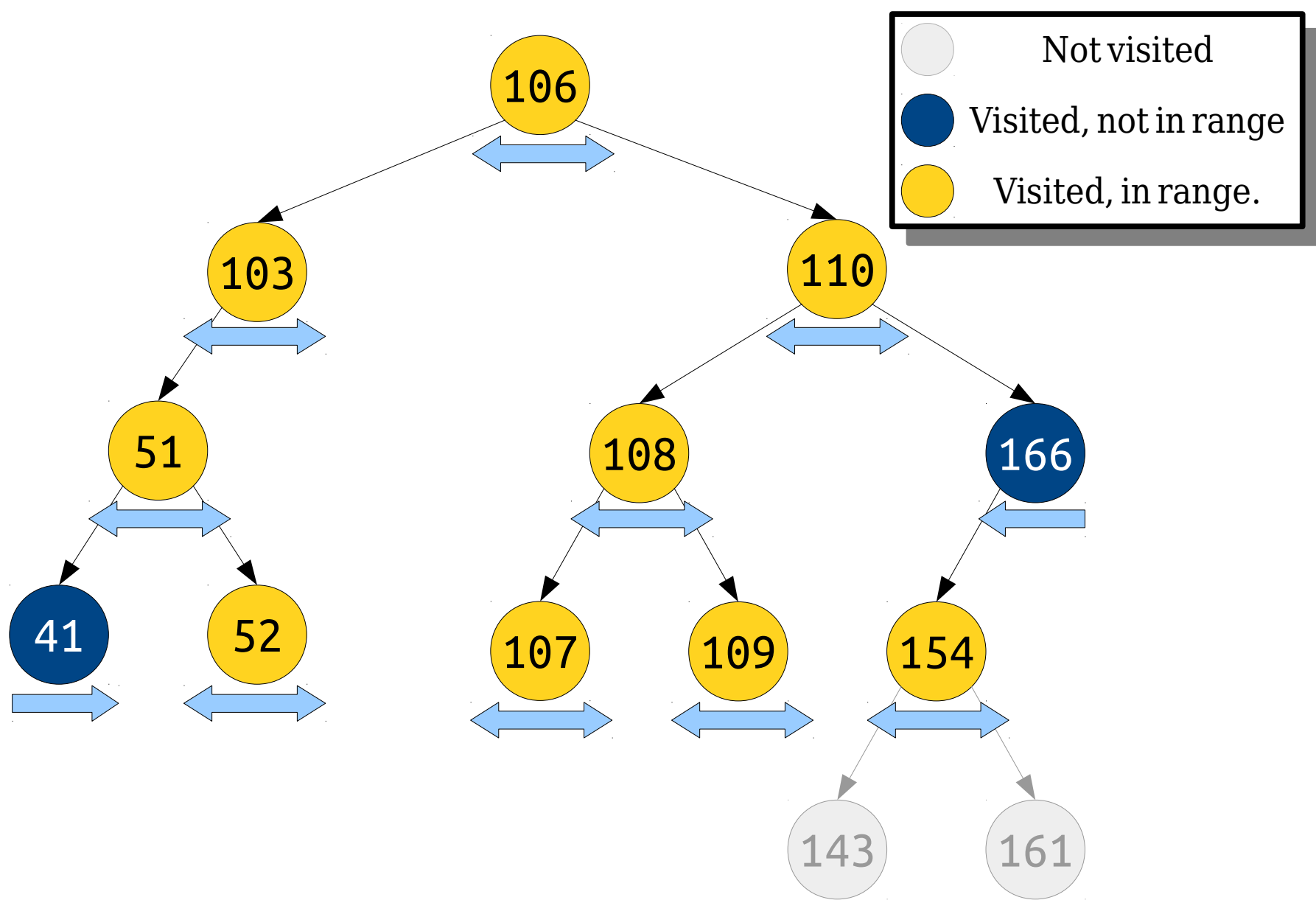
Find all elements in this tree in the range **[42, 165]**.



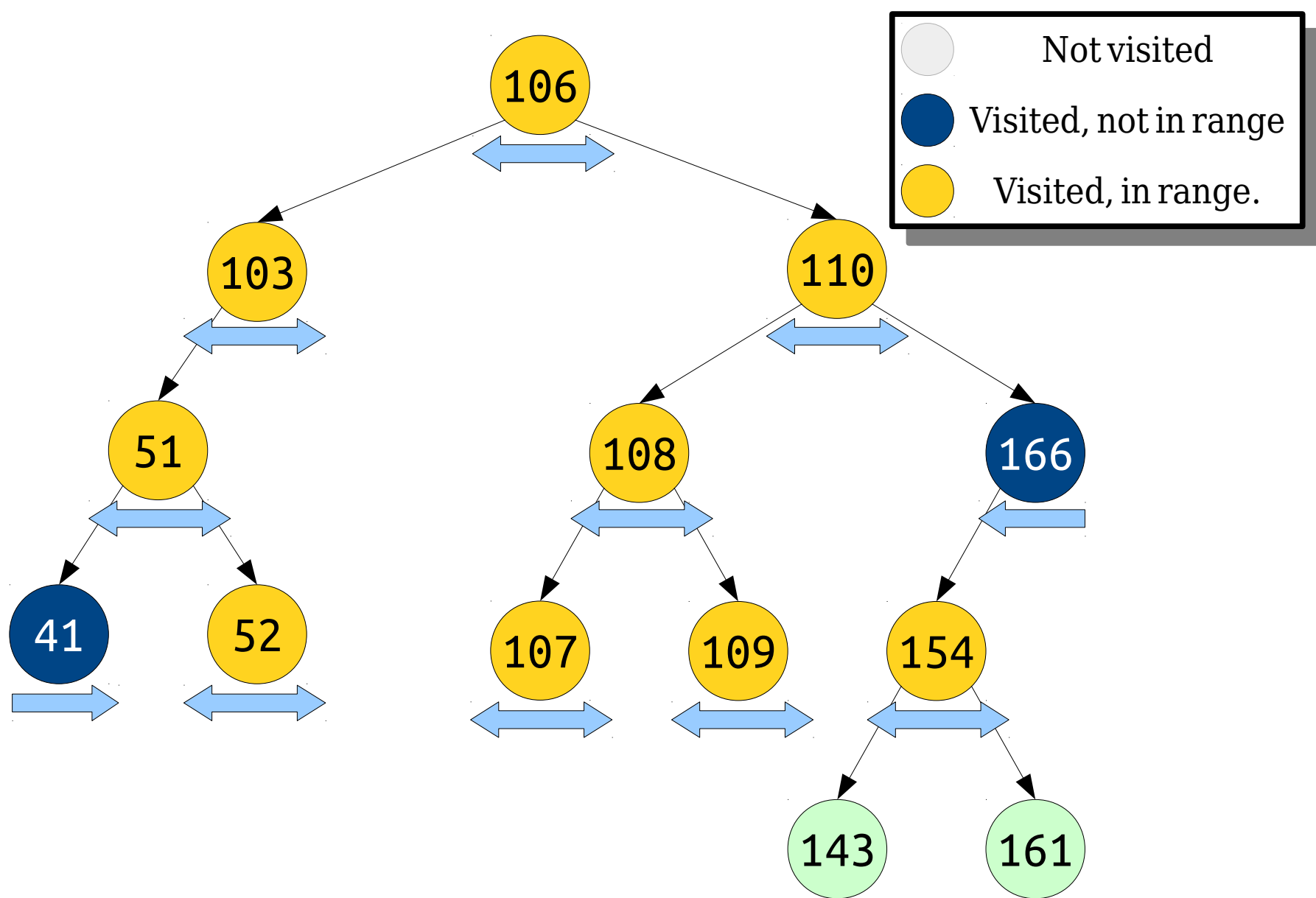
Find all elements in this tree in the range **[42, 165]**.



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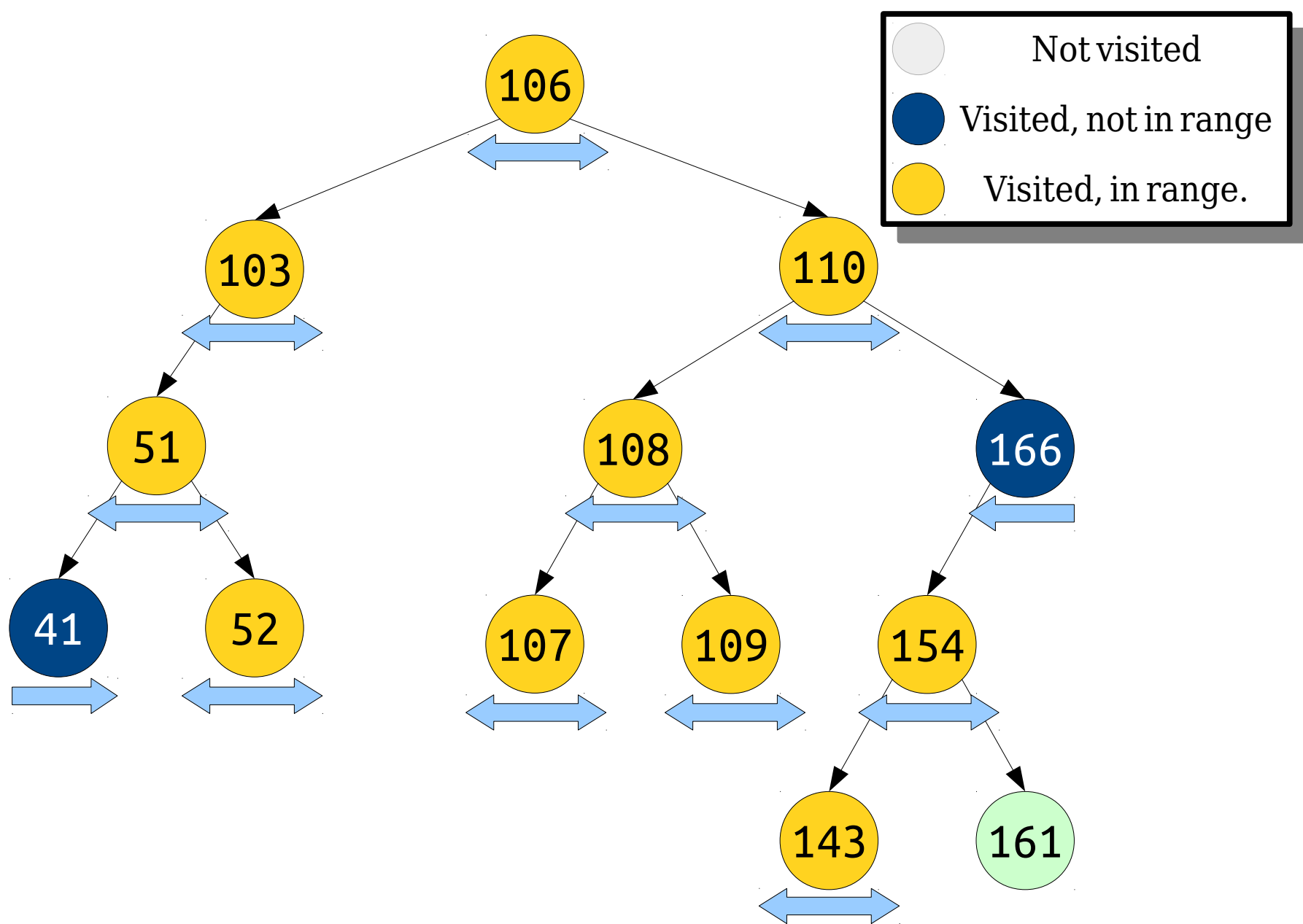


Find all elements in this tree in the range **[42, 165]**.

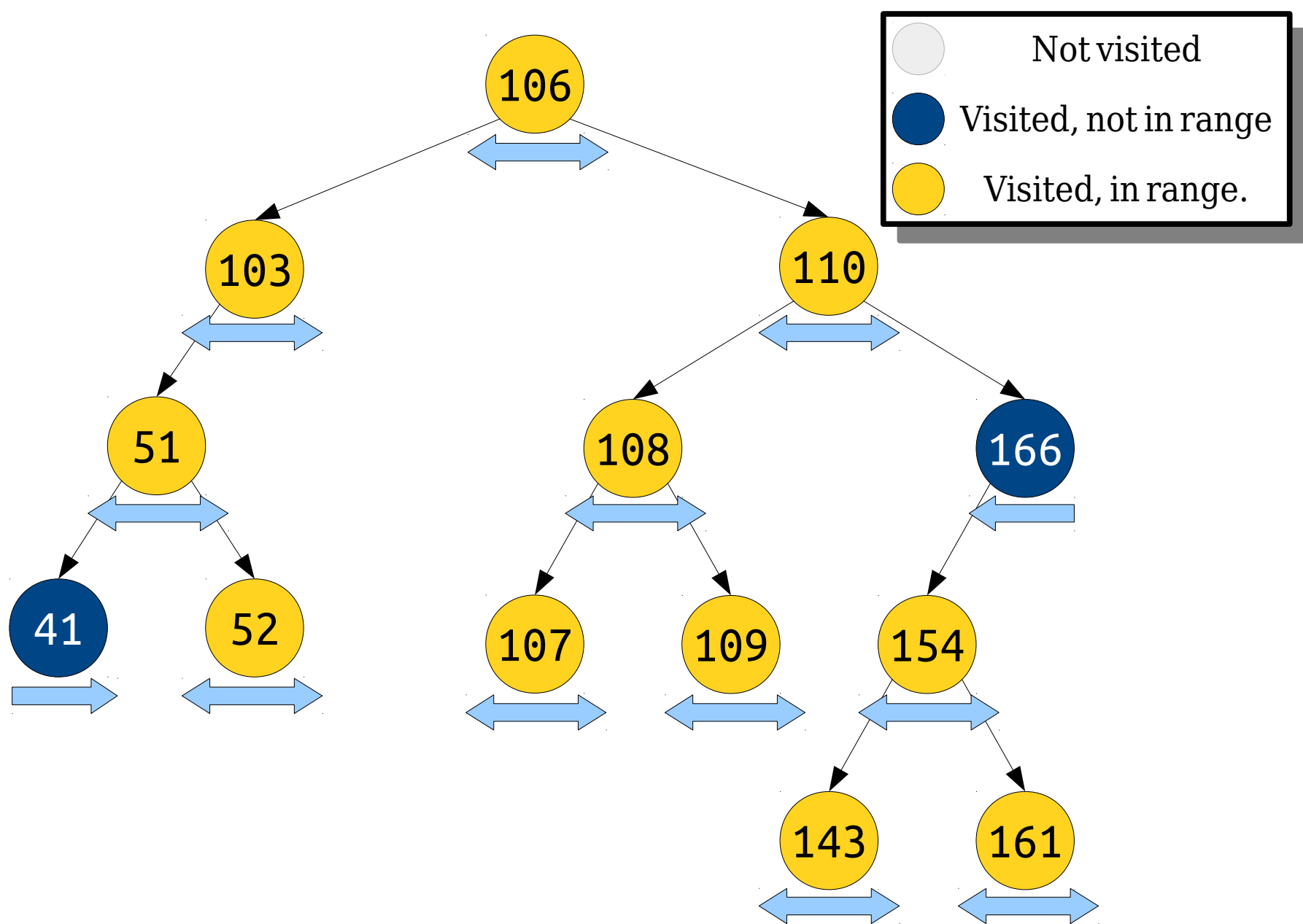


Find all elements in this tree in the range **[42, 165]**.

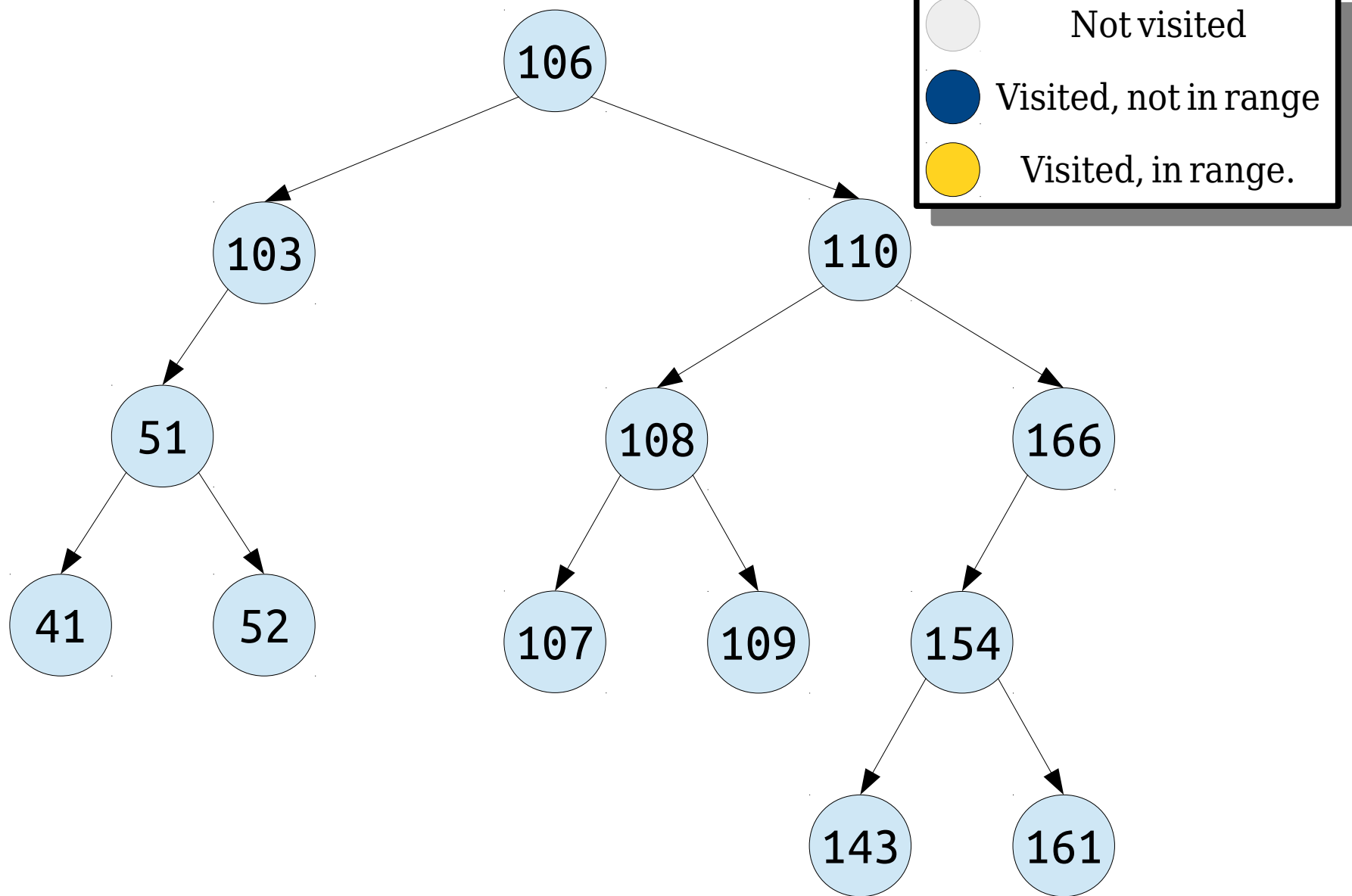




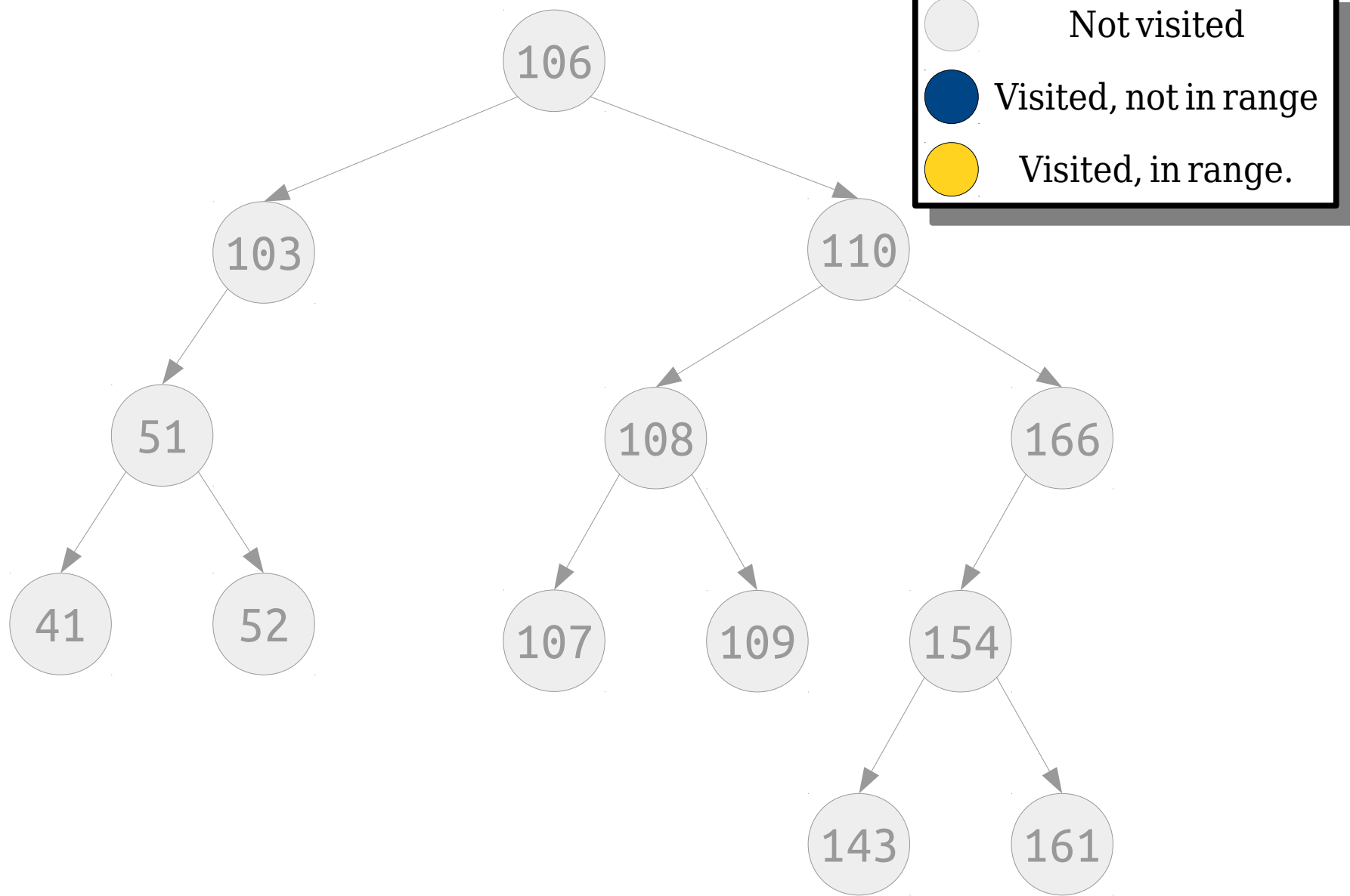
Find all elements in this tree in the range **[42, 165]**.



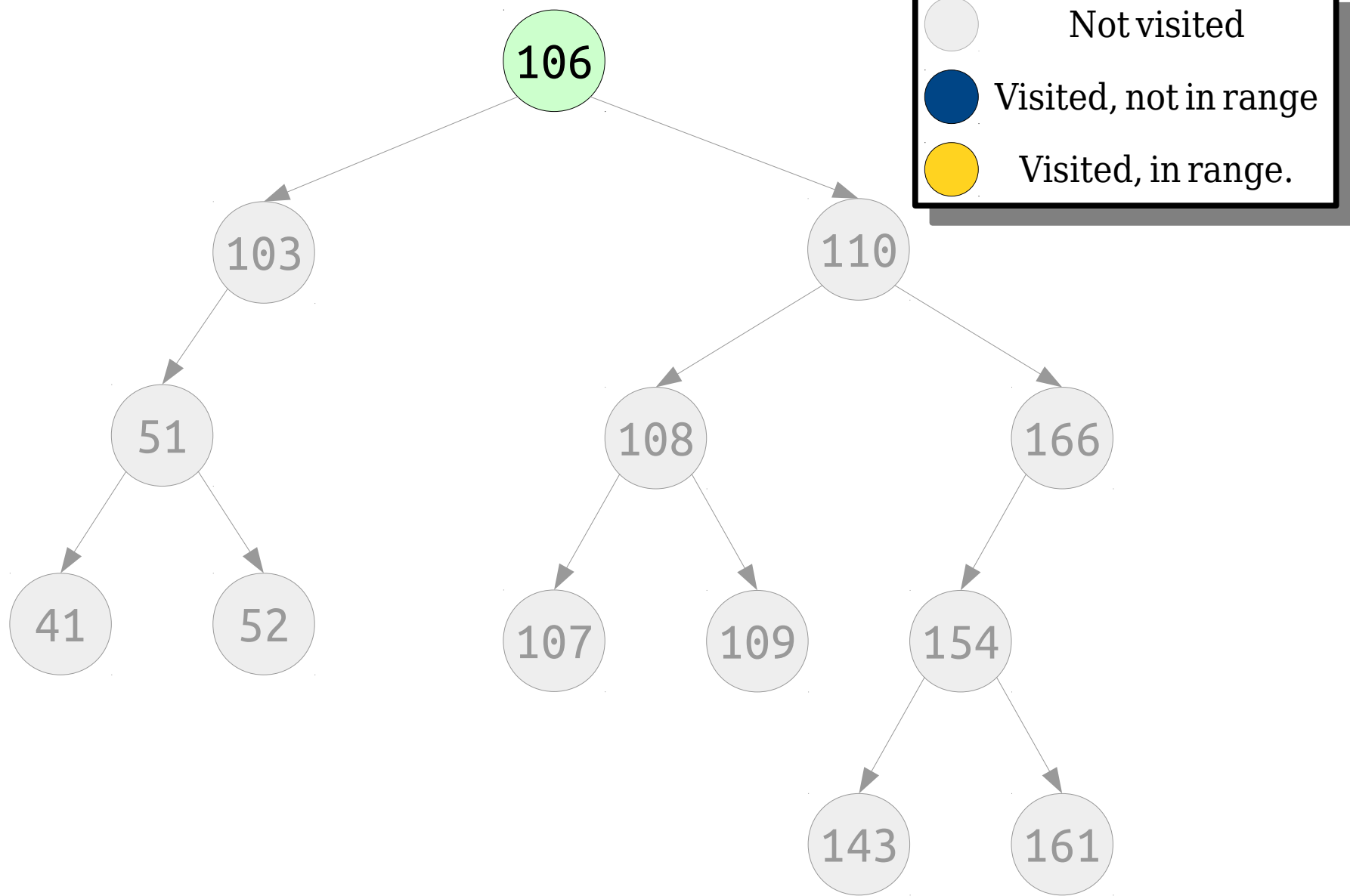
Find all elements in this tree in the range **[42, 165]**.



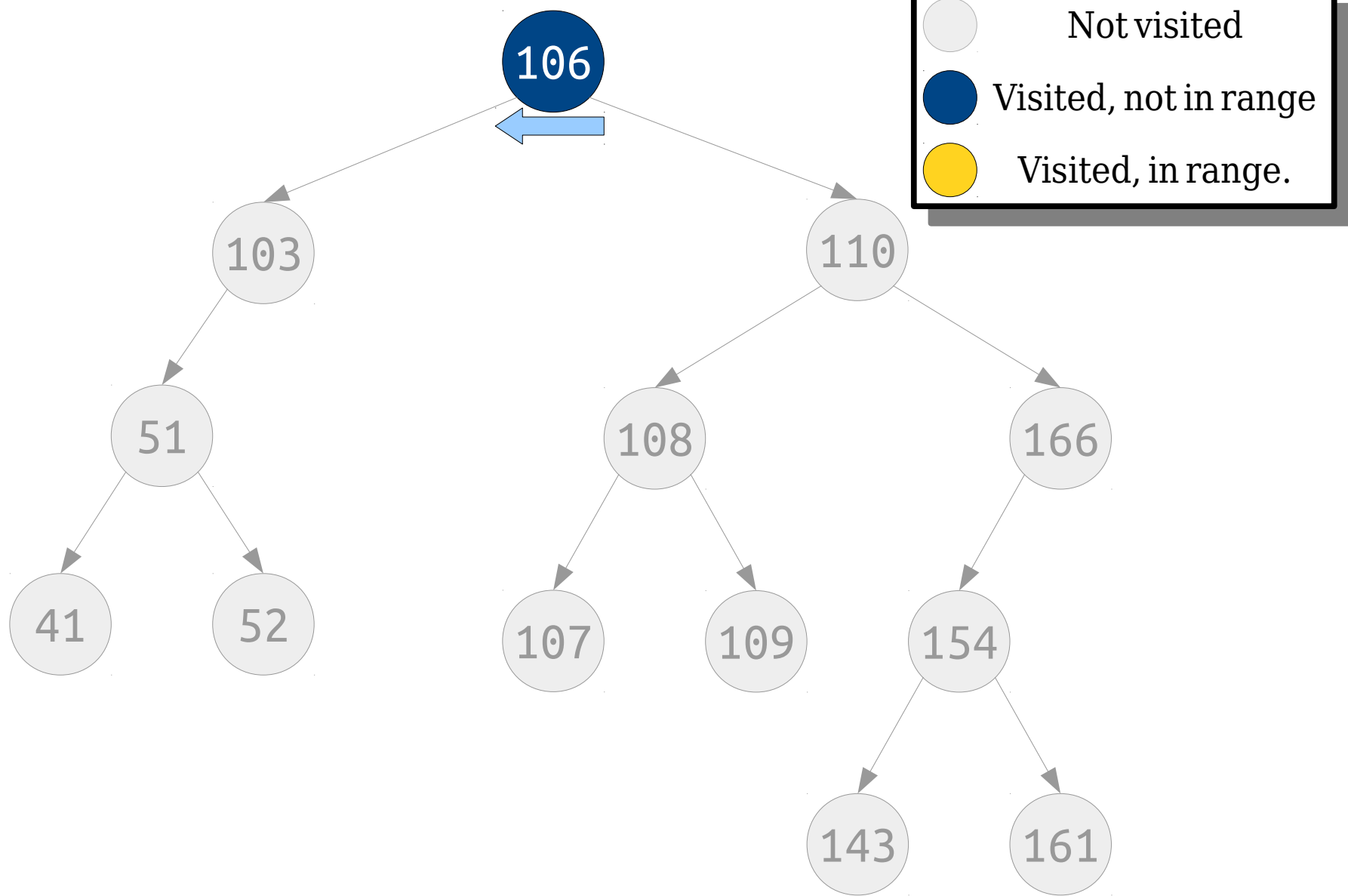
Find all elements in this tree in the range **[49, 50]**.



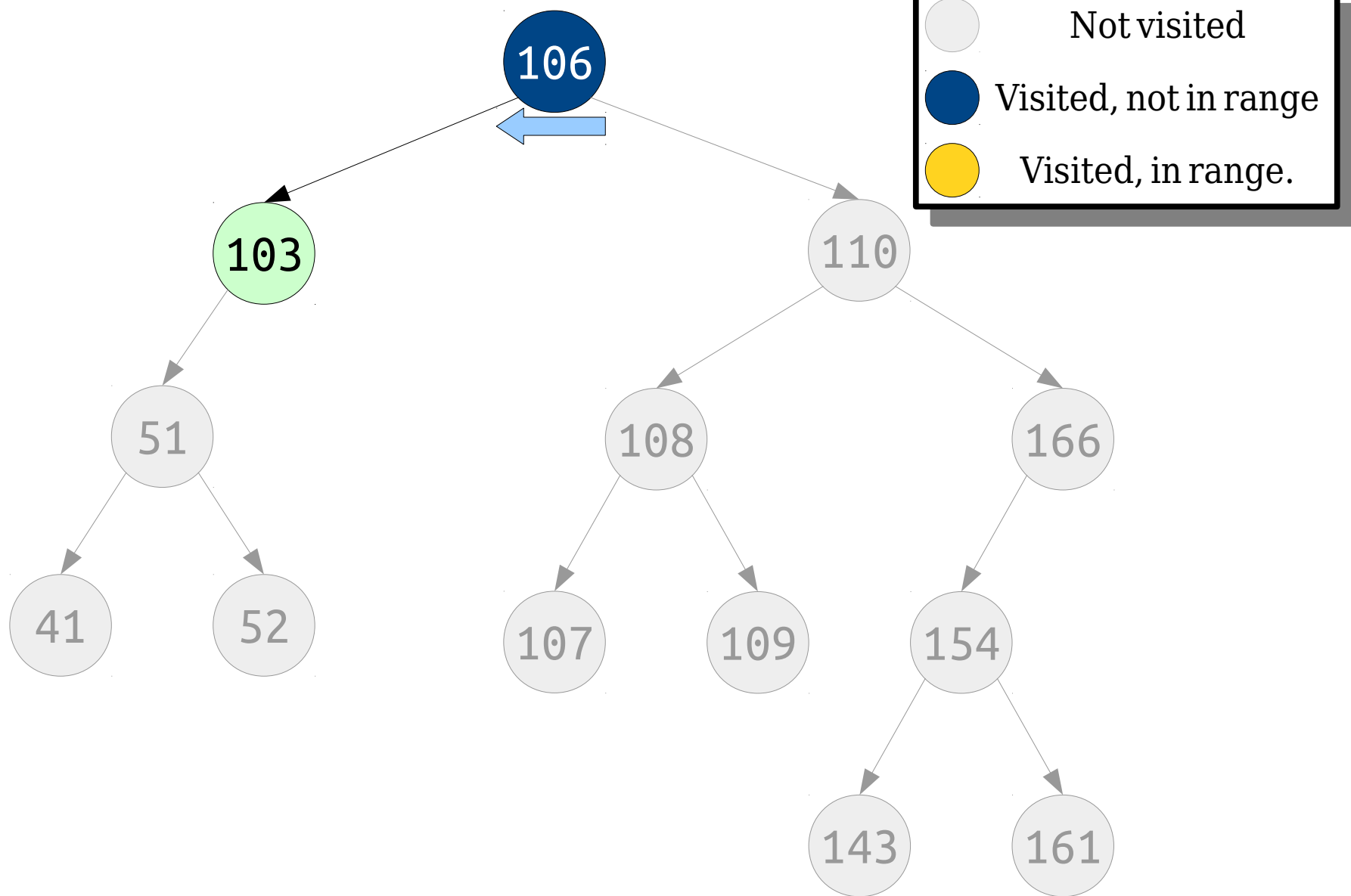
Find all elements in this tree in the range **[49, 50]**.



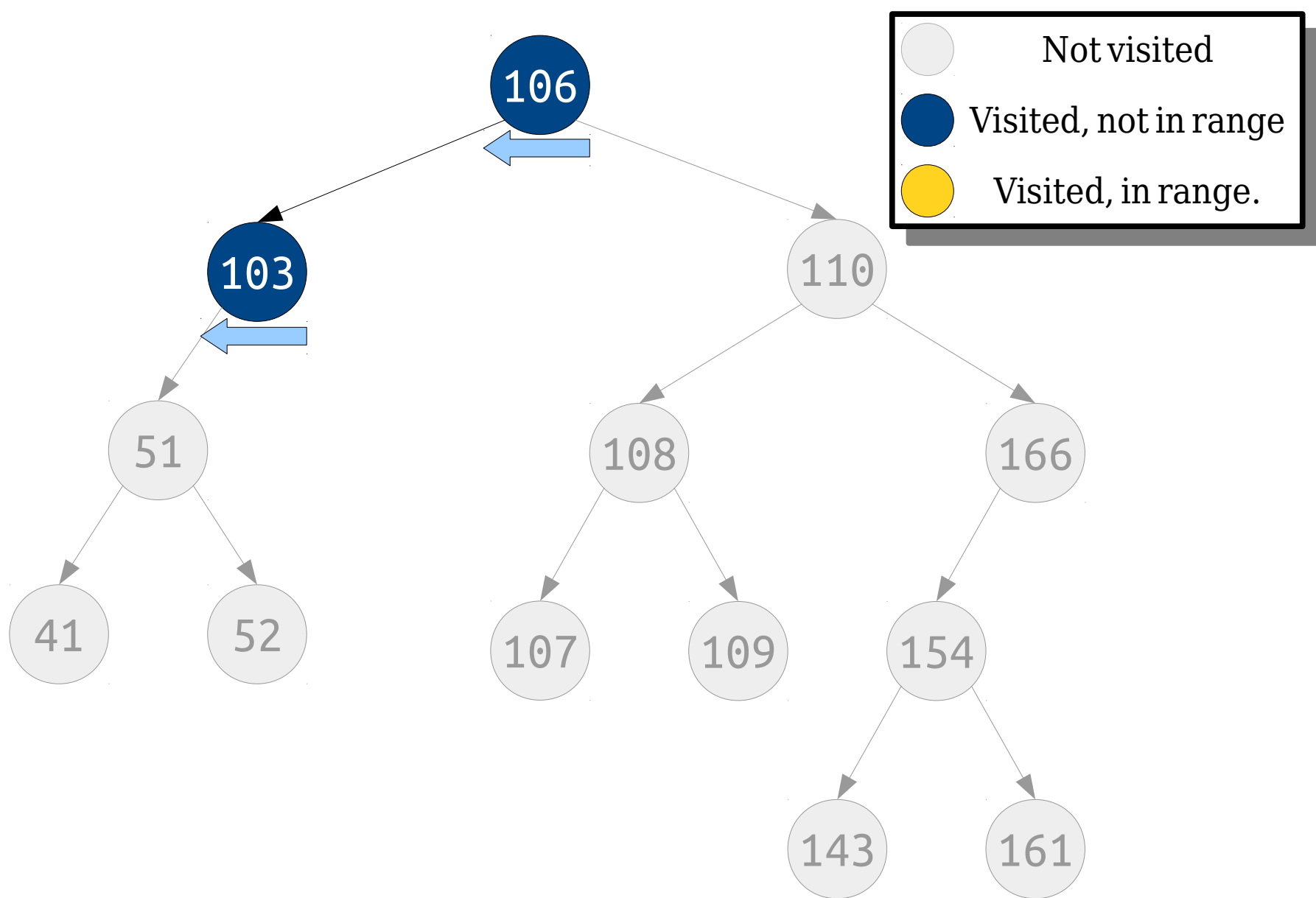
Find all elements in this tree in the range **[49, 50]**.



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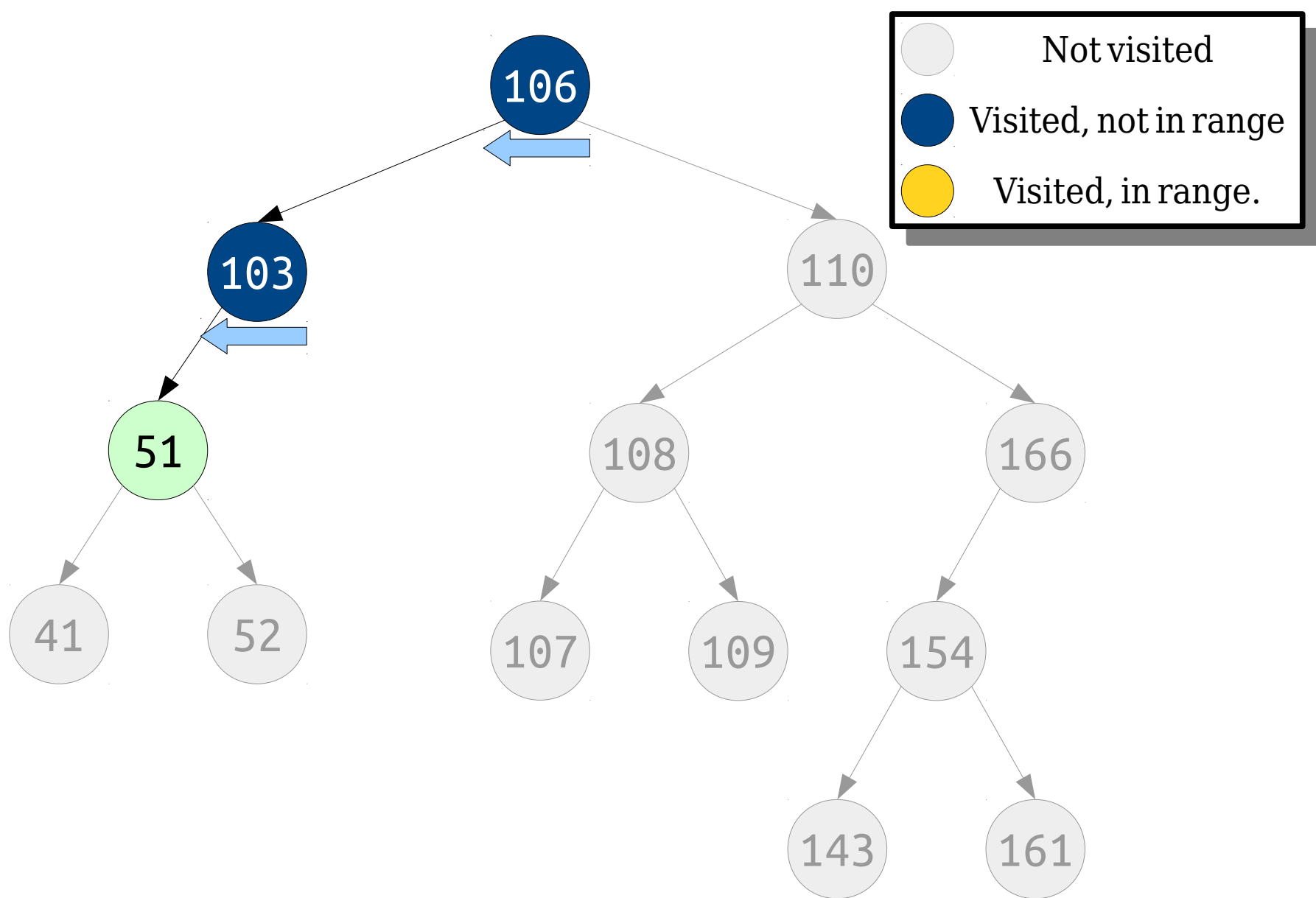


Find all elements in this tree in the range **[49, 50]**.

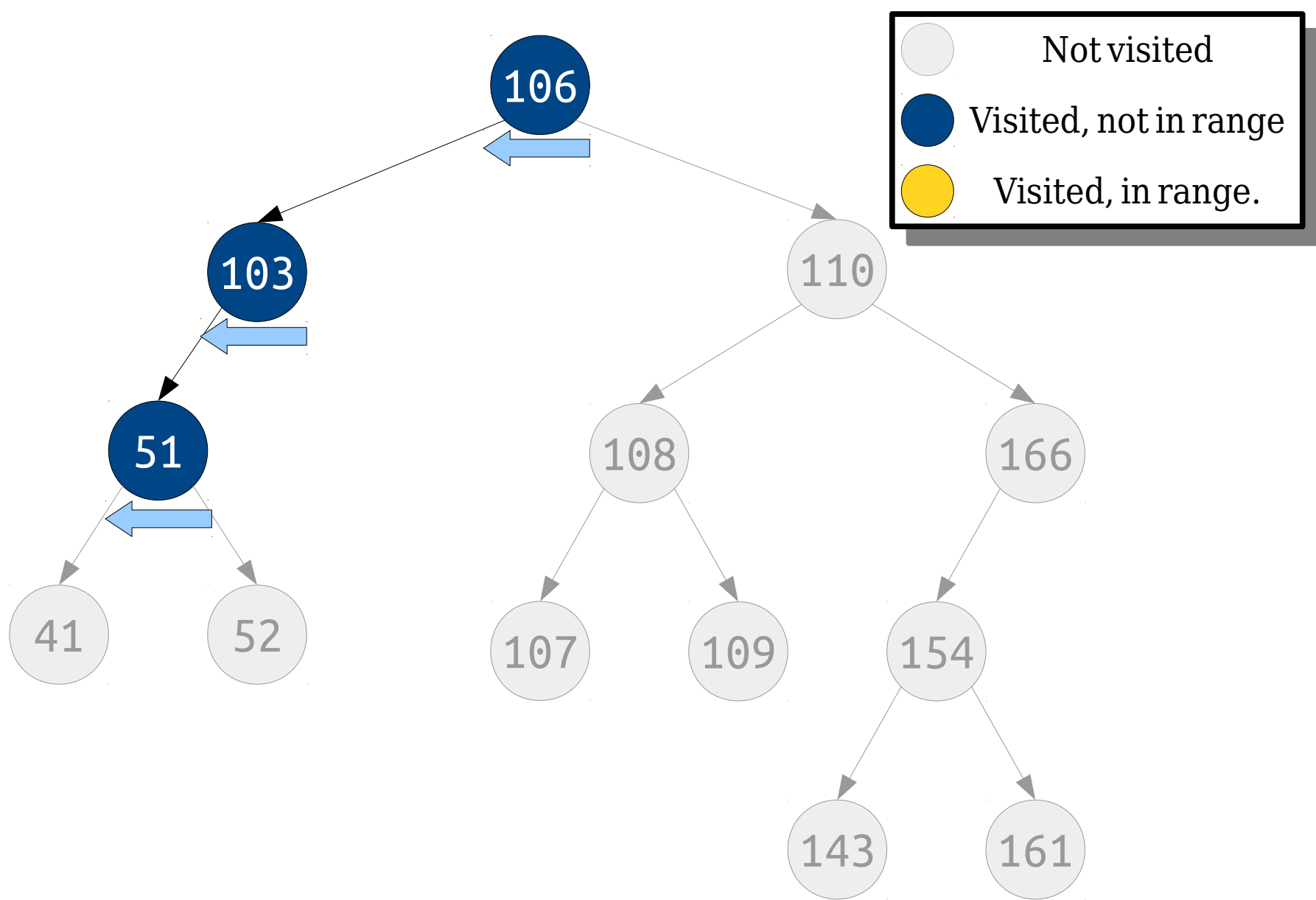


Find all elements in this tree in the range **[49, 50]**.

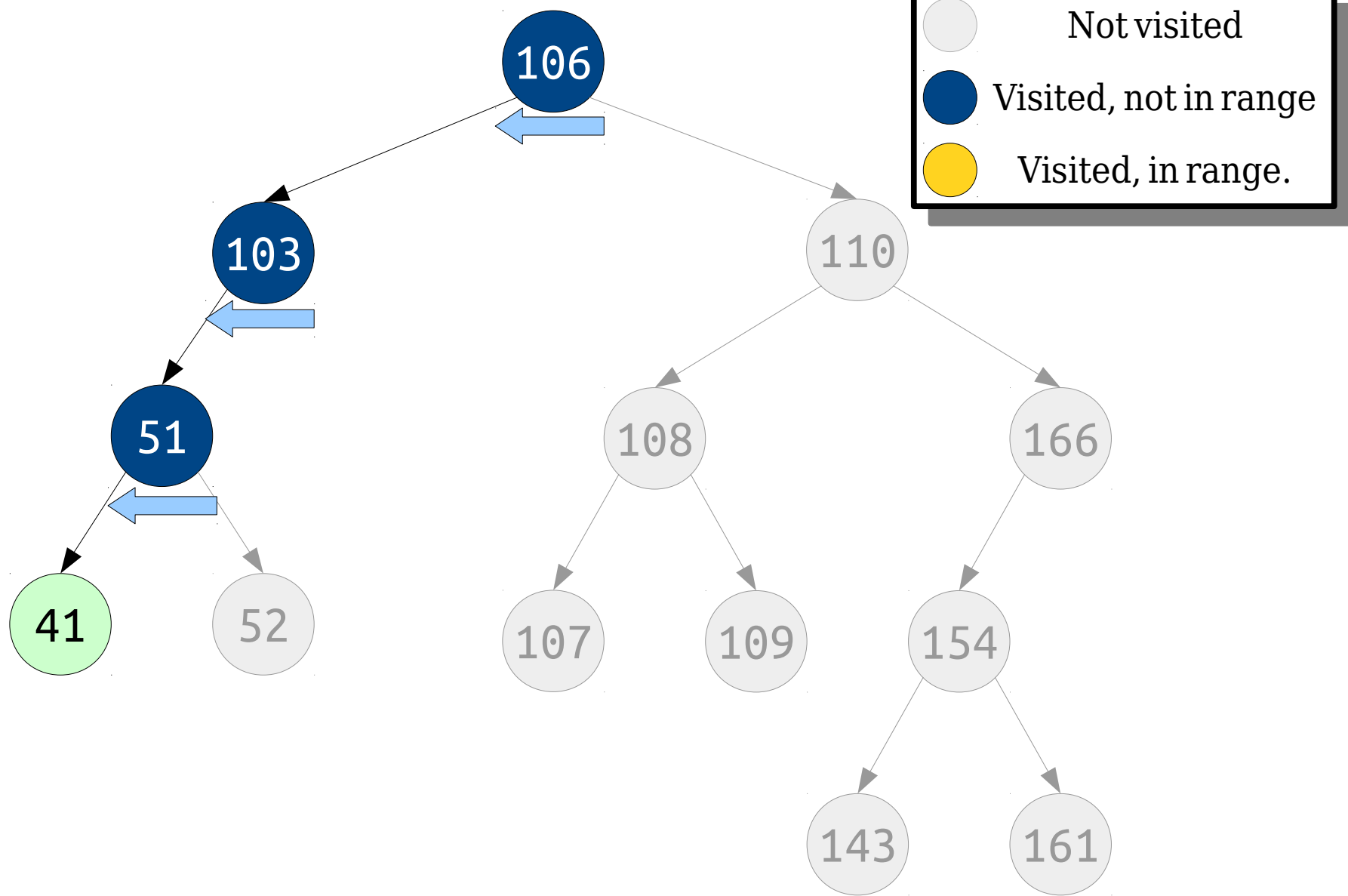




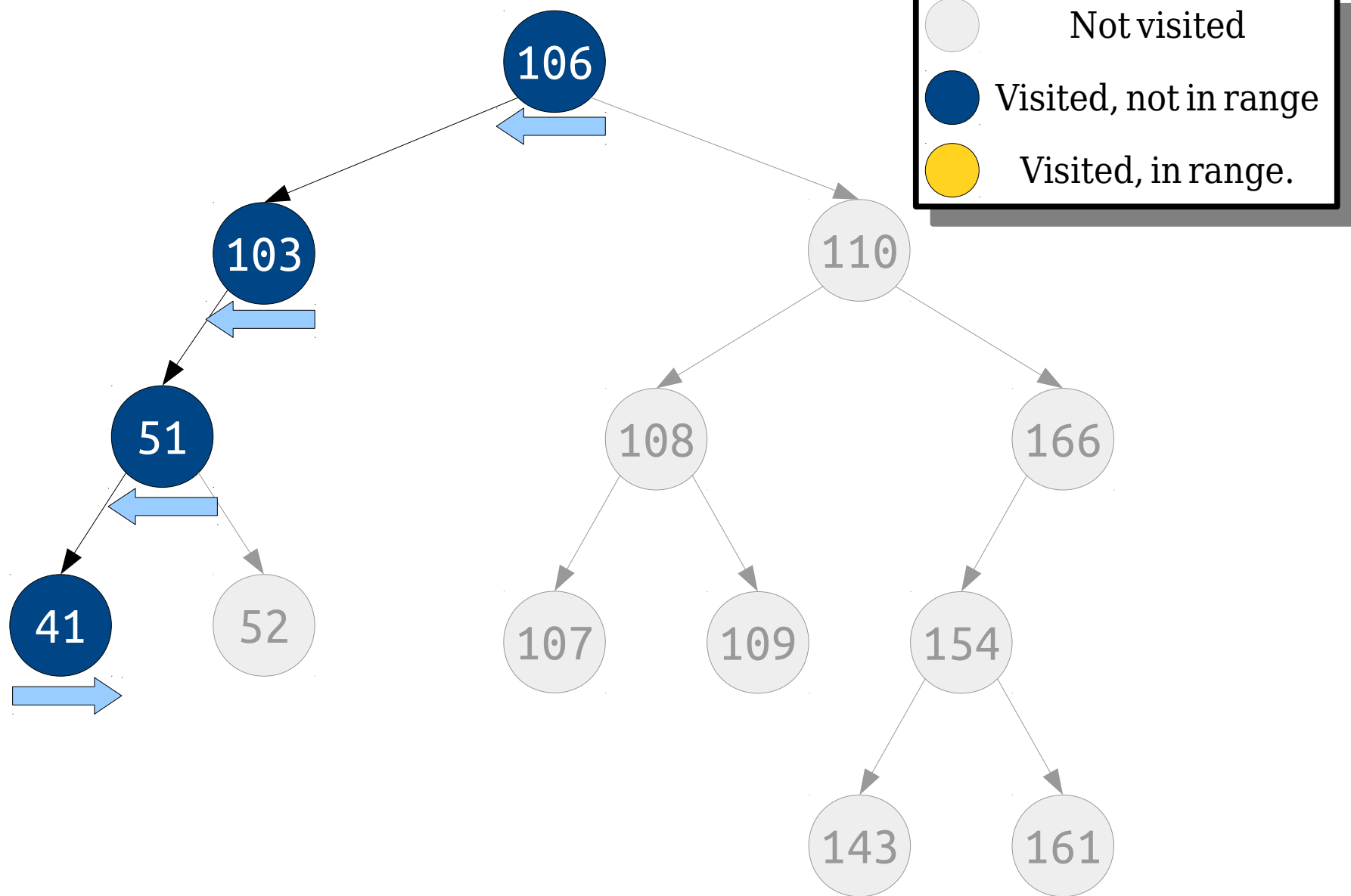
Find all elements in this tree in the range **[49, 50]**.



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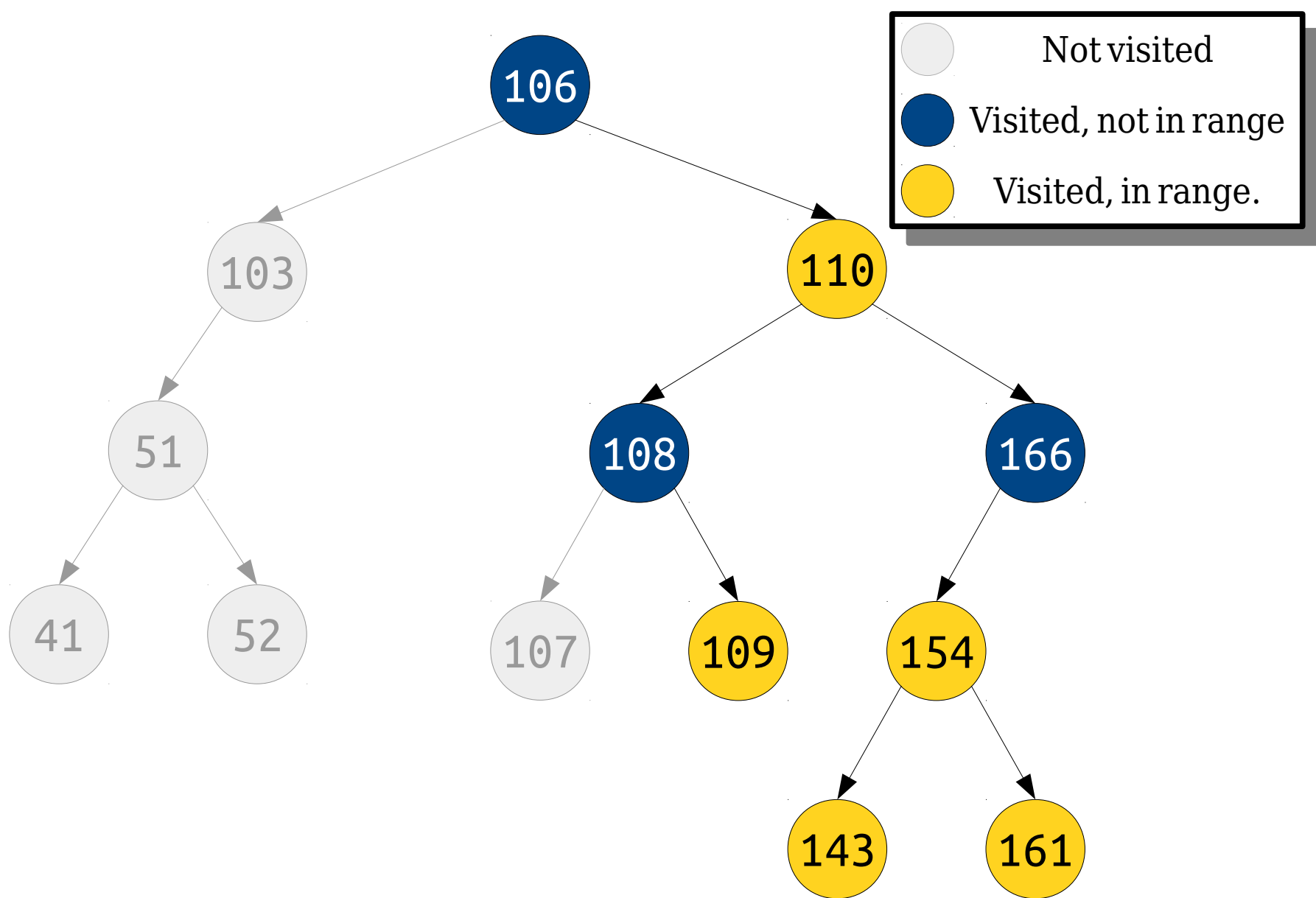


Find all elements in this tree in the range **[49, 50]**.

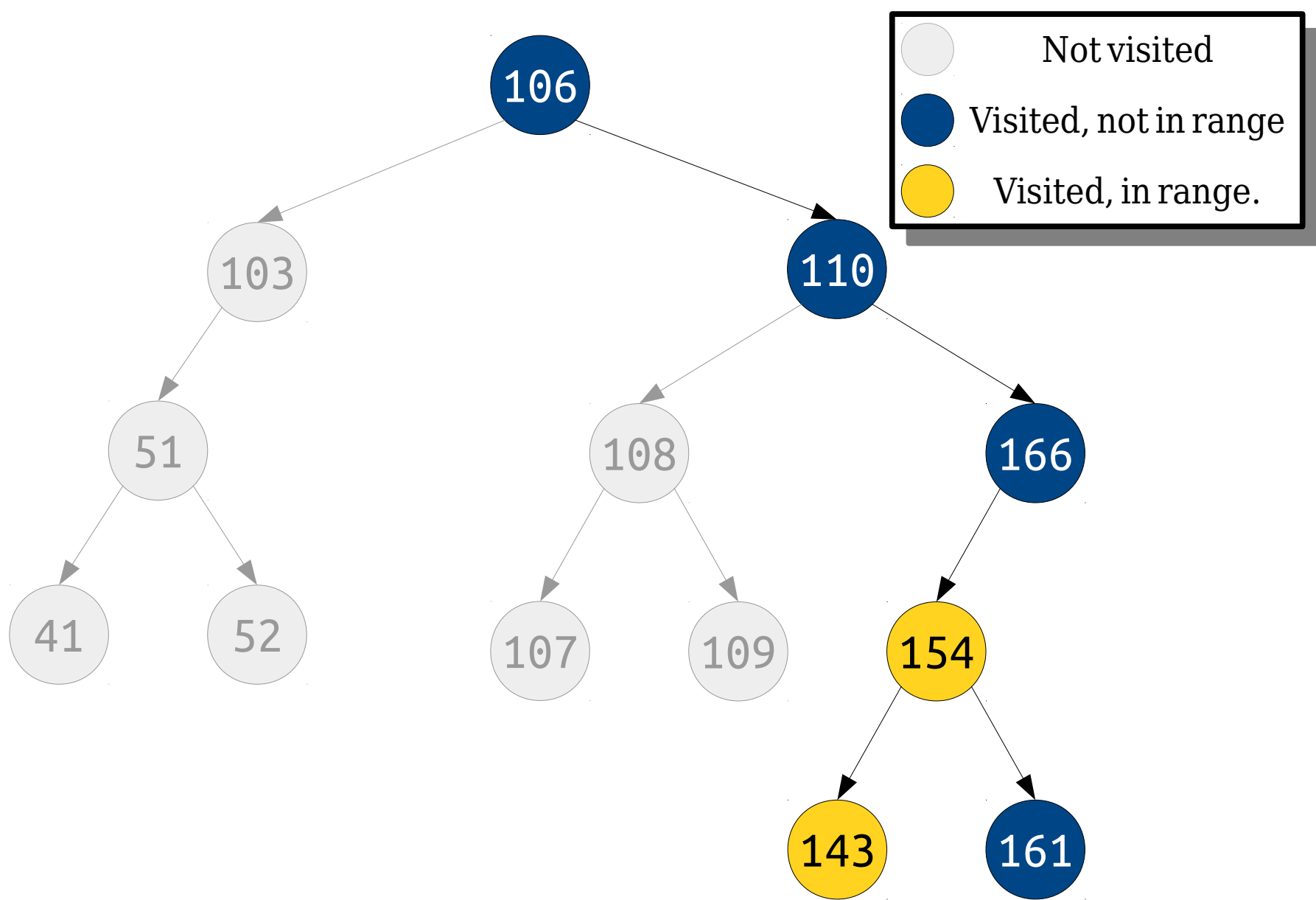
# Range Searches

- A hybrid between an inorder traversal and a regular BST lookup!
- The idea:
  - If the node is in the range being searched, add it to the result.
  - Recursively explore each subtree that could potentially overlap with the range.
- ***Question to Ponder:*** Given how an inorder traversal works, how would you code this up if you wanted the values back in sorted order?

How efficient is a range search?

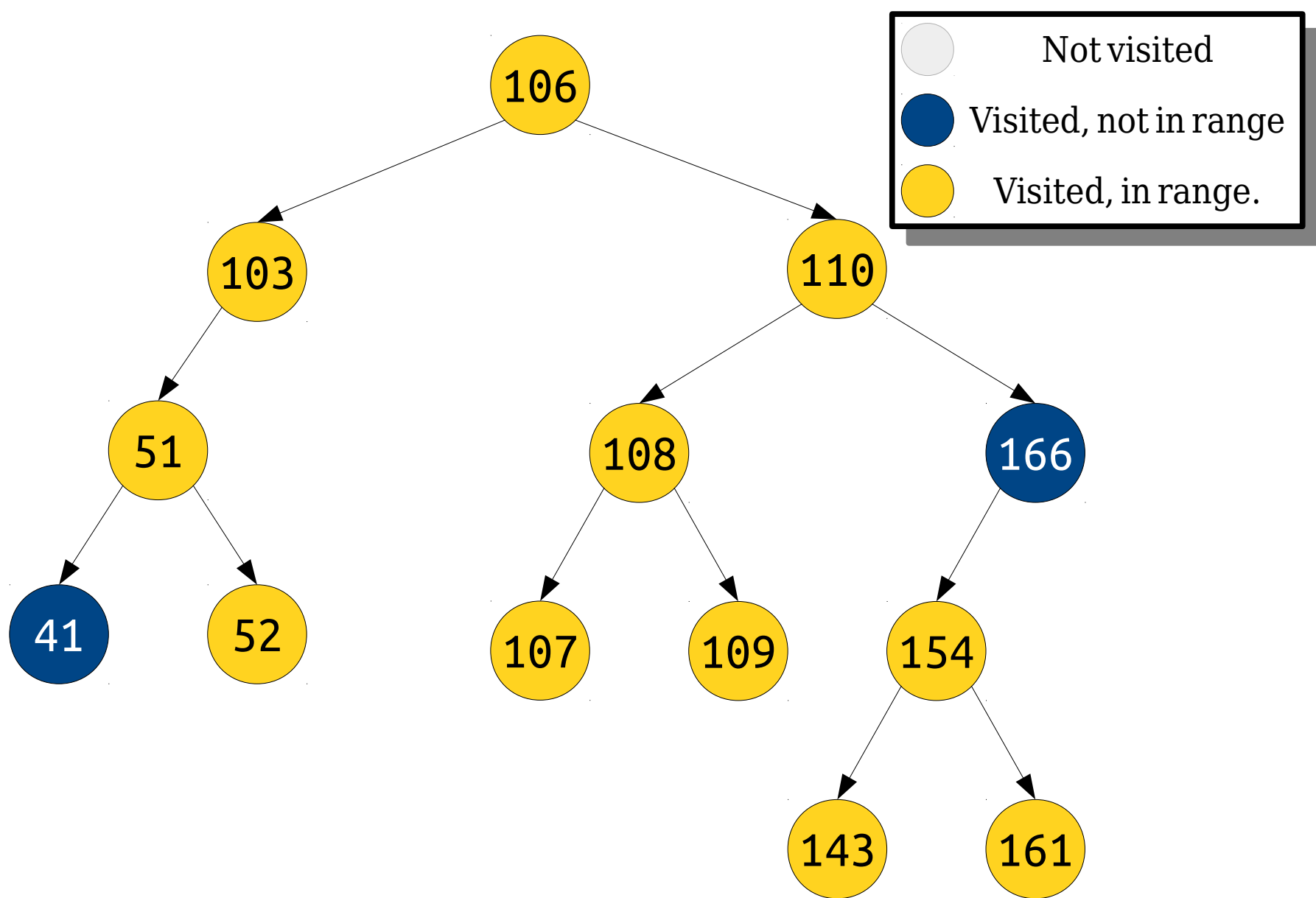


Find all elements in this tree in the range **[109, 163]**.

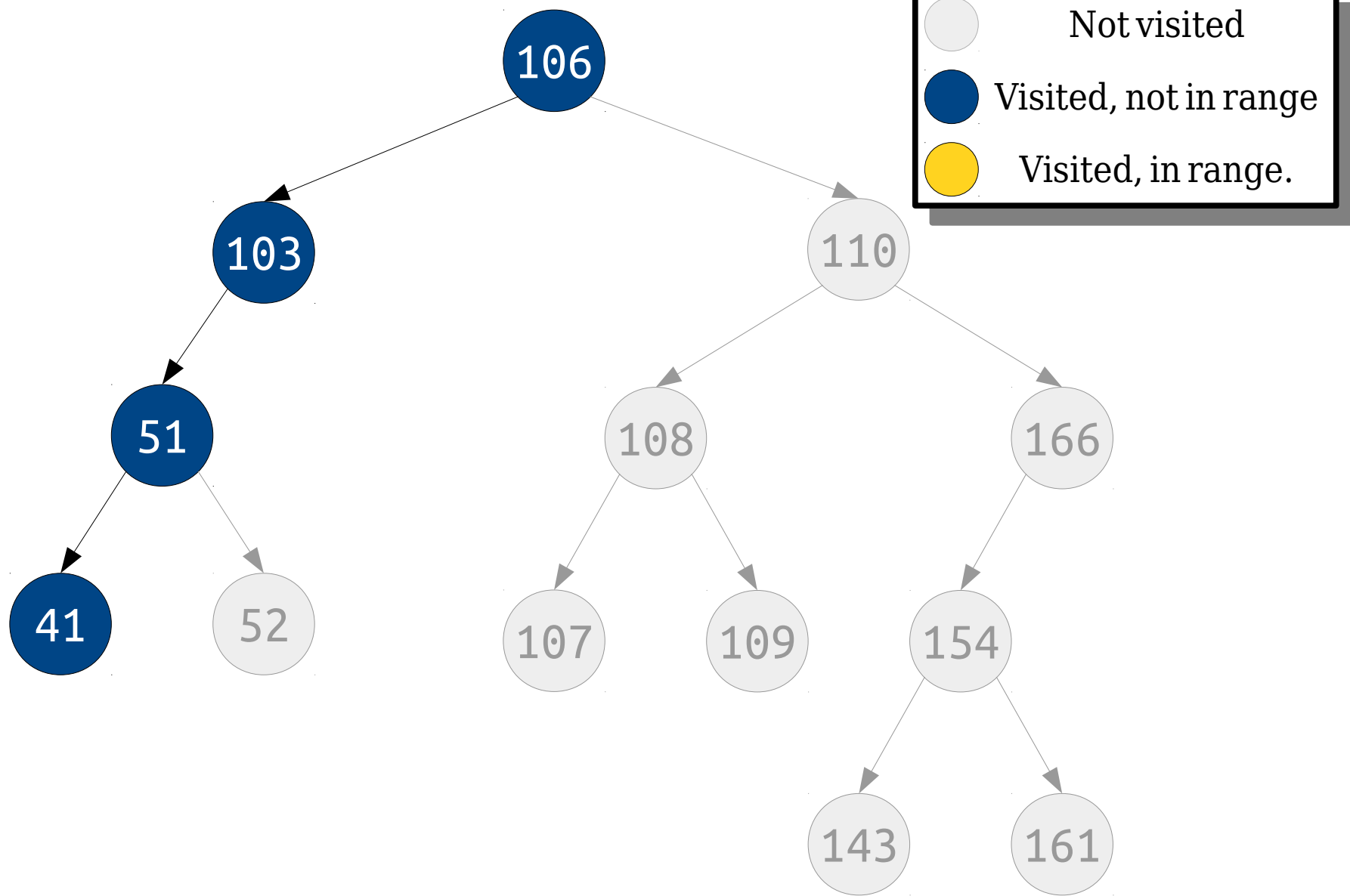


Find all elements in this tree in the range **[124, 155]**.

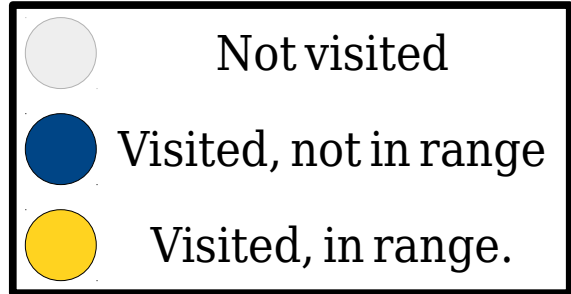
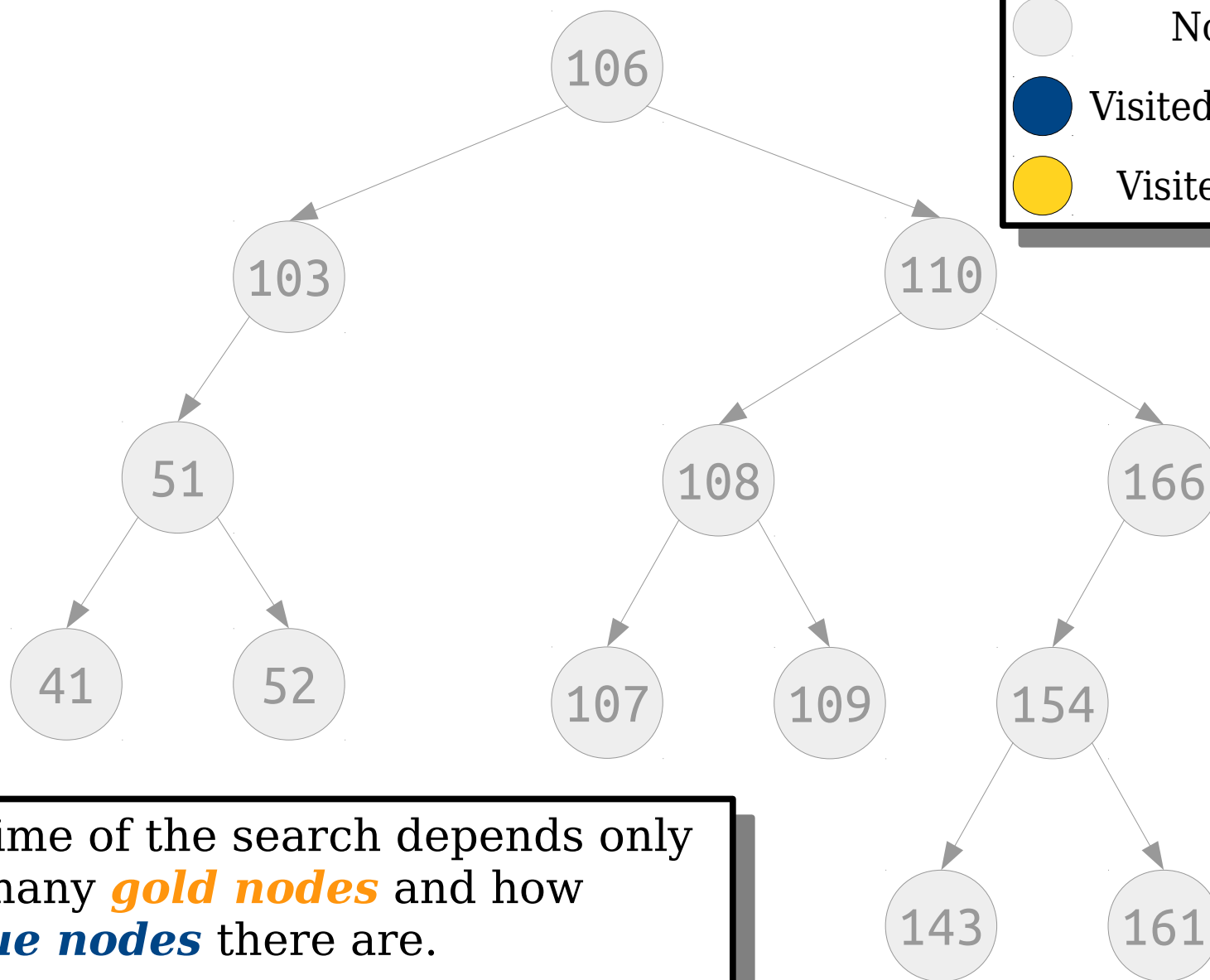




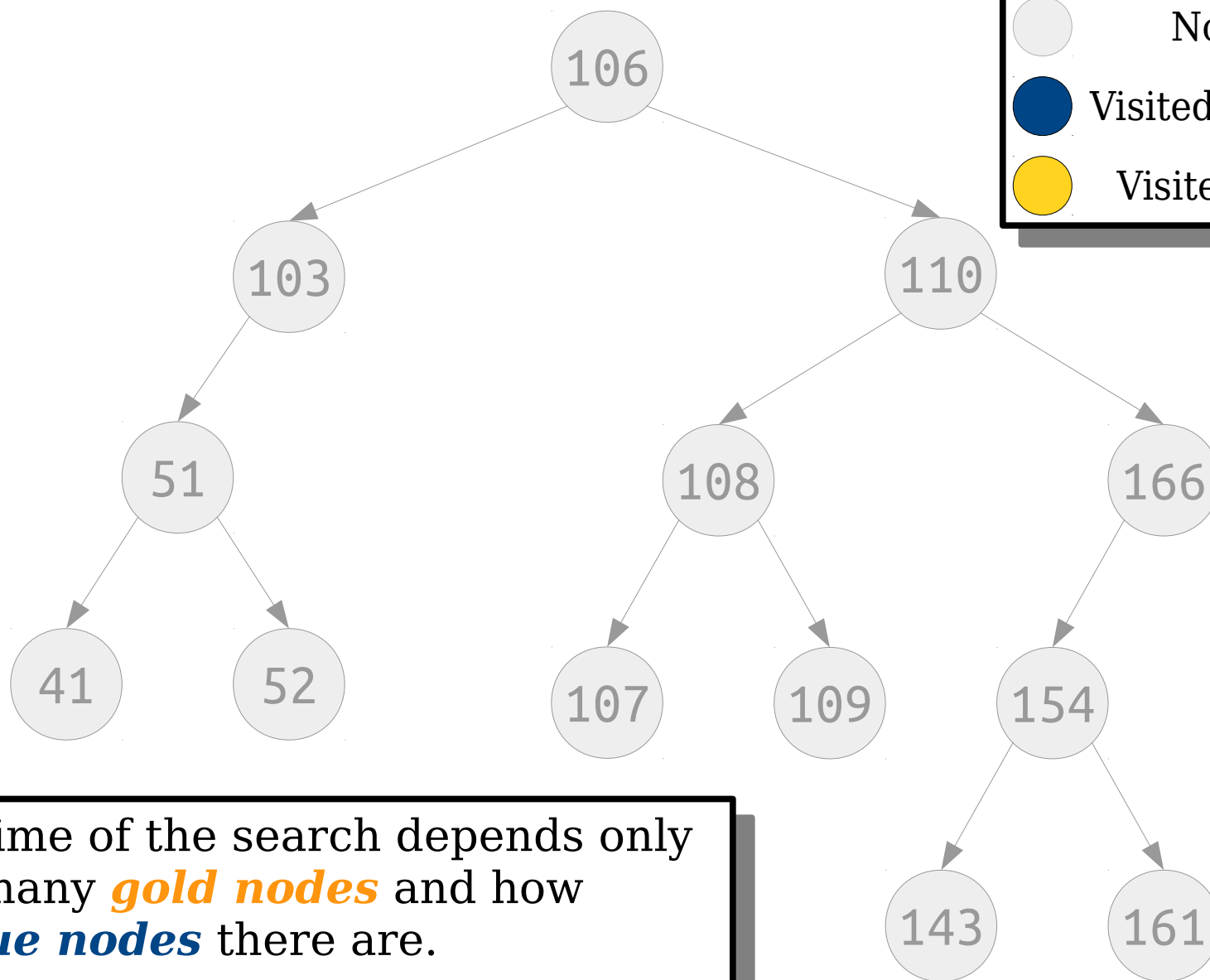
Find all elements in this tree in the range **[42, 165]**.



Find all elements in this tree in the range **[49, 50]**.

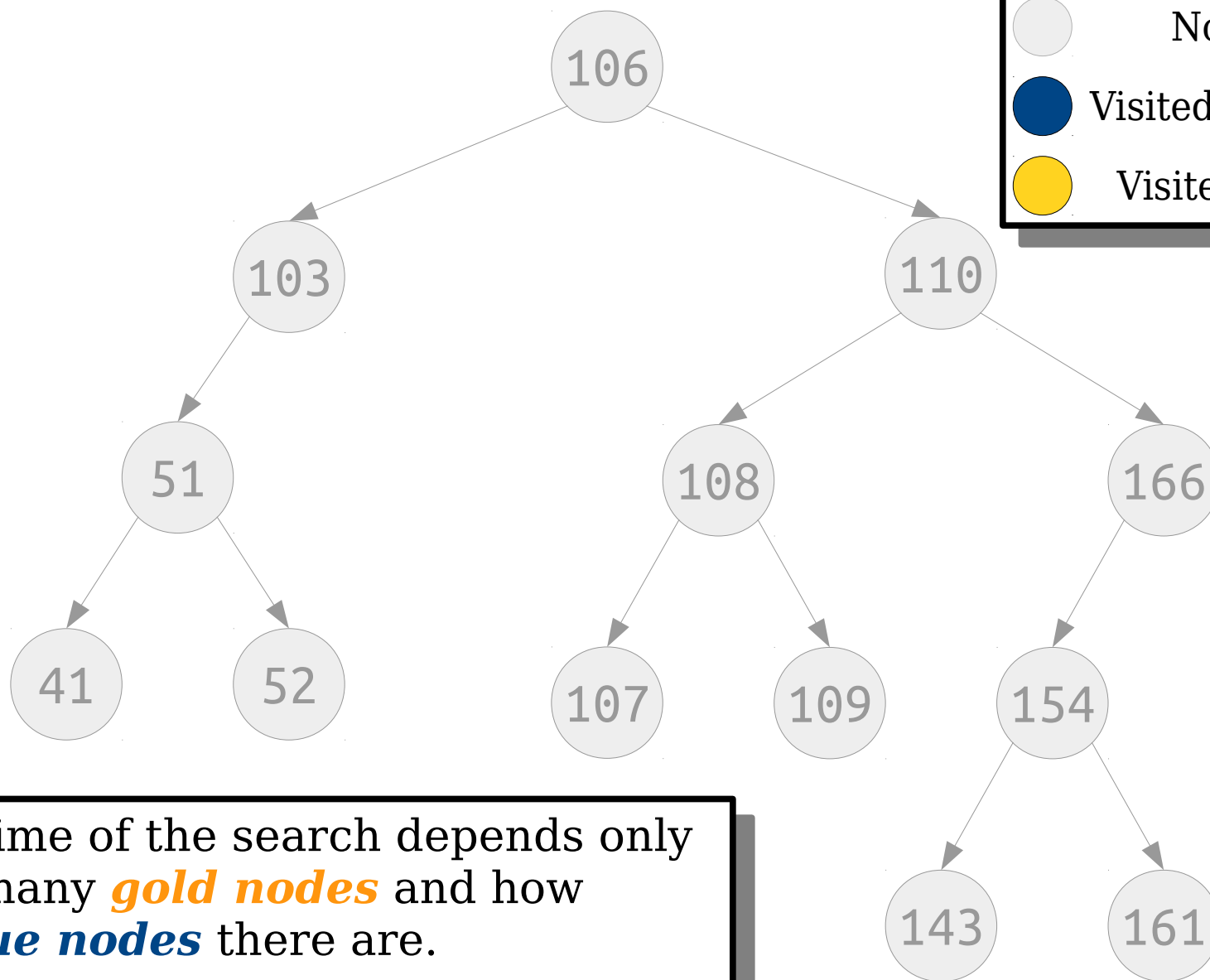


The runtime of the search depends only on how many **gold nodes** and how many **blue nodes** there are.



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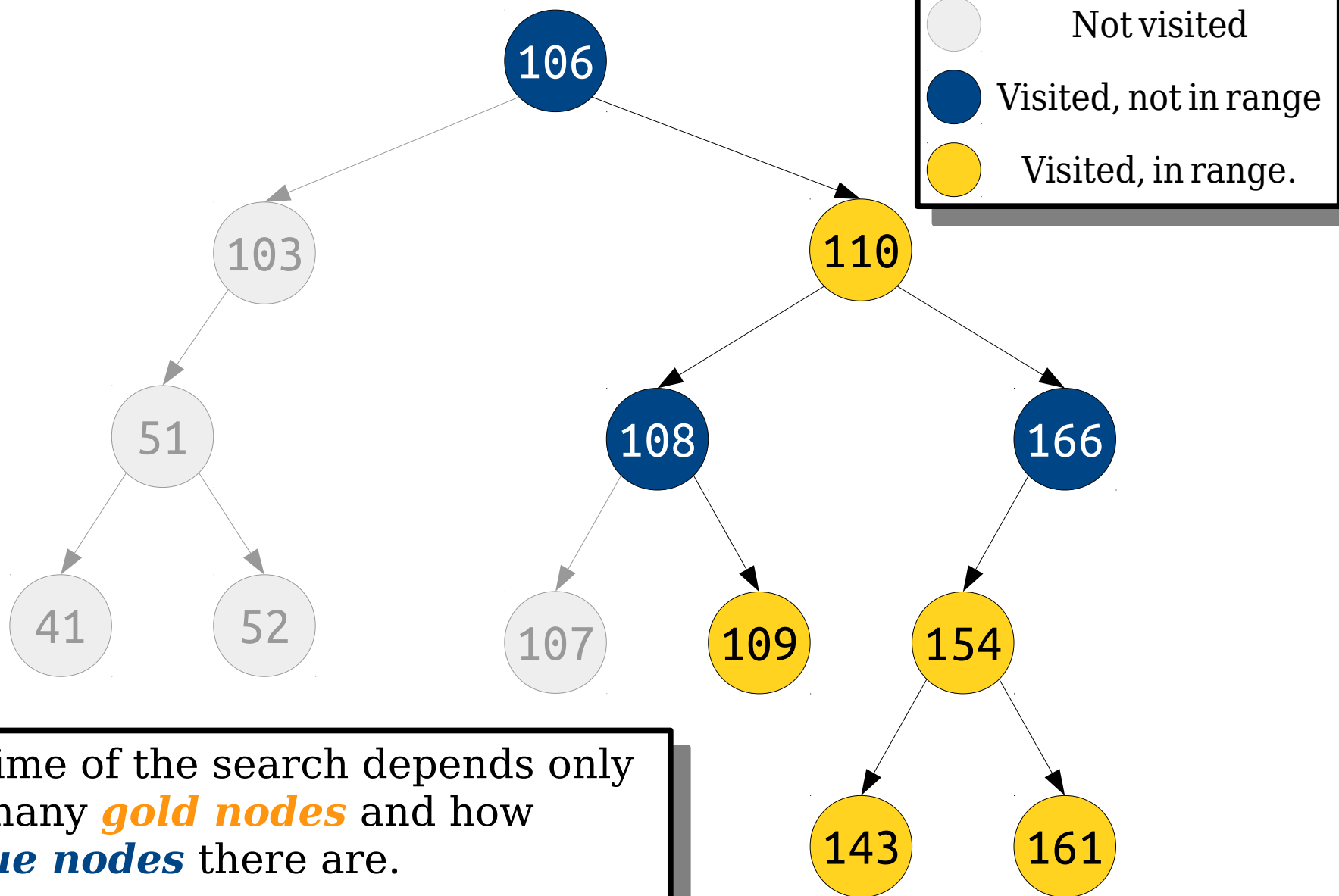
The number **gold nodes** is equal to the number of elements in the range.



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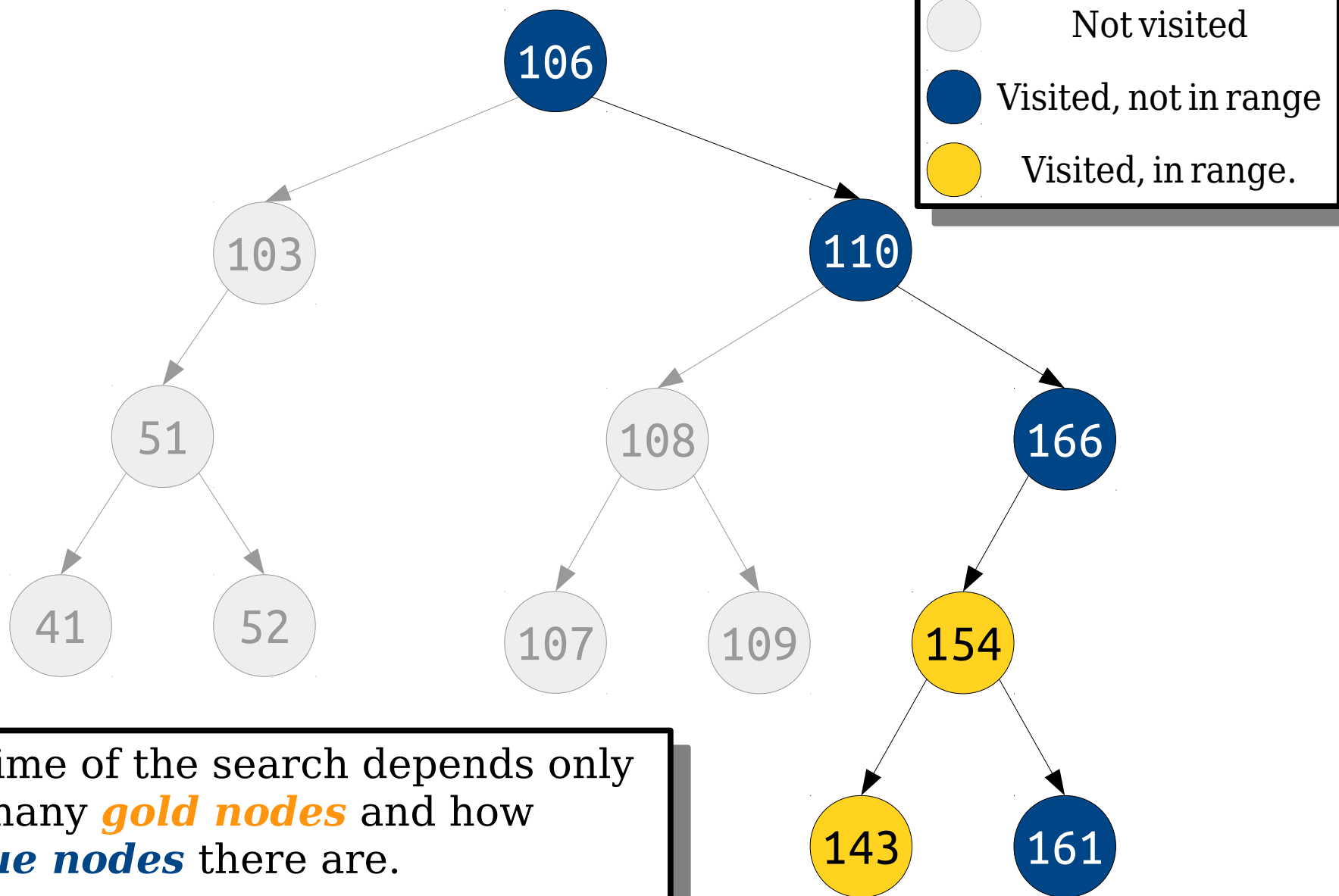
The number of **blue nodes** in each layer of the tree is at most two.



The runtime of the search depends only on how many **gold nodes** and how many **blue nodes** there are.

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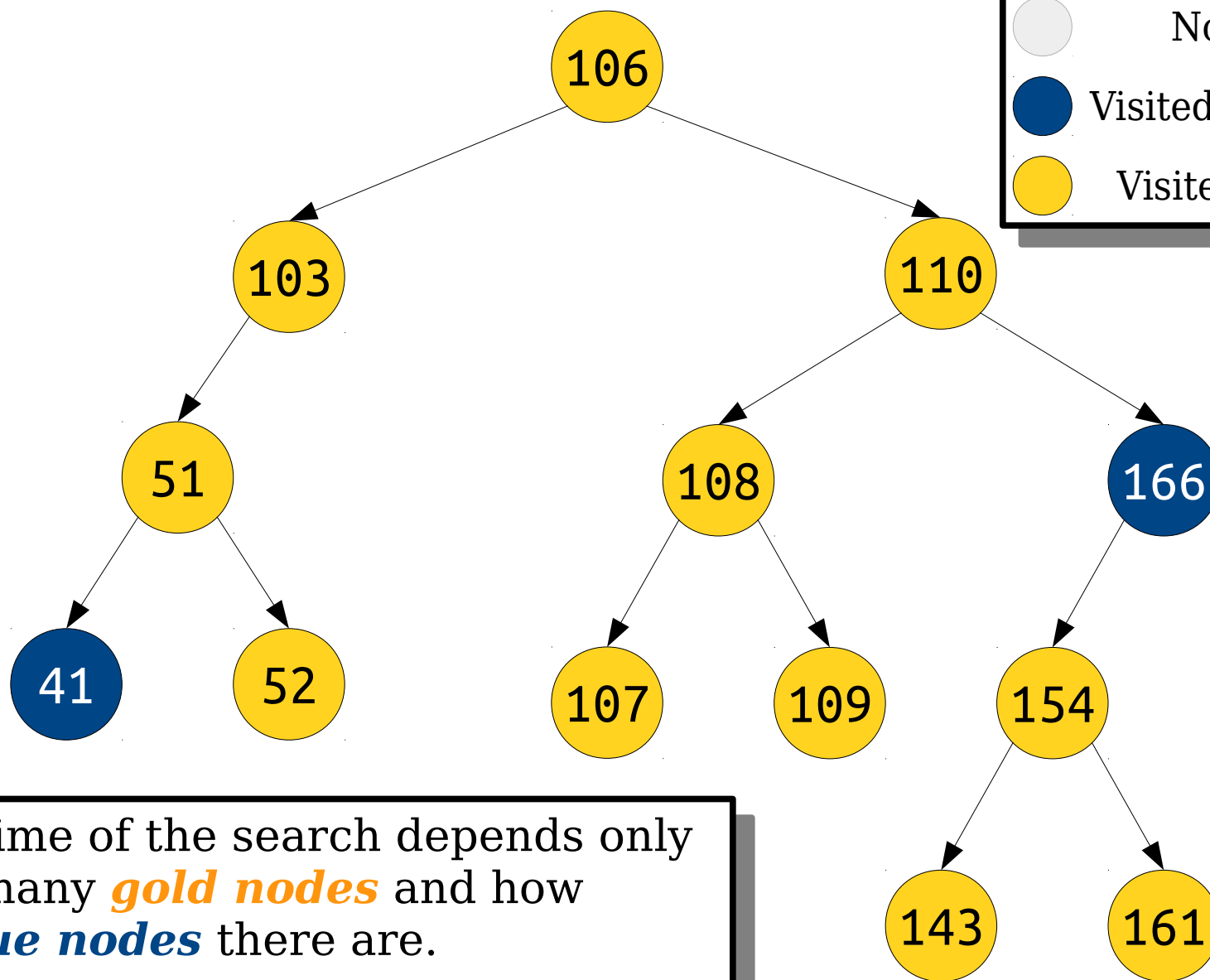
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The runtime of the search depends only on how many **gold nodes** and how many **blue nodes** there are.

The number **gold nodes** is equal to the number of elements in the range.

The number of **blue nodes** in each layer of the tree is at most two.

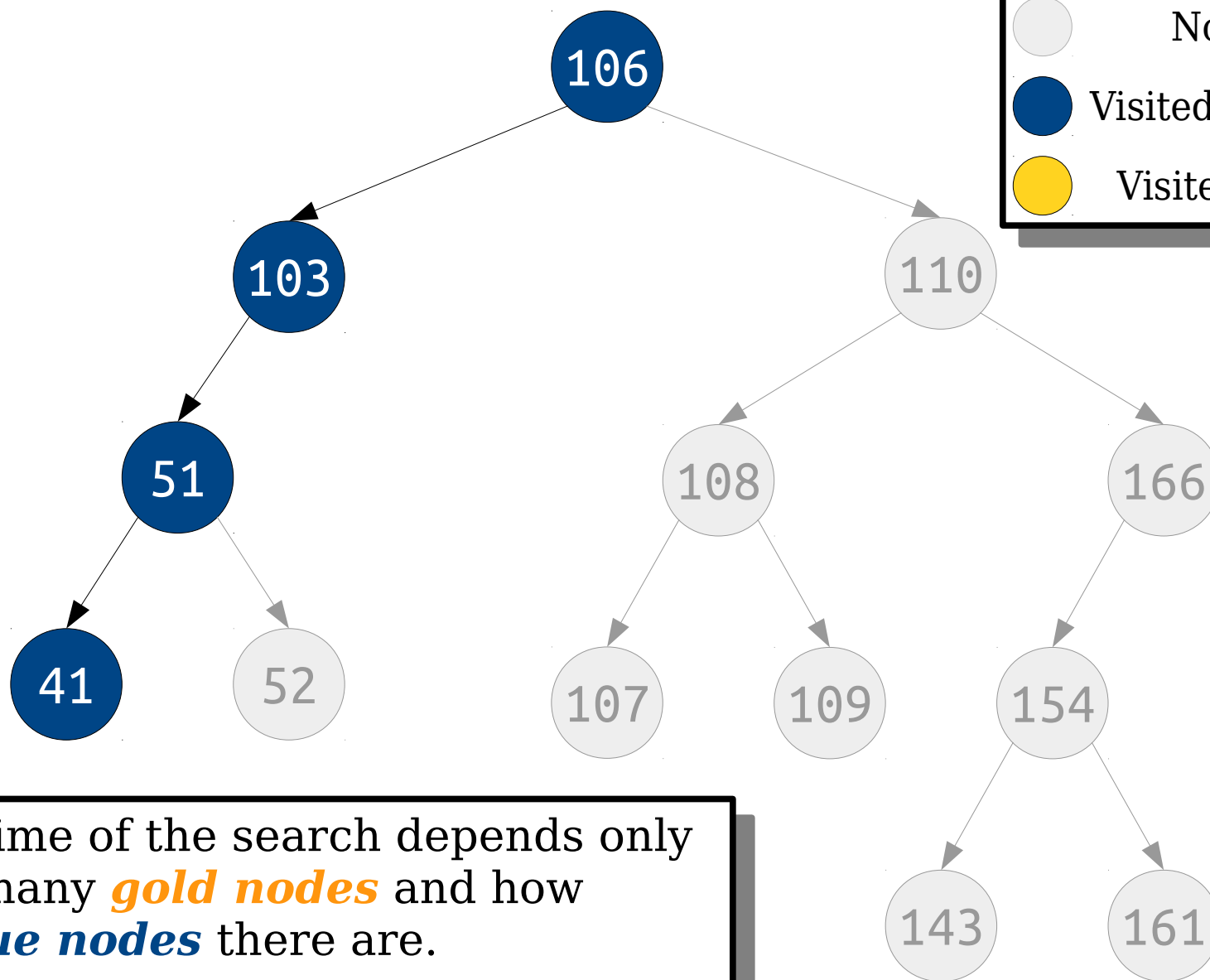


The runtime of the search depends only on how many **gold nodes** and how many **blue nodes** there are.

The number **gold nodes** is equal to the number of elements in the range.

The number of **blue nodes** in each layer of the tree is at most two.





The runtime of the search depends only on how many **gold nodes** and how many **blue nodes** there are.

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# Range Searches

- The cost of a range search in a BST of height  $h$  is  $O(h + k)$ ,  
where  $k$  is the number of matches reported.
- Notice that
  - if  $k$  is low (close to 0), then the runtime is close to  $O(h)$ , the cost of a single search; and
  - if  $k$  is high (close to  $n$ ), then the runtime is close to  $O(n)$ , the cost of an inorder traversal.
- This is called an ***output-sensitive algorithm***.