Diffrentiability

(x_1, y_1) and (x_2, y_2)

$$\frac{y_2 - y_1}{x_2 - x_1}$$

$(a+h, f(a+h))$

$a+h$

$f(a+h)$

$f(a-h)$ $a-h$

$f(a)$ $(a, f(a))$

$a+h$

a
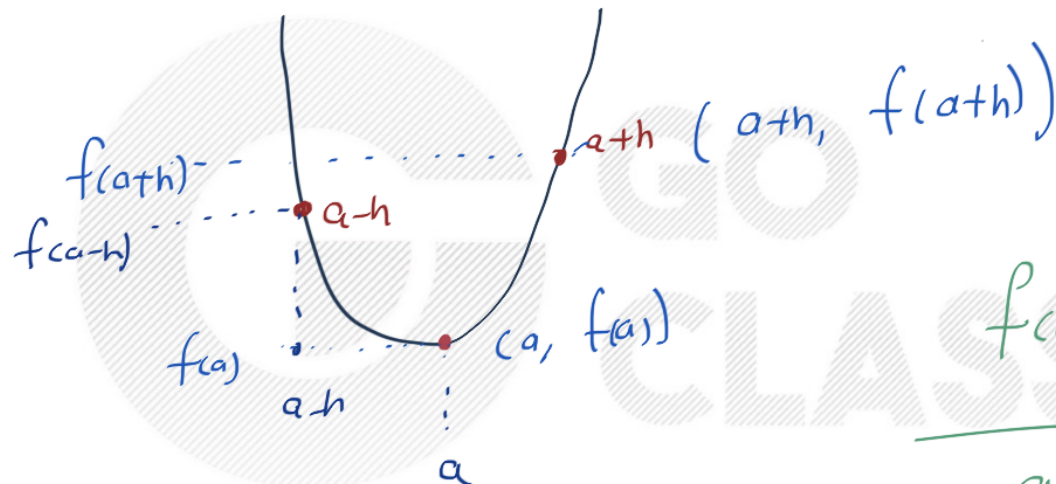
$$\frac{f(a+h) - f(a)}{a+h - a} = \boxed{\frac{f(a+h) - f(a)}{h}}$$
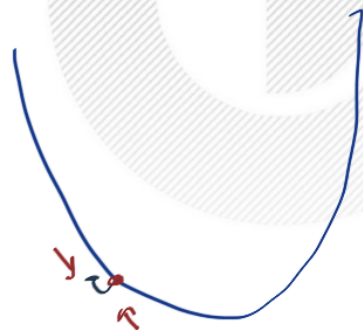
$$\frac{f(a-h) - f(a)}{a-h - a} = \boxed{\frac{f(a-h) - f(a)}{-h}}$$

$$\left[\lim_{h \to 0} \frac{f(a-h) - f(a)}{-h} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \text{finite} \right]$$

LHD

RHD

$$f(x) = \begin{cases} x & x < 1 \\ 2-x & 1 \le x \le 2 \\ -2+3x-x^2 & x > 2 \end{cases}$$

$x = 1, 2$?

$2-(1+h)$
$= 2-1-h$
$= 1-h$

$x = 1$

$LHD = \lim_{h \to 0} \dfrac{f(1-h) - f(1)}{-h}$

$= \lim_{h \to 0} \dfrac{f(1+h) - f(1)}{h}$

$\boxed{1} \quad \lim_{h \to 0} \dfrac{\cancel{1}-h \cancel{\nearrow \cancel{1}}}{-h}$

$= \lim_{h \to 0} \dfrac{\cancel{1}-h -\cancel{1}}{h} \quad \boxed{-1}$

function is NOT differntiable at $x = 1$

$x = 2$

LHD

$$\frac{f(2-h) - f(2)}{-h} = \frac{f(2+h) - f(2)}{h}$$

$$\frac{-h^2 + h - 0}{h} = \frac{-h \cdot 1}{}$$

$$\underbrace{h}_{\downarrow \atop -1}$$

$$= \frac{4 + 3h \cdot 4 - h^2 - 4h}{}$$

$$= -h^2 + h$$

$-2 + 3x - x^2$

$-2 + 3(2+h) - (2+h)^2$

$2 - (2-h)$

$= 2 - 2 + h$

$$\frac{h - 0}{-h} \to -1 =$$

$= -2 + 6 + 3h - (4 + h^2 + 4h)$

$$f'(x) = \begin{cases} 1 & x < 1 \\ -1 & 1 \le x \le 2 \\ 3 - 2x & x > 2 \end{cases}$$

differentiable at $x = 2$

$$\left[\begin{array}{c} f'(1^-) = f'(1) \\ 1 \neq -1 \end{array} \right]$$

$f'(2^-)$

$f'(2^+)$

$3 - 4$

-1

$= -1$

$$f(x) = \begin{cases} k\sqrt{x+1} & 0 \leq x \leq 3 \\ \\ mx+2 & 3 < x \leq 5 \end{cases}$$

is **differentiable**

$$f(3) = f(3^+) = f(3)$$
$$ \nearrow 2k \quad \nearrow 3m+2$$

$k+m$?

$$f'(3^-) = f'(3^+)$$

at $x = 3$

LHD

$$f'(x) = \begin{cases} \dfrac{k}{2\sqrt{x+1}} & 0 \leq x \leq 3 \\ \\ m & 3 < x \leq 5 \end{cases}$$

$$\dfrac{k}{4} = m$$

$$\Rightarrow \boxed{k = 4m}$$

$$2k = 3m+2 \longrightarrow \text{Continuity}$$

$$\begin{cases} k = 4m \\ 2k = 8m \end{cases}$$

$$8m = 3m+2$$

$$\Rightarrow 5m = 2$$

$$m = \frac{2}{5} \qquad\qquad k+m = \frac{10}{5} = 2$$

$$k = \frac{8}{5}$$