Arithmetic-Geometric Progressions(AGP)

GATE PYQs

GATE CSE 2018 | Question: 1

Which one of the following is a closed form expression for the generating function of the sequence $\{a_n\}$, where $a_n = 2n + 3$ for all $n = 0, 1, 2, \ldots$?

A. $\frac{3}{(1-x)^2}$
B. $\frac{3x}{(1-x)^2}$
C. $\frac{2-x}{(1-x)^2}$
D. $\frac{3-x}{(1-x)^2}$

Given that $a_n = 2n + 3$

Let $G(x)$ be the generating function for the sequence $\{a_n\}$.

So, $G(x) = \sum_{n=0}^{\infty} a_n x^n$

$$= \sum_{n=0}^{\infty}(2n+3)x^n$$
$$= \sum_{n=0}^{\infty}(2n)x^n + \sum_{n=0}^{\infty}(3)x^n$$
$$= 2\sum_{n=0}^{\infty} nx^n + 3\sum_{n=0}^{\infty} x^n$$
$$= 2A + 3B$$

Now, $A = \sum_{n=0}^{\infty} nx^n$. By expanding, it will look like: $0 + 1x + 2x^2 + 3x^3 + \ldots$ which is an AGP series with first term, $(a) = 0$, common difference, $(d) = 1$, ratio, $(r) = x$.

Sum of infinite AGP series $= \boxed{\frac{a}{1-r} + \frac{dr}{(1-r)^2}.}$

So, $A = \frac{0}{1-x} + \frac{x}{(1-x)^2} = \frac{x}{(1-x)^2}$

and $B = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \ldots = \frac{1}{1-x}$

Therefore, $2A + 3B = \frac{2x}{(1-x)^2} + \frac{3}{1-x}$

$$= \frac{2x+3-3x}{(1-x)^2} = \frac{3-x}{(1-x)^2}$$

Option (D) is correct.

$$A = \sum_{n=0}^{\infty} nx^n$$
$$= 1\cdot x + 2\cdot x^2 + 3x^3 + 4x^4 + \cdots$$
$$= x\left[1\cdot 1 + 2x + 3x^2 + 4x^3 + \cdots\right]$$
$$= x\left[\frac{1}{1-x} + \frac{1\times x}{(1-x)^2}\right]$$

GO Classes

GATE CSE 2002 | Question: 2.10

Consider the following algorithm for searching for a given number x in an unsorted array $A[1..n]$ having n distinct values:

1. Choose an i at random from $1..n$
2. If $A[i] = x$, then Stop else Goto 1;

Assuming that x is present in A, what is the expected number of comparisons made by the algorithm before it terminates?

A. n
B. $n - 1$
C. $2n$
D. $\frac{n}{2}$

GATE CSE 2002 | Question: 2.10

Expected number of comparisons $(E) = 1\times$ Probability of find on first comparison $+2\times$Probability of find on second comparison $+\ldots+i\times$ Probability of find on ith comparison $+\ldots$

$$= 1 \times \frac{1}{n} + 2 \times \frac{n-1}{n^2} + 3 \times \frac{(n-1)^2}{n^3} + \ldots$$

$$= \frac{(1).(1/n)}{1-\frac{n-1}{n}} + \frac{(1).(1/n)((n-1)/n)}{\left(1-\frac{n-1}{n}\right)^2} \text{(Sum to infinity of aritmetico-geometric series with } a = d = 1, r = \frac{n-1}{n} \text{ and } b = \frac{1}{n}) = 1 + n - 1 = n$$