

# Ratio & Proportion

T

Relation

Comparision

$$\frac{A}{B}$$

$$\sqrt{1:2}$$

$$A:B$$

$$1:2$$

$$C:D$$

$$2:4$$

$$\frac{A}{B} = \frac{1}{2} - \frac{2}{4} = \frac{3}{6}$$

$$\begin{matrix} 15 & 30 \\ 10 & 20 \end{matrix}$$

x Exact Value

$$1:2 :: 2:4$$

$$3:4$$

$$C:D$$

$$3:4$$

$$9:12$$

$$A : B$$

$$B : C$$

$[1 : 2] \times 3$

$1 : 2$        $3 : 4$

$$A : B : C = ?$$

$$\begin{matrix} \\ \diagdown \\ \text{II}^{\text{nd}} \end{matrix}$$

$$A : B : C$$

$$1 : 2 : 3$$

$$3 : 6$$

$$6 : 8$$

~~$A : B : C$~~

~~$3 : 6 : 8$~~

$\checkmark$   $\text{II}^{\text{rd}}$   $A : B : C$

$1 : 2 : 3$

$3 : 6 : 8$

$3 : 6 : 8$

$A : B$  $1 : 2$  $B : C$  $3 : 4$  $C : D$  $5 : 6$  $D : E$  $7 : 8$ 

$\square A : B : C : D : E$

 $1 : 2 : 2 : 2 : 2$  $3 : 3 : 4 : 4 : 4$  $5 : 5 : 6 : 6 : 6$  $7 : 7 : 7 : 7 : 8$

Eg.1 If  $\underline{\underline{A}} : \underline{\underline{B}}$  is  $2 : 3$  then find  $\underline{(2A+3B)} : \underline{(5A+2B)}$

$$2A + 3B : 5A + 2B$$

$$\frac{4+9}{13} : \frac{10+6}{16}$$

Ans

$$A : B \Rightarrow 2 : 3$$

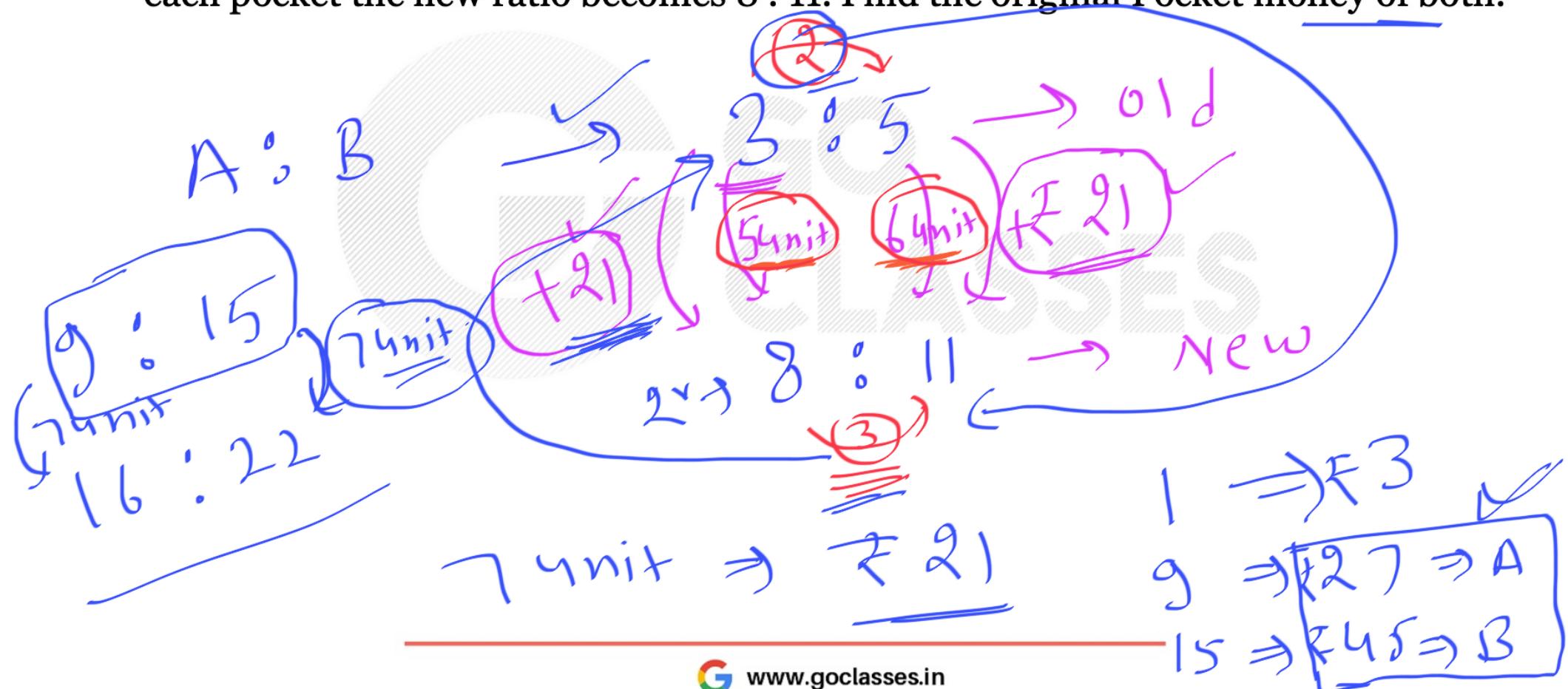
$$\begin{cases} A \Rightarrow 2x \\ B \Rightarrow 3x \end{cases} \quad \begin{matrix} A : B \\ 2x : 3x \\ 2 : 3 \end{matrix}$$

$$\frac{2A+3B}{5A+2B} = \frac{2[2x] + 3[3x]}{5[2x] + 2[3x]}$$

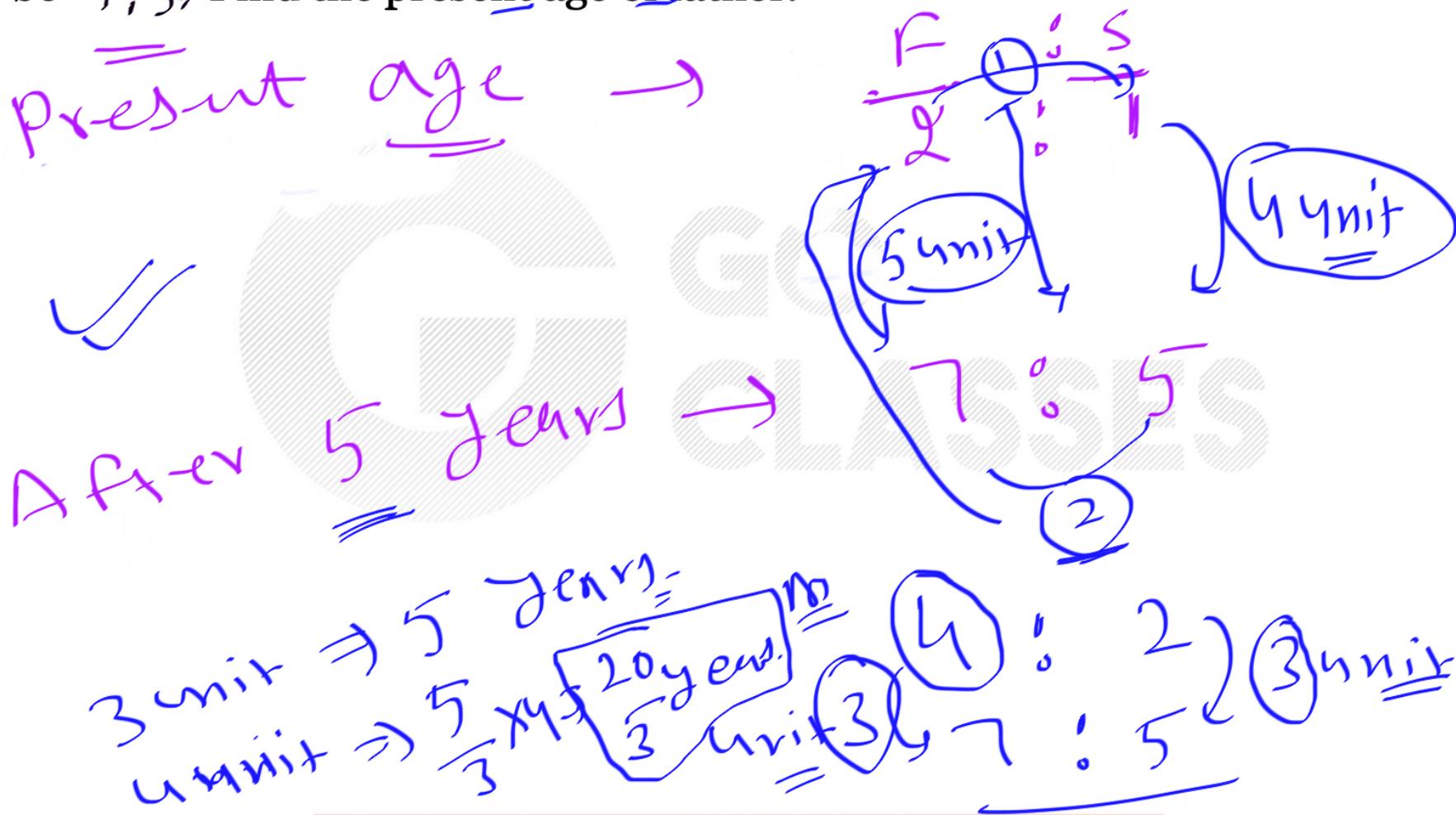
$$= \frac{x[4+9]}{x[10+6]} = \frac{13}{16}$$

## Equate the difference of ratios

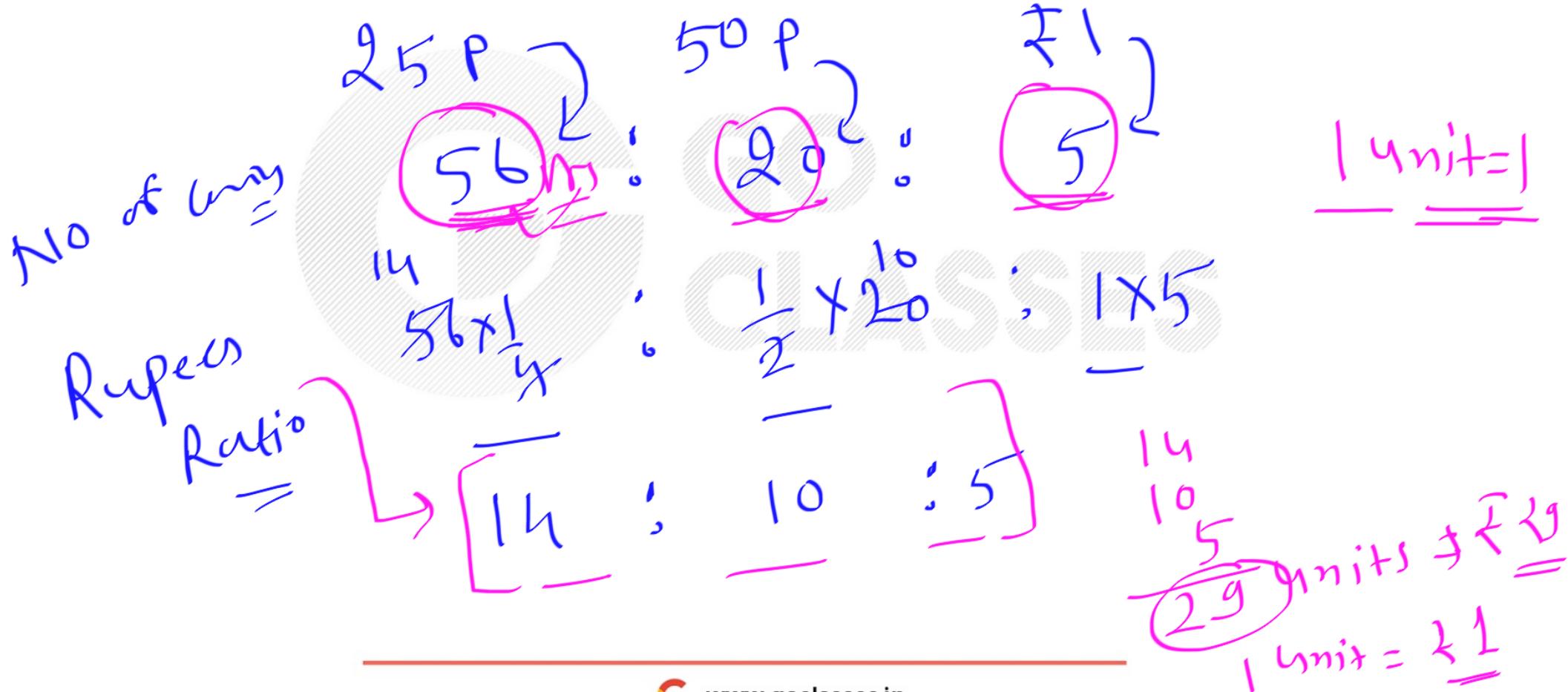
Eg.2 A & B has the money in their pocket in the ratio of  $3 : 5$ , if adding 21 rupees in each pocket the new ratio becomes  $8 : 11$ . Find the original Pocket money of both.



Eg.3 Present age of father is doubled of his son. And after 5 years the ratio of their age will be  $7:5$ . Find the present age of father.



Eg.4 In a wallet the ratio of 25p coins, 50p coins and 1-rupee coins is 56:20:5. And the total rupees in that beg are 29. Find number of coins of each type.



## Types of Ratio

If two quantities A & B have their ratio as A:B = x:y then

**Duplicate ratio** of A:B is  $x^2 : y^2$

**TriPLICATE ratio** of A : B is  $x^3 : y^3$

**Sub-duplicate ratio** of A : B is  $\sqrt{x} : \sqrt{y}$

**Sub-triplicate** of A: B is  $\sqrt[3]{x} : \sqrt[3]{y}$

Eg.5 IF A:B is 81:100. Find triplicate of its sub-duplicate

$$\begin{array}{l} 81 : 100 \\ \downarrow \\ 9 : 10 \end{array}$$

$$\begin{array}{l} 9^3 : 10^3 \\ 729 : 1000 \end{array}$$

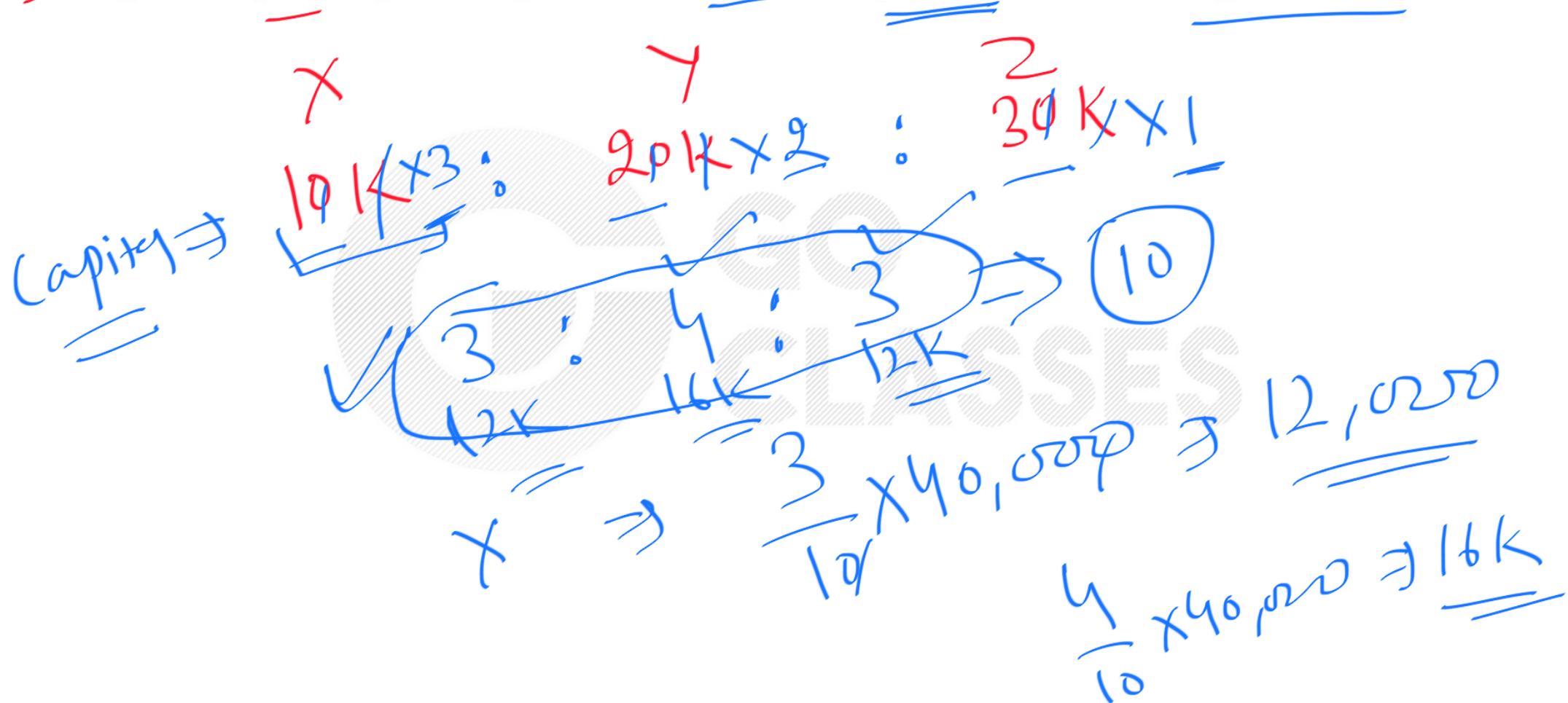
Eg.6 Divide **3000** rupees among A,B & C. If B gets half of A&C together and C gets same as A&B together.

$$\begin{aligned} B &= \frac{1}{2}(A+C) \\ 2B &= A+C \\ C &= A+B \end{aligned}$$

$$\begin{aligned} B &= \frac{1}{2}(A+C) \\ B &= \frac{1}{2} \times 2 = 1 \\ C &= A+B \\ C &= 1+2 = 3 \\ A+B+C &= 1+2+3 = 6 \\ A:B:C &= 1:2:3 \\ C &= 3 \\ B &= 2 \\ A &= 1 \end{aligned}$$

$$\begin{aligned} 2B + f &= 2A + f + 3 \\ 2B - B &= 2A \\ B &= 2A \\ B &= \frac{1}{2}A \\ C &\Rightarrow 1+2=3 \\ A:B:C &= 1:2:3 \end{aligned}$$

Eg.7 x,y and z invested 10,000, 20,000 and 30,000 in a business for 3 years, 2 years and 1 year respectively. If the profit after 3 years is 40,000 then find the profit share of each.



Eg.8 A, B and C invested 500, 2,000 and 1,500 in a business for 3 years. But A added 1000 in his investment after 1 year and B & C each withdrew 500 from their investment after 2 years. If the profit after 3 years is 1800 then find the profit share of each.

A

$$500 \times 1$$

$$1000 \times 2$$

$$\frac{25}{5}$$

5

B

$$2000 \times 2$$

$$1500 \times 1$$

$$\frac{5500}{11}$$

11

$$5 : 11 : 3$$

C

$$1500 \times 2$$

$$1000 \times 1$$

$$\frac{4000}{4}$$

4

$$24 : 5 : 1800$$



## Product of two numbers = LCM \* HCF

Eg. A : B

$\frac{3}{4}$

$$\begin{aligned}A &\Rightarrow 3x \\B &\Rightarrow 4x\end{aligned}$$

LCM  $\Rightarrow \underline{\underline{48}}$   
HCF  $\Rightarrow \underline{\underline{x}}$

Find

$$3x + 4x = 16$$

$$3x + 4x = 48 \times \text{HCF}$$

$$\left[ \text{HCF} = \frac{x^2}{4} \right] \Rightarrow$$

$$x = \frac{x^2}{4}$$

$$x = 4 \quad \boxed{y}$$

$A \times B = \text{their LCM} \times \frac{\text{HCF}}{\downarrow}$   
is divisible

HCF = Maximum Common  
LCM  $\Rightarrow$  Lowest number that  
both numbers.



✓ Forth proportion  $\boxed{A : B : C : D}$   $\underline{3, 10, 27}$  find 4<sup>th</sup> prop

$$3 : 10 :: 27 : D \Rightarrow \frac{3}{10} = \frac{27}{D} \Rightarrow D = 90$$

✓ Third Proportion  $\boxed{A : B : B : C}$   $\underline{12 \& 18}$  asking third prop

$$12 : 18 :: 18 : C \Rightarrow \frac{12}{18} = \frac{18}{C} \Rightarrow C = 27$$

Second Proportion/Mean Proportion  $\boxed{A : B : B : C} \Rightarrow$  find mean  $\circled{B} \& \circled{C}$

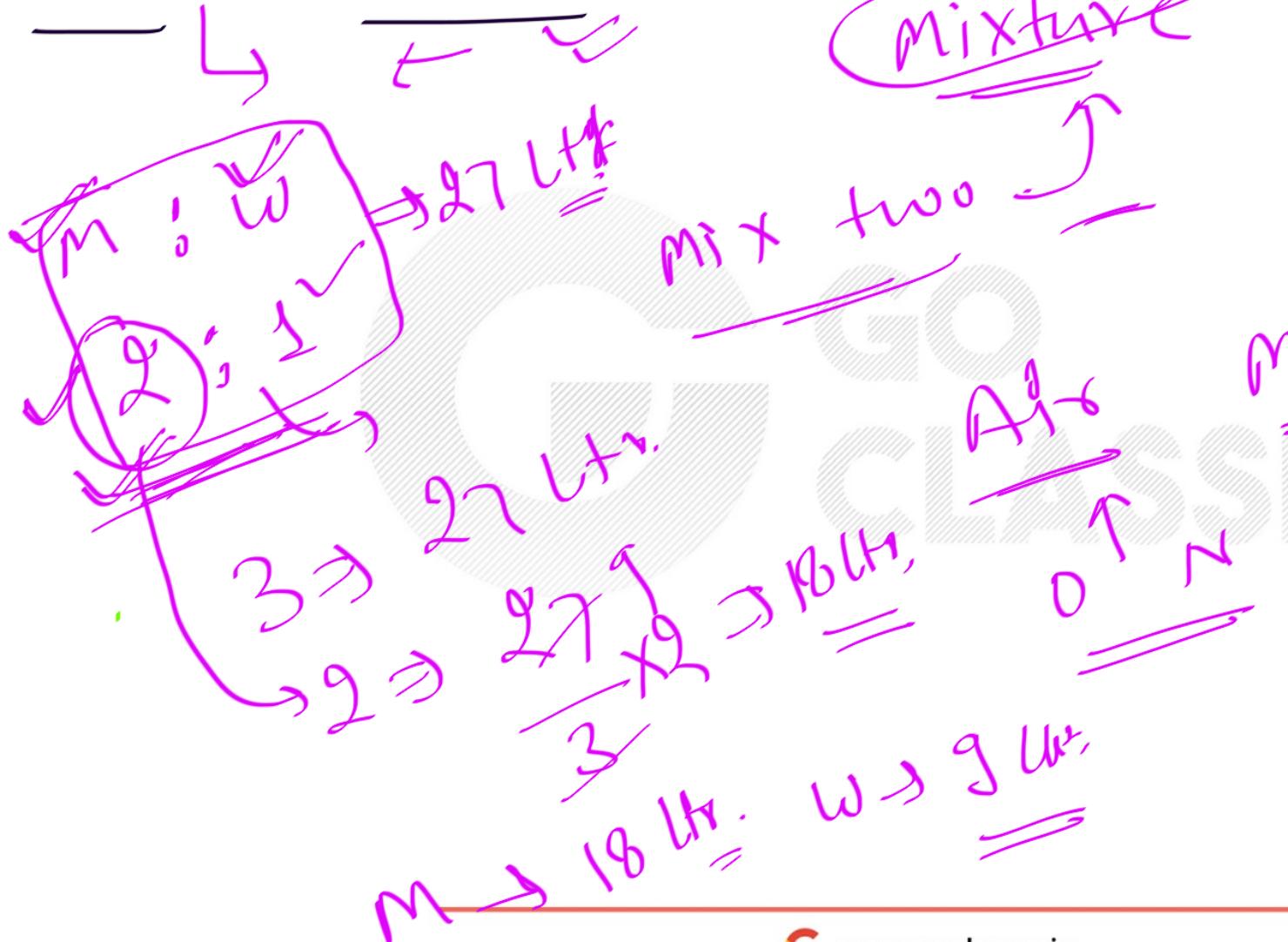
$$B = \frac{81 + 10}{9 + 10} \Rightarrow B = 90$$

$$81 : B :: B : 100$$

$$\frac{81}{B} = \frac{B}{100} \Rightarrow B^2 = 81 \times 100$$

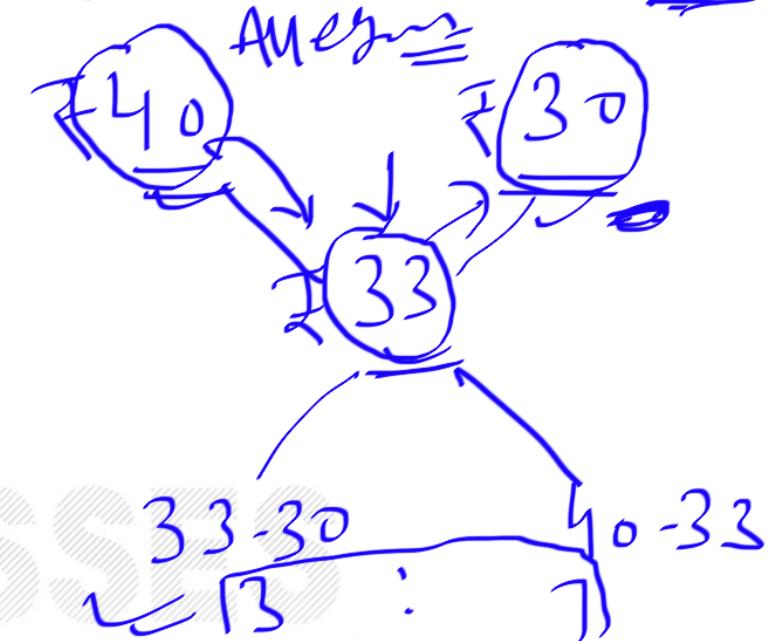
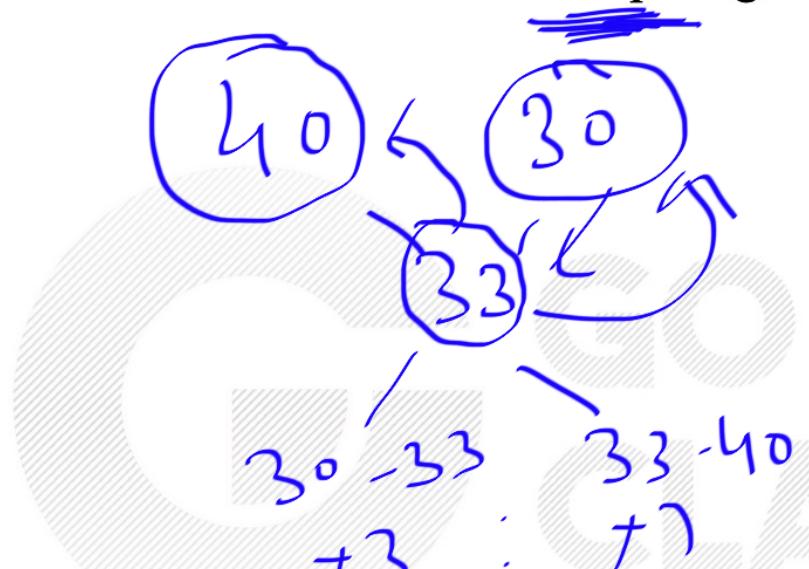
$$\underline{B = 9 \times 10 = 90}$$

# Mixture & Allegation



Eg.9 In what ratio, a shopkeeper should mix the rice of worth ₹40 per kg in the rice of worth ₹30 per kg so that the mixture of rice become of ₹ 33 per kg?

$$3 : 7$$



Eg 10. In what ratio, the water should be added to milk of ₹36 per Ltr so that the value of mixture become of ₹28 per Ltr?  $\frac{M}{W} = \frac{36}{28}$

$$\frac{M}{W} = \frac{36}{28}$$

$$\frac{M}{W} = \frac{9}{7}$$

$$\frac{28}{8} = \frac{7}{2}$$

**Eg.11** In a mixture of 40 litres, the ratio of milk and water is  $4 : 1$ . How much water must be added to this mixture so that the ratio of milk and water becomes  $2 : 3$ ?



$m : w$   
 $(4; 1)$   
 ~~$32 \text{ Nut}$~~  :  $8 \text{ Lts}$   
 ~~$2 ; 3$~~   $w$   
 ~~$32 \text{ Lts}$~~   $48 \text{ Nut}$   
 $40 \text{ Lts}$   
 $5 \Rightarrow 40 \text{ Nut}$   
 $4 \Rightarrow 40$   
 ~~$\frac{4}{5} \times 40 = 32$~~   
 $2 \Rightarrow 32 \text{ Nut}$   
 $3 \Rightarrow 32 \text{ Nut}$   
 ~~$\frac{3}{5} \times 32 = 16$~~   
 $13 \Rightarrow 48$

Eg 12. If container A & B has mixture of milk and water in 1:5 and 5:7 ratio respectively. If both container mixes together in other container C then the ratio of milk and water in C becomes 1:3. Find if the container B has 5Ltr mixture then what was the quantity in A.

- A. 18  
B. 30  
C. 10  
D. 9

$$\begin{array}{c}
 \text{A : B} \\
 \boxed{\frac{6}{12} - \frac{5}{12}} \\
 \text{C} \\
 \boxed{\frac{13}{12} - \frac{4}{12}} \\
 \frac{1}{0} + 2 \\
 \boxed{2 : 1} \\
 \end{array}
 \quad
 \begin{array}{c}
 \text{A} \\
 \boxed{1:5} \\
 \text{M:W} \\
 \text{B} \\
 \boxed{5:7} \\
 \text{M:W} \\
 \text{C} \\
 \boxed{1:3} \\
 \text{M:W} \\
 \end{array}
 \quad
 \begin{array}{c}
 1 \Rightarrow 5 \\
 \boxed{2 \Rightarrow 10}
 \end{array}$$

$$p^{-x} = \frac{1}{q} \Rightarrow q = \frac{1}{p^{-x}}$$

$$\begin{aligned} q &= p^x \\ r &= q^y \\ p &= r^z \end{aligned}$$

**Q.1.** If  $pqr \neq 0$  and  $p^{-x} = \frac{1}{q}, q^{-y} = \frac{1}{r}, r^{-z} = \frac{1}{p}$ , what is the value of the product  $xyz$ ? [ ] =

- A.  $-1$
- B.  $\frac{1}{pqr}$
- C.  $1$
- D.  $pqr$

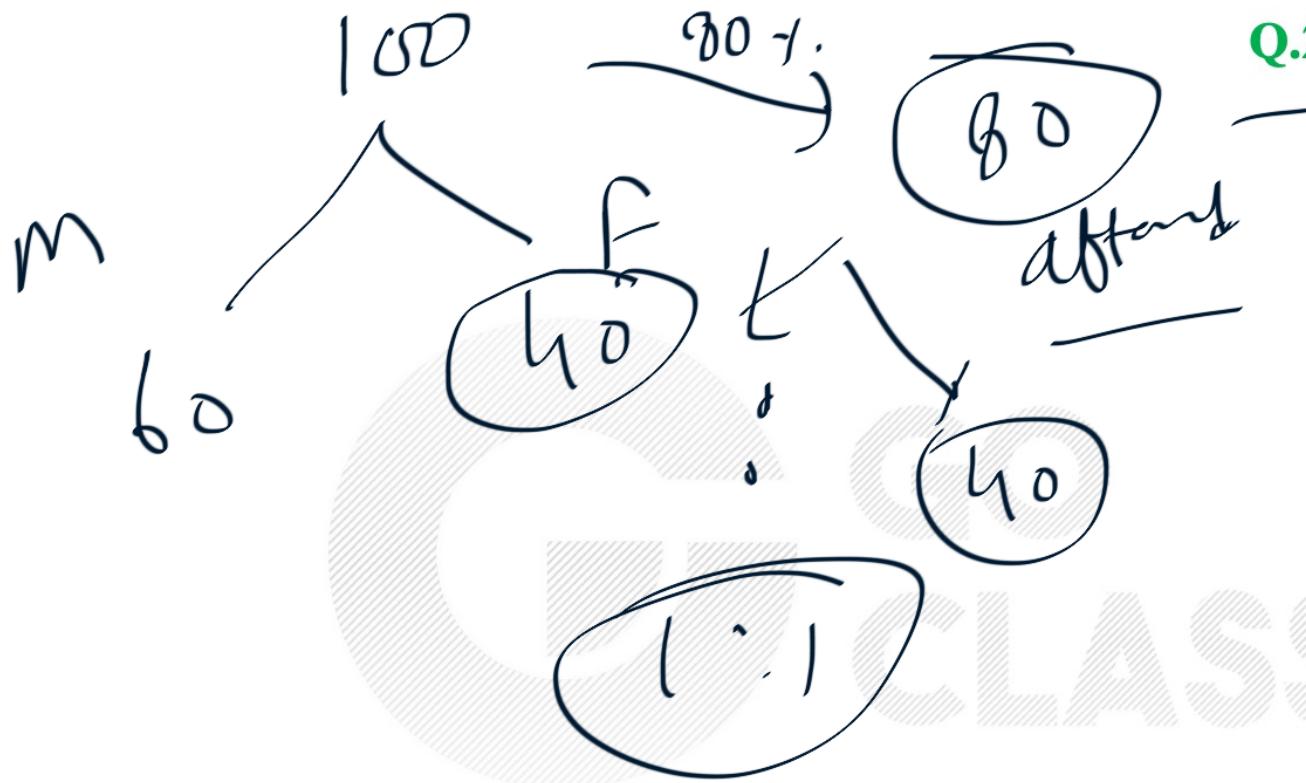
**Gate CSE 2018**

$$q = (\gamma^2)^{-x}$$

$$q = \gamma^{-x}$$

$$\gamma = (\gamma^{x_2})^{-y}$$

$$\gamma = \gamma^{x_2 y_2}$$

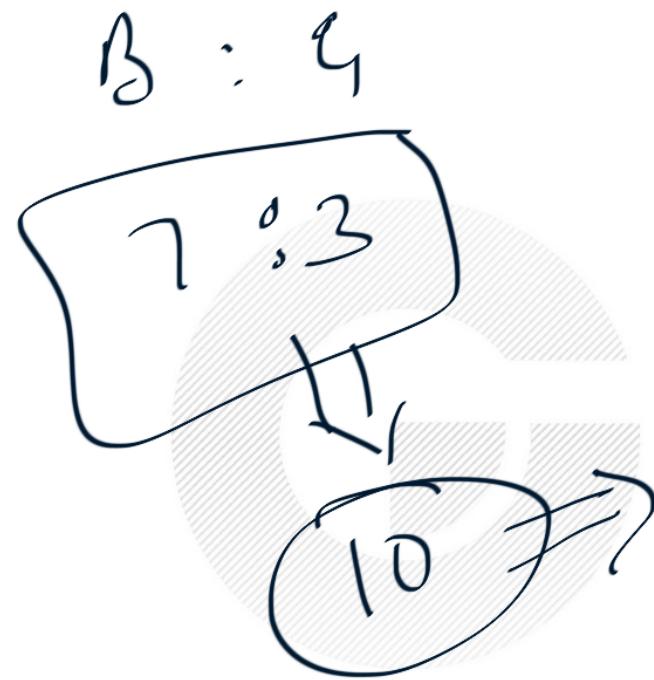


**Q.2.**

In a party, 60% of the invited guests are male and 40% are female. If 80% of the invited guests attended the party and if all the invited female guests attended, what would be the ratio of males to females among the attendees in the party?

- A. 2:3
- B. 1:1
- C. 3:2
- D. 2:1

**Gate CSE 2018**

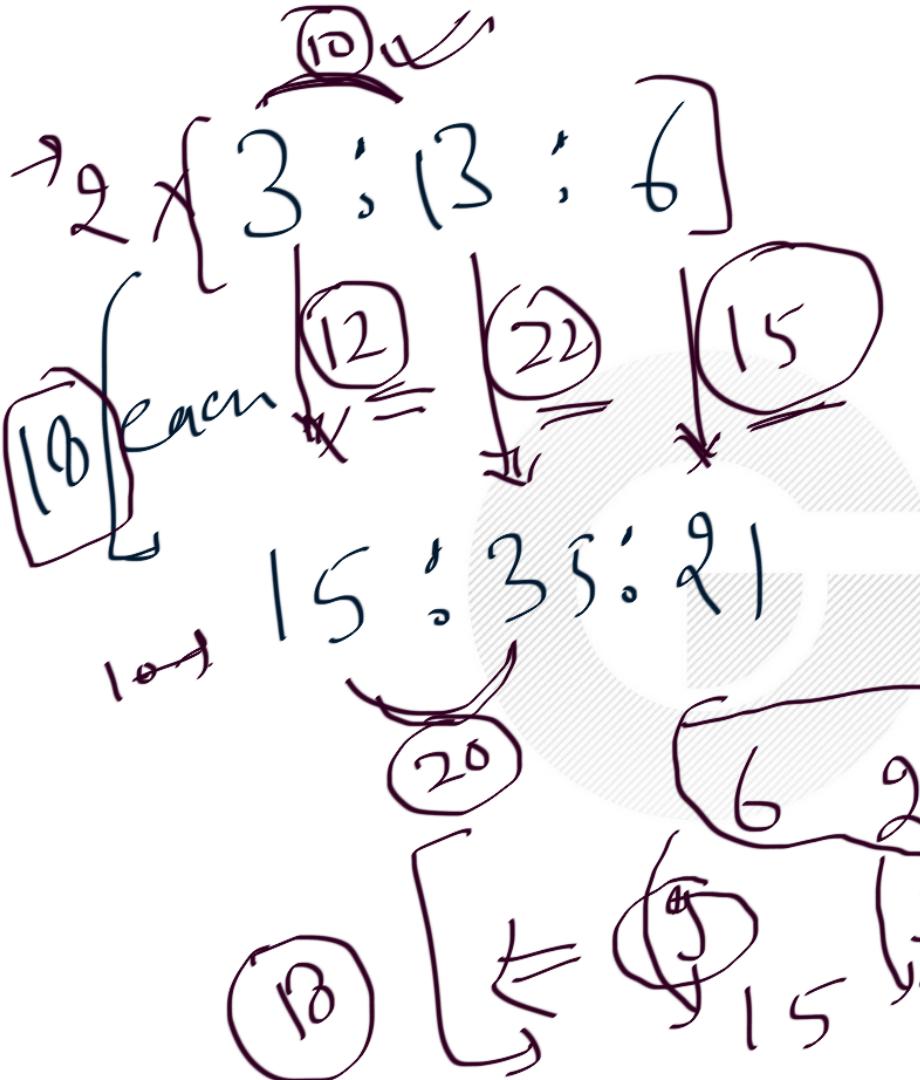


**Q.3.** The ratio of boys to girls in a class is 7 to 3. Among the options below, an acceptable value for the total number of students in the class is:

- A. 21
- B. 37
- C. 50
- D. 73

**Gate CSE 2021**

GO  
CLASSES



**Q.4.** The number of students in three classes is in the ratio  $3:13:6$ . If 18 students are added to each class, the ratio changes to  $15:35:21$ . The total number of students in all the three classes in the beginning was:

- A. 22
- B. 66
- C. 88
- D. 10

**Gate CSE 2021**

g unit  $\rightarrow$  18 st  
 $\frac{1}{4}$  unit  $\rightarrow$  2 st  
 $\frac{1}{44}$  unit  $\rightarrow$  88 st

$$\textcircled{n^2}$$

$$\frac{625}{= 1-100}$$

$$\frac{n(n+1)}{=}$$

$$1-100$$

$$\textcircled{50}$$

$$2500$$

$$1-149$$

$$\frac{74 \times 75}{5550}$$

Q.5.

The ratio of 'the sum of the odd positive integers from 1 to 100' to 'the sum of the even positive integers from 150 to 200' is

- A. 45:95  
B. 1 : 2  
 C. 50:91  
D. 1:1

Gate Civil 2020

$$\frac{2500}{250} ; \frac{4550}{91}$$

$$\frac{250}{250} ; \frac{91}{10100}$$

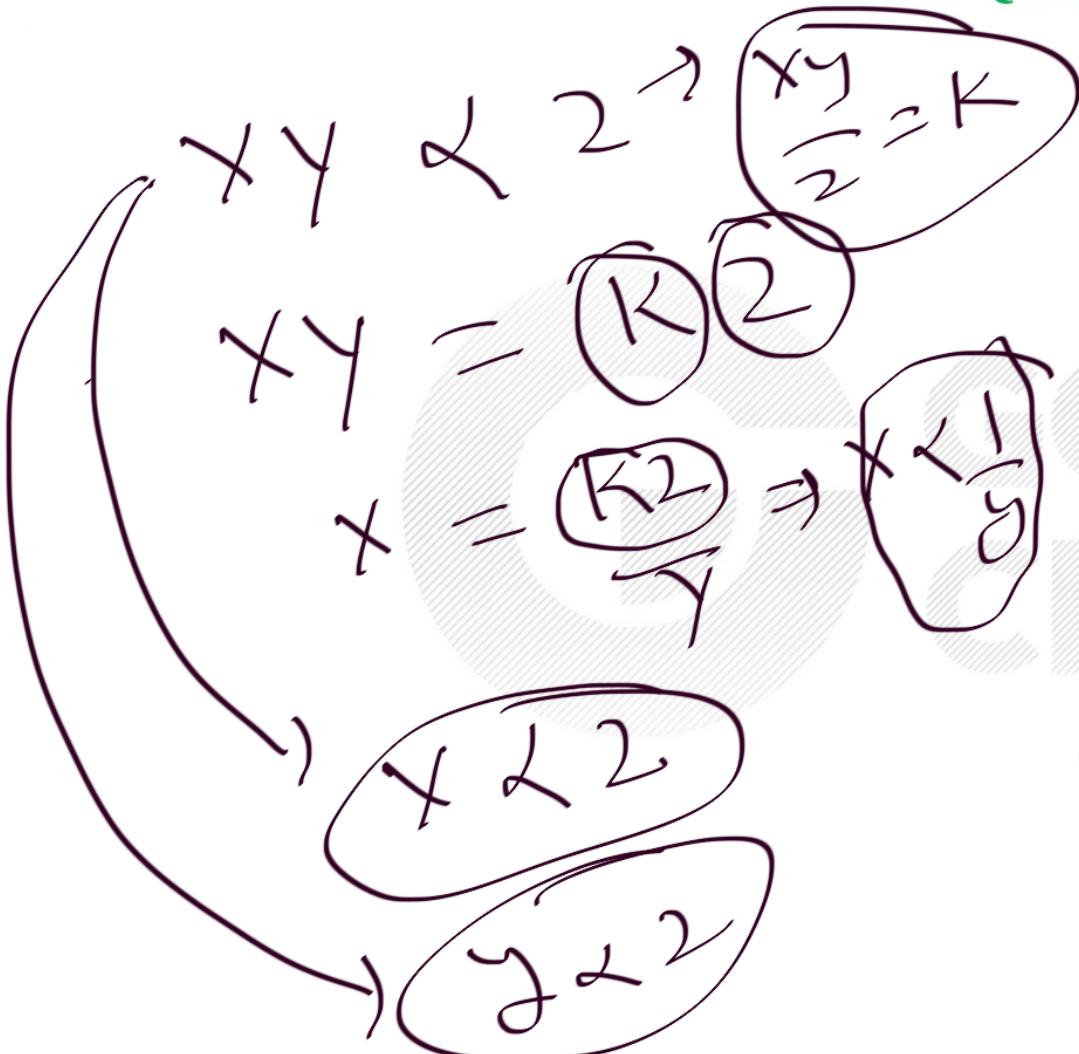
$$- 5550$$

$$\frac{4550}{4550}$$

$$\frac{100 \times 101}{10100}$$

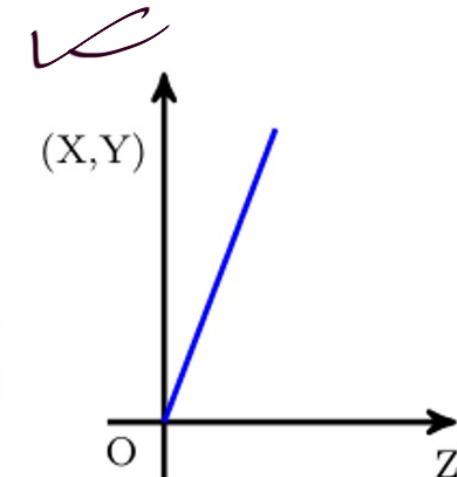
**Q.6.**

An engineer measures THREE quantities,  $X$ ,  $Y$  and  $Z$  in an experiment. She finds that they follow a relationship that is represented in the figure below: (the product of  $X$  and  $Y$  linearly varies with  $Z$ )



Then, which of the following statements is FALSE?

- A. For fixed  $Z$ ;  $X$  is proportional to  $Y$
- B. For fixed  $Y$ ;  $X$  is proportional to  $Z$
- C. For fixed  $X$ ;  $Z$  is proportional to  $Y$
- D.  $XY/Z$  is constant



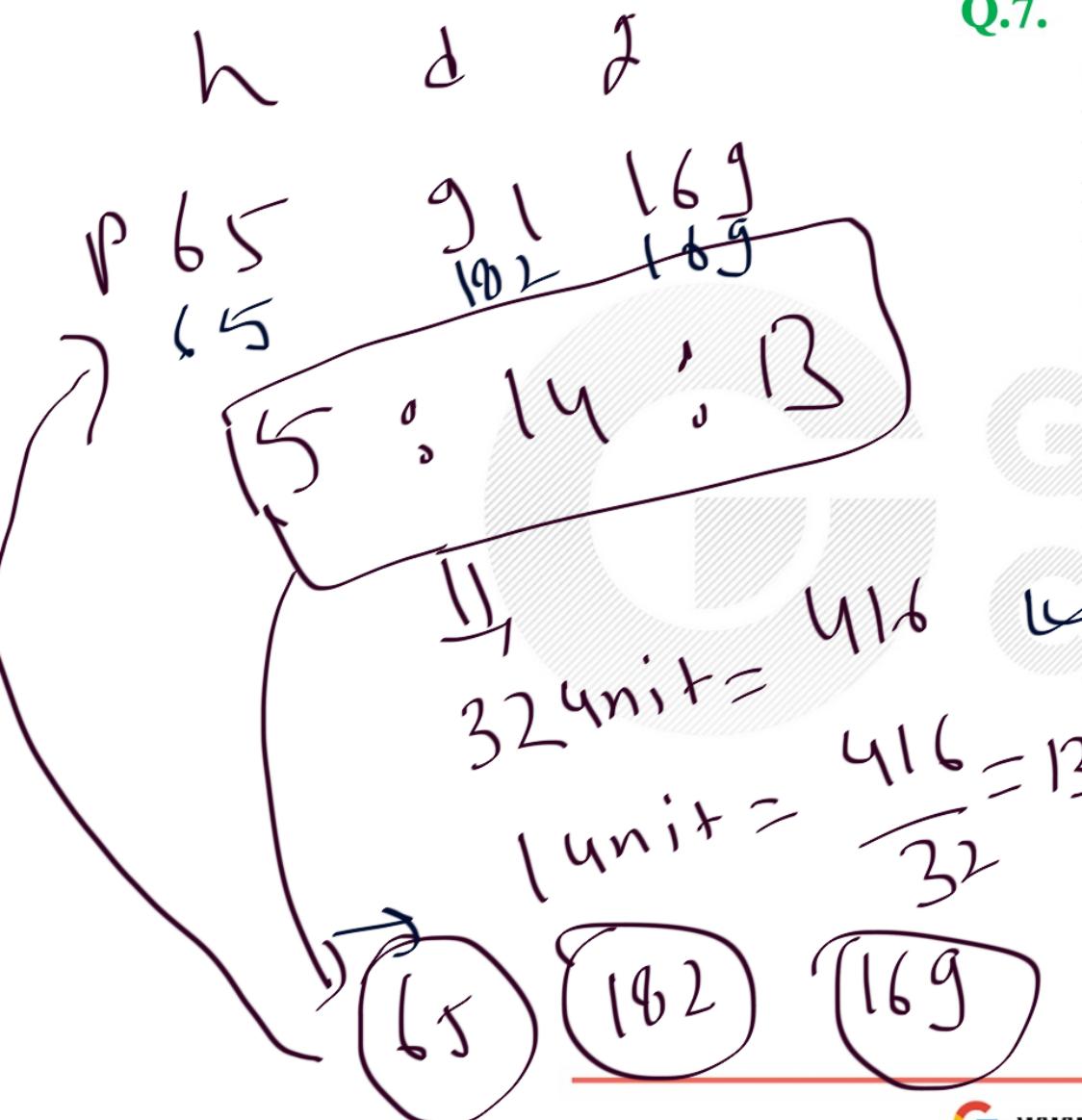
**Gate Mechanical 2020**

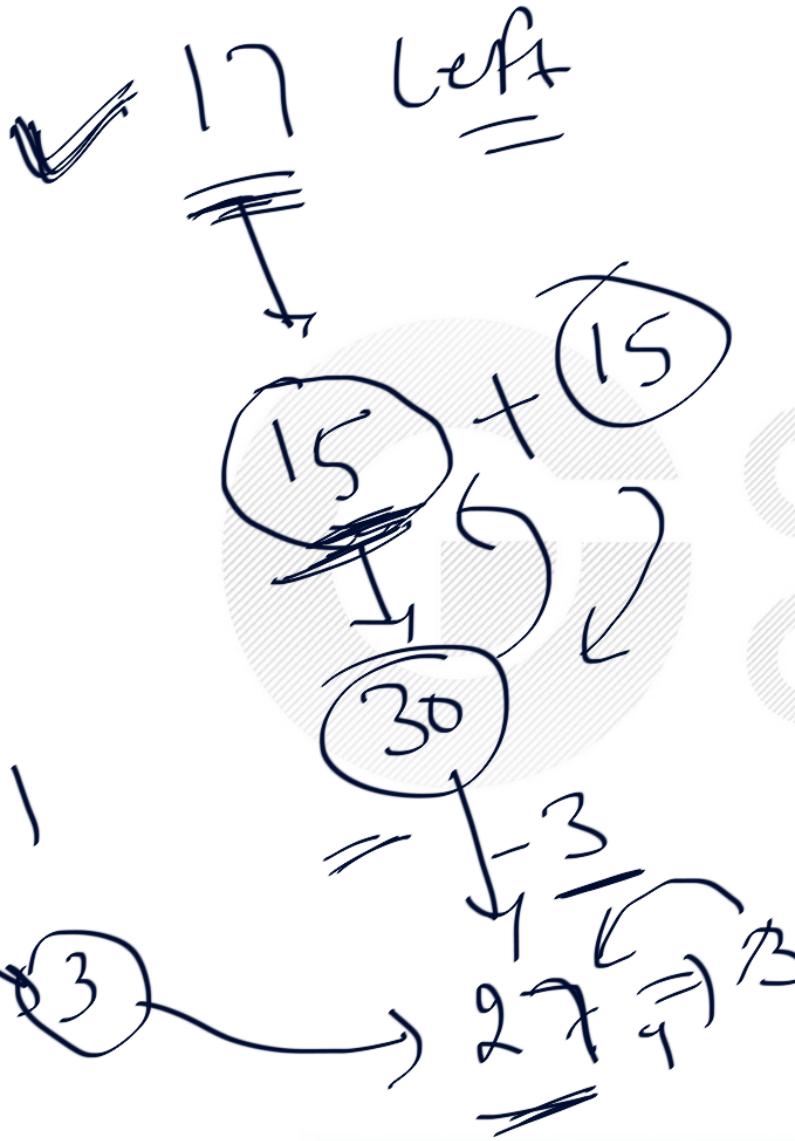
**Q.7.** The number of hens, ducks and goats in farm  $P$  are 65, 91 and 169, respectively. The total number of hens, ducks and goats in a nearby farm  $Q$  is 416. The ratio of hens : ducks : goats in farm  $Q$  is 5:14:13. All the hens, ducks and goats are sent from farm  $Q$  to farm  $P$ .

The new ratio of hens : ducks : goats in farm  $P$  is \_

- A. 5:7:13
- B. 5:14:13
- C. 10:21:26
- D. 21:10:26

**Gate Mechanical 2020**

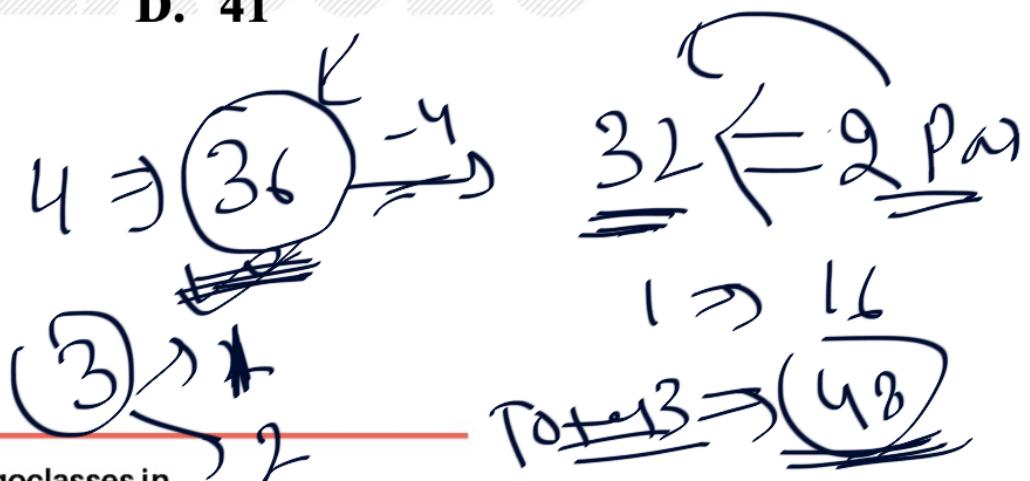




**Q.8.** Three friends,  $R$ ,  $S$  and  $T$  shared toffee from a bowl.  $R$  took  $\frac{1}{3}$ rd of the toffees, but returned four to the bowl.  $S$  took  $\frac{1}{4}$ th of what was left but returned three toffees to the bowl.  $T$  took half of the remainder but returned two back into the bowl. If the bowl had 17 toffees left, how many toffees were originally there in the bowl?

- A. 38
- B. 31
- C. 48
- D. 41

Gate 2021 AG



**Q.9.**

If  $m$  students require a total of  $m$  pages of stationery in  $m$  days, then 100 students will require 100 pages of stationery in

- 
- A. 100 days
  - B.  $m/100$  days
  - C.  $100/m$  days
  - D.  $m$  days

**Gate 2011 GG**

$$\frac{m \times m}{m} = \frac{100 \times D}{100}$$

$$\cancel{D} = m \text{ days}$$

**Q.10.**

If  $3 \leq X \leq 5$  and  $8 \leq Y \leq 11$  then which of the following options is TRUE?

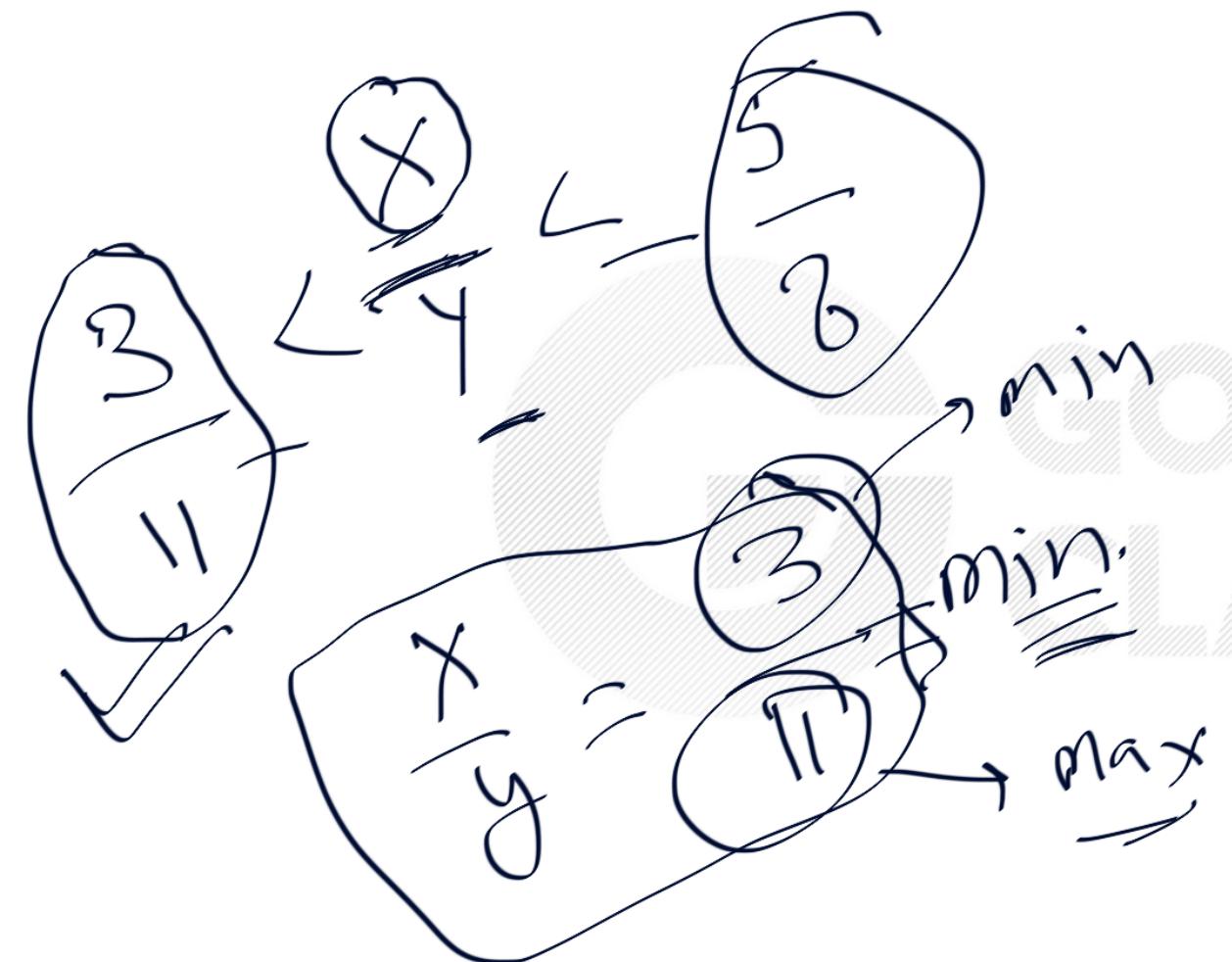
A.  $\frac{3}{5} \leq \frac{X}{Y} \leq \frac{8}{5}$

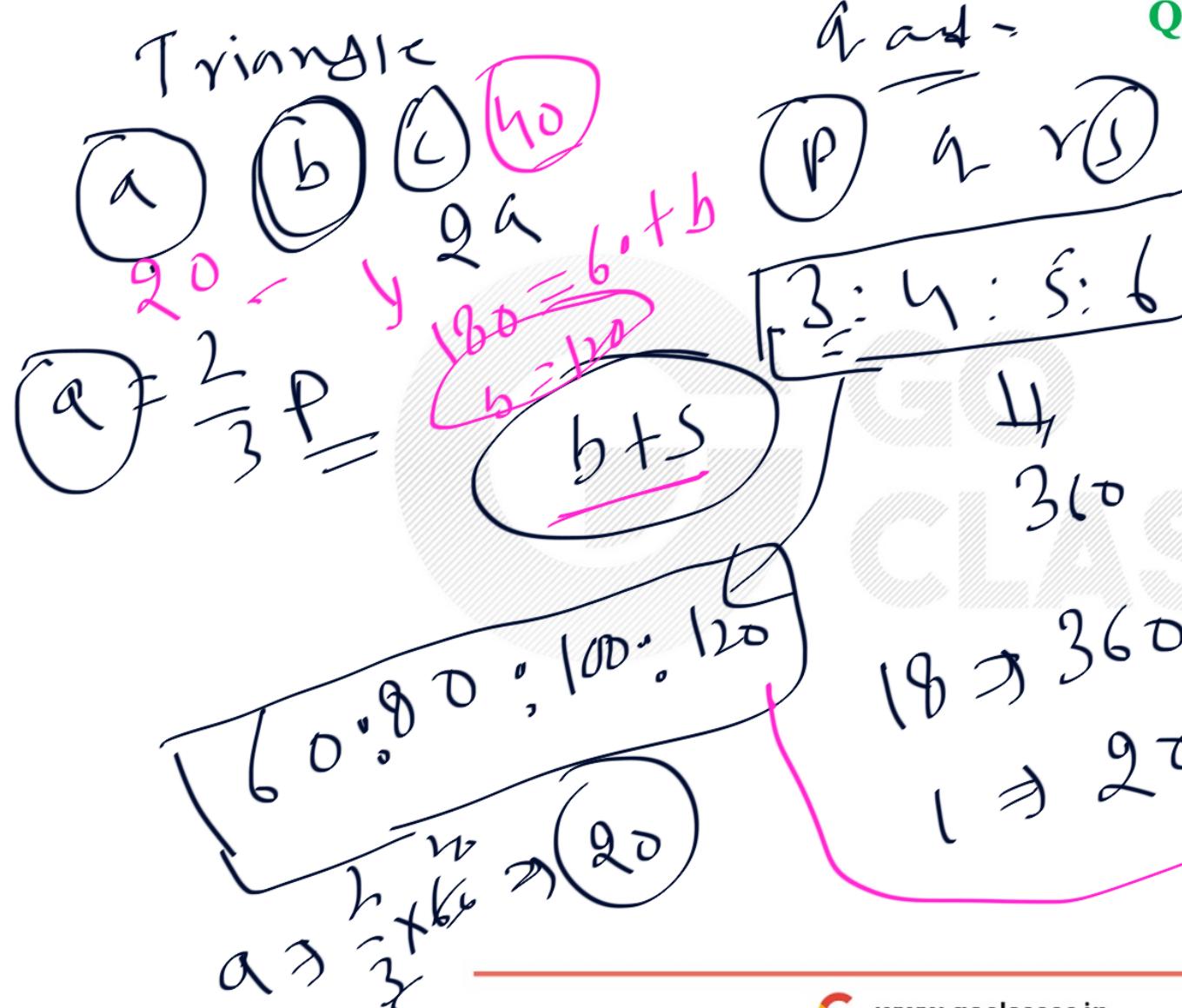
B.  $\frac{3}{11} \leq \frac{X}{Y} \leq \frac{5}{8}$

**Gate 2011 GG**

C.  $\frac{3}{11} \leq \frac{X}{Y} \leq \frac{8}{5}$

D.  $\frac{3}{5} \leq \frac{X}{Y} \leq \frac{8}{11}$

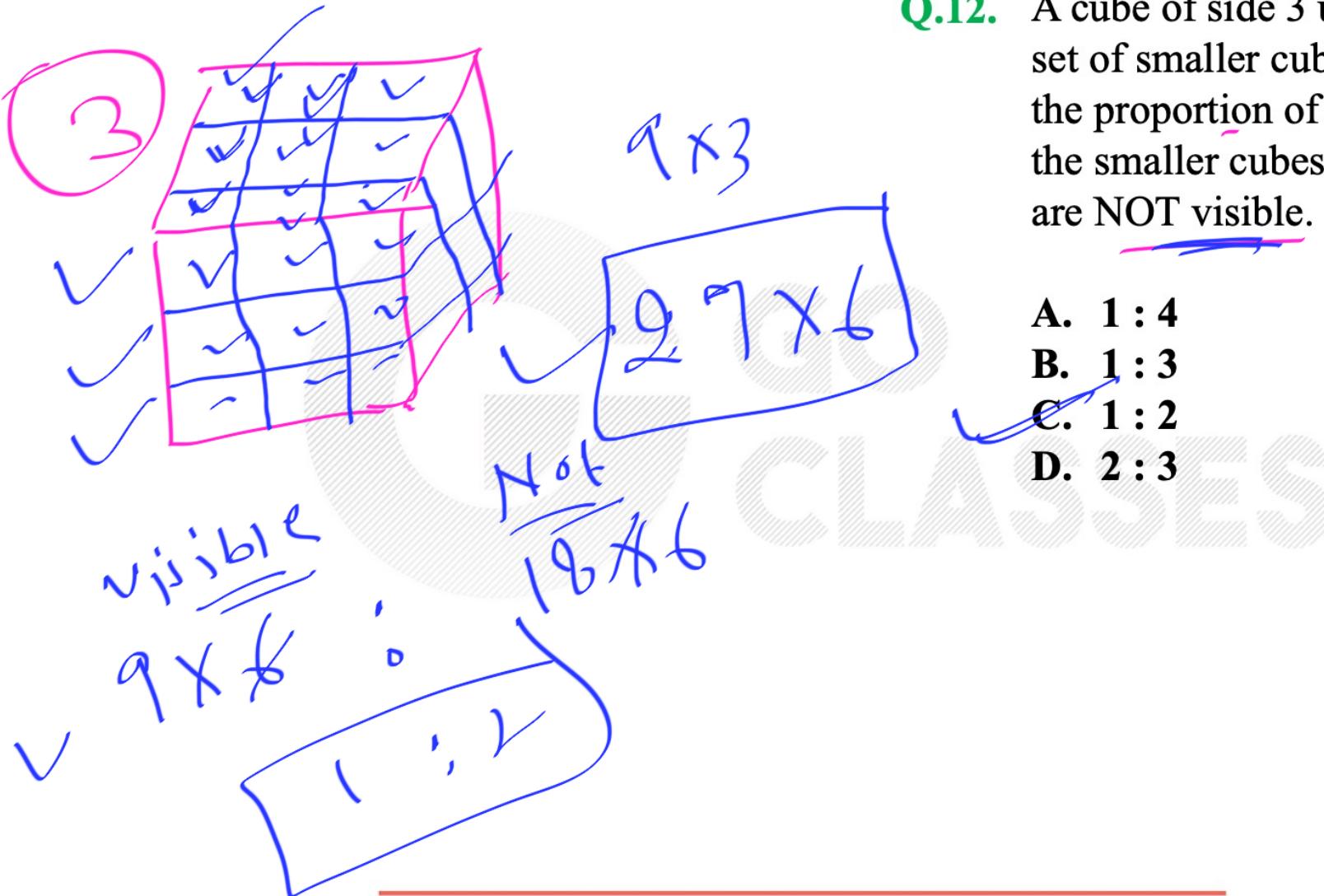




**Q.12.** A cube of side 3 units is formed using a set of smaller cubes of side 1 unit. Find the proportion of the number of faces of the smaller cubes visible to those which are NOT visible.

- A. 1 : 4
- B. 1 : 3
- C. 1 : 2
- D. 2 : 3

**Gate 2015 EC**



**Q.13.** What is the value of  $x$  when  $81 \times \left(\frac{16}{25}\right)^{x+2} \div \left(\frac{3}{5}\right)^{2x+4} = 144$ ?

Gate 2017 CE

- A. 1
- B. -1
- C. -2
- D. Can not be determined

$$81 \times \left(\frac{16}{25}\right)^{x+2} \div \left(\frac{3}{5}\right)^{2x+4} = 144$$

$$9^2 \left(\frac{16}{25}\right)^2 \cdot \left(\frac{16}{25}\right)^x = 12^2 \times \left(\frac{3}{5}\right)^4 \left(\frac{3}{5}\right)^{2x}$$

$$\cancel{\left(9 \times \frac{16}{25}\right)^2} \cdot \left(\frac{16}{25}\right)^x = 12^2 \left(\frac{9}{25}\right)^2 \left(\frac{9}{25}\right)^x$$

$$\left(\frac{16}{9}\right)^x = \frac{9}{16}$$

$$\left(\frac{16}{25} \times \frac{25}{9}\right)^x = \left(12 \times \frac{9}{25} \times \frac{25}{16} \times \frac{1}{9}\right)^x$$

**Q.14.** The price of a wire made of a super alloy material is proportional to the square of its length. The price of 10m length of the wire is Rs. 1600. What would be the total price (in Rs.) of two wires of length 4m and 6m ?

$$P \propto L^2$$

$$P = KL^2$$

$$1600 = K(10)^2$$

$$K = 16$$

- A. 768
- B. 832
- C. 1440
- D. 1600

**Gate 2018 CE**

$$P = 16L^2$$

$$P_u \rightarrow 16 \times 16 \Rightarrow 256$$

$$P_L \rightarrow 16 \times 36 \Rightarrow \underline{576}$$

832

$$\text{Loss} \propto d^2$$

$$\text{Loss} = K D^2$$

$$4900 = K (7)^2$$

K = 100

$$\text{Loss} \rightarrow 100 \times 4^2$$

$$= 1600$$

**Q.15.** In manufacturing industries, loss is usually taken to be proportional to the square of the deviation from a target. If the loss is Rs. 4900 for a deviation of 7 units, what would be the loss in Rupees for a deviation of 4 units from the target?

- A. 400
- B. 1200
- C. 1600
- D. 2800

**Gate 2018 CE**

$\text{G : C}$   
 $(2, 3)$

$\text{G : C}$   
 $(5, 1)$

$\text{G : C}$   
 $3 : 7$

**Q.16.** Two alloys  $A$  and  $B$  contain gold and copper in the ratios of  $2 : 3$  and  $3 : 7$  by mass, respectively. Equal masses of alloys  $A$  and  $B$  are melted to make an alloy  $C$ . The ratio of gold to copper in alloy  $C$  is  $=$ .

- A.  $5 : 10$
- B.  $7 : 13$
- C.  $6 : 11$
- D.  $9 : 13$

**Gate 2018 EC**

$$G_C = \frac{2}{5} + \frac{3}{10}$$
$$G_C = \frac{7}{10}$$
$$G_C = \frac{7}{20}$$

$A : B$   
 $1 : 1$

$G_C : C$   
 $7 : 13$

**Q.17.** The ratio of the number of boys and girls who participated in an examination is 4 : 3. The total percentage of candidates who passed the examination is 80 and the percentage of girls who passed the exam is 90. The percentage of boys who passed is \_\_\_\_\_.

- A. 55.50
- B. 72.50
- C. 80.50
- D. 90.00

**Gate 2019 EE**

