Continuity

- A function $f(x)$ is continious at $x=a$

if

$$\lim_{x \to a} f(x) = f(a) = \text{value of function at } a$$

or

$$LHL = RHL$$

$$f(x) = \begin{cases} \dfrac{\sqrt{1+kx} - \sqrt{1-kx}}{x} & -1 \le x < 0 \\ \dfrac{2x+1}{x-1} & 0 \le x < 1 \end{cases}$$

find k, if f(x) is cont^n at $x = 0$.

$f(0) = -2$

LHL :- $\lim_{x \to 0^-} f(x)$

LHL

$$\lim_{x \to 0^-} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x}$$

$x = 0 - h$

$kx = -kh$

$$\lim_{h \to 0} \frac{\sqrt{1-kh} - \sqrt{1+kh}}{-h} \times \frac{\sqrt{1-kh} + \sqrt{1+kh}}{-h}$$

$$= \frac{1-kh - (1+kh)}{-h} = \frac{\cancel{1}-kh - \cancel{1}-kh}{-h}$$

$$= \frac{-2kh}{-h} = 2k$$

LHL $2k = f(0) = -2$

$$\boxed{k = -1}$$

$$f(x) = \begin{cases} 3ax + b & x > 1 \\ 11 & x = 1 \\ 5ax - 2b & x < 1 \end{cases}$$

$$\lim_{x \to 1^-} 5ax - 2b$$

$5a - 2b$

given :- $f(x)$ is cont^n at $x = 1$, then $a, b = ?$

$LHL = RHL = f(1)$

$5a - 2b = 3a + b = 11$

$5a - 2b = 11$

$3a + b = 11$

a, b ?

f and g are cont^n at $x = a$

(1) $f(x) \pm g(x)$ is also cont^n

(2) $f(x) \cdot g(x)$ or $\frac{f(x)}{g(x)}$ are cont^n at $x = a$

$g(a) \neq 0$

$$f(x) = \begin{cases} |x| + 3 & x \leq -3 \\ -2x & -3 < x < 3 \\ 6x + 2 & x \geq 3 \end{cases}$$

x is cont^n at $x = 3$? $x = -3$?

not cont^n at $x = 3$

$f(-3) = 6$

$f(-3^+) = 6$

$f(-3^-) = 6$

cont^n $x = -3$

$f(3) = 20$

$f(3^-) = -6$

$f(3^+) = 20$

Composite f^n : $f(g(x))$

is $cont^n$ at $x = a$

if $g(x)$ is con^n at $x = a$

$f(x)$ is $cont^n$ at $g(a)$