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1.1.1 Continuity And Differentiability: GO Classes 2024 | Weekly Quiz 14 | Calculus | Question: 13 top◁

The function

$$f(x) = \begin{cases} e^x & \text{if} \quad x \leq 1 \\ mx + b & \text{if} \quad x > 1 \end{cases}$$

is continuous and differentiable at $x = 1$.

Find the value of $m - b$?

A. e B. $-e$ C. $e - 1$ D. $1 - e$

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Answer key⊡

1.1.2 Continuity And Differentiability: GO Classes 2024 | Weekly Quiz 14 | Calculus | Question: 15 top◁

Find the values of A and B that make

$$f(x) = \begin{cases} x^2 + 1 & \text{if} \quad x \geq 0 \\ A \sin x + B \cos x & \text{if} \quad x < 0 \end{cases}$$

differentiable at $x = 0$.

A. $A = 0, B = 1$ B. $A = 1, B = 0$ C. $A = 0, B = -1$ D. $A = -1, B = 0$

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Answer key⊡

1.1.3 Continuity And Differentiability: GO Classes 2024 | Weekly Quiz 14 | Calculus | Question: 18 top◁

If g is continuous (but not differentiable) at $x = 0, g(0) = 8$, and $f(x) = xg(x)$, find $f'(0)$.

A. 0   B. 8

C. 1   D. $f(x)$ is also not differentiable at $x = 0$.

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Answer key⊡

1.1.4 Continuity And Differentiability: GO Classes 2024 | Weekly Quiz 14 | Calculus | Question: 20 top◁

Suppose $f(x)$ is continuous and diffrentiable for all x. And $-1 \leq f'(x) \leq 3$ fo all x. Which of the following is/are ALWAYS true?

A. $f(5) \leq f(3) + 6$ B. $f(5) \geq f(3) - 2$

C. $f(5) \leq f(3) + 10$ D. $f(5) \geq f(3) - 10$

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Answer key⊡

1.1.5 Continuity And Differentiability: GO Classes 2024 | Weekly Quiz 14 | Calculus | Question: 22 top◁

Which of the following is/are TRUE?

A. $\displaystyle\int_{-5}^{5} \left(ax^2 + bx + c\right) dx = 2\int_{0}^{5} \left(ax^2 + c\right) dx$

B. If f and g are continuous and $f(x) \geqslant g(x)$ for $a \leqslant x \leqslant b$, then

$$\int_{a}^{b} f(x)dx \geqslant \int_{a}^{b} g(x)dx$$

C. If f and g are differentiable, then

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

D. If f is differentiable, then $\frac{d}{dx}\sqrt{f(x)} = \frac{f'(x)}{2\sqrt{f(x)}}$.

goclasses2024_wq14   goclasses   calculus   continuity-and-differentiability   multiple-selects   2-marks

Answer key⬚

1.2   Definite Integral (3) top⬚

1.2.1 Definite Integral: GO Classes 2024 | Weekly Quiz 14 | Calculus | Question: 1top⬚

Let f be a continuous function defined for all real numbers. Suppose

$$\int_0^3 f(x)dx = 2, \quad \int_1^2 f(x)dx = 7, \quad \int_3^6 f(x)dx = -5, \quad \text{and} \quad \int_9^{18} f(x)dx = -3$$

Find $\int_3^6 f\left(\frac{1}{3}t\right)dt$

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Answer key⬚

1.2.2 Definite Integral: GO Classes 2024 | Weekly Quiz 14 | Calculus | Question: 2top⬚

Which of the following represents the value of $\int_1^e \frac{\ln(x)}{x}dx$?

A. 1 B. 2 C. $1/2$ D. e

goclasses2024_wq14   goclasses   calculus   integration   definite-integral   1-mark

Answer key⬚

1.2.3 Definite Integral: GO Classes 2024 | Weekly Quiz 14 | Calculus | Question: 21top⬚

Define

$$g(x) = \begin{cases} -6x + 2 & x \le 2 \\ x - 2 & x > 2 \end{cases}$$

Find $\int_1^4 g(x)dx$.

goclasses2024_wq14   numerical-answers   goclasses   calculus   definite-integral   2-marks

Answer key⬚

1.3   Differentiation (13) top⬚

1.3.1 Differentiation: GO Classes 2024 | Weekly Quiz 14 | Calculus | Question: 11top⬚

Suppose f is a function such that $f'(x) = 4x^3$ and $f''(x) = 12x^2$.

Which of the following is /are true?

A. f has a local maximum at $x = 0$ by the first derivative test
B. f has a local minimum at $x = 0$ by the first derivative test
C. f has a local maximum at $x = 0$ by the second derivative test
D. f has a local minimum at $x = 0$ by the second derivative test

Answer key⬚

1.3.2 Differentiation: GO Classes 2024 | Weekly Quiz 14 | Calculus | Question: 12top⬚

The derivative of $g(x) = e^{\sqrt{x}}$ is

A. $\sqrt{x}e^{\sqrt{x}-1}$

B. $2e^{\sqrt{x}}x^{-05}$

C. $\dfrac{0.5e^{\sqrt{x}}}{\sqrt{x}}$

D. $e^{\sqrt{x}}$

1.3.3 Differentiation: GO Classes 2024 | Weekly Quiz 14 | Calculus | Question: 14top⬚

Given $F(x) = (f(g(x)))^2, g(1) = 2, g'(1) = 3, f(2) = 4$, and $f'(2) = 5$, find $F'(1)$

Answer key⬚

1.3.4 Differentiation: GO Classes 2024 | Weekly Quiz 14 | Calculus | Question: 19top⬚

Suppose that $f(x)$ and $g(x)$ are differentiable functions and that $h(x) = f(x)g(x)$. You are given the following table of values:
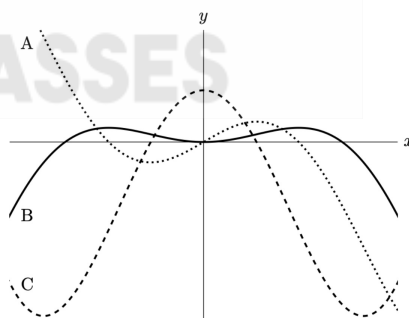
$h(1)$	24
$g(1)$	6
$f'(1)$	-2
$h'(1)$	20

Using the table, find $g'(1)$.

Answer key⬚

1.3.5 Differentiation: GO Classes 2024 | Weekly Quiz 14 | Calculus | Question: 23top⬚

Shown below are portions of the graphs of $y = f(x), y = f'(x)$, and $y = f''(x)$.
Determine which graph is which.

A. $A - f(x), B - f'(x), C - f''(x)$
B. $A - f'(x), B - f(x), C - f''(x)$
C. $A - f''(x), B - f(x), C - f'(x)$
D. $A - f'(x), B - f''(x), C - f(x)$

The function $f(x)$ is defined as follows:

$$f(x) = \begin{cases} \dfrac{x}{x^2+1} & x \leq 0 \\ ? & x > 0 \end{cases}$$

Note that the formula for $f(x)$ for $x > 0$ is unknown.

However, it is known that $f(x)$ is differentiable at each point in its domain $(-\infty, \infty)$, and that $f'(x) > 0$ for all $x \geq 0$. Which of the following option is/are TRUE?
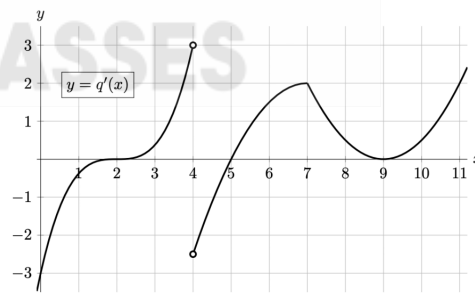
A. $x = 0$ has global minima
B. $x = 0$ has global maxima
C. $x = -1$ has global maxima
D. $x = -1$ has global minima

goclasses2024_wq14   goclasses   calculus   differentiation   maxima-minima   multiple-selects   2-marks

Answer key

Let $q(x)$ be a continuous function which is defined for all real numbers. A portion of the graph of $q'(x)$, the derivative of $q(x)$, is shown below.

On which of the following interval(s) is $q''(x)$ positive?

A. $(0, 2)$ B. $(2, 4)$ C. $(7, 9)$ D. $(5, 7)$

goclasses2024_wq14   goclasses   calculus   differentiation   maxima-minima   multiple-selects   2-marks

Answer key

We consider a function $f(x)$ defined for all real numbers. We suppose that the first and second derivatives $f'(x)$ and $f''(x)$ are also defined for all real numbers. Below we show the graph of the second derivative of f. You may assume that $f''(x)$ is decreasing outside of the region shown.
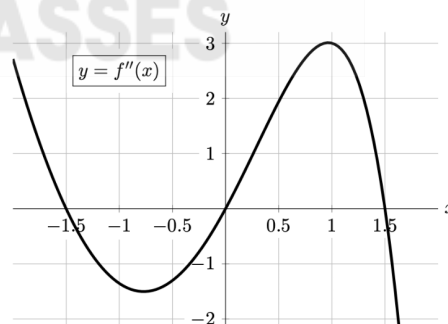
Suppose that $f'(0) = 5$. How many critical points does f have?

goclasses2024_wq14   numerical-answers   goclasses   calculus   differentiation   maxima-minima   2-marks

Answer key

1.3.9 Differentiation: GO Classes 2024 | Weekly Quiz 14 | Calculus | Question: 28 _{top}

Consider a continuous function $f(x)$, and suppose that $f(x)$ and its first derivative $f'(x)$ are differentiable everywhere. Suppose we know the following information about $f(x)$ and its first and second derivatives.

- On the interval $(-\infty, -2)$, we have $f(x) = 2^{-x}$.
- $\lim\limits_{x \to \infty} f(x) = 6$
- $f(2) = -5, f(3) = 7,$ and $f(4) = 8$
- $f'(x)$ is equal to 0 at $x = -1, 2, 4,$ and not at any other x-values.
- $f''(x) < 0$ on the intervals $-1 < x < 0$ and $3 < x < 5$, and not on any other interval.

Find the global minimum of $f(x)$ on $(-\infty, \infty)$? (minimum value of $f(x)$)

Answer key _[a]

1.3.10 Differentiation: GO Classes 2024 | Weekly Quiz 14 | Calculus | Question: 29 _{top}

If $f(x) = (1+x)\left(1+x^2\right)\left(1+x^3\right)\left(1+x^4\right)$, then $f'(0) =?$

Answer key _[a]

1.3.11 Differentiation: GO Classes 2024 | Weekly Quiz 14 | Calculus | Question: 5 _{top}

Some information about the derivative $p'(x)$ and the second derivative $p''(x)$ of a function $p(x)$ is provided in the table below.

x	-4	-3	-2	-1	0	1	2
$p'(x)$	1	0	-2	0	-1	0	2
$p''(x)$	-1	0	0	0	0	2	1

At which of the following values of x must $p(x)$ have a local minimum?

A. $x = -3$ B. $x = -2$ C. $x = -1$ D. $x = 1$

Answer key _[a]

1.3.12 Differentiation: GO Classes 2024 | Weekly Quiz 14 | Calculus | Question: 7 _{top}

The function $g(x)$ is given by the equation

$$g(x) = \begin{cases} ax^2 & x \le 1 \\ b - \ln(3x) & x > 1 \end{cases}$$

where a and b are constants.
Find the value of a such that function is differentiable at $x = 1$.

A. $1/2$ B. $-1/2$ C. 2 D. -2

Answer key _[a]

1.3.13 Differentiation: GO Classes 2024 | Weekly Quiz 14 | Calculus | Question: 9 _{top}

The following are tables of values for two differentiable functions $f(x)$ and $g(x)$ and their derivatives. Missing values are denoted by a "?". Assume that each of these functions is defined for all real numbers, that $f'(x)$ and $g'(x)$ are continuous.

x	0	2	3	6	9
$f(x)$	-1	?	0	-2	?
$f'(x)$	1	4	-1	?	1

x	-1	1	3	7	11
$g(x)$	-4	1	2	6	7
$g'(x)$	7	?	3	4	?

Let $z(x) = f(g(x))$. Find $z'(3)$

goclasses2024_wq14   numerical-answers   goclasses   calculus   differentiation   1-mark

Answer key⬚

1.4 Integration (1) top⬚

1.4.1 Integration: GO Classes 2024 | Weekly Quiz 14 | Calculus | Question: 3 top⬚

Find $\displaystyle\int xe^{-3x}\,dx$

A. $\dfrac{1}{2}e^{-3x}(x-3)+C$

B. $-\dfrac{1}{9}e^{-3x}(3x+1)+C$

C. $-\dfrac{1}{6}x^2e^{-3x}+C$

D. $-\dfrac{1}{3}e^{-3x}(x+1)+C$

goclasses2024_wq14   goclasses   calculus   integration   1-mark

Answer key⬚

1.5 Limits (6) top⬚

1.5.1 Limits: GO Classes 2024 | Weekly Quiz 14 | Calculus | Question: 10 top⬚

Which of the following is/are TRUE?

A. If $\displaystyle\lim_{x\to 5} f(x) = 0$ and $\displaystyle\lim_{x\to 5} g(x) = 0$, then $\displaystyle\lim_{x\to 5} \frac{f(x)}{g(x)}$ does not exist.

B. If f, g, are any two functions which are continuous for all x, then $\dfrac{f}{g}$ is continuous for all x.

C. It is possible that functions f and g are not continuous at a point x_0, but $f + g$ is continuous at x_0.

D. If $f'(c) = 0$ then $f(x)$ has a local maximum or a local minimum at $x = c$.

goclasses2024_wq14   goclasses   calculus   limits   multiple-selects   1-mark

Answer key⬚

1.5.2 Limits: GO Classes 2024 | Weekly Quiz 14 | Calculus | Question: 16 top⬚

Is there a number b such that $\displaystyle\lim_{x\to -2} \frac{bx^2 + 15x + 15 + b}{x^2 + x - 2}$ exists? If so, find the value of b and the value of the limit.

A. -1

B. -2

C. 1

D. There is no such b for that above limit exist

goclasses2024_wq14   goclasses   calculus   limits   2-marks

Answer key⬚

1.5.3 Limits: GO Classes 2024 | Weekly Quiz 14 | Calculus | Question: 17 top⬚

$\displaystyle\lim_{x\to\infty} (x + \sin x)^{\frac{1}{x}}$

goclasses2024_wq14   numerical-answers   goclasses   calculus   limits   2-marks

Answer key⬚

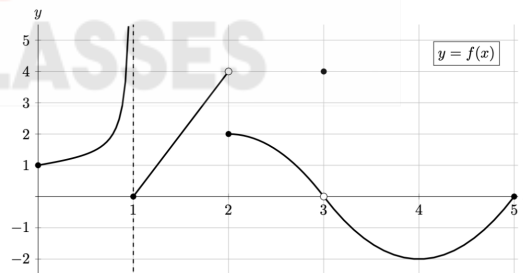Let $M = \lim\limits_{x \to 0^+} \left(e^x + 3x\right)^{1/x}$.

Find the value of $\log_e M$.

goclasses2024_wq14 numerical-answers goclasses calculus limits 2-marks

Answer key◁

Below is a portion of the graph of an even function $f(x)$, which has domain $(-\infty, \infty)$ even though the graph below only shows the function on the interval $[0, 5]$. Note that $f(x)$ has a vertical asymptote at $x = 1$.

Find

$$\lim_{h \to 0} \frac{f(1.5 + h) - f(1.5)}{h}$$

goclasses2024_wq14 numerical-answers goclasses calculus limits 1-mark

Answer key◁

Consider the piecewise function

$$q(x) = \begin{cases} 7e^{x-C} + \dfrac{x}{3x - 2} & x < 0 \\[3mm] \dfrac{6 + 5x}{2 + 3x + 4^x} & x \geq 0 \end{cases}$$

where C is a constant.
Find $\lim\limits_{x \to \infty} q(x)$

goclasses2024_wq14 numerical-answers goclasses calculus limits 1-mark

Answer key◁

Consider the family of functions

$$g(x) = a \ln(x) + \frac{b}{x}$$

defined for $x > 0$, where a and b are positive constants.

Any function $g(x)$ in this family has only one critical point. In terms of a and b, what is the x-coordinate of that critical point?

A. $x = b/a$ is a critical point where the function shows local minima
B. $x = b/a$ is a critical point where the function shows local maxima
C. $x = a/b$ is a critical point where the function shows local minima
D. $x = a/b$ is a critical point where the function shows local maxima

goclasses2024_wq14   goclasses   calculus   maxima-minima   1-mark

Answer key

1.7   Mean Value Theorem (1) top

1.7.1 Mean Value Theorem: GO Classes 2024 | Weekly Quiz 14 | Calculus | Question: 25 top

Suppose $h(x)$ is a function such that $h(x)$ has exactly three critical point. Two of which are shown in the table below.
Assume that both $h(x)$ and $h'(x)$ are differentiable on $(-\infty, \infty)$

x	0	3	5	7
$h(x)$	2	?	4	4
$h'(x)$	-1	0	0	?

Further using Lagrange mean value theorem in the interval $[5, 7]$, we can determine the interval of the third critical point.
On which of the following intervals must $h(x)$ be increasing on the entire interval?

A. $(0, 3)$ B. $(3, 5)$ C. $(5, 6)$ D. $(6, 7)$

goclasses2024_wq14   goclasses   calculus   mean-value-theorem   multiple-selects   2-marks

Answer Keys

1.1.1	A	1.1.2	A	1.1.3	B	1.1.4	A;B;C;D	1.1.5	A;B;C
1.2.1	21	1.2.2	C	1.2.3	-5	1.3.1	B	1.3.2	C
1.3.3	120	1.3.4	8	1.3.5	B	1.3.6	D	1.3.7	A;B;D
1.3.8	2	1.3.9	-5	1.3.10	1	1.3.11	D	1.3.12	B
1.3.13	12	1.4.1	B	1.5.1	C	1.5.2	A	1.5.3	1
1.5.4	4	1.5.5	4	1.5.6	0	1.6.1	A	1.7.1	B