



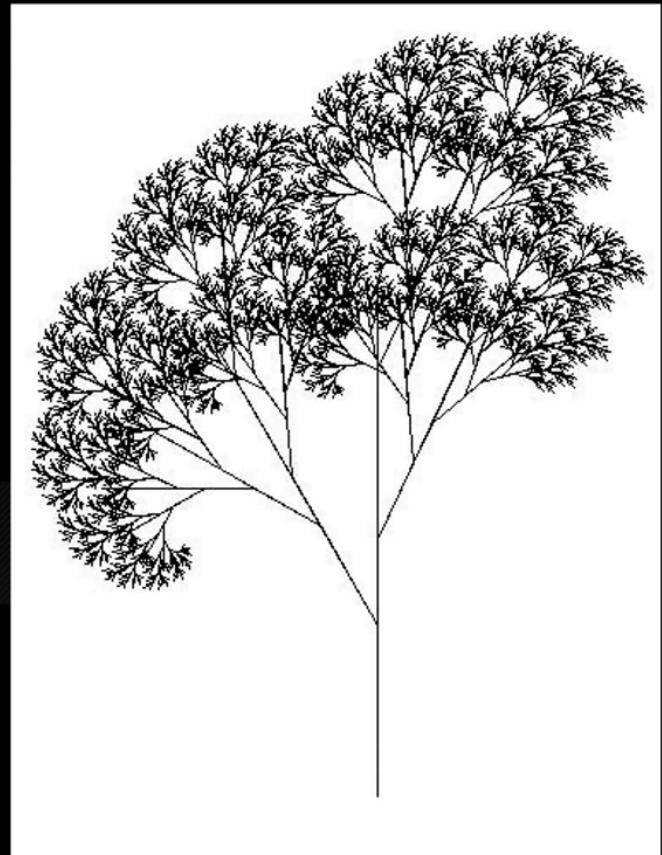
# Recursion

The Loopless Loop



# Recursion

Functions defined in terms of itself are called recursive functions





$$n! = \underline{n} \times \underline{n-1} \times \underline{n-2} \times \underline{n-3} \cdots \times \underline{1}$$

$$n! = n \times (n-1)!$$

$$\text{fact}(n) = n \times \text{fact}(n-1)$$





# C Programming

```
factorial(int n)
```

$$\text{factorial}(n) = \begin{cases} 1 & \underline{n=1 \text{ or } 0} \\ n \times \text{factorial}(n-1) & \text{Recursive step} \end{cases}$$

Base case



# C Programming

```
factorial(int n)
{
    int fact;
    if (n==1)
        return(1);
    else
        fact = n*factorial(n-1);
    return(fact);
}
```

*Base case*

*factorial (n) =*

*1    n = 1*

*n \* factorial(n-1)*

*Recursive case*



# C Programming

```
factorial(int n)
{
    int fact;
    if (n==1 || n==0)
        return(1);
    else
        fact = n*factorial(n-1);
    return(fact);
}
```

How does this work?

factorial(2) → 1

2 ↗ factorial(2)

↗ 2\*factorial(1)  
↗ ,



# C Programming

```
factorial(int n)
{
    int fact;

    if (n==1 || n==0)
        return(1);
    else
        fact = n*factorial(n-1);

    return(fact);
}
```

How does this work?

factorial(3)

fact = 3 \* factorial(2)  
return fact

factorial(2)

fact = 2 \* factorial(1)

factorial(1)

fact



# C Programming

```
factorial(int n)
{
    int fact;

    if (n==1 || n==0)
        return(1);
    else
        fact = n*factorial(n-1);

    return(fact);
}
```

How does this work?

factorial(3)

fact = 3 \* factorial(2)  
return fact

factorial(2)

fact = 2 \* factorial(1)

factorial(1)

fact

factorial(1)

factorial(2)

factorial(3)

main



# C Programming

```
factorial(int n)
{
    int fact;

    if (n==1 || n==0)
        return(1);
    else
        fact = n*factorial(n-1);

    return(fact);
}
```

How does this work?

factorial(3)

fact = 3 \* factorial(2)  
return fact

factorial(2)

fact = 2 \* factorial(1)

factorial(1)

fact

factorial(1)

factorial(2)

factorial(3)

main



# C Programming

```
factorial(int n)
{
    int fact;

    if (n==1 || n==0)
        return(1);
    else
        fact = n*factorial(n-1);

    return(fact);
}
```

How does this work?

factorial(3)

fact = 3 \* factorial(2)  
return fact

factorial(2)

fact = 2 \* factorial(1)

factorial(1)

fact

factorial(1)

factorial(2)

factorial(3)

main



# C Programming

```
factorial(int n)
{
    int fact;

    if (n==1 || n==0)
        return(1);
    else
        fact = n*factorial(n-1);

    return(fact);
}
```

How does this work?

factorial(3)

fact = 3 \* factorial(2)  
return fact

factorial(2)

fact = 2 \* factorial(1)

1

1

factorial(1)

fact

factorial(1)

factorial(2)

factorial(3)

main

$$3 \times 2 \times 1 = 6$$

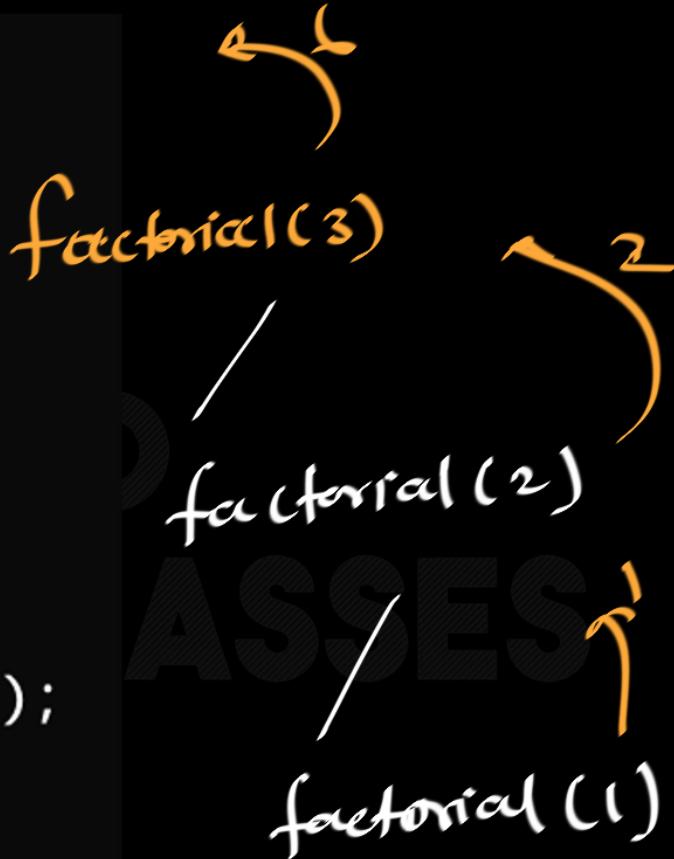


# C Programming

```
factorial(int n)
{
    int fact;

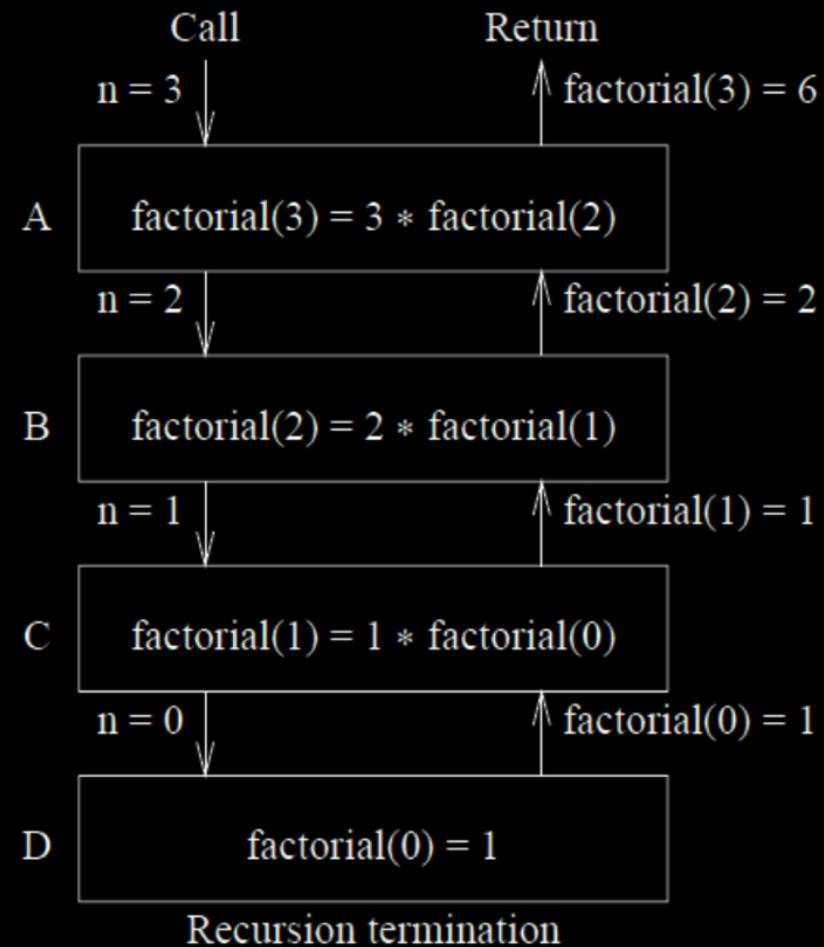
    if (n==1)
        return(1);
    else
        fact = n*factorial(n-1);

    return(fact);
}
```





# C Programming



(a)

Activation record for A
Activation record for B
Activation record for C
Activation record for D

(b)

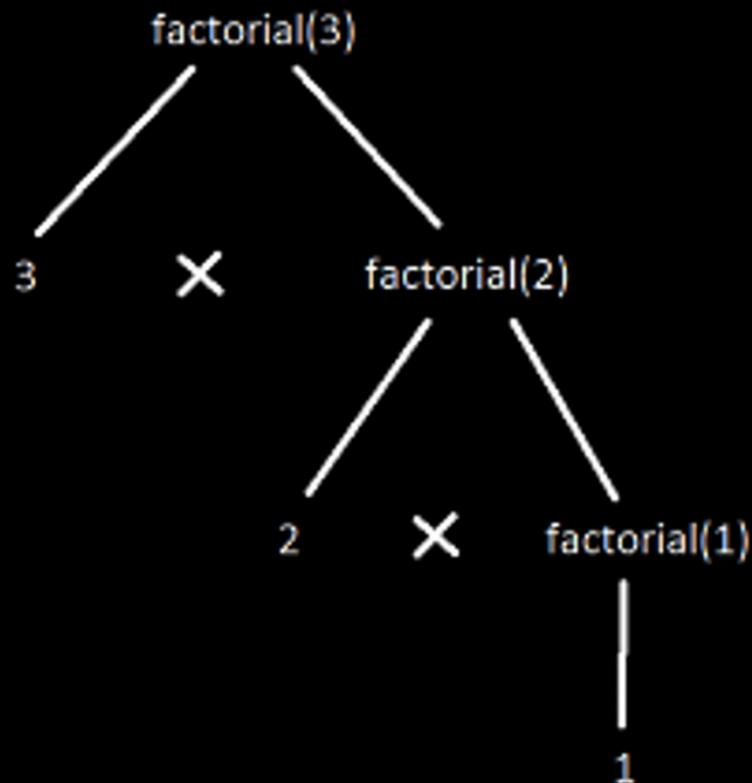


```
factorial(int n)
{
    int fact;

    if (n==1)
        return(1);
    else
        fact = n*factorial(n-1);

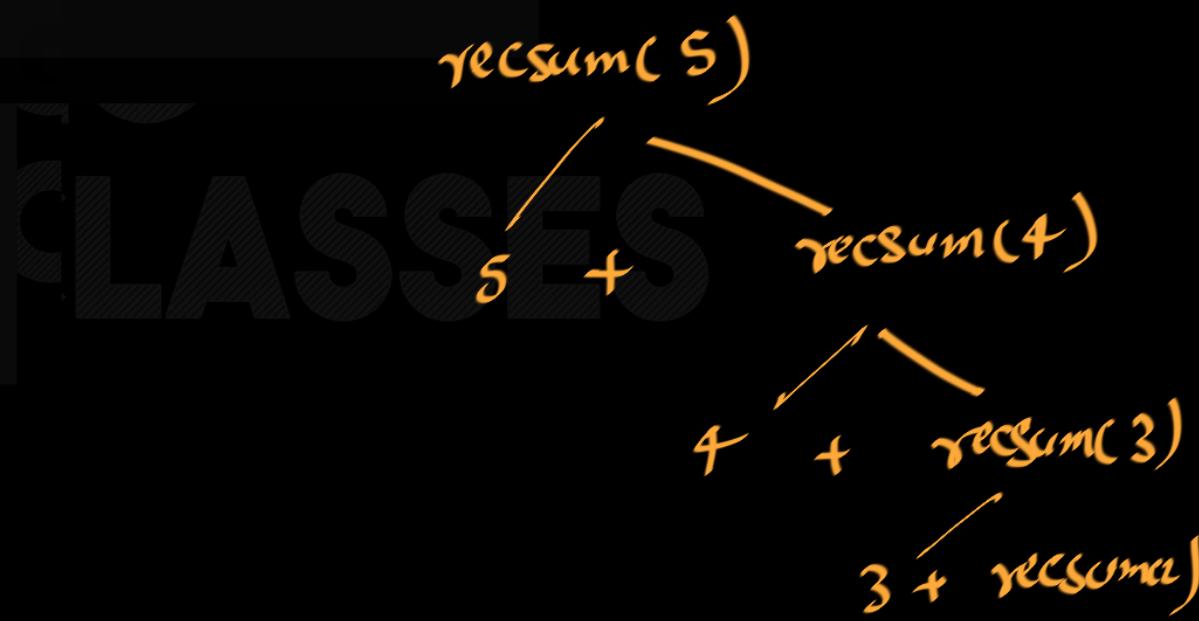
    return(fact);
}
```

Preferred



```
int recsum(x) {  
    if (x === 1) {  
        return x;  
    } else {  
        return x + recsum(x - 1);  
    }  
}
```

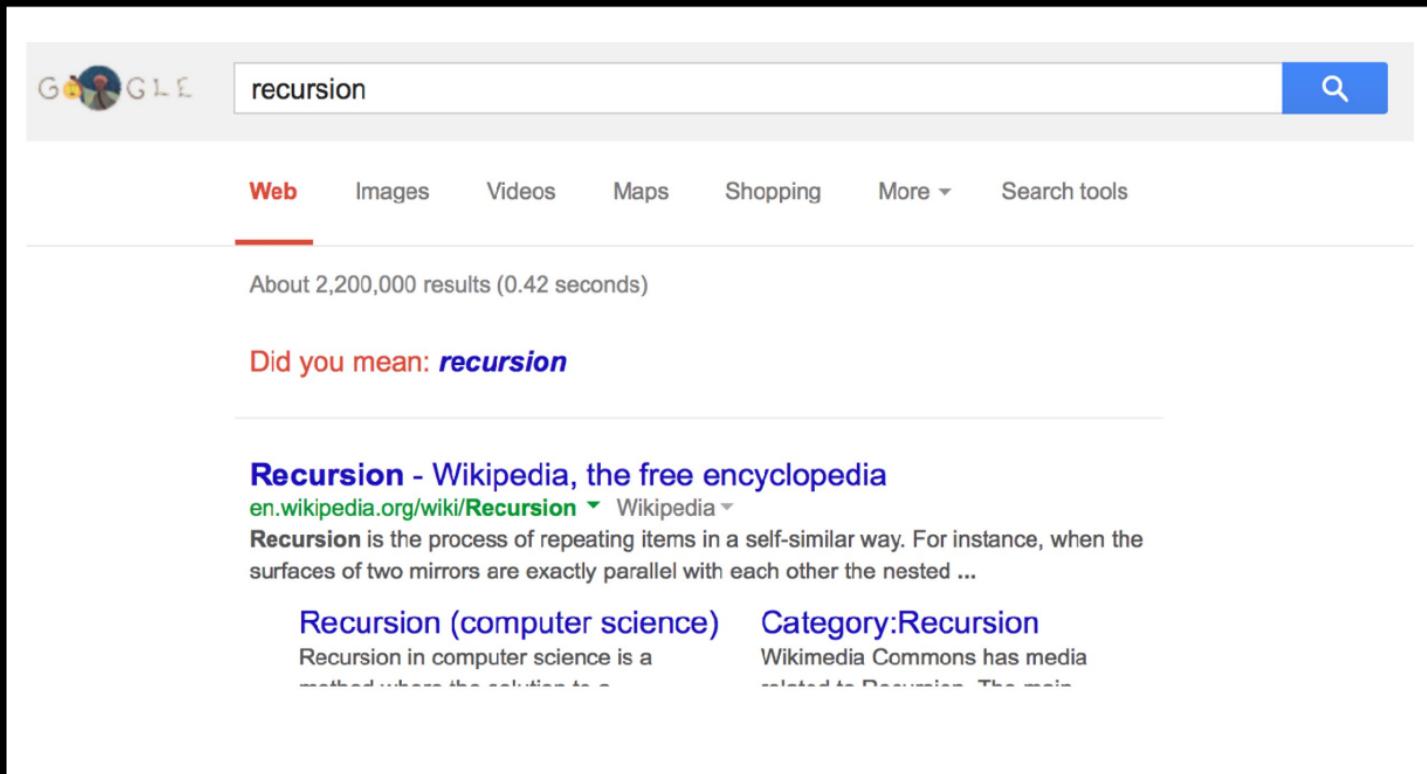
```
recsum(5)  
5 + recsum(4)  
5 + (4 + recsum(3))  
5 + (4 + (3 + recsum(2)))  
5 + (4 + (3 + (2 + recsum(1))))  
5 + (4 + (3 + (2 + 1)))  
15
```





# C Programming





A screenshot of a Google search results page for the query "recursion". The search bar at the top contains "recursion". Below the search bar, the "Web" tab is selected, followed by "Images", "Videos", "Maps", "Shopping", "More", and "Search tools". A message indicates "About 2,200,000 results (0.42 seconds)". A "Did you mean: recursion" suggestion is shown in red. The first result is a link to the Wikipedia article on Recursion, titled "Recursion - Wikipedia, the free encyclopedia". The snippet below the title reads: "Recursion is the process of repeating items in a self-similar way. For instance, when the surfaces of two mirrors are exactly parallel with each other the nested ...". To the left of the snippet is a link to "Recursion (computer science)" and to the right is a link to "Category:Recursion". Both links are in blue.



Let's recurse on you.

How many students total are directly behind you  
in your "column" of the classroom?

Rules:

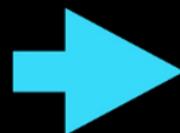
1. You can see only the people directly in front and behind you.  
So, you can't just look back and count.
2. You are allowed to ask questions of the people in front /  
behind you.

How can we solve this problem *recursively*?

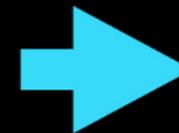


# Recursive Student Counting

CS220 students  
in the front row



Professor with a question



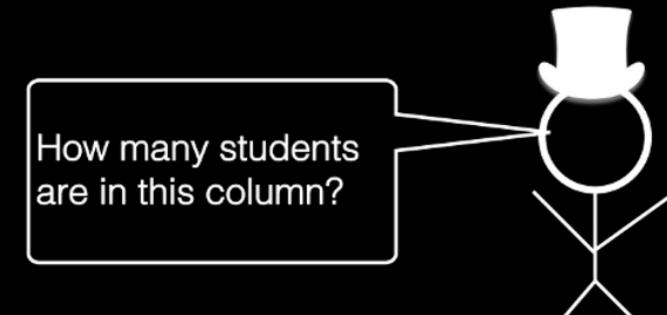


# Recursive Student Counting

Constraints:

- You can only talk to the student behind / in front of you

**What should each student ask the person behind them?**





# Recursive Student Counting

Strategy: **reframe** question as “how many students are behind you?”

how many are behind you?



Process:

if nobody is behind you: say 0  
else: ask them, say their answer+1



# Recursive Student Counting

**Strategy:** reframe question as “how many students are behind you?”

**Process:**

**if nobody is behind you:** say 0

**else:** ask them, say their answer+1

how many are behind you?





# Recursive Student Counting

Strategy: **reframe** question as “how many students are behind you?”

Process:

if nobody is behind you: say 0  
else: ask them, say their answer+1

Observations:

- Each student runs the **same** “code”
- Each student has their **own** “state”

Aha! Clearly there must be 25 students in this column





*Hi, I am Jake  
the Time traveller*

BROOKLYN  
99 - HOLTPOSTING

*This*

*Oh, really?  
then tell me something  
that happens in the future*

*Hi, I am Jake  
the Time traveller*

BROOKLYN  
99 - HOLTPOSTING

*This*

*Oh, really?  
then tell me something  
that happens in the future*

*Hi, I am Jake  
the Time traveller*

BROOKLYN  
99 - HOLTPOSTING

*This*

*Oh, really?  
then tell me something  
that happens in the future*

*This*

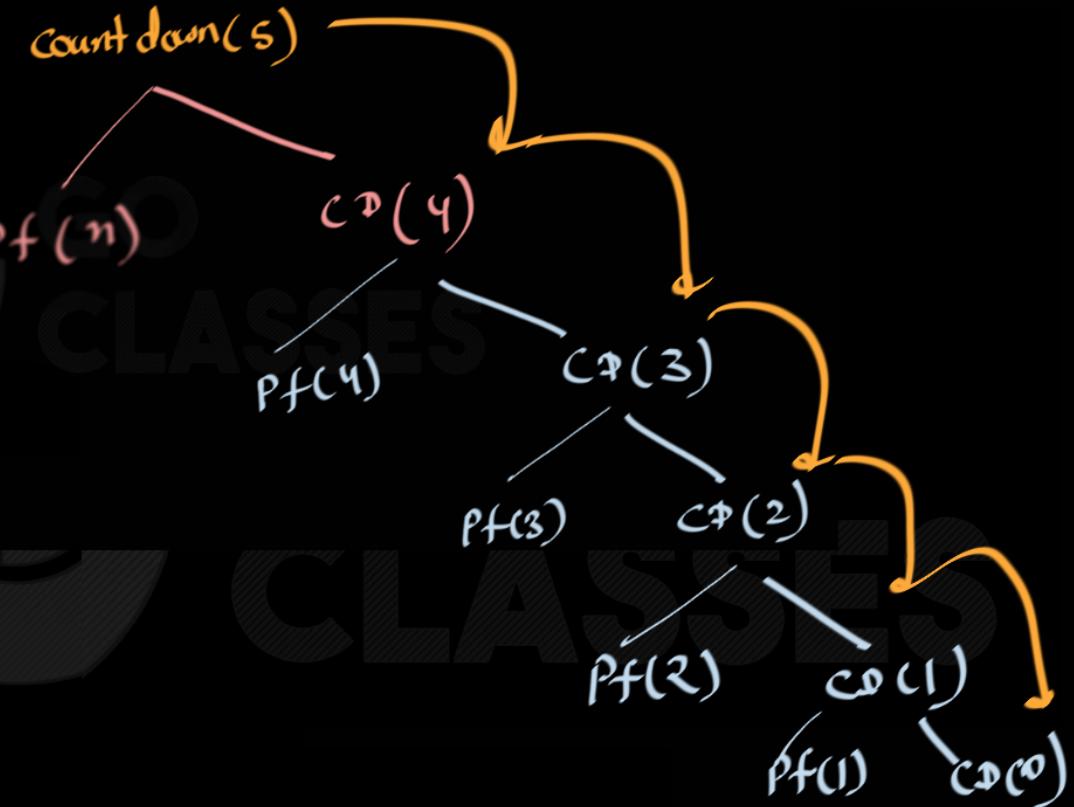
## Question 1

```
#include<stdio.h>

void countdown(int n){
    if (n==0)
        return;

    printf("%d ", n);
    countdown(n-1);
}

int main()
{
    countdown(5);
}
```



5 4 3 2 1

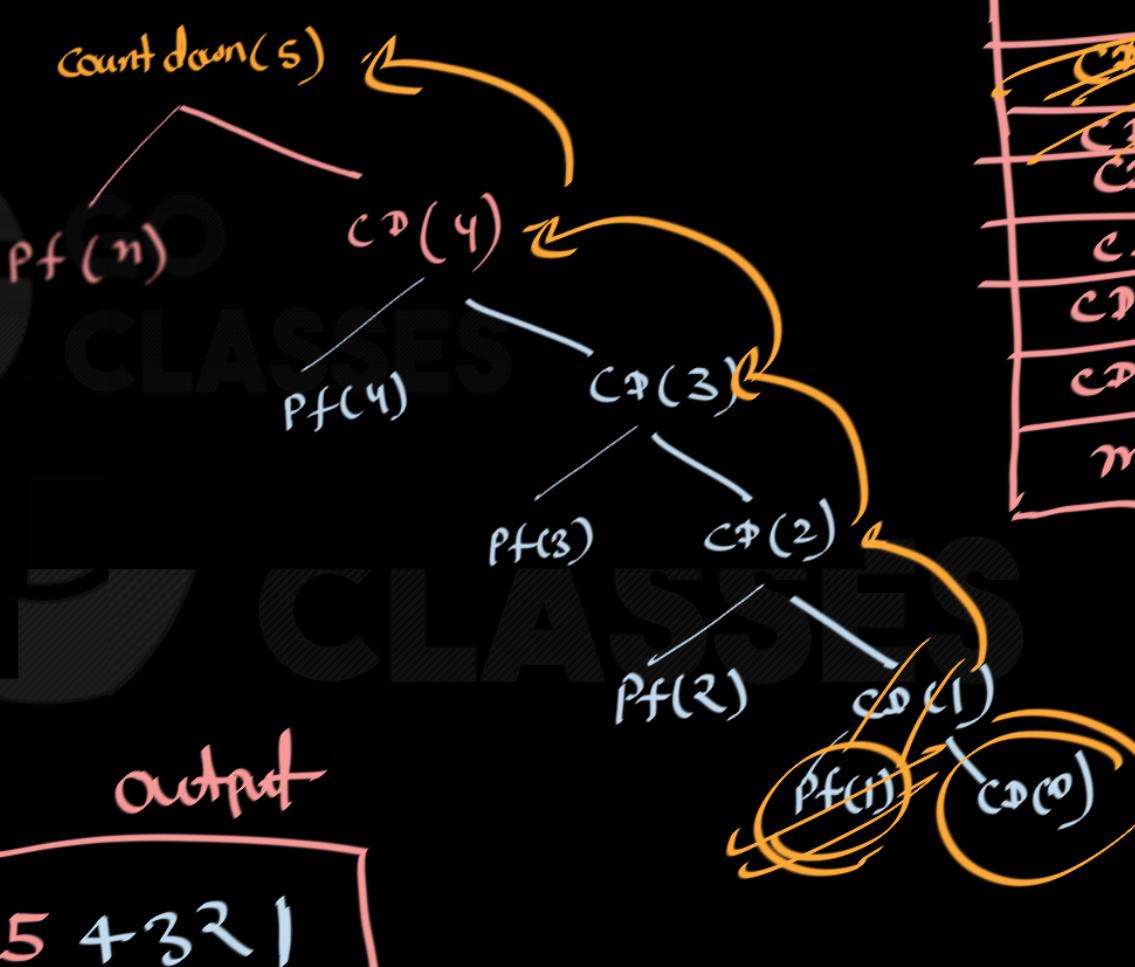
## Question 1

```
#include<stdio.h>

void countdown(int n){
    if (n==0)
        return;

    printf("%d ", n);
    countdown(n-1);
}

int main()
{
    countdown(5);
}
```

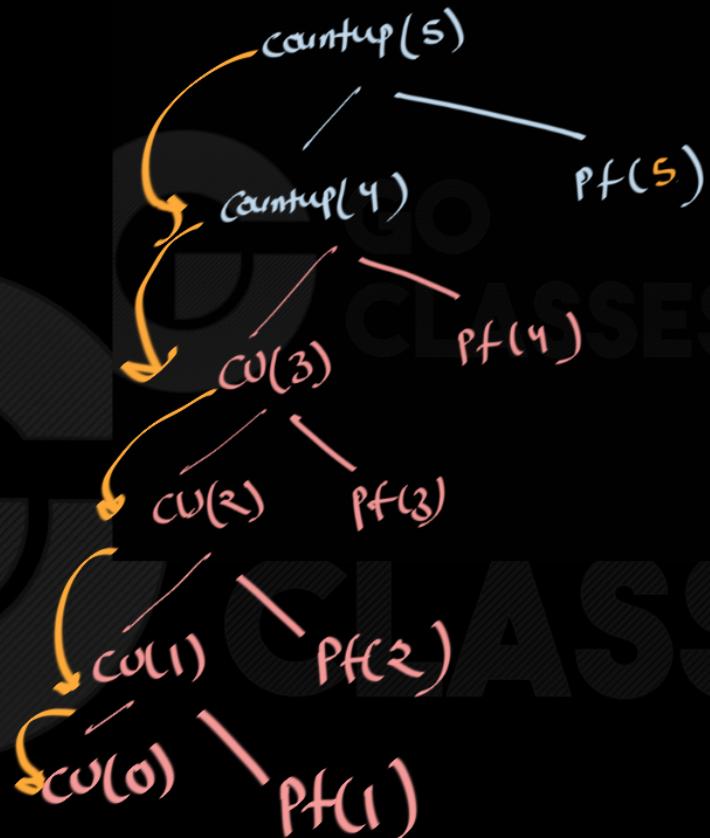




## Question 2

```
#include<stdio.h>
```

```
void countup(int n){  
    if (n==0)  
        return;  
  
    countup(n-1);  
    printf("%d ", n);  
}  
  
int main()  
{  
    countup(5);  
}
```

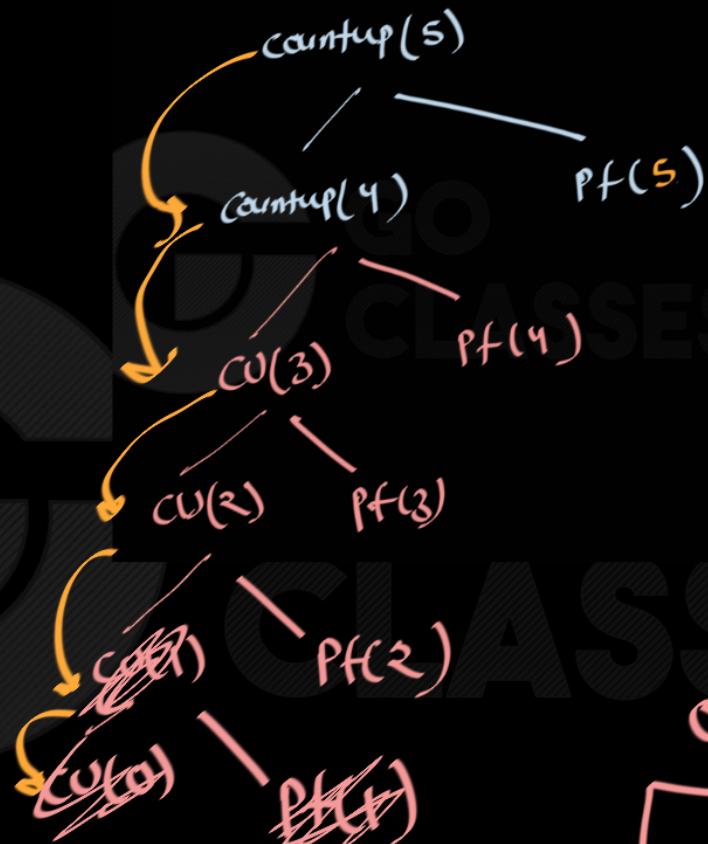


1

## Question 2

```
#include<stdio.h>
```

```
void countup(int n){  
    if (n==0)  
        return;  
  
    countup(n-1);  
    printf("%d ", n);  
}  
  
int main()  
{  
    countup(5);  
}
```



output:

1 2 3 4 5

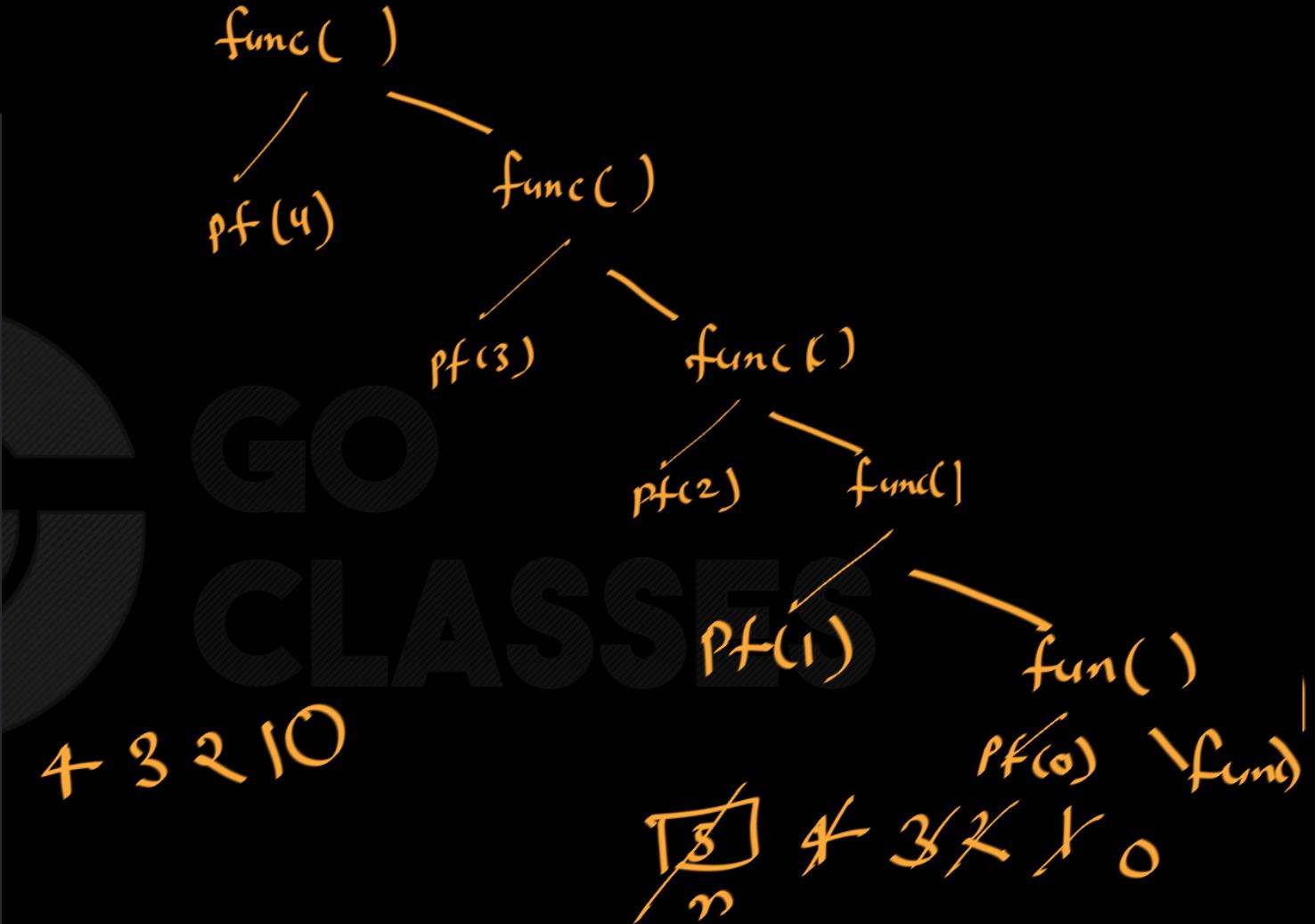


```
#include<stdio.h>

void func(){
    static int n = 5;

    if (n==0)
        return;
    n--;
    printf("%d ", n);
    func();
}

int main()
{
    func();
}
```



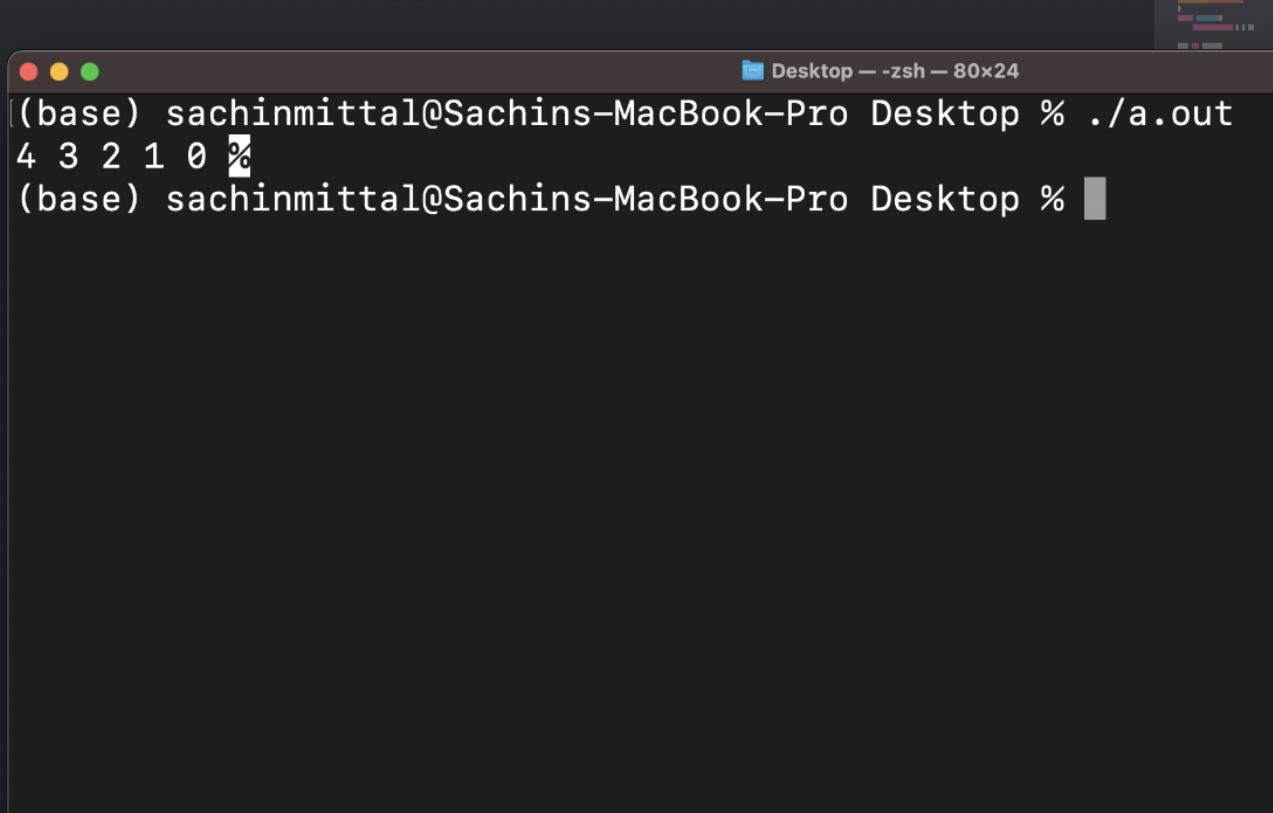


```
#include<stdio.h>

void func(){
    static int n = 5;

    if (n==0)
        return;
    n--;
    printf("%d ", n);
    func();
}

int main()
{
    func();
}
```

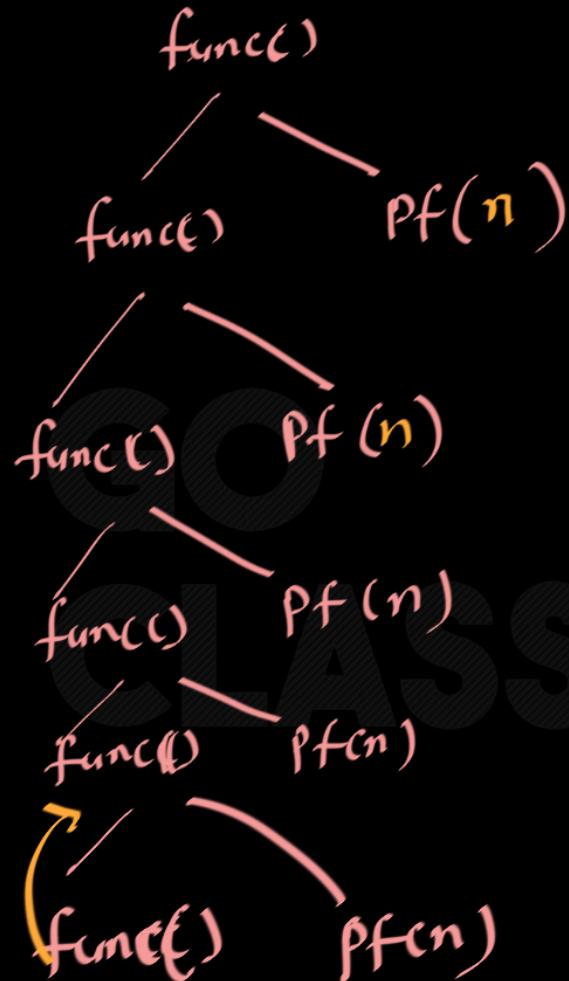


A screenshot of a terminal window titled "Desktop — zsh — 80x24". The window shows the output of a C program named "a.out". The program prints the numbers 4, 3, 2, 1, and 0 on separate lines, followed by a percentage sign (%). The terminal has a dark theme with light-colored text and a dark background. The window title bar includes the path "Desktop" and the command ". ./a.out".

```
(base) sachinmittal@Sachins-MacBook-Pro Desktop % ./a.out
4 3 2 1 0 %
(base) sachinmittal@Sachins-MacBook-Pro Desktop %
```

```
1 #include<stdio.h>
2
3 void func(){
4     static int n = 5;
5
6     if (n==0)
7         return;
8     n--;
9     func();
10    printf("%d ", n);
11
12 }
13
14 int main()
15 {
16     func();
17 }
```

ANSWER





GO  
CLASSES



## Question 3

```
int recursiveFunc(int n)
{
    static int count = 0;

    if (n <= 0)
        return count;

    count++;
    return recursiveFunc(n - 1);
}

int main()
{
    int result = recursiveFunc(5);
    printf("Result: %d\n", result);

}
```

What will be the output of the above code?

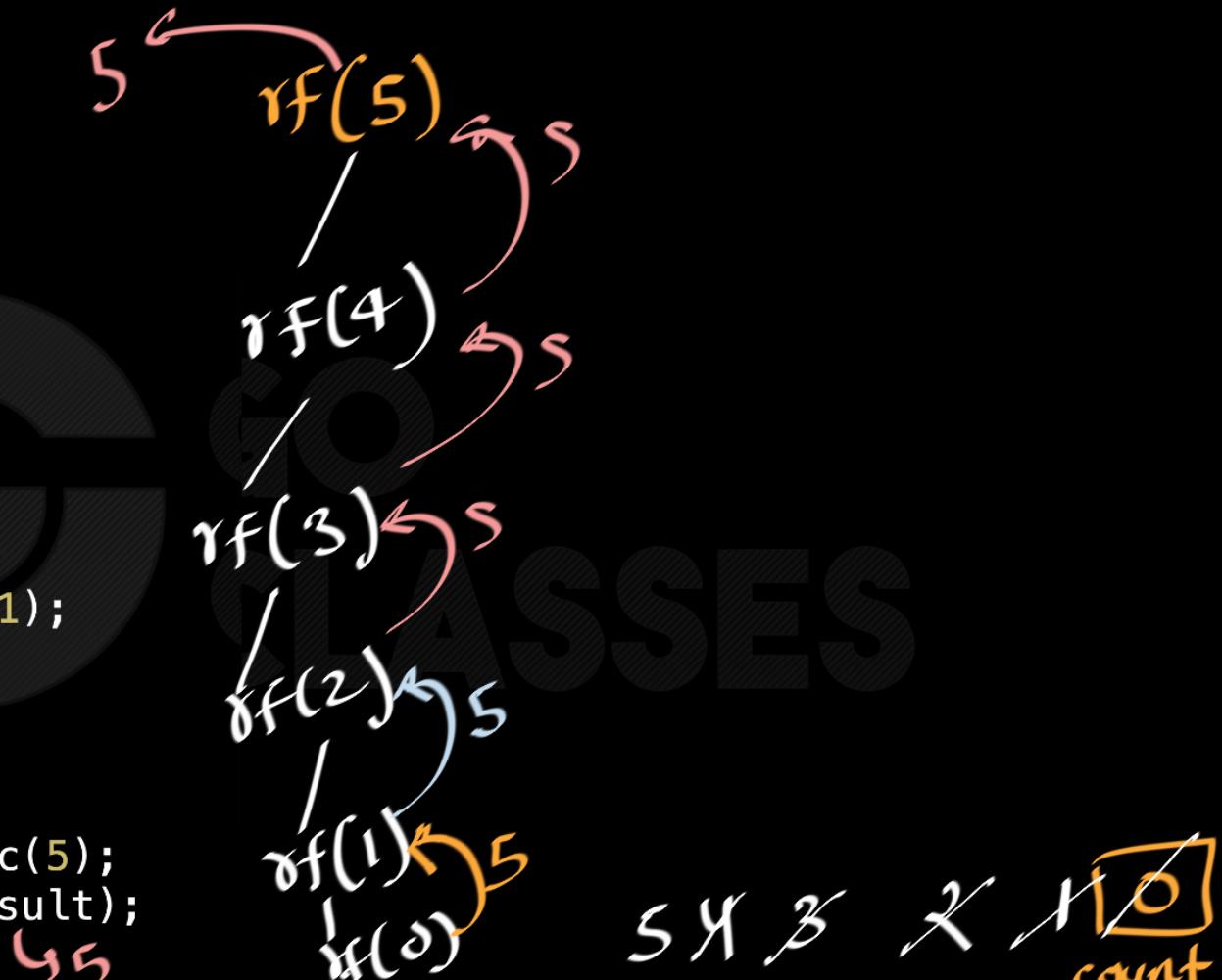
- A) Result: 0
- B) Result: 1
- C) Result: 4
- D) Result: 5



## Question 3

```
int recursiveFunc(int n)
{
    static int count = 0;
    if (n <= 0)
        return count;
    count++;
    return recursiveFunc(n - 1);
}

int main()
{
    int result = recursiveFunc(5);
    printf("Result: %d\n", result);
}
```





## Question 4

```
int recursiveFunc(int n)
{
    static int count = 0;

    if (n <= 0)
        return count;

    count += n;
    return recursiveFunc(n - 1);
}

int main()
{
    int result = recursiveFunc(3);
    printf("Result: %d\n", result);
}
```

What will be the output of the above code?

- A) Result: 0
- B) Result: 3
- C) Result: 6
- D) Result: 10



## Question 4

```
int recursiveFunc(int n)
{
    static int count = 0;
    if (n <= 0)
        return count;
    count += n;
    return recursiveFunc(n - 1);
}

int main()
{
    int result = recursiveFunc(3);
    printf("Result: %d\n", result);
}
```

f(3)

f(2)

f(1)

f(0)

6    5    3  
count  0

6



## Question 5

```
#include <stdio.h>
void increase(int x)
{
    static int i = 0;
    x = i + x;
    i++;
    printf("x=%d, i=%d\n", x, i);
}

int main(void)
{
    int i = 0;
    for ( ; i < 4; ++i)
    {
        increase(i);
    }
}
```

H.W.

GO  
CLASSES



## Question 6



```
int func(int a)
{
    static int j=0;
    j++;
    return a*j;
}

int main(void)
{
    int x=10;

    while(x>0){
        printf("%d\n",func(x));
        --x;
    }

    return (0);
}
```

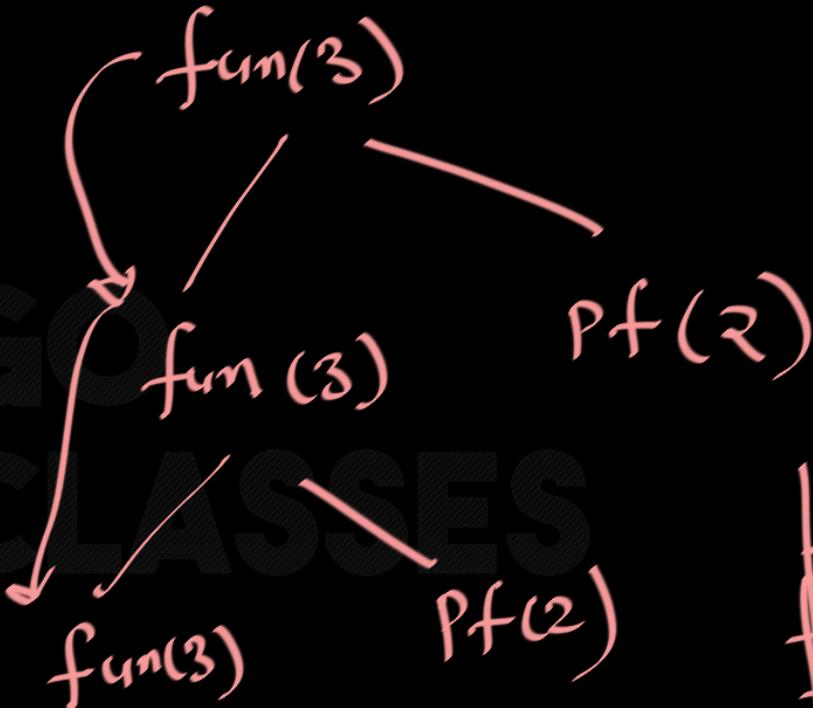


## Question 7

```
void fun(int n)
{
    if(n==0) return;
    else {
        fun(n--);
        printf("%d ", n);
    }
}

main()
{
    fun(3);
}
```

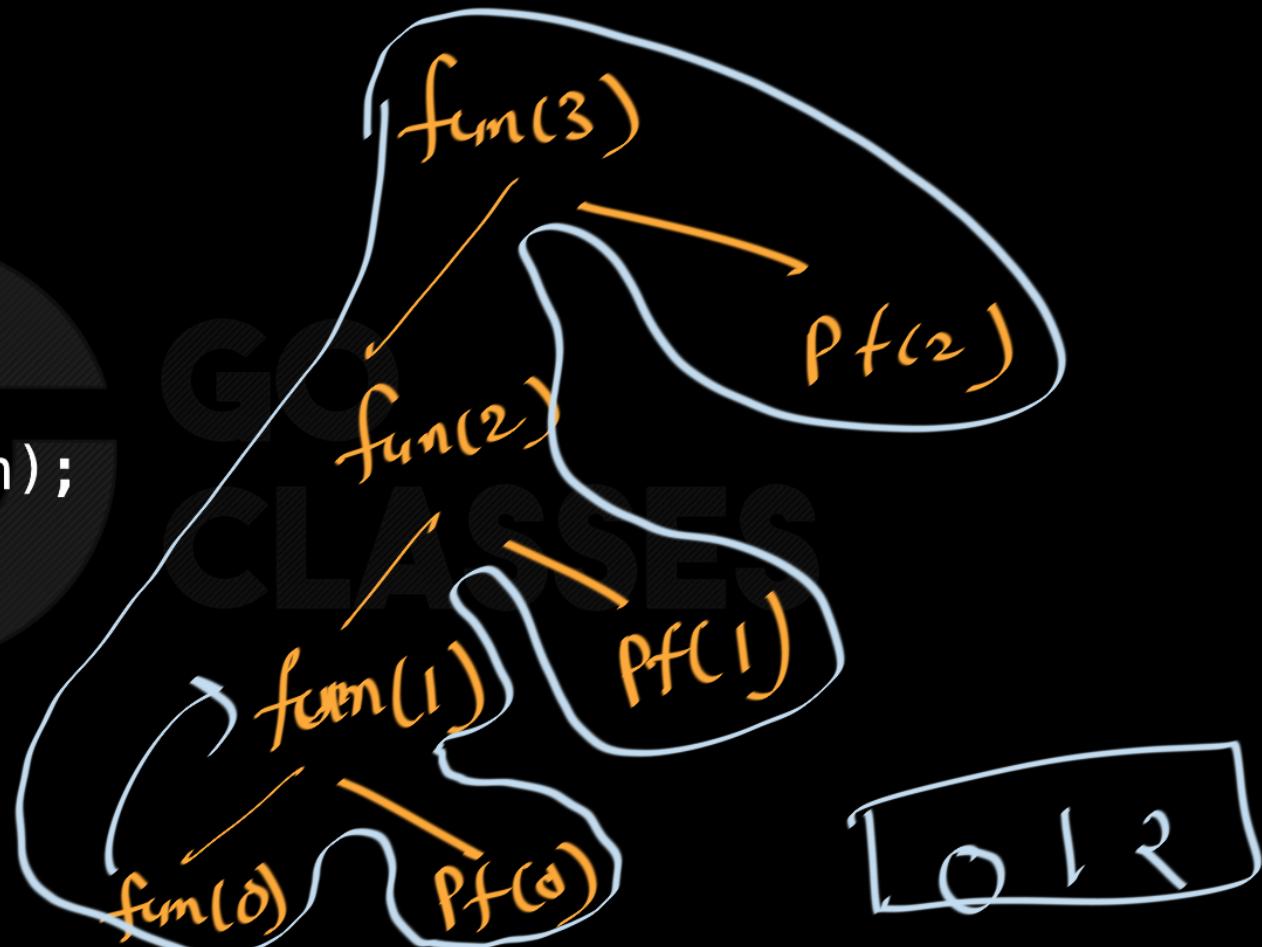
Stack overflow



## Question 8

```
void fun(int n)
{
    if(n==0) return;
    else {
        fun(--n);
        printf("%d ", n);
    }
}

main()
{
    fun(3);
}
```





# C Programming

## Question 9-12

### Program 1

```
void recurse() {  
    static int i = 4;  
    if (--i) {  
        recurse();  
        printf("%d", i);  
    }  
}
```

### Program 2

```
void recurse() {  
    static int i = 4;  
    printf("%d", i);  
    if (--i) {  
        recurse();  
    }  
}
```

### Program 3

```
void recurse(int i) {  
    if (--i) {  
        printf("%d", i);  
        recurse(i);  
    }  
}
```

### Program 4

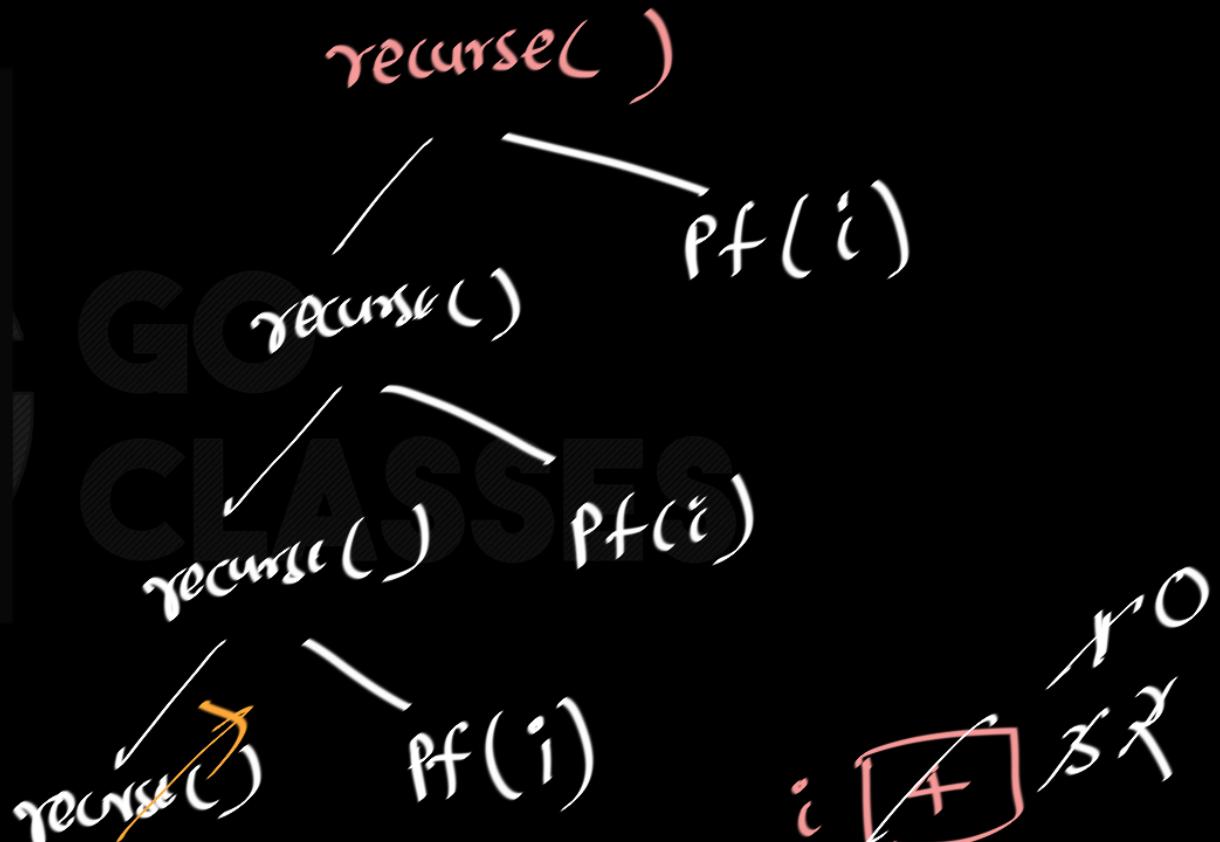
```
void recurse(int i) {  
    printf("%d", i);  
    if (--i) {  
        recurse(i);  
    }  
}
```



## Program 1

```
void recurse() {  
    static int i = 4;  
    if (--i) {  
        recurse();  
        printf("%d", i);  
    }  
}
```

output:  
 0 0 0

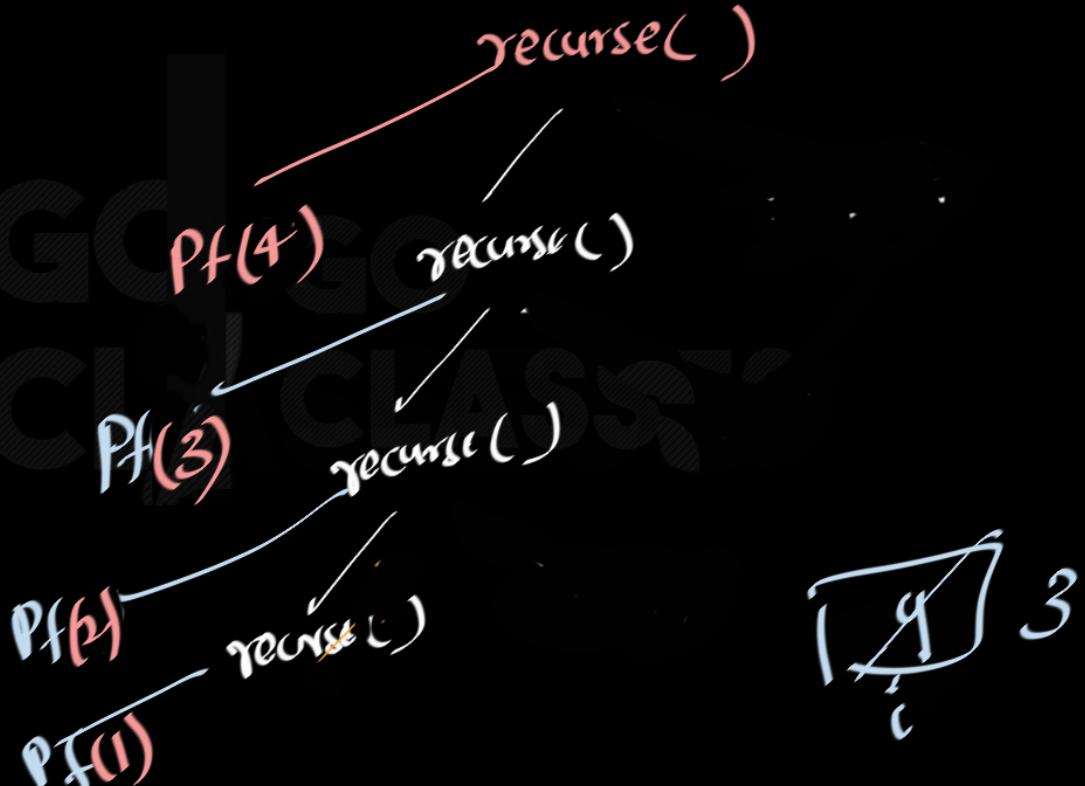




## Program 2

```
void recurse() {  
    static int i = 4;  
    printf("%d", i);  
    if (--i) {  
        recurse();  
    }  
}
```

(4 3 2)

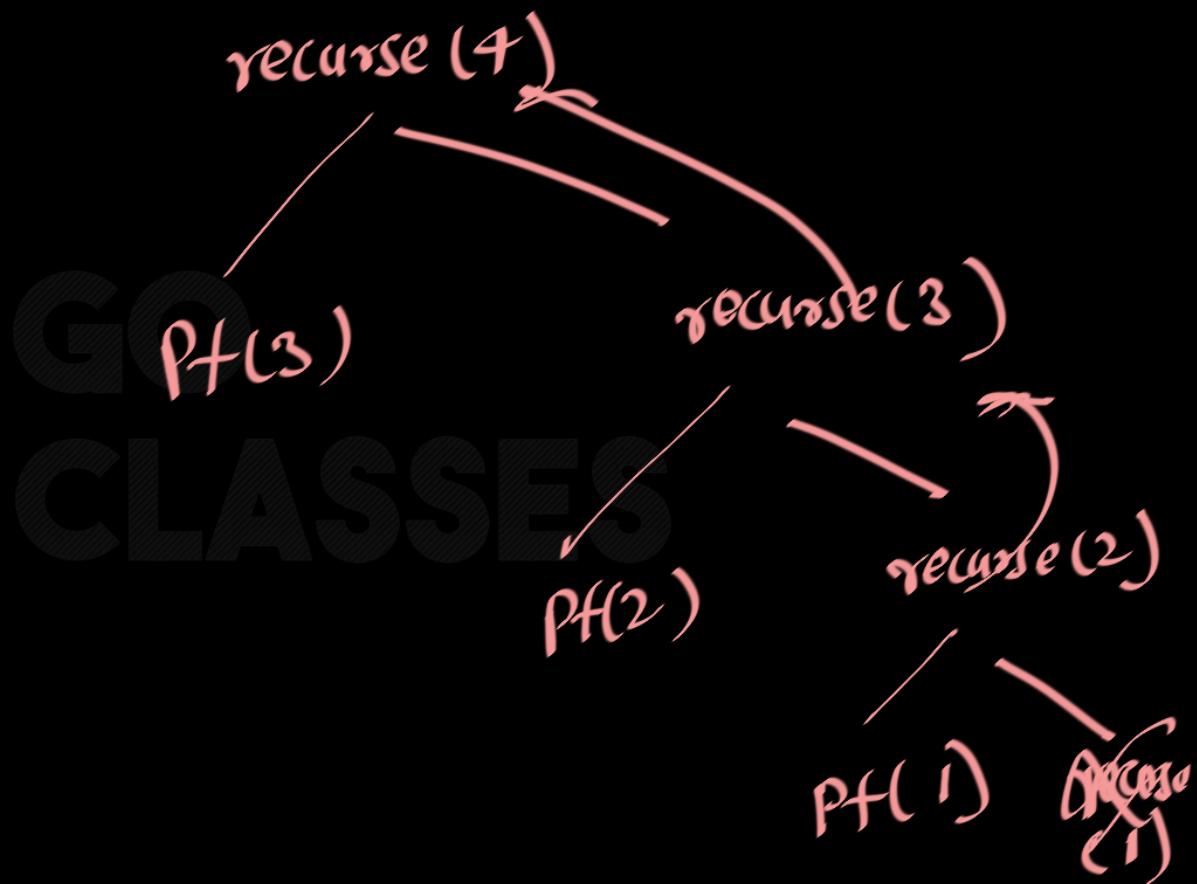




## Program 3

```
void recurse(int i) {  
    if (--i) {  
        printf("%d", i);  
        recurse(i);  
    }  
}
```

3 2 1

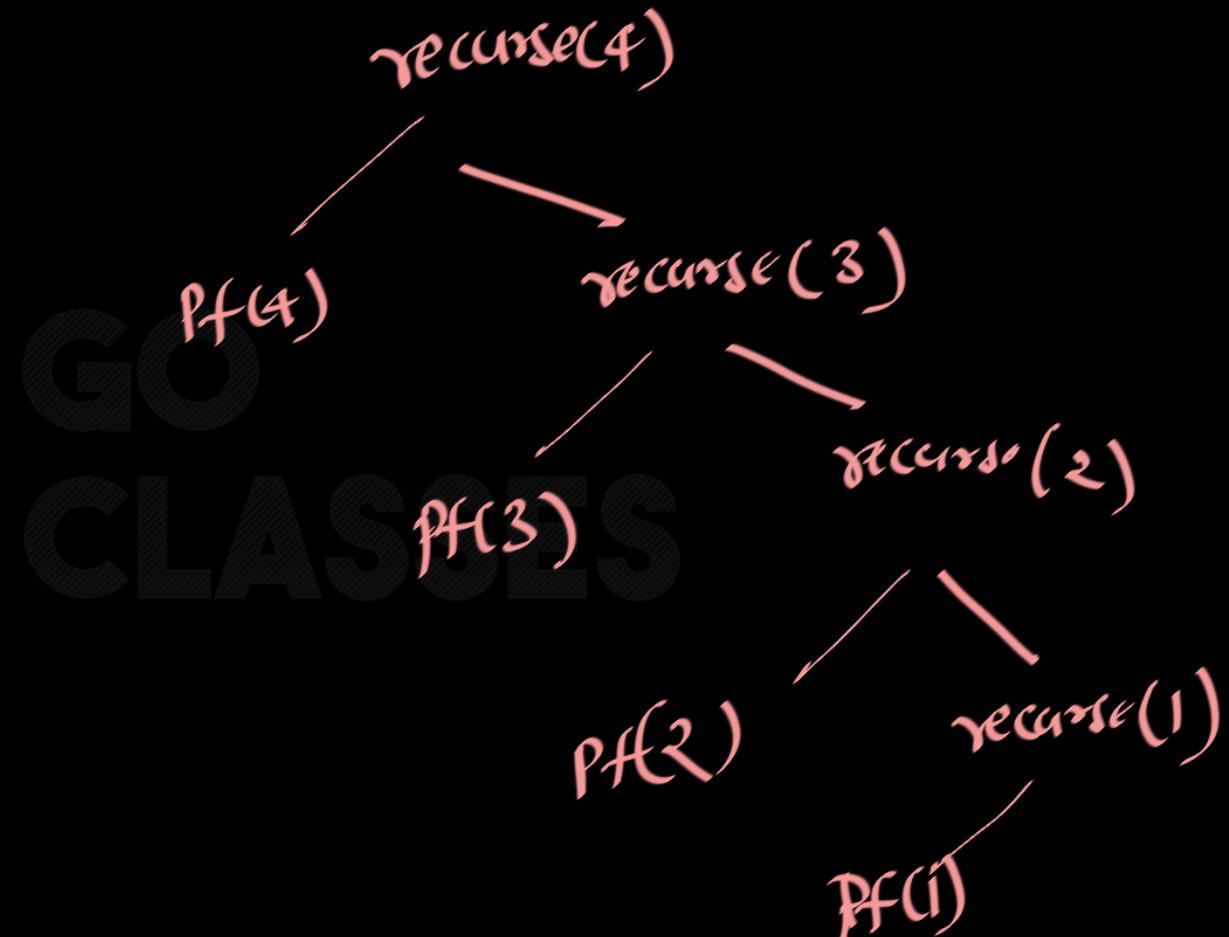




## Program 4

```
void recurse(int i) {  
    printf("%d", i);  
    if (--i) {  
        recurse(i);  
    }  
}
```

4 3 2 1



## Program 1

000

```
void recurse() {  
    static int i = 4;  
    if (--i) {  
        recurse();  
        printf("%d", i);  
    }  
}
```

## Program 2

4321

```
void recurse() {  
    static int i = 4;  
    printf("%d", i);  
    if (--i) {  
        recurse();  
    }  
}
```

## Program 3

321

```
void recurse(int i) {  
    if (--i) {  
        printf("%d", i);  
        recurse(i);  
    }  
}
```

## Program 4

4321

```
void recurse(int i) {  
    printf("%d", i);  
    if (--i) {  
        recurse(i);  
    }  
}
```



## Question 13

```
double power(double x, int n)
{
    if (n == 1)
        return x;
    else
        return x * power(x, n - 1);
    --n;
}
```

```
int main()
{
    int a = power(2, 5);
    printf("%d", a);
}
```

[http://web.mit.edu/10.001/Web/Homework/Solutions/10\\_001\\_F02midterm\\_soln.pdf](http://web.mit.edu/10.001/Web/Homework/Solutions/10_001_F02midterm_soln.pdf)



# C Programming

## Question 14

```
#include <stdio.h>
void func2(int n); // Forward declaration
void func1(int n)
{
    if (n > 0)
    {
        printf("func1: %d\n", n);
        func2(n - 1);
    }
}
void func2(int n)
{
    if (n > 0)
    {
        printf("func2: %d\n", n);
        func1(n - 1);
    }
}
int main()
{
    func1(3);
    return 0;
}
```



# C Programming

## Question 15

```
#include <stdio.h>
void r();
int i = 20;

int main() {
    int i = 5;
    printf("%d ", i);
    r();
    printf("%d ", i);
    r();
    return 0;
}

void r() {
    static int j;
    if (i < 10) {
        i = 3;
        printf("%d ", i);
    }
    printf("%d ", i + j);
    i++;
    j++;
}
```