1 Alternate Solution

Consider a 4-qubit system. Its a 16 dimensional vector space. Let each orthogonal basis vector (E.G. $|0000\rangle$) be associated to each tile on the grid.

We simultaneously use 4 separate 4-qubit systems to solve simultaneously 4 subproblems (A,B,C and D).

A corresponds to finding the positions of the 1s on the grid, B corresponds to finding positions of 2s on the grid etc.

We run the problems simultaneously.

We use a superposition of all vector basis vector as the input to each of the 4 sub prolem solvers.

As step 1: Each of the subproblem solvers then projects on the subspace spanned by the basis vectors that do NOT correspond to the row and column of the first known entry. This gives us then vectors v_{A1}, v_{B1}... made up of basis vectors that give us the "possible places" where 1,2,3,4 could be.

Then take v_{A1} and project it on the orthogonal complement of the subspace spanned by the basis vectors that make up v_{B1}, v_{C1}, v_{D1}. This set of projections will leave a singular basis vector, which will be a guaranteed position of where a 1 could be.

We repeat this for all other numbers and get at least one positions where each ith number is (v_{i1}).

We then project v_{A1} onto the orthogonal complement of the subspace spanned by v_{11} to obtain v_{A2}. We then proceed with the process until we get $v_{i1}, v_{i2}, v_{i3}, v_{i4}$ for all i. These will all guaranteed to be made up of single basis vectors that will be the possible positions of the number i.

We can then encode this in some quantum circuit.