PHY 385: Invesigation of laser polarization via polarizers and waveplates

Meet Chaudhari

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1 Introduction and Theory

Electromagnetic waves have a special property called polarization which contains the details of the orientation of the electric field vector with respect to the transverse plane perpendicular to the wave vector $\vec{k}$. The polarization of the electric field component of the EM wave then is represented by a vector called the polarizing vector. For a fully general electric field in the EM wave of angular frequency ω, wave vector $\vec{k}$, and phase offset ϕ_1:

$$\vec{E}(\vec{r},t) = \begin{pmatrix} E_x \\ E_y \cdot e^{i\Delta\phi} \end{pmatrix} \cdot e^{i(\vec{k}\cdot\vec{r}-\omega t+\phi_1)} \tag{1}$$

the polarizing vector is given by $\vec{P} = \begin{pmatrix} E_x \\ E_y \cdot e^{i\Delta\phi} \end{pmatrix}$ where $E_x, E_y \in \mathbb{R}$. The first component is chosen to be real because only a phase difference between the two independent components matters. This is because a power detector would only record $P \propto |\vec{E}|^2 = \vec{E}^* \cdot \vec{E}$ where an overall phase factor would be reduced to 1.

Now, the phase difference between the two components of the polarizing vector determines the kind of polarization the electric field has. If $\Delta\phi = n\pi, n \in \mathbb{N}$, we have $e^{i\Delta\phi} = \pm 1 \Rightarrow \vec{P} = (E_x, \pm E_y)$. This means the electric field wave is linearly polarized. Instead, if $e^{i\Delta\phi}$ is complex, we get elliptical polarization where the electric field vector's tip traces an ellipse in the transverse planes.

We used linear polarizers and waveplates to study the properties of laser polarization. A linear polarizer is an optic that has a transmission axis angled at say θ such that the component of EM wave aligned along that axis is transmitted whereas the component perpendicular to the transmission axis is removed from the input wave via power loss. Therefore, generally for a polarizer, $|\vec{E}_{out}|^2 \leq |\vec{E}_{in}|^2$. The Jones matrix for such a polarizer is given by[1]:

$$P_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} \cos\theta^2 & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin\theta^2 \end{pmatrix} \tag{2}$$

On the other hand, a waveplate is an optic which adds a phase equal to its retardance ϕ distributed (possibly unevenly) between the components of $\vec{P}$ when an EM wave passes through. The phase added to each component depends on their projection on the axis of the waveplate given by its angle θ. Thus, when $\vec{P}$'s angle is aligned with this axis, it is unaffected. By convention, the second component of $\vec{P}$ (see eq. 3) is given the phase difference. Therefore, the waveplate is used to create a phase difference between the components of the polarizing vector. Importantly, an ideal waveplate preserves the power of the input beam. Thus, $|\vec{E}_{out}|^2 = |\vec{E}_{in}|^2$. The Jones matrix of a waveplate of retardance ϕ oriented at angle θ is:

$$W_\theta(\phi) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{3}$$

When there are multiple polarizers and waveplates in an optical setup, the output electric field, $\vec{E}_{out}$, is calculated by applying the Jones matrices of the the optical components on the electric field. For example, $\vec{E}_{in}$ passing through an arbitary linear polarizer and then a waveplate is transformed to $\vec{E}_{out}$ as follows:

$$\vec{E}_{out} = (W_{\theta_w}(\phi)P_{\theta_p}) \cdot \vec{E}_{in} \tag{4}$$

[1] Astronomy, BYU Physics and. "Optics Textbook." Optics Textbook, optics.byu.edu/textbook.

The predictions of experimental results by using Jones matrices were verified in the series of experiments conducted. This was done by taking multiple polarizers and different waveplates to create optical system whose Jones matrices could be calculated. This could then be compared to experimental data produced by altering say the orientation of the polarizers and waveplates. The corrresponding procedure and method is described below.

2 Procedure

2.1 Materials/Apparatus Used

For polarizers and waveplates, I used two economy linear polarizers[2] by *Thorlabs* of unknown transmission axes and 2 *newlight* waveplates[3] of retardance $\phi_1 = \frac{\pi}{2}$ (quarter wave plate) and $\phi_2 = \pi$ (half wave plate) operating at 532 nm. The light source was *Thor Labs'* Helium Neon laser operating at 0.8 mW 632.8 nm [4]. We also used a photo detector by *Thor labs*[5] to record power of the EM wave in terms of voltages and a *Keysight* multimeter to observe the voltage recorded. In addition, we used Neutral Density Filters of OD: 0.7 and 2.0 to prevent saturation of the photo diode. We also used a round silver mirror was also used to direct the laser into the setup. Finally, we used an optical breadboard, rotatory mounts, and posts to hold the above components in place.

2.2 Setups and Method

2.2.1 2 Linear Polarizers

Before setting up the experiment, I handled the polarizer to check out its properties. By rotating one of the polarizers, I first observed the surrounding light sources such as digital screens of various colors and ceiling lamps. I was able to observe a change in color of white computer screen to a shade of violet. Contrarily, I saw a distinct circular rainbow spectrum on a white phone screen. By rotating the polarzier, I noted that the intensity of light changed periodically while never quite reaching zero intensity. However, no special observations were made when looking at ceiling lamps. This indicates that the polarization of light from various common sources of light are quite different. The ceiling lamps seem to emit completely unpolarized light whereas that emitted by digital screens seem quite polarized.

Now, my group[6] and I wanted to record how changing the orientation of one polarizer with respect to another of fixed orientation changes the intensity of the output light. We wanted the maximum intensity of laser power input through the first polarizer of fixed but unknown transmission axis in the experiment to not record very small voltages, which would be indistinguishable from noise. Since we knew that the laser was linearly polarized, we created a setup with just the first polarizer P_1 (see figure 1) where it was mounted in a rotatory mount. The photo detector's output was read via a multimeter connected to it. We then started rotating the first polarizer and quickly found that the photo diode was getting saturated at a voltage reading of 12.3 V. Thus, we added 2 Neutral Density Filters with their total OD = 2.7 in front of the photo diode which would lower the intensity of light that reaches the sensor in the detector. Then by trial and and error, we found the orientation that allows maximum intensity of light to go through. It corresponded to about 5.2 ± 0.2 V. This polarizer then should be aligned with the polarizing vector of the input beam and we recorded the rotatory mount's reading from the top of about $(268 \pm 1)^\circ$ for reference. Then, we added another polarizer P_2 (see figure 1) between the photo diode and the first polarizer P_1. For convention, we rotated the second polarizer till we had the maximum possible intensity recorded in the multimeter, which was about (4.2 ± 0.2) V. Then we rotated the second polarizer in 10° increments to cover a range of 180° from the initial arrangement and recorded the voltage and angle measurements of the second polarizer. Because the voltage values were constantly fluctuating, we waited for them to go through the range of values they could take and recorded the average. The plot of the data and corresponding calculations are shown in the results section. We noted that the least possible

[2] "Economy Film Polarizers With Windows." Economy Film Polarizers With Windows, www.thorlabs.com.

[3] "Single Plate Low-Order Waveplates". Newlightphotonics.Com, 2023, https://www.newlightphotonics.com/Waveplates/Single-Plate-Low-Order-Waveplates. Accessed 15 Feb 2023.

[4] "HeNe Lasers: Red." HeNe Lasers: Red, www.thorlabs.com.

[5] "Thorlabs - DET36A2 Si Detector, 350 - 1100 Nm, 14 Ns Rise Time, 13 Mm2, Universal 8-32 / M4 Mounting Holes." Thorlabs - DET36A2 Si Detector, 350 - 1100 Nm, 14 Ns Rise Time, 13 Mm2, Universal 8-32 / M4 Mounting Holes, www.thorlabs.com.

[6] Alex Zheng, Meet Chaudhari, and Shatho

intensity corresponded to about 0.01 V and the maximum corresponded to 4.2 ± 0.2 V. We recorded the positions of the rotatory mounts for these values of voltages for the next experiment.

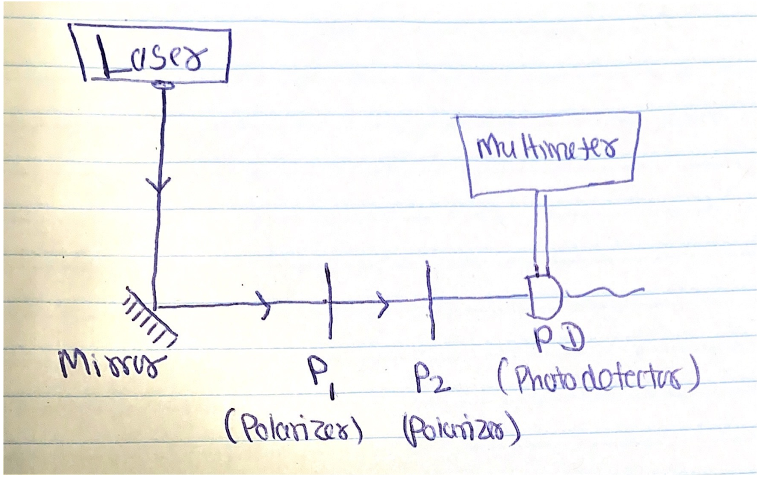

Figure 1: Schematic of experiment for investigating changes in light intensity with 2 polarizers. Initially only the polarizer P_1 is placed and aligned with the polarization of the light from laser. Then the second polarizer P_2 is added and rotated to record its orientation vs the voltage measured through the multimeter. To prevent oversaturation, 2 ND filters are placed with a total OD = 2.7

2.2.2 2 Polarizers and waveplates

From the previous experiment, we fixed the positions of the rotatory mounts of both polarizers such that the minimum intensity of light was passing through and kept the exact same setup otherwise. Then, we added a quarter wave plate ($\phi_1 = \frac{\pi}{2}$) between the two polarizers by attaching it to a rotatory mount and fixing it on a post (see figure 2). Again, we did not know the characteristic angle θ corresponding to the waveplate because it was just a circular piece of optic that was fitted into the mount. So, we used the angle markings in the rotatory mount again. We then rotated the waveplate's rotatory mount in increments of 10° from 0° to 180° and recorded the corresponding averaged voltmeter readings. Note that these angle readings are for the outer rotatroy mount and may not correspond to the angle θ in the Jones matrix in eq. 3. We then replaced the quarter wave plate with a half wave plate ($\phi_2 = \pi$) and repeated the same procedure for data collection. The corresponding calculations and plots are shown in the results section below.

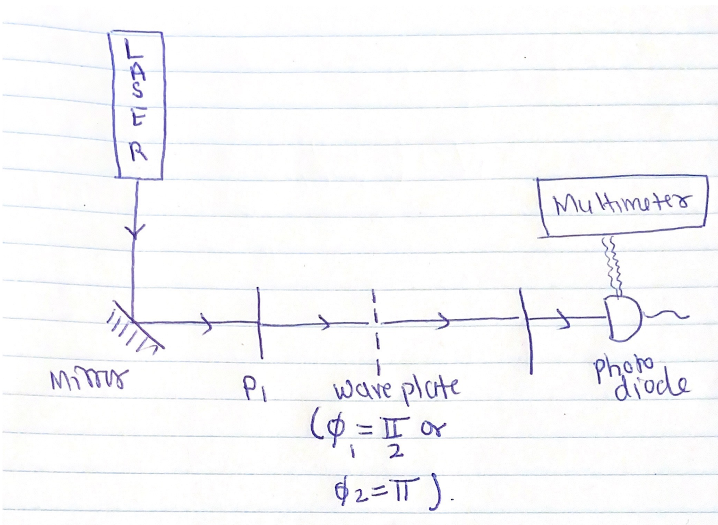

Figure 2: Schematic of experiment for investigating changes in light intensity with 2 polarizers and a waveplate in between. The two polarizers are setup to allow the minimum intensity of light to reach the photo diode. The waveplate between them is rotated to record data of voltage readings from multimeter and the orientation of the rotatory mount.

3 Results and Analysis

3.1 2 Polarizers Experiment

We had polarizers without their transmission axes marked, so we recorded the positions of the rotatory mount of the second polarizer (P_2). We had aligned the first polarizer exactly with the polarizing vector of the laser and then found that the rotatroy mount angle / position of $(110 \pm 1)^\circ$ made the most amount of laser intensity reach the photo diode which was around $(4.2 \pm 0.2)V$. This means that if we define the fixed angle of the first polarizer P_1 to be 0° in some new basis, then the angle $(110 \pm 1)^\circ$ of the second polarizer is also equivalent to 0° in that new basis. Thus, the orientation of the second polarizer relative to the first polarizer in the new basis is given by $\theta_2 = \theta_{2,R} - 110^\circ$, where $\theta_{2,R}$ is the angle of the rotatory mount of the second polarizer we recorded in the experiment. Also, in the new basis, $\vec{E}_{in}$ only has one component $E_x\vec{e}_1$ with the other being $0\vec{e}_2$ because it is aligned with the first polarizer. Then, using Jones matrices for both polarizers, where the first is at $\theta_1 = 0$ and the other is at θ_2, we get:

$$\vec{E}_{out} = P_{\theta_2} P_{0^\circ} \vec{E}_{in} \tag{5}$$

$$\vec{E}_{out} = \begin{pmatrix} \cos{\theta_2}^2 & \sin\theta_2\cos\theta_2 \\ \sin\theta_2\cos\theta_2 & \sin{\theta_2}^2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} E_x \\ 0 \end{pmatrix} \tag{6}$$

$$= \begin{pmatrix} \cos{\theta_2}^2 & 0 \\ \sin\theta_2\cos\theta_2 & 0 \end{pmatrix} \begin{pmatrix} E_x \\ 0 \end{pmatrix} = E_x \begin{pmatrix} \cos{\theta_2}^2 \\ \sin\theta_2\cos\theta_2 \end{pmatrix}$$

Since the photo diode measures power, where power $\propto |\vec{E}_{out}|^2$, using eq. 6, we get:

$$\text{power} \propto |E_x|^2(\cos{\theta_2}^4 + \sin{\theta_2}^2\cos{\theta_2}^2) = |E_x|^2\cos{\theta_2}^2(\cos{\theta_2}^2 + \sin{\theta_2}^2) \tag{7}$$

$$= |E_x|^2\cos{\theta_2}^2$$

Since it is assumed that the power of the laser incident on the photo diode is proportional to its voltage output that we measured, we should anticipate a $\cos{\theta_2}^2$ relationship in the collected data. The uncertainty in the angle was calculated by propagating the uncertainty in radians with the least count of 1° in the rotatory mount by $\Delta\theta_2$ (in rad) $= \frac{\pi}{180} \cdot 1$ rad. The uncertainty in the voltmeter was taken to be 0.2 V because although the voltage fluctuated in a larger range, we thought that a major contribution to the voltage fluctuations was due to other light sources. Also, in some cases where the voltage stabilized, it showed fluctuations of around 0.2 V. The data was plotted and a line of best fit was fitted to it. These were plotted using python's matplotlib, scipy, and numpy libraries and the coding was a collective group effort. The results are shown below in figure 3.

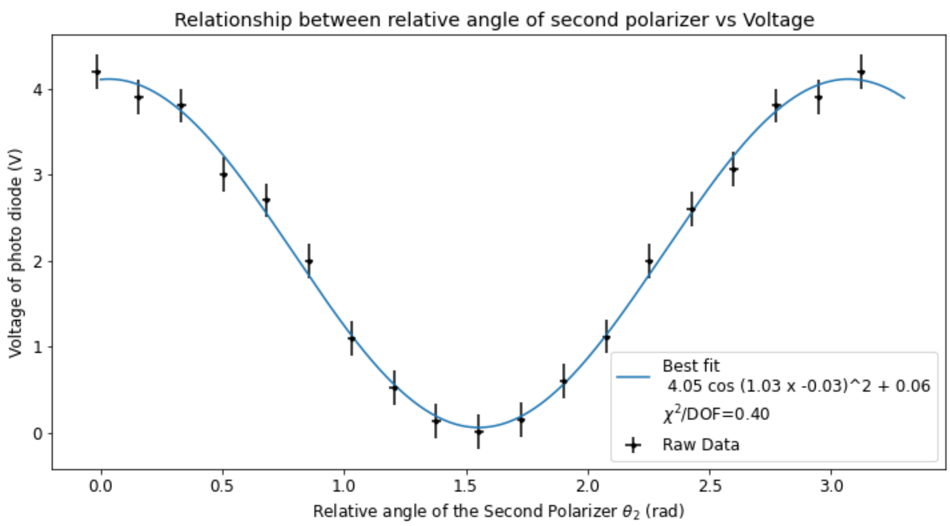

Figure 3: Graph of voltage (in V) recorded by a photodiode versus the relative angle (in rad) between two linear polarizers. The line of best fit is shown in the graph above which is a $|E_x|^2 \cdot \cos{\theta_2}^2$ graph given the very negligible phase and base offsets. This agrees with the calculation using Jones matrices that power detected $\propto |E_x|^2 \cdot \cos{\theta_2}^2$

Figure 3 clearly demonstrates the relationship derived in equation 7, wherer we can see that the data follows a $|E_x|^2 \cdot \cos\theta_2{}^2$ trend with a peak of $(4.2 \pm 0.2)V$ which was noted in the initial adjustments. Further, the minimum point of the plots is at around $\theta_2 \approx \frac{\pi}{2}$ which is what we expect as according to equation 7, power = 0 there. We can see that the plot doesn't quite reach 0 V. This is because there were other light sources in the room where the experiment was conducted which would have served as a constant background for the photodiode. Furthermore, there were reflections from the mirror and the polarizers in front of the photo diode which could be depolarized and re-enter the system again to contribute to the intensity. Moreover, the reduced chi squared for the fit was $\chi^2_{red} = 0.40$ which shows that the curve has been overfitted. But this was anticipated as we collected around 20 measurements. Overall, these results agree very well with the theory.

3.2 2 Polarizers and Waveplate Experiment

In the experiment, the first polarizer is oriented at 0° while the second one is oriented at $90° = \frac{\pi}{2}$ rad. Thus, for an arbitary waveplate between the two polarizers in the experiment, the Jones matrix of the whole setup is given by:

$$O = P_{90^\circ} W_\theta(\phi) P_{0^\circ} \tag{8}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} cos\theta & -sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & e^{i\phi} \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0.5\sin 2\theta(1 - e^{i\phi}) & 0 \end{pmatrix}$$

Thus, the output beam from the optical setup given the input beam $\vec{E}_{in} = \begin{pmatrix} E_x \\ 0 \end{pmatrix}$ is:

$$\vec{E}_{out} = O \cdot \vec{E}_{in} = E_x \begin{pmatrix} 0 \\ 0.5\sin 2\theta(1 - e^{i\phi}) \end{pmatrix} \tag{9}$$

Thus, using equation 9, the power detected in this case would be given by:

$$\text{power} \propto |\vec{E}_{out}|^2 = |E_x|^2 \cdot 0.25 \sin 2\theta^2 (2 - 2\cos\phi) \tag{10}$$

$$= \frac{|E_x|^2 (1 - \cos\phi)}{2} \sin 2\theta^2$$

Therefore, for a **quarter waveplate** ($\phi_1 = \frac{\pi}{4}$), we have

$$\text{power} \propto \frac{|E_x|^2(\sqrt{2}) - 1}{2\sqrt{2}} \sin 2\theta^2 \approx 0.15 |E_x|^2 \sin 2\theta^2 < |\vec{E_x}|^2 \tag{11}$$

This indicates that no matter what angle θ we orient the quarter wave plate, we cannot get to make the whole optical system transparent as we were able to with the 2 polarizers in previous experiment (with $\theta_2 = n\pi$). We also verified this experimentally and the fact that the power deposited or voltage behaves like $\sin^2 2\theta$ in the figure 4. We can see that the voltage peak is less than 3.5 V whereas the voltage corresponding to transparent system is given by about 4.2 ± 0.2 V as measured in the polarization experiment (see figure 1). The data points were fitted with a line of best fit of form $y = A\sin 2\theta + \alpha^2 +$ base using python plotting libraries. The line of best fit was calculated as

$$y = (3.25 \sin^2 (2.0 \cdot \theta + 2.02) + 0.04) \text{ V} \tag{12}$$

which has the required factor of 2 in the frequency term as eq. 11 predicts. There is also a phase factor of $\alpha = 2.02$ which is expected because the angle measurements used to calculate the fit were of the mount's and not the waveplate itself. The reduced chi-squared value for this fit was calculated as $\chi^2_{red} = 0.14$ which I think shows how good the fit is even if it indicates that the data has been overfitted.

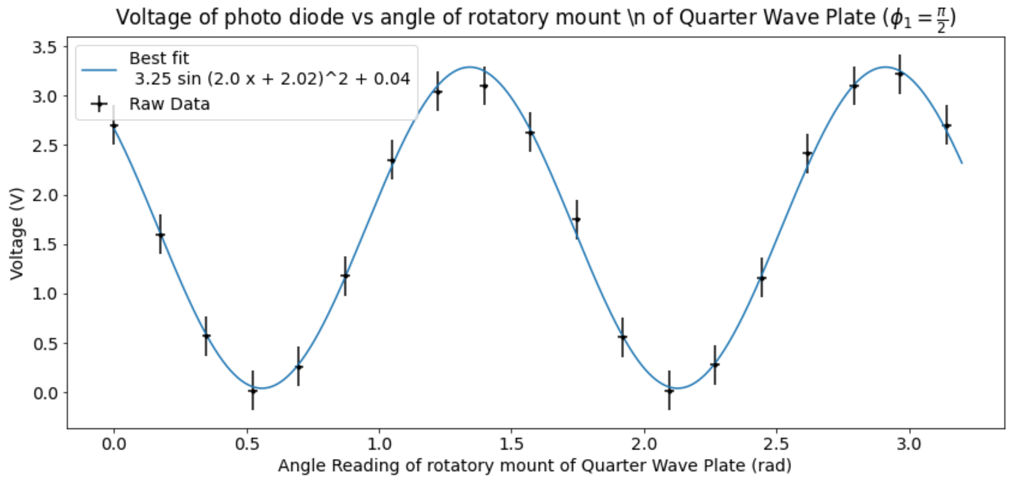

Figure 4: Graph of voltage (in V) recorded by a photodiode versus the angle (in rad) of the rotatory mount in which a quarter wave plate ($\phi_1 = \frac{\pi}{2}$) is mounted. The line of best fit is shown in the graph above which is of form a $\sin^2 2\theta$ but with a phase shift because the angle on the x-axis of the plot is that of the mount not the waveplate itself as in the equation. The fit gives very negligible base offset and has a frequency of 2.0. This agrees with the calculation using Jones matrices that power detected for a quarter wave plate is $\propto 0.15|E_x|^2 \sin^2 2\theta$

Doing similar analysis for **half wave plate** ($\phi_2 = \pi$), we can plug $\phi = \pi$ in eq. 10 to get that the power detected by the photodiode when a half plate is introduced is:

$$\text{power} \propto |E_x|^2 \sin 2\theta^2 \tag{13}$$

Here, unlike the case of quarter wave plate, if $\sin 2\theta^2 = 1$, then power $\propto |E_x|^2$ which means that the entire system can be made transparent by setting the half wave plate to an angle $\theta = (2k+1)\frac{\pi}{4}, k \in \mathbb{N}$. Experimentally, this is what we observed and is seen in figure 5 where the peak of the voltage vs angle of rotator curve reaches the maximum voltage of $(4.2 \pm 0.2)V$ as was possible for the 2 polarizers. Also, a best fit line was fitted to the data points, which has the equation:

$$y = y = (4.05 \sin^2 (2.03 \cdot \theta + 1.90) + 0.06) \text{ V} \tag{14}$$

which has the desired features of the frequency of the curve being $2.03 \approx 2.0$ and a negligible base offset of the sinusoid $0.06V \approx 0V$.

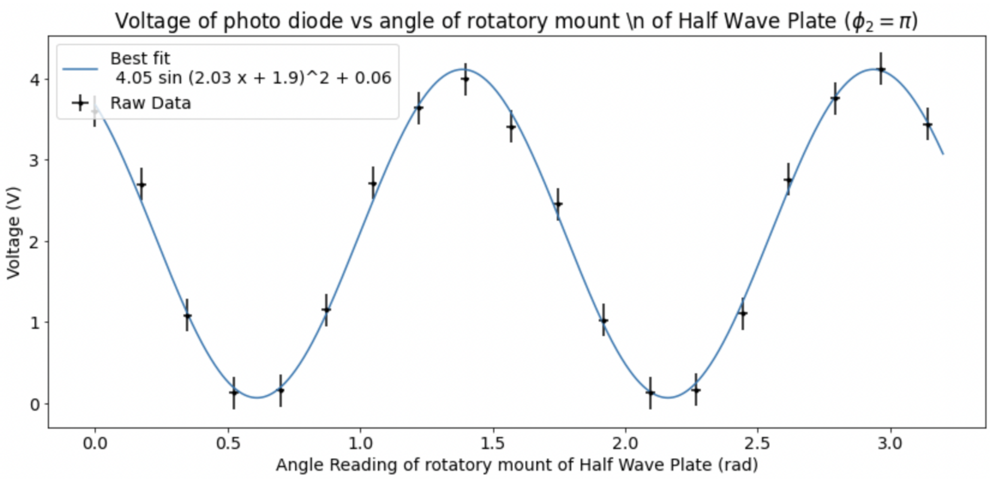

Figure 5: Graph of voltage (in V) recorded by a photodiode versus the angle (in rad) of the rotatory mount in which a half wave plate ($\phi_2 = \pi$) is mounted. The line of best fit is shown in the graph above which is of form a $\sin 622\theta$ but with a phase shift because the angle on the x-axis of the plot is that of the mount not the wave plate itself as in the equation. The fit gives very negligible base offset and has a frequency of 2.03. This agrees with the calculation using Jones matrices that power detected for a quarter wave plate is $\propto |E_x|^2 \sin^2 2\theta$

The chi-squared fit value for this fit was again $\chi^2_{red} = 0.36$ which in my interpretation shows that the data collected fits the proposed model very well. This is because the setup was very sensitive and hence we were able to make very accurate measurements.

Conclusion

In a series of experiment, my group and I investigated many properties of polarizers, waveplates, and theb behavior of polarised light from a Helium Neon Laser. The experiemntal results matched incredibly well with the predictions made through the Jones matrcies for polarizers and waveplates. We observed how a half wave plate can make an optical setup consisting of orthagonally arranged polarizers transparent while a quarter wave plate cannot. The fits matched really well with the data and hence led to reduced chi squared values than 1 which was expected given the precision of the setup.

References

1. Astronomy, BYU Physics and. “Optics Textbook.” Optics Textbook, optics.byu.edu/textbook.