PHY 224: Internal Resistance of Cell Battery and DC Voltage source

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Abstract

The purpose of the experiment was to measure the internal resistance of 2 power sources: a cell battery and an external DC voltage source. We found that one particular circuit construction is better for the cell battery while all circuit constrictions seem to give appropriate relationships for the DC regulated power supply.

1 Introduction

The objective of the report is to investigate the current voltage relationship of 2 power sources, a cell battery and an external DC source, when an external resistance is connected to them to draw power. When no external voltage source is connected to a power source, a condition called open circuit, one can measure the open circuit voltage V_∞ across its terminals. But, when an external load, like a resistor, draws current from the voltage source, the voltage across the source decreases. This is caused by voltage drops across the components of the voltage source itself. Now, *Thevenin's Theorem* states that "any linear circuit containing several voltages and resistances can be replaced by just one single voltage in series with a single resistance connected across the load."[1]

Assuming the circuit inside the voltage source to be linear, the internal components could be represented by a single resistor of resistance R_{int} (see figure 1), which can vary with the magnitude of the current.

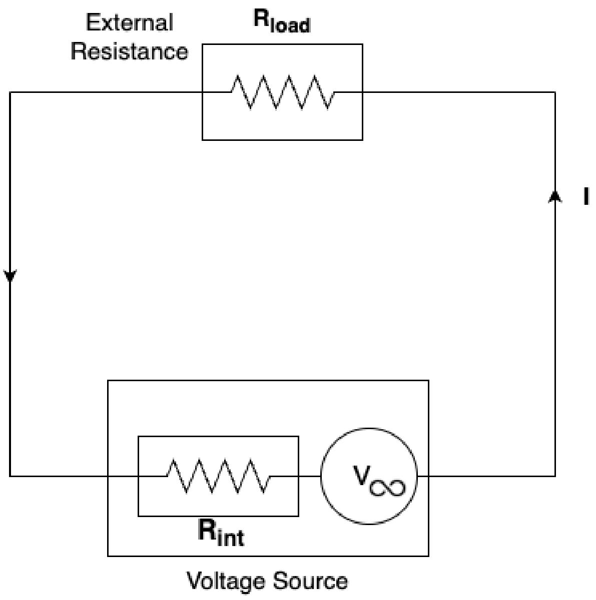

Figure 1: Current (in mA) vs voltage (in V) across the bulb with uncertainties.

[1]Storr, Wayne. "Thevenins Theorem Tutorial for DC Circuits." Basic Electronics Tutorials, 6 Aug. 2022, https://www.electronics-tutorials.ws/dccircuits/dcp$_7$*.html*.

Since this simplifies the circuit to a series connection between the internal resistor and the external resistor, we can apply Ohm's law, $V = I \cdot R$, to the whole circuit. This yields us:

$$V = V_{\infty} - I \cdot R_{int} \tag{1}$$

where V is the voltage across the power source and I the current in the circuit.

By attaching different external resistors to a voltage source and recording the voltage across the voltage source and the current in the circuit, we can calculate the internal resistance R_{int} of the source. Also, since the ammeters and multimeters are not ideal, we would use two different experimental setups to make measurements for each of the voltage sources. This would allow us to determine which method gives better results. Further, we would be able to combine data from both setups to create a a more accurate data set of voltage current values. This would lead to a more accurate calculation of internal resistances.

2 Procedure

2.1 Material Required

To conduct both experiments, we needed a cell battery, an external DC voltage source, a voltmeter, an ammeter, 5 different resistors, and connecting wires. Therefore, we used EnerSys LEAD-6-10 Lead Acid Battery-6V, Keysight's E36103B DC power supply for external voltage, 2 Keysight U1272 "True RMS" multimeters (one acting as voltmeter and other as an ammeter), Pomona 36 wires for connecting the components, and an electrical board consisting of resistors

2.2 Method

First, we decided that we would choose resistors of resistances 100 Ω, 220 Ω, 470 Ω, $2.7k$ Ω, $27k$ Ω, and $100k$ Ω from the component board as the external loads to measure the current-voltage relationship for both the voltage sources. This decision was based on the high range of the resistances available. We could vary the current in the circuits significantly by altering R_{load} from 100 Ω to $100k$ Ω. Although there were potentiometers available, we would have required to know the resistance corresponding to the knob positon. Thus, we agreed to just use the 6 component resistors. Secondly, we guessed that the open circuit voltage of the cell battery, V_{∞}^{C}, would be approximately around 6 V as the battery was explicitly rated 6 V. We confirmed this by measuring V_{∞}^{C} before each V-I measurement was done on the battery (see table A2 in appendix). The average value was found as $\mathbf{V_{\infty}^{C} = 6.45\ V}$. Since we wanted to compare the magnitudes of the internal resistances of both the voltage sources, we decided to set the open circuit voltage of the Keysight DC voltage source explicitly at 6.5 V i.e. $\mathbf{V_{\infty}^{K} = 6.5\ V}$.

Note that we call the multimeter used for measuring current as "ammeter" and the one used for measuring voltage as "voltmeter". As can be seen in figure 2, there are 2 possible ports to connect the ammeter (the 2 ports on red) in addition to the COM port for grounding. Since we expected the current I to be in the mA range, we decided to use the right port labelled μ A as it gave a precision of 10^{-3} mA in the 30 mA range whereas the left port gave a precision of 10^{-1} mA.

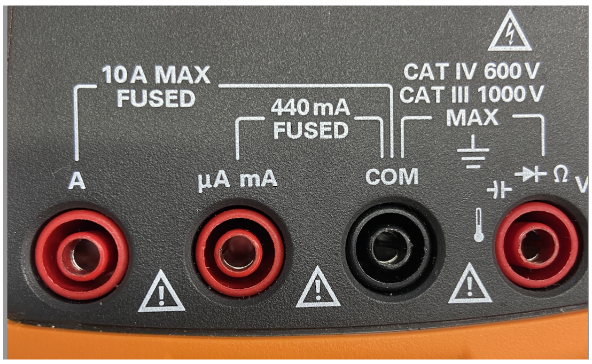

Figure 2: Ports of Ammeter. Left most port gives reading in Amps with precision of 10^{-1} mA while the one beside it gives reading with a precision of 10^{-3} mA

Then, we had two options of connecting the ammeter and voltmeter in the circuit as shown in Figure 3. Option 1 assumes that the voltmeter is perfect, i.e. no current passes through it. This means that we can place the Ammeter directly in series with the load resistor and assume that the measured current is the same throughout the circuit. Meanwhile, the ammeter is imperfect i.e. has a non-negligible resistance which creates a voltage drop worth accounting for. On the other hand, option 2 assumes that the ammeter is perfect, which means there is essentially no voltage drop to be measured across it. Whereas, it also assumes that the voltmeter might be imperfect as some current might pass through it which must be measured by the ammeter outside the parallel sub-circuit to measure the total current I in the circuit. So, given a non-ideal multimeter, we could either use it as an ammeter in the option 1 setup or as the voltmeter in option 2 setup by the above arguments. This assumes that the other multimeter is perfect.

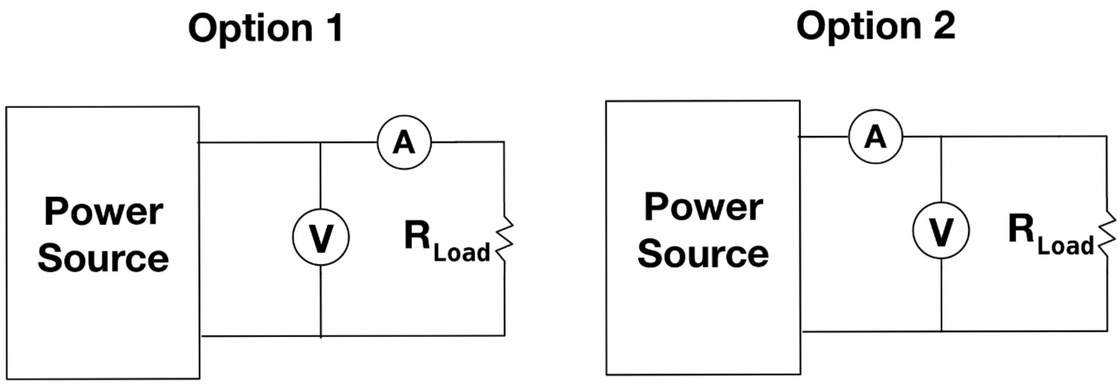

Figure 3: Two possible circuit setups to conduct the experiment. Option 1 assumes the ammeter is not perfect, has non-negligible resistance, and voltmeter is perfect, has very high resistance and vice-versa for option 2. Image courtesy of PHY-224 instructors

Since we didn't know whether any of the multimeters was perfect, both the options seemed equally valid initially. So, we decided to employ both circuit constructions for the cell battery and external DC source. We first started the experiment with the cell battery. Since the cell battery's emf or open circuit voltage seemed a bit volatile, we decided to measure it before each trial in both circuit structures. Then, we connected the cell battery to a 100 Ω resistor in the option 1 circuit at the place of the power source (see figure 3). We noticed that the voltmeter reading was fluctuating leading to a fluctuating ammeter reading. Thus, we decided to instantaneously take pictures of the voltmeter and ammeter after completing the circuit to both once their values seemed to stabilize for a while. This helped us to get both the proper readings without the resistors heating up or showing any non-linear behavior. Then, we disconnected the circuit and recorded the voltmeter and current reading in an excel file. Next, we measured V_{∞}^{C} again and repeated the experiment with other resistors. We repeated the same procedure for option 2 experiments.

Finally, we repeated essentially the same procedure as above for both the circuit designs (see figure 3) with the Keysight DC power source. But this time we didn't keep measuring the open circuit voltage because the power source was good enough that it remained constant through trials. So, we first set the first circuit arrangement, and then took measurements for the 5 external load resistors with the emfs as 6.5V, 10V, 15V, and 20V. Then, we did the same for 2nd circuit design. The data is shown in the tables in the results section.

3 Results

The uncertainties in the potential difference and the current were calculated using the Keysight U1270 Multimeter manual (see appendix for sample calculations). Then, all the V vs I data sets were plotted.

The following figures are error bar plots of Tables 1-10 (see appendix A). Along with each plot, we have the measured value of V_∞, R_{int}, and I_{max}.

$$V_\infty = 6.442 \pm 0.003 \text{ V}, R_{int} = 0.27 \pm 0.09 \ \Omega, I_{max} = 24 \pm 8 \text{ A}$$

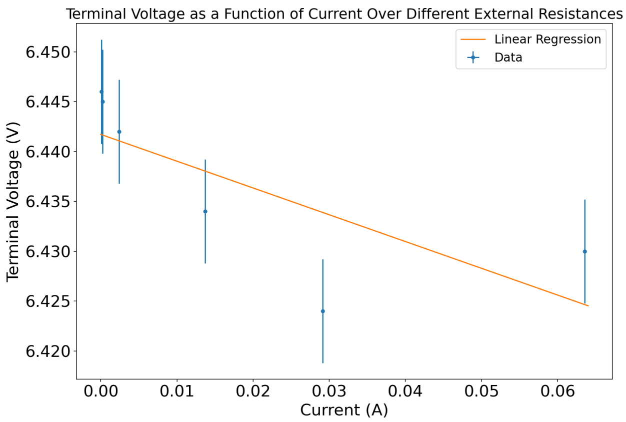

Figure 4: Terminal Voltage as a Function of Current Over Different External Resistances for the Cell Battery using Option 1 when $V_\infty = 6.5$ V

$$V_\infty = 6.451 \pm 0.003 \text{ V}, R_{int} = 1.87 \pm 0.09 \ \Omega, I_{max} = 3.5 \pm 0.2 \text{ A}$$

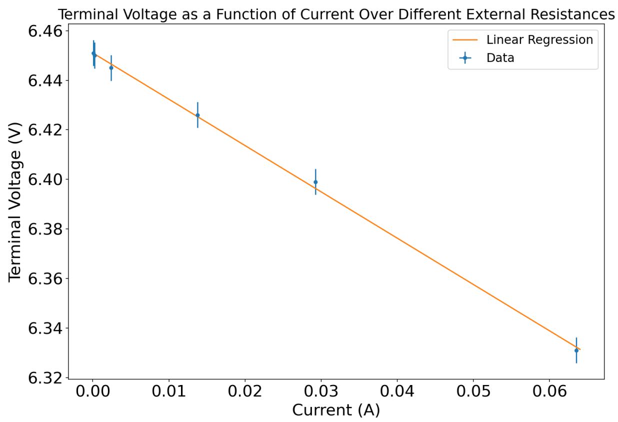

Figure 5: Terminal Voltage as a Function of Current Over Different External Resistances for the Cell Battery using Option 2 when $V_\infty = 6.5$ V

$$V_\infty = 6.502 \pm 0.003 \text{ V}, R_{int} = 0.05 \pm 0.09 \ \Omega, I_{max} = 100 \pm 300 \text{ A}$$

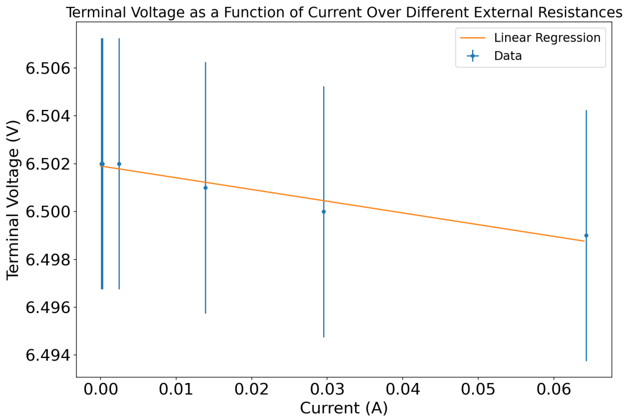

Figure 6: Terminal Voltage as a Function of Current Over Different External Resistances for the DC Power Supply using Option 1 when $V_\infty = 6.5$ V

$$V_\infty = 6.502 \pm 0.003 \text{ V}, R_{int} = 1.49 \pm 0.09 \ \Omega, I_{max} = 4.4 \pm 0.3 \text{ A}$$

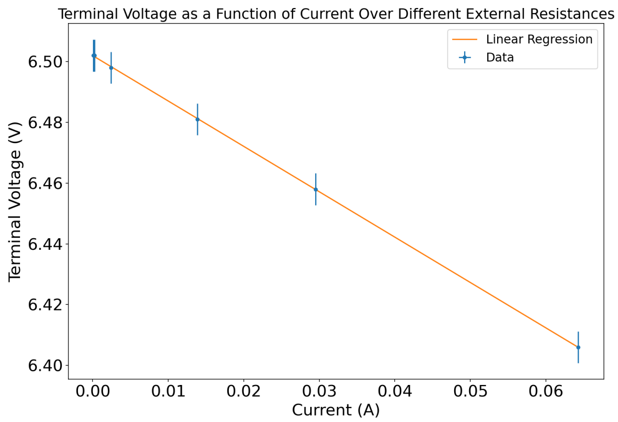

Figure 7: Terminal Voltage as a Function of Current Over Different External Resistances for the DC Power Supply using Option 2 when $V_\infty = 6.5$ V

$$V_\infty = 10.002 \pm 0.004 \text{ V}, R_{int} = 0.05 \pm 0.08 \ \Omega, I_{max} = 200 \pm 400 \text{ A}$$

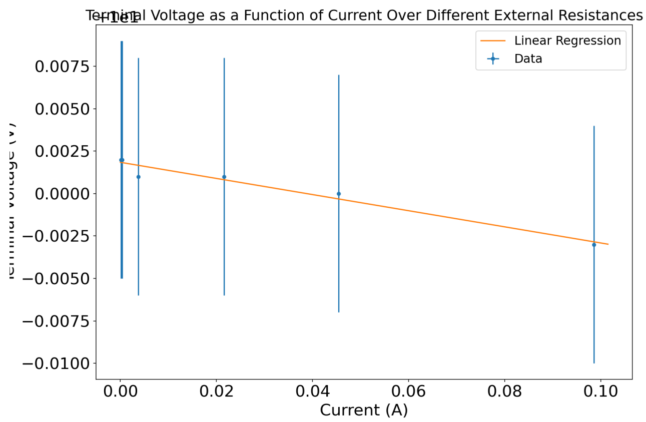

Figure 8: Terminal Voltage as a Function of Current Over Different External Resistances for the DC Power Supply using Option 1 when $V_\infty = 10$ V

$$V_\infty = 10.002 \pm 0.004 \text{ V}, R_{int} = 1.50 \pm 0.08\ \Omega, I_{max} = 6.7 \pm 0.4 \text{ A}$$

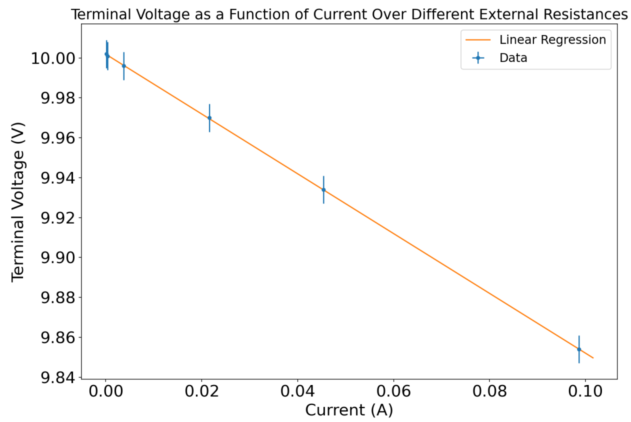

Figure 9: Terminal Voltage as a Function of Current Over Different External Resistances for the DC Power Supply using Option 2 when $V_\infty = 10$ V

$$V_\infty = 15.003 \pm 0.005 \text{ V}, R_{int} = 0.04 \pm 0.07\ \Omega, I_{max} = 300 \pm 600 \text{ A}$$

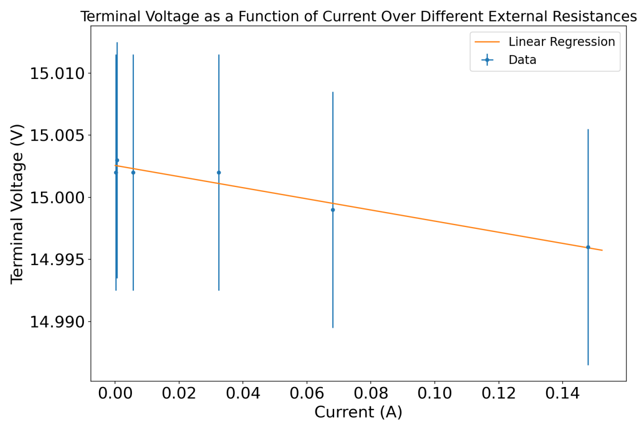

Figure 10: Terminal Voltage as a Function of Current Over Different External Resistances for the DC Power Supply using Option 1 when $V_\infty = 15$ V

$$V_\infty = 15.002 \pm 0.005 \text{ V}, R_{int} = 1.50 \pm 0.07 \ \Omega, I_{max} = 10.0 \pm 0.5 \text{ A}$$

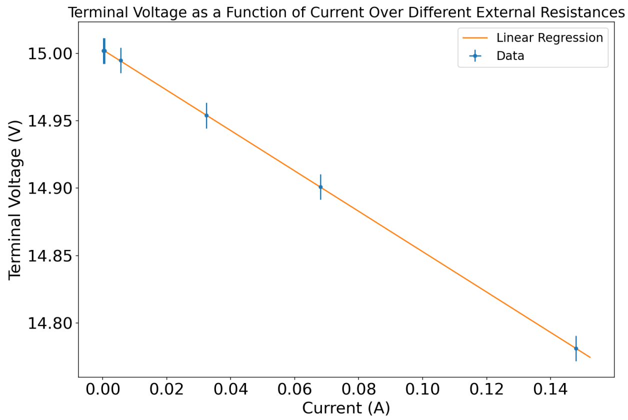

Figure 11: Terminal Voltage as a Function of Current Over Different External Resistances for the DC Power Supply using Option 2 when $V_\infty = 15$ V

$$V_\infty = 20.004 \pm 0.065 \text{ V}, R_{int} = 0.04 \pm 0.07 \ \Omega, I_{max} = 400 \pm 700 \text{ A}$$

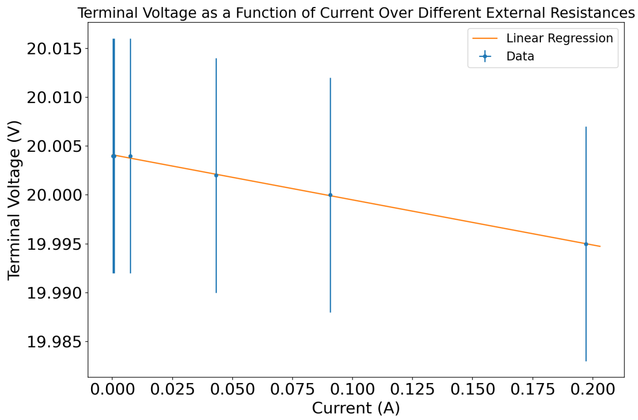

Figure 12: Terminal Voltage as a Function of Current Over Different External Resistances for the DC Power Supply using Option 1 when $V_\infty = 20$ V

$$V_\infty = 20.003 \pm 0.006 \text{ V}, R_{int} = 1.50 \pm 0.07 \ \Omega, I_{max} = 13.4 \pm 0.6 \text{ A}$$

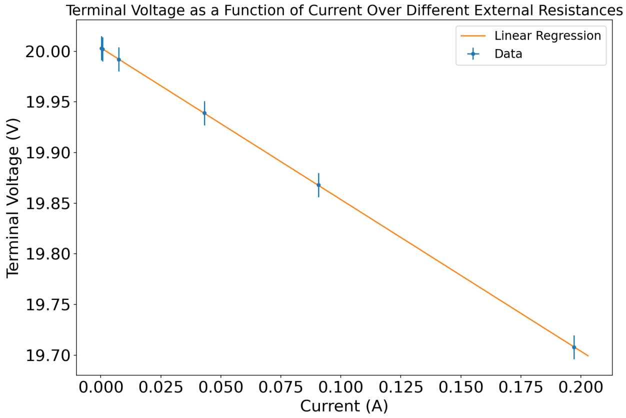

Figure 13: Terminal Voltage as a Function of Current Over Different External Resistances for the DC Power Supply using Option 2 when $V_\infty = 20$ V

The measured voltage is more reliable from Option 1 than Option 2, and the measured current is more reliable from Option 2 than Option 1, as discussed in the Analysis section. Therefore, we have included here figures which are the error bar plots of the cell battery and DC power supply using the voltage from the associated Option 1 circuit and the current from the associated Option 2 circuit.

$$V_\infty = 6.442 \pm 0.003 \text{ V}, R_{int} = 0.27 \pm 0.09 \ \Omega, I_{V=0} = 24 \pm 8 \text{ A}$$

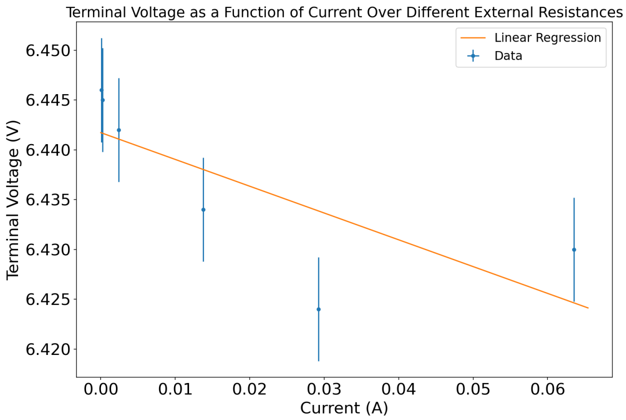

Figure 14: Terminal Voltage as a Function of Current Over Different External Resistances for the Cell Battery when $V_\infty = 6.5$ V using Voltage Measurements from Option 1 and Current Measurements from Option 2

$$V_\infty = 6.502 \pm 0.002 \text{ V}, R_{int} = 0.05 \pm 0.09 \ \Omega, I_{V=0} = 100 \pm 300 \text{ A}$$

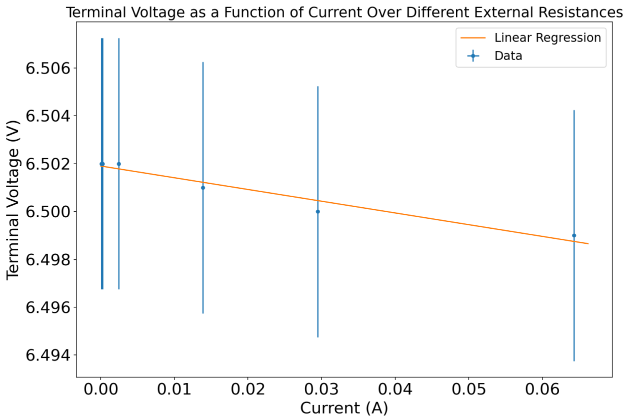

Figure 15: Terminal Voltage as a Function of Current Over Different External Resistances for the DC Power Supply when $V_\infty = 6.5$ V using Voltage Measurements from Option 1 and Current Measurements from Option 2

$$V_\infty = 10.002 \pm 0.003 \text{ V}, R_{int} = 0.05 \pm 0.08 \ \Omega, I_{V=0} = 200 \pm 400 \text{ A}$$

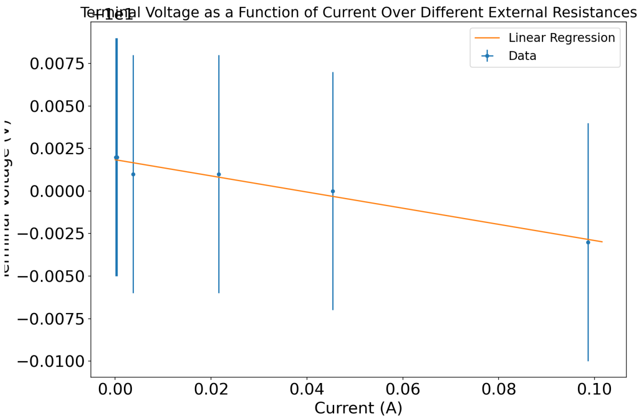

Figure 16: Terminal Voltage as a Function of Current Over Different External Resistances for the DC Power Supply when $V_\infty = 10$ V using Voltage Measurements from Option 1 and Current Measurements from Option 2

$$V_\infty = 15.003 \pm 0.005 \text{ V}, R_{int} = 0.04 \pm 0.07 \ \Omega, I_{V=0} = 300 \pm 500 \text{ A}$$

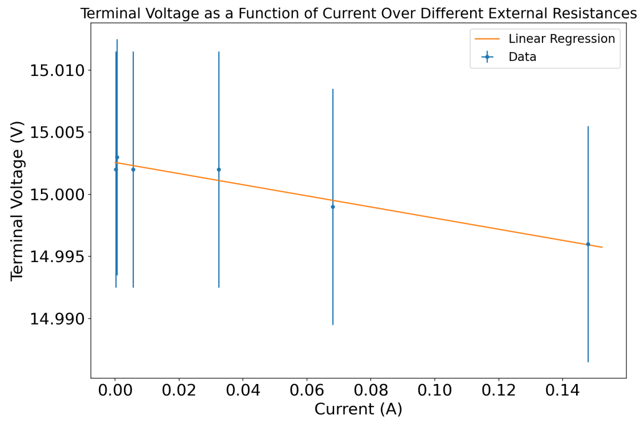

Figure 17: Terminal Voltage as a Function of Current Over Different External Resistances for the DC Power Supply when $V_\infty = 15$ V using Voltage Measurements from Option 1 and Current Measurements from Option 2

$$V_\infty = 20.004 \pm 0.006 \text{ V}, R_{int} = 0.05 \pm 0.07 \ \Omega, I_{V=0} = 400 \pm 700 \text{ A}$$

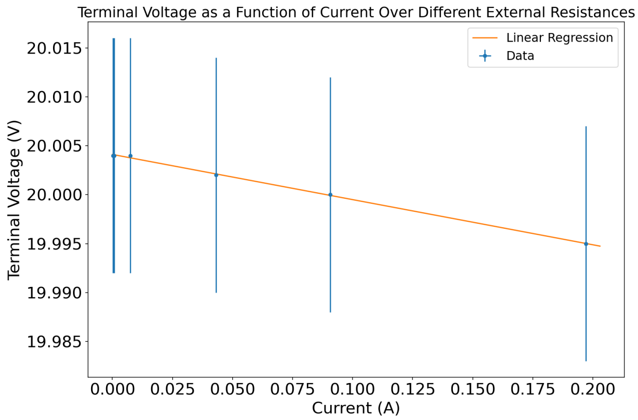

Figure 18: Terminal Voltage as a Function of Current Over Different External Resistances for the DC Power Supply when $V_\infty = 20$ V using Voltage Measurements from Option 1 and Current Measurements from Option 2

The following figure shows the maximum current value as the calculated open-circuit voltage increases.

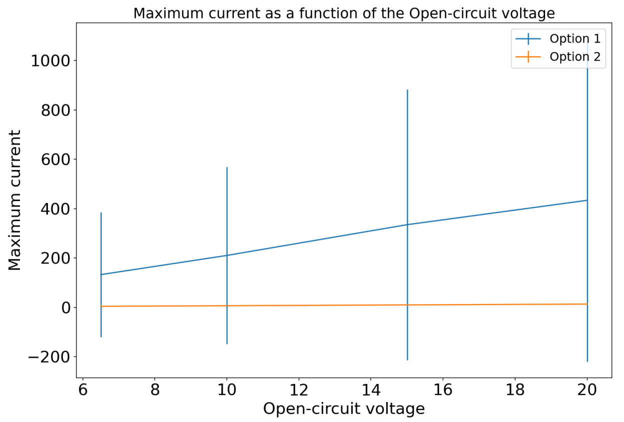

Figure 19: Maximum current as a function of the Open-circuit voltage. Horizontal error bars for option 1 points and all error bars for option 2 points are too small to see.

4 Analysis

We can clearly see that the measured voltages in option 2 are more far apart compared to those in option 1 for both the cell battery and the DC battery (see corresponding figures in Results section). This might be because of the imperfectness of the ammeter and voltmeter. Furthermore, as can be seen in figure 4, the data points at higher currents do not seem to follow a linear relationship. This might be due to the cell battery draining due to a high current caused by a low resistance resistor. However, this is not the case when the same experiment is performed with Option 2 circuit design. (see figure 3). So, we can conclude that option 2 is at least better for the cell battery in allowing us to get a linear relationship in the terminal voltage.

However, both the options seem appropriate for the DC regulated voltage source. This implies that this source is better than the cell battery. Also, our procedure of taking photos of the readings seems to have paid off as the graphs seem appropriate. Moreover, we found the internal resistance of the cell battery to about $R_{int} = 6.5$ V whereas it fluctutated for the DC regulated supply. This can be interpreted by thinking that the internal resistance is very small.

For Figures 4-18, the x-intercept of these plots is the maximum current possible at a certain V_∞. Figure 19 shows the relationship between V_∞ and the maximum current for both options. Option 1 shows that the maximum current increases as V_∞ increases, and for option 2, the maximum current is roughly constant. This is expected because the currents measured in Option 2 are the current from the whole circuit which should not change, but the currents from Option 1 could change if the voltmeter is not perfect.

A Data

Table 1: Car Battery Option 1: Voltage-Current Data

R_{load} (in Ω)	V_{∞}^{C}(in V)	$u(V_{\infty}^{C})$ (in V)	V (in V)	$u(V)$	I (in mA)	$u(I)$ (in mA)
100	6.460	0.052	6.430	0.002026	63.57	0.17714
220	6.414	0.112	6.424	0.002056	29.149	0.063298
470	6.437	0.237	6.434	0.0021185	13.729	0.032458
2700	6.441	1.352	6.442	0.002676	2.384	0.009768
27000	6.443	13.502	6.445	0.008751	0.237	0.005474
100000	6.446	50.002	6.446	0.027001	0.059	0.005118

Table 2: Car Battery Option 2: Voltage-Current Data

R_{load} (in Ω)	V_{∞}^{C}(in V)	$u(V_{\infty}^{C})$ (in V)	V (in V)	$u(V)$	I (mA)	$u(I)$ (mA)
100	6.451	0.052	6.331	0.0051655	63.52	0.17704
220	6.445	0.112	6.399	0.0051995	29.229	0.063458
470	6.448	0.237	6.426	0.005213	13.752	0.032504
2700	6.449	1.352	6.445	0.0052225	2.388	0.009776
27000	6.450	13.502	6.450	0.005225	0.238	0.005476
100000	6.451	50.002	6.451	0.0052255	0.06	0.00512

Table 3: Voltage-Current Data for DC Voltage Source Option 1: $V_{\infty}^{K} = 6.5$ V

R_{load} (in Ω)	V (in V)	$u(V)$	I (in mA)	$u(I)$ (in mA)
100	6.499	0.0052495	64.30	0.1786
220	6.500	0.00525	29.497	0.063994
470	6.501	0.0052505	13.871	0.032742
2700	6.502	0.005251	2.406	0.009812
27000	6.502	0.005251	0.239	0.005478
100000	6.502	0.005251	0.060	0.00512

Table 4: Voltage-Current Data for DC Voltage Source Option 2: $V_{\infty}^{K} = 6.5$ V

R_{load} (in Ω)	V (in V)	$u(V)$	I (in mA)	$u(I)$ (in mA)
100	6.406	0.005203	64.28	0.17856
220	6.458	0.005229	29.499	0.063998
470	6.481	0.0052405	13.872	0.032744
2700	6.498	0.005249	2.407	0.009814
27000	6.502	0.005251	0.24	0.00548
100000	6.502	0.005251	0.061	0.005122

Table 5: Voltage-Current Data for DC Voltage Source Option 1: $V_{\infty}^{K} = 10$ V

R_{load} (in Ω)	V (in V)	$u(V)$	I (in mA)	$u(I)$ (in mA)
100	9.997	0.006999	98.55	0.2471
220	10	0.007	45.4	0.1408
470	10.001	0.007001	21.561	0.048122
2700	10.001	0.007001	3.705	0.01241
27000	10.002	0.007001	0.368	0.005736
100000	10.002	0.007001	0.097	0.005194

Table 6: Voltage-Current Data for DC Voltage Source Option 2: $V_{\infty}^{K} = 10$ V

R_{load} (in Ω)	V (in V)	$u(V)$	I (in mA)	$u(I)$ (in mA)
100	9.854	0.006927	98.61	0.24722
220	9.934	0.006967	45.39	0.14078
470	9.97	0.006985	21.563	0.048126
2700	9.996	0.006998	3.706	0.012412
27000	10.001	0.007001	0.369	0.005738
100000	10.002	0.007001	0.098	0.005196

Table 7: Voltage-Current Data for DC Voltage Source Option 1: $V_{\infty}^{K} = 15$ V

R_{load} (in Ω)	V (in V)	$u(V)$	I (in mA)	$u(I)$ (in mA)
100	14.996	0.009498	147.88	0.34576
220	14.999	0.0095	68.09	0.18618
470	15.002	0.009501	32.34	0.06968
2700	15.002	0.009501	5.557	0.016114
27000	15.003	0.009502	0.551	0.006102
100000	15.002	0.009501	0.145	0.00529

Table 8: Voltage-Current Data for DC Voltage Source Option 2: $V_{\infty}^{K} = 15$ V

R_{load} (in Ω)	V (in V)	$u(V)$	I (in mA)	$u(I)$ (in mA)
100	14.781	0.009391	147.88	0.34576
220	14.901	0.009451	68.08	0.18616
470	14.954	0.009477	32.341	0.069682
2700	14.995	0.009498	5.558	0.016116
27000	15.002	0.009501	0.553	0.006106
100000	15.002	0.009501	0.147	0.005294

Table 9: Voltage-Current Data for DC Voltage Source Option 1: $V_{\infty}^{K} = 20$ V

R_{load} (in Ω)	V (in V)	$u(V)$	I (in mA)	$u(I)$ (in mA)
100	19.995	0.011998	197.09	0.44418
220	20	0.012	90.75	0.2315
470	20.002	0.012001	43.1	0.1362
2700	20.004	0.012002	7.409	0.019818
27000	20.004	0.012002	0.735	0.00647
100000	20.004	0.012002	0.193	0.005386

Table 10: Voltage-Current Data for DC Voltage Source Option 1: $V_{\infty}^{K} = 20$ V

R_{load} (in Ω)	V (in V)	$u(V)$	I (in mA)	$u(I)$ (in mA)
100	19.708	0.011854	197.04	0.44408
220	19.868	0.011934	90.75	0.2315
470	19.939	0.01197	43.11	0.13622
2700	19.992	0.011996	7.411	0.019822
27000	20.002	0.012001	0.736	0.006472
100000	20.003	0.012002	0.195	0.00539

B Plots of Data

The following figures are error bar plots of Tables 1-10. Along with each plot, we have the measured value of V_∞, R_int, and I_{max}.

$$V_\infty = 6.442 \pm 0.003 \text{ V}, R_{int} = 0.27 \pm 0.09 \ \Omega, I_{max} = 24 \pm 8 \text{ A}$$

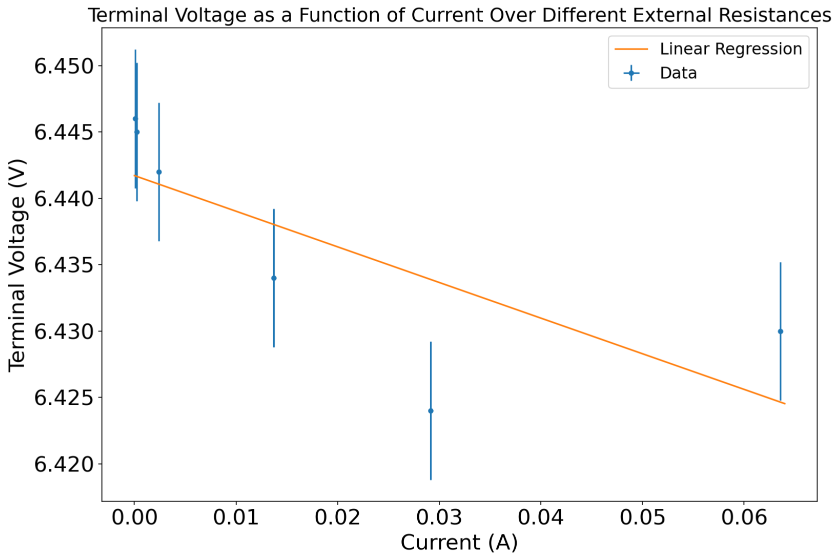

Figure B.20: Terminal Voltage as a Function of Current Over Different External Resistances for the Cell Battery using Option 1 when $V_\infty = 6.5$ V

$$V_\infty = 6.451 \pm 0.003 \text{ V}, R_{int} = 1.87 \pm 0.09 \ \Omega, I_{max} = 3.5 \pm 0.2 \text{ A}$$

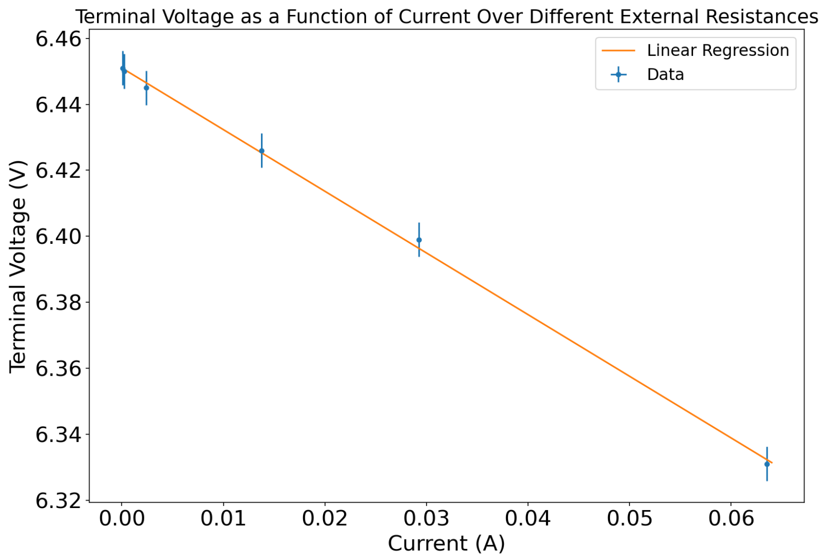

Figure B.21: Terminal Voltage as a Function of Current Over Different External Resistances for the Cell Battery using Option 2 when $V_\infty = 6.5$ V

$$V_\infty = 6.502 \pm 0.003 \text{ V}, R_{int} = 0.05 \pm 0.09 \ \Omega, I_{max} = 100 \pm 300 \text{ A}$$

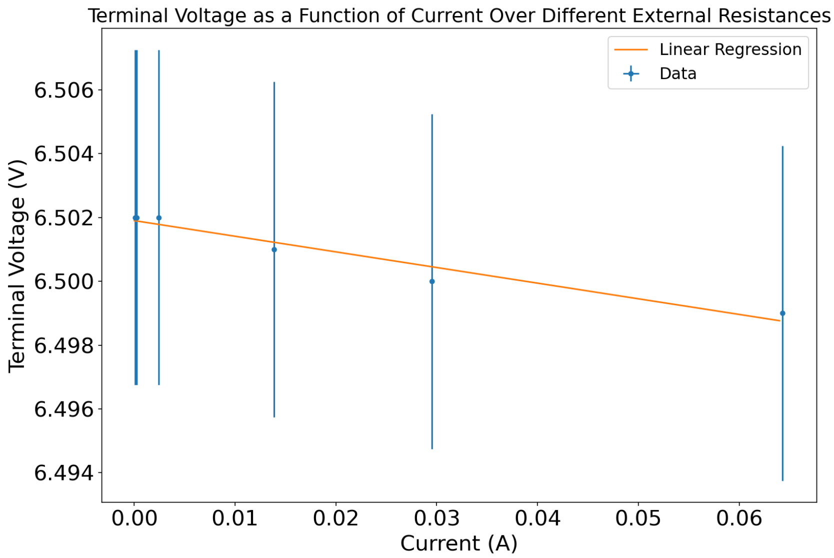

Figure B.22: Terminal Voltage as a Function of Current Over Different External Resistances for the DC Power Supply using Option 1 when $V_\infty = 6.5$ V

$$V_\infty = 6.502 \pm 0.003 \text{ V}, R_{int} = 1.49 \pm 0.09 \ \Omega, I_{max} = 4.4 \pm 0.3 \text{ A}$$

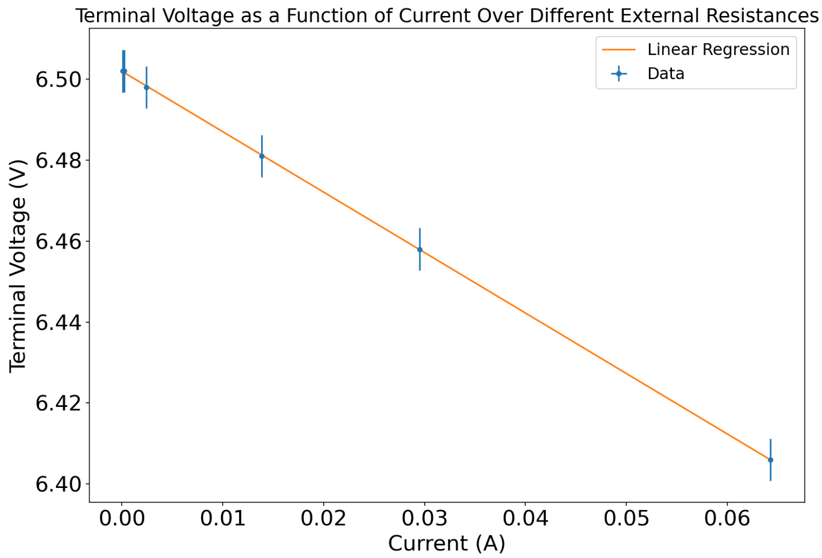

Figure B.23: Terminal Voltage as a Function of Current Over Different External Resistances for the DC Power Supply using Option 2 when $V_\infty = 6.5$ V

$$V_\infty = 10.002 \pm 0.004 \text{ V}, R_{int} = 0.05 \pm 0.08 \ \Omega, I_{max} = 200 \pm 400 \text{ A}$$

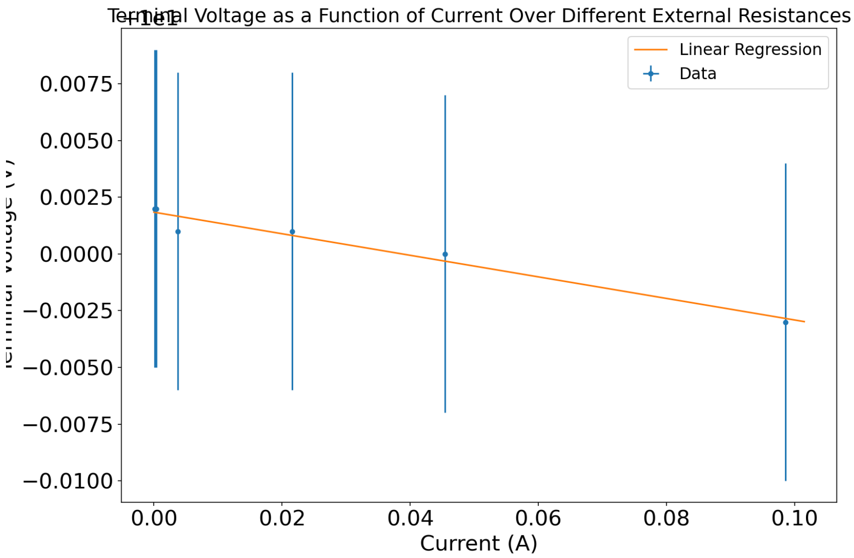

Figure B.24: Terminal Voltage as a Function of Current Over Different External Resistances for the DC Power Supply using Option 1 when $V_\infty = 10$ V

$$V_\infty = 10.002 \pm 0.004 \text{ V}, R_{int} = 1.50 \pm 0.08\ \Omega, I_{max} = 6.7 \pm 0.4 \text{ A}$$

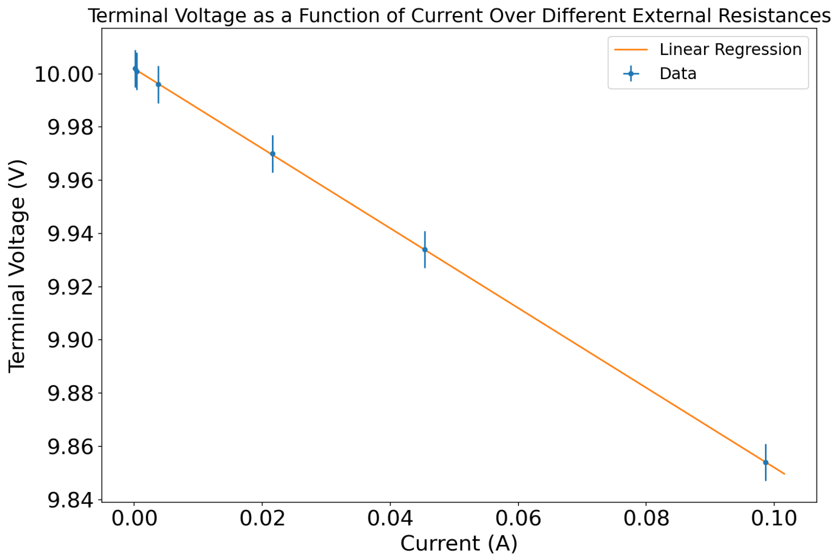

Figure B.25: Terminal Voltage as a Function of Current Over Different External Resistances for the DC Power Supply using Option 2 when $V_\infty = 10$ V

$$V_\infty = 15.003 \pm 0.005 \text{ V}, R_{int} = 0.04 \pm 0.07\ \Omega, I_{max} = 300 \pm 600 \text{ A}$$

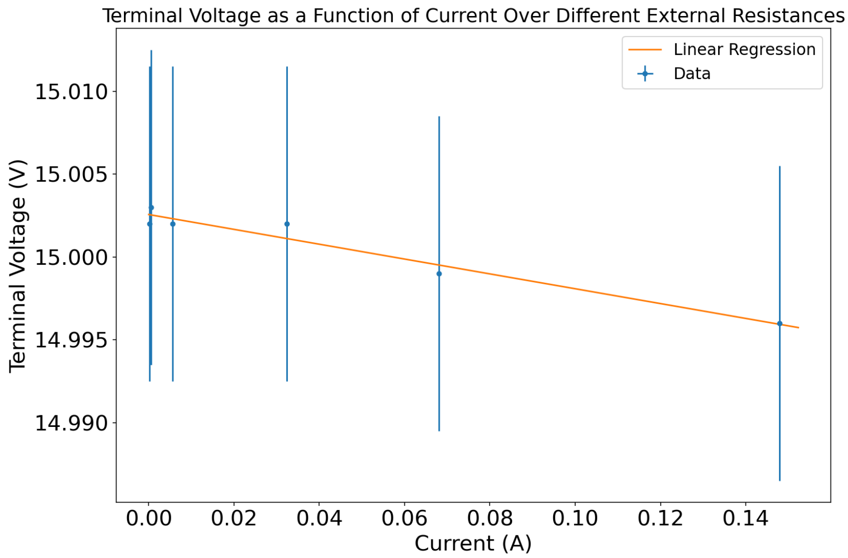

Figure B.26: Terminal Voltage as a Function of Current Over Different External Resistances for the DC Power Supply using Option 1 when $V_{\infty} = 15$ V

$$V_{\infty} = 15.002 \pm 0.005 \text{ V}, R_{int} = 1.50 \pm 0.07\ \Omega, I_{max} = 10.0 \pm 0.5 \text{ A}$$

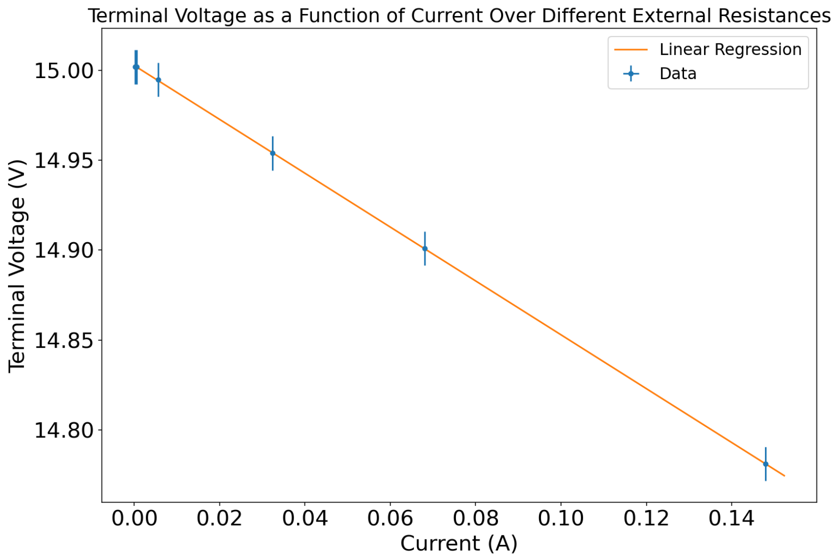

Figure B.27: Terminal Voltage as a Function of Current Over Different External Resistances for the DC Power Supply using Option 2 when $V_{\infty} = 15$ V

$$V_{\infty} = 20.004 \pm 0.065 \text{ V}, R_{int} = 0.04 \pm 0.07\ \Omega, I_{max} = 400 \pm 700 \text{ A}$$

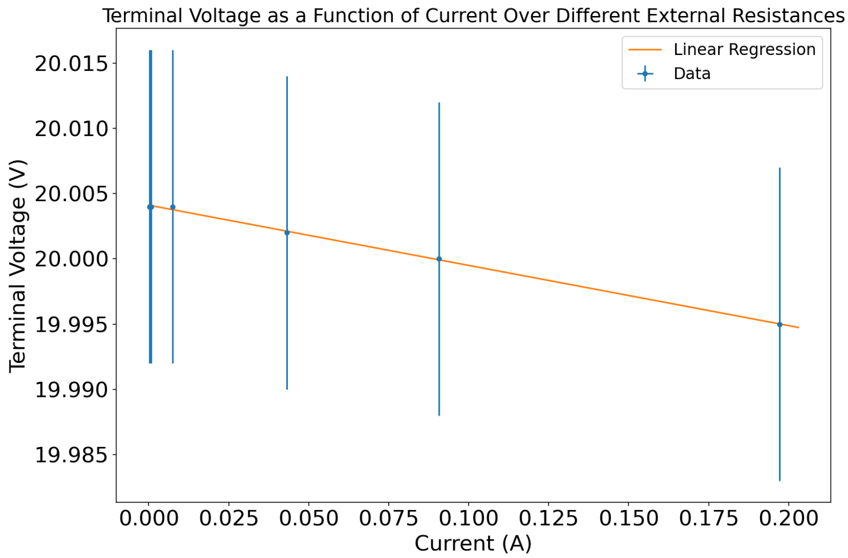

Figure B.28: Terminal Voltage as a Function of Current Over Different External Resistances for the DC Power Supply using Option 1 when $V_\infty = 20$ V

$$V_\infty = 20.003 \pm 0.006 \text{ V}, R_{int} = 1.50 \pm 0.07 \ \Omega, I_{max} = 13.4 \pm 0.6 \text{ A}$$

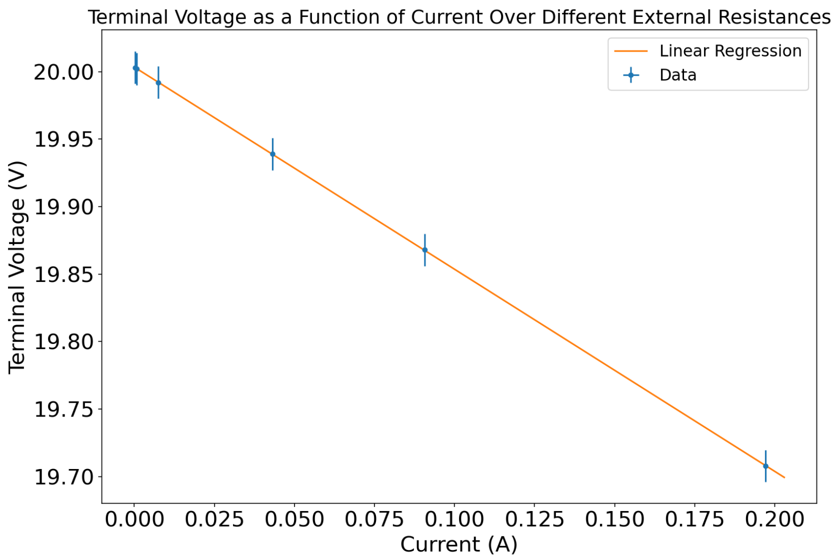

Figure B.29: Terminal Voltage as a Function of Current Over Different External Resistances for the DC Power Supply using Option 2 when $V_\infty = 20$ V

C Sample Calculations

All the voltage measurements were done in the 30 V range. So, the formula for calculating uncertainty was found from the Keysight multimeter manual as:

$$u(V) = 0.05\% \times reading + 2 \cdot 0.001 \text{ V}$$

For example, for the first voltage measurement in Table 1, $V = 6.430$ V. So,

$$u(V)\Big|_{V=6.430\text{V}} = \frac{0.05}{100} \cdot 6.430 + 0.002 \text{ V} = 0.005215 \text{ V}$$

The values of the current were found to be sometimes in the 30 mA range and sometimes in 300 mA range. The uncertainty calculation was done similar as above.

D Indication of who did what?

Both Meet and Sam collaborated on all aspects of the lab. Coding and LaTeX work was shared equally. Sam lead the work on Part 1 of the lab and Meet lead the work on Part 2 of the lab.