Measurement of laser wavelength using diffraction via circular aperture and diffraction gratings.

Meet Chaudhari

March 21, 2023

1 Introduction and Theory

Diffraction of light waves involves the light waves originating from a finite region in space which due to the uncertainty principle spread away from their natural linear path. The initial electric field in the electromagnetic wave can be Fourier transformed to express it in plane waves. Each of the plane waves is then propagated to a point on a screen and then their sum is inverse Fourier transformed to yield the net electric field at that point on the screen. Clearly, the electric field contributions from the plane wave components interfere with each other at the various points. The light scattered from the screen then forms a diffraction pattern that we can see.

The positions of maxima and minima in the diffraction pattern depends on the distance between the point of origin of the wave and the screen among other things. We performed the experiment with monochromatic red laser light which would have the well defined wavelength that we wish to measure.

We first performed the experiment with a circular aperture. In the far-field approximation, where the distance D between the aperture and the screen is very large compared to the distance between the central maxima and a point of minimum intensity y, we have the formula for the wavelength as:

$$\lambda_a = \frac{yd}{mD} \tag{1}$$

where m is the order of the point of minimum intensity (1 for the first minimum on either side, 2 for the ones next to them, etc.) and d is the diameter of the aperture[1].

We then performed the experiment with transmission and reflecting diffraction gratings. Each has closely spaced groves that when reflecting or transmitting light, act as multiple slits with finite length.

For the transmission grating, the wavelength is expressed as[2]:

$$\lambda_T = \frac{a(\sin\theta_m - \sin\theta_i)}{m} \tag{2}$$

where a is the spacing between the groves in the grating, $m = 0, 1, 2, ...$ is the order of the bright spot that appears on the screen, θ_i is the incident angle of the laser on the grating, and θ_m is the smaller angle of the m^{th} order bright spot relative to the normal of the grating.

Similarly, for the reflecting grating, the corresponding formula for the wavelength of light is [3]:

$$\lambda_R = \frac{a(\sin\theta_m + \sin\theta_i)}{m} \tag{3}$$

where all the variables have the same meaning as before. Now we describe the procedure for all the measurements of $\lambda_a, \lambda_T, \lambda_R$.

[1] “Fraunhofer Single Slit Diffraction.” Hyperphysics.phy-Astr.gsu.edu, hyperphysics.phy-astr.gsu.edu/hbase/phyopt/sinslit.html.

[2] “Visible Transmission Gratings.” Www.thorlabs.com, www.thorlabs.com/newgrouppage9.cfm?objectgroup$_i d = 1123pn = GT13 - 03.Accessed21Mar.2023.$

[3] see footnote 2

2 Procedure

2.1 Materials/Apparatus Used

For the laser, we used *Thor Labs'* Helium Neon laser operating at 0.8 mW 632.8 nm [4]. We used *Thor Labs'* circular iris[5] with an apperture diameter variable from $1 - 12$ mm with an uncertainty of ± 0.13 mm. For diffraction gratings, we used *Thor Labs'* holographic reflective grating with 2400 groves per mm and a visible Transmission grating with 300 groves per mm. We also used round silver mirrors to direct the laser into the setup. Finally, we used an optical breadboard, rotatory mounts, and 3" (75 mm) Long Posts to hold the above components in place. We also used a steel measuring tape with least count of 1 mm and white sheets of paper for screen.

2.2 Method and Results

For observing diffraction of the laser with a circular aperture, we setup our instruments as in figure 1.

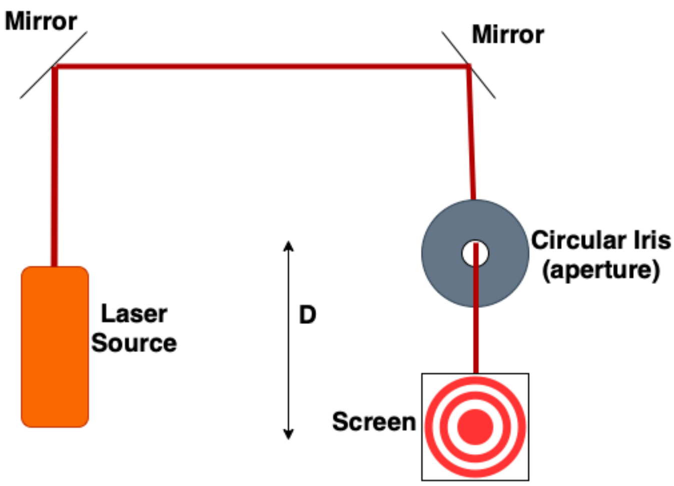

Figure 1: Setup for diffraction of red laser through a circular aperture.

We adjusted the beam path through the knobs on the mirrors such that the beam passed through the centre of the iris. To achieve maximum diffraction possible, we used the mechanical lever on the iris to have the smallest aperture possible which from the specification is $d = (1.00 \pm 0.13)mm$. We also made sure the beam was travelling parallel to our optical breadboard so that the distance measured from our aperture to our screen was in agreement with the actual path length travelled by the laser emerging from the iris. We first obtained the diffraction pattern on a white sheet of paper. To be able to mark the fringe positions clearly to calculate fringe separation, we made measurements at $D = 1.31 \pm 0.01$ m, 2.50 ± 0.01 m, 3.80 ± 0.01 m which are large enough to observe distinct fringes. We took photos of the fringes with a grid marked on the screen to later find out the fringe spacing y (see figure 4 in appendix) using an image analysis tool[6]. We measured the distance between the screen and the aperture via the ruler, which unfortunately slightly bent in the middle due to its weight giving us a higher D measurement. Thus, using equation 1, we calculated the wavelength of the laser for the 3 measurements as $\lambda_a = 1100 \pm 70$ nm, 1300 ± 200 nm, 800 ± 200 nm. Except the last measurement for $D = 3.80 \pm 0.01$ mm, the other two measurements of λ_a do not contain the original wavelength $\lambda_0 = 632.8$ nm within their respective uncertainties. The average of the 3 measurements is then

$$(\lambda_a)_{avg} = 1100 \pm 70 \text{ nm} \tag{4}$$

For the gratings, we first used the reflecting grating. We fixed it in a holder and post making sure the arrow marked on top was horizontal which indicated that the groves were perpendicular

[4] "HeNe Lasers: Red." HeNe Lasers: Red, www.thorlabs.com.
[5] "Thorlabs - ID12 Mounted Standard Iris, Ø12 Mm Max Aperture, TR3 Post." Www.thorlabs.com, www.thorlabs.com/thorproduct.cfm?partnumber=ID12. Accessed 21 Mar. 2023.
[6] "Measure in Photo Online - Eleif.net." Eleif.net, eleif.net/photomeasure.

to the optical breadboard and our setup. This is needed so that the fringe orders lie in the plane parallel to the breaboard. In our case, the grating was such that we could only measure the angular position of the $m = 1$ fringe. The setup was as shown in figure 2.

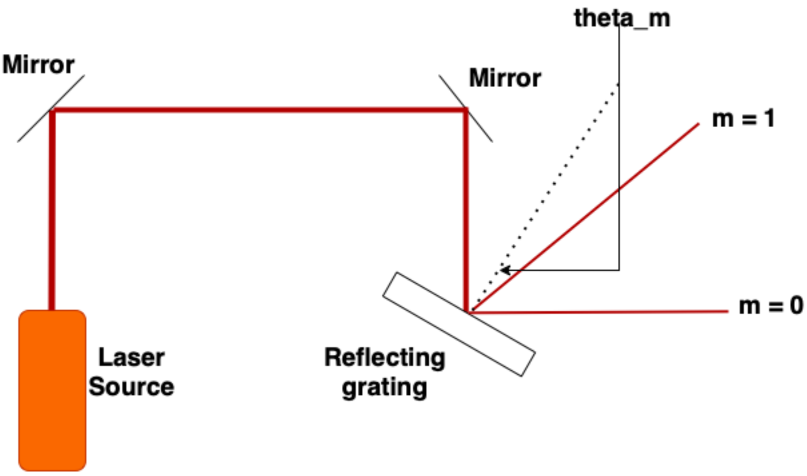

Figure 2: Setup for diffraction of red laser through a reflecting diffraction grating.

To measure $\theta_{m=1} = \theta_1$, we used the grid markings on the breadboad and used trignometry. We found $\theta_1 = (43 \pm 2)°$. Also, we observed that the indident angle θ_i was nearly equal to the θ_1 as the first order fringe seemed to just have reflected back. The grove spacing was calculated

$$a = \frac{1 \text{ mm}}{2400} \approx 4.2 \cdot 10^{-7} \text{ m}. \tag{5}$$

Then using equation 3, we calculated the wavelength to be

$$\lambda_R = 560 \pm 30 \text{ nm} \tag{6}$$

Compared to the wavelength obtained through the circular aperture, the mean value of this measurement is more closer to the true value of $\lambda_0 = 632.8$ nm.

Finally, we used the transmission diffraction grating by having the laser beam incident at $\theta_i = 0°$ on the it as seen in figure 3. Again, we fixed the grating in a holder such that the arrow on its top face was horizontal.

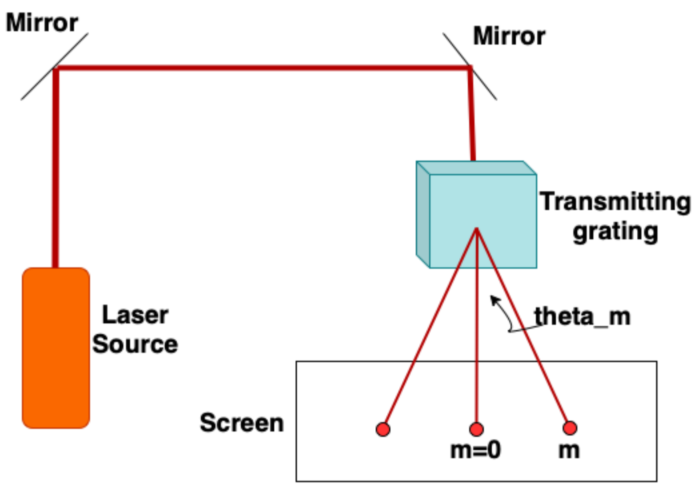

Figure 3: Setup for diffraction of red laser through a transmitting diffraction grating.

In this experiment, we conducted 4 trials at difference distances of the screen from the transmitting grating with $D = (7.6 \pm 0.2) \cdot 10^{-2}$ m, $(1.52 \pm 0.02)10^{-1}$ m, $(2.29 \pm 0.02)10^{-1}$ in, $(3.05 \pm 0.02)10^{-1}$ in which were converted from inches on the breadboard. For the first two distances, we observed fringes upto order $m = 3$ on each side of the central bright fringe and for the last two we observed till only $m = 2$ on either side. These fringe positions were marked on a white sheet of paper along with the distance D at which they were measured (see figure 7 in appendix). Again, the separation of the fringes was calculated from an image analysis software using a ruler's markings as reference.

Then, using equation 2 for each value of m that was accessible for each D, and $a = \frac{1\text{mm}}{300}$, I calculated the average wavelength for each D in increasing order as:

$$\lambda_T = 612.1 \pm 0.8 \text{ nm}, 661.3 \pm 0.4 \text{ nm}, 649.6 \pm 0.3 \text{ nm}, 649.6 \pm 0.3 \text{ nm} \tag{7}$$

Then, taking average over all D measurements above, I found the average wavelength via transmission diffraction grating as:

$$\lambda_T = 643.2 \pm 0.3 \text{ nm} \tag{8}$$

This is even closer to $\lambda_0 = 632.8$ nm than that found via the reflecting grating.

Conclusion

In a series of experiment, my group and I measured the wavelength of a laser with a well defined wavelength $\lambda_0 = 632.8$ nm via diffraction of the laser beam via a circular aperture, a reflecting diffraction grating, and a transmitting diffraction grating. I calculated the corresponding averaged wavelength via each method as $\lambda_a = 1100 \pm 70$ nm, $\lambda_R = 560 \pm 30$ nm, and $\lambda_T = 643.2 \pm 0.3$ nm. Among all three, the one calculated by the circular aperture was the most inaccurate and imprecise and the one calculated by transmitting refractive grating was the most accurate and precise. Thus, I think wavelength measurements should be done via transmitting diffraction gratings.

References

1. Astronomy, BYU Physics and. "Optics Textbook." Optics Textbook, optics.byu.edu/textbook.

A Circular aperture / iris images

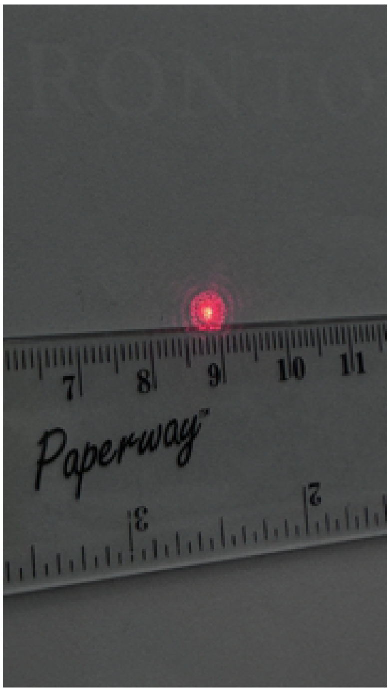

Figure 4: D = 1.31 m

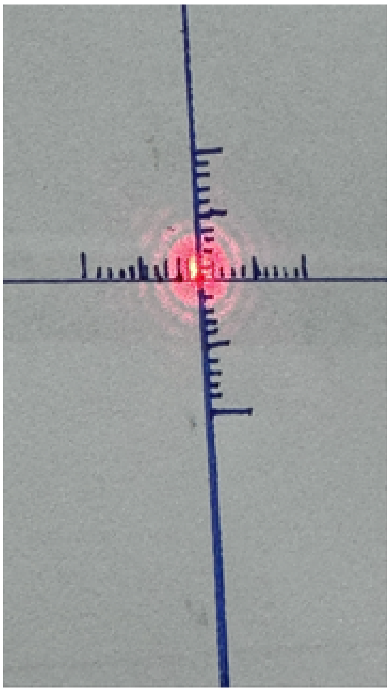

Figure 5: D = 2.50 m

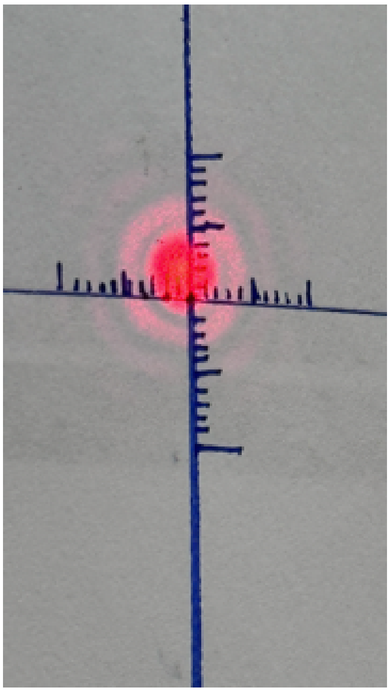

Figure 6: D = 3.80 m

B Transmitting grating: exemplar observation

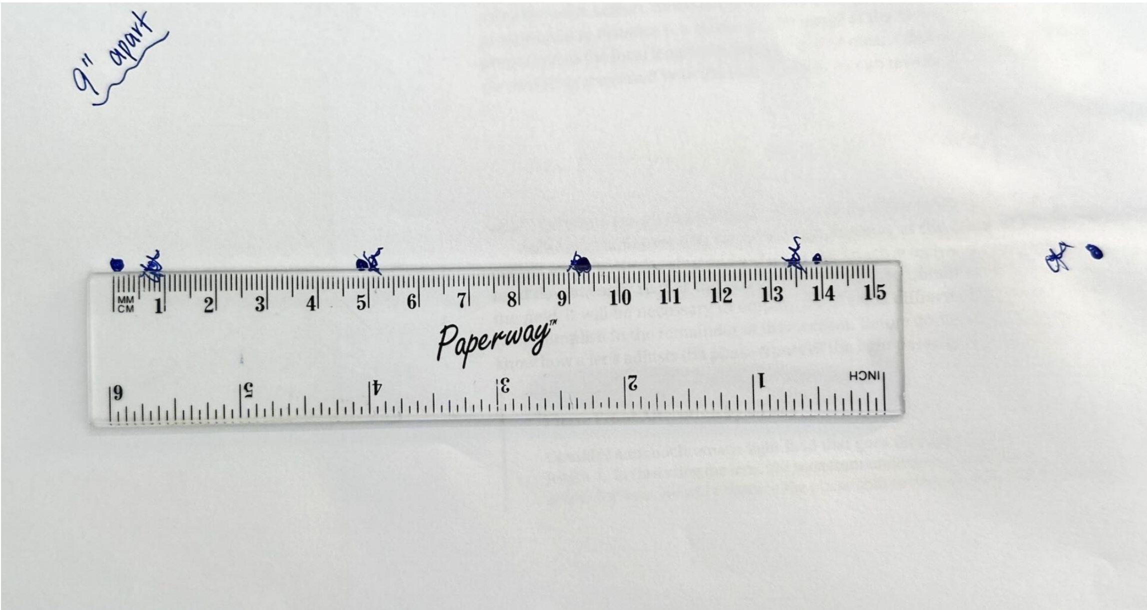

Figure 7: $D \approx 9$ inches. The points marked with pen are the positions where the brigh fringe orders were observed.