PHY 224: Radioactivity Experiments

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1 Procedure

1.1 Material Required

The experiment was performed with a Geiger counter, an orange Fiesta plate containing Uranium, and a sample of meta-stable Barium (Ba 137m).

1.2 Method

Firstly, the Geiger counter was turned on without adding a sample of radioactive material beneath it to measure background radiation for about 20 minutes. The recorded "clicks" or emission events were recorded through a computer to which the counter was connected and the data was plotted on a graph of counts vs time for visualization. Then, the Geiger counter was placed right above the sample of meta stable barium and it was allowed to record data for 20 minutes. The counter was set to record emission events in intervals of 20s. Similarly, the counter was placed above the Fiesta plate to count emission rates of uranium in the plate. Using the collected data, the results were plotted and from the best fit parameters resulting from numerical techniques. Note that from now on, the decay of Ba 137m would be referred to as fast decay while that of the uranium sample would be called slow decay.

2 Results

2.1 Fast Decay

Since the Geiger counter only counts emission events, the first step was to eliminate background radiation from the radiation of the source. This was done by subtracting the mean value background radiation counts collected before the experiment from the total counts: $N_S = N_T - N_B$. Then, the emission count of the source (Ba 137m) was converted into a rate: $R_S = \frac{N_S}{\Delta t}$, where $\Delta t = 20s$. However, it was found that the R_s was negative for an instance in the sampling period. This might be due to the mean background radiation counts being larger than the total counts measured by the Geiger counter ($N_T < N_B \Rightarrow N_S < 0$). Since the logarithms only accept positive arguments, we adjusted this minor anomaly by setting $R_S = |\frac{N_S}{\Delta t}|$ so that the emission rate of the source always remain non-negative.

The uncertainty in the emission rate was calculated as $u(R_S) = \frac{u(N_S)}{\Delta t} = \frac{\sqrt{N_T+N_B}}{\Delta t}$. See sample uncertainty calculation in appendix. Now, using the rate of emission $R_S(t) = R_S \pm u(R_S)$ and the expected decay relation $R_S(t) = R_S(0) \cdot (\frac{1}{2})^{\frac{t}{t_{1/2}}}$, we performed linear and non-linear regression on the experimental values of $R_S(t)$ to find the best fit parameters for each regression: Initial emission rate = $R_S(0)$ and half-life of Ba 137m = $t_{1/2}$. Furthermore, to calculate the theoretical curve corresponding to the standard value of half life = 153 s, we performed a non-linear regression again to calculate the initial emission rate. See appendix for calculations.

The corresponding equations of best fit are as follows (see Appendix for sample calculations):

- Linear regression:

$$R_S(t) = 35.0 \pm 0.6 \cdot (\frac{1}{2})^{\frac{t}{t_{1/2}}}\ s^{-1} \text{ with } t_{1/2} = 153 \pm 2 \text{ s} \tag{1}$$

- Non-Linear regression:

$$R_S(t) = 35.6 \pm 0.6 \cdot (\frac{1}{2})^{\frac{t}{t_{1/2}}}\ s^{-1} \text{ with } t_{1/2} = 149 \pm 2 \text{ s} \tag{2}$$

- Theoretical Curve:

$$R_S(t) = 35.2 \pm 0.3 \cdot (\frac{1}{2})^{\frac{t}{t_{1/2}}}\ s^{-1} \text{ with } t_{1/2} = 153s \tag{3}$$

These were plotted in figure 1 along with the experimental data.

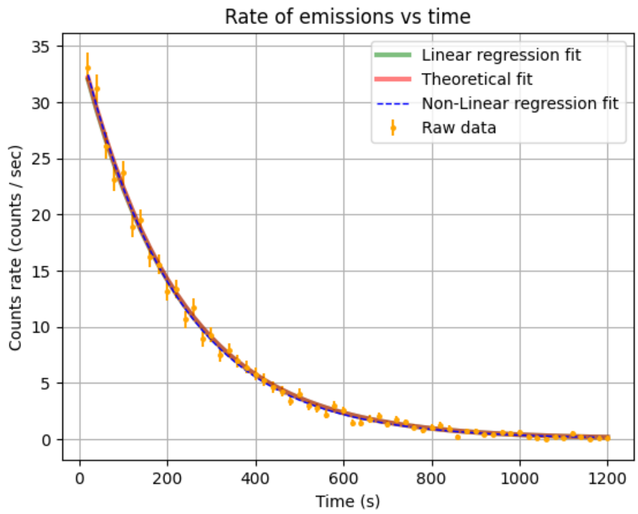

Figure 1: Plot of raw data with error bars, curves of linear and non linear regressions and the theoretical curve representing a half life of 153 s.

As can be seen in Figure 1, all the three curves and the data points a clear trend of exponential decay. Furthermore, the three curves almost coincide with each other with the non-linear regression getting slightly out of alignment.

To see the exponential decay more unambiguously, these were plotted on a semi log-plot with the y-axis being the log-scale. See figure 2.

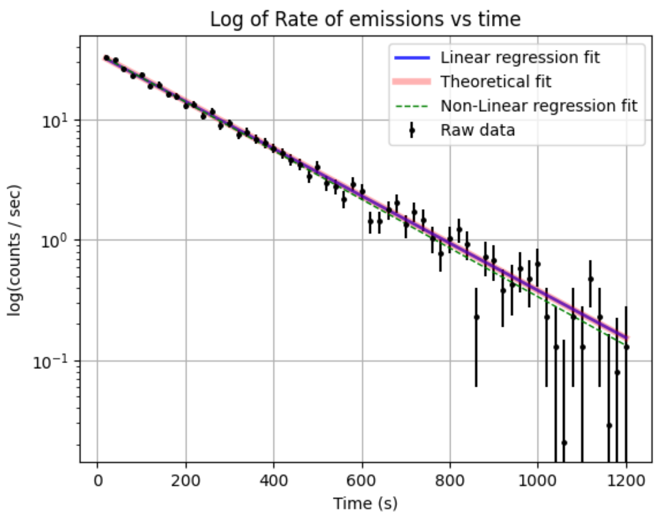

Figure 2: Plot of raw data with error bars, curves of linear and non linear regressions and the theoretical curve representing a half life of 153 s on a semi log plot

Here, we can clearly see that the non-linear regression fit deviates a bit from the theoretical curve. Moreover, the striking feature of figure 2 is that the uncertainties get very large as the emission rate approaches 0. Note that uncertainties going below 0 aren't relevant as the reaction rate is always negative.

Finally, to analyze the accurateness and the preciseness of the linear and non-linear regressions, we plotted the residual plots of both the regressions when compared to the experimental data along with the uncertainties. These are shown in figure 4 and figure 5.

Figure 3: Residual plot of linear fit

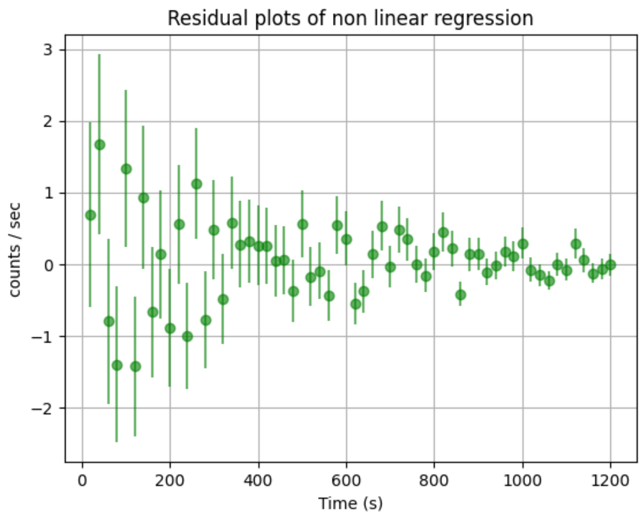

Figure 4: Residual plot of non-linear fit.

From the residual plots, we can see that under experimental uncertainty, almost all the points in both the residual plots pass through 0. This means that the linear and non-linear fits match the data very closely. Furthermore, the points get closer to 0 as time increases which indicates that the fits better match the data after a long period of time.

2.2 Slow Decay

The slow decay of the Fiesta Plate occurs as random events that have a mean emission rate based on the half-life of the Uranium Oxide in the plate. Random events like this should follow a Poisson distribution, where the parameter, μ, of the Poisson distribution is the expected average number of counts in a given time interval.

The Geiger counter detects emissions from the Fiesta Plate and emissions from the background radiation. To analyze emissions from the Fiesta plate, we must first remove emissions from the background radiation.

We can check that the background radiation also follows a Poisson Distribution. Figure 5 shows that the background radiation follows a Poisson Distribution. For the Poisson Distribution, we used the mean background count for the parameter $\mu = 3.4$. For the Gaussian distribution, we used the mean and standard deviation for the parameters $\mu = 3.4, \sigma = 2.044$. Other similar histograms are shown in the Appendix.

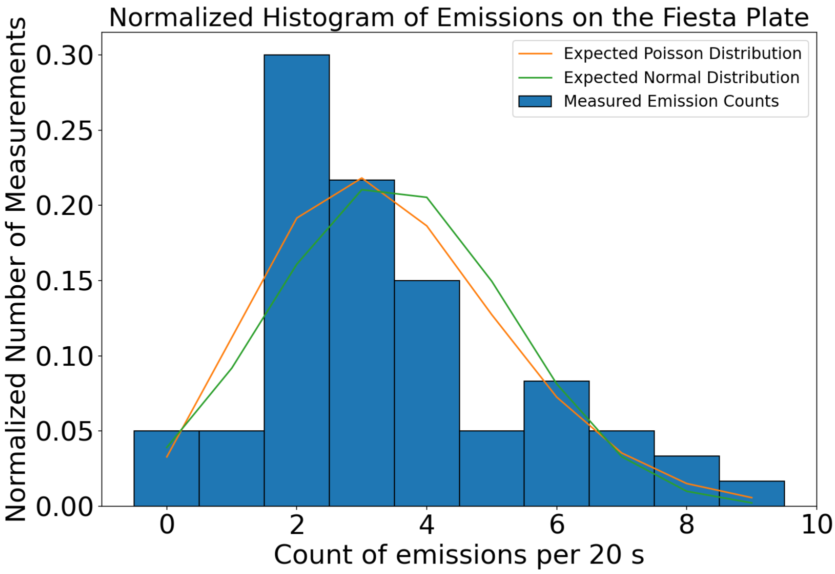

Figure 5: Histogram of Background radiation on the Geiger counter with the expected Poisson and Gaussian Distribution. The histogram is normalized to fit the expected probability distributions.

Since the histogram roughly follows the expected Poisson Distribution, the maxiumum likelihood estimator for the background radiation count is the mean. The time interval for data collection for the background radiation is 20 seconds, so we found that the mean background radiation causes detections on the Geiger counter at a rate of 0.17 emissions per second. We subtracted the appropriate number of emissions from each measurement sample of the Fiesta plate to cancel out the background radiation detected on the Geiger counter.

Figure 6 shows the count of emissions from the Fiesta plate with the expected Poisson and Gaussian distributions on top. For the Poisson Distribution, we used the mean background count for the parameter $\mu = 42.7$. For the Gaussian distribution, we used the mean and standard deviation for the parameters $\mu = 42.7, \sigma = 6.65$. Other similar histograms are shown in the Appendix.

The time interval for each measurement sample of the Fiesta plate is 3 seconds. This shows that the mean rate of emissions from the Fiesta plate is 14 ± 2 emissions per second.

3 Analysis

3.1 Fast Decay

The linear regression found the half-life of Ba-137m to be 153 ± 2 s. This is very close to the expected value of 2.6 minutes = 156 seconds. The non-linear regression found the half-life of Ba-137m to be 149 ± 2 s. This is also close to the expected value, but it is farther from the expected value than the value from the linear fit. The linear regression seems like a better fit than the non-linear regression.

We measure the goodness of fit of the two regressions by calculating their reduced chi-squared values. This value for the linear regression is 1.0196, and for the non-linear regression, it is 1.1183. Since the reduced chi-squared value measures the error of the fit, values close to 1 indicate better fits. This further supports the claim that the linear regression is a better fit than the non-linear regression.

The error bars on the residual plots tend to pass through 0 or are near zero, and there are only very minor differences in the residual plots between linear and non linear regressions. This suggests

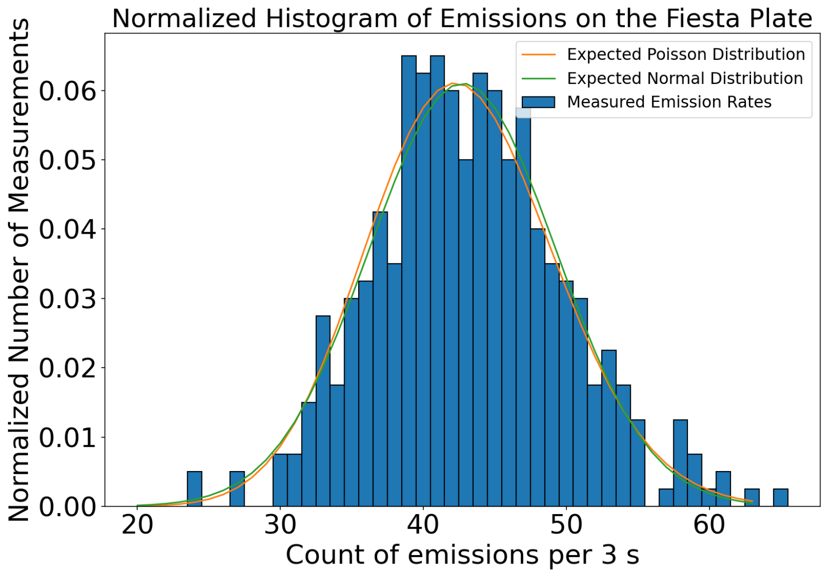

Figure 6: Histogram of Background radiation on the Geiger counter with the expected Poisson and Gaussian Distribution. The histogram is normalized to fit the expected probability distributions.

that both linear and non linear regressions are appropriate for the data.

3.2 Slow Decay

Plotting the background emission histogram with its expected Poisson and Gaussian distribution shows that the background radiation roughly follows a Poisson distribution, as expected. However, it is not a perfect fit. Possible sources of error are discussed in section 3.3.

The Poisson Distribution is similar to the Gaussian distribution for the background radiation. This occurs when the Poisson distribution has a large number of data points or a large mean. The mean of the background radiation ($\mu = 3.4$) is quite small, but the number of measurements, 60, is large which is why the Poisson distribution and the Gaussian distribution are so close.

The histogram of Fiesta plate submissions and the expected Poisson and Gaussian Distribution show that the Fiesta plate emission count follows a Poisson Distribution. This result can also be seen with the mean and standard deviation of the data. The mean and standard deviation of a Poisson distribution should exhibit the following relation:

$$\sqrt{\text{mean}} = \text{standard deviation}$$

The mean of the Fiesta plate count is 42.74, and the standard deviation is 6.65. Since $\sqrt{42.74} \approx 6.65$, we have further evidence that the Fiesta plate count follows a Poisson Distribution, as expected.

The rate of the Fiesta plate count was found to be 14 ± 2 emissions per second. The expected Gaussian Distribution and Poisson Distribution are very similar for the Fiesta plate. When the number of measurements and the mean of a Poisson Distribution increase, the overall probability distribution becomes closer to a Gaussian distribution. The mean ($\mu = 42.74$) and the number of measurements (60) are both large, which explains why the Poisson distribution is very close to the Gaussian distribution.

3.3 Sources of Error

For the fast decay, statistical errors could be from the curve fitting algorithm and uncertainty in the count measurements. An experimental error could be that the barium sample may not be pure barium-137m, and there could be other radioactive substances in the sample. Also, as the cesium-137 is decaying into barium-137, there could be detections from the cesium, not the barium. Additionally, we assume we do not gain any new barium over the course of the experiment, but the cesium could still decay into barium as the experiment is carried out. This error could be minimized by choosing a material that gets produced faster, and by using a detector that only counts emissions at the specific energy expected.

For the slow decay, statistical errors could be that the mean is small ($\mu = 3.4$), or that there are not many observations per bin (see Figure 12). An experimental error could be that we assume the rate of emissions is constant over time. Realistically, the rate of emissions decreases over time as the radioactive substance decays. However, the assumption of constant rate is appropriate because the half life of Uranium Oxide is about 500 days (Schieferdecker). Over 20 minutes, the Uranium Oxide decays a very small amount, meaning the rate decreases by a very small amount. In contrast, Ba-137m has a half-life of 2.6 minutes. Over 20 minutes, the rate of decay changes drastically. Choosing a substance that has a longer half-life could minimize the error for the slow decay, although a half-life of 500 days is already large enough for this error to be negligible.

Statistical sources of error present for both fast and slow decay could be the time interval that we collected data over. For the fast decay, we collected data in time intervals of 20 seconds. If we collected data in smaller intervals, we would collect the same data with more precision and more data points. This would also decrease the error in the statistical analyses. For the slow decay, the Poisson Distribution would more closely match the Gaussian Distribution due to a higher number of measurements.

A limitation of the experiment is that we only collect data on a plate. Collecting data on a spherical shell around the sample will allow us to collect the total number of emissions from the sample. This could allow us to determine other properties of the sample. Also, the sample might not emit particles equally in all directions. This means that the intensity over a given plate may not be the intensity in other directions.

4 Conclusion

The linear regression was the most appropriate fit for the fast decay with a reduced chi-squared value of 1.0196. The half life was determined to be 153 ± 2 s. The initial intensity is 35.0 ± 0.6 counts per second. The main sources of error were the statistical errors in the curve fitting program. The analysis of the slow decay found that the background radiation and the emissions from the Fiesta plate follow a Poisson Distribution. The mean rate of emissions from the Fiesta plate is 14 ± 2 emissions per second.

5 References

Schieferdecker H, Dilger H, Doerfel H, Rudolph W, Anton R. Inhalation of U aerosols from UO2 fuel element fabrication. Health Phys. 1985 Jan;48(1):29-48. doi: 10.1097/00004032-198501000-00003. PMID: 3967974.

6 Appendix

6.1 Plots

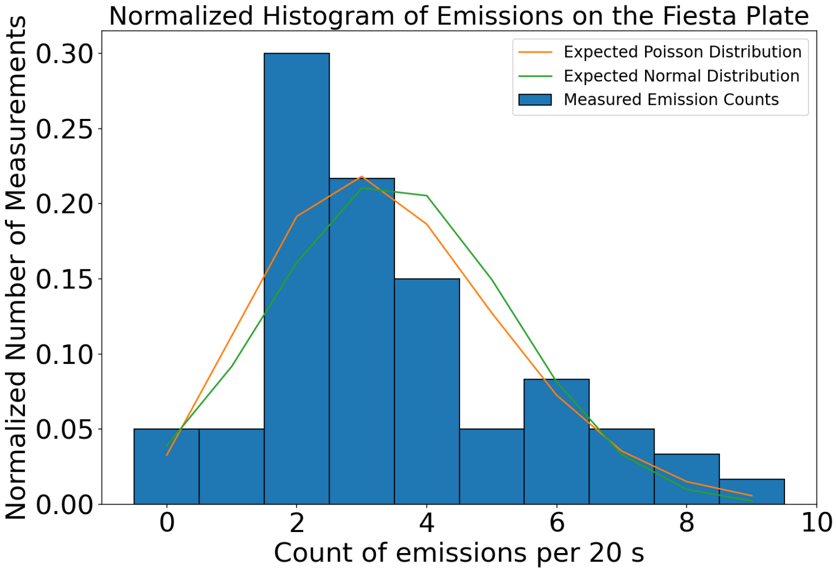

Figure 7: Histogram of Background radiation on the Geiger counter with the expected Poisson and Gaussian Distribution. The histogram is normalized to fit the expected probability distributions.

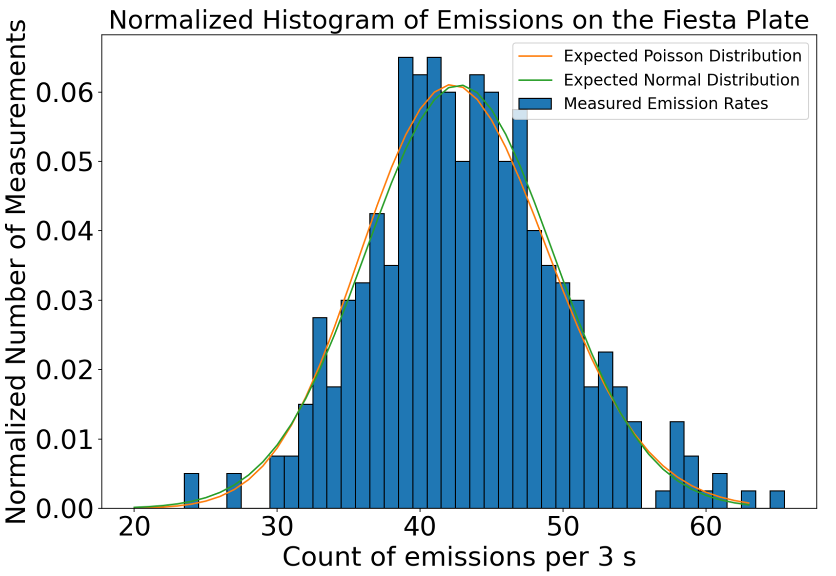

Figure 8: Histogram of Fiesta radiation on the Geiger counter with the expected Poisson and Gaussian Distribution. The histogram is normalized to fit the expected probability distributions.

6.1.1 Slow Decay

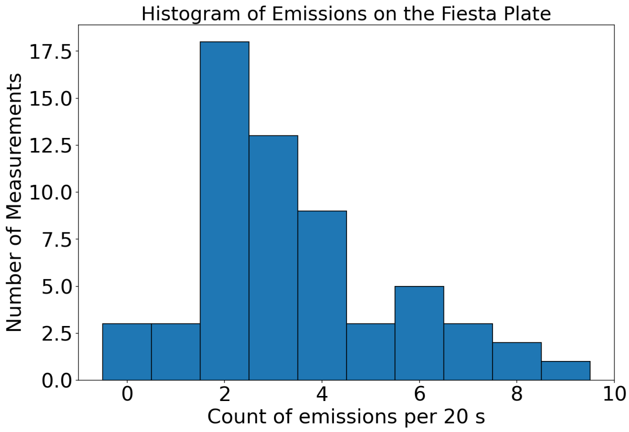

Figure 9: Histogram of Background radiation on the Geiger counter.

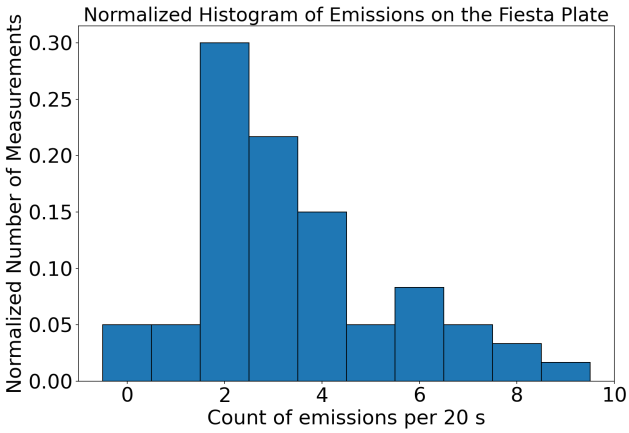

Figure 10: Histogram of Background radiation on the Geiger counter. The histogram is normalized to fit the expected probability distributions.

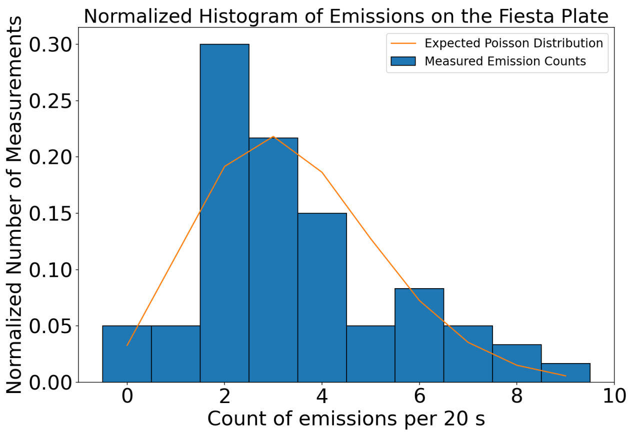

Figure 11: Histogram of Background radiation on the Geiger counter with the expected Poisson Distribution. The histogram is normalized to fit the expected probability distributions.

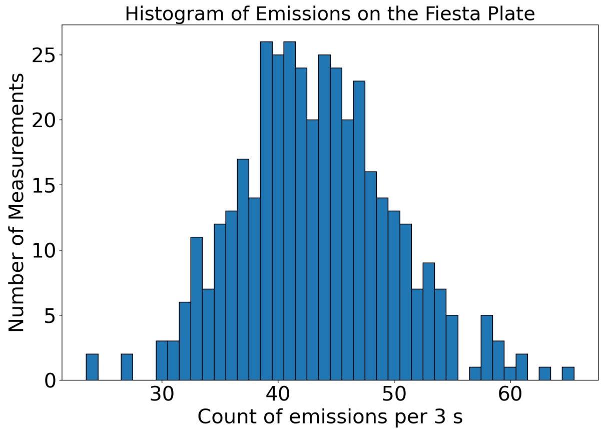

Figure 12: Histogram of Fiesta radiation on the Geiger counter.

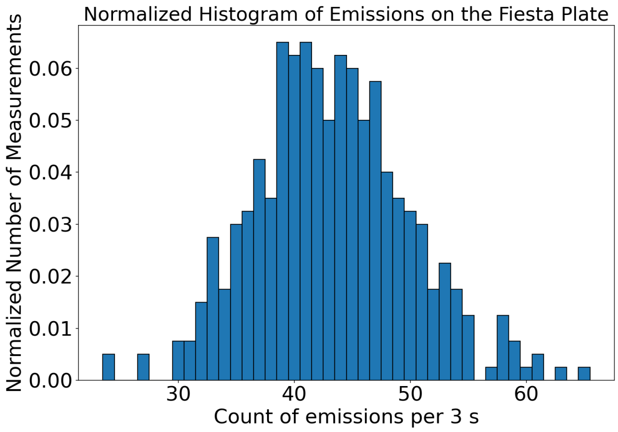

Figure 13: Histogram of Fiesta radiation on the Geiger counter. The histogram is normalized to fit the expected probability distributions.

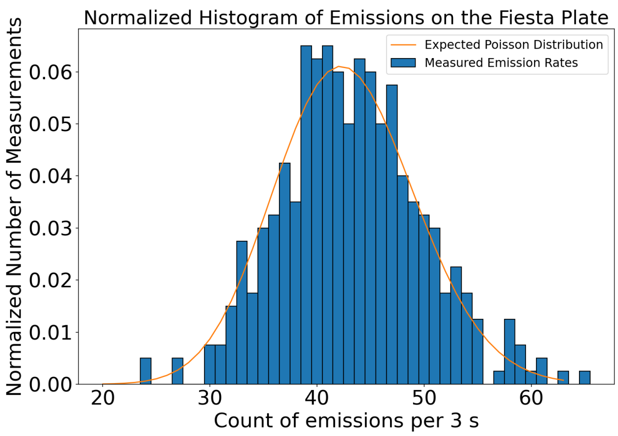

Figure 14: Histogram of Fiesta radiation on the Geiger counter with the expected Poisson Distribution. The histogram is normalized to fit the expected probability distributions.

6.2 Sample Calculations

6.2.1 Fast Decay

u(R$_\mathbf{S}$) $\Rightarrow$ We know the variance of results of counting experiments = no. of experiments = N. Since the standard deviation is = $u(x) = \sqrt{Var(x)}$, if we treat the standard deviation as uncertainty than we get that for an experiment in which we count N times, the uncertainty is $\sqrt{N}$.

Now, by definition, $N_S = N_T - N_B$.

So, $u(N_S) = u(N_T - N_B) = \sqrt{(u(N_T))^2 + (u(N_B))^2} = \sqrt{N_T + N_B}$ follows from uncertainty propagation and the fact that $u(N) = \sqrt{N}$ for any counting data. Furthermore, since $R_S = \frac{N_S}{\Delta t}$ and if we treat Δt as a constant with 0 uncertainty, then:

$$u(R_S) = \sqrt{(\frac{d}{dN_S}(\frac{N_S}{\Delta t}) \cdot u(N_S))^2} = |\frac{u(N_S)}{\Delta t}| = \frac{\sqrt{N_T + N_B}}{\Delta t}$$

For example, for the first sampling period of 20s,

$$u(R_S) = \frac{\sqrt{N_T + N_B}}{\Delta t} = \frac{\sqrt{666+3}}{20} = 33.45 \text{ counts/sec}$$

Sample calculations for linear regression $\Rightarrow$ For linear regression, logarithms were used to linearize the equation $R_S(t) = R_S(0) \cdot (\frac{1}{2})^{\frac{t}{t_{1/2}}}$. The linear equation obtained was:

$$\log R_S(t) = \log R_S(0) - \frac{\log 2}{t_{1/2}} \cdot t$$

Since we wanted to take uncertainty into account while performing the linear regression, we calculated $u(\log R_S(t))$ by uncertainty propagation as:

$$u(\log R_S(t)) = \sqrt{(\frac{d(\log R_S(t))}{dI} \cdot u(R_S(t)))^2} = |\frac{u(R_S(t))}{R_S(t)}|$$

For example, for the first sampling time period, we have: $u(R_S(t)) = 33.45$ and $R_S(t) = \frac{N_T - N_B}{\Delta t} = \frac{666-3}{20} = 33.15$ counts/sec. So, $u(\log R_S(t)) = |\frac{u(R_S(t))}{R_S(t)}| = |\frac{33.45}{33.15}| = 1.00$ counts/sec.

Furthermore, the linear regression returned the best fit parameters $(a, b) = (-\frac{\log 2}{t_{1/2}}, \log R_S(0))$ and their uncertainties $(u(a), u(b))$. So, the proper uncertainties $u(R_S(0))$ and $u(t_{1/2})$ were calculated as:

$$b = \log R_S(0) \Rightarrow R_S(0) = e^b$$

and therefore, the uncertainty $u(R_S(0))$ could be calculated as

$$\Rightarrow u(R_S(0)) = \sqrt{\frac{d(e^b)}{db} \cdot u(b)^2} = e^b \cdot u(b)$$

Furthermore, $t_{1/2}$ could be expressed as

$$a = -\frac{\log 2}{t_{1/2}} \Rightarrow t_{1/2} = -\frac{\log 2}{a}$$

and its uncertainty as:

$$\Rightarrow u(t_{1/2}) = \sqrt{(\frac{[-d(\frac{\log 2}{a})}{da}) \cdot u(a)]^2} = |\frac{u(a) \cdot \log 2}{a^2}|$$

For the linear regression, we got (a,b) = (-0.0045, 3.56) and $(u(a), u(b)) = (6.34 \cdot 10^{-5}, 1.79 \cdot 10^{-2})$. So,

$$\overline{R_S(0)} = e^b = 35.036 \text{ counts/sec and } u(R_S(0)) = e^b \cdot u(b) = 0.626 \text{ counts/sec} \Rightarrow \boxed{R_S(0) = 35.0 \pm 0.6 \text{ counts/sec}}$$

and, the half time for linear regression is calculated as:

$$\overline{t_{1/2}} = -\frac{\log 2}{a} = 153.223 \text{ sec and } u(t_{1/2}) = |\frac{u(a) \cdot \log 2}{a^2}| = 2.147 \text{ sec} \Rightarrow \boxed{t_{1/2} = 153 \pm 2 \text{ sec}}$$

Sample calculations for non-linear regression $\Rightarrow$ Similar uncertainty transformations were done for non-linear regression. For example, for the parameter $a = \frac{1}{t_{1/2}}$:

$$t_{1/2} = \frac{1}{a} \Rightarrow u(t_{1/2}) = \sqrt{[\frac{d(\frac{-1}{a})}{da} \cdot u(a)]^2)} = |\frac{u(a)}{a^2}|$$

The value $R_S(0)$ and uncertainty $u(R_S(0))$ were directly extracted from a parameter in non-linear regression. So, using non-linear regression, we got the best fit parameter values as $(a, \overline{R_S(0)}) = (6.723 \cdot 10^{-3}, 3.561 \cdot 10)$ and their uncertainties as $(u(a), u(R_S(0))) = (9.439 \cdot 10^{-5}, 6.438 \cdot 10^{-1})$ Thus,

$$\boxed{R_S(0) = 35.6 \pm 0.6 \text{ counts/sec}}$$

and,

$$\overline{t_{1/2}} = \frac{1}{6.723 \cdot 10^{-3}} = 148.73 \text{ sec and } u(t_{1/2}) = |\frac{u(a)}{a^2}| = 2.01 \text{ sec}$$

$$\Rightarrow \boxed{t_{1/2} = 148 \pm 2 \text{ sec}}$$

6.2.2 Slow Decay

Calculation and uncertainty calculation for the mean rate of Fiesta plate emissions:

Let R_{mean} be the rate of emissions,

$$\begin{aligned}
R_{mean} &= \frac{N_{mean}}{\Delta t_{\text{Fiesta}}} \\
&= \frac{\text{mean(Fiesta Count)}}{\Delta t_{\text{Fiesta}}} \\
&= \frac{1}{\Delta t_{\text{Fiesta}}} \times \text{mean}\big(\text{Total Count - mean(Background Count} \cdot \frac{\Delta t_{\text{Fiesta}}}{\Delta t_{\text{Background}}})\big) \\
&= \frac{1}{3} \times \text{mean}\big(\text{Total Count - mean(Background Count} \cdot \frac{3}{20})\big) \\
&= \frac{1}{3} \times \text{mean(Total Count - 0.5125)} \\
&= 14.2467
\end{aligned}$$

Let $u(R_{mean})$ be the uncertainty in the rate of emissions.

$$\begin{aligned}
u(R_{mean}) &= \frac{\sqrt{N_{mean}}}{\Delta t_{\text{Fiesta}}} \\
&= \frac{1}{3} \times \sqrt{\text{mean(Total Count - 0.5125)}} \\
&= 2.1792
\end{aligned}$$