PHY 224: Introductory Report

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Exercise 1

In the first exercise, a circuit (figure 1) was created on the provided online laboratory (see PHET).It consists of an unknown resistor connected with a power source (a battery), a voltmeter connected in parallel with the resistor and an ammeter in series. The aim of this simulation was to deduce a

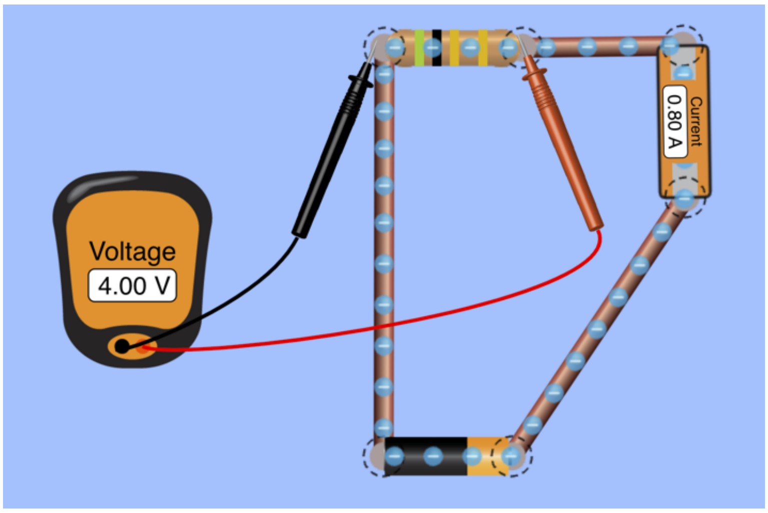

Figure 1: Circuit Schematic for Exercise 1 with unknown resistor

relationship between the voltage across the resistor and the current through it. The ammeter was connected in series with the resistor because it is known that the current (rate of flow of charge) is constant in a series connection due to the law of conservation of charge (and hence current). However, the voltmeter is connected in parallel with the resistor because we only intend to measure the potential difference across the resistor and not any additional segment of the wire. Therefore, the voltage of the battery was altered to 6 different values and the corresponding voltage across the resistor and the current through it were measured. The corresponding data is in table 1.

Table 1: Voltage and current across unknown resistance

Battery Voltage	Voltage from Voltmeter (V in Volts)	Uncertainty in Voltmeter reading ($u(V)$ in Volts)	Current from Ammeter (I in Amps)	Uncertainty in Current ($u(I)$ in Amps)	Resistance of resistor ($R = \frac{V}{I}$ in Ω)
1.0	1.00	0.02	0.20	0.01	5.0
2.0	2.00	0.02	0.40	0.01	5.0
3.0	3.00	0.02	0.60	0.01	5.0
4.0	4.00	0.02	0.80	0.01	5.0
5.0	5.00	0.02	1.00	0.01	5.0
6.0	6.00	0.02	1.20	0.01	5.0

Sample Uncertainty Calculation

The uncertainty in each voltmeter and ammeter reading was calculated using data sheet of the Keysight-U1272 multimeter set at the ranges of 300 V and 10 A respectively. For example, at the battery voltage of 4.0 volts, the voltage V across the resistor was measured to be 4.00 V and the current from ammeter to be 0.80 A.

According to the data sheet, at the 300 V level, the uncertainty in measurement is

$$u(V) = 0.05\% \cdot \ \textit{reading} + 2 \cdot \ \textit{precision}$$

At the 300 V level, the precision of the voltmeter was read from data sheet as 0.01 V.
Therefore, the uncertainty in voltage was calculated as

$$u(V)\Big|_{V=4.0V} = \frac{0.05}{100} \cdot 4.0 \text{ V} + 2 \cdot 0.01 \text{ V} = 0.0205 \text{ V} \approx 0.02 \text{ V (in 1 s.f.)}$$

Similarly, the precision of the ammeter at 10 A, assuming it being the Keysight-U1272 multimeter, was found to be 0.001 A, with the uncertainty being

$$u(I) = 0.3\% \cdot \ \textit{reading} + 10 \cdot \ \textit{precision}$$

So the particular uncertainty at $I = 0.80$ A is

$$u(I)\Big|_{I=0.80A} = \frac{0.3}{100} \cdot 0.8 \text{ A} + 10 \cdot 0.001 \text{ V} = 0.0124 \text{ A} \approx 0.01 \text{ A (in 1 s.f.)}$$

Graphical Analysis

Using the pairs of voltage and current values and their uncertainties, a graph of current vs voltage across the unknown resistor was plotted (see figure 2)

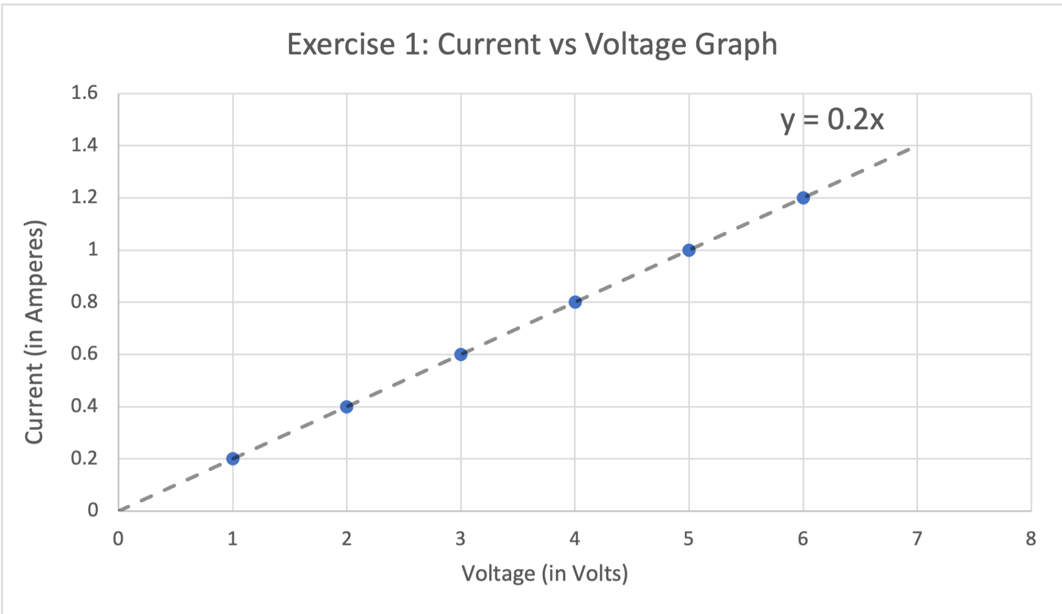

Figure 2: Current (I) through the unknown resistance plotted against different voltage values (V) across it. The points are marked as blue dots and the dotted line is the line of best fit.

The uncertainties in both the current and voltage are so small that they cannot be seen on the graph. Nevertheless, we get the line of best fit as $y = 0.2 \cdot x$, which means the slope is $\frac{I}{V} = 0.2$. Because we define resistance of a resistor as $\frac{V}{I}$, we get the best fit value of the resistance from the slope as

$$R = \frac{V}{I} = (\frac{I}{V})^{-1} = \frac{1}{0.2} \ \Omega = 5 \ \Omega$$

Now, I decided to calculate the standard deviation in the value of the resistance to get a measure of the uncertainty in its value. This is because it records the spread of values around the mean. So, the mean was calculated as:

$$\overline{R} = \frac{\sum_{i=1}^{6} R_i}{6} = 5.0\ \Omega$$

, where R_i corresponds to the value of resistance for the i-th pair of voltage-current values in table 1. Using this, we can calculate the standard deviation as:

$$s = \sqrt{\frac{1}{6-1}\sum_{i=1}^{6}(R_i - \overline{R})^2}\ \Omega = 0\ \Omega$$

The standard deviation is zero because all the R_is are exactly equal to $\overline{R}$. So, we can just conclude that the resistance of the unknown resistor is found to be $R = 5\ \Omega$. This in fact matches with the value of the resistor derived from its color code. The color code (green-black-gold-gold) yields a resistance value of $50 \cdot 0.1 \pm 5\%(50 \cdot 0.1)\ \Omega = 5.0 \pm 0.3\ \Omega$ which corresponds to our simulation result, thus providing confidence in the experiment.

Exercise 2

In this exercise, I manually changed the value of the given resistor to $R = 77.7\ \Omega$. The circuit looked like that in figure 3. As can be seen, the color codes of the resistor have changed.

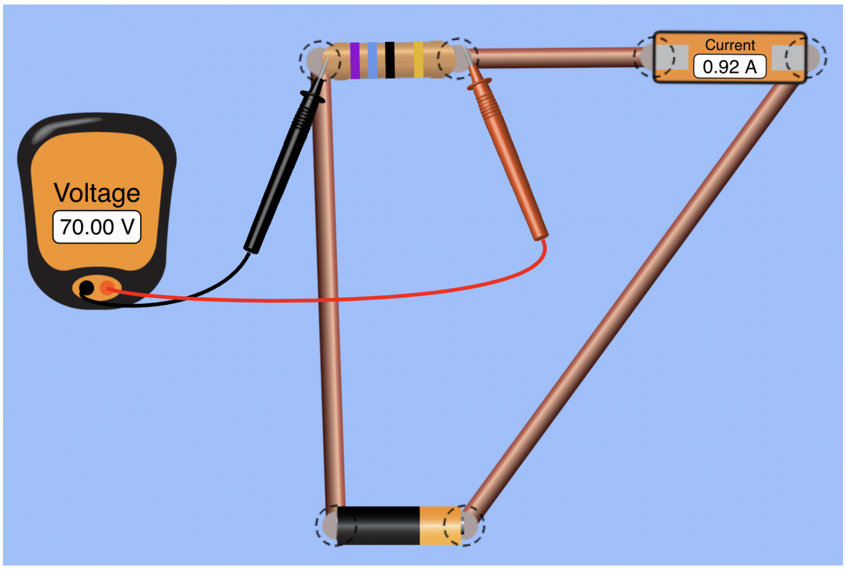

Figure 3: Circuit Schematic for Exercise 2 with a known resistor of resistance 77.7 Ω

Then, repeating the same procedure as in Exercise 1, I collected the voltage and current across the known resistor in table 2:

Table 2: Voltage and current across known resistance

Battery Voltage	Voltage from Voltmeter (V in Volts)	Uncertainty in Voltmeter reading ($u(V)$ in Volts)	Current from Ammeter (I in Amps)	Uncertainty in Current ($u(I)$ in Amps)	Resistance of resistor ($R = \frac{V}{I}$ in Ω)
20.0	20.00	0.03	0.26	0.01	77
30.0	30.00	0.04	0.39	0.01	77
40.0	40.00	0.04	0.51	0.01	78
50.0	50.00	0.05	0.64	0.01	78
60.0	60.00	0.05	0.77	0.01	78
70.0	70.00	0.06	0.90	0.01	78

Sample Uncertainty Calculation

The uncertainty in each voltmeter and ammeter reading was determined again using the formulas provided in Keysight-U1272 multimeter datasheet at the ranges of 300 V and 10 A respectively. For example, at the battery voltage of 70.0 volts, the voltage V across the resistor was measured to be 70.00 V and the current from ammeter to be 0.90 A.

According to the data sheet,the uncertainty in measurement is

$$u(V) = 0.05\% \cdot \ reading + 2 \cdot \ precision$$

At the 300 V level, the precision of the voltmeter was read from data sheet as 0.01 V. Therefore, the uncertainty in voltage was calculated as

$$u(V)\Big|_{V=70.00V} = \frac{0.05}{100} \cdot 70.00 \text{ V} + 2 \cdot 0.01 \text{ V} = 0.055 \text{ V} \approx 0.06 \text{ V (in 1 s.f.)}$$

Similarly, the precision of the ammeter at 10 A was found to be 0.001 A, with the uncertainty being

$$u(I) = 0.3\% \cdot \ reading + 10 \cdot \ precision$$

So the particular uncertainty at $I = 0.90$ A is

$$u(I)\Big|_{I=0.90A} = \frac{0.3}{100} \cdot 0.90 \text{ A} + 10 \cdot 0.001 \text{ V} = 0.0127 \text{ A} \approx 0.01 \text{ A (in 1 s.f.)}$$

Graphical Analysis

Using the pairs of voltage and current values and their uncertainties from table 2, a scatter graph of current vs voltage across the resistor of 77.7 Ω was plotted along with line of best fit (see figure 4)

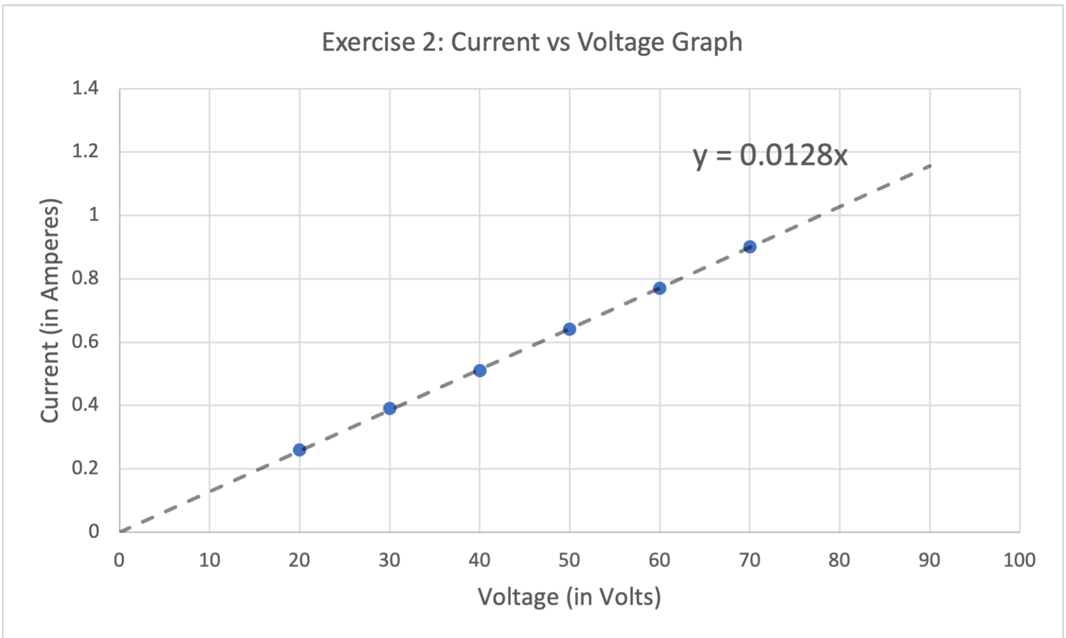

Figure 4: Current (I) through the resistor of 77.7 Ω plotted against different voltage values (V) across it. The points are marked as blue dots and the dotted line is the line of best fit.

The uncertainties along both axes are too small to be seen on the graph. The equation of line of best fit we get is $y = 0.0128 \cdot x$, which means the slope is $\frac{I}{V} = 0.0128$. Because we define resistance of a resistor as $\frac{V}{I}$, we get the best fit value of the resistance from the slope as

$$R = \frac{V}{I} = (\frac{I}{V})^{-1} = \frac{1}{0.0128} \ \Omega = 78.1 \ \Omega$$

Again, calculating the standard deviation in the value of the resistance was chosen as the way to measure the uncertainty in the resistance. For this, the mean resistance value must be calculated as:

$$\overline{R} = \frac{\sum_{i=1}^{6} R_i}{6} = 78.0\ \Omega$$

, where R_i corresponds to the value of resistance for the i-th pair of voltage-current values in table 2. Using this, the standard deviation in R is:

$$s = \sqrt{\frac{1}{6-1}\sum_{i=1}^{6}(R_i - \overline{R})^2}\ \Omega = 0.6\ \Omega$$

So, the resistance of the resistor is found to be $R = 78.0 \pm 0.6\ \Omega$. The value of the resistor from its color code (violet-silver-black-gold) yields a resistance value of $78 \cdot 1 \pm 5\%(78 \cdot 1)\ \Omega = 78 \pm 4\ \Omega$. Therefore, the resistance derived from the simulation is completely under the uncertainty budget provided by the resistor's color code.

Exercise 3

The goal of this exercise was to calculate the resistance of the wires connecting the circuit. For this, the known resistor from exercise 2 with resistance $R = 78.0 \pm 0.6\ \Omega$ was connected again to a battery and an ammeter in series (see figure 5). However, this time the voltmeter was connected across the battery to measure the exact voltage coming out of it. Before setting different voltages,

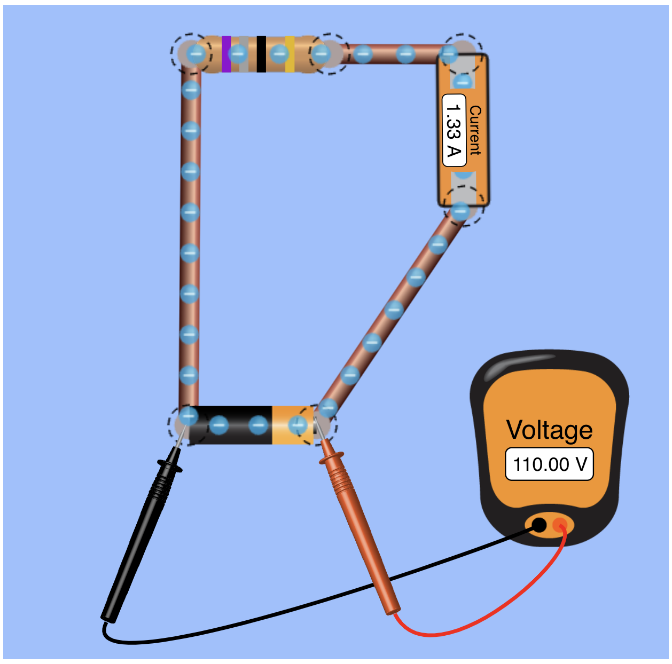

Figure 5: Circuit Schematic for Exercise 3 with the known resistor ($78.0 \pm 0.6\ \Omega$) connected with wires of increased resistivity

the resistivity of the connecting wires was set to somewhere between on the scale "tiny to lots" as it appears in the simulation lab. Note that we don't connect the voltmeter across our known resistance anymore because we already know it's voltage-current relationship (it's resistance).

We can simplify the task of measuring the wire's resistance by imagining it as a resistor of unknown resistance r connected with our known resistor R in series (see figure 6). The idea is to vary voltage of the battery and record the current. But this time, we will use the known resistance to get a sense of how the resistance of the system has changed.

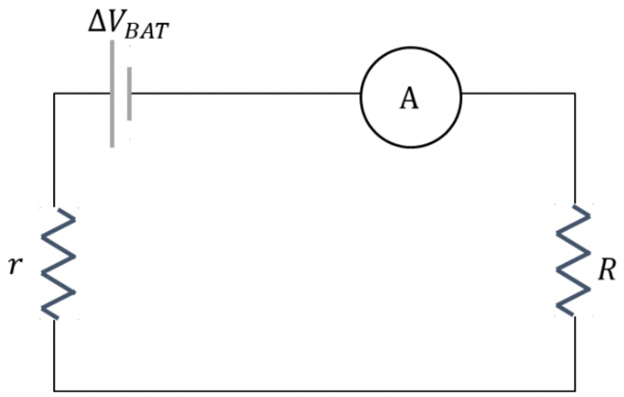

Figure 6: Circuit Schematic with the known resistor ($R = 78.0 \pm 0.6\ \Omega$) connected with wires of increased resistivity idealised as resistance r along with Ammeter (A) and battery (ΔV_{BAT}) (image courtesy of PHY 224 faculty)

The existing theory about circuits says that resistances add in a series arrangement. Then, according to Ohm's law, it must be that $V_{Net} = I_{Net} \cdot (R + r) \Rightarrow r = \frac{V_{Net}}{I_{Net}} - R$. Thus, we can repeat the same experiment to calculate r. After repeating the same procedure of altering battery voltage and recording current through ammeter, the recorded data is in table 3:

Table 3: Voltage and current data points with increased wire resistivity

Battery Voltage	Voltage across battery from Voltmeter (V_{Net} in Volts)	Uncertainty in Voltmeter reading ($u(VNet)$ in Volts)	Current from Ammeter ($INet$ in Amps)	Uncertainty in Current ($u(INet)$ in Amps)	Total resistance of system ($R_{Net} = \frac{VNet}{INet}$ in Ω)
60.0	60.00	0.05	0.73	0.01	82.2
70.0	70.00	0.06	0.85	0.01	82.4
80.0	80.00	0.06	0.97	0.01	82.5
90.0	90.00	0.07	1.09	0.01	82.6
100.0	100.00	0.07	1.21	0.01	82.6
110.0	110.00	0.08	1.33	0.01	82.7

Sample Uncertainty Calculation

The uncertainty in each voltmeter and ammeter reading was determined again using the formulas provided in Keysight-U1272 multi meter data-sheet at the ranges of 300 V and 10 A respectively. For example, at the battery voltage of 110.0 volts, the voltage V across the battery was measured to be 110.00 V and the current from ammeter to be 1.33 A.

According to the data sheet,the uncertainty in measurement is

$$u(V_{Net}) = 0.05\% \cdot \ reading + 2 \cdot \ precision$$

At the 300 V level, the precision of the voltmeter was read from data sheet as 0.01 V.
Therefore, the uncertainty in voltage was calculated as

$$u(V_{Net})\Big|_{V_{Net}=110.00V} = \frac{0.05}{100} \cdot 110.00\text{ V} + 2 \cdot 0.01\text{ V} = 0.075\text{ V} \approx 0.08\text{ V (in 1 s.f.)}$$

Similarly, the precision of the ammeter at 10 A was found to be 0.001 A, with the uncertainty being

$$u(I_{Net}) = 0.3\% \cdot \ reading + 10 \cdot \ precision$$

So the particular uncertainty at $I_{Net} = 1.33$ A is

$$u(I_{Net})\Big|_{I_{Net}=1.33A} = \frac{0.3}{100} \cdot 1.33 \text{ A} + 10 \cdot 0.001 \text{ V} = 0.014 \text{ A} \approx 0.01 \text{ A (in 1 s.f.)}$$

Graphical Analysis

Using the pairs of voltage and current values and their uncertainties from table 3, a scatter graph of current vs voltage was plotted along with line of best fit (see figure 7)

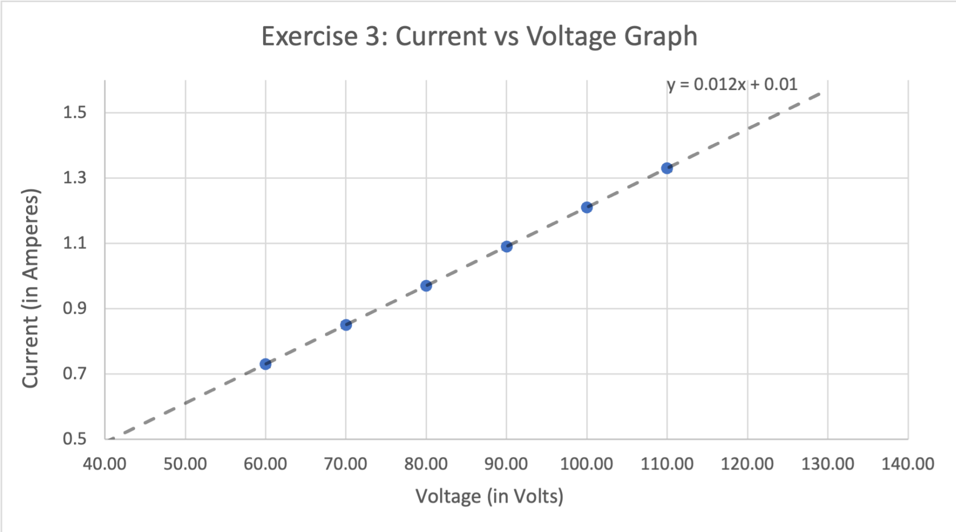

Figure 7: Current (I) through the whole circuit plotted against different net voltage values (V) across the system. The points are marked as blue dots and the dotted line is the line of best fit.

The uncertainties along both axes are too small to be seen on the graph. The equation of line of best fit we get is $y = 0.012 \cdot x + 0.01$, which means the slope is $\frac{I_{Net}}{V_{Net}} = 0.012$ and the y-intercept is 0.01. Ideally, the y-intercept should be 0 for Ohm's law to hold exactly. However, since we chose random values of voltage, it seems the slight alteration in the y-intercept might be due to statistical fluctuations in the simulation, which physically might be due to heating of resistor. Because total resistance is $\frac{V_{Net}}{I_{Net}}$, we get the best fit value of the net resistance from the slope as

$$R_{Net} = \frac{V_{Net}}{I_{Net}} = (\frac{I_{Net}}{V_{Net}})^{-1} = \frac{1}{0.012} \ \Omega = 83.3 \ \Omega$$

Again, calculating the standard deviation in the value of the resistance was chosen as the way to measure the uncertainty in the resistance. For this, the mean resistance value must be calculated as:

$$\overline{R_{Net}} = \frac{\sum_{i=1}^{6} R_{i,Net}}{6} = 82.5 \ \Omega$$

, where $R_{i,Net}$ corresponds to the value of resistance for the i-th pair of voltage-current values in table 3. Using this, the standard deviation in R_{Net} is:

$$s = \sqrt{\frac{1}{6-1}\sum_{i=1}^{6}(R_{i,Net} - \overline{R_{Net}})^2} \ \Omega = 0.19 \ \Omega \approx 0.2 \ \Omega$$

So, the net resistance in the system is found to be $R_{Net} = 82.5 \pm 0.2 \ \Omega$. From previous discussion, $r(R_{Net}, R) = R_{Net} - R$. So, $\overline{r} = (82.5 - 78.0) \ \Omega = 4.5 \ \Omega$

$$u(r) = \sqrt{(\frac{\partial r}{\partial R_{Net}} \cdot s)^2 + (\frac{\partial r}{\partial R} \cdot u(R))^2} = \sqrt{s^2 + u(R)^2} = \sqrt{0.19^2 + 0.6^2} \ \Omega = 0.62 \ \Omega$$

Thus, the wire's resistance is calculated as $r = 4.5 \pm 0.6 \ \Omega$