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(1) $\begin{bmatrix} x-1 & x+1 & x \\ -1 & x & 0 \\ 0 & 1 & 1 \end{bmatrix}$

It is given that x is real

$|A| = \begin{vmatrix} x-1 & x+1 & x \\ -1 & x & 0 \\ 0 & 1 & 1 \end{vmatrix}$

$= x^2 - x + x + 1 - x = x^2 - x + 1$

if $|A| = 0$, then $x^2 - x + 1 = 0$

$\therefore x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$

$x = \dfrac{1 \pm \sqrt{-3}}{2}$

$x = \dfrac{1}{2} \pm \dfrac{i\sqrt{3}}{2}$

$\therefore$ It is given x is real

Hence $|A| \neq 0$

$\therefore$ Rank $|A| = 3$

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(2) $\lambda = ?$

$x_1 + 2x_2 + x_3 = 3$

$x_1 + x_2 + x_3 = \lambda$

$3x_1 + x_2 + 3x_3 = \lambda^2$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 3 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ \lambda \\ \lambda^2 \end{bmatrix}$$

$[A|B]$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 1 & 1 & 1 & \lambda \\ 3 & 1 & 3 & \lambda^2 \end{array}\right]$$

$R_2 - R_1$, $R_3 - 3R_1$

$$[A|B] \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 0 & \lambda - 3 \\ 0 & -5 & 0 & \lambda^2 - 9 \end{array}\right]$$

$R_3 - 5R_2$

$$[A|B] \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 0 & \lambda - 3 \\ 0 & 0 & 0 & \lambda^2 - 5\lambda + 6 \end{array}\right] \rightarrow (1)$$

Rank $[A] = 2$

$\therefore$ The system will be consistent if

Rank $[A|B]$ = Rank $[A] = 2$

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$\Rightarrow \lambda^2 - 5\lambda + 6 = 0$

$\Rightarrow \lambda = 2, 3$

$\therefore$ The system is consistent when $\lambda = 2$, $\lambda = 3$

For $\lambda = 2$ using 1

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x + 2y + z_3 = 3$

$-x_2 = -1$

$\Rightarrow x_2 = 1$

rank $= 2 < 3$

$\therefore$ infinetly many solutions

no. of parameters $\Rightarrow 3 - 2 = 1$

$x_1 + 2 + z_3 = 3$

$x_1 + x_3 = 1$

let $x_3 = t \Rightarrow x_1 = 1 - t$

The sol^n is $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1-t \\ 1 \\ t \end{bmatrix}$ is infinitely no. of solutions.

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For $\lambda = 3$, using (1)

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

rank $= 2 < 3 \Rightarrow$ infinite no. of solutions

no. of parameters $= n - r = 1$

$x_1 + 2x_2 + x_3 = 3$

$-y = 0 \quad \Rightarrow \boxed{y = 0}$

$\Rightarrow x_1 + x_3 = 3$ let $x_3 = t \Rightarrow x_1 = 3 - t$

$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3-t \\ 0 \\ t \end{bmatrix}$ is the infinite no. of solutions.

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(3) let $k_1 x_1 + k_2 x_2 + k_3 x_3 = 0 \rightarrow (1)$

$k_1 [2 \; 3 \; 4 \; -2] + k_2 [-1 \; -2 \; -2 \; 1] + k_3 [1 \; 1 \; 2 \; -1] = 0$

$= [2k_1 \; +3k_1 \; 4k_1 \; -2k_1] + [-k_2 \; -2k_2 \; -2k_2 \; k_2]$
$+ [k_3 \; k_3 \; 2k_3 \; -k_3] = 0$

$= [2k_1 - k_2 + k_3 \quad 3k_1 - 2k_2 + k_3 \quad 4k_1 - 2k_2 + 2k_3 \quad -2k_1 + k_2 - k_3] = 0$

$2k_1 - 2k_2 + k_3 = 0$
$3k_1 - 2k_2 + k_3 = 0$ $\Rightarrow$ homogenous system of equations.
$4k_1 - 2k_2 + 2k_3 = 0$
$-2k_1 + k_2 - k_3 = 0$

$$\begin{bmatrix} 2 & -1 & 1 \\ 3 & 2 & 1 \\ 4 & -2 & 2 \\ -2 & 1 & -1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \rightarrow \frac{1}{2} R_1$

$$\begin{bmatrix} 1 & -1/2 & 1/2 \\ 3 & -2 & 1 \\ 4 & -2 & 2 \\ -2 & 1 & -1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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$R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 4R_1$, $R_4 \to R_4 + 2R_1$

$$\begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & -1/2 & -1/2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore$ rank$(A) = 2 < 3$

$\therefore$ There are infinitely many non-trivial sol^n.

$\therefore$ The given vectors are linearly dependent

$k_1 - \frac{1}{2}k_2 + \frac{1}{2}k_3 = 0$

$-\frac{1}{2}k_2 - \frac{1}{2}k_3 = 0$

$\therefore$ let $k_3 = t$

$-\frac{1}{2}k_2 = \frac{1}{2}t \quad \therefore k_2 = -t$

$k_1 - \frac{1}{2}(-t) + (t)(-\frac{1}{2}) = 0$

$k_1 + \frac{t}{2} + \frac{t}{2} = 0 \quad \therefore k_1 + t = 0$

$\therefore k_1 = -t$

Substituting in eq (1)

$-t x_1 + (-t)x_2 + (t)x_3 = 0$

$\therefore t x_3 = t x_2 + t x_1$

$\therefore x_3 = x_2 + x_1$

$\therefore$ The relation is $x_3 = x_2 + x_1$