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TUT 3 Complex No.

Q1) Prove :- $\left(\frac{-1+i\sqrt{3}}{2}\right)^n + \left(\frac{-1-i\sqrt{3}}{2}\right)^n$

is $= -1$ if $n = 3k \pm 1$

is $= 2$ if $n = 3K$

($k \rightarrow$ integer)

$\Rightarrow (-1)^n\left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)^n + (-1)^n\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right)^n$ (taking negative sign common)

modulus $\Rightarrow \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2} = 1$

$(-1)^n\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)^n + (-1)^n\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^n$

$(-1)^n\left(\cos\frac{\pi}{3} - i\sin\frac{n\pi}{3}\right) + (-1)^n\left(\cos\frac{n\pi}{3} + i\sin\frac{\pi}{3}\right)^n$

$n = 3k \pm 1$

$k = 0, 1, 2, 3 \ldots 9$

if $k = 0$

$n = 3(0) \pm 1$

$n = \pm 1$

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$n = 1$

$$(-1)\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right) + (-1)\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

$$-\cos\frac{\pi}{3} + \cancel{i\sin\frac{\pi}{3}} - \cos\frac{\pi}{3} - \cancel{i\sin\frac{\pi}{3}}$$

$$= -2\cos\frac{\pi}{3}$$

$$= -2 \times \frac{1}{2}$$

$$= -1$$

$n = -1$

$$(-1)\left(\cos(-1)\left(\frac{\pi}{3}\right) - i\sin(-1)\left(\frac{\pi}{3}\right)\right) + (-1)\left(\cos(-1)\left(\frac{\pi}{3}\right) + i\sin(-1)\left(\frac{\pi}{3}\right)\right)$$

$$(-1)\cos\left(-\frac{\pi}{3}\right) + (-1)\left(\cos\left(-\frac{\pi}{3}\right)\right)$$

$$= -2 \times \cos\frac{\pi}{3}$$

$$= -2 \times \frac{1}{2}$$

$$= 1$$

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~~if n = 3~~

if $n = 3k$
taking $k = 1$
$\therefore n = 3$

$$(-1)^3\left(\cos\frac{\pi(3)}{3} - i\sin(3)\left(\frac{\pi}{3}\right)\right) + (-1)^3\left(\cos 3\left(\frac{\pi}{3}\right) + i\sin 3\left(\frac{\pi}{3}\right)\right)$$

$$(-1)(\cos\pi - i\sin\pi) + (-1)(\cos\pi + \sin\pi)$$

$$(-1)((-1)-0) + (-1)((-1)-0)$$

$$= 1+1$$

$$= 2$$

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Q2) Given; $\cos(u+iv) = x+iy$

To proove

i) $(1+x^2)+y^2 = (\cosh v+\cos u)^2$

ii) $(1-x^2)+y^2 = (\cosh v-\cos u)^2$

Proof $x+iy = \cos(u+iv)$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ (identity)

$x+iy = \cos u\cos(hv) - \sin u\sin(hv)$

$x+iy = \cos u\cosh v + i(-\sin u\sinh v)$

$\to x = \cos u.\cosh v$ and $y = \sin u\sinh v$

Comparing real and imaginary parts we get

i) $L.H.S = (1+x)^2+y^2$

$= (1+\cos u.\cosh v)^2 + (-\sin u\sinh v)^2$

$= 1+\cos^2 u\cosh^2 v + 2\cos u\cosh v + \sin^2 u\sin^2 hv$

$= 1+\cos^2 u(1+\sinh^2 v) + 2\cos u\cosh v + \sin^2 u\sinh^2 v$

$= 1+\cos^2 u + \cos^2 u\sinh^2 v + \sin^2 u\sinh^2 v + 2\cos u\cosh v$

$= 1+\cos^2 u + \sinh^2 v(\cos^2 u+\sin^2 u) + 2\cos u\cosh v$

$= (1+\cos^2 u+\sinh^2 v) + 2\cos u\cosh v$

$= (1+\sinh^2 v)+\cos^2 u + 2\cos u.\cosh v$

$= \cosh^2 v+\cos^2 u + 2\cos u.\cosh v$

$= \cosh^2 v+\cos^2 u + 2\cos u.\cosh v$

$= (\cos v+\cos u)^2$

$= R.H.S$

Hence proved.

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ii) $L.H.S = (1-x)^2 + y^2$

$= (1-\cos u\cosh v)^2 + (-\sin u\sinh v)^2$

$= 1 + \cos^2 u\cosh^2 v - 2\cos u\cosh v + \sin^2 u\sinh^2 v$

$= 1 + \cos^2 u(\sinh^2 v) + \sin^2 u\sinh^2 v - 2\cos u\cosh v$

$= 1 + \cos^2 u + \cos^2 u\sinh^2 v + \sin^2 u\cdot\sinh^2 v - 2\cos u\cosh v$

$= 1 + \cos^2 u + (\cos^2 u + \sin^2 u)\sinh^2 v - 2\cos u\cosh v$

$= 1 + \cos^2 u + \sinh^2 v - 2\cos u\cdot\cosh v$

$= 1 + \sinh^2 v + \cos^2 u - 2\cos u\cdot\cosh v$

$= \cosh^2 v + \cos^2 u - 2\cos u\cdot\cosh v$

$= (\cosh v - \cos u)^2$

$= R.H.S$

Hence proved

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Q3) $\tan\left(\frac{\pi}{4} + iv\right) = re^{i\theta}$

$= r(\cos\theta + i\sin\theta)$

i) $\frac{\tan(\pi/4)\tan(iv)}{1 - \tan(\frac{\pi}{4})\tan(iv)} = r(\cos\theta + i\sin\theta)$

$\therefore \frac{1 + \tan(iv)}{1 - \tan(iv)} = r(\cos\theta + i\sin\theta)$

$\therefore \frac{1 + i\tanh v}{1 - i\tanh v} = r(\cos\theta + i\sin\theta) \rightarrow (A)$

$\therefore \frac{(1 + i\tanh v)}{(1 - i\tanh v)} \cdot \frac{(1 + i\tanh v)}{(1 + i\tanh v)}$ {multiplying by it component on numerator & denominator}

$= r(\cos\theta + i\sin\theta)$

$\therefore \frac{(1 - \tanh^2 v + 2i\tanh v)}{(1 + \tanh^2 v)} = r(\cos\theta + i\sin\theta)$

$\therefore \frac{(\mathrm{sech}^2 v + 2i\tanh v)}{(1 + \tanh^2 v)} = r(\cos\theta + i\sin\theta)$

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On comparing Real and Imaginary parts

$r\cos\theta = \frac{(\operatorname{sech}^2 v)}{(1+\tanh^2 v)} \rightarrow (a)$ and $r\sin\theta = \frac{(2\tanh v)}{(1+\tanh^2 v)} \rightarrow (b)$

Squaring both (a) and (b) and adding we get:

$$r^2(\cos^2\theta + \sin^2\theta) = \frac{(\operatorname{sech}^4 v) + 4\tanh^2 v}{(1+\tanh^2 v)^2}$$

$$\therefore r^2 = \frac{(1-\tanh^2 v)^2 + 4\tanh^2 v}{(1+\tanh^2 v)^2}$$

$$\therefore r^2 = \frac{(1+\tanh^2 v)^2}{(1+\tanh^2 v)^2}$$

Hence $r = 1$

ii) Dividing (b) by (a)
we get

$$\tan\theta = \frac{2\tanh v}{\operatorname{sech}^2 v} = 2\tanh v \cosh^2 v = \sinh 2v$$

Hence $\tan\theta = \sinh 2v$

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iii) Using (A)
by putting $r = 1$

$$= \frac{1 + i\tanh v}{1 - i\tanh v} = e^{i\theta}$$

Applying Componendo and Dividendo on both sides

$$\therefore \frac{2}{2i\tanh v} = \frac{e^{i\theta}+1}{e^{i\theta}-1}$$

$$= \frac{e^{i\theta/2} + e^{-i\theta/2}}{e^{i\theta/2} - e^{-i\theta/2}}$$

$$= \frac{2\cos(\theta/2)}{2i\sin\theta/2}$$

$\therefore$ Hence $\tanh v = \tan\left(\frac{\theta}{2}\right)$.