Batch: A3 Roll No.: 16010121051
Experiment / assignment / tutorial No. 4
Grade: AA / AB / BB / BC / CC / CD / DD

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Meet Gala

Somaiya TRUST (1)

Q1) $y - \cos x \frac{dy}{dx} = y^2(1-\sin x)\cos x$ $\left\{ \begin{array}{l} y=2 \\ x=0 \end{array} \right\}$

This can be written as $\cos x \frac{dy}{dx} - y = -y^2(1-\sin x)\cos x$

Dividing by $-y^2\cos x$

$\therefore \; -\frac{1}{y^2}\frac{dy}{dx} + \frac{\sec x}{y} = 1-\sin x$

putting $\frac{1}{y} = v$

$-\frac{1}{y^2}\frac{dy}{dx} = \frac{dv}{dx}$, we get $\frac{dv}{dx} + \sec x + v = 1-\sin x$

This is linear equation of form $\frac{dy}{dx} + Py = Q$

$\therefore$ I.F $= e^{\int p\,dx}$

I.F $= e^{\int \sec x\,dx}$

I.F $= e^{\log(\sec x + \tan x)}$

I.F $= \sec x + \tan x$

Solution:-

$v\,(\sec x + \tan x) = \int(\sec x + \tan x)(1-\sin x)\,dx + c$

$= \int \frac{(1+\sin x)}{\cos x}\cdot(1-\sin x)\,dx$

$= \int \frac{(1-\sin^2 x)\,dx}{\cos x}$

$$= \int \frac{\cos^2 x}{\cos x}\,dx = \int \cos x\,dx$$

$$= \sin x + c$$

$$\therefore \frac{\tan x + \sec x}{y} = \sin x + c$$

When $x = 0$

$y = 2$

i.e $\frac{1}{2} = c$

$$\therefore \tan x + \sec x = y\left(\sin x + \frac{1}{2}\right)$$

Q2) $xy(1 + xy^2)\frac{dy}{dx} = 1$

$$\frac{dx}{dy} = xy + x^2 y^3$$

$$\therefore \frac{1}{x^2}\frac{dx}{dy} - \frac{1}{x}\,y = y^3$$

putting $\frac{-1}{x} = v$

$$\frac{1}{x^2}\frac{dx}{dy} = \frac{dv}{dy}$$

we get $\frac{dv}{dy} + vy = y^3$

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This is linear differential equation

$\therefore e^{\int P\cdot dy} = e^{\int y\,dy} = e^{y^2/2}$

$\therefore$ solution $= v e^{y^2/2} = \int e^{y^2/2} \cdot y^3 dy + c$

$= e^{y^2/2}(y^2-2)$ $\left\{ \text{put } \frac{y^2}{2} = t \right)$

The solution is

$v \cdot e^{y^2/2} = e^{y^2/2}(y^2-2) + c$

$= \frac{-1}{x} e^{y^2/2} = e^{y^2/2}(y^2-2) + c$

$\therefore \frac{-1}{x} = y^2 - 2 + c e^{-y^2/2}$

$\therefore \frac{1}{x} = 2 - y^2 + c e^{-y^2/2}$

Q3) $\lim\limits_{x\to 0} \frac{x^{1/2}\tan x}{(e^x-1)^{3/2}}$

Let $l = \lim\limits_{x\to 0} \frac{\sqrt{x}\tan x}{(e^x-1)^{3/2}}$ $\left[\frac{0}{0} \text{ form}\right]$

$= \lim\limits_{x\to 0} \frac{x\sqrt{x}}{(e^x-1)^{3/2}} \cdot \frac{\tan x}{x}$

$= \lim\limits_{x\to 0} \frac{x\sqrt{x}}{(e^x-1)^{3/2}} \cdot \lim\limits_{x\to 0}\left(\frac{\tan x}{x}\right)$

$$= \lim_{x \to 0} \left(\frac{x}{e^{x}-1}\right)^{3/2} \quad \left[\because \lim_{x \to 0} \frac{\tan x}{x} = 1\right]$$

Now, $\lim_{x \to 0} \frac{x}{e^{x}-1} = \lim_{x \to 0} \frac{1}{e^{x}} = 1$

[Applying L'Hospital's rule]

$$\therefore \lim_{x \to 0} \left(\frac{x}{e^{x}-1}\right)^{3/2} = (1)^{3/2} = 1$$

Hence, $\underline{\underline{l = 1}}$

Q4) $\lim_{x \to \infty} \frac{e^{x}}{\left[\left(x+\frac{1}{x}\right)^{x}\right]^{x}} = e^{1/2}$

Let $L = \lim_{x \to \infty} \frac{e^{x}}{\left[\left(1+\frac{1}{x}\right)^{x}\right]^{x}} = \lim_{x \to \infty} \frac{e^{x}}{\left(1+\frac{1}{x}\right)^{x^2}}$

$$\therefore \log L = \lim_{x \to \infty} \log . \frac{e^{x}}{\left(1+\frac{1}{x}\right)^{x^2}}$$

$$= \lim_{x \to \infty} \left[\log e^{x} - \log \left(1+\frac{1}{x}\right)^{x^2}\right]$$

$$= \lim_{x \to \infty} \left[x - x^2 \log\left(1+\frac{1}{x}\right)\right] \Rightarrow [\infty - \infty]$$

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(3)

$$\log L = \lim_{x\to\infty} x\left[1 - x\log\left(1+\frac{1}{x}\right)\right]$$

$$= \lim_{x\to\infty} \frac{\left[1 - x\log\left(1+\frac{1}{x}\right)\right]}{\left(\frac{1}{x}\right)}$$

By applying L'Hospital's rule:

$$\log L = \lim_{x\to\infty} \frac{0 + \left[(1)\log\left(1+\frac{1}{x}\right) + x\left(\frac{1}{1+\frac{1}{x}}\right)\left(\frac{-1}{x^2}\right)\right]}{\left(\frac{1}{x^2}\right)}$$

$$= \lim_{x\to\infty} \frac{\log\left(1+\frac{1}{x}\right) + \left(\frac{x}{x-1}\right)\left(\frac{-1}{x}\right)}{\left(\frac{1}{x^2}\right)}$$

$$= \lim_{x\to\infty} \frac{\log\left(1+\frac{1}{x}\right) - \frac{1}{(x+1)}}{\left(\frac{1}{x^2}\right)}$$

Again by L'Hospital rule:

$$\log L = \lim_{x\to\infty} \frac{\left(\frac{1}{(1+\frac{1}{x})}\right)\left(\frac{-1}{x^2}\right) + \frac{1}{(x+1)^2}}{\left(\frac{-2}{x^3}\right)}$$

$$= \lim_{x\to\infty} \frac{1}{\left(1+\frac{1}{x}\right)} \frac{\left[-1 + \frac{1}{(1+1/x)}\right]}{\left(-2/x\right)}$$

$$= \lim_{x\to\infty} \frac{1}{(1+\cancel{1/x})^2}\ 0 \ \frac{\left(-1 - 1/x + 1\right)}{\left(-2/x\right)}$$

$$= \lim_{x\to\infty} \frac{\left(-1/x\right)}{\left(-2/x\right)}$$

$$\log L = \frac{1}{2}$$

$$\boxed{\therefore L = e^{1/2} \quad = R.H.S}$$

Hence proved.