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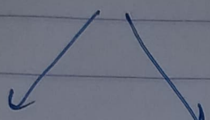
EM - SHORT NOTES

(1) Coplanar & Concurrent Forces:-

* Resultant of Forces:- Force or a couple that will have the same effect to the body, both in translation & rotation



$R_x = \Sigma F_x$ $R_y = \Sigma F_y$

$\tan \theta = \frac{R_y}{R_x}$

* Coplanar Forces :- Forces which occur in the same plane

Concurrent Forces:- Forces such that their lines of action meet at a single point

* Polygon Law → It states that if number of coplanar - concurrent forces acting simultaneously on a body be represented as sides of a polygon then the closing side of polygon will represent the resultant force

(This can be verified easily by vector addition) or any such graphical method.

* Resultant of Forces Theorem can be held true by various examples one being the Universal Force Table. Exp: We keep various ~~rati~~. Forces on various angles and add a resultant weight in a direction where we can acehieve equilibrium. The particular magnitude + direction is called the Resultant Force.

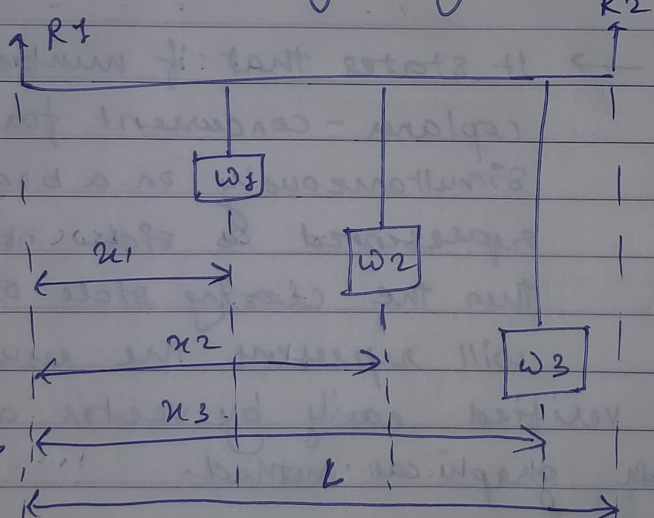
(2) Support Reaction Beam :-

* Beams :- Beams are structural members which are generally horizontal. They are subjected to lateral forces which act ~~ho.~~ orthogonally to the length of member.

* Supports :- There are various types of mechanisms used for supporting the beams. At the supports. The reactive forces are developed which are determined by using the concept of equilibrium.

* FBD :- A free body daigram is a graphical illustration used to visualize ~~to~~ applied forces ~~on~~ any body

* eg



$F_1 = w_1 g$

$F_2 = w_2 g$

$F_3 = w_3 g$

equilibrium

(1) $-w_1 x_1 - w_2 x_2 - w_3 x_3 + R_2 L = 0$

(2) $R_1 + R_2 = F_1 + F_2 + F_3.$

$\therefore \Sigma M = 0$

$\therefore$ This is because sum of moments about a reaction point is equal to zero. $(\Sigma M = 0)$

* Equilibrium :- A condition in which all influences acting upon a body are cancelled by others resulting in a stable, balanced or unchanging system.

* Types of Sapports :-

ⓐ Hinge Support :- → Rotational Motion ✓
→ linear Motion ✗

ⓑ Roller Support :- It is free to roll on a surface on which it rests. Reaction is in the direction of the normal to the support.

ⓒ Fixed Support :- → Rotational motion ✗
→ Linear motion ✗
But its forces can be split up in various axes/ components.

ⓓ Smooth Surface Support :- Very much similar to Roller support just that frictional forces are not offered by the resting plane

ⓔ Rope / string / Cable Support :- It offers a pull force called as Tensional Force caused by Gravity. Away from the Gravitational force direction.

* Types of Loads :-

(a) Point Load :- Forces applied on a single point

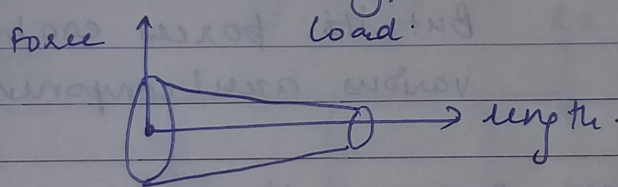
(b) Uniformly distributed Load :- Load is uniformly spread over length

→ * Equivalent Load = Load Intensity × Length.

(c) Uniformly varying Load :- Load is uniformly varied over a length.

→ * We need to integrate Intensity × Length to get the final equivalent load.

(d) Trapezoidal Load :- A type of Uniformly varying Load.

→ * 

(e) Couple Load :- If a couple load acts on any Body a rotation motion is caused in the body.

(f) Varying Load :- Load varies with some relation and equilibrium load must be found using integration.

* Newton's Laws of Motion:-

① An objects motion is uniform until acted on by a force externally.

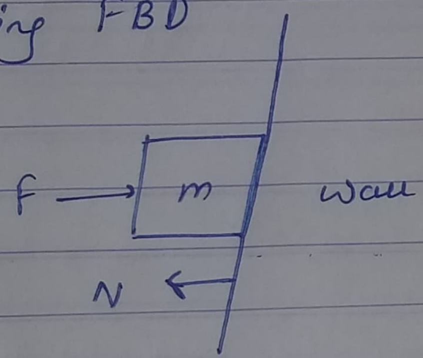
② Acceleration of an object is directly proportional to mass and force

⟶ $f = m \cdot a$

↓ force ↘ mass ⟶ Acceleration.

③ For every action there is an equal and opposite reaction.

* Using FBD

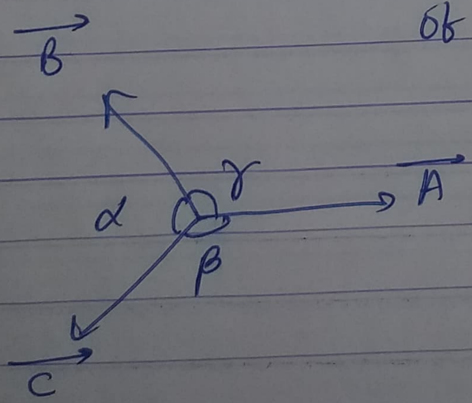


⟶ here; $f = m \cdot a = N$

$f = N$

$a = \frac{f}{m} = \frac{N}{m}.$

* LAMIS THM ⟶ It states that if the resultant of three vectors is 0 then the magnitude of a vector is directly proportional to the sine of angle between other two vectors.

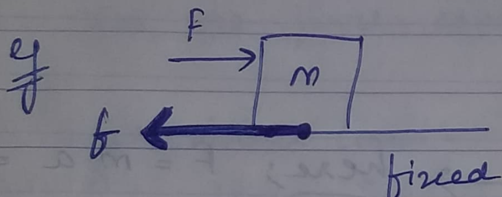


⟶ $\frac{|\vec{A}|}{\sin(\alpha)} = \frac{|\vec{B}|}{\sin(\beta)} = \frac{|\vec{C}|}{\sin(\gamma)}$

③ Friction :-

* Frictional Force :- A force that is created whenever 2 forces / surfaces move or try to move across each other.

* Friction always opposses the motion of one surface across another surface
* Friction is dependant on Both Ammount of Contact Forces / mass as well as Texture of the surface.

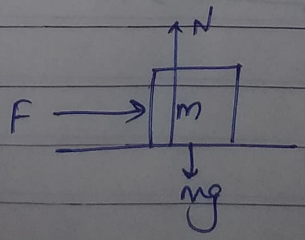


* Friction → Static (μ_s) → B/w two or more given surfaces that are not moving respective to each other
* Friction → Kinetic (μ_K) → B/w two or more moving surfaces.

* COF (μ) → Coefficient of Friction. is basically the measure of amount of interaction between any two surfaces mayb or may not be moving respective to each other.

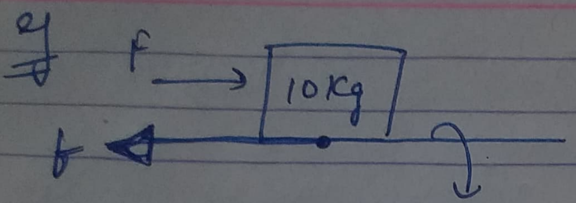
* $0 \leq \mu \leq 1$

$\mu_s > \mu_K$.



$$\boxed{\mu = \frac{F}{N}}$$

Eg



$\mu_s = 0.5\ N$

$\mu_k = 0.4\ N$

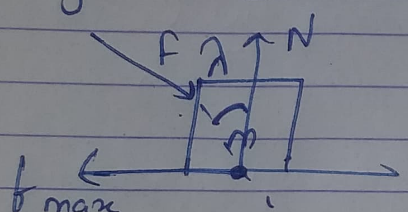
$f_{max} = \mu_s \times N$

$f_{max} = 0.5 \times (10 \times g)$

$\underline{f_{max} = 50N.}$

This means that the block is ~~kit~~ at rest until $F > 50N$ and frictional force ~~of~~ acting will be equal to F if $F \leq 50N$.

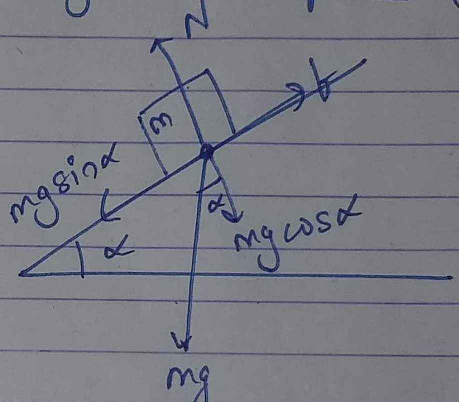
★ Angle of Friction (λ)



★ $\tan(\lambda) = \frac{f_{max}}{N} = \frac{\mu_s \times \cancel{N}}{\cancel{N}}$

$\lambda = \tan^{-1}(\mu_s)$

Angle of Repose (α)



★ $mg\sin(\alpha) = f = \mu_s(N)$

$\cancel{mg}\sin(\alpha) = f = \mu_s(\cancel{mg}\cos(\alpha))$

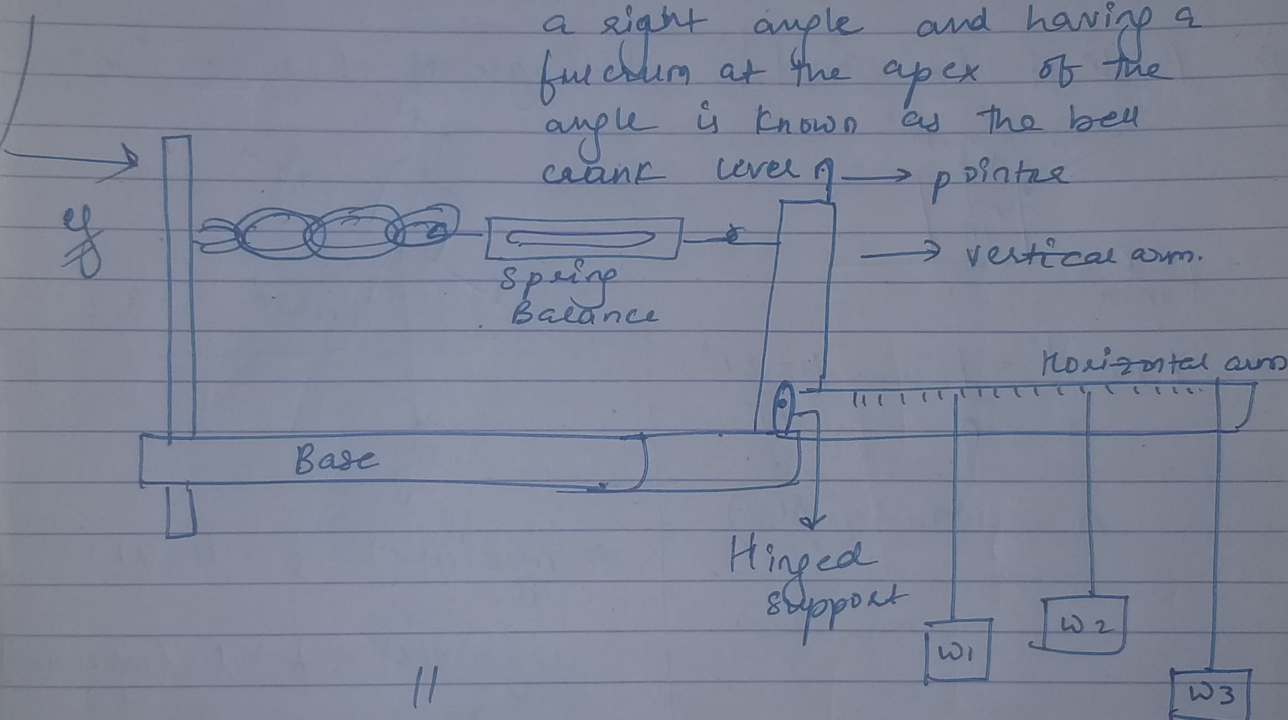
$\tan(\alpha) = \mu_s$

$\alpha = \tan^{-1}(\mu_s)$

(4) Bell Crank Lever:-

* Principle of Moments:- The algebraic sum of moments of a system of a coplanar forces about any point is equal to the moment of the resultant of a force of the system about the same point.

* Bell Crank lever:- A lever who's two arms form a right angle and having a fulcrum at the apex of the angle is known as the bell crank lever



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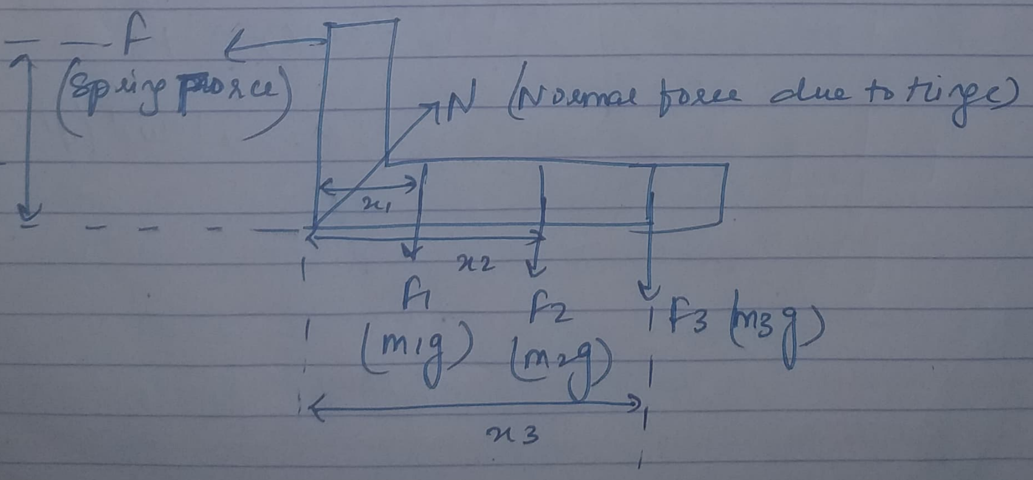


$T_1 \rightarrow$ Initial spring reading

$T_2 \rightarrow$ Final spring reading on adding weights.

$\therefore$ Tensile Force of Vertical Arm $= T_2 - T_1$
(F)

$$\Sigma M_0 = F \times y - W_1 x_1 - W_2 x_2 - W_3 x_3$$

or

$$= (T_2 - T_1) y - F_1 x_1 - F_2 x_2 - F_3 x_3$$

(5) Newton's Second Law :-

* Force applied to a body produces a proportional acceleration

$$\therefore F_1 + F_2 + \cdots + F_n = F_{net} = m \times a$$

$$\rightarrow F = m \times a$$

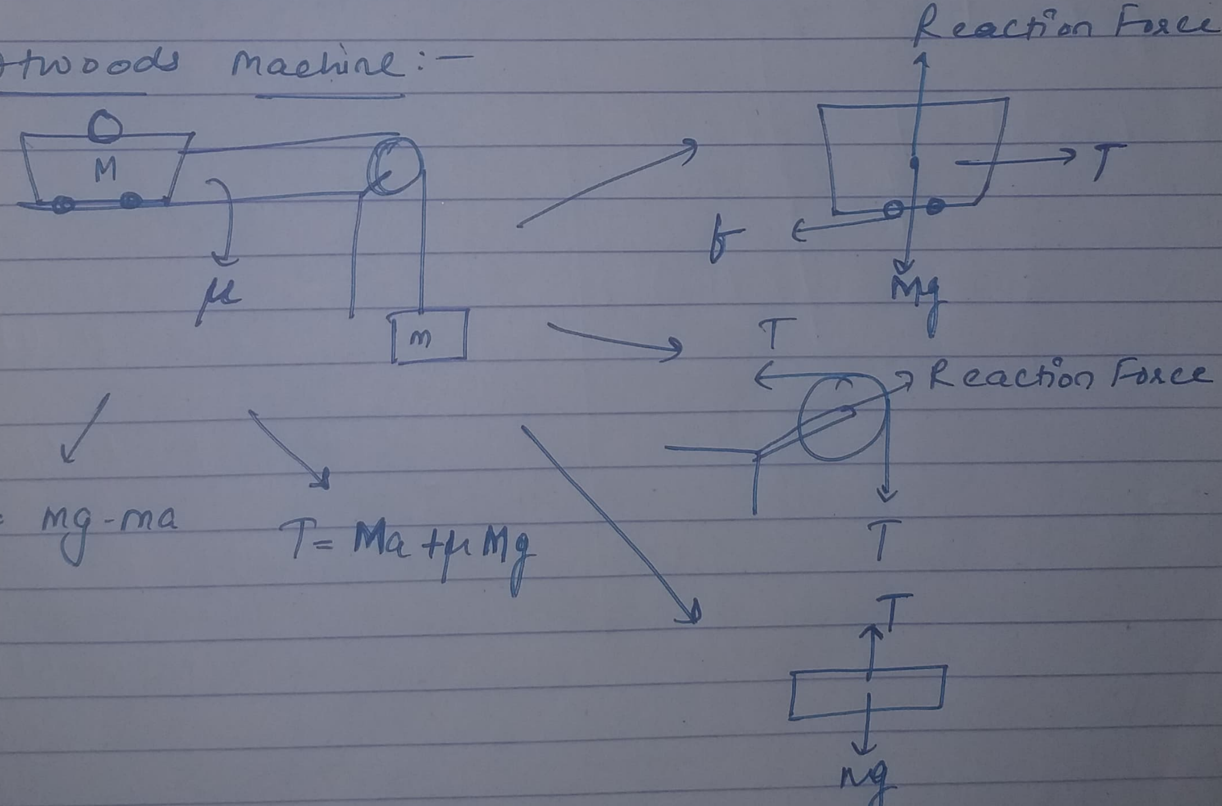
$$= m \times \frac{dv}{dt} \qquad \left(\because dp = m\,dv\right)$$

$$= \frac{m \times dv}{dt}$$

$$F = \frac{dp}{dt}$$

$\therefore$ Force is defined as rate of change of momentum.

* Atwoods Machine :-



$T = mg - ma$ $\qquad$ $T = Ma + \mu Mg$

$$acc = \frac{mg - \mu Mg}{M + m}$$

D'AlemBerts Principle :- For any body the algebraic sum of externally applied forces and the forces resisting motion in any given direction is zero.

To calculate distance travelled by cart :-

(1) $a = \frac{mg - \mu Mg}{M + m}$

(2) $S = \cancel{ut}^{0} + \frac{1}{2} at2$ $(\because u = 0)$

$S = \frac{1}{2} at^2$

(6) Moment of Inertia of fly wheel :-

→ $PE_{loss} = m \times g \times h$

$m \downarrow$ (mass of weight hanger)

→ $KE_{rotational} = \frac{1}{2}(I)\omega^2$ → (angular velocity)

$I \downarrow$ (moment of Inertia)

→ $KE_{translation} = \frac{1}{2} m \times v^2$

→ $Work_{friction} = n \times W_f$ → (work done to overcome frictional torque)

$n \downarrow$ (no. of ~~cords~~ times the cord is wrapped around the axle)

→ $PE_{loss} = KE_{Rotational} + KE_{translational} + Work_{friction}$

$$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 + nW_f$$

(∵ Kinetic energy of the flywheel assembly is expended in rotating 'N' times against the same frictional torque.)

↳ $NW_f = \frac{1}{2}(I)\omega^2$

$$W_f = \frac{1}{2N}(I)\omega^2 \quad \text{——— (i)}$$

also; $v = \omega r$ ——— (2)

→ $$\boxed{I = \frac{Nm}{N+n}\left(\frac{2gh}{\omega^2} - r^2\right)}$$

* $N \rightarrow$ no. of times flywheel rotated before stopping
  $n \rightarrow$ no. of windings of the string on axle.

* $$\omega_{average} = \frac{2\pi N}{t}$$

* Moment of Inertia :- The measurement of Rigid Bodies to Resist the change in their state of Rotational Motion.

(7) Elastic & Inelastic Collision :-

* **Collission** :- The abrupt change caused in the path of any moving body or bodies due to interaction with the other body is called collission.

* **Linear Momentum ~~Consersion~~ Conservation** :- Total linear momentum of a system of a particle is conserved if there is no external force acting on the system.

* **Conservation of Energy** :- Energy can neither be destroyed nor can be created but can be converted from one form of energy to another.

* **Collission**

→ Elastic :- Conservation of momentum ✓
Conservation of KE ✓
$e = 1$

→ Inelastic :- Conservation of momentum ✓
Conservation of KE ✗
$e < 1$

* **Solving Elastic collission Q's:**

(1) $p^1 + p^2 = p'_1 + p'_2$

$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$ ← (*)

② $KE_1 + KE_2 = KE_1' + KE_2'$

$$\frac{1}{2}m_1(v_1)^2 + \frac{1}{2}m_2(v_2)^2 = \frac{1}{2}m_1(v_1')^2 + \frac{1}{2}m_2(v_2')^2 \leftarrow ★$$

→ we can calculate v_1' and v_2' using these two formullas:-

→ if $m_1 = m_2$ then $V_1' = V_2$, $V_2' = V_1$ } exchange of velocities on colission.

→ if $V_2 = 0$ then:

$$V_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) V_1$$

$$V_2' = \left(\frac{2m_1}{m_1 + m_2}\right) V_2$$

★ Solving Inelastic Collision Q's :-

Coefficient of Restitution (e) :- the ratio of the final to initial velocity between 2 objects after they collide

★ $e \geq 0$ and $e \leq 1$

★ e → 1 → perfectly elastic
→ 0 → perfectly inelastic
→ < 1 → inelastic

① $p_1 + p_2 = p_1' + p_2'$

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \leftarrow ★$$

② $e = \frac{v_2' - v_1'}{v_1 - v_2}$

* Impulse :- Ammount of change in an objects momentum.

$$(J) = \Delta p = P_f - P_i$$

$$\Delta p = m(v_f - v_i)$$

$$\Delta p = F(\Delta T)$$ ← product of force applied to an object and the ammount of time applied.

(8) Projectile Motion :-

* **Projectile** :- • Any body thrown with some initial velocity which is then allowed to move under the action of gravity alone.

- The followed path is called Trajectory.

- Horizontal component of speed doesn't change (V_x)

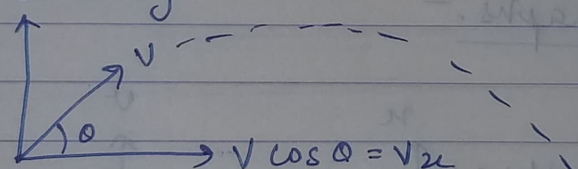
  ∴ Horizontal distance travelled :-

  $$S_x = V_x \times t$$

- $$V'_y = V_y + at$$

  $$S_y = V_y t + \frac{1}{2}at^2$$

- $v \sin\theta = V_y$



- Range :- $\underbrace{v \cos\theta}_{\text{Horizontal speed}} \times \underbrace{t}_{\text{time of flight}}$

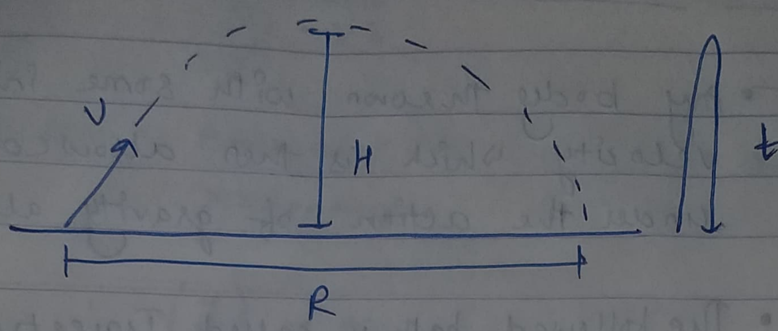
  max when $\theta = 45^\circ$

- time of flight :- $t = \dfrac{2u\sin\theta}{g}$

- max height :- $H = \dfrac{u^2 \sin^2\theta}{2g}$

- Range :- $R = \dfrac{u^2 \sin 2\theta}{g}$

diagram



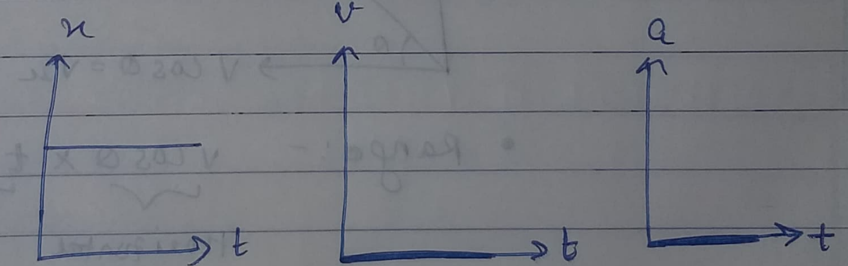
Basic Kinematic Equations:-

1. $v = u + at$
2. $v^2 = u^2 + 2as$
3. $s = ut + \frac{1}{2}at^2$

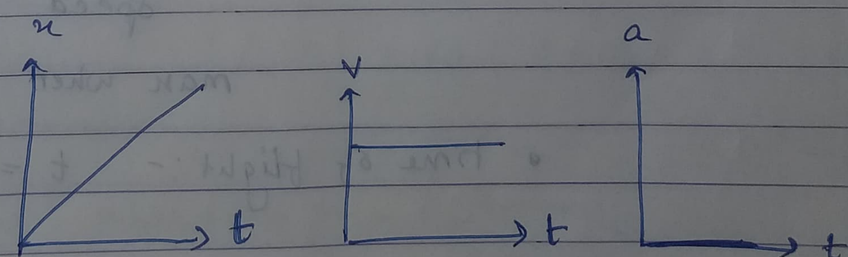
only when acc is constant or else use diff / integration.

Kinematics graphs:-

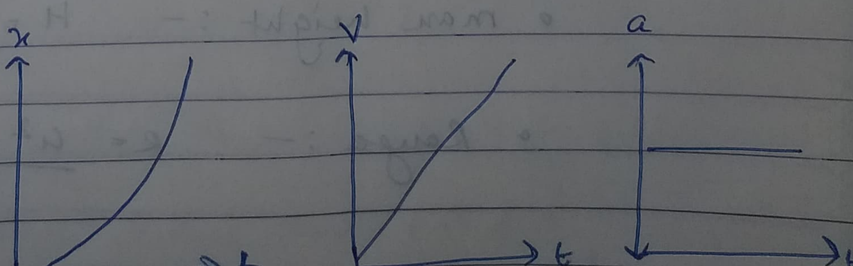
Stationary object:-



uniform motion:-



motion under constant acc →



* $V = \frac{ds}{dt}$ → slope of s-t ~~gtg~~ graph

$a = \frac{dv}{dt}$ → slope of v-t graph.

$a = \frac{v\,dv}{ds}$

$V = \int a\,dt$ → area under a-t graph.

$S = \int v\,dt$ → area under v-t graph

(9) Compound Pendulum :-

* Work Energy Theorem :- The work done by the sum of all forces acting on a body equals to the net change in the kinetic energy of the particle.

* Impulse - Momentum Principle :- The Impulse acting on an object produces a change in momentum of the object that is equal in both magnitude and direction to the impulse.

if $\Delta P = 0$ or $F = 0$ then total momentum has no change as a fn of time.

or else :- $\boxed{F_{net} \Delta t = m \Delta V}$

* K = radius of Gyration :-

$$T = 2\pi \left\{ \frac{(h^2 + K^2)}{gh} \right\}^{1/2}$$

$$T^2 h = \left(\frac{4\pi^2}{g}\right) h^2 + \left(\frac{4\pi^2}{g}\right) K^2$$

(converting to $y = mx + c$ for hT^2 v/s h^2)

$$m = 4\pi^2 / g$$

$$c = \left(\frac{4\pi^2}{g}\right) k^2$$

Hence by solving & observing values of 'c' and 'm' we can find radius of gyration. 'K'

(10) Jib Cranes:-

* **Crane :-** A crane is a type of machine, generally equipped with a hoist, ropes or chains and sheaves, that can be used both to lift and lower materials and to move them horizontally

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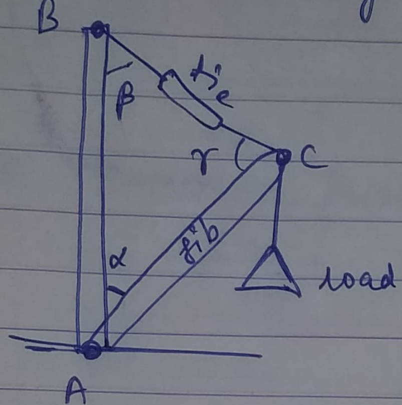

$$\frac{F_{AB}}{\sin\alpha} = \frac{F_{BC}}{\sin\beta} = \frac{F_{AC}}{\sin\gamma}$$

* **Centre of gravity :-** Centre of gravity or centre of mass is basically the average location of the weight of an object or a location where all the weight can be assumed to be accumulated.

* **Centroid:-** The centroid is the centre of mass of an object, the average position of all points would be in the object.

* **Varignon's Thm :-** Suppose two concurrent coplanar forces act on a point. The moment of forces about a point is equal to the sum of the moments of the forces about the point.

* Inertia of rest :- The resistance of a body to change its state of rest is called inertia of rest and the vice - versa is called Inertia of motion.

* Ballistic pendulum :- A device used for measuring a bullet's momentum. from which it's possible to calculate the velocity and kinetic energy.

* point of transmissibility :- It states that the point of application of a force can be moved anywhere along its line without changing the external reaction forces on a rigid body.

* Velocity & Acceleration in coordinate system :-

$$\vec{V} = \frac{d\vec{r}}{dt}$$

$$\vec{V}_x = \frac{dx}{dt} = \int a_x \, dt$$

$$\vec{V}_y = \frac{dy}{dt} = \int a_y \, dt$$

$$x = \int V_x \, dt \qquad y = \int V_y \, dt.$$

* centroid of lamina :- The point on which it would balance perfectly when placed on a needle.