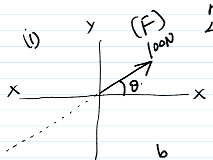
Module 1

Tuesday, October 12, 2021 10:23 AM

https://www.youtube.com/watch?v=ksmsp9OzAsI

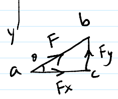
Force
= point of Application
Magnitude
Line of action

(i)



$\theta = 30^\circ$

Resolution of force

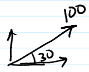


$\cos\theta = \frac{F_x}{F}$

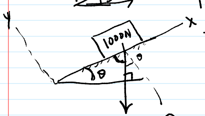
$F_x = F\cos\theta$

$\sin\theta = \frac{F_y}{F}$

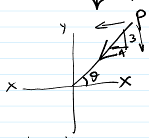
$F_y = F\sin\theta$



$\vec{F} = (100\cos 30)(\hat{i}) + (100\sin 30)\hat{j}$

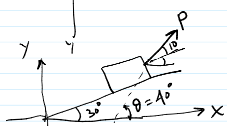


$\vec{W} = (-1000\sin\theta)\hat{i} + (-1000\cos\theta)\hat{j}$

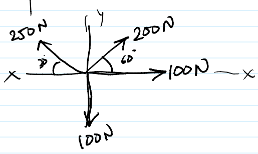


$\bar{P} = (-P\cos\theta)\hat{i} + (-P\sin\theta)\hat{j}$

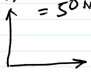
$= -P(0.8)\,\hat{i} - P(0.6)\hat{j}$



$\bar{P} = (P\cos 40)\,\hat{i} + (P\sin 40)\hat{j}$

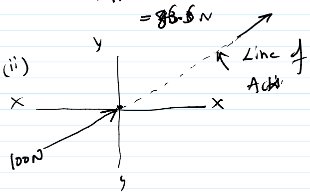


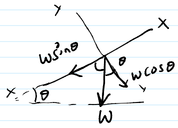
$F_y = 100\sin 30$
$= 50\text{ N}$



$F_x = 100\cos 30$
$= 86.6\text{ N}$

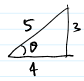
(ii)





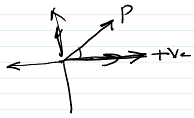
$W\sin\theta = W\cos(90-\theta)$

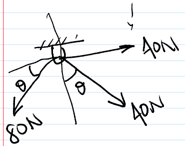
$W\cos\theta = W\sin(90-\theta)$



$\cos\theta = \frac{4}{5} = 0.8$

$\sin\theta = \frac{3}{5} = 0.6$

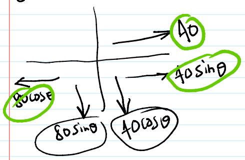




Find the angle θ if the resultant of 3 forces is vertical and also find the resultant.

Since R is vertical $\therefore R_x = \Sigma F_x = 0$

$R = \sqrt{R_x^2 + R_y^2} = R_y$



$R_x = \Sigma F_x = 0$

$0 = \cancel{40} + \cancel{40}\sin\theta - \cancel{80}^{2}\cos\theta$

$2\cos\theta = 1 + \sin\theta$

$\cos\theta = 0.5 + 0.5\sin\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + [0.5 + 0.5\sin\theta]^2 = 1$

$\sin^2\theta + (0.5)^2 + 2\times 0.5\times\sin\theta\times 0.5 + (0.5\sin\theta)^2 = 1$

$\sin^2\theta\,[1 + 0.5^2] + 0.5\sin\theta + [0.5^2 - 1] = 0$

$ax^2 + bx + c = 0 \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \sin\theta \qquad a = 1 + 0.5^2, \quad b = 0.5, \quad c = (0.5^2 - 1)$

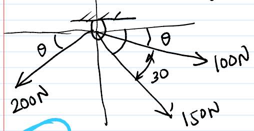
$\sin\theta = 3/5 = 0.6 \qquad \theta = \sin^{-1}\left(\frac{3}{5}\right) = 36.87^\circ$

$\cos\theta = \frac{4}{5} = 0.8$

$R = R_y = 80\sin\theta + 40\cos\theta$

$= 80\times\frac{3}{5} + 40\times\frac{4}{5} = 80\text{N}$

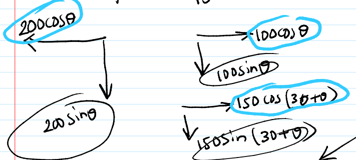
$R = 80\text{N} \downarrow$



What is the value of angle θ if R is vertical and also find R.

$R_x = \Sigma F_x = 0$ [∵ R is vertical]

$0 = 100\cos\theta + 150\cos(30+\theta) - 200\cos\theta$



$10\cancel{0}\cos\theta = 15\cancel{0}\cos(30+\theta)$

$\frac{10\cos\theta}{15} = \cancel{15}[\cos 30\cos\theta - \sin 30\sin\theta]$

on simplification $\tan\theta = \frac{30}{75}$

$\theta = 21.8^\circ$

$\frac{10}{15}\cos\theta = \frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta$

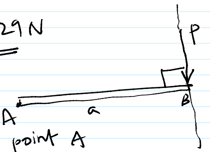
$\frac{1}{2}\sin\theta = \cos\theta\left(\frac{10}{15} + \frac{\sqrt{3}}{2}\right)$

$R = R_y = (\Sigma F_y) = 200\sin\theta + 100\sin\theta + 150\sin(30+\theta)$

$= 229.29\text{N}$

Moment of force

$= 229.29\,N$

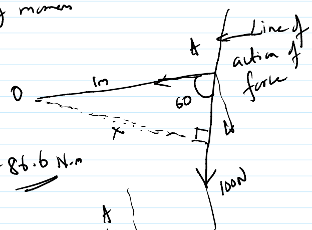


Moment of force

M_A = moment of force about point A

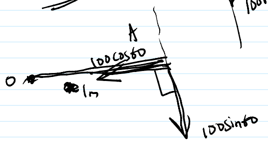
= Force × (⊥r distance betn line of action of force & the point of moment)

$= Pa$ N-m



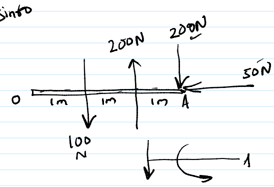
$M_O = 100(x)$ $\quad x = 1\sin 60$

$M_O = 100 \times 1\sin 60 = 100\sin 60 = 86.6$ N-m

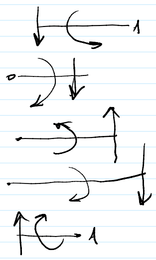


$M_O = (100\cos 60)(0) + (100\sin 60)(1)$

$= 100\sin 60$



20/10/21 Find moment of all the forces (i) @ O & (ii) @ A



(i) moment of all the forces about O.

$\circlearrowleft\!+ \ \Sigma M_O = +(100)(1) - (200)(2) + 200(3) + (50)(0)$

$= 100 - 400 + 600$

$= 300$ N-m

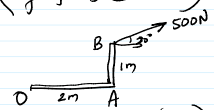
$= 300$ N-m $\circlearrowleft$

$\circlearrowleft\!+ \ \Sigma M_A = +(200)(1) - 100(2) = 0$

Varignon's Law of moments

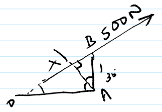
$\Sigma M_O = M_O^R$

(Summation of moments of all the forces @ point O) = (moment of Resultant of forces @ O)



Find moment of 500 @ A

①

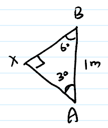


$\Sigma M_A = (500)(Ax)$

$= 500(1\cos 30)$

$= 500 \times \frac{\sqrt{3}}{2}$

$= 250\sqrt{3}$ N-m

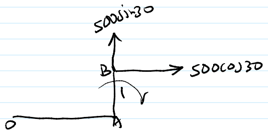


$\cos 30 = \frac{AX}{AB}$

$AX = AB\cos 30$

$= 1\cos 30$

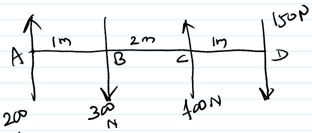
②

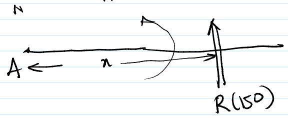


$\Sigma M_A = (500\sin 30)(0) + (500\cos 30)(1)$

$= 250\sqrt{3}$

Ex





Find the resultant of forces

$R_x = \Sigma F_x = 0$

$R_y = +\uparrow \Sigma F_y$

$= +200 - 300 + 400 - 150$

$R_y = 150\,N$

i.e $R_y = 150\,N \uparrow$

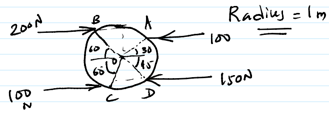
According to Law of moments

$\circlearrowleft\!+ \ \Sigma M_A = M_A^R$

$(300)(1) - (400)(3) + (150)(4) = -150(x)$

$x = 2$ m

Find the resultant of parallel force system.

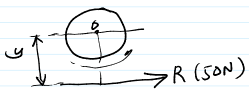


$R_y = 0 \ (\Sigma F_y)$

$R_x = \overrightarrow{+\Sigma F_x}$

$= 200 + \cancel{100} - \cancel{100} - 150$

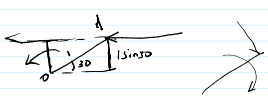
$= 50\,N \ (\rightarrow)$



According to Law of moments

$\circlearrowleft\!+ \ \Sigma M_O = M_O^R$

$-100 \times 1\sin 30 + 200 \times 1\sin 60 - 100 \times 1\sin 60$ ... $(50)(y)$



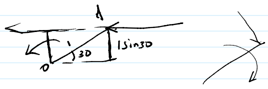
According Law of moments

$+\circlearrowright \sum M_O = M_O^R$

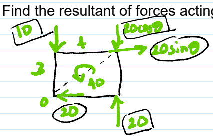
$-100\times 1\sin 30 + 200\times 1 \sin 60 - 100\times 1\sin 60$

$+150\times 1 \sin 45 = (50)(y)$

$\boxed{y = -2.85}$



Find the resultant of forces acting on the plate as shown in the figure



$R_x = \vec{\sum} F_x$

$= -20 + 20\sin\theta$

$= -20 + 20\times 0.6$

$= -8$

$|R_x| = 8 \leftarrow$

$\tan\theta = \frac{3}{4}$

$\sin\theta = \frac{3}{5} = 0.6$

$\cos\theta = \frac{4}{5} = 0.8$

$R_y = +\uparrow \sum F_y$

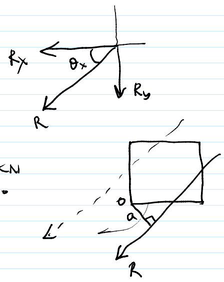
$= -10 - 20\cos\theta + 20 = -10 - 16 + 20$

$= -6$

$|R_y| = 6 \downarrow$

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{8^2 + 6^2} = 10\,kN$

$\theta_x = \tan^{-1}\left(\left|\frac{R_y}{R_x}\right|\right) = \tan^{-1}\left(\frac{6}{8}\right) = 36.87^\circ$



According to Law of moments

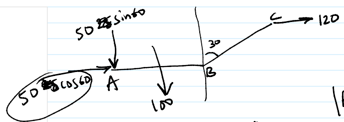
$+\circlearrowright \sum M_O = M_O^R$

$(20)(5) - 20(4) - 40 = (10)(a)$

$100 - 80 - 40 = 10a$

$a = -\frac{20}{10} = -2m$

Find the resultant of the force system acting on a body OABC, shown in figure. Also find the points where the resultant will cut the X and Y axis. What is the distance of resultant from O?



$R_x = \vec{\sum} F_x$

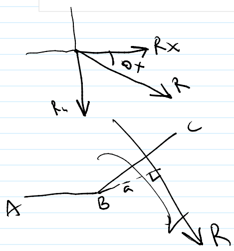
$= 25 + 120 = 145$

$|R_x| = 145 \rightarrow$

$R_y = \uparrow \sum F_y$

$= -50\sin 60 - 100 = -143.3$

$|R_y| = 143.3 \downarrow$



$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(145)^2 + (143.3)^2} = 203.86\,N$

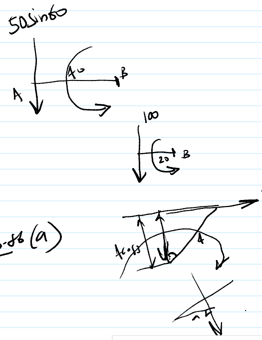
$\theta_x = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{143.3}{145}\right) = 44.66$

According to Law of moments

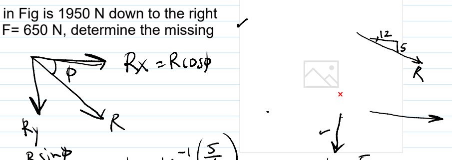
$+\circlearrowright \sum M_B = M_B^R$

$(-50\sin 60)(40) - 100(20) + (120)(40\cos 30) = 203.86(a)$

$a = \cancel{-14.91}\ cm = 2.0837\,cm$



The resultant R of four forces, of which three are shown in Fig is 1950 N down to the right with a slope of 5 to 12 through point A. if P= 750 N, and F= 650 N, determine the missing force T and its X intercept.



$R_x = 1950 \cos\phi \rightarrow$

$= 1800$

$R_y = 1950 \sin\phi \downarrow$

$= 750$

$R_x = R\cos\phi$

$R_y = R\sin\phi$

$\phi = \tan^{-1}\left(\frac{5}{12}\right) = 22.62$

$\tan\phi = \frac{5}{12}$

$R_x = \vec{\sum} F_x$

$1800 = 750 + 500\cos\alpha + T\cos\theta$

$T\cos\theta = 750$

$\alpha = \tan^{-1}\left(\frac{4}{3}\right) = 53.13$

$R_y = \uparrow \sum F_y$

$-750 = +500\sin\alpha - 650 - T\sin\theta \qquad T\sin\theta = 500$

$\frac{T\sin\theta}{T\cos\theta} = \frac{500}{750} \qquad \tan\theta = \frac{50}{75} \Rightarrow \theta = \tan^{-1}\left(\frac{50}{75}\right) = 33.69^\circ$

$T\sin\theta = 500 \qquad T = \frac{500}{\sin(33.69)} = 901.389\,N$

$\circlearrowright \sum M_O = M_O^R$

$(750)(4) + 650(2) + (T\sin\theta)(x)$

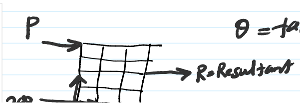
$= (1800)(2) + (750)(3)$

$x = \frac{1550}{500} = 3.1$

According to Law of moments

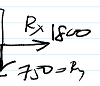
$\sum M_O = M_O^R$

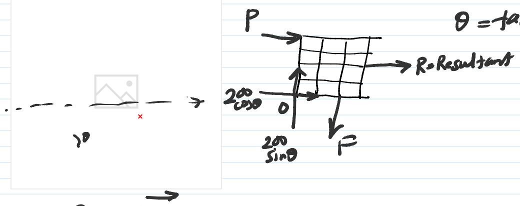
Three forces produce a horizontal resultant force through the point A. Find the magnitude and sense of force P and F



$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13 \qquad R = \sqrt{R_x^2 + R_y^2}$

$\uparrow R_y = \sum F_y = 0$ [Because R is horizontal]


Rx 1800
750=Ry



$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13$ $\quad R = \sqrt{R_x^2 + R_y^2}$

$\uparrow R_y = \Sigma F_y = 0$ [Because R is horizontal]

$0 = 200\sin\theta - F$

$F = 200\sin\theta = 200\sin(53.13) = 160 \downarrow$

According to Law of moments

$+\circlearrowright \Sigma M_O = M_O^R$

$P(4) + (160)(2) = (R)(2)$

$P(4) - R(2) = -320$ — ②

$R_x = \overset{+\rightarrow}{\Sigma} F_x$

$R = P + 200\cos\theta = P + 120$ — ①

$R - P = 120$ — ①

$$\begin{bmatrix} a_1 x + b_1 y = c_1 \\ a_2 x + b_2 y = c_2 \end{bmatrix}$$

Here $x = P$ $\quad a_1 = -1 \quad | \quad a_2 = 4$

$y = R$ $\quad b_1 = 1 \quad | \quad b_2 = -2$

$c_1 = 120 \quad | \quad c_2 = -320$

$x = -40 = P$

$y = 80 = R$

$\therefore P = 40 \leftarrow$ $\quad R = 80 \rightarrow$

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$[A]\ [x] = [C]$$

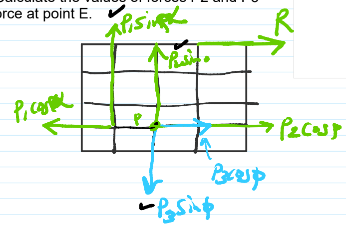
$$[x] = \text{inv}(A) \times C$$

A square lamina is subjected to a force of P1 = 1580 N as shown. Calculate the values of forces P2 and P3 such that the resultant of the system of forces will be a horizontal force at point E.

$P_1 = 1580$ $\quad \alpha = \tan^{-1}\left(\frac{3}{1}\right) = 71.56$

$P_2 = ?$ $\quad \beta = \tan^{-1}\left(\frac{3}{2}\right) = 56.31$

$P_3 = ?$ $\quad \phi = \tan^{-1}\left(\frac{1}{2}\right) = 26.56$



According to Law of moments

$+\circlearrowright \Sigma M_P = M_P^R$

$(P_1 \sin\alpha)(1) = (R)(3)$ $\quad R = \frac{1580 \sin 71.56}{3} = 500 \rightarrow$

$R_x = \overset{\rightarrow}{\Sigma} F_x$

$R = 500 = -P_1\cos\alpha + P_2\cos\beta + P_3\cos\phi$

$P_2\cos\beta + P_3\cos\phi = 500 + 1580\cos(71.56)$ — ①

$R_y = \overset{+\uparrow}{\Sigma} F_y$

$0 = P_1\sin\alpha + P_2\sin\beta - P_3\sin\phi$ $\Rightarrow$ $P_2\sin\beta - P_3\sin\phi = -1580\sin(71.56)$ — ②

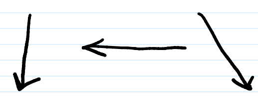
Solving equations ① & ② we get

$P_2 = -900.625$

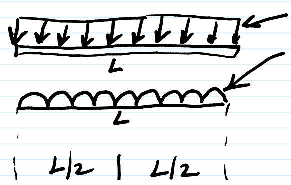
$P_3 = 1636.248$

Types of Loads

① point or concentrated Load

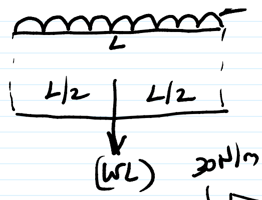


② uniformly distributed load (udl) or Rectangular Load



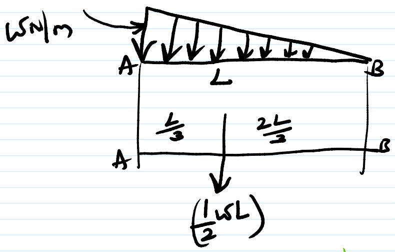
③ Uniformly Varying load (uvl)

or Rectangular Load

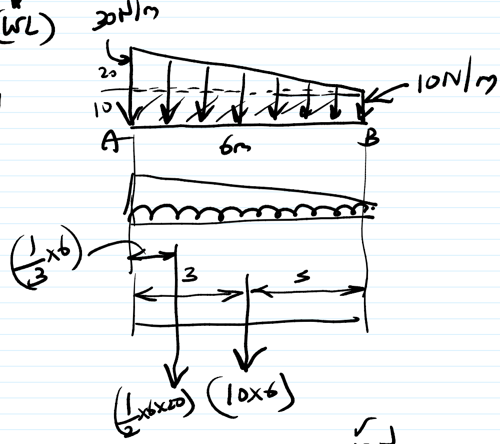


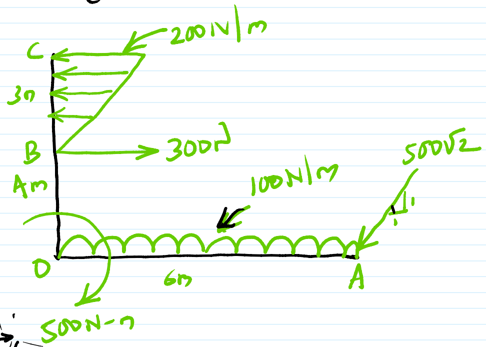
(3) Uniformly Varying Load (UVL)

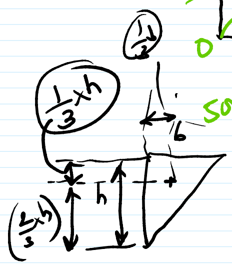
(i) Triangular Load



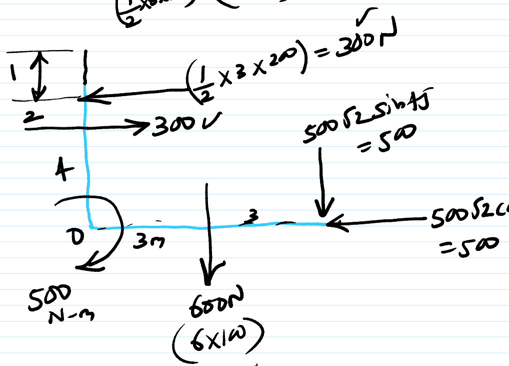
(ii) Trapezoidal







$A = \frac{1}{2} \times b \times h$



$R_x = \overset{+\rightarrow}{\sum F_x} = -\cancel{300} + \cancel{300} - 500$
$= -500$

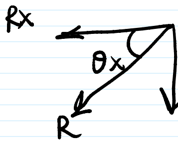
$|R_x| = 500 \leftarrow$

$R_y = +\uparrow \sum F_y$
$= -600 - 500 = -1100$

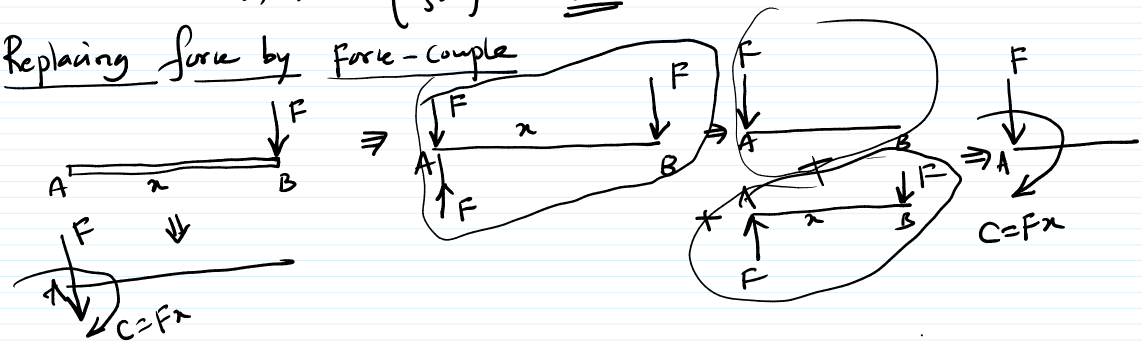
$|R_y| = 1100 \downarrow$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(500)^2 + (1100)^2} = \underline{\underline{1208.304}}\ N$$

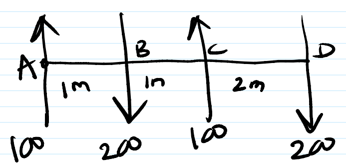
$$\theta_x = \tan^{-1}\left(\frac{1100}{500}\right) = \underline{\underline{65.55^\circ}}$$



Replacing force by Force-couple



Ex

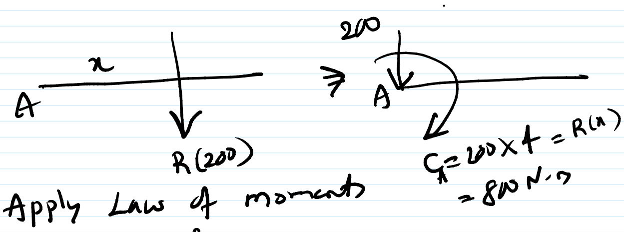


$R_x = 0 = \sum F_x$

$R_y = +\uparrow \sum F_y = \cancel{100} - \cancel{200} + \cancel{100} - 200$

$R_y = -200$

$|R_y| = 200 \downarrow$

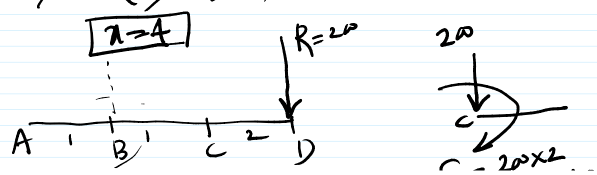


Apply Law of moments

$\Rightarrow \sum M_A = M_A^R$

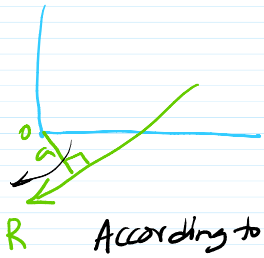
$200(1) - (100)(2) + (200)(4) = 200(x)$

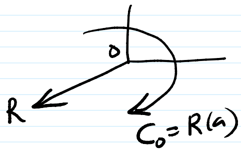
$\boxed{x = 4}$



8 NOV 2021 Space forces

Vector representation of a force





According to Law of moments

$\circlearrowright + \quad \sum M_O = M_O^R$

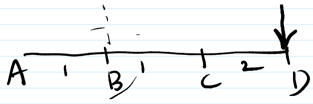
os45

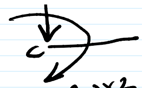
$$\left\{ (500) + (600)(3) + (500)(6) + (300)(4) - (300)(6) \right\} = 1208.304\,(a)$$

$$\boxed{a = 3.88 \text{ m}}$$

Ry

$|R_y| = 200$ ✓





$C_t = 200 \times 2$
$= 400$ N-m

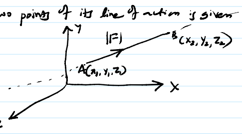
8 NOV 2021 Space forces

Vector representation of a force

① If magnitude of force & two points of its line of action is given

$\bar{F} = |F|\, \bar{\lambda}_{AB}$

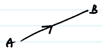
$\bar{\lambda}_{AB}$ = unit vector of line AB ($\bar{e}_{AB}$)



$$\bar{\lambda}_{AB} = \frac{(x_2-x_1)i+(y_2-y_1)j+(z_2-z_1)k}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}}$$

$$\bar{F} = F_m\left[(x_2-x_1)i+(y_2-y_1)j+(z_2-z_1)k\right]$$

$$F_m = \text{Force multiplier} = \frac{|F|}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}}$$



Ex A force of magnitude 700N passes thro point A (0,3,0) & B (5,0,4)
write equation of the force in vector form

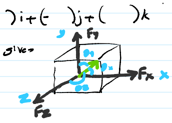
$|F| = 700$N

$$\bar{\lambda}_{AB} = \frac{(x_B-x_A)i+(y_B-y_A)j+(z_B-z_A)k}{\sqrt{(x_B-x_A)^2+(y_B-y_A)^2+(z_B-z_A)^2}}$$

$$= \frac{(5-0)i+(0-3)j+(4-0)k}{\sqrt{(5)^2+(-3)^2+(4)^2}} = \frac{5i+(-3j)+4k}{\sqrt{50}}$$

$$\bar{F} = |F|\,\bar{\lambda}_{AB} = \frac{700}{\sqrt{50}}\left[5i-3j+4k\right] = (\quad)i+(-\quad)j+(\quad)k$$

② If magnitude of force & direction angles are given

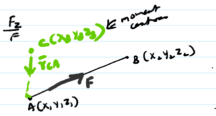


$$\bar{F} = F_x i + F_y j + F_z k$$
$$= (F\cos\theta_x)\,i + (F\cos\theta_y)\,j + (F\cos\theta_z)\,k$$

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

$$\cos\theta_x = \frac{F_x}{F} \qquad \cos\theta_y = \frac{F_y}{F} \qquad \cos\theta_z = \frac{F_z}{F}$$

Moment of a force about a point



Steps

① Represent the force in vector form
$\bar{F} = F_x i + F_y j + F_z k$

② Find the position vector extending from moment centre to any point on the line of action
$\bar{r} = r_x i + r_y j + r_z k$

③ perform cross product of the position vector & force vector

$$\bar{M}_C^F = \bar{r}\times\bar{F} = \begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$\left.\begin{array}{l} \bar{F} = |F|\left[\dfrac{(x_2-x_1)i+(y_2-y_1)j+(z_2-z_1)k}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}}\right] \\ F_x = \dfrac{|F|(x_2-x_1)}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}},\quad F_y = \dfrac{|F|(y_2-y_1)}{\sqrt{\ }},\quad F_z = \dfrac{|F|(z_2-z_1)}{\sqrt{\ }} \end{array}\right\} \text{Force vector step ①}$$

$$\bar{r}_{CA} = (x_1-x_3)i + (y_1-y_3)j + (z_1-z_3)k \Big\} \text{position vector step ②}$$

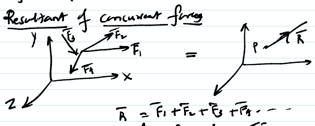
$$\bar{M}_C^F = \bar{r}_{CA}\times\bar{F} = \begin{vmatrix} i & j & k \\ (x_1-x_3) & (y_1-y_3) & (z_1-z_3) \\ F_x & F_y & F_z \end{vmatrix}$$

We can also take $\bar{r}_{CB}$ instead of $\bar{r}_{CA}$

$$\bar{r}_{CB} = (x_2-x_3)i + (y_2-y_3)j + (z_2-z_3)k$$

If the point c is origin then $x_3 = y_3 = z_3 = 0$

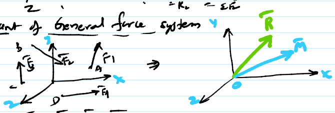
Resultant of concurrent forces



$$\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \bar{F}_4 \cdots$$

x component of R $= R_x = \Sigma F_x$
y " " $= R_y = \Sigma F_y$
z " " $= R_z = \Sigma F_z$

Resultant of General force system



$$\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \bar{F}_4$$
$$\bar{M}_O = \bar{M}_O^{F_1} + \bar{M}_O^{F_2} + \bar{M}_O^{F_3} + \bar{M}_O^{F_4}$$

Varignon's theorem
The moment of resultant of force systems about a point is equal to the sum of moments of varying forces about the same point

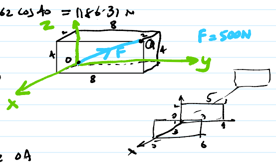
A force acts at origin in a direction defined by angles $\theta_y = 65^\circ$ and $\theta_z = 40^\circ$
If $F_x = -750$N find (i) the other components of force
(ii) magnitude of force (iii) θ_x

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$
$$\cos^2\theta_x + (\cos 65)^2 + (\cos 40)^2 = 1$$
$$\cos\theta_x = \pm 0.4843$$

$\cos\theta_x = 0.4843$ & $\cos\theta_x = -0.4843$
$\theta_x = 61.43$ & $(180 - 61.43)$

$F_x = |F|\cos\theta_x$ [$\because F_x$ is -ve]
$-750 = |F|\cos(180 - 61.43)$
$|F| = 1548.62$ N

$F_y = |F|\cos\theta_y = 1548.62 \cos 65 = 654.47$ N
$F_z = |F|\cos\theta_z = 1548.62 \cos 40 = 1186.31$ N



What are the components of force F(500N)
The coordinates of O (0, 0, 0)
coordinates of A ($x_A = 2$
$y_A = 8$
$z_A = 4$

λ_{OA} = unit vector along OA

$$= \frac{(2-0)i + (8-0)j + (4-0)k}{\sqrt{(2-0)^2+(8-0)^2+(4-0)^2}} = \frac{2i+8j+4k}{\sqrt{84}}$$

$z_A = 4$

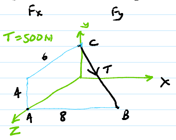
λ_{OA} = unit vector along OA

$$= \frac{(2-0)i + (8-0)j + (4-0)k}{\sqrt{(2-0)^2 + (8-0)^2 + (4-0)^2}} = \frac{2i + 8j + 4k}{\sqrt{84}}$$

∴ Vector representation of force F $= |F| \bar{\lambda}_{OA}$

$$= 500\left[\frac{2i + 8j + 4k}{\sqrt{84}}\right] = \underbrace{(109.10)}_{F_x} i + \underbrace{(436.43)}_{F_y} j + \underbrace{(218.21)}_{F_z} k$$

Determine the components of the force T = 500N



The coordinates of C
$x_C = 0, \ y_C = 4, \ z_C = 0$

The coordinates of B
$x_B = 8, \ y_B = 0, \ z_B = 6$

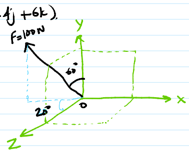
The unit vector along CB $\bar{\lambda}_{CB} = \frac{(x_B - x_C)i + (y_B - y_C)j + (z_B - z_C)k}{\sqrt{(x_B - x_C)^2 + (y_B - y_C)^2 + (z_B - z_C)^2}}$

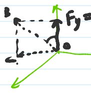
$$\bar{\lambda}_{CB} = \frac{(8-0)i + (0-4)j + (6-0)k}{\sqrt{8^2 + (-4)^2 + 6^2}} = \frac{8i - 4j + 6k}{\sqrt{116}}$$

Vector representation of force T(=500) is

$$= |T| \bar{\lambda}_{CB} = \frac{500}{\sqrt{116}}(8i - 4j + 6k)$$

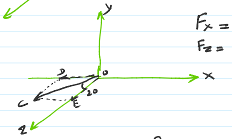
Determine the components of force F along x, y, z axes.





$F_y = 100\cos 60$

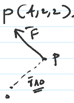
$F_{xz} = 100\sin 60$
$= 50\sqrt{3}$



$F_x = -50\sqrt{3}\sin 20 = -29.61$ N
$F_z = +50\sqrt{3}\cos 20 = +81.37$ N

Example on moment of force

A force $\bar{F} = 10i + 4j + 2k$ is acting at point P(4, 2, 2).
Find the moment of the force @ origin



$\bar{M}_O^F = \bar{r}_{AO} \times \bar{F}$

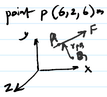
$\bar{r}_{OA} = (4-0)i + (2-0)j + (2-0)k$

$$\bar{M}_O^F = \begin{vmatrix} i & j & k \\ 4 & 2 & 2 \\ 10 & 4 & 2 \end{vmatrix} = (i)[2\times2 - 2\times4] - j[4\times2 - 10\times2] + k[4\times4 - 10\times2]$$

$= (-4i + 12j - 4k)$ N-m

10/11/21

Force $(80i + 50j - 60k)$ N passes through a point P(6, 2, 6)m. Find moment of this force about Q(8, 1, 4)m.



$$M_Q^F = \bar{r}_{PQ} \times \bar{F} = \begin{vmatrix} i & j & k \\ (x_P - x_Q) & (y_P - y_Q) & (z_P - z_Q) \\ F_x & F_y & F_z \end{vmatrix}$$

$$= \begin{vmatrix} i & j & k \\ (6-8) & (2-1) & (6-4) \\ 80 & 50 & -60 \end{vmatrix} = \begin{vmatrix} i & j & k \\ -2 & 1 & 2 \\ 80 & 50 & -60 \end{vmatrix} = (-160i + 40j - 180k) \text{ N-m}$$

A force of 200N acts through point A(4, 5, −8)m to B(10, −15, 20)m.
Find the moment of this force @ a point C(2, 3, −4).

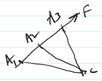
Vector representation of force F $= |F| \bar{\lambda}_{AB}$

$$= (200)\left[\frac{(10-4)i + (-15-5)j + (20+8)k}{\sqrt{(10-4)^2 + (-20)^2 + (28)^2}}\right] = 34.4i - 114.5j + 160.3k$$

Moment of force $\bar{F}$ about point C

$\bar{M}_C^F = \bar{r}_{AC} \times \bar{F}$ or $\bar{r}_{BC} \times \bar{F}$

$$= \begin{vmatrix} i & j & k \\ (x_A - x_C) & (y_A - y_C) & (z_A - z_C) \\ F_x & F_y & F_z \end{vmatrix} \text{ or } \begin{vmatrix} i & j & k \\ (x_B - x_C) & (y_B - y_C) & (z_B - z_C) \\ F_x & F_y & F_z \end{vmatrix}$$



$$= \begin{vmatrix} i & j & k \\ 2 & 2 & -4 \\ 34.4 & -114.5 & 160.3 \end{vmatrix} \text{ or } \begin{vmatrix} i & j & k \\ 8 & -18 & 24 \\ 34.4 & -114.5 & 160.3 \end{vmatrix}$$

$= i(2\times160.3 - 4\times114.5)$ OR $(-137.4i + 456.8j - 296.8k)$
$= (-137.4i + 458.2j - 297.8k)$

A force of 80N acts through point A(2, 0, 11) towards point B(6, 8, 3)
Find the moment of this force about (i) x-axis (ii) y-axis (3) z-axis.

Vector form of force $= \bar{F} = |F| \bar{\lambda}_{AB}$

$$= (80)\left[\frac{(6-2)i + (8-0)j + (3-11)k}{\sqrt{(6-2)^2 + (8-0)^2 + (3-11)^2}}\right] = (26.67i + 53.34j - 53.34k) \text{ N}$$

(i) To find moment about x-axis

$$\bar{M}_O^F = \bar{r}_{AO} \times \bar{F} = \begin{vmatrix} i & j & k \\ 2 & 0 & 11 \\ 26.67 & 53.34 & -53.34 \end{vmatrix} = -586.74i + 400.05j + 106.67k$$

magnitude of moment about x-axis

$= \bar{M}_O^F \cdot \bar{\lambda}_{ox} = (-586.74i + 400.05j + 106.67k) \cdot (i)$ $[i \cdot i = 1, \ j \cdot j = 0, \ i \cdot k = 0]$

$= -586.74$ N-m

moment vector about x-axis $= \bar{M}_{ox}^F = \cancel{(\ldots)} = -586.74i$

(ii) To find moment @ y-axis

magnitude of moment @ y-axis $= \bar{M}_O^F \cdot \bar{\lambda}_{oy}$

$= (-586.74i + 400.05j + 106.67k)(j)$

$= (400.05)$ N-m

moment vector @ y-axis $= (400.05j)$ N-m

15/11/21

A force of 20 kN acts at a point A(3, 4, 5) and has its line of action passing through B(5, −3, 4). Find the moment of this force about CD C(2, −5, 3) and D(−3, 4, 6)

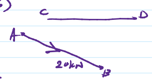
C D

Force in vector form

A

15/11/21

A force of 20 kN acts at a point A (3,4,5) and has its line of action passing through B (5,−3,4). Find the moment of this force about CD C(2,−5,3) and D(−3,4,6)



Force in vector form

$$\bar{F} = |F|\,\bar{\lambda}_{AB} = 20\left[\frac{(5-3)i+(-3-4)j+(4-5)k}{\sqrt{(5-3)^2+(-3-4)^2+(4-5)^2}}\right]$$

$$= \frac{20}{\sqrt{54}}(2i-7j-1k) = 5.44i - 19.05j - 2.72k.$$

Moment of force F about any point of the line CD

$$\bar{M}_C^F = \bar{r}_{A/C} \times F = \begin{vmatrix} i & j & k \\ x_A-x_C & y_A-y_C & z_A-z_C \\ F_x & F_y & F_z \end{vmatrix} = \begin{vmatrix} i & j & k \\ (3-2) & (4+5) & (5-3) \\ 5.44 & -19.05 & -2.72 \end{vmatrix}$$

$$= (13.62i + 13.6j - 68.0k)\ kN\text{-}m$$

Unit vector of line CD

$$\bar{\lambda}_{CD} = \frac{(x_D-x_C)i+(y_D-y_C)j+(z_D-z_C)k}{\sqrt{(x_D-x_C)^2+(y_D-y_C)^2+(z_D-z_C)^2}}$$

$$= -0.47i + 0.84j + 0.28k$$

magnitude of moment about line CD

$$|\bar{M}_{CD}^F| = \bar{M}_C^F \cdot \bar{\lambda}_{CD} = (13.67i + 13.6j - 68.01k)\cdot(-0.47i + 0.84j + 0.28k)$$

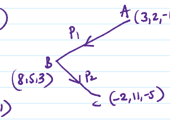
$$= -14.02\ kN\text{-}m$$

Moment of force in vector from (@ line CD)

$$\bar{M}_{CD}^F = |M_{CD}^F| \cdot \bar{\lambda}_{CD} = (-14.02)(-0.47i + 0.84j + 0.28k)$$

$$= (6.61i - 11.78j - 3.93k)$$

A force $P_1 = 10N$ $\overline{AB}$, $P_2 = 5N$ $\overline{BC}$



Find
(i) Resultant of P_1 & P_2 in vector form
(ii) moment of Resultant @ D (1,1,1)
(iii) The magnitude of the components of the resultant along line BK K(5,8,3)

$$\bar{P}_1 = |P_1|\,\bar{\lambda}_{AB} = (10)\left[\frac{(8-3)i+(5-2)j+(3+1)k}{\sqrt{(8-3)^2+(5-2)^2+(3+1)^2}}\right] = 7.07i + 4.24j + 5.65k$$

$$\bar{P}_2 = |P_2|\,\bar{\lambda}_{BC} = (5)\left[\frac{(-2-8)i+(11-5)j+(-5-3)k}{\sqrt{(-2-8)^2+(11-5)^2+(-5-3)^2}}\right] = -3.54i + 2.12j - 2.82k$$

$$\bar{R} = \bar{P}_1 + \bar{P}_2 = 3.54i + 6.36j + 2.83k$$

$$|R| = \sqrt{(3.54)^2+(6.36)^2+(2.83)^2} = 7.81$$

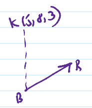
$$\theta_x = \cos^{-1}\left(\frac{R_x}{R}\right) = \cos^{-1}\left(\frac{3.54}{7.81}\right) =$$

$$\theta_y = \cos^{-1}\left(\frac{R_y}{R}\right) = \cos^{-1}\left(\frac{6.36}{7.81}\right) =$$

$$\theta_z = \cos^{-1}\left(\frac{R_z}{R}\right) = \cos^{-1}\left(\frac{2.83}{7.81}\right) =$$

(ii) $$\bar{M}_D^R = \bar{r}_{D/B} \times \bar{R} = \begin{vmatrix} i & j & k \\ (x_B-x_D) & (y_B-y_D) & (z_B-z_D) \\ R_x & R_y & R_z \end{vmatrix} =$$

(iii)

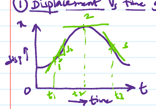


$$\bar{\lambda}_{BK} = \frac{(x_K-x_B)i+(y_K-y_B)j+(z_K-z_B)k}{\sqrt{(x_K-x_B)^2+(y_K-y_B)^2+(z_K-z_B)^2}}$$

$$|R_{BK}| = \bar{R}\cdot\bar{\lambda}_{BK} =$$

motion curves

① Displacement Vs time graph (x V t or S V t)

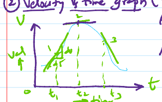


At $t = t_1$ Vel $= V_1 = \frac{dx}{dt} > 0$ (+ve) slope

At $t = t_2$ Vel $= V_2 = \frac{dx}{dt} = 0$

At $t = t_3$ Vel $= V_3 = \frac{dx}{dt} < 0$ (−ve slope)

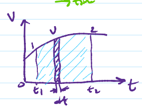
② Velocity V time graph (V V t)



At $t = t_1$ slope $= \frac{dv}{dt}(>0) = acc^n$

At $t = t_2$ slope $= 0$

At $t = t_3$ slope < 0 $\left(\frac{dv}{dt} = a < 0\right)$

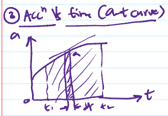


$V = \frac{dx}{dt}$ $\quad dx = V\,dt = \text{Area of strip} = dA$

$$\int_{x_i}^{x_f} dx = \int_{t_1}^{t_2} V\,dt = \int_{t_1}^{t_2} dA$$

$$\boxed{x_f - x_i = \text{Area under V-t curve}}$$

③ Acc^n V time (a-t curve)

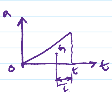


slope $= \frac{da}{dt}$ = Jerk or pulse (m/s^3)

$a = \frac{dv}{dt}$ $\quad dv = a\,dt$ = Area of strip

$$\int_{V_i}^{V_f} dv = \int_{t_1}^{t_2} a\,dt$$

$$\boxed{V_f - V_i = \text{Area under a-t curve}}$$



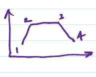
$$\boxed{x_t = x_0 + V_0 t + \text{Area} \times \bar{t}}$$

few important points

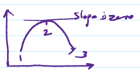
① If the displacement is a polynomial of degree(n). then the degree of velocity is (n−1)
" " acc^n is (n−2)

$x = t^n$
$V = n\,t^{(n-1)}$
$a = n(n-1)\,(t)^{(n-2)}$

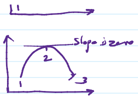
② for line 12 slope is +ve is const
line 23 " " zero
line 34 " " −ve is const



③ slope for curve 12 → +ve
The magnitude slope ~~decrease~~
slope at 2 = 0

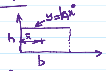


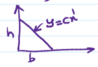
line 23 " " zero
line 34 " " -ve is constt

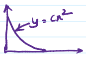


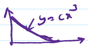
③ slope for curve 12 ⇒ +ve
The magnitude slope decreases
slope at 2 = 0
slope 23 ⇒ −ve

Formulae for Area

① Rectangle  $A = \frac{bh}{1}$ $\bar{x} = \frac{b}{2}$

② Triangle  $A = \frac{bh}{2}$ $\bar{x} = \frac{b}{3}$

③ parabola  $A = \frac{bh}{3}$ $\bar{x} = \frac{b}{4}$

④ cubic  $A = \frac{bh}{4}$ $\bar{x} = \frac{b}{5}$

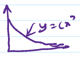
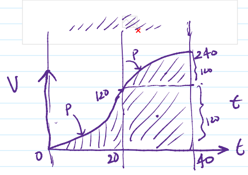
n^{th}  $A = \frac{bh}{n+1}$ $\bar{x} = \frac{b}{n+2}$

Fig shows a - t diagram for a particle moving along X -axis for a time interval of 0 to 40 seconds. Plot V -t & S -t diagrams. Particle starts from rest & from origin.

$V_0 = 0$ (vel at $t=0$)
$S_0 = 0$ (disp at $t=0$)

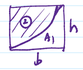
$\boxed{t = 0 \text{ to } 20}$

$V_{20} - V_0 = $ Area under a-t
$V_{20} - 0 = \frac{1}{2} \times 20 \times 12$
$V_{20} = 120$ m/s



$\boxed{t = 20 \text{ to } 40}$

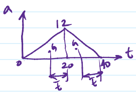
$V_{40} - V_{20} = $ Area under a-t
$V_{40} - 120 = \frac{1}{2} \times 20 \times 12 = 120$
$V_{40} = 240$ m/s



$A_1 = \frac{1}{3} bh$
$A_2 = bh - \frac{1}{3} bh$
$= \frac{2}{3} bh$

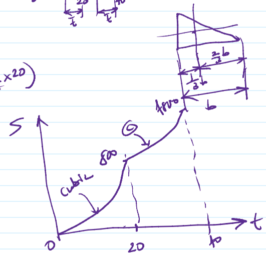
S-t
$\boxed{\text{method ①}}$
$t = 0$ to 20
$S_{20} - S_0 = $ Area under V-t
$S_{20} - 0 = \frac{1}{3} \times 20 \times 120$
$S_{20} = 800$ m

$t = 20$ to 40
$S_{40} - S_{20} = $ Area under V-t
$S_{40} - 800 = (20 \times 120) + \frac{2}{3} \times 20 \times 120$
$S_{40} = 4800$ m



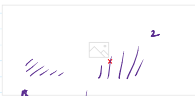
method②
$t = 0$ to 20
$S_{20} = S_0 + V_0 t + A\bar{t}$
$S_{20} = 0 + 0 + \left(\frac{1}{2} \times 20 \times 12\right)\left(\frac{1}{3} \times 20\right) = 800$ m

$t = 20$ to 40
$S_{40} = S_{20} + V_{20} \times 20 + \left(\frac{1}{2} \times 20 \times 12\right)\left(\frac{2}{3} \times 20\right)$
$= 800 + (120 \times 20) + (20 \times 4 \times 20)$
$= 4800$ m

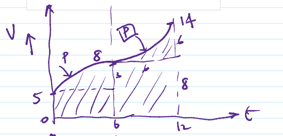


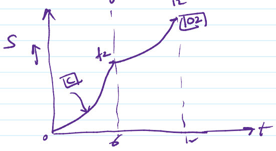
The acceleration-time diagram for the linear motion in figure. Construct velocity – time and Displacement- time diagrams for the motion assuming that the motion starts with initial velocity of 5 m/s from starting point.

$V_0 = 5$ m/s, $S_0 = 0$



$\boxed{t = 0 \text{ to } 6}$
$V_6 - V_0 = $ Area under a-t
$V_6 - 5 = \frac{1}{2} \times 6 \times 1$
$V_6 = 3 + 5 = 8$ m/s
$S_6 - S_0 = $ Area under V-t
$S_6 - 0 = (6 \times 5) + \left(\frac{2}{3} \times 6 \times 3\right) =$
$S_6 = 42$



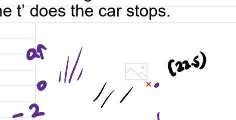


$\boxed{t = 6 \text{ to } 12}$
$V_{12} - V_6 = $ Area under a-t
$V_{12} - 8 = \frac{1}{2} \times 6 \times 2$
$V_{12} = 14$ m/s

$S_{12} - S_6 = $ Area under V-t
$S_{12} - 42 = \left(\frac{1}{3} \times 6 \times 6\right) + (6 \times 8)$
$S_{12} = 42 + 12 + 48$
$= 102$ m

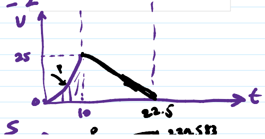
Fig shows a - t diagram for a car. Plot V -t & S -t diagrams. If car starts from rest & from origin, at what time t' does the car stops.

$V_0 = 0$, $S_0 = 0$, $V_{t'} = 0$



$\boxed{t = 0 \text{ to } 10}$

$V_{10} - V_0 = $ Area under a-t
$V_{10} - 0 = \frac{1}{2} \times 10 \times 5$
$V_{10} = 25$ m/s



$S_{10} - S_0 = $ Area under V-t

S

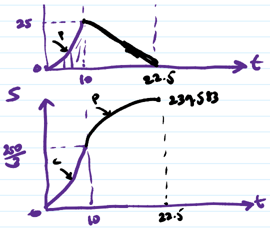
$V_{10} - 0 = \frac{1}{2} \times 10 \sim$

$V_{10} = 25\ m/s$

$S_{10} - S_0 =$ Area under V vs t

$S_{10} - 0 = \frac{1}{3} \times 10 \times 25$

$S_{10} = 250/3 =$



$\boxed{t = 10 \text{ to } t'}$

$V_{t'} - V_{10} = -2(t' - 10)$

$0 + 25 = +2(t' - 10)$

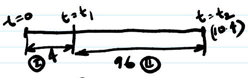
$t' = 12.5 + 10 = 22.5$

$S_{22.5} - S_{10} =$ Area under V-t

$S_{22.5} - \left(\frac{250}{3}\right) = \frac{1}{2} \times (22.5 - 10)(25)$

$\boxed{S_{22.5} = 239.583\ m}$

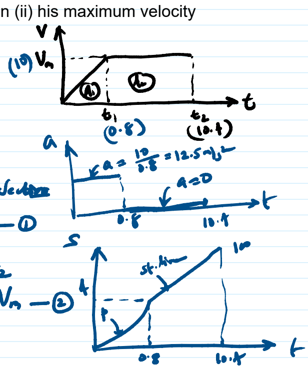
In an Asian Games of 100 m event an athlete accelerates uniformly from the start to his maximum velocity in a distance of 4 m and runs the remaining distance with that velocity. If the athlete finishes the race in 10.4 sec, determine (i) his initial acceleration (ii) his maximum velocity



① → Acc^n

$A_1 = 4m =$

$A_2 = 96m$



$A_1 =$ Area under V-t from 0 to t_1 sec

$4 = \frac{1}{2} \times t_1 \times V_m \Rightarrow V_m t_1 = 8$ — ①

$A_2 =$ Area under V-t from t_1 to t_2

$96 = (t_2 - t_1) V_m = (10.4 - t_1) V_m$ — ②

$10.4 V_m - V_m t_1 = 96$

$10.4 V_m - 8 = 96$

$V_m = \frac{(96+8)}{10.4} = 10\ m/s$ = max vel

$t_1 = \frac{8}{10} = 0.8$ sec

Initial acc^n $= a = \frac{10}{0.8}$

$= 12.5\ m/s^2$

$\boxed{S\text{-}t}$ $S_{0.8} - S_0 =$ Area under V-t

$S_{0.8} - 0 = 4$

$S_{0.8} = 4m$

$S_{10.4} - S_{0.8} = 96$

$S_{10.4} - 4 = 96$

$S_{10.4} = 100$

A car travels along a straight road with the speed shown by V -t graph. Find the total distance the car travels until it stops when t = 48 s. Also plot s-t & a-t graphs.

$\boxed{t = 0 \text{ to } 30}$

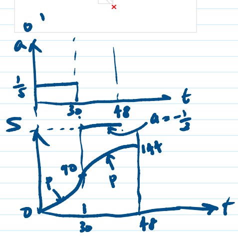
$a =$ slope of V-t

$= \frac{6}{30} = \frac{1}{5}\ m/s^2$

$S_{30} - S_0 =$ Area under V-t

$S_{30} - 0 = \frac{1}{2} \times 30 \times 6$

$S_{30} = 90m$



$\boxed{t = 30 \text{ to } 48}$

$a =$ slope $= -\frac{6}{18} = -\frac{1}{3}\ m/s^2$

$S_{48} - S_{30} =$ Area under V-t

$S_{48} - 90 = \frac{1}{2} \times 18 \times 6$

$S_{48} - 90 = 54$

$S_{48} = 54 + 90 = 144m$

A bicycle moves along a straight road such that its position is described by the graph as shown. Draw V-t & a-t graphs.

$\boxed{t = 0 \text{ to } 10}$

$S = t^2$

$V = \frac{ds}{dt} = 2t$ At t = 10 $V_{10} = 2 \times 10 = 20\ m/s$

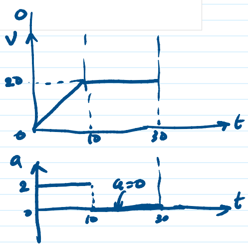
$a = \frac{dv}{dt} = 2$

$\boxed{t = 10 \text{ to } 30}$

$S = 20t - 100$

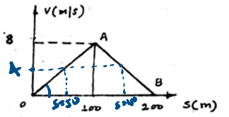
$V = \frac{ds}{dt} = 20$

$a = \frac{dv}{dt} = 0$



The V-S graph for cart traveling on a straight road is shown. Determine the acceleration of the cart at S = 50m and S = 150m.

The V-S graph for cart traveling on a straight road is shown. Determine the acceleration of the cart at S = 50m and S = 150m.



$$a = \frac{dv}{dt} = \frac{dv}{dt} \times \frac{ds}{ds}$$

$$= \frac{ds}{dt}\frac{dv}{ds}$$

$$a = V\frac{dv}{ds} \qquad \frac{dv}{ds} \Rightarrow \text{slope of } V \text{ vs } S \text{ graph}$$

$$a_{(s=50)} = \left(V\frac{dv}{ds}\right)_{s=50} = (4)\left(\frac{8}{200}\right) = \frac{32}{200} = 0.16\ m/s^2$$

$$a_{(s=150)} = \left(V\frac{dv}{ds}\right)_{s=150} = (4)\left(\frac{-8}{200}\right) = \frac{-32}{200} = -0.16\ m/s^2$$

↑ Negative slope

Variable Accⁿ

$x = f(t)$ $\quad V = f(t)$, $a = f(t)$

$a = f(V)$ $\quad a = f(x)$

$V = f(x)$

① The velocity of particle is defined as $V = t^3 - 5t^2 + 3t + 4$. If initial displacement is 2m find

(a) Initial velocity (b) Initial accⁿ (c) time at which $a = 0$

(d) displacement in first 4 seconds (e) displacement in 6^{th} second.

Ans $V = t^3 - 5t^2 + 3t + 4$

(a) $t = 0$ $\quad V = 0 - 0 + 0 + 4 = V_{initial}$ or $U_0 = 4\ m/s$

(b) $a = \frac{dv}{dt} = 3t^2 - 10t + 3$

at $t = 0$, $\quad a = 0 - 10(0) + 3 = 3$ or $a_0 = 3\ m/s^2$

(c) $a = 0 = 3t^2 - 10t + 3$ $\qquad \left[ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$

$t = \frac{1}{3}$ or 3

∴ accⁿ is zero at $t = \frac{1}{3}$ sec & $t = 3$ sec

(d) $V = \frac{dx}{dt} = t^3 - 5t^2 + 3t + 4$ $\qquad dx = (t^3 - 5t^2 + 3t + 4)dt$

Integrating dx

$\int dx = \int (t^3 - 5t^2 + 3t + 4)\,dt$

$x = \frac{t^4}{4} - \frac{5t^3}{3} + \frac{3t^2}{2} + 4t + C$

At $t = 0$, $x = x_0 = $ intial displacement $= 2$

$2 = 0 - 0 + 0 + 0 + C$ $\quad$ ∴ $C = 2$

$x = \frac{t^4}{4} - \frac{5}{3}t^3 + \frac{3}{2}t^2 + 4t + 2$

$x(\text{at } t = 4) = \frac{(4)^4}{4} - \frac{5}{3}(4)^3 + \frac{3}{2}(4)^2 + 4(4) + (2) = -0.67$

(e) Find displacement in 6^{th} second

$=$ Displacement in 6 seconds $-$ Displacement in 5 secs

$= x_6 - x_5 = 44 - 7.42 = 36.58m$

$x_6 = \frac{(6)^4}{4} - \frac{5}{3}(6)^3 + \frac{3}{2}(6)^2 + 4(6) + 2 = 44$

$x_5 = \frac{(5)^4}{4} - \frac{5}{3}(5)^3 + \frac{3}{2}(5)^2 + 4(5) + 2 = 7.42$

The motion of a particle is defined by $a = 0.8t\ m/s^2$. If At $t = 2$, $x = 5m$, $V = 12\ m/s$ find the position & velocity at $t = 6$

$a = \frac{dv}{dt} = 0.8t$

$dv = 0.8t\ dt$

Integrating above equation

$\int dv = \int 0.8t\ dt$

$V = \frac{0.8t^2}{2} + c_1$

$\boxed{V = 0.4t^2 + 10.4}$

At $t = 2$, $V = 12$

$12 = \frac{0.8(2)^2}{2} + c_1$

$c_1 = 12 - 0.8 \times 2$

$= 12 - 1.6$

$= 10.4$

$V = \frac{dx}{dt} = 0.4t^2 + 10.4$

$dx = (0.4t^2 + 10.4)dt$

Integrating

$\int dx = \int (0.4t^2 + 10.4)\,dt$

$x = \frac{0.4t^3}{3} + 10.4t + c_2$

$\boxed{x = \frac{0.4t^3}{3} + 10.4t - \frac{253}{15}}$

At $t = 2$, $x = 5$

$5 = \frac{0.4}{3}(2)^3 + 10.4(2) + c_2$

$c_2 = -\frac{253}{15} = -16.867$

At $t = 6$ $\qquad V = 24.8\ m/s$

$x = 74.33\ m/s$

At $t=6$ $\quad V = 24.8$ m/s

$x = 74.33$ m/s

The accⁿ of an oscillating particle is defined by $a = -kx$
find (i) k if $V = 15$ m/s at $x = 0$, & $V = 0$ at $x = 3$m
(ii) The speed of the particle when $x = 2$m

$$a = \frac{dv}{dt} = V\frac{dv}{dx}$$

$$-kx = V\frac{dv}{dx} \Rightarrow \int V\,dv = \int -kx\,dx$$

Integrate

$$\int_{V=15}^{V=0} V\,dv = \int_{x=0}^{x=3} -kx$$

$k =$

OR

$$\frac{V^2}{2} = \frac{-kx^2}{2} + C_1$$

Apply BCs

(i) At $x=0$, $V=15$

$$\frac{15^2}{2} = \frac{-k(0)^2}{2} + C_1 \qquad \boxed{C_1 = 112.5}$$

(ii) $$\frac{(0)^2}{2} = \frac{-k(3)^2}{2} + C_1$$

$$0 = -4.5k + 112.5$$

$$\boxed{k = \frac{112.5}{4.5} = 25}$$

(ii) find V at $x = 2$

$$\frac{V^2}{2} = \frac{-25}{2}x^2 + 112.5$$

$$\frac{V^2}{2} = -12.5(2)^2 + 112.5$$

$$\boxed{V = 11.18 \text{ m/s}}$$

30/1/21

The velocity of the body is defined by $V = (6 - 0.03x)$ m/s. If $x=0$ at $t=0$
find (a) the distance travelled by body when it comes to rest
(b) accⁿ at $t=0$ (c) time when $x = 100$m

$V = 6 - 0.03x$

(a) $V = 0 = 6 - 0.03x \quad \therefore x = \frac{6}{0.03} = 200$m

(b) $V = (6 - 0.03x)$

$$\frac{dv}{dt} = 0 - 0.03\frac{dx}{dt} \Rightarrow a = -0.03\frac{dx}{dt} = -0.03V$$

$$a = -0.03[6 - 0.03x]$$

accⁿ at $t=0$ (is also accⁿ at $x=0$)

$$a = -0.03[6 - 0.03\times 0] = -6\times 0.03 = -0.18 \text{ m/s}^2$$

(c) find t when $x = 100$m

$$V = 6 - 0.03x = \frac{dx}{dt}$$

$$dt = \frac{dx}{6 - 0.03x}$$

Integrating $\int dt = \int \frac{dx}{6 - 0.03x}$

$$t = -\frac{1}{0.03}\log_e(6 - 0.03x) + C_1$$

We know that At $t=0$, $x=0$ $\Rightarrow C_1 = 59.725$

In calculator
$\log \Rightarrow \log_{10}$
$\ln = \log_e$

$$t = -\frac{1}{0.03}\log_e(6 - 0.03\times 100) + C_1$$

$= 23.105$ seconds

$\boxed{\text{OR}}$

$$\int_{x=0}^{x=100} \frac{dx}{6 - 0.03x} = \int_{t=0}^{t=t} dt$$

$t = 23.105$ days

The accⁿ of a vehicle at any instant is given by $a = \frac{8}{(V^2+1)}$ m/s
If the vehicle starts from rest find its
(a) velocity when $x = 20$m
(b) displacement when $V = (54 \text{ kpH} \times \frac{5}{18})$ m/s

$$a = V\frac{dv}{dx} = \frac{8}{(V^2+1)}$$

$$(V^3 + V)\,dv = 8\,dx$$

Integrating

$$\int (V^3 + V)\,dv = \int 8\,dx$$

$$\frac{V^4}{4} + \frac{V^2}{2} = 8x + C_1$$

At $t=0$, $V=0$, $x=0$ $\quad C_1 = 0$

$$8x = \frac{V^4}{4} + \frac{V^2}{2}$$

(a) find V at $x = 20$

$$8\times 20 = \frac{V^4}{4} + \frac{V^2}{2}$$

$$\frac{V^4}{4} + \frac{V^2}{2} - 160 = 0 \qquad ax^2 + bx + c$$

$V^2 = a_1$

..2

$$\frac{v^4}{4}+\frac{v^2}{2}-160=0 \qquad ax^2+bx+c$$

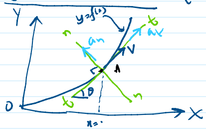
$$v^2=a_1$$

$$\frac{a_1^2}{4}+\frac{a_1}{2}-160=0 \qquad a_1=v^2= \qquad v=4.93 \text{ m/s}$$

(b) find x at $V=54\times\frac{5}{18}=15$ m/s

$$\frac{15^4}{4}+\frac{15^2}{2}=8(x) \Rightarrow x=1596.1\text{ m}$$

Curvi-linear motion (2-D motion)

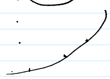


$\theta=\tan^{-1}\left(\frac{dy}{dx}\right) \Rightarrow$ slope

Velocity V $\quad V_y \quad V_x$

$V_x=V\cos\theta \qquad V_y=V\sin\theta$

a_x & a_y



a_t = tangential acc^n = $(R\alpha)=\frac{dv}{dt}$

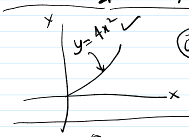
a_n = normal acc^n $=\frac{V^2}{R}$

$$R = \text{Radius of curvature} = \left|\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}\right|$$

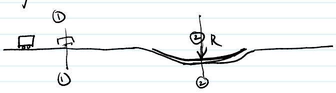
$$=\left|\frac{(V_x^2+V_y^2)^{3/2}}{V_xa_y-V_ya_x}\right| = \left|\frac{V^3}{V_xa_y-V_ya_x}\right|$$

$$V_x=\frac{dx}{dt} \qquad a_x=\frac{d^2x}{dt^2}=\frac{dV_x}{dt}$$

$$V_y=\frac{dy}{dt} \qquad a_y=\frac{d^2y}{dt^2}=\frac{dV_y}{dt}$$



OR $\quad x=4t^2 \qquad y=\frac{5}{t^2}$



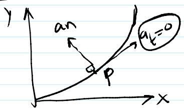
Acc^n at ① ①

$R_1=\infty$

$a_{n_1}=\frac{V_1^2}{\infty}=0$

At section ②

$R_2\neq\infty \qquad a_{n_2}=\frac{V_2^2}{R_2}\neq 0$



If car moves with const vel of 100 kph

$$a_t=\frac{dv}{dt}=\frac{d[\text{const}]}{dt}=0$$

But $a_n\neq 0 \qquad a_n=\frac{V^2}{R}$

$R\neq 0$

$\neq\infty$

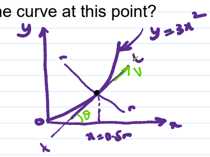
$$\boxed{a=\sqrt{a_n^2+a_t^2}=\sqrt{a_x^2+a_y^2}}$$

A car travels along a vertical curve on a road, the equation of the curve being $3x^2=y$
(x - horizontal and y - vertical distances in m). The speed of the car is constant and equal to 3 m/s.
(a) Find its acceleration when the car is at x=0.5 m.
(b) What is the radius of curvature of the curve at this point?

$V=3$ m/s const $\therefore a_t=0$

$y=3x^2$

$$\frac{dy}{dx}=6x, \quad \frac{d^2y}{dx^2}=6$$



At $x=0.5$

$\frac{dy}{dx}=6\times0.5=3=\tan\theta$ (slope)

$\theta=\tan^{-1}(3)=71.56$

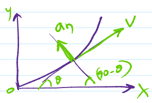
$$R=\left|\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}\right|_{x=3} = \left|\frac{[1+(3)^2]^{3/2}}{6}\right| = 5.27\text{ m}$$

$$a_n=\frac{V^2}{R}=\frac{(3)^2}{5.27}=1.71 \text{ m/s}^2$$

$a_t=0 \quad [\because V$ is const$]$

The resultant acc^n $a=\sqrt{a_n^2+a_t^2}$

$=1.71$ m/s^2



A particle moves on a circular path with its position from rest defined by S = t^3 + 5 t, where the arc length s and time t are measured in meter and second respectively. It is observed that at t = 0.66 seconds the magnitude of the acceleration is 8.39 m/s². What is the diameter of the path.

S = Arc AB = t^3+5t

At $t=\frac{2}{3}=0.66$ sec $\qquad a=8.39$ m/s^2

find dia

$S=R\theta=(t^3+5t)$ | At $t=\frac{2}{3}$ seconds

At $t = \frac{2}{3} = 0.66$ sec $\quad a = 8.39\ m/s^2$

find dia

$S = R\theta = (t^3 + 5t)$

$\theta = (t^3 + 5t)/R$

$\omega = \frac{d\theta}{dt} = \frac{(3t^2+5)}{R}$

$\alpha = \frac{d\omega}{dt} = \frac{1}{R}(6t)$

At $t = \frac{2}{3}$ seconds

$\omega = \frac{1}{R}\left[3 \times \frac{4}{9} + 5\right] = \frac{6.33}{R}$

$\alpha = \frac{1}{R} 6 \times \frac{2}{3} = \frac{4}{R}$

$a_n = \frac{V^2}{R} = \frac{(R\omega)^2}{R} = \omega^2 R = \frac{(6.33)^2}{R^{\cancel{2}}} \times \cancel{R} = \frac{40.11}{R} = a_n$

$a_t = R\alpha$

$= \cancel{R} \times \frac{4}{\cancel{R}}$

$a_t = 4$

$a^2 = a_n^2 + a_t^2$

$(8.39)^2 = \left(\frac{40.11}{R}\right)^2 + (4)^2$

$R^2 = \left(\frac{40.11^2}{54.39}\right)$

$R = 5.44\ m$

A rocket follows the path such that its acceleration is given by â =(4i + t j) m/s². At r = 0 it starts from rest. At t = 10 seconds , find (a) speed of rocket (b) radius of curvature of its path.

$a_x = 4\ m/s^2 \quad a_y = t\ m/s^2 \quad$ At $r=0,\ t=0,\ V=0$

find (i) V (ii) R at $t = 10$ seconds

$a_x = \frac{dV_x}{dt} = 4$

$dV_x = 4\,dt$

Integrating

$\int dV_x = \int 4\,dt$

$V_x = 4t + C_1$

At $t=0,\ V_x=0$

$0 = 4(0) + C_1 \quad \therefore C_1 = 0$

$\boxed{V_x = 4t}$

At $t = 10$ seconds

$V_x = 4 \times 10 = 40\ m/s$

$a_y = t$

$\frac{dV_y}{dt} = t$

$dV_y = t\,dt$

Integrating

$\int dV_y = \int t\,dt$

$V_y = \frac{t^2}{2} + C_2$

At $t=0,\ V_y=0$

$\boxed{C_2 = 0}$

$\boxed{V_y = \frac{t^2}{2}}$

At $t=10$

$V_y = \frac{10^2}{2} = 50\ m/s$

$\therefore$ At $t=10 \quad V = \sqrt{V_x^2 + V_y^2} = \sqrt{(40)^2 + (50)^2} = 64.03\ m/s$

At $t=10 \quad a_x = 4\ m/s^2 \quad a_y = t = 10\ m/s^2$

$a = \sqrt{a_x^2 + a_y^2} = \sqrt{4^2 + 10^2} = 10.77\ m/s^2$

$R = \left|\frac{V^3}{V_x a_y - V_y a_x}\right| = \left|\frac{(64.03)^3}{(40)(10) - (50)(4)}\right| = 1312.56\ m$

$a_n = \frac{V^2}{R} = \frac{(64.03)^2}{1312.56} = 3.12\ m/s^2$

$a^2 = a_t^2 + a_n^2 = a_x^2 + a_y^2$

$a_t = \sqrt{a^2 - a_n^2} = \sqrt{(10.77)^2 - (3.12)^2} = 10.3\ m/s^2$

A Particle moves in the X – Y plane with velocity components, $V_x = 8t - 2$ & $V_y = 2$. If it passes through the point (x,y) = (14,4) at t = 2 seconds, find the equation of the path traced by the particle. Also find resultant acceleration at t = 2 seconds.

$V_x = 8t - 2$

$a_x = \frac{dV_x}{dt} = 8$

$V_x = \frac{dx}{dt} = 8t - 2$

$dx = (8t-2)\,dt$

Integrating

$\int dx = \int (8t-2)\,dt$

$x = \frac{8t^2}{2} - 2t + C_1$

At $t=2 \quad x = 14$

$14 = 4(2)^2 - 2(2) + C_1$

$C_1 = 2$

$\boxed{x = 4t^2 - 2t + 2}$ — (1)

$V_y = 2$

$a_y = \frac{dV_y}{dt} = 0$

$V_y = 2 = \frac{dy}{dt}$

$dy = 2\,dt$

Integrating

$\int dy = \int 2\,dt$

$y = 2t + C_2$

At $t=2 \quad y = 4$

$4 = 2 \times 2 + C_2$

$\boxed{C_2 = 0}$

$\boxed{y = 2t}$ — (2)

$a = \sqrt{a_x^2 + a_y^2}$

$= 8\ m/s^2$

put (2) in (1)

$x = (2t)^2 - 2t + 2 = y^2 - y + 2$

$\boxed{x = y^2 - y + 2} \Leftarrow$ Eqn of path

A particle moves along a hyperbolic path x²/16 - y² = 28. If the x - component of velocity is V_x = 4 m/s and remains constant, determine the magnitudes of particle's velocity and acceleration when it is at point (32 m, 6 m).

$V_x = 4\ m/s$ const $\quad \therefore a_x = 0$

$\frac{x^2}{16} - y^2 = 28 \Rightarrow$ Eqn of path

Diff. w.r.t time t

$\frac{1}{16} 2x \left(\frac{dx}{dt}\right) - 2y\frac{dy}{dt} = 0 \Rightarrow \frac{x V_x}{16} = y V_y \Rightarrow \frac{x \times 4}{16} = y V_y$

$V_y = \frac{32}{} m/s$

$\frac{x^2}{16} - y^2 = 28$ ...

Diff. w.r.t time t

$\frac{1}{16} 2x \left(\frac{dx}{dt}\right) - 2y \frac{dy}{dt} = 0 \Rightarrow \frac{x V_x}{16} = y V_y \Rightarrow \frac{x \times 4}{16} = y V_y$

$\boxed{\frac{x}{4} = y V_y}$ At $x = 32,\ y = 6$ $\frac{32}{4} = 6 V_y$ $V_y = \frac{32}{24}$ m/s

$V_y = 1.33$ m/s

$V = \sqrt{V_x^2 + V_y^2} = \sqrt{(4)^2 + (1.33)^2}$

$= 4.22$ m/s

Diff w.r.t time t

$\frac{V_x}{4} = y \frac{d(V_y)}{dt} + V_y \frac{dy}{dt} \Rightarrow \frac{4}{4} = y a_y + V_y^2$

At $x = 32,\ y = 6,\ V_y = 1.33$

$1 = (6) a_y + (1.33)^2 \Rightarrow a_y = -0.128$ m/s²

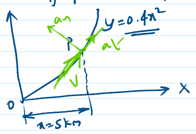
$a_x = 0$

$a = \sqrt{a_x^2 + a_y^2} = 0.128$ m/s²

A particle travels along the parabolic path as shown in fig. When it is at point P, it has a speed of 200 m/s which is increasing at the rate of 0.8 m/s². Find acc^n of particle at P.

$a_t = 0.8$ m/s² $V = 200$ m/s

$a = \sqrt{a_n^2 + a_t^2}$ $a_n = \frac{V^2}{R}$



$y = 0.4x^2$

$\frac{dy}{dx} = 0.8x$ $\frac{d^2y}{dx^2} = 0.8$

At $x = 5$ km $\frac{dy}{dx} = 0.8 \times 5 = 4$

$R = \left| \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2} \right|_{x=5} = \left| \frac{\left[1 + (4)^2\right]^{3/2}}{0.8} \right| = 87.62$ km

$R = 87.62 \times 10^3$ m

$a_n = \frac{V^2}{R} = \frac{(200)^2}{87.62 \times 10^3} = 0.456$ m/s²

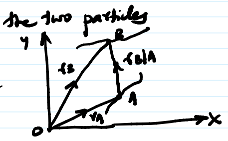
$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(0.8)^2 + (0.456)^2} = 0.92$ m/s²

Relative motion

Relative motion between the two particles

$r_B = r_A + r_{B/A}$

$r_{B/A}$ = position of B w.r.t A



$\boxed{r_{B/A} = r_B - r_A}$

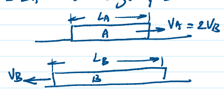
Diff. w.r.t time t $r_A = x_A i + y_A j$

$\boxed{V_{B/A} = V_B - V_A}$ $r_B = x_B i + y_B j$

Diff. w.r.t time t

$\boxed{a_{B/A} = a_B - a_A}$

Ex① Two Trains A & B are moving on a parallel track in opposite dir^n. $V_A = 2V_B$. They take 20 seconds to pass each other. Find V_A & V_B if length of A = 240m & length of B = 300m



$V_{B/A} = V_B - V_A$

$= -V_B - 2V_B$

$= -3V_B$

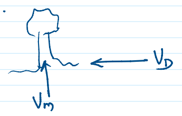
$r_{B/A} = r_B - r_A = -300 - 240 = -540$

$V_{B/A} = \frac{r_{B/A}}{t}$

$-3V_B = \frac{-540}{20}$ $\therefore V_B = \frac{540}{3 \times 20} = 9$ m/s (←)

$\therefore V_A = 2V_B = 2 \times 9 = 18$ m/s (→)

② A monkey is climbing Tree vertically up with vel. of 10 m/s while dog is running towards the tree chasing monkey with a vel. of 15 m/s along a st. line. Find the velocity of dog w.r.t monkey.



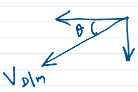
$V_m = 0i + 10j$

$V_D = -15i + 0j$

$V_{D/m} = V_D - V_m$

$= (-15i + 0j) - (+0i + 10j)$

$= -15i - 10j$



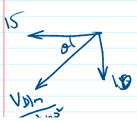
$|V_{D/m}| = \sqrt{15^2 + 10^2} = 18.02$ m/s

$\theta = \tan^{-1}\left(\left|\frac{10}{15}\right|\right) = 33.69°$

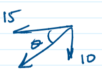
OR

$V_{D/m} = V_D - V_m = (\vec{V_D}) + (-\vec{V_m})$

$= \overleftarrow{15} + (\downarrow 10)$



$=$



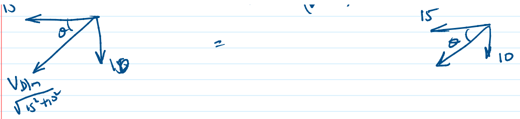
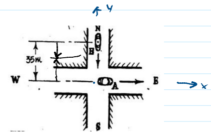


Figure shows two cars A and B at a distance of 35m. Car A is traveling east at a constant speed of 36 KPH. Car B starts from rest and moves south with a constant acceleration of 1.2 m/s². Determine the (1) position (ii) velocity and (iii) acceleration of car B relative to car A six seconds after car A crosses the intersection of roads.



Kine: $r_{B|A}$, $V_{B|A}$, $a_{B|A}$

motion of A

$V_A = (36\,kpH \times \frac{5}{18})\,m/s = 10\,m/s \rightarrow$ (const)

After $t = 6$ seconds

$V_A = 10\,m/s = 10i + 0j$, $a_A = 0$

$r_A = s = ut + \frac{1}{2}at^2 = 10 \times 6 = 60\,m$

$r_A = 60i + 0j$

$a_A = 0i + 0j$

motion of B

After $t = 6$ seconds

$V_B = u_B + a_B t = 0 + 1.2 \times 6 = 7.2\,m/s \downarrow$

$V_B = 0i - 7.2j$

$S_B = (ut + \frac{1}{2}at^2)_B = 0 + \frac{1}{2} \times 1.2 \times 6^2$

$= 21.6\,m$

$r_B = 35 - 21.6 = 13.4$

$r_B = 0i + 13.4j$

$a_B = 0i - 1.2j$

$r_{B|A} = r_B - r_A$

$= (0i + 13.4j) - (60i + 0j)$

$r_{B|A} = -60i + 13.4j$

$= \sqrt{(60)^2 + (13.4)^2} =$

$\theta_1 = \tan^{-1}\left(\frac{13.4}{60}\right) =$

$V_{B|A} = V_B - V_A$

$= (0i - 7.2j) - (10i + 0j)$

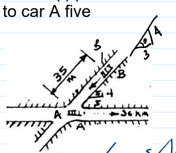
$= -10i - 7.2j$

$= \sqrt{10^2 + 7.2^2}$

$\theta_2 = \tan^{-1}\left(\frac{7.2}{10}\right) =$

$a_{B|A} = a_B - a_A = -1.2j$

Figure below shows cars A and B at a distance of 35 m. Car A moves with a constant speed of 36 KPH and car B starts from rest with an acceleration of 1.5 m/s². Determine relative (I) position (ii) velocity (iii) acceleration of car B with respect to car A five seconds after car A crosses the intersection.



motion of A $t = 5$

$u_A = (36 \times \frac{5}{18})\,m/s = 10\,m/s$ (const)

$a_A = 0$, $V_A = 10i + 0j$

$S_A = r_A = ut + \frac{1}{2}at^2 = 10 \times 5 = 50\,m \rightarrow$

$r_A = 50i + 0j$

$a_A = 0i + 0j$

$\cos\theta = \frac{3}{5} = 0.6$

$\sin\theta = \frac{4}{5} = 0.8$

motion of B

$u_B = 0$, $a_B = 1.5\,m/s^2$

$V_B = u_B + a_B t = 0 + 1.5 \times 5 = 7.5\,m/s$

$V_B = -7.5\cos\theta i - 7.5\sin\theta j = -7.5 \times 0.6i - 7.5 \times 0.8j = -4.5i - 6j$

$a_B = -1.5\cos\theta i - 1.5\sin\theta j = -1.5 \times 0.6i - 1.5 \times 0.8j = -0.9i - 1.2j$

$S_B = u_B t + \frac{1}{2}a_B t^2 = 0 + \frac{1}{2} \times 1.5 \times 5^2 = 18.75$

$r_B = 35 - 18.75 = 16.25$

$r_B = 16.25\cos\theta i + 16.25\sin\theta j = 16.25 \times 0.6i + 16.25 \times 0.8j$

$r_B = 9.75i + 13j$

$r_{B|A} = r_B - r_A = (9.75i + 13j) - (50i + 0j) = -40.25i + 13j$

$= \sqrt{(40.25)^2 + (13)^2} = 42.3$

$\theta_1 = \tan^{-1}\left(\frac{13}{40.25}\right) = 17.9°$

$V_{B|A} = V_B - V_A = (-4.5i - 6j) - (10i + 0j) = -14.5i - 6j$

$= \sqrt{(14.5)^2 + (6)^2} = 15.7\,m/s$

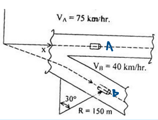
$\theta_2 = \tan^{-1}\left(\frac{6}{14.5}\right) = 22.47°$

$a_{B|A} = a_B - a_A = (-0.9i - 1.2j) - (0i + 0j) = -0.9i - 1.2j$

$= \sqrt{(0.9)^2 + (1.2)^2} = 1.5$

$\theta_3 = \tan^{-1}\left(\frac{1.2}{0.9}\right) = 53.13$

Automobile A is traveling along a straight highway, while B is moving along a circular curve of 150 m radius. The speed of A is increased at the rate of 1.5 m/s² and the speed at B is being decreased at the rate of 0.9 mls². For the position shown in figure, determine the velocity of A relative to B and the acceleration of A relative to B. At this instant the speed of A is 75 KPH and that of B is 40 KPH.



$V_A = (75\,kpH \times \frac{5}{18})\,m/s = 20.83\,m/s \rightarrow$

$\boxed{V_A = 20.83i + 0j}$

$V_B = (40\,kpH \times \frac{5}{18})\,m/s = 11.11\,m/s$

$V_B = (11.11\cos 30)i - (11.11\sin 30)j$

$\boxed{V_B = 9.62i - 5.56j}$

$V_{A|B} = V_A - V_B = (20.83i + 0j) - (9.62i - 5.56j)$

$= 11.21i + 5.56j$

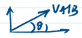
$= \sqrt{(11.21)^2 + (5.56)^2} = 12.51\,m/s$

$\theta_1 = \tan^{-1}\left(\frac{5.56}{...}\right) = 26.4°$

$= 11.21i + 8.56j$

$= \sqrt{(11.21)^2 + (8.56)^2} = 12.51$ m/s

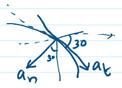
$\theta_1 = \tan^{-1}\left(\frac{8.56}{11.21}\right) = 26.4^\circ$



$a_A = 1.5i + 0j$

a_B

$a_{tB} = -0.9$ m/s² (deceleration)



a_{tBx}

a_{tBy}

$a_{tB} = -0.9\cos 30\, i - (-0.9\sin 30)j$

$a_{tB} = -0.78i + 0.45j$

$a_{nB} = \frac{V_B^2}{R} = \frac{(11.11)^2}{150} = 0.823$ m/s²

$a_{nB} = -0.823\cos 60\, i - 0.823\sin 60\, j = -0.411i - 0.712j$

$\bar{a}_B = \bar{a}_{tB} + \bar{a}_{nB} = (-0.78i + 0.45j) + (-0.411i - 0.712j)$

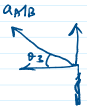
$\boxed{a_B = -1.191i - 0.262j}$

$a_{A/B} = a_A - a_B = (1.5i + 0j) - (-1.191i - 0.262j)$

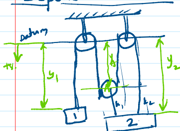
$a_{A/B} = 2.691i + 0.262j$

$= \sqrt{(2.691)^2 + (0.262)^2} = 2.703$ m/s²

$\theta_2 = \tan^{-1}\left(\frac{0.262}{2.691}\right) = 5.56^\circ$



Dependent motion



Find a_1 if $a_2 = 6$ m/s² ↑

Also find $a_{1/2}$

Here single rope/cord is used to connect block 1 & 2.

$y_1 + 3y_2 + k_1 + k_2 = $ const

Diff w.r.t time t

$V_1 + 3V_2 - 0 - 0 = 0$ — ①

Diff above equation again w.r.t time t

$a_1 + 3a_2 = 0$

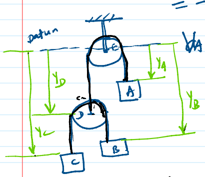
$a_2 = +6$ ↑ ∴ $a_1 + 3 \times 6 = 0$ $a_1 = -18$ m/s²

$a_1 = 18$ ↓

$a_{1/2} = a_1 - a_2$

~~$= 6 - (-18) = 24$ m/s²~~

$= -18 - (6) = -24$ m/s²



$V_A = 5$ m/s ↓, $V_C = 2$ m/s ↑ Find V_B

Length of the string which connects A & D = const

$L_1 = y_D + c_1 + y_A = $ const

Diff $= V_D + 0 + V_A = 0$

$V_A + V_D = 0$ —①

Length of the string (or cord) which connects C & B = const

↓ +ve

$(y_C - y_D) + c_2 + (y_B - y_D) = $ const

Diff w.r.t time t

$V_C - V_D + 0 + V_B - V_D = 0$

$V_C + V_B = 2V_D$ —②

$V_A + V_D = 0$ ①

$V_A = 5$ m/s ↓

$V_C = 2$ m/s ↑

$V_C = -2$ m/s

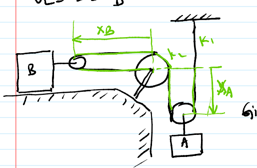
From eqⁿ ① $5 + V_D = 0$ ∴ $V_D = -5$ m/s

From eqⁿ ② $V_C + V_B = 2V_D$

$-2 + V_B = 2(-5)$

$V_B = -10 + 2 = -8$ m/s

$V_B = 8$ m/s ↑



Given $V_A = 18$ m/s ↓ Find V_B &

$a_A = 2$ m/s² ↑ a_B

$k_1 + k_2 + 2X_B + 2X_A = $ const

$0 + 2V_B + 2V_A = 0$

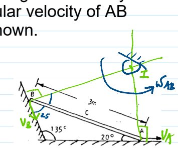
$V_B = -V_A$

$a_B = -a_A$

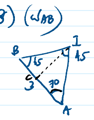
A bar AB, 3 m long slides down the plane shown in Fig. The velocity of end A is 3.6 m/s to the right. Determine the angular velocity of AB and velocity of end B and center C at the instant shown.

I = ICR of AB

$V_A = (IA)(\omega_{AB})$, $V_B = (IB)(\omega_{AB})$



By sine rule



$\frac{BI}{\sin 70} = \frac{3}{\sin 45} = \frac{AI}{\sin 65}$

$AC = 1.5$, $AI = 3.85$

$BI = 4$ m

$AI = 3.85$ m

$V_A = IA\ \omega_{AB}$ ∴ $\omega_{AB} = \frac{V_A}{IA} = \frac{3.6}{3.85} = 0.935$ r/s

∴ $V_B = (IB)(\omega_{AB}) = (4)(0.935) = 3.74$ m/s

By cosine rule $IC^2 = AC^2 + IA^2 - 2\, AC\, IA \cos 70$

$= (1.5)^2 + (3.85)^2 - 2 \times 1.5 \times 3.85 \times \cos 70$

$IC = 3.622$

$V_C = (IC)(\omega_{AB}) = (3.622)(0.935) = 3.386$ m/s

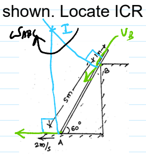
By cosine rule $IC^2 = BC^2 + IB^2 - 2\,IB\cdot BC\cos$

$= (1.5)^2 + (3.85)^2 - 2\times 1.5\times 3.85\times\cos 30$

$IC = 3.622$

$V_C = (IC)(\omega_{AB}) = (3.622)(0.935) = 3.386$ m/s

Velocity point A on a rod is 2 m/s at the instant shown. Locate ICR and determine velocity of pt B on the rod.



$\tan 30 = \frac{BI}{5}$

$BI = 5\tan 30 = 2.89$ m

$AI^2 = AB^2 + BI^2 = 5^2 + 2.89^2$

$AI = 5.78$ m

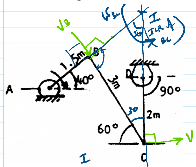
$V_A = (IA)(\omega_{ABC}) \quad \therefore \omega_{ABC} = \frac{V_A}{IA} = \frac{2}{5.78} = 0.346$ r/s

$V_B = (IB)(\omega_{ABC})$

$= (2.89)(0.346) = 1$ m/s

$V_C = (IC)(\omega_{ABC}) =$

A crank AB of a mechanism as shown in Fig above is moving with angular velocity of 2 rev/mm in a clockwise direction. Find the angular velocity of the arm CD when AB makes an angle of 40° with the horizontal.



$N_{AB} = 2$ RPM $\quad \omega_{AB} = \frac{2\pi N_{AB}}{60} = \frac{2\times\pi\times 2}{60} = \left(\frac{\pi}{15}\right)$ r/s

$V_B = (r\omega)_{AB} = (r\omega)_{BC}$

$= (AB)(\omega_{AB}) = (BI)(\omega_{BC}) \quad -①\quad \Rightarrow \omega_{BC} =$

$V_C = (r\omega)_{BC} = (r\omega)_{CD} \quad -② \quad \Rightarrow \omega_{CD} =$

$= (CI)\,\omega_{BC} = (CD)(\omega_{CD})$

By sine rule

$\frac{BI}{\sin 30} = \frac{3}{\sin 50} = \frac{CI}{\sin 100}$

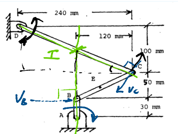
$BI = 1.958$ m

$CI = 3.857$ m

① $\omega_{BC} = \frac{AB\times\omega_{AB}}{BI} = \frac{1.5\times\pi}{1.958\times 15} = 0.16$ r/s

② $\omega_{CD} = \frac{(CI)(\omega_{BC})}{CD} = \frac{3.857\times 0.16}{2} = 0.31$ r/s

AB rotates with constant angular velocity of 15 rad / sec CW. Find angular velocity of CD & velocity of E if E being mid point of BC.



I is the ICR of BC

$\omega_{AB} = 15$ r/s

$V_B = (r\omega)_{AB} = (r\omega)_{BC}$

$(AB)(\omega_{AB}) = (BI)(\omega_{BC}) \Rightarrow ①$

$V_C = (r\omega)_{BC} = (r\omega)_{CD}$

$(CI)(\omega_{BC}) = (CD)(\omega_{CD}) \Rightarrow ②$

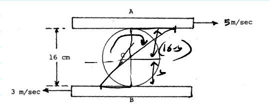
$CI = \sqrt{50^2 + 120^2} = 130$

$CD = 2\times CI = 260$

① $(30)(15) = (100)(\omega_{BC}) \Rightarrow \omega_{BC} = \frac{30\times 15}{100} = 4.5$ r/s

② $(130)(4.5) = (260)(\omega_{CD}) \Rightarrow \omega_{CD} = 2.25$ r/s

A roller of radius 8 cm rides between two horizontal bars moving in the opposite directions as shown in the figure. Locate the instantaneous centre of velocity and give its distance from B. Assume no slip conditions at the points A and B.



$V_A = (16-y)(\omega) \quad -①$

$V_B = (y)(\omega) \quad -②$

$\frac{①}{②} = \frac{V_A}{V_B} = \frac{5}{3} = \frac{(16-y)(\omega)}{(y)(\omega)}$

$5y = (16-y)(3)$

$3y + 5y = 48 \quad y = \frac{48}{8} = 6$ cm

$V_B = y\,\omega$

$300 = (6)(\omega) \quad \omega = \left(\frac{300}{6}\right)$ r/s

If both plates move in same dir^n

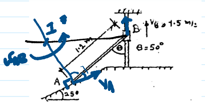
$V_A = (16+y)(\omega) = 5$

$V_B = (y)(\omega) = 3$

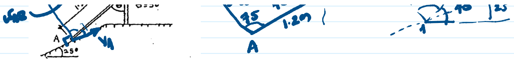
$y =$

Figure below shows a collar B which moves upwards with a constant velocity of 1.5 m/s. At the instant when θ = 50º, determine (i) the angular velocity of rod AB which is pinned at B and freely resting at A against 25° slopping ground and (ii) the velocity of end A of the rod.

I is the ICR of link AB.



According to sine rule ($\triangle ABI$)



According to sine rule ($\triangle ABI$)

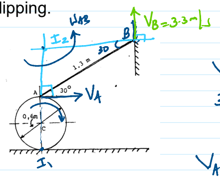
$$\frac{AI}{\sin 40} = \frac{1.2}{\sin 65} = \frac{BI}{\sin 75} \qquad AI = 0.85m \quad BI = 1.28m$$

$V_B = (BI)(\omega_{AB})$ $\quad \therefore \omega_{AB} = \frac{1.5}{1.28}$ r/s $= 1.172$ r/s

$1.5 = 1.28\ \omega_{AB}$

$V_A = (r\omega) = (AI)(\omega_{AB}) = 0.85 \times 1.172 = 1$ m/s

A uniform cylinder C to which is pinned a rod AB at A with other end B moving along Vertical wall as shown in Fig. If the end B of the rod is moving upward along the vertical wall at a speed of 3.3 m/s., find the angular velocity of the cylinder assuming the cylinder is rolling without slipping.



I_1 = ICR of cylinder

I_2 = ICR of Rod AB

$BI_2 = 1.3\cos 30$

$V_B = (r\omega)_{AB} = (BI_2)(\omega_{AB})$

$3.3 = (1.3\cos 30)(\omega_{AB})$

$\therefore \omega_{AB} = 2.93$ r/s

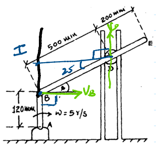
$V_A = (r\omega)_{cyl} = (r\omega)_{AB}$

$= (AI_1)\ \omega_{cyl} = (AI_2)(\omega_{AB})$

$(0.6)\ \omega_{cyl} = (1.3\sin 30)(2.93)$

$\omega_{cyl} = 3.174$ r/s

Rod BDE is partially guided by a roller at D which moves in a vertical track. Knowing that the instant shown the angular velocity of AB is 5 rad/sec clockwise and β = 25°, determine (i) angular velocity of rod BE (ii) velocity of point E.



$I \Rightarrow$ ICR of the link BDE

$V_B = (r\omega)_{AB} = (AB)(\omega_{AB})$

$= (120)(5)$ mm/s

$= 600$ mm/s

$V_B = (r\omega)_{BDE} = (BI)(\omega_{BDE}) \Rightarrow 600 = 211.31(\omega_{BDE})$

$\omega_{BDE} = 2.84$ r/s

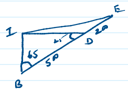
$V_D = (r\omega)_{BDE} = (DE)(\omega_{BDE}) \rightarrow V_D = (453.15)(2.84)$

$= 1286.95$ mm/s

$V_E = (EI)(\omega_{BDE}) =$

$V_E = (640.02)(2.84)$

$= 1817.66$ mm/s



$BI = 500\sin 25 = 211.31$

$DE = 500\cos 25 = 453.15$

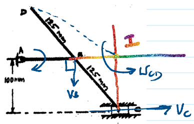
using cosine rule $\triangle BIE$

$EI^2 = BI^2 + BE^2 - (2\cos 65)(BI)(BE)$

$= (211.31)^2 + (\cancel{453.15}\ 700)^2 - 2 \times 211.31 \times \cancel{453.15}\ 700 \times \cos 65$

$EI = 640.02$ mm

At the position shown in figure, the crank AB has angular velocity of 3 rad/s clockwise. Find the velocity of slider C and the point D at the instant shown.



$I \Rightarrow$ ICR of CD

$V_B = (r\omega)_{AB} = (r\omega)_{CD}$

$(AB)(\omega_{AB}) = (BI)(\omega_{CD}) \Rightarrow \omega_{CD} =$ r/s

$V_C = (r\omega)_{CD} = (CI)(\omega_{CD}) =$

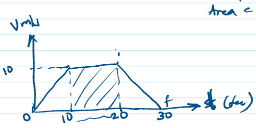
$V_D = (ID)(\omega_{CD}) =$

AB
BI
CI
DI

Revision class 20/12/2021

motion curves ① x-t ② v-t ③ a-t

slope = vel; slope = a, + Area = change in displacement; slope = jerk, + Area = dv, + $\frac{da}{dt}$



Find

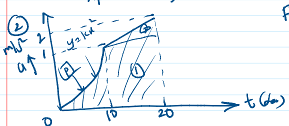
(i) change in displacement from t = 10 to 20

$X_{20} - X_{10}$ = Area under v-t

$dx = X_{20} - X_{10} = 10 \times 10 = 100$ m

(ii) what will be the acc^n from t = 20 to 30 secs

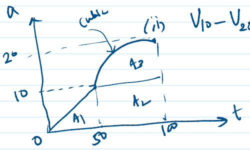
$$a = \frac{y_f - y_i}{x_f - x_i} = \frac{0 - 10}{30 - 10} = \frac{-10}{10} = -1\ \text{m/s}^2$$



Find change in velocity from (i) t = 0 to 10 secs (ii) t = 10 to 20 s

(i) $V_{10} - V_0 = dv$ = Area under a-t

$(dv)_{t=0 \text{ to } 10} = \frac{1}{3} \times 10 \times 1 = \frac{10}{3}$ m/s

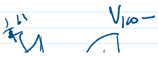


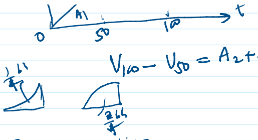
(ii) $V_{10} - V_{20}$ = Area under a-t

$= \frac{1}{2} \times 10 \times [1+2] = \frac{30}{2} = 15$ m/s

Find change in velocity from t = 50 to 100 sec

$V_{100} - V_{50} = A_2 + A_1 = (50 \times 10) + (\frac{3}{4} \times 50 \times 10) = 875$ m

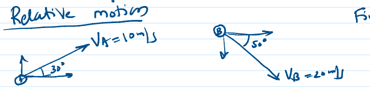




$t = 50 \rightarrow 100$

$$V_{100} - V_{50} = A_2 + A_3 = (50 \times 10) + \left(\tfrac{2}{3} \times 50 \times 10\right) = 833 \text{ m}$$

Relative motion



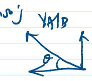
Find $V_{A/B}$

$V_A = (10\cos 30)\,i + (10\sin 30)\,j = 8.66i + 5j$

$V_B = (20\cos 50)\,i - (20\sin 50)\,j =$

$V_{A/B} = V_A - V_B = (8.66i + 5j) - 20\cos 50\, i + 20\sin 50\, j$

$= -4.2i + 20.32j$

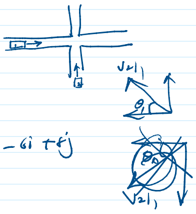


$|V_{A/B}| = \sqrt{(-4.2)^2 + (20.32)^2} = 20.7 \text{ m/s}$

$\theta = \tan^{-1}\left|\frac{20.32}{4.2}\right| = 78.32^\circ$

$V_1 = 6 \text{ m/s} \rightarrow, \quad V_2 = 8 \text{ m/s} \uparrow$

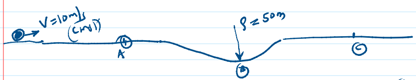
Find $|V_{2/1}| = ?$



$V_1 = 6i + 0j, \quad V_2 = 0i + 8j$

$V_{2/1} = V_2 - V_1 = (8j) - (6i) = -6i + 8j$

$|V_{2/1}| = \sqrt{(6^2) + (8)^2} = 10 \text{ m/s}$



Find acc^n of particle when it is at (1) A (2) B (3) C

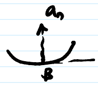
At A: $a_t = 0$ [v is const]

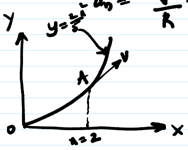
$a_n = \frac{V^2}{R} = \frac{(10)^2}{\infty} = 0$

At C: $a_t = 0$, $a_n = 0$

At B: $a_t = 0$ [v is const]

$a_n = \frac{V^2}{R} = \frac{(10)^2}{50} = \frac{100}{50} = 2 \text{ m/s}^2$





$V = 100 \text{ m/s}$ (const) find acc^n at A ($x = 2$ m)

$a_t = 0$

$a_n = \frac{V^2}{R}$

$y = \frac{2}{5}x^2 \quad \frac{dy}{dx} = \frac{2 \times 2x}{5} \quad \frac{d^2y}{dx^2} = \frac{4}{5}$

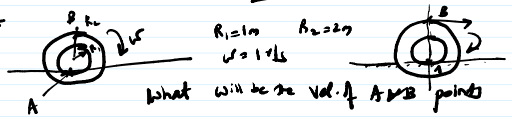
At $x = 2$, $\frac{dy}{dx} = \frac{8}{5}$, $\frac{d^2y}{dx^2} = \frac{4}{5}$

$$R = \left|\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}\right|_{x=2} = 8.4 \text{ m}$$

$a_n = \frac{V^2}{R} = \frac{(100)^2}{8.4} \text{ m/s}^2 = 1190.48 \text{ m/s}^2$

$a = \sqrt{a_n^2 + a_t^2} = 1190.48 \text{ m/s}^2$

ICR



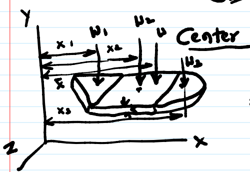
$R_1 = 1$ m, $R_2 = 2$ m, $\omega = 1$ r/s

What will be the vel. of A & B points

$A \Rightarrow$ ICR ($I_C = 0$)

$V_B = (AB)(\omega)$

$= (3)(1) = 3 \text{ m/s} \rightarrow$



Center of gravity [10, 20, 30]

$W = W_1 + W_2 + W_3 =$

According to the Law of moments

$W\bar{x} = W_1x_1 + W_2x_2 + W_3x_3$

$$\bar{x} = \frac{W_1x_1 + W_2x_2 + W_3x_3}{W_1 + W_2 + W_3} = \frac{\Sigma Wx}{\Sigma W}$$

$|||^{ly}$ $\bar{y} = \frac{\Sigma Wy}{\Sigma W}$ $\quad \bar{z} = \frac{\Sigma Wz}{\Sigma W}$

ρ = density = $\frac{\text{mass}}{\text{vol}}$

mass = density × Vol

$m = \rho V$

$W = mg = (\rho V g)$

$V_1 = A_1 t$ $\quad m_1 = \rho V_1 = (\rho A_1 t)$

$V_2 = A_2 t$

$V_3 = A_3 t$

$\bar{x} = \frac{\Sigma Wx}{\Sigma W}$

$= \frac{W_1x_1 + W_2x_2 + W_3x_3}{W_1 + W_2 + W_3}$

$\bar{x} = \frac{m_1 g x_1 + m_2 g x_2 + m_3 g x_3}{m_1 g + m_2 g + m_3 g}$

$= \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{\Sigma mx}{\Sigma m}$

$\bar{x} = \frac{(\rho A_1 t)x_1 + (\rho A_2 t)x_2 + (\rho A_3 t)x_3}{(\rho A_1 t) + (\rho A_2 t) + (\rho A_3 t)}$

If the plate is of same material & thickness

$\rho_1 = \rho_2 = \rho_3 = \rho \qquad t_1 = t_2 = t_3 = t$

say

$(\gamma A_1 t) + (\gamma A_2 t) + (\gamma A_3 t)$

If the plate is of same material & thickness

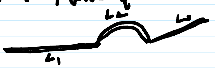
$\rho_1 = \rho_2 = \rho_3 = \rho \qquad t_1 = t_2 = t_3 = t$

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3} = \frac{\Sigma Ax}{\Sigma A} \qquad \bar{y} = \frac{\Sigma Ay}{\Sigma A} \text{ — 2D}$$

If the plate is of same material but of diff thick

$$\bar{x} = \frac{V_1x_1 + V_2x_2 + V_3x_3}{V_1 + V_2 + V_3} = \frac{\Sigma Vx}{\Sigma V}, \quad \bar{y} = \frac{\Sigma Vy}{\Sigma V} \text{ — 3D}$$

If the rod / wire of same material & same C.S. Area



$A_1 = A_2 = A_3 = A$

$\rho_1 = \rho_2 = \rho_3 = \rho$

$(V = AL_1)$

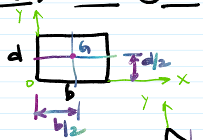
$$\bar{x} = \frac{(\cancel{A}L_1)x_1 + (\cancel{A}L_2)x_2 + (\cancel{A}L_3)x_3}{\cancel{A}L_1 + \cancel{A}L_2 + \cancel{A}L_3} = \frac{L_1x_1 + L_2x_2 + L_3x_3}{L_1 + L_2 + L_3}$$

$$\bar{x} = \frac{\Sigma Lx}{\Sigma L} \qquad \bar{y} = \frac{\Sigma Ly}{\Sigma L} \text{ — 1D}$$

Formulae of Area & their c.g Location

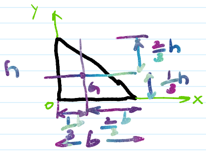
① Rectangle

$A = bd$



② Triangle

$A = \frac{1}{2}bh$

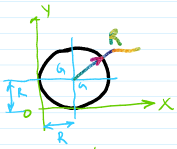


③ Circle

$A = \pi R^2$

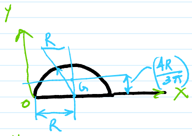
$= \pi\left(\frac{D}{2}\right)^2$

$= \frac{\pi D^2}{4}$



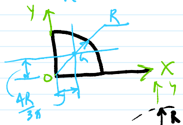
④ Semi-circle

$A = \frac{\pi R^2}{2} = \frac{\pi D^2}{8}$



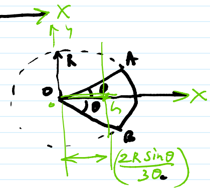
⑤ Q. circle

$A = \frac{\pi R^2}{4} = \frac{\pi D^2}{16}$



⑥ circular Sector

$A = R^2\theta_c \qquad (\theta_c = \text{in radians})$

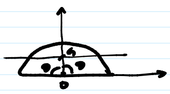


Semi-circle

$2\theta = 180$

$\theta = 90$

$\theta_c = \frac{\pi}{2}$

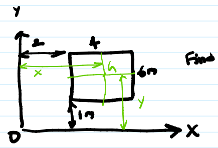


$A = R^2\theta_c = R^2 \times \frac{\pi}{2} = \frac{\pi R^2}{2}$

$OG = \frac{2R\sin\theta}{3\theta_c} = \frac{2R\sin 90}{3 \times \frac{\pi}{2}}$

$= \frac{4R}{3\pi}(1)$

Ex



Find A, x_g, y_g

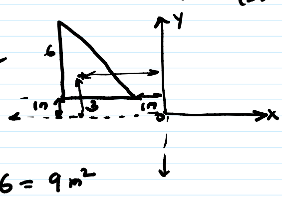
$A = 4 \times 6 = 24\,m^2$

$x_g = 2 + \left(\frac{4}{2}\right) = 4m$

$y_g = 1 + \left(\frac{6}{2}\right) = 4m$

②

Find Area & coordinates of cg.

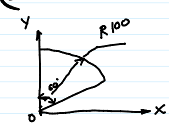


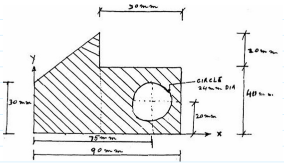
$A = \frac{1}{2} \times 3 \times 6 = 9\,m^2$

$x_g = -(1+2) = -3, \quad y_g = 1 + \left(\frac{1}{3} \times 6\right) = 1 + 2 = 3$

$= -(1 + \frac{2}{3} \times 3) = -3$

Find Area & coordinates of centroid

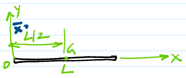




C.G of lines | Rod | 1-D

$\bar{x} = \frac{\Sigma Lx}{\Sigma L}$ $\bar{y} = \frac{\Sigma Ly}{\Sigma L}$

① Straight line

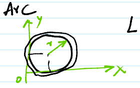


$L = L$
$\bar{x} = L/2 ,\ \bar{y} = 0$

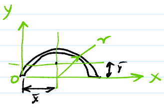
② Inclined line

$L = L$ $\bar{x} = \frac{L}{2}\cos\theta ,$ $\bar{y} = \frac{L}{2}\sin\theta$

③ circular Arc

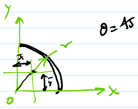


$L = 2\pi r ,\ \bar{x} = \bar{y} = r$



④ Semi-circular Arc

$L = \pi r$
$\bar{x} = r \ ,\ \bar{y} = \frac{2r}{\pi}$

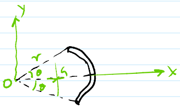


$\theta = 45 = \frac{\pi}{4}$

$\bar{x} = \frac{r\sin\theta}{\alpha} = oc$

$= \frac{r\sin 45}{\frac{\pi}{4}} = oc$

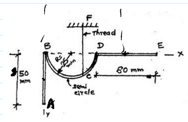
⑤ Q. circle
$L = \frac{r\pi}{2}$
$\bar{x} = \bar{y} = \frac{2r}{\pi}$



⑥ Arc of a circle

$L = 2r\theta$
$\bar{y} = 0$ $\bar{x} = \frac{r\sin\theta}{\alpha}$

Find the position of C (x_c, y_c) for suspending the wire of uniform weight of 4 N/m which is bent as shown in the figure so that the portion DE remains horizontal in equilibrium position.

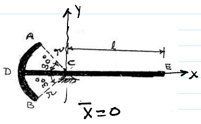


$L_1 = AB = 50 ,\ x_1 = 0 ,\ y_1 = -25$
$L_2 = BC = \pi(25) ,\ x_2 = 25 ,\ y_2 = -\frac{2\times 25}{\pi}$
$L_3 = DE = 60 ,\ x_3 = 50 + \frac{60}{2} = 80 ,\ y_3 = 0$

$\bar{x} = \frac{L_1x_1 + L_2x_2 + L_3x_3}{L_1 + L_2 + L_3} = \frac{0 + (\pi\times 25)(25) + (60)(80)}{50 + \pi(25) + 60} = \underline{\underline{35.873}}$

$\bar{y} = \frac{L_1y_1 + L_2y_2 + L_3y_3}{L_1 + L_2 + L_3} = \frac{(50)(-25) + (\pi(25))\left(\frac{-50}{\pi}\right) + 0}{50 + \pi(25) + 60} = \underline{\underline{13.26}}$

The figure below shows is formed of a thin homogeneous wire. Find the length 'L' of portion CE of the wire for which the centre of gravity of the figure is located at C.



$\bar{x} = 0$
$\bar{y} = 0$

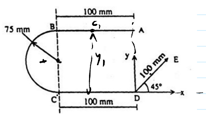
$L_1 = CE = L \ ,\ x_1 = \frac{L}{2}$
$L_2 = CD = r ,\ x_2 = -0.5r$
$L_3 = \text{Arc}AB = \left(60\times\frac{\pi}{180}\right)(r) ,\ x_3 = \frac{-r\sin 30}{\left(\frac{30\times\pi}{180}\right)}$

$0 = \frac{L^2}{2} - r^2\times 0.5 - 2r^2\times 0.5$
$\frac{L^2}{2} = r^2\times 1.5$ $\boxed{L = \sqrt{3}r}$

$\bar{x} = 0 = \frac{L_1x_1 + L_2x_2 + L_3x_3}{L_1 + L_2 + L_3} \Rightarrow 0 = L_1x_1 + L_2x_2 + L_3x_3$

$\boxed{L = \sqrt{3}r}$

Locate C.g



$L_1 = AB = 100$
$x_1 = -50 ,\ y_1 = 150$
$L_2 = \text{Arc } BC = (\pi)(75)$
$x_2 = -\left(100 + \frac{2\times 75}{\pi}\right) ,\ y_2 = 75$
$L_3 = CD = 100$
$x_3 = -50 ,\ y_3 = 0$

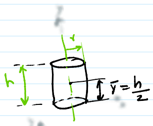
$L_4 = DE = 100$
$x_4 = 50\cos 45 ,\ y_4 = 50\sin 45$

$\bar{x} = \frac{L_1x_1 + L_2x_2 + L_3x_3 + L_4x_4}{L_1 + L_2 + L_3 + L_4}$
$= -77.05$
$\bar{y} = 67.59$

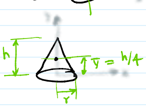
C.g of Volumes

Formulae of primitive volumes

① A solid cylinder
Volume $= V_c = \pi r^2 h ,\ \bar{y} = \frac{h}{2}$



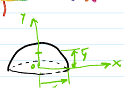
② A solid right circular cone
$V = \frac{1}{3}\pi r^2 h ,\ \bar{x} = 0 ,\ \bar{y} = \frac{h}{4}$



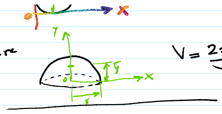
③ Sphere

$V = \frac{4}{3}\pi r^3$ $\bar{x} = r ,\ \bar{y} = r$

④ Hemi-sphere

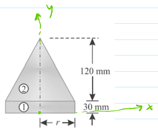


$V = \frac{2\pi r^3}{3} ,$ $\bar{x} = 0 ,\ \bar{y} = \frac{3r}{8}$



④ Hemi-sphere $V = \frac{2\pi r^3}{3}, \quad \bar{x} = 0, \quad \bar{y} = \frac{3r}{8}$

① A solid body formed by joining the base of a right circular cone of height H to the equal base of a right circular cylinder of height h. Calculate the distance of the centre of mass of the solid from its plane face, when H = 120 mm and h = 30 mm



$\bar{x} = 0$

$V_1 = (\text{cylinder}) = \pi(r)^2(30) \ mm^3$

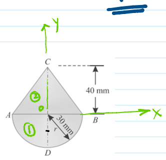
$y_1 = \frac{30}{2} = 15 \ mm$

$V_2 = (\text{cone}) = \frac{\pi}{3} r^2 (120) = 40\pi r^2 \ mm^3$

$y_2 = 30 + \frac{120}{4} = 60 \ mm$

$\bar{y} = \frac{V_1 y_1 + V_2 y_2}{V_1 + V_2} = \frac{(30\pi \cancel{r^2})(15) + (40\pi \cancel{r^2})(60)}{70\pi \cancel{r^2}} = \mathbf{40.7 \ mm}$

A body consists of a right circular solid cone of height 40 mm and radius 30 mm placed on a solid hemisphere of radius 30 mm of the same material. Find the position of centre of gravity of the body.



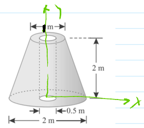
$V_1 = \frac{2\pi(30)^3}{3}, \quad y_1 = -\frac{3(30)}{8}$

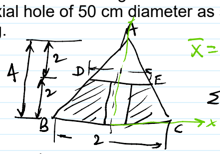
$V_2 = \frac{1}{3}\pi(30)^2 40, \quad y_2 = +\frac{40}{4}$

$\bar{y} = \frac{V_1 y_1 + V_2 y_2}{V_1 + V_2} =$

$\bar{y} = \frac{-\frac{2\pi(30)^3}{3} \times \frac{3(30)}{8} + \frac{\pi(30)^2 40}{3} \times 10}{V_1 + V_2} = \frac{-82500\pi}{30000\pi} = -2.75$

A frustum of a solid right circular cone has an axial hole of 50 cm diameter as shown in Fig.





$\bar{x} = 0$

$\Sigma V = V_1(ABC) - V_2(ADE) - V_3(\text{hole})$

$\bar{y} = \frac{V_1 y_1 - V_2 y_2 - V_3 y_3}{V_1 - V_2 - V_3}$

$V_1 = \frac{\pi}{3}(1)^2 \times 4, \quad y_1 = \frac{4}{4} = 1$

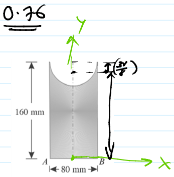
$V_2 = \frac{\pi}{3}(0.5)^2 2, \quad y_2 = 2 + \frac{2}{4} = 2.5$

$V_3 = \pi(0.25)^2(2), \quad y_3 = \frac{2}{2} = 1$

$\bar{y} = \frac{(\frac{\pi}{3} \times 4)(1) - (\frac{\pi(0.5)^2 2}{4})(2.5) - \pi(2)(0.25)^2(1)}{(\frac{\pi}{3} \times 4) - \frac{2\pi}{3}(0.5)^2 - (\pi)(0.25)^2(2)}$

$= 0.76$

A hemisphere of radius 40 mm is cut out from a right circular cylinder of diameter 80 mm and height 160 mm as shown in Fig. Find the centre of gravity of the body from the base AB.



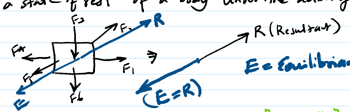
$V_1 = \text{cylinder} = \pi \times 40^2 \times 160, \quad y_1 = \frac{160}{2} = 80$

$V_2 = \text{hemisphere} = \frac{2\pi(40)^3}{3}, \quad y_2 = 160 - \frac{3(40)}{8}$

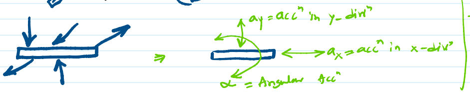
$\bar{y} = \frac{V_1 y_1 - V_2 y_2}{V_1 - V_2} = \frac{(\pi \times 40^2 \times 160)(80) - (\frac{2\pi \times 40^3}{3})(145)}{670208.4328} = 67$

Equilibrium of forces

– It is a state of rest of a body under the action of forces



E = Equilibriant of forces



DOF 3

Rest

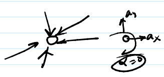
① No motion in x-dir^n ($a_x = 0$) $\quad \Sigma F_x = m\cancel{a_x}^{\text{zero}} = 0$

② No motion in y-dir^n ($a_y = 0$) $\quad \Sigma F_y = m\cancel{a_y}^{\text{zero}} = 0$

③ No rotation ($\alpha = 0$) $\quad \Sigma M = I\cancel{\alpha}^{\text{zero}} = 0$

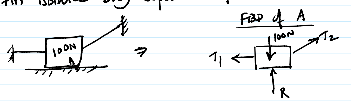
} Equations of equilibrium for coplanar force (Non-concurrent)

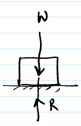
For coplanar concurrent forces $\quad \Sigma F_x = 0$, $\Sigma F_y = 0$



Free Body Diagram (FBD)

An isolated body seperated from all other connected bodies or surfaces.





Types of Equilibrium (to be studied)

① Equilibrium of collinear force system

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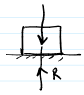
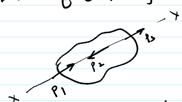
① Equilibrium of collinear force system



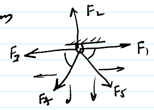


$\Sigma F_x = 0$

$P_1 - P_2 + P_3 = 0$

② Equilibrium of concurrent force system

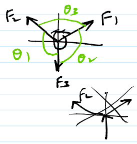
$\Sigma F_x = 0$ & $\Sigma F_y = 0$



③ Equilibrium of 3 concurrent forces

Lami's theorem

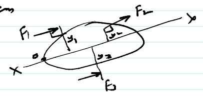
$$\frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}$$



④ Equilibrium of parallel force system

$\Sigma F_x = 0$

$\Sigma M_o = 0$

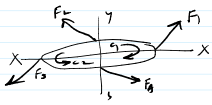


⑤ Equilibrium of general force system

$\Sigma F_x = 0$

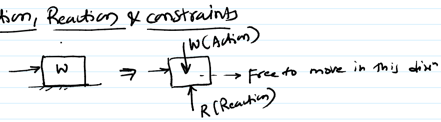
$\Sigma F_y = 0$

$\Sigma M = 0$

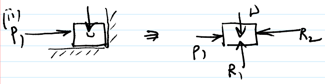


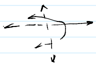
Action, Reaction & constraints

(i)



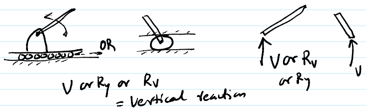
(ii)





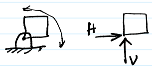
Types of Supports & Reactions

① Roller Support

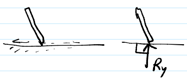


V or R_y or R_V = Vertical reaction

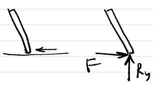
② pin or Hinge Support



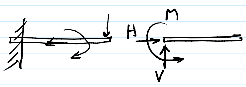
③ Smooth Surface



④ Rough Surface



⑤ Fixed or Built-in Support



⑥