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Q1)

$$\begin{bmatrix} 121051 & 0 & 0 \\ 0 & 121051 & 0 \\ 0 & 0 & 121051 \end{bmatrix} = A$$

i) The characteristic equation is $\begin{vmatrix} 121051-\lambda & 0 & 0 \\ 0 & 121051-\lambda & 0 \\ 0 & 0 & 121051-\lambda \end{vmatrix} = 0$

$\therefore (121051-\lambda)\left[(121051-\lambda)(121051-\lambda) - 0\right] + 0\left[(121051-\lambda)(0) - 0\right]$
$+ 0\left[(0) - (121051-\lambda)(0)\right]$

$\therefore (121051-\lambda)\left[(121051-\lambda)(121051-\lambda)\right] + 0$

$\therefore (121051-\lambda)^3 = 0$

$\therefore \lambda = 121051, 121051, 121051$

ii) For $\lambda = 121051$,

$[A - \lambda_2 I]X = 0$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$0x_1 + 0x_2 + 0x_3 = 0$

Rank $= 0$

$\therefore$ no. of eigen vectors $\Rightarrow 3 - 0 = 3$

$\therefore$ We have 3 linearly independent solution.

let the parameters be a, b, c

$\therefore \lambda = 1, 2, 1, 0, 5, 1$ | 1601012105T A-3
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the eigenvectors are = $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$

$= a\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + b\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

iii) To verify that the eigenvectors are linearly independent

$k_1 X_1 + k_2 X_2 + k_3 X_3 = 0$

$k_1 [1\ 0\ 0] + k_2 [0\ 1\ 0] + k_3 [0\ 0\ 1] = 0$

$\therefore k_1 + 0 + 0 = 0 \rightarrow (1)$

$\therefore 0 + k_2 + 0 = 0 \rightarrow (2)$

$\therefore 0 + 0 + k_3 = 0 \rightarrow (3)$

$\therefore$ from the above equations

$k_1 = k_2 = k_3 = 0$

$\therefore$ The vectors are linearly independent.

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Q2) $a = 51$

$b = 52$

$c = 53$

i) $P = \begin{bmatrix} 51 & 52 & 53 \\ 52 & 53 & 51 \\ 53 & 51 & 52 \end{bmatrix}$

ii) To prove $(a+b+c)$ i.e (156), $\sqrt{3}$, $-\sqrt{3}$ are eigen values of P

characteristic eq^n $\Rightarrow$ $(P - \lambda I) = 0$

$$= \begin{vmatrix} 51-\lambda & 52 & 53 \\ 52 & 53-\lambda & 51 \\ 53 & 51 & 52-\lambda \end{vmatrix} = 0$$

$$\Rightarrow 51-\lambda\left[(53-\lambda)(52-\lambda) - (51)^2\right] + 52\left[52(52-\lambda) - (53)(51)\right] + 53\left[(52)(51) - 53(53-\lambda)\right]$$

$\Rightarrow -\lambda^3 + 156\lambda^2 + 3\lambda - 468$

$\Rightarrow -(\lambda - 156)\cdot(\lambda^2 - 3)$

$\Rightarrow -(\lambda - 156)(\lambda - \sqrt{3})(\lambda + \sqrt{3}) = 0$

$\therefore \lambda_1 = 156$

$\lambda_2 = \sqrt{3}$

$\lambda_3 = -\sqrt{3}$

$\therefore$ 156, $\sqrt{3}$, $-\sqrt{3}$ are eigen values that is $(a+b+c)$, $\sqrt{3}$, $(-\sqrt{3})$

Hence proved

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iii) Verification of Cayley Hamilton theorem.

Characteristic equation is

$$\lambda^3 - 156\lambda^2 - 3\lambda + 468 = 0$$

~~$\lambda^3 + 156\lambda^2$~~

According to Cayley Hamilton theorem;
'P' satisfies the equation.

$$P^3 - 156P^2 - 3P + 468 = 0 \rightarrow \text{I}$$

$$P^2 = \begin{bmatrix} 51 & 52 & 53 \\ 52 & 53 & 51 \\ 53 & 51 & 52 \end{bmatrix} \begin{bmatrix} 51 & 52 & 53 \\ 52 & 53 & 51 \\ 53 & 51 & 52 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 8114 & 8111 & 8111 \\ 8111 & 8114 & 8111 \\ 8111 & 8111 & 8114 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 8114 & 8111 & 8111 \\ 8111 & 8114 & 8111 \\ 8111 & 8111 & 8114 \end{bmatrix} \begin{bmatrix} 51 & 52 & 53 \\ 52 & 53 & 51 \\ 53 & 51 & 52 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 1265469 & 1265472 & 1265475 \\ 1265472 & 1265475 & 1265469 \\ 1265475 & 1265469 & 1265472 \end{bmatrix}$$

putting value of I, P^2, P^3 in (I)

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$$P^3 - 156P^2 - 3P + 468 = 0$$

$$\begin{bmatrix} 1265469 & 1265472 & 1265475 \\ 1265472 & 1265475 & 1265469 \\ 1265475 & 1265469 & 1265472 \end{bmatrix} - 156 \begin{bmatrix} 8114 & 8111 & 8111 \\ 8111 & 8114 & 8111 \\ 8111 & 8111 & 8114 \end{bmatrix}$$

$$- 3 \begin{bmatrix} 51 & 52 & 53 \\ 52 & 53 & 51 \\ 53 & 51 & 52 \end{bmatrix} + \begin{bmatrix} 468 & 0 & 0 \\ 0 & 468 & 0 \\ 0 & 0 & 468 \end{bmatrix}$$

$= 0$

Hence proved