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Roll No :- 16010121051 A3
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Q1) $u = \log(x^3+y^3-x^2y-xy^2)$

i) To prove :- $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$

$\rightarrow \frac{\partial u}{\partial x} = \frac{1}{x^3+y^3-x^2y-xy^2}(3x^2-2yx-y^2)$

$$x\frac{\partial u}{\partial x} = \frac{3x^3-2yx^2-y^2x}{x^3+y^3-x^2y-xy^2} \quad -(1)$$

$$\frac{\partial u}{\partial y} = \frac{1}{x^3+y^3-x^2y-xy^2}(3y^2-x^2-2xy)$$

$$y\frac{\partial u}{\partial y} = \frac{3y^3-x^2y-2xy^2}{x^3+y^3-3x^2y-xy^2} \quad -(2)$$

Adding (1) and (2)

$$x\frac{\partial u}{\partial x} + y\frac{\partial y}{\partial y}$$

$$= \frac{3x^3-2yx^2-y^2x+3y^3-x^2y-2xy^2}{x^3+y^3-x^2y-xy^2}$$

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$$= \frac{3x^3 + 3y^3 - 3yx^2 - 3xy^2}{x^3 + y^3 - x^2y - xy^2}$$

$$= \frac{3(x^3 + y^3 - yx^2 - xy^2)}{x^3 + y^3 - x^2y - xy^2}$$

$= 3$ Hence proved

ii) $\frac{\partial^2 u}{\partial x^2} + \frac{2\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = \frac{-4}{(x+y)^2}$

$u = \log(x^3 + y^3 - x^2y - xy^2)$

$u = \log(x^3 - x^2y + y^3 - xy^2)$

$u = \log((x^3 - x^2y) + y^3 - xy^2))$

$u = \log(x^2(x-y) - y^2(x-y)]$

$u = \log[(x^2-y^2)(x-y)]$

$u = \log[(x-y)^2(x+y)]$

$u = \log(x+y) + \log(x-y)^2$

$u = \log(x+y) + 2\log(x-y)$

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$$\frac{\partial u}{dx} = \frac{1}{x+y} + \frac{2}{x-y}$$

$$\frac{\partial^2 u}{dx^2} = \frac{-1}{(x+y)^2} - \frac{2}{(x-y)^2} \rightarrow (1)$$

$$\frac{\partial^2 u}{dxdy} = \frac{-1}{(x+y)^2} + \frac{2}{(x-y)^2}$$

$$\frac{du}{dy} = \frac{1}{x+y} - \frac{2}{(x-y)}$$

$$\frac{d^2u}{dy^2} = \frac{-1}{(x+y)^2} - \frac{2}{(x-y)^2} \rightarrow (2)$$

$$\frac{d^2u}{dxdy} = \frac{-1}{(x+y)^2} + \frac{2}{(x-y)^2} \rightarrow (3)$$

From (1), (2), (3)

$$\frac{\partial^2 u}{dx^2} + 2\frac{d^2u}{dxdy} + \frac{d^2u}{dy^2}$$

$$= \frac{-1}{(x+y)^2} - \frac{2}{(x-y)^2} + 2\left(\frac{-1}{(x+y)^2} + \frac{2}{(x-y)^2}\right)$$

$$+ \left(\frac{-1}{(x+y)^2} - \frac{2}{(x-y)^2}\right)$$

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$$\boxed{\frac{\partial^2 y}{dx^2} + \frac{2\,\partial^2 y}{dx \cdot dy} + \frac{d^2 y}{dy^2} = \frac{-4}{(x+y)^2}}$$

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Q2) $u = \frac{1}{r}$, $r = \sqrt{x^2+y^2+z^2}$

To prove :- $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{d z^2} = 0$

$\Rightarrow$ $\frac{\partial r}{\partial x} = \frac{x}{r}$, $\frac{\partial r}{\partial y} = \frac{y}{r}$, $\frac{\partial r}{\partial z} = \frac{z}{r}$

$\therefore \frac{\partial u}{\partial x} = \frac{du}{dr} \cdot \frac{\partial r}{\partial x} = \frac{-1}{r^2} \cdot \frac{x}{r} = \frac{-x}{r^3}$

and $\frac{\partial^2 u}{\partial x^2} = \frac{-1}{r^3} + \frac{3x}{r^4} \cdot \frac{\partial x}{\partial x} = \frac{-1}{r^3} + \frac{3x^2}{r^5}$

Similarly, $\frac{\partial^2 u}{\partial y^2} = \frac{-1}{r^3} + \frac{3y^2}{r^5}$

$\frac{\partial^2 u}{\partial z^2} = \frac{-1}{r^3} + \frac{3z^2}{r^5}$

$\therefore \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$

$= \frac{-3}{r^3} + \frac{3(x^2+y^2+z^2)}{r^5} = \frac{-3}{r^3} + \frac{3}{r^3} = 0$ //

Hence proved

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Q3) $u = x^2 + y^2 + z^2$; $x = e^t$, $y = e^t \sin t$, $z = e^t \cos t$

To prove :- $\frac{du}{dt} = 4e^{2t}$

Ans → $\frac{du}{dt} = \frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial u}{\partial z} \cdot \frac{dz}{dt}$

$\frac{\partial u}{\partial x} = 2x$, $\frac{\partial u}{\partial y} = 2y$, $\frac{\partial u}{\partial z} = 2z$

$\frac{dx}{dt} = e^t$, $\frac{dy}{dt} = e^t(\sin t + \cos t)$, $\frac{dz}{dt} = e^t(-\sin t + \cos t)$

$\frac{du}{dt} = \frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial u}{\partial z} \cdot \frac{dz}{dt}$

$= 2x(e^t) + 2y(e^t(\sin t + \cos t)) + 2z(e^t(-\sin t + \cos t))$

$= 2x(x) + 2y(y+z) + 2z(z-y)$

$= 2x^2 + 2y^2 + 2z^2 - 2yz + 2yz$

$= 2x^2 + 2y^2 + 2z^2$

$= 2(x^2+y^2+z^2)$

$= 2u$ → (1)

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substituting value of u in equation(1)

$$\frac{du}{dr} = 2u = 2(2e^{2t}) = 4e^{2t}$$

Hence proved

$$\frac{Du}{dr} = 4e.$$