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Q1)

$$L.H.S = \frac{1+\cos 7\theta}{1+\cos\theta}$$

$$= \frac{2\cos^2(7\theta/2)}{2\cos^2(\theta/2)} \quad \left[(1+\cos\theta) = 2\cos^2(\tfrac{\theta}{2}) \right]$$

$$= \left[\frac{\cos(7\theta/2)}{\cos(\theta/2)}\right]^2$$

$$= \frac{2\cos 7\theta/2}{2\cos\theta/2} \cdot \frac{\sin\theta/2}{\sin\theta/2} \quad \{\text{multiplying } \sin\theta/2 \text{ in numerator and denominator}\}$$

$$= \left(\frac{\left[\sin\left(\frac{7\theta}{2}+\frac{\theta}{2}\right) - \sin\left(\frac{7\theta}{2}-\frac{\theta}{2}\right)\right]}{\sin^2\theta}\right)^2$$

$$= \left(\frac{\sin 4\theta - \sin 3\theta}{\sin\theta}\right)^2 \rightarrow (1)$$

By De Movere's Theorem we get

$$\cos 4\theta + i\sin 4\theta = (\cos\theta + i\sin\theta)^4$$

$$= \cos^4\theta + 4\cos^3\theta(i\sin\theta) + 6(\cos\theta)^2(i\sin\theta)^2 + 4\cos\theta(i\sin\theta)^3 + i^4\sin^4\theta$$

$$= \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta + i(4\cos^3\theta\sin\theta) - i(4\cos\theta\sin^3\theta)$$

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$= (\cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta) + i(4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta)$

By equating imaginary part we get;

$$\sin 4\theta = 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta \quad \rightarrow (2)$$

consider,

$$\cos 3\theta + i\sin 3\theta = (\cos\theta + i\sin\theta)^3$$

$$= \cos^3\theta + 3\cos^2\theta(i\sin\theta) + 3\cos\theta(i\sin\theta)^2 + (i\sin\theta)$$

$$= \cos^3\theta - 3\cos\theta\sin^2\theta + i(3\cos^2\theta\sin\theta) - i(\sin^3\theta)$$

$$= (\cos^3\theta - 3\cos\theta\sin^2\theta) + i(3\cos^2\theta\sin\theta - \sin^3\theta)$$

Equating imaginary parts;

$$\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta \quad \rightarrow (3)$$

Put (2) & (3) in (1)

$$\frac{1+\cos 7\theta}{1+\cos\theta} = \frac{(\sin 4\theta - \sin 3\theta)^2}{(\sin\theta)^2}$$

$$= (4\cos^3\theta\sin\theta - 4\cos\theta\sin^2\theta - 3\cos^2\theta\sin^2\theta)^2 / \sin\theta)$$

$$= (4\cos^3\theta - 4\cos\theta\sin^2\theta - 3\cos^2\theta + \sin^2\theta)^2$$

$$= [4\cos^3\theta - 4\cos\theta(1-\cos^2\theta) - 3\cos^2\theta + (1-\cos^2\theta)]^2$$

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$= [4\cos^3\theta - 4\cos\theta + 4\cos^3\theta - 3\cos^2\theta + 1 - \cos^2\theta]^2$

$= (8\cos^3\theta - 4\cos^2\theta - 4\cos\theta + 1)^2 \rightarrow (4)$

Now we know

$x = 2\cos\theta \rightarrow$ (given)

Substitute $2\cos\theta$ as x in (4)

$$\frac{1+\cos 7\theta}{1+\cos\theta} = (x^3 - x^2 - 2x + 1)^2$$

Hence proved

Q2) $\cos^8\theta = \frac{1}{2^7} \cos 8\theta + 8\cos 6\theta + 28\cos 4\theta + 56\cos^2\theta$

let $x = \cos\theta + i\sin\theta$

$\frac{1}{x} = \cos\theta - i\sin\theta$

$x^n = (\cos n\theta + i\sin n\theta) = (\cos\theta + i\sin\theta)^n \rightarrow (1)$

$\frac{1}{x^n} = (\cos n\theta - i\sin n\theta) = (\cos\theta - i\sin\theta)^n \rightarrow (2)$

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Adding (1) and (2)

$$x^n + \frac{1}{x^n} = 2\cos n\theta$$

$$\left(x + \frac{1}{x}\right)^n = (2\cos\theta)^n$$

Put $n = 8$

$$(2\cos\theta)^8 = \left(x + \frac{1}{x}\right)^8$$

$$= x^8 + 8x^7 \cdot \frac{1}{x} + 28x^6 \frac{1}{x^2} + 56x^5 \frac{1}{x^3} + 70x^4 \frac{1}{x^4} + 56x^2 \frac{1}{x^2}$$

$$+ 28x^2 \frac{1}{x^6} + 8x \frac{1}{x^7} + \frac{1}{x^8}$$

$$= \left(x^8 + \frac{1}{x^8}\right) + 8\left(x^6 + \frac{1}{x^6}\right) + 28\left(x^4 + \frac{1}{x^4}\right) + 56\left(x^2 + \frac{1}{x^2}\right) + 70$$

$$= 2^8 \cos^8\theta = 2\left[\cos 8\theta + 8\cos 6\theta + 28\cos 4\theta + 56\cos 2\theta\right.$$

$$\cos^8\theta = \frac{1}{2^7}\left(\cos 8\theta + 8\cos 6\theta + 28\cos 4\theta + 56\cos 2\theta + 35\right)$$

Hence proved..

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Q3) $S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$\therefore |S| = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$

$\therefore |S| = -1(0-1) + 1(1-0)$

$\therefore |S| = 2$

$|S| \neq 0$

$\therefore S^{-1}$ exists

Now,

$S_{11} = (-1)^{1+1} \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = (0-1) = -1$ $\qquad S_{31} = (-1)^{3+1} = 1$

$S_{12} = (-1)^{1+2} \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = 1$ $\qquad S_{32} = (-1)^{3+2} \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = 1$

$S_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1$ $\qquad S_{33} = (-1)^{3+3} \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$

$S_{21} = (-1)^{2+1} \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = 1$ $\qquad S_{22} = (-1)^{2+2} \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$

$S_{23} = (-1)^{2+3} \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = 1$

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$$\operatorname{adj} S = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

Now we know

$$S^{-1} = \frac{\operatorname{adj} S}{|S|} = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} -1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 \end{bmatrix}$$

Also,

$$SA = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \times \frac{1}{2}\begin{bmatrix} b+c & c-a & b-a \\ c-b & c+a & a-b \\ b-c & a-c & a+c \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0+c-b+b-c & 0+c+a+a-c & a+c-b+a+b \\ b+c+0+b-c & c-a+0+a-c & b-c+0+a+b \\ b+c+c-d+0 & c-a+c+a+0 & b-a+a-b+0 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & 2a & 2a \\ 2b & 0 & 2b \\ 2c & 2c & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & a & a \\ b & 0 & b \\ c & c & 0 \end{bmatrix}$$

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$$\therefore SAS^{-1} = \frac{1}{2}\begin{bmatrix} 0 & a & a \\ b & 0 & b \\ c & c & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0+a+c & 0-a+a & 0+a-a \\ -b+0+b & b+0+b & b+0-b \\ -c+c+0 & c-c+0 & c+c-0 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 2a & 0 & 0 \\ 0 & 2b & 0 \\ 0 & 0 & 2c \end{bmatrix}$$

$$= \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$\therefore SAS^{-1}$ is a diagonal matrix.

Hence poored.