PRACTICE PROBLEMS ON RECTIFICATI

TYPE – I

1. Find the circumference of the circle of radius a.

2. Find the length of the arc of the parabola $y^2 = 8x$ cut off by the latus rectum.

3. Find the arc length of $x^2 = 4y$ cut off by its latus rectum.

4. Show that the length of the parabola $y^2 = 4ax$ from the vertex to the end of the latus rectum is $a\left[\sqrt{2} + log\left(1 + \sqrt{2}\right)\right]$. Find the length of arc cut off by the line $3y = 8x$

5. Find the arc length of $y^2 = 4x$ cut off by the line $y = 2x$.

6. Find the length of arc of parabola $y^2 = 4a(a - x)$ cut off by the y-axis.

7. Draw the curve $y = x(2 - x)$ and find the length of an arc from $x = 0$ to $x = 2$.

8. Find the length of the parabola $x^2 = 4y$ which lies inside the circle $x^2 + y^2 = 6y$

9. Find the length of the arc of parabola $y^2 = 4x$ which lies inside the curve $x^2 + y^2 = 5$

10. Show that the length of the arc of the curve $ay^2 = x^3$ from the origin to the point whose abscissa is b is $\frac{8a}{27}\left[\left(1 + \frac{9b}{4a}\right)^{3/2} - 1\right]$

11. Find the arc length of $ay^2 = x^3$ from $(0,0)$ to (a, a)

12. Find the length of the arc of $y = e^x$ from $(0, 1)\ to\ (1, e)$

13. Prove that the length of the arc of the curve $y = log\left(\frac{e^x-1}{e^x+1}\right)$ from $x = 1$ and $x = 2$ is $log\left(e + \frac{1}{e}\right)$

14. Find the length of the arc of the curve $y = log\left(\tan h\frac{x}{2}\right)$ from $x = 1\ to\ x = 2$

15. Find the total length of the curve $x^{2/3} + y^{2/3} = a^{2/3}$

16. For the astroid $x^{2/3} + y^{2/3} = a^{2/3}$, show that $s^3 \propto x^2$ where s is the length of the arc, measured from the cusp which lies on y – axis to any point (x, y)

17. Show that if s is the arc of the curve $9y^2 = x(3-x)^2$ measured from the origin to the point $P(x, y)$ then $3S^2 = 3y^2 + 4x^2$

18. Prove that the length of the arc of the curve $y^2 = x\left(1 - \frac{1}{3}x\right)^2$ from the origin to the point $P(x, y)$ is given by $S^2 = y^2 + \frac{4}{3}x^2$. Hence, rectify the loop

19. If S is the length of the curve $y^2 = x\left(1 - \frac{x}{3}\right)^2$ measured from the origin to the point whose abscissa is a then prove that $9s^2 = a(a+3)^2$. Hence, find the perimeter $(0 < a < 3)$

20. Find the length of the loop of the curve $3ay^2 = x(x-a)^2$

21. Show that the length of the loop $3ay^2 = x^2(a - x) is\ 4a/\sqrt{3}$

22. Show that the length of the loop $9ay^2 = x(x - 3a)^2\ is\ 4\sqrt{3}.a$

23. Find the total length of the loop of the curve $9y^2 = (x + 7)(x + 4)^2$

24. Find the perimeter of the loop of the curve $9ay^2 = (x - 2a)(x - 5a)^2$

25. For the catenary $y = c\cos h\frac{x}{c}$, prove that If S is arc length measured from the vertex to (x, y) then **(i)** $S = csin\,h\frac{x}{c}$ **(ii)** $y^2 = c^2 + s^2$ **(iii)** $s = ctan\Psi$ where Ψ is the angle between x axis and tangents drawn at point (x, y).

26. Show that the length of the arc of the curve $4ax = y^2 - 2a^2 log.\frac{y}{a} - a^2$ from $O(0, a)$ to any point $P(x, y)$is $\frac{y^2}{2a} - \frac{a}{2} - x$

ANSWERS

1.	$2\pi a$	**2.**	$4[\sqrt{2} + log(1 + \sqrt{2})]$	**3.**	$2\sqrt{2} + 2log(\sqrt{2} + 1)$
4.	$a\left[\log 2 + \frac{15}{16}\right]$	**5.**	$\sqrt{2} + log(1 + \sqrt{2})$	**6.**	$2a[\sqrt{2} + log(1 + \sqrt{2})]$
7.	$\sqrt{5} + \frac{1}{2}\log(2 + \sqrt{5})$	**8.**	$2[\sqrt{6} + log(\sqrt{2} + \sqrt{3})]$	**9.**	$2\sqrt{2} + 2log(1 + \sqrt{2})$
11.	$\frac{a}{27}(13\sqrt{13} - 8)$	**12.**	$\sqrt{1 + e^2} - \sqrt{2} - log\left[\frac{1+\sqrt{1+e^2}}{e(1+\sqrt{2})}\right]$	**14.**	$log\left(e + \frac{1}{e}\right)$
15.	$6a$	**18.**	$4\sqrt{3}$	**19.**	$4\sqrt{3}$
20.	$\frac{4a}{\sqrt{3}}$	**23.**	$4\sqrt{3}$	**24.**	$4\sqrt{3}a$

TYPE - II

1. For the curve $x = (a + b)\cos\theta - bcos\left(\frac{a+b}{b}.\theta\right), y = (a + b)sin\theta - b\,sin\left(\frac{a+b}{b}.\theta\right)$, show that $s = \frac{4b}{a}(a + b)cos\left(\frac{a\theta}{2b}\right)$ where s is measured from $\theta = \pi b/a\ to\ \theta$.

2. Find the length of one arc of the cycloid $x = a(\theta - sin\theta), y = a(1 + cos\theta)$.

3. Find the length of the cycloid $x = a(\theta + sin\theta), y = a(1 - cos\theta)$ from one cusp to another cusp. If s is the length of the arc from the origin to a point $P\ (x, y)$ show that $s^2 = 8ay$

5. Trace the curve $x = a\ (\theta - sin\theta), y = a(1 - cos\theta)$ as θ varies from 0 to 2π. Show that the line $\theta = {2\pi}/{3}$ divides it in ratio 1 : 3.

6. For the curve $x = a(2\cos t - \cos 2t), y = a(2\sin t - \sin 2t)$, show that the length of the arc of the curve measured from $t = 0$ to the point where the tangent makes an angle ψ with the tangent at $t = 0$ is given by $s = 16a\,sin^2\frac{\Psi}{6}$.

7. Prove that the length of the arc of the curve $x = a\,sin2\theta(1 + cos2\theta), y = a\cos 2\theta(1 - cos2\theta)$ measured from the origin to (x, y) is $\frac{4}{3}a\sin 3\theta$.

8. Find the length of the loop of the curve. $x = t^2, y = t\left(1 - \frac{t^2}{3}\right)$

9. Show that the length of the tractrix $x = a[\cos t + \log\tan(t/2)], y = a\sin t\ from\ t = \pi/2$ to any point t is a $log\ (sin\ t)$

10. Show that in the curve $8a^2y^2 = x^2(a^2 - x^2), s = \frac{a}{2\sqrt{2}}(2\theta + sin\theta\ cos\theta)$. where, $x = a\ sin\ \theta$, and that the perimeter of one loop is $\pi\ a/\sqrt{2}$

OR Prove that the entire length of the curve $8a^2y^2 = x^2(a^2 - x^2)\ is\ \ \pi\ a\sqrt{2}$

11. Prove that the length of the curve $x = e^{\theta}\left[\sin\frac{\theta}{2} + 2\ cos\frac{\theta}{2}\right], y = e^{\theta}\left[cos\frac{\theta}{2} - 2\ sin\frac{\theta}{2}\right]$ measured from $\theta = 0\ to\ \theta = \pi\ is\ \frac{5}{2}[e^{\pi} - 1]$

12. Find the length of the astroid $x = a\cos^3 t\,, y = a\sin^3 t$

13. Find the total length of the curve $(x/a)^{2/3} + (y/b)^{2/3} = 1$. Hence, deduce the total length of the curve $x^{2/3} + y^{2/3} = a^{2/3}$. Also show that the line $\theta = \pi/3$ divides the length of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ in the first quadrant in the ratio 1: 3

14. Find the length of the following curves:

(i) $x = a(2\ cos\theta + cos2\theta), y = a(2\ sin\theta + \sin 2\theta)\ from\ \theta = 0$ to any point θ.

(ii) $x = a(\theta - sin\theta), y = a(1 - cos\theta)$ from $\theta = 0$ to $\theta = 2\pi$

(iii) $x = a(cos\ \theta + \theta\ sin\theta), y = a(sin\ \theta - \theta cos\ \theta)$ from $\theta = 0$ to $\theta = 2\pi$

(iv) $x = ae^{\theta}\ sin\theta,\ y = ae^{\theta}\ cos\theta$ from $\theta = 0$ to $\theta = 2\pi$

(v) $x = a(3cos\theta - cos3\theta), y = a(3sin\theta - sin3\theta)$ from $\theta = \pi/2$ to any point θ

(vi) $x = a(\theta + sin\ \theta), y = a(1 + cos\theta)$ between two consecutive cusps.

(vii) $x = log(sec\theta + tan\theta) - sin\theta, y = cos\theta$ from $\theta = 0$ to any point θ

(viii) $x = a(t - \tan ht), y = a\sec ht$ from $t = 0$ to any point t.

(ix) $x = 1 - \cos t + (3/5)t, y = (4/5)\sin t$ from $t = 0$ to $t = \pi$

(x) $x = a\ cost + at\ sin\ t, y = a\ sin\ t - at\ cos\ t$ from $t = 0$ to $t = \pi/2$

ANSWERS

2.	$8a$	**3.**	$8a$	**4.**	$8a$	**8.**	$4\sqrt{3}$
12.	$6a$	**13.**	$\frac{4(a^2+ab+b^2)}{(a+b)}, 6a$				

14.

(i)	$8a\sin\left(\frac{\theta}{2}\right)$	**(ii)**	$8a$	**(iii)**	$2\pi^2 a$	**(iv)**	$\sqrt{2}(e^{\pi/2} - 1)a$
(v)	$6a\cos\theta$	**(vi)**	$8a$	**(vii)**	$logsec\theta$	**(viii)**	$a\log\cos h\ t$
(ix)	$\pi + (\frac{6}{5})$	**(x)**	$\pi^2 a/8$				

TYPE – III

1. Find the length of the cardioide $r = a(1 + \sin\theta)$

2. Find the length of the perimeter $r = a(1 + \cos\theta)$. Prove also that the upper half of cardiode is bisected by the line $\theta = \pi/3$.

3. Show that upper half of $r = 2a\ cos^2\left(\frac{\theta}{2}\right)$ is bisected by the line $\theta = \pi/3$.

4. Find the perimeter of the cardioide $r = a(1 - \cos\theta)$ and prove that the line $\theta = 2\pi/3$ bisects the upper half of the cardioide.

5. Find the length of the arc of the curve $r = a\sin^2\left(\frac{\theta}{2}\right)$ from $\theta = 0$ to any point $P(\theta)$

6. Find the length of the cardioide $r = a(1 - \cos\theta)$ lying outside the circle $r = a\cos\theta$

7. Find the length of the cardioide $r = a(1 + \cos\theta)$ which lies outside the circle $r + a\cos\theta = 0$

8. Find the length of the cardioide $r = a(1 - \cos\theta)$ lying inside the circle $r = a\cos\theta$

9. Show that the length of the arc of that part of cardioide $r = a(1 + \cos\theta)$ which lies on the side of the line $4r = 3\, asec\theta$ away from the pole is $4a$

OR Show that the perimeter of cardioid $r = a(1 + \cos\theta)$ is bisected by the line $4\,r = 3a\sec\theta$

10. Show that for the parabola $\frac{2a}{r} = 1 + cos\theta$, the arc intercepted between the vertex and the extremity of the latus rectum is $a\left[\sqrt{2} + log\left(1 + \sqrt{2}\right)\right]$

11. Find the length of the arc of the parabola $r = \frac{6}{1+\cos\theta}$ from $\theta = 0$ to $\theta = \pi/2$

12. Find the length of the Cissoid $r = 2a\tan\theta\sin\theta$ from $\theta = 0\ to\ \theta = \pi/4$

13. Find the length of the upper arc of one loop of Lemniscate $r^2 = a^2\cos 2\theta$

14. Show that the total perimeter of $r^2 = a^2\cos 2\theta\ \ is\ \frac{a}{\sqrt{2\pi}}\left(\left|\overline{1/4}\right.\right)^2$

15. Find the total length of the curve $r = a\ sin^3(\theta/3)$

16. Prove that the length of the spiral $r = a\ e^{\theta\cot\alpha}$ as r increases from r_1 to r_2 is given by $(r_2 - r_1)\sec\alpha$

OR Prove that the length of arc of equi-angular spiral $r = ae^{\theta\cot\alpha}$ varies as difference of radii vectors of extremities of the arc.

17. Taking $s = 0$ at $\theta = 0$, find the length of the arc OP of the spiral $r = ae^{\theta\cot\alpha}$ from 0 to $P(\theta)$

18. Find the length of the spiral $r = a^{m\theta}$ lying inside the circle $r = a$.

ANSWERS

1. $8a$ **2.** $8a$ **4.** $8a$

5. $4asin^2\left(\frac{\theta}{4}\right)$ **6.** $4a\sqrt{3}$ **7.** $4a\sqrt{3}$

8. $8a\left(1 - \frac{\sqrt{3}}{2}\right)$ **12.** $2a\left(\sqrt{5} - 2\right) + a\sqrt{3}log\left(\frac{4-\sqrt{15}}{7-4\sqrt{3}}\right)$ **13.** $\frac{a}{4\sqrt{2\pi}}\left(\left|\overline{1/4}\right.\right)^2$

15. $\frac{3}{2}\pi a$ **17.** $a\sec\alpha\left(e^{\theta cot\alpha} - 1\right)$ **18.** $\frac{a}{m}\sqrt{1 + m^2(log\ a)^2}$