# Quick Sort :-

## Worst case

$$T(n) = T(n-1) + n-1$$

**Telescoping :-**

$$T(n) - T(n-1) = n-1$$

$$T(n) + T(n-1) + T(n-2) + T(n-3) + \dots T(3) + T(2)$$
$$- T(n-1) - T(n-2) - T(n-1) - T(n-3)$$
$$- T(2) - T(1)$$

$$= (n-1) + (n-2) + (n-3) + \dots 2 + 1$$

$$= \frac{(n-1)(n)}{2} = \frac{n^2 - n}{2} = O(n^2)$$

## Average Case :-

Pivot element at position "$i$"
where $i \in \{0, 1, 2, \dots n-1\}$

Time for partitioning an array : $cn$

Head subarray contain $i$ subarray
& Tail subarray contain $n-1-i$ subarray.

$$\boxed{T(n) = T(i) + T(n-1-i) + cn}$$

Average running time for sorting.

$$T(n) = \frac{1}{n}\sum_{i=0}^{n-1}\left(T(i) + T(n-1-i) + cn\right)$$

$$= \frac{1}{n}\Big[ T(0) + T(1) + T(2) + T(3) \ldots\ldots T(n-1)$$
$$+ T(n-1) + T(n-2) + T(n-3) \ldots\ldots$$
$$T(2) + T(1) + T(0)\Big] + cn$$

$$= \frac{2}{n}\left[T(0) + T(1) + T(2) \ldots\ldots T(n-1)\right] + cn$$

$$nT(n) = 2\left[T(0) + T(1) + T(2) + \ldots T(n-1)\right] + cn^2 \quad \text{--- } (1)$$

for $n = n-1$

$$(n-1)T(n-1) = 2\left[T(0) + T(1) + \ldots\ldots T(n-2)\right] + c(n-1)^2 \quad \text{--- } (2)$$

$(1) - (2)$

$$nT(n) - (n-1)T(n-1)$$

$$= 2T(n-1) + cn^2 - c(n-1)^2$$

$$= 2T(n-1) + \cancel{cn^2} - \cancel{cn^2} \cancel{+} 1 \cancel{+} 2cn$$

$$\cong 2T(n-1) + 2cn$$

$$nT(n) = 2T(n-1) + 2cn + (n-1)T(n-1)$$

$$nT(n) = (n+1)T(n-1) + 2cn$$

$$\frac{T(n)}{n+1} = \frac{T(n-1)}{n} + \frac{2c}{n+1}$$

$$\frac{T(n)}{n+1} - \frac{T(n-1)}{n} = \frac{2c}{n+1} \quad \text{--- } 3$$

Telesoping $\frac{T(n)}{n+1} - \frac{T(n-1)}{n} = \frac{2c}{n+1}$

to get explicit form.

$$\frac{T(n)}{n+1} + \frac{T(n-1)}{n} + \frac{T(n-2)}{n-1} + \cdots \frac{T(2)}{3} + \frac{T(1)}{2}$$

$$- \frac{T(n-1)}{n} - \frac{T(n-2)}{n-1} - \frac{T(n-3)}{n-2} - \frac{T(2)}{3} - \frac{T(1)}{2}$$

$$- \frac{T(0)}{1}$$

$$= \frac{2c}{n+1} + \frac{2c}{n} + \frac{2c}{n-1} + \text{——} + \frac{2c}{2}$$

$$\frac{T(n)}{n+1} - \frac{T(0)}{1} = 2c\left(\frac{1}{n+1} + \frac{1}{n} + \frac{1}{n-1} + \cdots \frac{1}{3} + \frac{1}{2}\right)$$

$$\frac{T(n)}{n+1} = 2c\left[1 + \frac{1}{2} + \frac{1}{3} + \cdots \frac{1}{n-1} + \frac{1}{n} + \frac{1}{n+1} - 1\right]$$

$$= 2c\left[\underline{\underline{H_{n+1}}} - 1\right]$$

$H_{n+1}$ = sum of $(n+1)$ harmonic Numbers

$$T(n) \simeq 2c(n+1)\left[H_{n+1} - 1\right]$$

$$= 2cn + 2c \left[H_{n+1} - 1\right]$$

$$= c'n + 2c\left[H_{n+1} - 1\right]$$

$$= n\left[H_{n+1} - 1\right]$$

$$= n \log n - n$$

$$= O(n \log n)$$

$$H_n = \sum_{k=1}^{n} \frac{1}{k} = \ln(n) + \gamma$$

[Standard Asymptotic Formula for Harmonic Sum]

where $\gamma = 0.5772$

→ Euler Mascheroni constant