① Probability Distribution Function :-

Probability Mass Function & Discrete Variable :

Discrete Variables :- variables which we can count.

e.g. :- no. of pencils in box
no. of votes in election

Probability Mass Function :

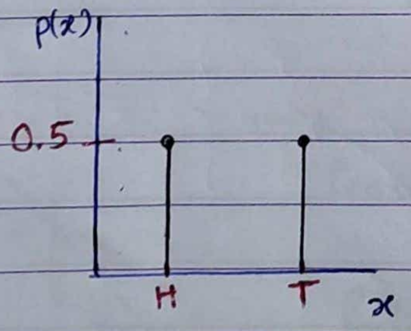
Function which describes the probability associated with random variable x (Discrete)

$P(X=x)$ = Probability of random variable x to an event.

Ex. if toss a coin, what will be probability of getting i) Head ii) Tail

i) $p(H) = P(X=H) = 0.5$
ii) $p(T) = P(X=T) = 0.5$

* Here, we can count, what are the possible outcomes there are 2 possible outcomes Head or Tail



Continuos Variables :- variables we can't count, but we can measure.

e.g. Height, weight

Probability Density Function :-

+ It is used for continuos random variables.

But, why we can't use probability mass function ?

Let's see, probability mass function $= P(X=x)$

Suppose we are finding probability taken by X on some given value x, but before this understand what is probability.

$$\text{probability of an event} = \frac{\text{no. of favorable outcomes}}{\text{no. of total possible outcomes}}$$

So, according to formula,

$$P(X=x) = \frac{\text{no. of favorable outcomes of } x}{\text{no. of total possible outcomes}}$$

In case of Discrete values, we can calculate total possible outcomes, But for continuos variables there are infinite possible outcomes. because of decimal values.

$$\therefore \quad P(X=x) = \frac{\text{no. of favorable outcomes of } x}{\infty}$$

and anything divided by ∞ is zero

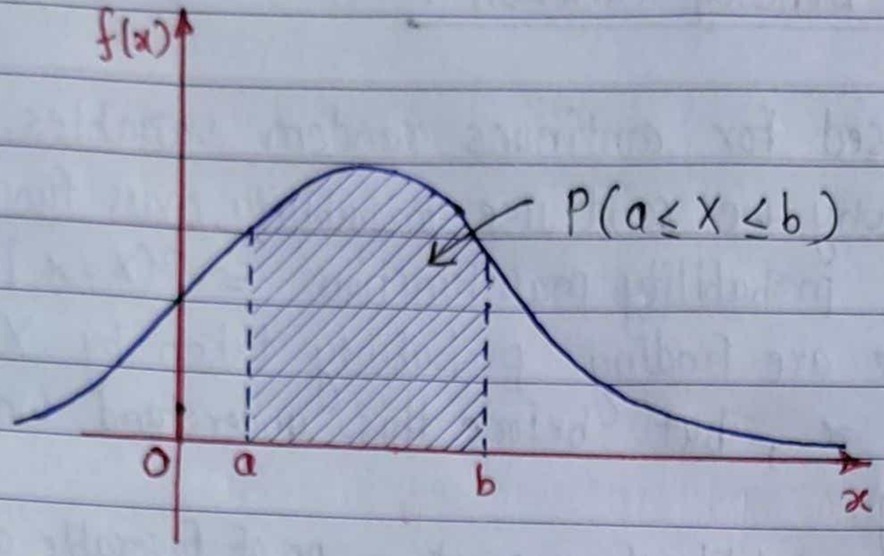
$$\therefore \quad P(X=x) = 0$$

$\therefore$ Hence we can't use probability mass function for continuos variables.

Instead of $P(X=x)$, we will calculate probability of X lying in an interval (a,b), probability density function formula is,

$$\boxed{P(a < X < b) = \int_a^b f(x) \cdot dx}$$

$$\boxed{P(a < X \le b) = P(a \le X \le b) = P(a \le X < b) = P(a < X < b)}$$



+ $\int_{-\infty}^{\infty} f(x)\,.dx = 1$

+ $f(x) \geq 0$, for all x

Ex. Let x be continuous random variables with the PDF given by,

$$f(x) = \begin{cases} x \; ; & 0 < x < 1 \\ 2-x \; ; & 1 < x < 2 \\ 0 \; ; & x > 2 \end{cases}$$

Find

$$P(0.5 < x < 1.5)$$

$\Rightarrow$

$$f(x) = \begin{cases} x \; ; & 0 < x < 1 \\ 2-x \; ; & 1 < x < 2 \\ 0 \; ; & x > 2 \end{cases}$$

$$P(0.5 < x < 1.5) = \int_{0.5}^{1} x \; dx + \int_{1}^{1.5} (2-x) \; dx$$

Integration formulae	$\int x \cdot dx = \frac{x^2}{2}$	$\int 1 \, dx = x$

$$= \int_{0.5}^{1} x \cdot dx + \int_{1}^{1.5} (2-x)\, dx$$

$$= \left(\frac{x^2}{2}\right)_{0.5}^{1} + \int_{1}^{1.5} 2 \cdot dx - \int_{1}^{1.5} x \cdot dx$$

$$= \left(\frac{x^2}{2}\right)_{0.5}^{1} + 2\int_{1}^{1.5} 1 \, dx - \left(\frac{x^2}{2}\right)_{1}^{1.5}$$

$$= \left(\frac{x^2}{2}\right)_{0.5}^{1} + 2\left(x\right)_{1}^{1.5} - \left(\frac{x^2}{2}\right)_{1}^{1.5}$$

$$\left(\frac{x^2}{2}\right)_{0.5}^{1} = \left[\frac{(1)^2}{2} - \frac{(0.5)^2}{2}\right] = \frac{1}{2} - \frac{0.25}{2} = \frac{0.75}{2} = \frac{3}{8}$$

$$2\left(x\right)_{1}^{1.5} = \left[2(1.5) - 2(1)\right] = 3 - 2 = 1$$

$$\left(\frac{x^2}{2}\right)_{1}^{1.5} = \left[\frac{(1.5)^2}{2} - \frac{(1)^2}{2}\right] = \frac{2.25}{2} - \frac{1}{2} = 0.625$$

$$= \frac{3}{8} + 1 - 0.625$$

$$\boxed{P(0.5 < 1.5) = \frac{3}{4} = 0.75}$$

Cumulative Distribution Function (CDF):

Cumulative Distribution Function, is the probability Function that X will take value less than or equal to x.

For Discrete distribution Functions,

CDF gives probability values till what we specify,

$$F_X(x) = P(X \le x)$$

For Continuous distribution Functions,

CDF gives the area under probability density function upto given value specified.

$$F(x) = P(X \le x)$$

$$F(x) = \int_{-\infty}^{x} f(x)\, dx$$