LO ASSIGNMENT

Group 7

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Q1)

We have written **Revised simplex Code** from scratch in **Matlab**. It is stored in the File **Q1_Revised_Simplex.M**

Here is a sample input and its output.

We have taken following LPP as sample:

Maximise, Z: 4x1 + 6x2

Subject To: -x1 + x2 <= 11,

x1 + x2 <= 27,

and, 2x1 + 5x2 <= 90.

x1, x2 >=0.

For our code,

Input:

```
a=[-1,1;1,1;2,5];
b= [11,27,90];
c = [4,6];
```

Output:

```
x1 = 15
x2 = 12
The optimum objective function value=132.
```

Q2)

For General Graph maximum matching problem can be formulated as given below.

Date

Page

(Q2)

Given Graph $G = (V; E)$

Matching $M \subset E$ of G is collection of vertex disjoint edges

Task is to maximize $\|M\| \rightarrow$ size of set M.

ILP Formulation:

· consider a ~~point~~ variable $X_e \in \mathbb{R}^E$ $(e \in E)$ s.t

$$X_e = 1, \text{ if the edge } e \in M$$
$$X_e = 0, \text{ otherwise}$$

Consider ILP:

(objective func^n to be maximised) $\max \sum X_e$ where $e \in E$

subject to

- $\forall e \in E$ $X_e \in \{0, 1\}$
- $\forall v \in V$ $\sum X_e \leq 1$, where $e \in \delta(v)$

where $\delta(v)$ is the set of edges incident on the vertex $v \in V$

Now this is integer Linear Programming as each X_e, $e \in E$ can take only two ~~variables~~ values $\{0, 1\}$ both integers.

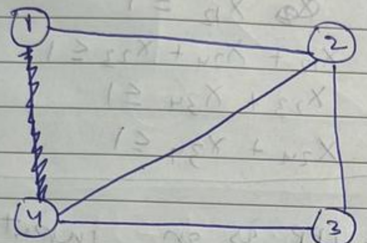
For given Graph Network 1, ILP is formulated as below

Now, Formulating this for the given Network 1

Network 1	1	2	3	4
1	0	1	0	0
2	1	0	1	1
3	0	1	0	1
4	0	1	1	0

i.e given graph is:

$V = \{1, 2, 3, 4\}$
$E = \{(1,2), (2,3), (2,4), (3,4)\}$



<u>Objective func^n</u>:

$$\max \; X_{12} + X_{23} + X_{24} + X_{34}$$

s.t

1. $X_e \in \{0,1\}$ $\quad \left(e \in E = \{(1,2), (2,3), (2,4), (3,4)\}\right)$

for $v=1$ ← 2. $X_{12} \le 1$

for $v=2$ ← 3. $X_{12} + X_{24} + X_{23} \le 1$

for $v=3$ ← 4. $X_{23} + X_{34} \le 1$

for $v=4$ ← 5. $X_{24} + X_{34} \le 1$

Now, to find the optimal value using the revised simplex method, we need relax ILP formed in LPP.

Now, to solve this problem using simplex method, we need change the condition little bit to obtain linear programming problem.

So, we change $X_e \in \{0,1\}$ to $X_e \in [0,1]$.

Thus LPP:

$$\max: Z = X_{12} + X_{23} + X_{24} + X_{34}$$

s.t

1. $0 \le X_e \le 1 \quad \forall e \in E$
2. $X_{12} \le 1$
3. $X_{12} + X_{24} + X_{23} \le 1$
4. $X_{23} + X_{34} \le 1$
5. $X_{24} + X_{34} \le 1$

On entering this LPP as an input to our simplex code, we get the following ans:

Optimum value of func^n as, $\boxed{Z = 2}$

when $x_{12} = 1$

$x_{23} = 0$

$x_{24} = 0$

$x_{34} = 1$

i.e $\|M\|_{max} = 2$

So, we gave this LPP in our revised Simplex method in the following way.

Input

```
c = [-1,-1,-1,-1];
b = [1,1,1,1,1,1,1];
a = [1,1,1,0;0,1,0,1;0,0,1,1;1,0,0,0;0,1,0,0;0,0,1,0;0,0,0,1];
```

Output

```
>> iitd
x1 = 1
x4 = 1
The optimum objective function value=2.
>>
```

So, our ans came out to be 2.

i.e., ||M||max = 2 for the given network graph 1.

Q3)

For General Graph minimum size vertex cover problem can be formulated as given below.

(Q3) Minimum vertex cover Problem.

~~This is~~

On observing, we see that, this problem is just the dual of the (Q2).

So, Given Graph $\boxed{G=(V,E)}$

Given Vertex-cover $V' \subset V$ of G is a collection of vertices s.t each edge in E is incident to atleast one of the vertex in V'.

Let $\|V'\|$ be the size of the vertex cover V'.
Task is to minimize $\|V'\|$.

Define variable $Y_v \in \mathbb{R}^V$ where $v \in V$

$$Y_v = 1 \text{ if vertex } v \in V' \rightarrow \text{(vertex cover)}$$
$$Y_v = 0 \text{, otherwise}$$

So, ILP is

$$\boxed{\begin{aligned} &\min \sum Y_v \quad \text{where } v \in V \\ &\text{Subject to:} \\ &\qquad Y_u + Y_v \geq 1 \qquad \forall\, e = (u,v) \in E \\ &\text{and} \quad Y_v \in \{0,1\} \qquad \forall\, v \in V \end{aligned}}$$

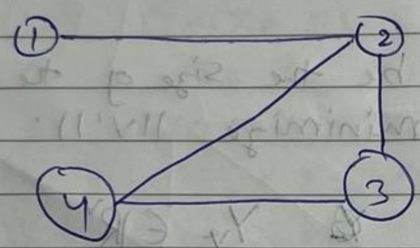
Now, this is an ILP as each variable $Y_v, v \in V$ can take only 2 values $\{0,1\}$ both integers

For given Graph Network 1, ILP is formulated as below

Now, for Network 1,

Network 1	1	2	3	4
1	0	1	0	0
2	1	0	1	1
3	0	1	0	1
4	0	1	1	0

i.e. graph is:



$V = \{1, 2, 3, 4\}$

$E = \{(1,2), (2,3), (2,4), (3,$

Objective func^n: min. $Y_1 + Y_2 + Y_3 + Y_4$

Subject to:

1. $Y_v \in \{0,1\}$ $\forall v \in V$
2. $Y_1 + Y_2 \geq 1$ $\rightarrow$ for $e = (1,2)$
3. $Y_2 + Y_3 \geq 1$ $\rightarrow$ for $e = (2,3)$
4. $Y_2 + Y_4 \geq 1$ $\rightarrow$ for $e = (2,4)$
5. $Y_3 + Y_4 \geq 1$ $\rightarrow$ for $e = (3,4)$

Now, to find the optimal value using the revised simplex method, we need relax ILP formed in LPP.

Similar to prev. part we need to change a little bit to obtain LPP:

i.e. we change $Y_v \in \{0,1\}$ to $Y_v \in [0,1]$

So, LPP:

$$\boxed{\begin{array}{lll} \text{min.} & Y_1 + Y_2 + Y_3 + Y_4 & \\ \text{s.t} & & \\ 1. & 1 \geq Y_v \geq 0 & \forall\ v \in V \\ 2. & Y_1 + Y_2 \geq 1 & \\ 3. & Y_2 + Y_3 \geq 1 & \\ 4. & Y_2 + Y_4 \geq 1 & \\ 5 & Y_3 + Y_4 \geq 1 & \end{array}}$$

On entering this LPP as an input we get the following result:

$$\boxed{\begin{array}{l} \text{optimum value of func}^n \text{ as } \boxed{Z=2} \\ \quad \text{when } y_1 = 0 \\ \quad\quad\quad\;\; y_2 = 1 \\ \quad\quad\quad\;\; y_3 = 1 \\ \quad\quad\quad\;\; y_4 = 0 \end{array}}$$

i.e. $\boxed{\|V'\| = 2}$

So, we gave this LPP in our revised Simplex method in the following way.

Input

```
% Question 3 input
  c = [1,1,1,1];
  b = [1,1,1,1,-1,-1,-1,-1];
  a = [1,0,0,0;0,1,0,0;0,0,1,0;0,0,0,1;-1,-1,0,0;0,-1,-1,0;0,-1,0,-1;0,0,-1,-1];
  revised(a,b,c)
```

Output

```
The optimum objective function value=2.
x2 = 1
x3 = 1
```

So, our ans came out to be 2.

i.e., ||V’||min = 2 for the given network graph 1.

Q4)

a)

We wrote Interior Point Method from scratch in Python named as **Interior Point Method Scratch.py.** We have used Kramarkar's algorithm (a type of IPOPT method) taught in class to solve a LPP.

Here is a sample input and its output.

We have taken following LPP as sample:

Minimise, Z: x1 – 3x2 + 3x3

Subject To: x1 - 3x2 + 2x3 = 0,

x1 + x2 + x3= 1,

and, x1, x2, x3 >=0.

For our code,

Input:

```
c = np.array([1, -3, 3])  # minimize
A = np.array([[1, -3, 2]])
```

Output:

```
Interior Point Method Scratch
C:\Users\SHOURYA\AppData\Local\Microsoft\WindowsApp
X: [7.49999056e-01 2.50000189e-01 7.54906486e-07]
optimal Value: 7.549027162760221e-07

Process finished with exit code 0
```

Although Optimal Ans is: X = [0.75,0.25,0] and Z = 0.

But since our code run for limited no of iterations ans is nearly close to the optimal ans. To enter input in our code it first must be converted to Karmarkar's special form called Canonical Form given as:

```
min cT.x
s.t. Ax = b
     eT.x = 1
     x ≥ 0
```

Any General LPP can be easily transformed into this canonical form by adding slack, surplus and artificial variables.

B) Formulating LPP for Network 2 to find out maximum flow from S to T:

(Q4)

(b) LP's for flow network 2 to find maximum flow from S to T.

$\boxed{\text{PRIMAL}}$:

$\max Z = x_1 + x_2 + x_3$

Subject to:

Arc Constraints:

~~A~~: S: $x_1 \le 11$, $x_2 \le 15$, $x_3 \le 10$

~~B~~: A: $x_4 \le 18$, $x_5 \le 4$

~~C~~: B: $x_6 \le 3$, $x_7 \le 8$, $x_8 \le 5$

~~D~~: C: $x_9 \le 6$, $x_{10} \le 3$, $x_{11} \le 11$

~~E~~: D: $x_{12} \le 4$, $x_{13} \le 17$, $x_{14} \le 6$

E: $x_{15} \le 3$, $x_{16} \le 16$, $x_{17} \le 13$

F: $x_{18} \le 12$, $x_{19} \le 4$, $x_{20} \le 21$

G: $x_{21} \le 4$, $x_{22} \le 9$, $x_{23} \le 4$, $x_{24} \le 3$

H: $x_{25} \le 4$, $x_{26} \le 5$, $x_{27} \le 4$

I: $x_{28} \le 7$, $x_{29} \le 9$

J: $x_{30} \le 2$, $x_{31} \le 15$

Non -ve Constraint:

$x_i \ge 0 \quad \forall\, i \in \{1, 2, \ldots, 30, 31\}$

Flow constraints:

$x_1 + x_6 + x_{18} - x_4 - x_5 = 0$

$x_2 - x_6 - x_8 = 0$

$x_3 + x_8 + x_{12} - x_9 - x_{10} - x_{11} = 0$

$x_9 + x_{15} + x_{19} - x_{12} - x_{13} - x_{14} = 0$

$x_4 - x_{15} - x_{17} = 0$

$x_5 - x_{18} - x_{19} - x_{20} = 0$

$x_{10} + x_{13} + x_{25} - x_{21} - x_{22} - x_{23} - x_{24} = 0$

$x_{11} + x_{21} + x_{23} + x_{30} - x_{25} - x_{26} - x_{27} = 0$

$x_{17} + x_{22} - x_{28} - x_{29} = 0$

$x_{23} + x_{26} - x_{30} - x_{31} = 0$

On solving it using IPOPT CODE and AMPL software we got the following answer:

On entering this LPP as input in our Interior Point Method code and AMPL Software answer come out to be $\boxed{29}$.

i.e, maximum flow from S to T in network2, that satisfies all the codition is $\boxed{29.}$

with

So, $Z = 29$

$x_1 = 11$, $x_2 = 8$, $x_3 = 10$, $x_4 = 10$, $x_5 = 4$

$x_6 = 3$, $x_7 = 0$, $x_8 = 5$, $x_9 = 6$, $x_{10} = 0$

$x_{11} = 9$, $x_{12} = 0$, $x_{13} = 3$, $x_{14} = 6$

$x_{15} = 3$, $x_{16} = 0$, $x_{17} = 7$, $x_{18} = 0$

$x_{19} = 0$, $x_{20} = 4$, $x_{21} = 0$, $x_{22} = 2$, $x_{23} = 0$

$x_{24} = 3$, $x_{25} = 2$, $x_{26} = 3$, $x_{27} = 4$

$x_{28} = 0$, $x_{29} = 9$, $x_{30} = 0$, $x_{31} = 3$

C) Taking Dual of the LP formed in part B, to find out min S-T cut for network 2:

(C) Taking Dual of the LP formed in prev. part of network 2.

$\boxed{\text{DUAL}}$:

$$\begin{aligned}\text{Min } Z = {} & 11y_1 + 15y_2 + 10y_3 + 18y_4 + 4y_5 + 3y_6 \\ & + 8y_7 + 5y_8 + 6y_9 + 3y_{10} + 11y_{11} \\ & + 4y_{12} + 17y_{13} + 6y_{14} + 3y_{15} \\ & + 16y_{16} + 13y_{17} + 12y_{18} + 4y_{19} + 21y_{20} \\ & + 4y_{21} + 4y_{22} + 4y_{23} + 3y_{24} \\ & + 4y_{25} + 5y_{26} + 4y_{27} + 7y_{28} \\ & + 9y_{29} + 2y_{30} + 15y_{31}\end{aligned}$$

Subject to:

$y_1 + y_{32} \geq 1$, $y_2 + y_{33} \geq 1$

$y_3 + y_{34} \geq 1$, $y_4 - y_{32} + y_{36} \geq 0$

$y_5 - y_{32} + y_{37} \geq 0$, $y_6 + y_{32} - y_{33} \geq 0$

$y_7 \geq 0$, $y_8 + y_{33} + y_{34} \geq 0$

$y_9 - y_{34} + y_{35} \geq 0$, $y_{10} - y_{34} + y_{38} \geq 0$

$y_{11} - y_{34} + y_{39} \geq 0$, $y_{12} + y_{34} - y_{35} \geq 0$

$y_{13} - y_{35} + y_{38} \geq 0$, $y_{14} - y_{35} \geq 0$

$y_{15} + y_{35} - y_{36} \geq 0$, $y_{16} \geq 0$

$y_{17} - y_{36} + y_{40} \geq 0$, $y_{18} + y_{32} - y_{37} \geq 0$

$y_{19} + y_{35} - y_{37} \geq 0$, $y_{20} - y_{37} \geq 0$

$y_{21} - y_{38} + y_{39} \geq 0$, $y_{22} - y_{38} + y_{40} \geq 0$

$y_{23} - y_{37} + y_{39} + y_{41} \geq 0$, $y_{24} - y_{38} \geq 0$

$y_{25} + y_{38} - y_{39} \geq 0$, $y_{26} - y_{39} + y_{41} \geq 0$

$y_{27} - y_{39} \geq 0$, $y_{28} - y_{40} \geq 0$

$y_{29} - y_{40} \geq 0$, $y_{30} + y_{39} - y_{41} \geq 0$

$y_{31} - y_{41} \geq 0$ and $\boxed{\begin{array}{l} y_i \geq 0 \ \forall i \in \{1, 2, \dots 31\} \\ y_i \in R \ \forall i \in \{32, \dots 41\} \end{array}}$

On solving it using IPOPT CODE and AMPL software we got the following answer:

On entering this LPP formulated as input in our IPOPT and AMPL codes, answer came out to be $\boxed{29}$.
i.e S-T min cut in network 2 is $\boxed{29}$

So, optimal $\boxed{Z = 29}$ with

$y_1 = 1$, $y_3 = 1$, $y_6 = 1$, $y_8 = 1$

$y_{33} = 1$ and

rest $y_i = 0$ $\forall\, i = \{1, 2, \ldots 41\}$ except $i \neq 1, 3, 6, 8, 33$

We find that Anwer of Q4 (b) and (c) is same. i.e.,

Max Flow from S to T = S-T Min – Cut = 29 for network 2.

Q5)

We solved all the 4 LP's formed in AMPL to verify the results we got from our codes written from scratch.

For Q2 LPP, we wrote the code stored in File Q2_LPP.run.

Output:

```
ampl: include 'C:\Users\SHOURYA\OneDrive\Desktop\LO Assignment\Q5_AMPL CODES\Q2_LPP.run';
CPLEX 22.1.1.0: optimal integer solution; objective 2
0 MIP simplex iterations
0 branch-and-bound nodes

Optimal Value Of The Objective Function
z = 2

Value of The Variables Giving Optimal Value
x12 = 1
x23 = 0
x24 = 0
x34 = 1
```

Optimal ans is 2, which matches the output of Q2.

For Q3 LPP, we wrote the code stored in File Q3_LPP.run.

Output:

```
ampl: include 'C:\Users\SHOURYA\OneDrive\Desktop\LO Assignment\Q5_AMPL CODES\Q3_LPP.run';
CPLEX 22.1.1.0: optimal integer solution; objective 2
0 MIP simplex iterations
0 branch-and-bound nodes

Optimal Value Of The Objective Function
z = 2

Value of The Variables Giving Optimal Value
y1 = 0
y2 = 1
y3 = 1
y4 = 0
```

Optimal ans is 2, which matches the output of Q3.

For Q4 (b) LPP Primal, we wrote the code stored in the file Q4_Primal_LPP.run.

Output:

```
ampl: include 'C:\Users\SHOURYA\OneDrive\Desktop\LO Assignment\Q5_AMPL CODES\Q4_Primal_LPP.run';
CPLEX 22.1.1.0: optimal solution; objective 29
8 dual simplex iterations (0 in phase I)

Optimal Value Of The Objective Function
z = 29

Value of The Variables Giving Optimal Value
x1 = 11
x2 = 8
x3 = 10
x4 = 10
x5 = 4
x6 = 3
x7 = 0
x8 = 5
x9 = 6
x10 = 0
x11 = 9
x12 = 0
x13 = 3
x14 = 6
x15 = 3
x16 = 0
x17 = 7
x18 = 0
x19 = 0
x20 = 4
x21 = 0
x22 = 2
x23 = 0
x24 = 3
x25 = 2
x26 = 3
x27 = 4
x28 = 0
x29 = 9
x30 = 0
x31 = 3
```

Optimal ans, i.e. Max Flow from S to T, for the given network 2 is 29, matching our above output.

For Q4 (c) LPP Primal, we wrote the code stored in the file Q4_Dual_LPP.run.

Output:

```
ampl: include 'C:\Users\SHOURYA\OneDrive\Desktop\LO Assignment\Q5_AMPL CODES\Q4_Dual_LPP.run';
CPLEX 22.1.1.0: optimal solution; objective 29
15 dual simplex iterations (0 in phase I)

Optimal Value Of The Objective Function
z = 29

Value of The Variables Giving Optimal Value
y1 = 1
y2 = 0
y3 = 1
y4 = 0
y5 = 0
y6 = 1
y7 = 0
y8 = 1
y9 = 0
y10 = 0
y11 = 0
y12 = 0
y13 = 0
y14 = 0
y15 = 0
y16 = 0
y17 = 0
y18 = 0
y19 = 0
y20 = 0
y21 = 0
y22 = 0
y23 = 0
y24 = 0
y25 = 0
y26 = 0
```

```
y26 = 0
y27 = 0
y28 = 0
y29 = 0
y30 = 0
y31 = 0
y32 = 0
y33 = 1
y34 = 0
y35 = 0
y36 = 0
y37 = 0
y38 = 0
y39 = 0
y40 = 0
y41 = 0
```

Optimal ans, i.e. S – T min - cut, for the given network 2 is 29, matching our above output.

Also we have verified that, for network 2,

<u>Max Flow From S to T (Q4 (b)) = S – T Min – Cut (Q4(c)) = 29, i.e. they both primal and dual LPP have same optimal.</u>