Meet Popat
2020320
SC

Homework-2

Prob-1:

a). For every term min of value = 0

$\therefore$ $x_1 + 0x_2 + 0x_3 = 0$ —(i)

$0x_1 + \sqrt{2}x_2 + 0x_3 = 0$ —(ii)

$0x_1 + 0x_2 + \sqrt{3}x_3 = 0$ —(iii)

$x_1 - x_2 + x_3 = 1$ —(iv)

$x_1 - 4x_2 = 2$ —(v)

ii) $\Rightarrow$ $0x_1 + x_2 + 0x_3 = 0$

iii) $\Rightarrow$ $0x_1 + 0x_2 + x_3 = 0$.

$\therefore$ Matrix form: $\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 1 \\ 1 & 4 & 0 \end{bmatrix}_{5\times 3}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times 1} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}_{5\times 1}}_{b}$

$\therefore$ $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 1 \\ 1 & 4 & 0 \end{bmatrix}$ & $b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}$

b). Similarly,

$\left.\begin{aligned} 0x_1 + 6x_2 + 0x_3 &= 4 \\ 4x_1 - 3x_2 + 0x_3 &= -1 \\ x_1 + 8x_2 + 0x_3 &= 3 \end{aligned}\right\}$ $\Rightarrow$ $\begin{aligned} 0x_1 + 6x_2 &= 4 \\ 4x_1 - 3x_2 &= -1 \\ x_1 + 8x_2 &= 3 \end{aligned}$

Matrix form: $\underbrace{\begin{bmatrix} 0 & 6 \\ 4 & -3 \\ 1 & 8 \end{bmatrix}_{3\times 2}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{2\times 1} = \underbrace{\begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}_{3\times 1}}_{b}$

$\therefore$ $A = \begin{bmatrix} 0 & 6 \\ 4 & -3 \\ 1 & 8 \end{bmatrix}$ & $b = \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}$

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Date

Page

c]. Rewriting the eq

Simplify :

$(6\sqrt{2}x_2 - 4\sqrt{2})^2 + (4\sqrt{3}x_1 - 3\sqrt{3}x_2)^2 + (2x_1 + 16x_2 - 6)^2$

Sq. term i.e for min each term 0.

$0x_1 + 6\sqrt{2}x_2 = 4\sqrt{2} \Rightarrow 0x_1 + 6x_2 = 4$

$4\sqrt{3}x_1 - 3\sqrt{3}x_2 = -\sqrt{3} \Rightarrow 4x_1 - 3x_2 = -1$

$2x_1 + 16x_2 = 6 \Rightarrow x_1 + 8x_2 = 3$

$\therefore$ Matrix form : $\underbrace{\begin{bmatrix} 0 & 6 \\ 4 & -3 \\ 1 & 8 \end{bmatrix}_{3\times 2}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{2\times 1} = \underbrace{\begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}_{3\times 1}}_{b}$

d]. Simplify :

$x^T x + \|Bx - d\|_2^2$ $\quad B \in \mathbb{R}^{p\times n}$, $d \in \mathbb{R}^p$

$\therefore$ we know that

if $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ $\quad x^T = [x_1\ x_2 \ldots x_n]$

$\Rightarrow x^T x = [x_1\ x_2 \ldots x_n]\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = x_1^2 + x_2^2 + \ldots x_n^2$

$\therefore x^T x = \sum_{i=1}^{n} x_i^2$

$\therefore x^T x = \|x\|_2^2$

$\therefore$ Hence, minimize

$\|x\|_2^2 + \|Bx - d\|_2^2$

By indentity & null matrices to minimize $\|x\|_2$ & $Bx = d$ to mini $\|Bx-d\|_2^2$ so,

$$\begin{array}{c} n \text{ rows} \\ \\ p \text{ rows} \end{array} \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ B_{11} & B_{12} & \cdots & \cdots & B_{1n} \\ \vdots & & & & \vdots \\ B_{p1} & B_{p2} & \cdots & \cdots & B_{pn} \end{bmatrix}_{(n+p)\times n} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ d_1 \\ \vdots \\ d_p \end{bmatrix}_{(n+p)\times 1}$$

$$A \quad x = b$$

$\therefore$ This is linear least squares problem

e] Min of

$$x^T D x + \|Bx-d\|_2^2 \qquad D \in \mathbb{R}^{n\times n} \text{ diagonal matrix}$$

(+ve diagonal elements)

similarly, as d, $\qquad x \in \mathbb{R}^{p\times n}$, $d \in \mathbb{R}^p$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1} \quad \& \quad D = \begin{bmatrix} D_1 & 0 & \cdots & 0 \\ 0 & D_2 & & \vdots \\ 0 & 0 & D_3 \cdots & \vdots \\ \vdots & & & \vdots \\ 0 & \cdots & \cdots & D_n \end{bmatrix}_{n\times n}$$

$$\text{so, } Dx = \begin{bmatrix} D_1 x_1 \\ D_2 x_2 \\ \vdots \\ D_n x_n \end{bmatrix}_{n\times 1}$$

&

$$x^T D x = D_1 x_1^2 + D_2 x_2^2 + \cdots + D_n x_n^2 \quad \text{---} (1)$$

(sum of linear least sq.)

$\therefore$

$$D_1 x_1^2 + D_2 x_2^2 + \cdots D_n x_n^2 + \|Bx - d\|_2^2$$

$$\therefore (\sqrt{D_1} x_1)^2 + (\sqrt{D_2} x_2)^2 \cdots\cdots (\sqrt{D} x_n)^2 + \|Bx - d\|_2^2$$

∴ Matrix form:

$$
\begin{array}{c} n \text{ rows} \left\{ \begin{array}{c} \\ \\ \\ \\ \end{array} \right. \\ p \text{ rows} \left\{ \begin{array}{c} \\ \\ \\ \end{array} \right. \end{array}
\underbrace{\begin{bmatrix} \sqrt{D_1} & 0 & \cdots & 0 \\ 0 & \sqrt{D_2} & \cdots & 0 \\ \vdots & & \ddots & \\ 0 & \cdots & & \sqrt{D_n} \\ B_{12} & B_{12} & \cdots & B_{1n} \\ \vdots & & & \\ B_{p1} & B_{p2} & & B_{pn} \end{bmatrix}_{(n+p)\times n}}_{A}
\underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \vdots \\ x_n \end{bmatrix}_{n\times 1}}_{x}
\underbrace{=}_{\approx}
\underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ d_1 \\ d_2 \\ \vdots \\ d_p \end{bmatrix}_{(n+p)\times 1}}_{b}
$$

$$Ax \approx \underline{b}$$

∴ Linear least square problem

Prob-2:

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}}_{b} \quad \Rightarrow Ax = b.$$

$\therefore \|b - Ax\|_2$ is the min, & # rows > # of columns of A

$\Rightarrow$ linear least sq. probm & minimize $\|b - Ax\|_2$

$\therefore$ Normal eqn $\Rightarrow A^T A x = A^T b$

(condition for existence of minimum x)

$$A^T A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & -1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\& \quad A^T b = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

so, $$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$R \Rightarrow \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 1 & 2 & 3 \end{array}\right]$$ Augmented matrix

$R_2 \rightarrow R_2 - R_1$

$$\Rightarrow \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 1 & 1 \end{array}\right] \Rightarrow \begin{aligned} x_1 + x_2 &= 2 \\ x_2 &= 1 \\ x_1 &= 1 \end{aligned}$$

$$\therefore x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$ soln of normal equation

For residual $(r) = b - Ax = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\therefore r = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$\therefore \|r\|_2 = \sqrt{0^2 + 0^2 + 1^2} = 1$ (Euclidean norm)

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Date

Page

To prove:

Consider any $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$\therefore r = b - Ax = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 - x_1 - x_2 \\ 1 - x_2 \\ 1 \end{bmatrix}$

For min differentiate w.r.t $x = 0$

$\therefore \min \|r\|_2$, $\nabla \|r\|_2 = 0$

And $\|r\|_2^2 = (2 - x_1 - x_2)^2 + (1 - x_2)^2 + 1^2 = f(x_1, x_2)$

$\therefore \dfrac{df(x_1, x_2)}{dx_1} = (2)(2 - x_1 - x_2)(-1) = 2x_1 + 2x_2 - 4$

$\dfrac{df(x_1, x_2)}{dx_2} = 2x_1 + 2x_2 - 4 + (2 - 2x_2)(-1) = 2x_1 + 4x_2 - 6$

$\therefore \nabla f = \begin{bmatrix} 2x_1 + 2x_2 - 4 \\ 2x_1 + 4x_2 - 6 \end{bmatrix}$

$\nabla f \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0$

$\therefore$ At $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\nabla f = 0$ (min cond)

$\therefore$ To be min it should if positive definite

$x^T A^T x > 0$

$\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 5 > 0$

$\therefore x^T A^T x > 0$

Hence, $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is minimum

Job 3 a) Let $P = \begin{bmatrix} I & A \\ A^T & 0 \end{bmatrix}$ where $A \in \mathbb{R}^{m \times n}$ with $m \geq n$

$$\Rightarrow P = \begin{array}{c} \\ m \\ \\ n \end{array} \begin{bmatrix} 1 & 0 & \cdots & 0 & a_{11} & a_{12} & \cdots & a_{1n} \\ 0 & 1 & \cdots & 0 & a_{21} & a_{22} & \cdots & a_{2n} \\ 0 & 0 & & & \vdots & \vdots & & \vdots \\ 0 & 0 & & & & & & \\ a_{11} & a_{21} & \cdots & a_{m1} & a_{m1} & a_{m2} & & a_{mn} \\ a_{12} & a_{22} & \cdots & a_{m2} & 0 & 0 & \cdots & 0 \\ \vdots & & & & 0 & & & 0 \\ \vdots & & & & \vdots & & & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} & 0 & 0 & \cdots & 0 \end{bmatrix}_{(m+n) \times (m+n)}$$

We will prove P is non singular by contradiction

Hence, Assume P is singular.
$\Rightarrow$ columns of P linearly dependent.
$\therefore$ Let P_i denote each column $i \in 1, 2 \ldots m+n$

$\therefore P = [P_1 \; P_2 \; \ldots \; P_{m+n}]$

For linearly independent $\Rightarrow$ $m+n$ variable in $x_i \in \mathbb{R}$ & $i \in \{1, 2, \ldots m+n\}$

s.t $\sum_{i=1}^{m+n} P_i x_i = 0$ $(x_i \neq 0)$

$\therefore$ The system of linear eqn form

$$\left.\begin{array}{l} x_1 + x_{m+1} a_{11} + x_{m+2} a_{12} + \cdots + x_{m+n} a_{1n} = 0 \\ x_2 + x_{m+1} a_{21} + \cdots + x_{m+n} a_{2n} = 0 \\ \vdots \\ x_m + x_{m+1} a_{m1} + \cdots + x_{m+n} a_{mn} = 0 \end{array}\right\} \text{1st } m \text{ columns}$$

In $Ax=b$ terms :

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & \vdots \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_{m+1} \\ x_{m+2} \\ \vdots \\ x_{m+n} \end{bmatrix} = -\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}$$

$$\quad A \qquad\qquad\qquad x \qquad\qquad b$$

$$\Rightarrow Ax = -b \quad \text{--- (1)}$$

Expressing in $Ax=b$ form

$$\begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$A^T \qquad\qquad b \qquad\qquad 0$$

$$\Rightarrow A^T b = 0 \quad \text{--- (2)}$$

$\therefore$ (I) $\Rightarrow Ax = -b \Rightarrow b = -Ax$

II $\Rightarrow A^T b = 0$

$-A^T A x = 0$

$\Rightarrow A^T A x = 0$

if $x \neq 0 \Rightarrow AA^T = 0$

$\Rightarrow$ col^n of A are L.D

But we known the col^n of A are linearly independ as null$\{A\} = \{0\}$ (null space of A has zero vector only)

If $Ax = 0$ then $A^T A x = A^T(0) = 0 \Rightarrow x \in Nul(A^T$

$\Rightarrow Nul(A) \subseteq Nul(A^T A)$ - (a)

If $x \in Nul(A^T A) \Rightarrow A^T A x = 0$

$\Rightarrow x^T A^T A x = 0$

$\Rightarrow (Ax)^T (Ax) = 0$

$= Ax = 0$

$\Rightarrow x \in Nul(A)$

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Date

Page

Hence $Null(A^TA) \subseteq Nul(A)$ ____ (b)

a & b $\Rightarrow$ columns of A are L.I

$\therefore$ contradiction

2) if $x = 0$

$Ax = 0 = b$

$\therefore b = 0$

$\therefore$ All x_i are zero

$\therefore$ our assumption that linear ~~i~~ dependent & any $x_i \neq 0$ is contradicted

Hence, from both the cases we arrived at a contraction & get that columns of P are linearly independent. $\therefore$

Hence, P is a non singular matrix

Hence proved.

Prob 3 b] $\begin{bmatrix} I & A \\ A^T & 0 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} b \\ 0 \end{bmatrix}$

$\Rightarrow I\hat{x} + A\hat{y} = b \quad \Rightarrow \hat{x} + A\hat{y} = b \quad — ①$

$A^T\hat{x} + 0\hat{y} = 0 \quad \Rightarrow A^T\hat{x} = 0 \quad — ②$

$① \times A^T \Rightarrow \cancel{A^T\hat{x}}^{0} + A^TA\hat{y} = A^Tb$

$\Rightarrow A^TA\hat{y} = A^Tb$

& also normal eqn $A^TAx = A^Tb$ can be ~~for~~ treated as CLSP problem

$\therefore A^TA\hat{y} = A^Tb = A^TAx$

$\Rightarrow \boxed{\hat{y} = x}$

$\therefore \hat{x} + Ax = b$ eq 1 substitute

$\boxed{\hat{x} = b - Ax}$

Hence $b - Ax = \hat{x}$ & $\hat{y} = x$ are the soln

Prob 5 d)

$$y = \frac{e^{\alpha t + \beta}}{1 + e^{\alpha t + \beta}}$$

$$\Rightarrow y(1 + e^{\alpha t + \beta}) = e^{\alpha t + \beta}$$

$$\therefore e^{\alpha t + \beta} = \frac{y}{1-y}$$

($y_i < 1$ $i \in \{1, 50$
$1 - y_i > 0$ (for plotti

Taking log

$$\Rightarrow \alpha t + \beta = \log\left(\frac{y}{1-y}\right)$$ (This is linear eq^n).

$$\Rightarrow \alpha t_i + \beta = \log\left(\frac{y_i}{1-y_i}\right)$$ where $i = 1, 2, \cdots 50$

Matrix form

$$\underbrace{\begin{bmatrix} t_1 & 1 \\ t_2 & 1 \\ \vdots & \\ t_{50} & 1 \end{bmatrix}_{50\times 2}}_{A} \underbrace{\begin{bmatrix} \alpha \\ \beta \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} \log(y_1/1-y_1) \\ \log(y_2/1-y_2) \\ \vdots \\ \log(y_{50}/1-y_{50}) \end{bmatrix}_{50\times 1}}_{b}$$

Hence $A = \begin{bmatrix} t_1 & 1 \\ t_2 & 1 \\ \vdots & \\ t_{50} & 1 \end{bmatrix}$ $x = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$ & $b = \begin{bmatrix} \log(y_1/1-y_1) \\ \log(y_2/1-y_2) \\ \vdots \\ \log(y_{50}/1-y_{50}) \end{bmatrix}$

in eq $Ax = b$ where $m > n$ $(m = 50, n = 2)$

$\therefore$ find x (α, β) so that $\|b - Ax\|_2$ is min

Hence, LLSP problem

Prob 5 b). Output images.

Prob 6). a) Normal eq^n $A^T A x = A^T b$

$$A^T A = \begin{bmatrix} 1 & 10^{-k} & 0 \\ 1 & 0 & 10^{-k} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 10^{-k} & 0 \\ 0 & 10^{-k} \end{bmatrix}$$

$$= \begin{bmatrix} 1+10^{-3k} & 1 \\ 1 & 1+10^{-2k} \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 1 & 10^{-k} & 0 \\ 1 & 0 & 10^{-k} \end{bmatrix} \begin{bmatrix} -10^{-k} \\ 1+10^{-k} \\ 1-10^{-k} \end{bmatrix} = \begin{bmatrix} 10^{-2k} \\ -10^{-2k} \end{bmatrix}$$

So,
$$\underbrace{\begin{bmatrix} 1+10^{-2k} & 1 \\ 1 & 1+10^{-2k} \end{bmatrix}}_{A^T A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 10^{-2k} \\ -10^{-2k} \end{bmatrix}}_{b'}$$

∴ solving we get
$$x = \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

b) : As k increases error increases bcoz 10^{-k} dec. small error in any digit results is total error in final output.

c). After k = 7 No output as ||A|| becomes singular matrix (|A| = 0), thus it gives out

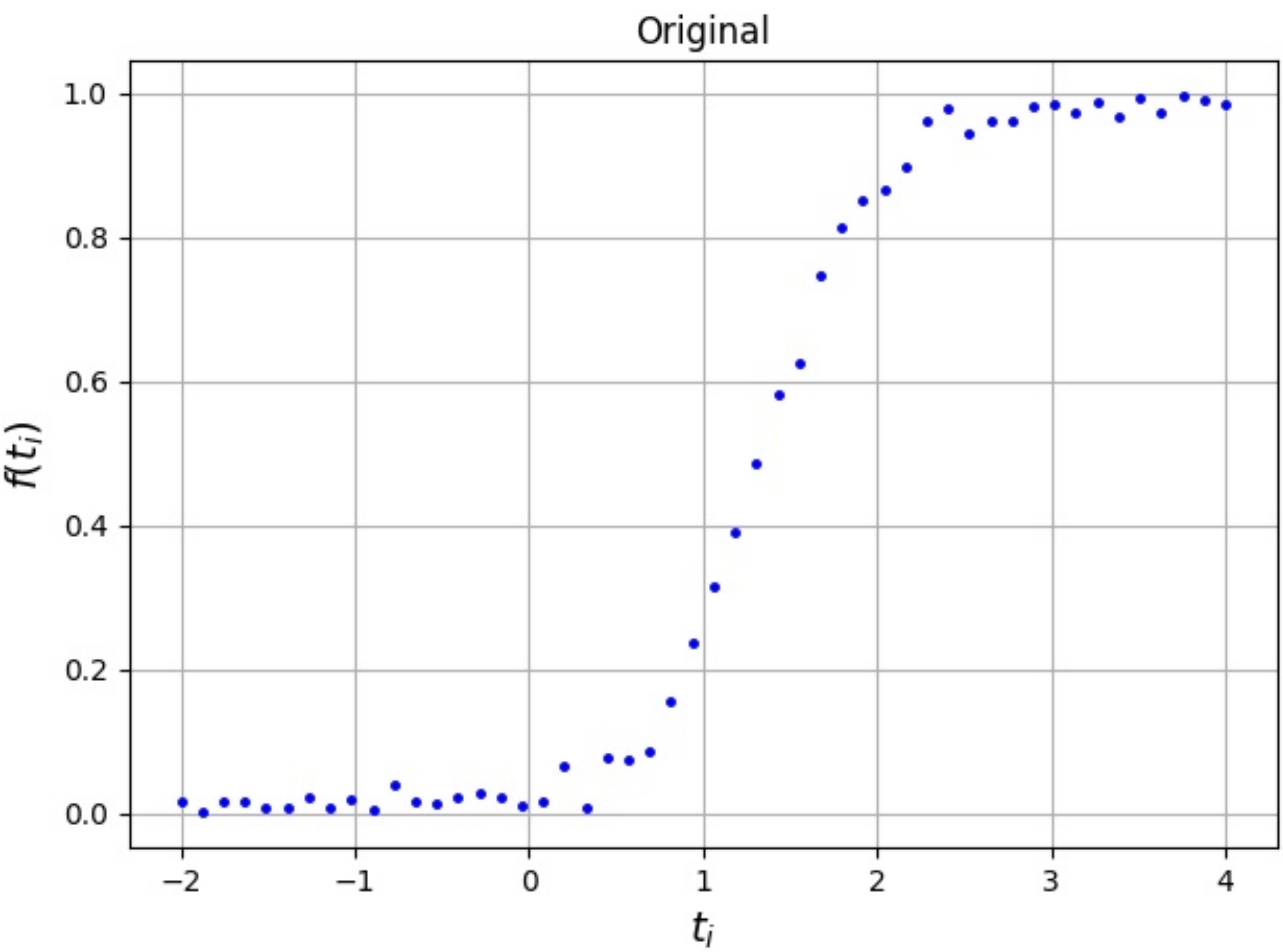
Prob 7) Explanation:

As the λ increase graph becomes continous

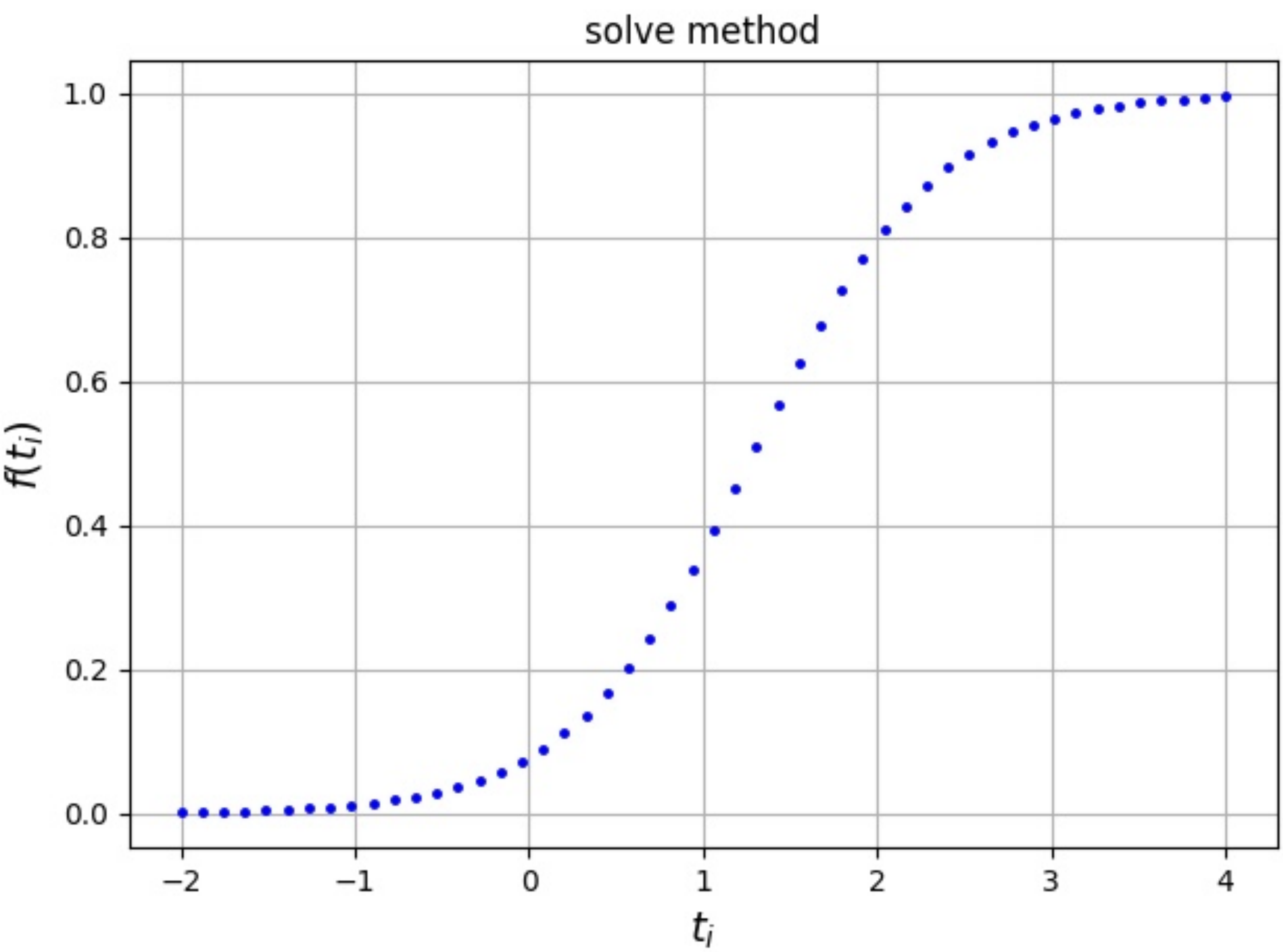
i.e from 1 => it is discrete at many points

& 10000 => it is almost continous

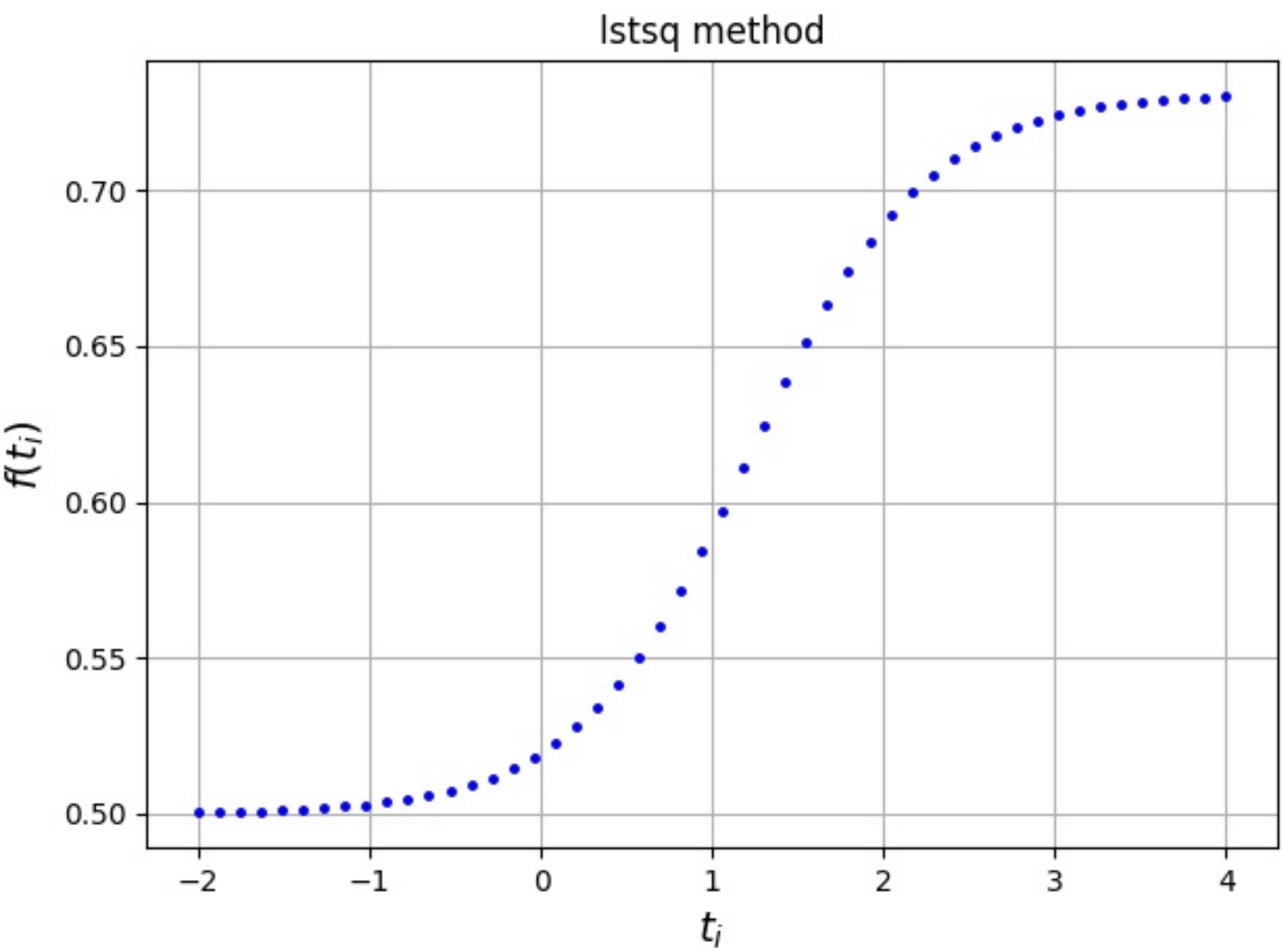
As $\lambda \uparrow$ more error.



Original
1.0
0.8
0.6
0.4
0.2
0.0
−2
−1
0
1
2
3
4
$f(t_i)$
t_i



solve method
1.0
0.8
0.6
0.4
0.2
0.0
$f(t_i)$
−2
−1
0
1
2
3
4
t_i

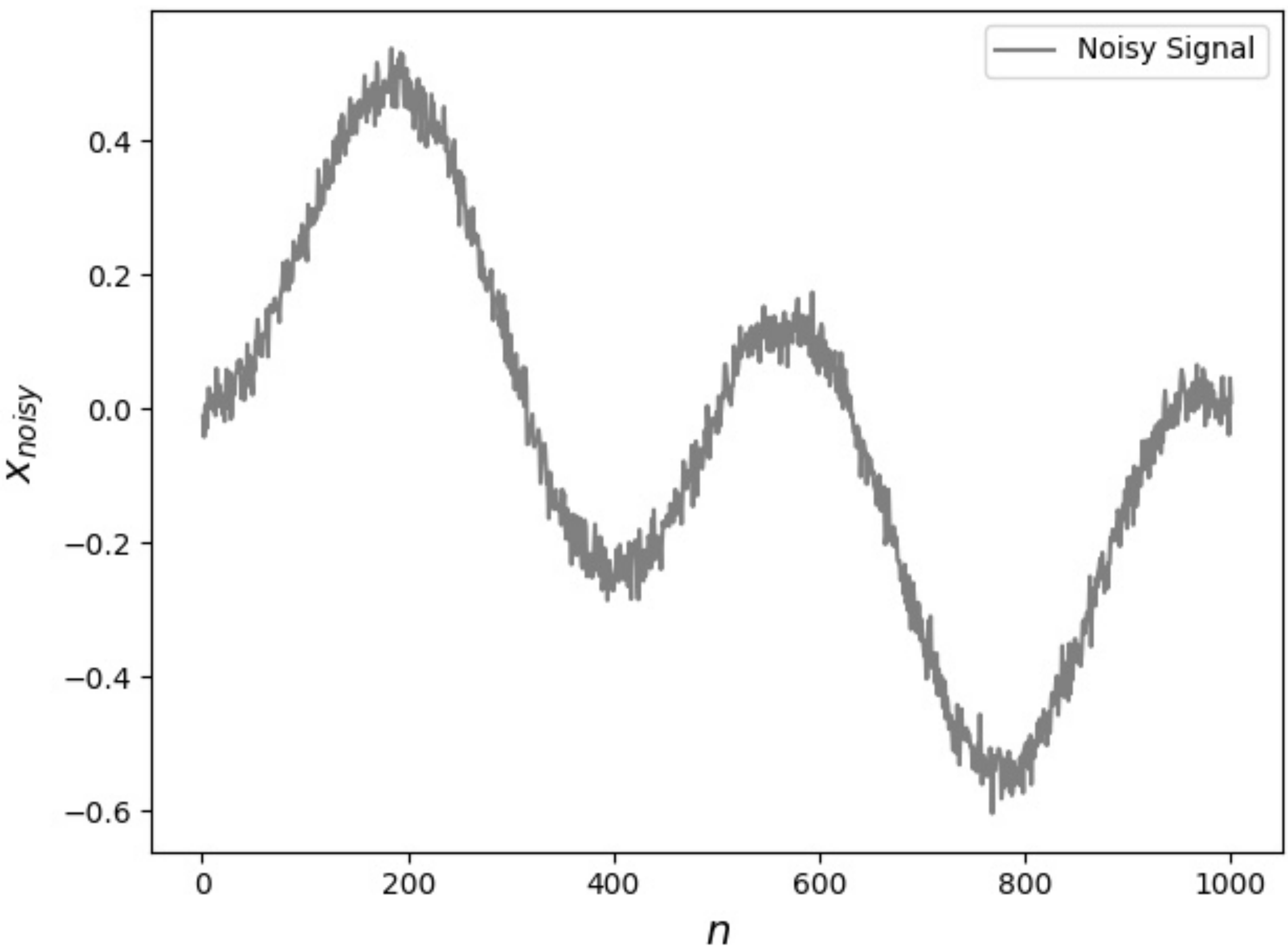

lstsq method
0.70
0.65
0.60
0.55
0.50
$f(t_i)$
−2
−1
0
1
2
3
4
t_i

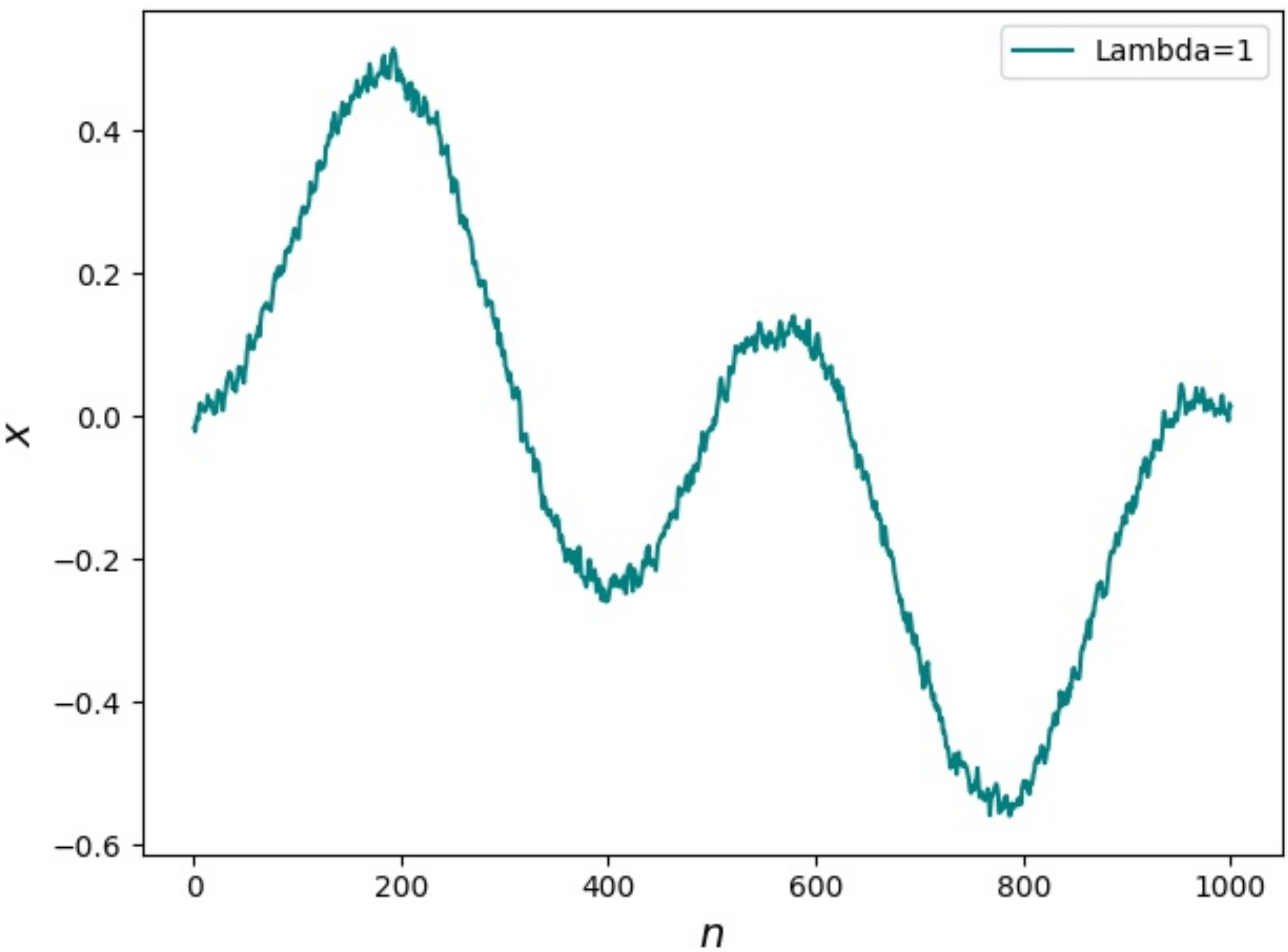
[2]:
```python
import numpy as np
import scipy as sp
##import matplotlib.pyplot as plt
import numpy.linalg as npla
import scipy.linalg as spla



# 6b
for k in range(6, 16):
    power = pow(10, -k)
    matrixA = np.array([[1,    1], [power, 0], [0, power]], dtype = float)
    b = np.array([[-power], [1+power], [1-power]], dtype = float)

    q_matrix, r_matrix = npla.qr(matrixA)
    x = spla.solve_triangular(r_matrix, np.matmul(q_matrix.T, b))
    print(x)
```

```
[[ 1.]
 [-1.]]
[[ 1.]
 [-1.]]
[[ 1.00000002]
 [-1.00000002]]
[[ 1.00000001]
 [-1.00000001]]
[[ 1.0000007]
 [-1.0000007]]
[[ 1.00001169]
 [-1.00001169]]
[[ 0.99998488]
 [-0.99998488]]
[[ 0.99944487]
 [-0.99944487]]
[[ 1.00202995]
 [-1.00202995]]
[[ 0.89212348]
 [-0.89212348]]
```

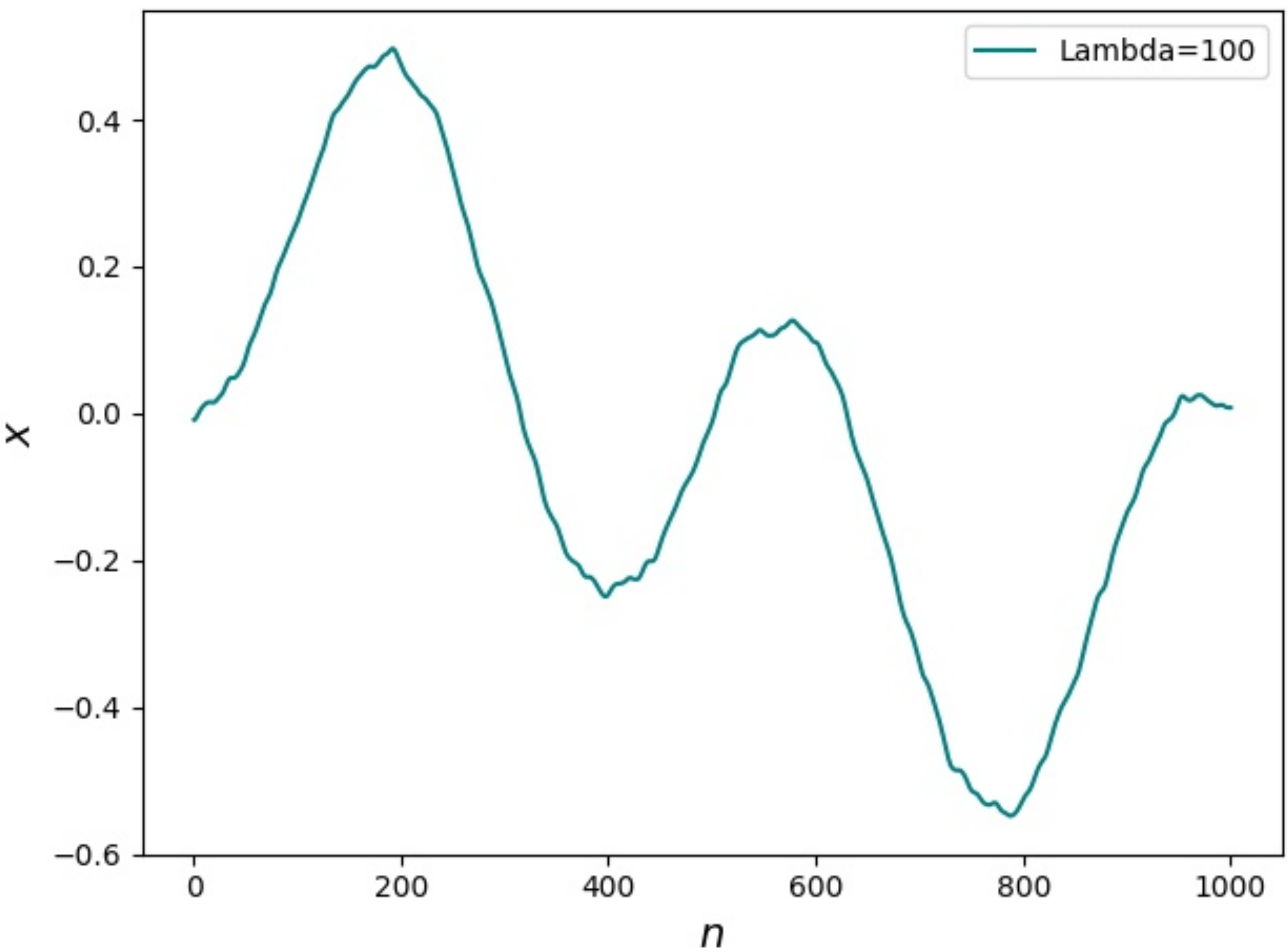
```
import scipy.linalg as spla

for k in range(6, 16):
    power = pow(10, -k)
    matrixA = np.array([[1,    1], [power, 0], [0, power]], dtype = float)
    b = np.array([[-power], [1+power], [1-power]], dtype = float)

    x = npla.solve(np.matmul(matrixA.T, matrixA), np.matmul(matrixA.T, b))
    print(x)
```

```
[[ 0.99991111]
 [-0.99991111]]
[[ 1.00079992]
 [-1.00079992]]
---------------------------------------------------------------------------
LinAlgError                               Traceback (most recent call last)
Input In [1], in <cell line: 6>()
      8 matrixA = np.array([[1,    1], [power, 0], [0, power]], dtype = float)
      9 b = np.array([[-power], [1+power], [1-power]], dtype = float)
---> 11 x = npla.solve(np.matmul(matrixA.T, matrixA), np.matmul(matrixA.T, b))
     12 print(x)

File <__array_function__ internals>:5, in solve(*args, **kwargs)

File ~/opt/anaconda3/lib/python3.9/site-packages/numpy/linalg/linalg.py:393, in solve(a, b)
    391 signature = 'DD->D' if isComplexType(t) else 'dd->d'
    392 extobj = get_linalg_error_extobj(_raise_linalgerror_singular)
--> 393 r = gufunc(a, b, signature=signature, extobj=extobj)
    395 return wrap(r.astype(result_t, copy=False))

File ~/opt/anaconda3/lib/python3.9/site-packages/numpy/linalg/linalg.py:88, in _raise_linalgerror_singular(err, f
lag)
     87 def _raise_linalgerror_singular(err, flag):
---> 88     raise LinAlgError("Singular matrix")

LinAlgError: Singular matrix
```

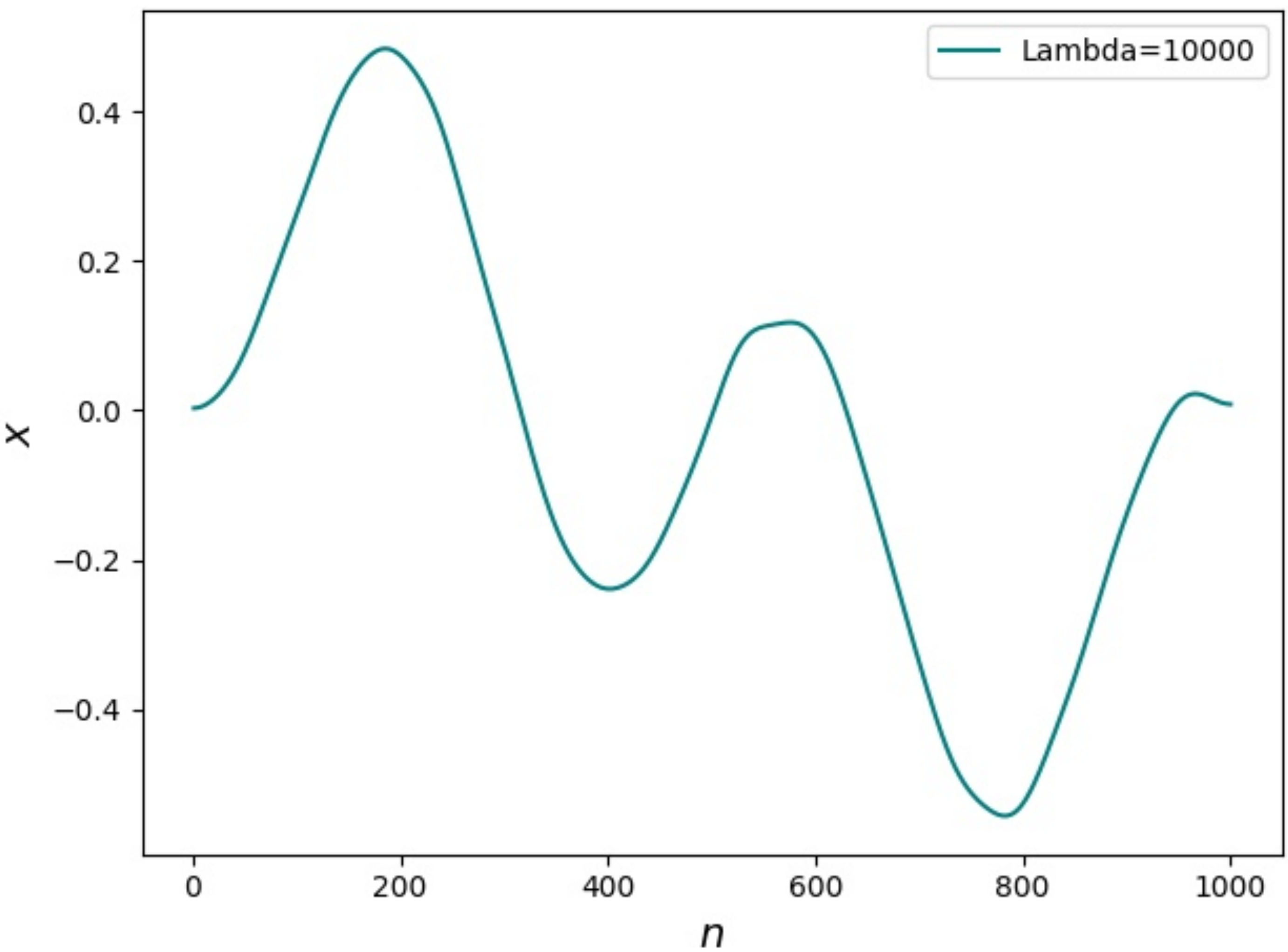

Noisy Signal
0.4
0.2
0.0
−0.2
−0.4
−0.6
x_{noisy}
0
200
400
600
800
1000
n



Lambda=1
0.4
0.2
0.0
−0.2
−0.4
−0.6
0
200
400
600
800
1000
x
n



Lambda=100
0.4
0.2
0.0
−0.2
−0.4
−0.6
x
0
200
400
600
800
1000
n


Lambda=10000
0.4
0.2
0.0
−0.2
−0.4
x
0
200
400
600
800
1000
n