Assignment 1

Name : Meet Popat
Roll No. : 2020320

Q1).For this we generated exponential data for different values of lambda = 1,2,3,4, using the function rexp(sample_size,lambda)

ASSIGNMENT-1

MEET POPAT
2020320

Q:1) $X_i \overset{iid}{\sim}$ Exponential(λ)

now, $\Rightarrow$ $X \sim$ Exponential(λ).

$\therefore$

$$f_X(x) = \lambda e^{-\lambda x}$$

$\therefore$ Likelihood function

$$L(\theta) = \prod_{j=1}^{n} f_X(x_j, \lambda)$$

$$= \prod_{j=1}^{n} \lambda e^{-\lambda x_j}$$

$$= \lambda^n e^{-\lambda \sum_{j=1}^{n} x_j}$$

Log-likelihood funtn $= \ln(L(\theta))$

$$= \ln(\lambda^n e^{-\lambda \sum_{j=1}^{n} x_j})$$

$$= \boxed{n \ln(\lambda) - \lambda \sum_{j=1}^{n} x_j}$$

$\therefore$ $\frac{dl}{d\lambda} = 0 \Rightarrow \frac{d(n\ln(\lambda) - \lambda \sum_{j=1}^{n} x_j)}{d\lambda} = 0$

$$\Rightarrow \frac{n}{\lambda} - \sum_{j=1}^{n} x_j = 0$$

$$\Rightarrow \hat{\lambda}_{MLE} = \frac{n}{\sum_{j=1}^{n} x_j}$$

$$\Rightarrow \boxed{\hat{\lambda}_{MLE} = \frac{1}{\bar{x}}}$$

For maximum,

$$\frac{dl^2}{d\theta^2} = \frac{d\left(n/\lambda - \sum_{i=1}^{n} x_i\right)}{d\lambda}$$

$$\frac{dl^2}{d\theta^2} = -\frac{n}{\lambda^2}$$

$$\therefore \frac{dl^2}{d\theta^2} < 0$$

$\therefore$ $l(\theta)$ is max at $\lambda = \frac{1}{\bar{x}}$.

5). Method of moments to find initial values

$X \sim Exp(\lambda)$

$$E[X] = 1/\lambda$$

$$\frac{\sum x_i}{n} = \frac{1}{\lambda}.$$

$$\hat{\lambda} = \frac{n}{\sum x_i} = \frac{1}{\bar{x}}$$

$\therefore$ Initial value of λ as $\frac{1}{mean(x)}$.

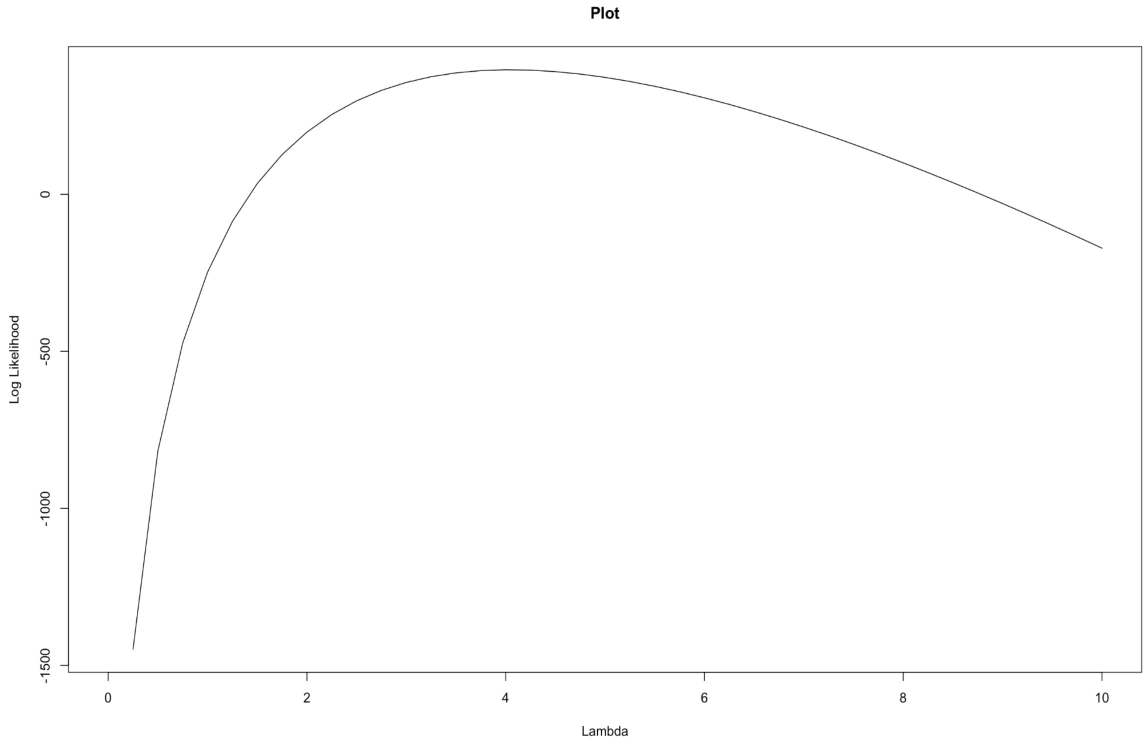
B. For the maximum likelihood estimate, R routine ‘optim()’ is used to maximize the log likelihood of the random sample drawn from the Exponential distribution.

For each value of lambda = 1,2,3,4; we used 3 initial values: 0.33, 0.65, 1/mean(data). Last one is obtained using the method of moments.

Estimated value lambda i.e. λMLE and its log likelihood value came out to be:

[1] "For Lamba ="
[1] 1
MLE with Initial Value = 0.33 is: 1.001428
MLE with Initial Value = 0.65 is: 1.001428
MLE with Initial Value = 1.001427 is: 1.001427
[1] "For Lamba ="
[1] 2
MLE with Initial Value = 0.33 is: 2.005771
MLE with Initial Value = 0.65 is: 2.005771
MLE with Initial Value = 2.005771 is: 2.005771
[1] "For Lamba ="
[1] 3
MLE with Initial Value = 0.33 is: 3.166751
MLE with Initial Value = 0.65 is: 3.166751
MLE with Initial Value = 3.166751 is: 3.166751
[1] "For Lamba ="
[1] 4
MLE with Initial Value = 0.33 is: 4.285989
MLE with Initial Value = 0.65 is: 4.285989
MLE with Initial Value = 4.285989 is: 4.285989

Q1c.Graphs came out to be:



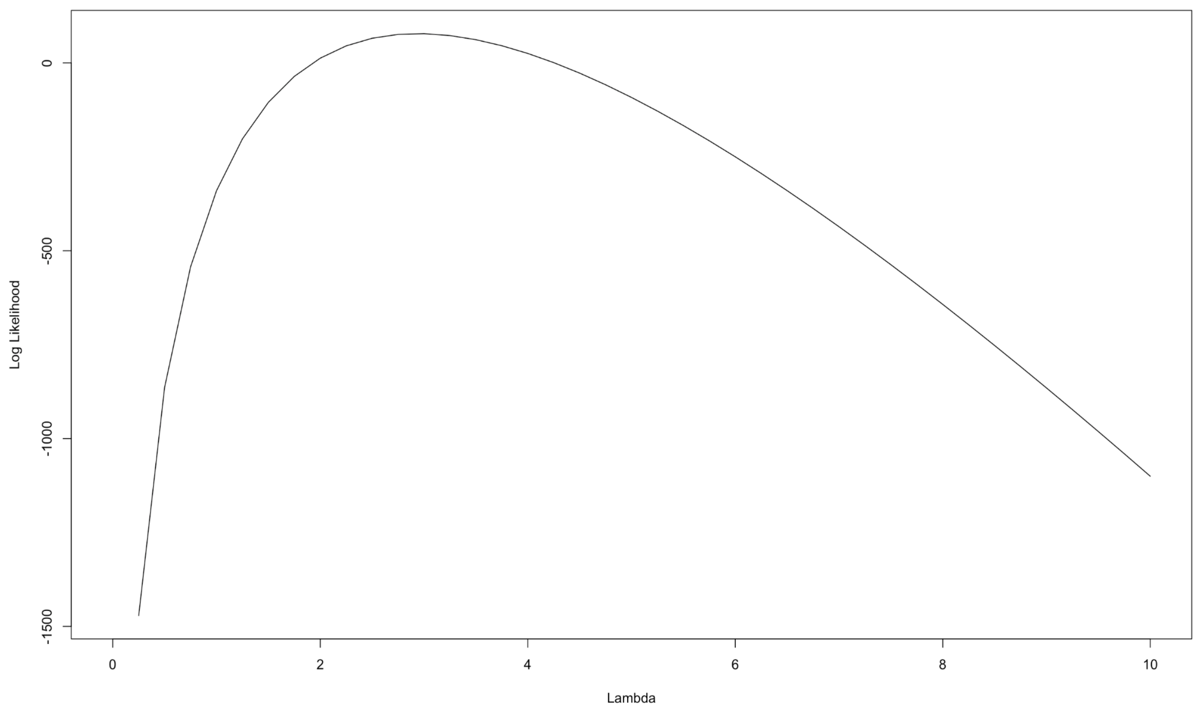
The plots are for the follows data with lambda as (1,2,3,4)
data <- rexp(1000,lambda)

Log likelihood is on Y-axis and different values of lambda are on x-axis.

Values are varied from 1 to 10 with steps of 0.25.

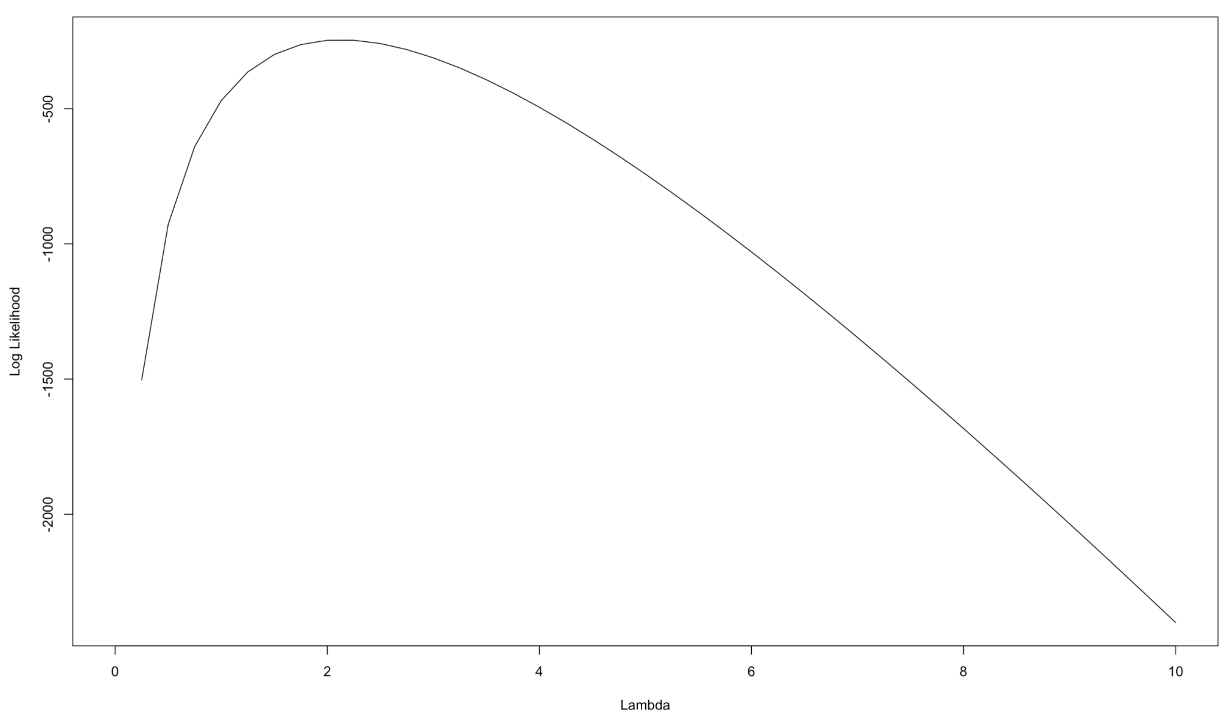
We observe that the log-likelihood function value at each is maximum at a lambda and also almost equal to the value calculated using optimiser(optim()).

Plot

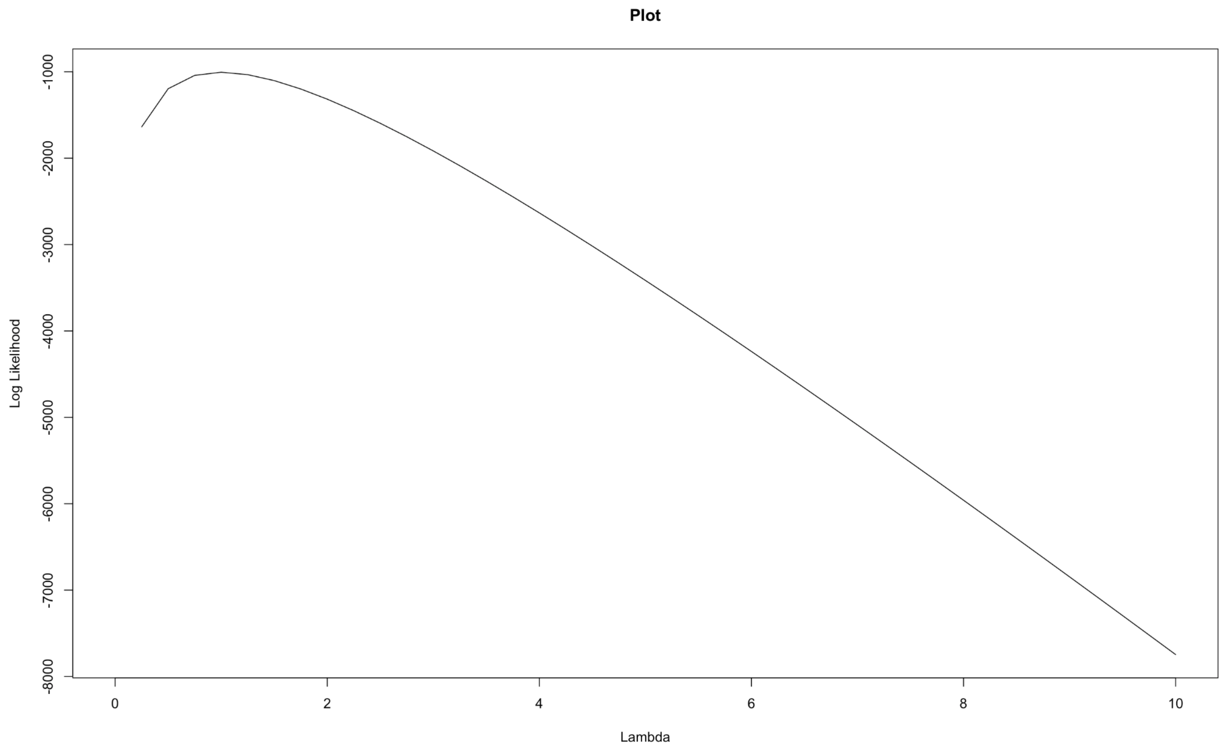


Log Likelihood
0
-500
-1000
-1500
0
2
4
6
8
10
Lambda

Plot



Log Likelihood
-500
-1000
-1500
-2000
0
2
4
6
8
10
Lambda



Hence the result are verified correctly.

Q2.

For the maximum likelihood estimate, R routine ‘optim()’ is used to maximize the log likelihood of the random sample drawn from the Normal distribution.

The following Maximum likelihood estimates for μ and σ 2 were obtained: μ ^ MLE = 4000.044 σ 2 MLE= 15.53298 .

Q:2 d). $X \overset{iid}{\sim}$ Normal (μ, σ)

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-1}{2} \frac{(x_i-\mu)^2}{\sigma^2}\right)$$

∴ Liklihood function

$$L(\theta) = \prod_{i=1} f(x_i)$$

$$= \frac{1}{(2\pi\sigma^2)^{n/2}} e^{-1/2\left(\sum_{i=1}^{n} \frac{(x_i-\mu)^2}{\sigma^2}\right)}$$

Log-likelihood $= \cancel{\log}\ \ln(L(\theta))$

$$= -\frac{n}{2} \log(2\pi\sigma^2) - \frac{1}{2\sigma^2} \sum_{i=1}^{n} (x_i-\mu$$

$$= \frac{n}{2} \log(2\pi) - \frac{n}{2} \log(\sigma^2) \ \frac{-1}{2\sigma^2} \sum_{i=1}^{n} (x_i-\mu)^2$$

∴ maximum

$$\frac{dl}{d\theta} = 0 \quad \& \quad \frac{dl}{d\sigma} = 0.$$

i) $\frac{dl}{d\sigma} = 0$

$$\Rightarrow 0 - \frac{n}{\sigma} + \frac{1}{\sigma^3} \Sigma (x_i-\mu)^2 = 0$$

$$\sigma = \sqrt{\frac{\Sigma(x_i-\mu)^2}{n}}$$

To check max/min

$$\frac{d^2 l}{d^2\sigma} = \frac{n}{\sigma^2} - \frac{3}{\sigma^4}\left(\Sigma (x_i-\mu)^2\right)$$

$$\therefore = \frac{n^2}{\Sigma(x_i-\mu)^2} - \frac{3n^2}{\Sigma(x_i-\mu)^2}$$

b) For fixed variance and fixed mean, 2 graphs were provided LogLikelihood function is plotted on the y-axis, μ is plotted on the x-axis varying between mu_hat-2 and mu_hat+2 with step = 0.2 and σ 2 is plotted on the y-axis varying between sigma_hat-2 and sigma_hat+2 with step = 0.2.

$$= \frac{-2n^2}{\Sigma(x_i-\mu)^2} < 0$$

$\therefore \frac{d^2 l}{d\sigma^2} < 0$. Hence maxima

2) $\frac{d\,l(\theta)}{d\mu} = 0$

Putting σ^2_{MLE} & $\frac{d\,l\theta}{d\mu} = 0$

$\Rightarrow \mu = \frac{\Sigma x_i}{n}$

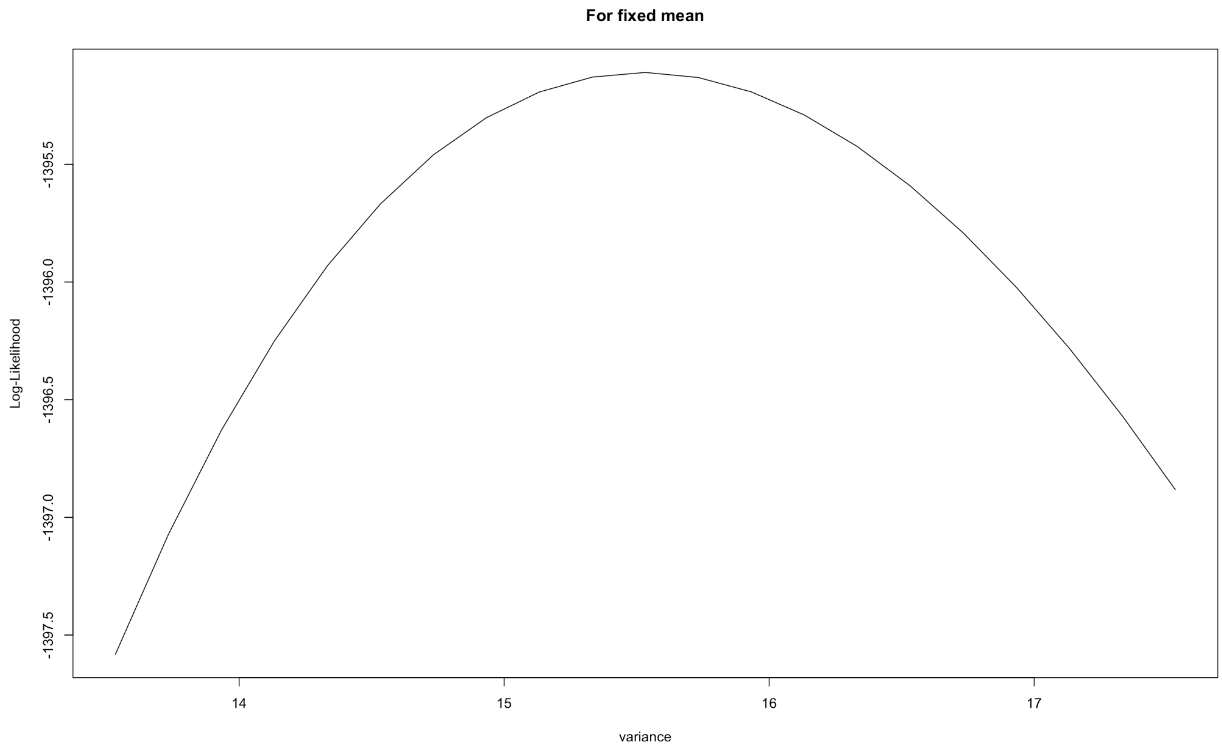
To check max

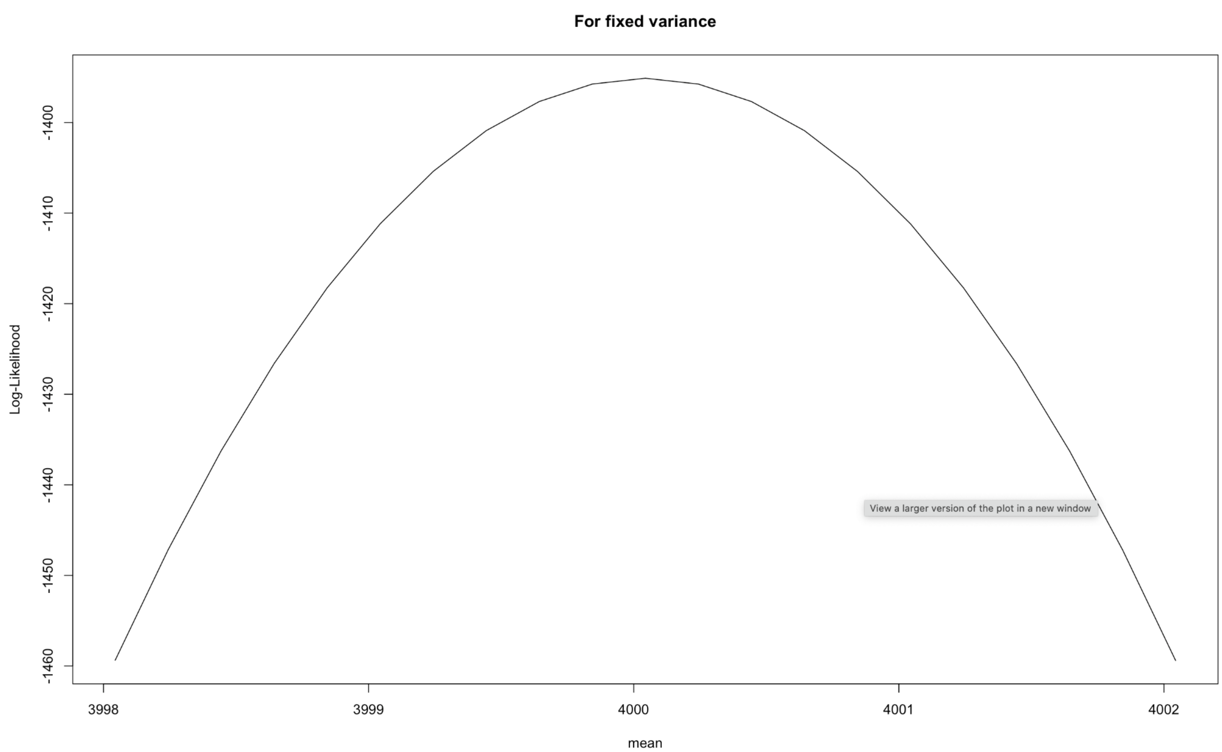
$$\frac{d^2 l}{d\mu^2} = \frac{-n}{\sigma^2}$$

$\therefore \frac{d^2 l}{d\mu^2} < 0$

Hence maxima

$$\boxed{\hat{\mu}_{MLE} = \frac{\Sigma x_i}{n}}$$





As we can see, we attain maximum at μ ≃ 4000.044 and sigma =15.53298 , consistent with the theoretical value.

C.
The find ML estimate of exp(-μ)
Substitute exp(-μ) = exp(-(1/n) summation xi)

Therefore ML estimate of exp(-μ) we just substitute mean with exp(mean)

Q.2) c) To find ML estimate of $e^{-\mu}$

∴ Likelihood function :

$$L(\theta) = \frac{1}{(\sqrt{2\pi\sigma^2})^n} \exp\left(-\frac{\sum (x_i-\mu)^2}{(2\sigma^2}\right)$$

∴ To maximize.

$$\frac{d\, \ln(L)}{d\mu} = \frac{-1}{(2\sigma^2)} \cdot \sum(x_i - \mu) = 0.$$

$$\therefore \mu = \frac{\sum x_i}{n}$$

& similarly $\sigma^2 = \frac{\sum}{n} (x_i - \mu)^2$

∴ To ML estimate of $\exp(-\mu)$ substitute μ into $\exp(-\mu)$

$$\exp(-\mu) = \exp(-(1/n) \times \sum x_i)$$

∴ ML estimate of

$$\exp\left(-\left(\frac{1}{n}\right) \times \sum x_i\right)$$