COMP-SCI 5552A Formal Software Specification

Solution for Home Work 1

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1. (20 points) Give a function that is not an injection and not a surjection.

An injective function or injection is a function that preserves distinctness: it never maps distinct elements of its domain to the same element of its codomain. In other words, every element of the function's codomain is the image of at most one element of its domain.

The function is surjective if every element of the codomain is mapped to by at least one element of the domain. (That is, the image and the codomain of the function are equal.) A surjective function is a surjection.

Example for a function which is neither injective nor surjective:

Let A be a set of all integers.

$$A = P = \{\dots\dots, -3, -2, -1, 0, 1, 2, 3, \dots\dots\}$$

Let f be a function on A such that

$$f(a) = |a| \; where \; a \in A$$

Let B be the codomain of f such that

$$B = Z = \{\dots\dots, -3, -2, -1, 0, 1, 2, 3, \dots\dots\dots\}$$

Consider the below scenario

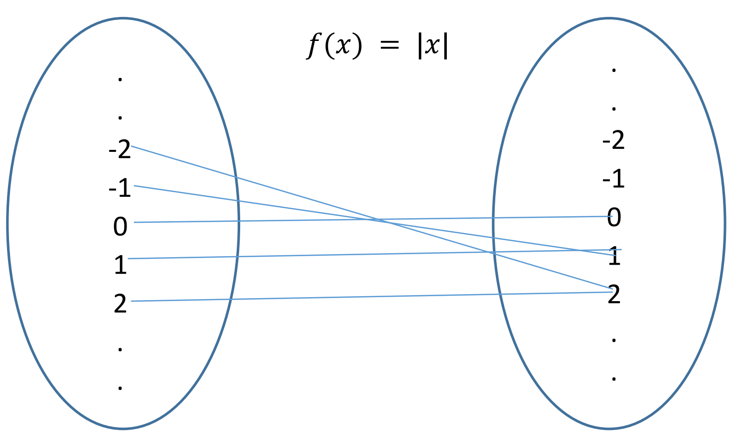
$$f(-3) = |-3| = 3$$

$$f(3) = |3| = 3$$

Here, both f(-1) and f(1) are mapped to same point in codomain B. Therefore the uniqueness mapping is not present.

In the similar way, all elements in B are not mapped as a result of A. The negative integers present in the codomain B are not mapped from the domain A. Therefore, we can consider the function f as non-surjective function.

Therefore, we can conclude that the function f(x) = |x| is neither an injection nor a surjection.



2. (20 points) How many different binary relations can be defined on finite sets A and B?

A binary relation on a set A is a collection of ordered pairs of elements of A. In other words, it is a subset of the Cartesian product $A^2 = A \times A$. More generally, a binary relation between two sets A and B is a subset of $A \times B$.

Let us consider that $|A|, |B|$ are the cardinalities of sets A and B respectively, then, the number of relations that can be defined on A and B is $2^{|A|*|B|}$.

Suppose

$$A = \{x, y\}$$

$$B = \{a, b\}$$

$$A\ X\ B = \{(x, a), (x, b), (y, a), (y, b)\}$$

$$|A| = 2, |B| = 2$$

Therefore, the binary relations that can be formed are:

1. $\emptyset$
2. $\{(a, 1)\}$
3. $\{(a, 2)\}$
4. $\{(b, 1)\}$
5. $\{(b, 2)\}$
6. $\{(a, 1), (a, 2)\}$
7. $\{(b, 1), (b, 2)\}$
8. $\{(a, 1), (b, 1)\}$
9. $\{(a, 1), (b, 2)\}$
10. $\{(a, 2), (b, 1)\}$
11. $\{(a, 2), (b, 2)\}$
12. $\{(a, 1), (a, 2), (b, 1)\}$
13. $\{(a, 1), (a, 2), (b, 2)\}$
14. $\{(a, 1), (b, 1), (b, 2)\}$
15. $\{(a, 2), (b, 1), (b, 2)\}$

16. $\{(a, 1), (a, 2), (b, 1), (b, 2)\}$

Therefore, the total number of relations are $2^4 = 16$

Since,

$$|A| = |B| = 2$$

$$|A| * |B| = 2 * 2 = 4$$

So, $2^4 = 16$.

Hence, we can declare that the number of binary relations on finite sets A & B is $2^{|A|*|B|}$.