a convenient form, and apply $\mathbb{E}[f(\pi')] \le s+1$, to get

$$\begin{aligned}
N(s) \ge \;& 1 + \sum_{n+1 \ge s' \ge s} (N(s') - n - 2 + s')\mathbb{P}\{f(\pi'\} = s'\} \\
& + \sum_{n+1 \ge s' \ge s} (n + 2 - s')\mathbb{P}\{f(\pi') = s'\} \\
=\;& 1 + \sum_{n+1 \ge s' \ge s} (N(s') - n - 2 + s')\mathbb{P}\{f(\pi'\} = s'\} \\
& + n + 2 - \sum_{n+1 \ge s' \ge s} s'\mathbb{P}\{f(\pi'\} = s'\} \\
=\;& \sum_{n+1 \ge s' \ge s} (N(s') - n - 2 + s')\mathbb{P}\{f(\pi') = s'\} \\
& + n + 3 - \mathbb{E}[f(\pi')] \\
\ge\;& \sum_{n+1 \ge s' \ge s} (N(s') - n - 2 + s')\mathbb{P}\{f(\pi') = s'\} \\
& + n + 2 - s.
\end{aligned}$$

By the induction hypothesis, $N(s') - n - 2 + s'$ is non-negative for $s' > s$. Therefore, after removing terms corresponding to $s' > s$, we get

$$N(s) \ge n + 2 - s + (N(s) - n - 2 + s)\mathbb{P}\{f(\pi') = s\},$$

which rearranges into

$$(N(s) - n - 2 + s)(1 - \mathbb{P}\{f(\pi') = s\}) \ge 0.$$

Now, $\mathbb{P}\{f(\pi') = s\}$ cannot be 1 because there is a policy $\pi'' = \mathsf{modify}(\pi, \{(s, a)\}) \in \Pi$, where $a \in \{0, 1\}$ and $a \ne \pi(s)$, such that $\mathbb{P}\{\pi' = \pi''\} > 0$ and $f(\pi'') > s$ (since s is not switchable in π''). Hence, we must have $N(s) \ge n + 2 - s$. $\square$

At this point, Theorem 9 follows as a corollary; the statement of the theorem is reproduced below.

Corollary 17. *Starting from $\pi^0 = 0^n$, the expected number of policies RPI evaluates on M_n before terminating is at least $\frac{n+1}{2}$.*

Proof. For $\pi^0 = 0^n$, $f(\pi^0) = 1$. Thus

$$L(\pi^0) \ge N(1) \ge n + 2 - 1 = n + 1.$$

In other words, RPI1 evaluates at least $n + 1$ policies in expectation, which implies RPI evaluates at least half that number of policies in expectation. $\square$