The "missing step" is as follows

$$V_a(t) = \iint E(l,m,t)\, dl\, dm$$

but we will express $E(l,m,t)$ as a function of its inverse Fourier transform

$$E(l,m,t) = \int E(l,m,\nu)\, e^{i2\pi\nu t}\, d\nu$$

$$\rightarrow V_a(t) = \iiint E(l,m,\nu)\, e^{i2\pi\nu t}\, dl\, dm\, d\nu$$

& we'll further write

$$E(l,m,\nu) = A(l,m,\nu)\, e^{i\phi(l,m,\nu)}$$

Now

$$V_a(t)\, V_b^*(t) = \iiiint\!\!\iint A(l,m,\nu) A^*(l',m',\nu')\, e^{i\phi(l,m,\nu)}\, e^{-i\phi(l',m',\nu')}$$

$$\cdot\, e^{i2\pi(x_a l + y_a m)}\, e^{-i2\pi(x_b l' + y_b m')}\, e^{i2\pi\nu t}\, e^{-i2\pi\nu' t}\, dl\, dl'\, dm\, dm'\, d\nu\, d\nu'$$

But if there is no spatial coherence the phase term $\phi(l,m,\nu)$ varies very rapidly (i.e., more than $A(l,m,\nu)$)

& we write

$$\int e^{i\phi(l,m,\nu)} e^{-i\phi(l',m',\nu')} dl'dm'd\nu' = \delta(l-l', m-m')\,\delta(\nu-\nu')$$

Then

$$V_a(t) V_b^*(\lambda) = \iiint |A(l,m,\nu)|^2 e^{i2\pi\nu(t-\lambda)} e^{i(\Delta x l + \Delta y m)} dl\,dm\,d\nu$$

If we set $\lambda = t - \tau$, with τ the time lag

$$V_a(t) V_b^*(t-\tau) = \iiint |A(l,m,\nu)|^2 e^{i2\pi\nu\tau} e^{i(\Delta x l + \Delta y m)} dl\,dm\,d\nu$$

Therefore

$$\langle V_a(t) V_b(t-\tau)\rangle = \lim_{T\to-\infty} \frac{1}{2T} \iiint\int_{-T}^{T} |A(l,m,\nu)|^2 e^{i2\pi\nu\tau} e^{i(\Delta x l + \Delta y m)} dl\,dm\,d\nu\,dt$$

$$= \iiint |A(l,m,\nu)|^2 e^{i2\pi\nu\tau} e^{i(\Delta x l + \Delta y m)} dl\,dm\,d\nu$$

$$= \iiint I(l,m,\nu)\, e^{i2\pi\nu\tau} e^{i(\Delta x l + \Delta y m)} dl\,dm\,d\nu$$

So, we can say that at a given frequency (which is how observations are made)

$$\langle V_a(t) V_b(t-\tau)\rangle_\nu = F\left[I(l,m,\nu) \right]$$

this subscript means "frequency of observations".