MAT3395 PROPOSAL
COOPERATION IN A SOCIETY OF INDIVIDUALS

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Objectives

The goal of the proposed project is to model a society of individiuals, be they human, animal, or otherwise, and to study the levels of cooperation in the society.

Background

That cooperation should exist in a society of beings that evolved through competition and survival of the fittest seems paradoxical. Cooperation often puts one's own survival in jeapordy, especially if the others don't cooperate with you. For example, if you share your limited food with another, who is either unwilling or unable to share their food with you. Yet cooperation exists in force, in the colonies of bees or ants, in shoals of fish, in business (through sponsorships and investment for example), in politics through alliances and treaties and so on.

The Prisoner's Dilemma is a game that is often used as a model for the emergence of cooperation.

In Prisoner's Dilemma, two robbers are caught and placed in separate interrogation rooms. The police have enough evidence to imprison both for 1 year, (say, they found a gun) but not enough evidence to convict for a maximum of 10 years. The police require a confession, so they tell each of the robbers that if they rat out the other (or *Defect*), they will get immunity. Now each robber can choose to either *Defect* to the police and rat out the other or *Cooperate* with their partner and not tell the police anything.

If both Defect, then the police didn't need the confession of one to convict the other, so neither get immunity. The judge would take into account that they confessed and gives both 8 years in jail.

If both Cooperate, then the police have no new evidence and can only convict for the evidence they have. Both get 1 year in jail.

If one Defects while the other Cooperates, then the defector gets immunity (0 years in jail!) while the other gets the maximum sentence of 10 years.

Imagine you're one of the robbers in the cell. You know that if the other Defects, you should Defect too, otherwise you'll get 10 years. You also know that if the other Cooperates then you should, again, defect, so that you get immunity. Regardless of what the other does you conclude that you should defect.

In this example, both defect and both get 8 years in jail each.

Now consider being put in this scenario multiple times. This is called the Iterated Prisoner's Dilemma. Eventually you notice that if you and your partner make the same decision you did the first time you will always end up with 8 years in jail. Collectively, this is 16 years in jail. However, if both choose to cooperate, then each get 1 year in jail. Even though you'd rather go free than be in jail, you'd rather be in jail for a year rather than 8. So, over time (over repeated plays) cooperating can be the better behaviour for oneself and for the common good.

To extend this idea to model a society, one can follow the steps of M. Nowak and R. May in "Evolutionary games and Spatial Chaos" (1992) (found here: http://projects.iq.harvard.edu/files/ped/files/nature92.pdf) where they created a two-dimensional grid and filled it with a population of individuals (or individual groups). In their paper each individual either always cooperates or always defects. Then, each turn, the individual plays Prisoner's Dilemma with all of it's neighbours, and gets a total score (or payoff). The next turn, each cell becomes occupied by the neighbour with the best payoff. Thus, they modelled a society as a two-dimensional grid of individuals and could observe the population of cooperators and defectors as the society evolved or "played".

Methods

The first step is to represent the possible outcomes using what's called a "payoff" matrix. That is, the 1 year in jail payoff for both robbers cooperating becomes the Reward $R = -1$, the 8 years for both robbers defecting is the Punishment $P = -8$, the immunity the defector gets is the temptation $T = 0$ and the robber who cooperates while the other defects gets the Sucker's Payoff $S = -10$.

If a robber cooperates, we denote it by C. If a robber defects, we denote it by D. So CC means both cooperate, CD, means one cooperated and the other defected, etc. The payoff matrix is this matrix

$$\begin{bmatrix} CC & CD \\ DC & DD \end{bmatrix} = \begin{bmatrix} R & S \\ T & P \end{bmatrix} = \begin{bmatrix} -1 & -10 \\ 0 & -8 \end{bmatrix}$$

In general you just need $T > R > P > S$.

In our model we will use Matlab or Javascript to create a two dimensional board where we can simulate the "society". Each cell will either be a cooperator or a defector. We will not study any of the many strategies (like tit-for-tat, etc.) that exist. Instead, a cell either always cooperates or always defects. The simulation will proceed as follows:

(1) Pick a cell. The cell plays the game with each of its 8 neighbours. The payoff the cell gets is determined by the payoff matrix. After playing with each neighbour one by one, the cell has a total payoff. Do this for each cell.
(2) Pick a cell. Out of itself and all of its neighbours, find the one that has the best payoff. If the one with the best payoff is a defector, the cell becomes a defector in the next round. If it is a cooperator, the cell becomes a cooperator in the next round.

This will give us an evolving grid of cooperators and defectors, and the populations will fluctuate.

To simplify our exploration we can set the cost matrix to

$$\begin{bmatrix} 1 & 0 \\ b & \epsilon \end{bmatrix}$$

Where $b > 1 = R$ and ϵ is an arbitrarily small number (so that $T > R > P > S$ is still true.). Then, b represents the advantage defectors have over cooperators. However, defectors get little to no payoff when playing with another defector. So defectors will always require some presence of a cooperator in the vicinity to survive. By playing with the value of b we explore how large a population of defectors can grow, and thus, how small a population of cooperators becomes. The analysis here will be similar to the analysis we did in class with the forest tent caterpillar.

Expected Outcome

Our methods will allow us to count the frequency of defectors and cooperators in the "society". For different values of b, we will see the dynamics over time of the two populations. Do they oscillate? Do they stabilise around a fixed value? Is there ever a case where the cooperators dominate and eliminate the defectors? Is there a b where the defectors eliminate the cooperators and thus worsen the overall situation for everyone?

In Nowak and May, they color coded their two-dimensional world and were able to find very interesting spatial structures. We should be able to reproduce these pretty pictures.

With our project we will also be able to observe what will happen if the initial population is predominantly defectors or predominantly cooperators. So for fixed values of b, we vary the starting proportions and analyse the resulting populations.

References

Nowak MA, RM May (1992). Evolutionary games and spatial chaos. Nature 359: 826-829. http://dx.doi.org/10.1038/359826a0. PDF

Nowak MA, RM May (1993). The spatial dilemmas of evolution. Int J Bifurcat Chaos 3: 35-78. http://dx.doi.org/10.1142/S0218127493000040. PDF

"Game Theoretical Models in Biology". Mark Broom. 2013. pg. 55-58