PROCEEDINGS OF SPIE

SPIEDigitalLibrary.org/conference-proceedings-of-spie

Adaptive Calibration Of Radio Interferometer Data

Frederic R. Schwab

SPIE.

Adaptive calibration of radio interferometer data

Frederic R. Schwab
National Radio Astronomy Observatory*, Edgemont Road, Charlottesville, Virginia 22901

Abstract

Radio interferometer observations are, essentially, samples of V, the Fourier transform of the radio brightness, I, on a patch of sky. The dominant errors usually are ascribable to individual elements of the array. Most damaging, often, are phase errors due to atmospheric refraction, but amplitude errors also may be serious. The sampling distribution improves as an observing run progresses since, as the earth rotates, any pair of elements samples V at different spatial frequencies lying along a curve. Adaptive calibration can be achieved as follows: Assumptions on the analytic properties of I lead to a measure of the consistency of the observations, from a given instant, with the total set of data; given an appropriate error model, an overdetermined system of equations in the unknown errors results; correcting the data accordingly, an improved estimate of I is derived; and the procedure may be repeated. This encompasses schemes proposed by Readhead and Wilkinson, and by Cotton, for compensation of phase errors in very-long-baseline interferometry (VLBI); similar methods are proposed by Muller and Buffington for optical telescopes. Results in analysis of Very Large Array (VLA) data are extremely encouraging. The computational expense is a few times that of standard methods.

Introduction

I would like to address the problem of adaptive compensation for systematic errors in radio interferometer data. The analysis here applies generally only to multi-element arrays in which the correlations among all antenna pairs are observed. We wish, given a set of assumptions on the analytic properties of the radio source brightness distribution, and given a mathematical or statistical model of the errors corrupting the data, to make an optimal estimate, in some well-defined sense, of the true source brightness distribution. According to a realistic error model, many measurements may have the same component of error; hence one might expect the problem of adaptive error compensation to be tractable, given a minimal set of assumptions.

There are certain subtleties involved, however, for even with errorless data the inverse problem in interferometry admits no unambiguous solution. Idealized, the problem is that of estimating the radio brightness distribution, I, from partial knowledge of the Fourier transform, V, of an (essentially known) tapering of I (the tapering is due to the primary antenna pattern of individual array elements and to non-monochromaticity, or finite bandwidth; the idealization is due, again, to finite bandwidth and to instrumental effects). Assumptions sufficient to guarantee a unique approximate solution to the inverse problem (that is, to select a particular solution from the many consistent with the data) are that I is smooth and of known, compact support, or that it is a linear combination of certain well-chosen functions, or that, considered as a statistical distribution, its most probable statistics are known. Standard estimation methods, in the presence of random errors, typically yield a least-squares solution consistent with the chosen assumptions. The curvature of the celestial sphere usually is ignored. For reasons of computational economy, only one method, based on the fast Fourier transform (FFT) and on the Högbom CLEAN algorithm,[1,2] is in widespread use in the mapping of complex radio source structure. The Very Large Array (VLA), a 27 element array under construction at NRAO, generates such quantities of data that often only an FFT-based digital processing technique is practical (Clark[3] has modified the Högbom algorithm to incorporate the FFT to advantage).

All standard estimation procedures perform unreliably in the presence of large systematic errors. At centimeter wavelengths the most damaging errors usually are long time-scale phase errors arising from large scale atmospheric irregularities. The calibration method in standard use is to observe a source of known structure, located near the source of interest, and to apply to the data those corrections which would yield approximately correct observations for the known source. Two tenuous assumptions are implicit: first, that over each element of the array the propagation effects in the direction to the calibrator are the same as those in the direction to the source, and, second, that the alternation of calibrator observations with source observations is appropriately matched to (i.e., shorter than) the time-scale of the atmospheric variations. An adaptive method

*The National Radio Astronomy Observatory is operated by Associated Universities, Inc. under contract with the National Science Foundation.

attempts to overcome these limitations by, in effect, using an unknown source as its own "calibrator". Figure 1 shows a map of a radio galaxy, M82, derived, by a standard estimation method, with standard calibration, from VLA observations at 6 cm. wavelength during adverse atmospheric conditions. One can see that systematic errors easily cause the data to lose their consistency with reasonable assumptions one might make about I. The dominant systematic errors present here are easily modelled; also, the error model is constraining in the type of correction it would permit.

A number of workers, concentrated in the VLBI community, have published work on so-called "phase-closure methods" dealing with phase error compensation: notably, Readhead and Wilkinson,[4,5] Cotton,[6] Fort and Yee,[7] Kosheleva,[8] and Kreinovič.[9] The problem is closely related to considerations involved in the design of control systems for real-time adaptive optics in new telescopes, including rubber-mirror telescopes, the Multiple Mirror Telescope, and aperture synthesis optical telescopes.[10-13] The method described below has yielded promising results in application to VLA data. It is very similar to the schemes of Readhead and Wilkinson and Cotton, though it is designed to compensate for amplitude errors as well as phase errors.

An error model

A fortunate circumstance, in the sense that some redundancy is introduced, is that the dominant components of error usually are ascribable to individual elements of the array. Further, each of these errors has a multiplicative action on the observed visibility (a phase error, additive in the observed phase, is multiplicative (by a complex exponential) in the observed complex visibility). Hence, to model the action on the visibility (V, above), we can lump the errors together and write

$$\tilde{V}_{ij}(t) \equiv \tilde{V}(u_{ij}(t),v_{ij}(t),w_{ij}(t)) = g_i(t)\bar{g}_j(t)V_{ij}(t) + N_{ij}(t) \qquad (1)$$

where i and j ($i<j$) index the array elements, $\tilde{V}_{ij}(t)$ is the visibility sample obtained on the i-j baseline at time t, V is the true visibility, g_k is the lumped error (a product of multiplicative errors) ascribable to element k, and N_{ij} is an additive error of unknown origin which is assumed to be small and well-behaved. The spatial frequency (u,v,w) has been parametrized by time.

Sources of phase error whose actions are modelled adequately in this manner include:

i) atmospheric and ionospheric refraction,
ii) inaccurate baseline determination,
and iii) local oscillator errors.

Adequately modelled amplitude errors might be due to:

iv) fluctuations in atmospheric attenuation,
v) receiver gain drifts,
vi) telescope shadowing,
or vii) receiver system temperature variations.

Propagation effects, i) and iv), due to atmospheric irregularities of very small spatial extent, smaller, say, than the antenna diameter, are not well-modelled, but neither do they seem to be very damaging in actual observations. Ionospheric refraction becomes significant at long observing wavelengths. iii) is a serious concern with nonconnected interferometers, as in VLBI. iv), at the VLA, becomes quite important at 2 cm. and 1.3 cm. wavelength. vi), telescope shadowing, can be serious in compact configuration arrays. Shadowing also introduces phase errors which are not poorly modelled by (1). v) and vii) are well compensated for at the VLA by electronic instrumentation, at least at 20 cm. and 6 cm. Errors which are not well modelled by (1) might be due to differing IF bandpass shapes, inaccurate telescope pointing, or errors in delay insertion.

A complete error model would include not only the action of the errors on the data, but the statistics of the error distributions as well. For example, phase errors due to atmospheric refraction could be characterized according to a knowledge of the spatial and temporal coherence of atmospheric fluctuations. An adaptive method could not realistically incorporate the statistics of each of the errors listed above, but, if one source of error were known to be strongly dominant, a model of its statistics might be used. Dyson[11] has proposed such a scheme for achieving diffraction-limited imaging with optical telescopes.

Possible assumptions

As a first step in considering a range of possible approaches to the problem, let us examine some assumptions that an adaptive method might incorporate:

Assumptions on the error model-

a) the action of the errors is $\tilde{V}_{ij} = g_i\bar{g}_jV_{ij}$

b) (phase errors only) the action is $\tilde{V}_{ij} = e^{i(\phi_i - \phi_j)} V_{ij}$
c) the errors are smoothly varying in time (and, for atmospheric errors, in space), say C^k (k continuous time derivatives)
d) the errors have a known distribution

Assumptions on the analytic properties of the brightness distribution I-
e) I is of compact support, and its approximate support, K, is known
f) I is smooth, perhaps C^∞ (real analytic, i.e., infinitely differentiable)
g) I is square-integrable
h) $I \geq 0$
i) I is supported on the sphere, S^2
j) I is approximately known, $I \approx I_o$
k) I can adequately be represented by a function given by a small number of parameters

Possible assumptions on I, if considered as a statistical distribution[10]-
l) (maximum entropy) from an appropriate function class, that the most probable I is selected by the criterion that $-\int_K I \log I$ be maximized
m) (minimum scatter of the intensity) similarly for $\int_K I^n$, $n \geq 2$
n) (smallest moment of inertia) similarly for $-\iint_K (x^2+y^2) I(x,y)\,dxdy$, assuming a shift of the centroid to (0,0)
o) (sharpness) similarly for $\iint_K \left| \frac{\partial^{m+n} I(x,y)}{\partial x^n \partial y^m} \right|^2 dxdy$

Assumption dependent on the geometry of the array-
p) there exist redundant spacings

Assumptions a) and b) are so often valid and would so serve to reduce the number of degrees of freedom, that usually one of them would be assumed. However, Okatan and Basart[14], given only c) and d), report apparent success with a method in which, by Kalman filtering applied before correlation, they treat errors due to ionospheric refraction in 26 MHz VLBI data. Given some of d), and making explicit use of the redundancy inherent in a) and b), an approach, in the spirit of Dyson's proposal, would be to formulate the problem in terms of multi-channel error prediction filtering, conditionally predicting, say, the behavior of the atmosphere based on a knowledge of its behavior in the recent past. Such a scheme could operate in real-time.

Assumption c) would be useful in extending the applicability of a method to cases of low signal-to-noise ratio (S/N), since it could serve to control the (fair number of) degrees of freedom. In my initial experimentation, in common with Readhead and Wilkinson and Cotton, I have made only implicit use of c); we assume that errors are constant during a sufficiently long time interval, the length of the interval depending on S/N. The main limitation of our methods is a requirement for relatively high S/N. Explicit use of c), achieved, perhaps, by computing simultaneous spline curves, might be better warranted than the current approach.

That I is at least rapidly decreasing is guaranteed by the tapering effects mentioned in the Introduction. Assumption e), that the support of I is compact and approximately known, is a strong assumption that is essential to most estimation methods. e) is used, for example, in the standard estimation procedure whenever a field of view for mapping is chosen intelligently.

Assumption f), that I is smooth, might be justified on physical grounds concerning the emission mechanisms. f) is incorporated in the Högbom deconvolution algorithm, where post-smoothing is used to obtain a realistic appearing, or appealing, solution. Though smoothing causes an approximate solution to lose its agreement with the data, it is often incorporated in order to render a method insensitive to small perturbations in the data. Some methods yield solutions in which the degree of smoothness is adaptive to well-behaved random errors - bright features might be "super-resolved" and dim ones not. This can be accomplished by choosing candidate solutions from a class of very smooth functions, using the estimated error in the data as a selection criterion. As the intent of an adaptive estimation method is to remove the bulk of the non-random errors, an adaptive smoothing mechanism might be desirable. As an aside, assuming that I is of compact support implies that V can be extended to a complex analytic function; this has been a basis of attempts to reconstruct I from $|V|$ alone.

Assumption g), that I is square-integrable, is important to the analytic derivation of an estimation method. g) enables one, for example, to express I as a linear combination of orthogonal basis functions or to express the solution as a Picard series. Choice of a basis can depend on K. Assumption h), that I is nonnegative, is a natural constraint in estimating the total intensity brightness distribution. We are not concerned with polarization mapping, since the systematic errors affecting polarization measurements split into two components - one of which is known once the errors in total intensity observations (usually of much higher S/N) are known, and the other of which (instrumental polarization) must be found by other means. To date, in most estimation methods, assumption h) has been incorporated, if at all, only in an ad hoc manner. However, it is a natural condition in the maximum entropy technique (MEM) outlined by Wernecke and D'Addario.[15] Assumption i), really a part of e), that I is concentrated on the unit sphere, is, again, a natural assumption. More owing to practical considerations (limited computing power) than to theoretical ones, i) is not incorporated by most standard methods. j) would be useful, but is not usually a valid assumption. Assumption k) is frequently made when dealing with sources of simple structure (model-fitting). Though k) is easily used in an adaptive method, its use is limited, since many sources are complex. k) is used in the initial stage of adaptive methods which have been applied to VLBI data.

The criteria for image sharpness of assumptions l-o) were introduced by Muller and Buffington.[10] They and Hamaker et al.[16] offer sufficiency proofs of m) (for n=2) and o) as suitable bases for certain phase error compensation methods. l) is the basis of the MEM, but I know of no standard estimation methods that assume any of m-o). n) would likely be inappropriate for sources consisting of a number of separated components, or[10] to estimate an I with a dim tail. A generalization of the criterion of o), known as a Sobolev norm, is used for similar purposes in practical numerical methods for the solution of BVP's; i.e., to choose the sharpest solution from a class of smooth candidates. o) is likely too cumbersome, however, for use in image reconstruction.

Assumption p), that there exist redundant spacings, generally is invalid because most radio astronomy arrays are designed for no redundancy; some nearly redundant observations, however, do occur, but randomly, at separated times.

Outline of concrete methods

Four schemes for adaptive estimation are outlined below. Some include, as a component, a standard estimation method (SM). SM's which incorporate some of e-o) and perform reliably in the presence of well-behaved random errors are not highly developed. Rigorous and nearly optimally efficient algorithms incorporating the natural assumptions h) and i), or incorporating l), are not known (Nor are SM's which acknowledge that the interferometer point source response, due to instrumental effects, is space-variant).

Method I combines some of a-o) directly. Wernecke and D'Addario's MEM, for example, might be modified to incorporate a) or b). Because of the difficulties just mentioned, an algorithm in which the uses of a-d) and e-o) are somehow logically separated might be preferred.

Method II incorporates a multi-channel filtering algorithm, as alluded to above, and attempts to achieve the separation by making full use of d); i.e., by making such a strong assumption on the statistical nature of the errors, that the errors may be determined under no further assumptions than that a source is being detected. Such an approach might be highly attractive if sufficiency of a valid error model were demonstrated.

Method III consists in optimizing "snapshots", or estimates of I obtained by simply computing the Fourier transform of the product of V and the error-corrupted sampling distribution corresponding to a brief observation. Given K, the maximization of a sharpness function given by one of l-o) yields estimates of the ψ_k. (Amplitude errors might be treated by constraining $\int |V|^2$ = constant.) Once each snapshot is optimized and its centroid computed, each segment of data is corrected accordingly, including for alignment of the centroids, and an SM is applied. Snapshots and snapshot partial derivatives are expensive to compute digitally, but the method might be suited to an optical computer.

Method IV is the most easily implemented. Like III, it incorporates a standard method, but uses the SM iteratively. It does not incorporate l-o) unless the SM does so. Beginning with data calibrated in the standard manner, SM is applied to obtain an estimate $I^{(0)}$ of I. Then using, say, a least-squares algorithm adapted to a) or b) and c) it corrects the data by the $g_k(t)$ or $\psi_k(t)$ which yield a best fit to $V^{(0)}$, the Fourier transform of $I^{(0)}$. The SM is then applied to generate a new estimate, $I^{(1)}$, and subsequent iterates, $I^{(n)}$, follow in the same manner.

Implementation of method IV

Readhead and Wilkinson (R&W), Cotton, and I have each implemented methods essentially of type IV. R&W and I use SM's which consist of interpolating the visibility observations, resampling them on a uniform grid, applying the FFT, and following this with the Högbom algorithm to deconvolve the point-spread function. (Cotton's SM uses a direct summation in lieu of the FFT, and omits the interpolation and resampling.) $V^{(n)}$ is given at each iteration as the Fourier transform of a linear combination of delta functions determined by the Högbom algorithm. In VLBI, the phase errors are so serious that the "standard" estimation method is not the SM just outlined, but usually is, instead, a model-fitting procedure. Hence, R&W and Cotton's adaptive methods usually start out with an $I^{(0)}$ derived by model-fitting; but the SM is used to generate subsequent iterates. Our error model is equally valid for VLBI and VLA data; the only substantial difference is that in VLBI the errors are of larger magnitude (and the inherent ambiguity arising from incomplete sampling usually is more serious).

R&W and Cotton treat only phase errors. These are always a more serious concern than amplitude errors, but the latter are, nevertheless, generally well-modeled by a). a) is easily incorporated: one can, say, constrain $\int |V|^2 dS$ = constant, where S is the sampling distribution. Phase error compensation alone often is justified in VLBI since amplitude error compensation, in addition, roughly doubles the number of degrees of freedom. Note that only relative position and intensity calibration is improved by the method; the absolute scales are set by the standard calibration method.

In the inner step of their algorithm, R&W do not use a least-squares method to solve for the $\psi_k^{(n)}$. Instead, given N antennas, they force exact agreement of phases on N-1 baselines with $\arg(\tilde{V}^{(n)})$. Then N-1 of the $\tilde{V}$ determine the $\psi_k^{(n)}$, according to b), up to an additive constant, α, and the remaining data are corrected accordingly (α is irrelevant). This procedure is likely not optimal, but may be satisfactory when observations on the N-1 selected baselines are those of highest S/N.

One can take logarithms in a) or b) and use linear regression to solve for the g_k or ψ_k. A straightforward such approach (not Cotton's linear method, which I have not analyzed) is failure-prone because, since Log is multi-valued, it is susceptible to ambiguities of 2π in the phase observations. E.g., fitting $\arg(\tilde{V}_{ij})-\psi_i+\psi_j$ to $\arg(V_{ij}^{(n)})$ often fails.

I have used an iterative nonlinear least-squares method to avoid such a problem. It has proven to be reliable and computationally inexpensive. To solve for errors g_k governed by a), I minimize

$$S_1 = \sum_{i<j} w_{ij} |\tilde{V}_{ij} - g_i \bar{g}_j V_{ij}^{(n)}|^2 , \tag{2}$$

subject to $\arg(g_r) = 0$ for some r. w_{ij} is a weight which might be chosen as the reciprocal of an estimate of $\mathrm{var}(|\tilde{V}_{ij}|)$. Assuming the g_k to vary slowly, the summation may extend over time (brief intervals of time) as well as over baseline. $\nabla S_1 = 0$ is rewritten as

$$\begin{pmatrix} g_1 \\ \cdot \\ \cdot \\ \cdot \\ g_N \end{pmatrix} = \begin{pmatrix} h_1(g_2,\ldots,g_N) \\ \cdot \\ \cdot \\ \cdot \\ h_N(g_1,\ldots,g_{N-1}) \end{pmatrix} . \tag{3}$$

Then a successive substitution method (the Jacobi method) is used to solve for a zero of the gradient of S_1. S_1 has no local maxima.

To solve for errors governed by b), I minimize

$$S_2 = \sum_{i<j} w_{ij} |\tilde{V}_{ij} - e^{i(\psi_i-\psi_j)} V_{ij}^{(n)}|^2 , \tag{4}$$

subject to $\psi_r = 0$ for some r. Again, a successive substitution method is used,[17] making use of the fact that S_2 is periodic of period 2π in each ψ_k. At each step, a correction $-\omega \tan^{-1}\left(\frac{\partial S_2}{\partial \psi_k} \Big/ \frac{\partial^2 S_2}{\partial \psi_k^2}\right)$ is added to ψ_k. $\omega \approx .8$ is a damping factor.

The rms difference between successive iterates, i.e., the Euclidean norm $\| I^{(n)} - I^{(n-1)} \|_2$, should be monitored as a test for convergence. Alternatively, the algorithm could terminate when the corrections to the data become acceptably small; e.g., when, for all t, $\| g_k^{(n)} - 1 \|_2 < \epsilon$ or $\| \psi_k^{(n)} \|_2 < \epsilon$.

In my implementation, at present, the recalibration step is run on a minicomputer with an array processor attachment, and the SM step of each iteration is run on a general purpose computer. This implementation is cumbersome; hence, though the programs have been run many times, systematic experimental studies have not yet been pursued. R&W and Cotton, in their papers, do report on tests to determine the sensitivity of their methods to perturbations in $I^{(0)}$.

Figures 1-4 illustrate an application of the method. Figure 1 is the result of the standard calibration, followed by the SM. Figures 2 and 3 show, respectively, the first and second iterates, $I^{(1)}$ and $I^{(2)}$, where only phase error compensation has been applied. A third iterate (photographed on a different scale) is shown in Figure 4 - here amplitude error compensation has been applied. The M82 data were obtained using 10 antennas of the VLA. The smallest features in the reconstructions have an angular size of roughly 1.2 seconds of arc.

Further investigations

Preliminary results obtained with the method described above suggest that it will be useful in many applications and that further study is merited. To facilitate the latter, and to make the method more readily available to observers, the present implementation will need to be streamlined, for ease of use, and automated, to remove obvious sources of human bias. The major problems in automation relate to the Högbom algorithm, a part of the SM. These problems are twofold: first, that no optimal stopping criterion is known for the Högbom algorithm, and, second, that it is often too expensive in computer time to run the algorithm to "convergence" (however defined). Clark's modification should alleviate the latter problem, and compromise solutions to the former are obvious (In common with similar methods, the Högbom method should not be carried "too far"; see Strand[18] for a good discussion).

After automating the procedure, but before modifying it drastically, a systematic study might be warranted. This could consist of case studies and of tests on artificial data. I have carried out only a few such tests, and the results are in accord with those of the more extensive testing reported by R&W and Cotton. Case studies would be useful in determining the limits of the procedure ($S/N > 2$ seems to be necessary) and in gaining a qualitative idea of the potential improvement to a radio map under various observing conditions.

Better use of assumption c) would, at least marginally, extend the applicability of the present method to cases of lower S/N. d), when valid, would likely be more useful than c), but it cannot readily be incorporated. Large fields with sources of complex structure are difficult to treat because of the inefficiency of the Högbom algorithm. Clark's modification should alleviate this problem. His improvement is possible because new hardware devices have made large multi-dimensional FFT's, and certain other computations, inexpensive. Other iterative algorithms which might be adapted to the problem could have FFT-based implementations, in particular, those due to Strand[18] and Graves and Prenter.[19]

Acknowledgements

I would like to thank Frazer Owen for suggesting this topic to me, as well as Ed Fomalont and Bill Cotton and others of the NRAO staff and user community for helpful discussions and encouragement.

References

1. Högbom, J. A., "Aperture Synthesis with a Non-regular Distribution of Interferometer Baselines", Astron. Astrophys. Suppl., Vol. 15, pp. 417-426. 1974.
2. Schwarz, U. J., "Mathematical-statistical Description of the Iterative Beam Removing Technique (Method CLEAN)", Astron. Astrophys., Vol. 65, pp. 345-356. 1978.
3. Clark, B. G., "An Implementation of CLEAN", VLA Computer Memo. No. 152, NRAO, Socorro, N. M. 1979.
4. Readhead, A. C. S. and Wilkinson, P. N., "The Mapping of Compact Radio Sources from VLBI Data", Astrophys. J., Vol. 223, pp. 25-36. 1978.
5. Wilkinson, P. N. and Readhead, A. C. S., "Map-making with Non-phase Stable Interferometers", in C. Schooneveld, ed., Image Formation from Coherence Functions in Astronomy, D. Reidel, Holland, pp. 83-91. 1979.
6. Cotton, W. D., "A Method of Mapping Compact Structure in Radio Sources Using VLBI Observations", Astron. J., Vol. 84, pp. 1122-1128. 1979.
7. Fort, D. N. and Yee, H. K. C., "A Method of Obtaining Brightness Distributions from Long Baseline Interferometry", Astron. Astrophys., Vol. 50, pp. 19-22. 1976.
8. Kosheleva, O. M., "Sufficiency Proof and Error Estimate for the Closure Phase Method", preliminary report appearing as abstract No. 79T-C45, Notices Amer. Math. Soc., Vol. 26, p. A-386. 1979.

9. Kreinovič, V. Ja., "Investigation of Convergence for Fort-Yee Method", preliminary report appearing as abstract No. 79T-C48, Notices Amer. Math. Soc., Vol. 26, p. A-386. 1979.

10. Muller, R. A. and Buffington, A., "Real-time Correction of Atmospherically Degraded Telescope Images through Image Sharpening", J. Opt. Soc. Am., Vol. 64, pp. 1200-1210. 1974.

11. Dyson, F. J., "Photon Noise and Atmospheric Noise in Active Optical Systems", J. Opt. Soc. Am., Vol. 65, pp. 551-558. 1975.

12. Buffington, A., Crawford, F. S., Pollaine, S. M., Orth, C. D., and Muller, R. A., "Sharpening Stellar Images", Science, Vol. 200, pp. 489-494. 1978.

13. Skalafuris, A. J., "A Survey of Image Compensation and Restoration Principles", Astrophys. Space Sci., Vol. 61, pp. 317-335. 1979.

14. Okatan, A. and Basart, J. P., "Reduction of Phase Noise in Interferometry with State-space Analysis", in C. Schooneveld, ed., op. cit., pp. 103-110.

15. Wernecke, S. J. and D'Addario, L. R., "Maximum Entropy Image Reconstruction", IEEE Trans. Comp., Vol. C-26, pp. 351-364. 1977.

16. Hamaker, J. P., O'Sullivan, J. D., and Noordam, J. E., "Image Sharpness, Fourier Optics, and Redundant-spacing Interferometry", J. Opt. Soc. Am., Vol. 67, pp. 1122-1123. 1977.

17. Brown, T. M., "Reconstruction of Turbulence-degraded Images Using Nonredundant Aperture Arrays", J. Opt. Soc. Am., Vol. 68, pp. 883-889. 1978.

18. Strand, O. N., "Theory and Methods Related to the Singular-function Expansion and Landweber's Iteration for Integral Equations of the First Kind", SIAM J. Num. Anal., Vol. 11, pp. 798-825. 1974.

19. Graves, J. and Prenter, P. M., "Numerical Iterative Filters Applied to First Kind Fredholm Integral Equations", Numer. Math., Vol. 30, pp. 281-299. 1978.

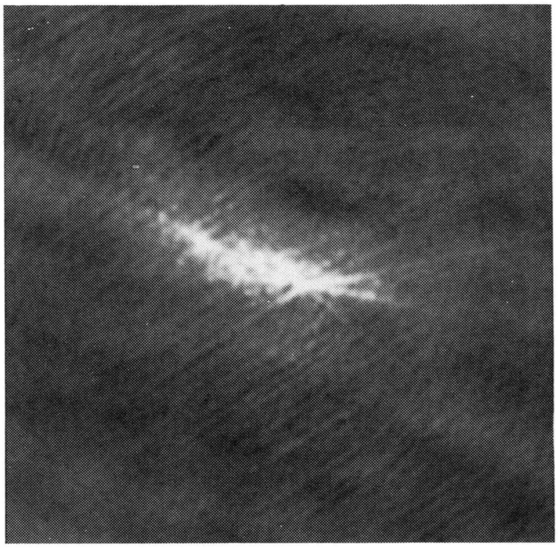

Figure 1. M82. A standard estimate derived from the standard calibration.

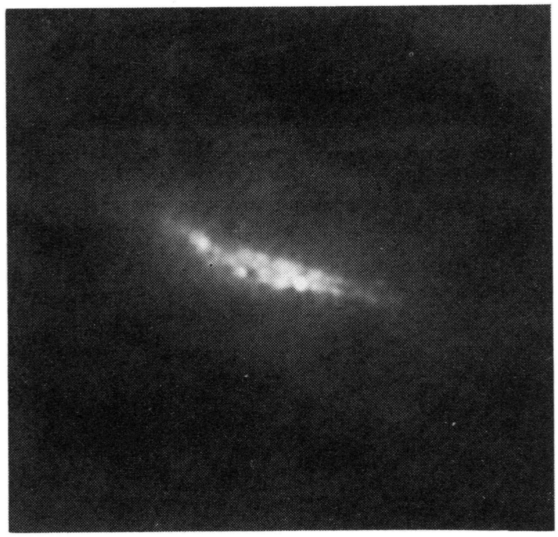

Figure 2. M82, via the adaptive method (phase error compensation), first iteration.

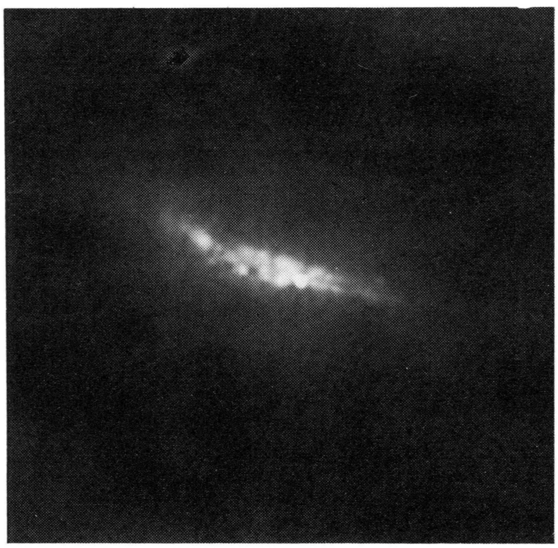

Figure 3. M82, second iteration.

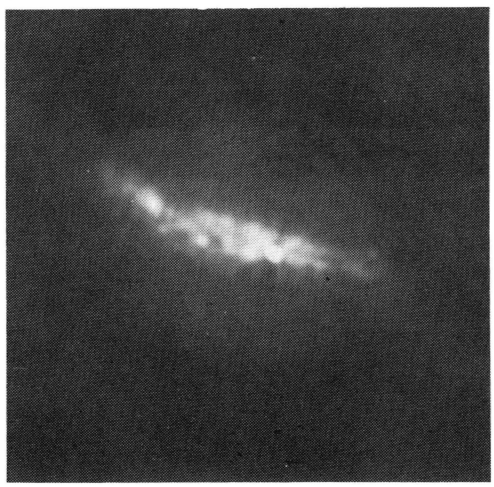

Figure 4. M82, third iteration, with both amplitude and phase error compensation.