PHY4311 ASSIGNMENT 6

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Assume $\frac{N_2}{N_T} \sim 1$. Assume a cavity with $r_1 = 1$ ($t_1 = 0$) and $r_2 = 0.99$ ($t_2 = 0.01$) with no losses. Assume finally that the cavity is $L = 50\text{cm}$ long and the gain medium filles the cavity length ($\ell = L$) having a small signalr gain of $g_0 = 0.01\text{cm}^{-1}$ and a saturation intensity of 10^4W/cm^2.

(a) Please calculate the output intensity in W/cm^2 of the laser.

$$\begin{aligned}
I^{out} &= tI^+ = \frac{t_2}{2} I^{sat}\Big(\frac{g_0}{g_t} - 1\Big) \\
&= \frac{t_2}{2} I^{sat}\Big(\frac{2\ell g_0}{1 - r_2} - 1\Big) \\
&= \frac{0.01}{2}(10^4)\Big(\frac{2(50)(0.01)}{1 - 0.99} - 1\Big) \\
&= 4950\text{W/cm}^2 \equiv P
\end{aligned}$$

(b) Please calculate the photon cavity lifetime and the cavity linewidth.

The cavity linewidth is

$$\begin{aligned}
\delta\nu_c &= \frac{1}{2\pi}\frac{c\ell g_t}{2L} \\
&= \frac{1}{2\pi}\frac{cg_t}{2} \\
&= \frac{1}{2\pi}\frac{c(1 - r_2)}{4\ell} \qquad \Big(g_t = \frac{1}{2\ell}(1 - r_2)\Big) \\
&= \frac{1}{2\pi}\frac{3 \times 10^8(1 - 0.99)}{4(0.5)} = 2.39 \times 10^5\text{Hz}
\end{aligned}$$

The photon cavity lifetime is

$$\begin{aligned}
\tau_c &= \frac{L}{\ell c g_t} \qquad (\ell = L) \\
&= \frac{1}{cg_t} = \frac{2\ell}{c(1 - r_2)} \\
&= \frac{2(0.5)}{3 \times 10^8(1 - 0.99)} = 330\text{ns}
\end{aligned}$$

(c) Assuming $\lambda = 500\text{nm}$, please calculate the Townes-Schawlow Limit $\Delta\nu$ of the laser.

$$
\begin{aligned}
\Delta\nu &= \frac{h\nu N_u}{\Delta N}\frac{4\pi(\delta\nu_c)^2}{P} \\
&= h\nu\frac{4\pi\delta\nu_c}{P} \qquad \left(\frac{N_u}{\Delta N} = \frac{N_u}{N_u - N_l} = 1 \,\text{since}\, N_l = 0\right) \\
&= h\frac{4\pi c\delta\nu_c}{P\lambda} \\
&= 6.63\times10^{-34}\frac{4\pi(3\times10^8)(2.39\times10^5)^2}{4950\times10^4\text{W/m}^2(500\times10^{-9})} \\
&= 5.77\times10^{-15}\text{Hz}
\end{aligned}
$$

(d) How far δL would you have to shift one mirror to move the cavity mode frequency by the Townes-Schawlow linewidth?

The change in length δL is the difference between the length L of the cavity that gives $\lambda = 500$nm with frequency ν and the length L' of the cavity that gives frequency $\nu' = \nu + \Delta\nu$.

$$\delta L = |L' - L|$$

We know that

$$
\begin{aligned}
\lambda &= \frac{2L}{n} \\
n &= \frac{2L}{\lambda} \\
&= \frac{2(0.5)}{500\times10^{-9}} \\
&= 2\times10^6
\end{aligned}
$$

So now

$$
\begin{aligned}
\lambda &= \frac{c}{\nu} = \frac{2L}{n} \\
\nu &= \frac{cn}{2L}
\end{aligned}
$$

And the new length can be found

$$
\begin{aligned}
\nu' &= \frac{cn}{2L'} \\
L' &= \frac{cn}{2\nu'} \\
&= \frac{cn}{2(\nu+\Delta\nu)} \\
&= \frac{3\times10^8(2\times10^6)}{2(\frac{3\times10^8}{500\times10^{-9}} + 5.77\times10^{-15})} \\
&= 0.5
\end{aligned}
$$

The difference is so small my calculator can't handle it

$$\delta L = 0$$

(e) How does δL compare with an atom

It's a loooot smaller than an atom