PHY3320 ASSIGNMENT 3

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Problem 1. Show that the Brewster's angle does not exist for S-polarized light for any n_1 and n_2. Sketch the ratios of reflected and transmitted fields with incident field as a function of the incident angle.

S-polarized light means the E field is perpendicular to the plane of incidence (the xz-plane) so it is pointing in the y direction. The B field lies in the xz-plane. We want the Fresnel equations for S-polarized light. The boundary conditions are

$$\begin{aligned}
\text{(i)} \qquad (E_{0I} + E_{0R})_y &= (E_{0T})_y \\
\text{(ii)} \qquad (B_{0I} + B_{0R})_z &= (B_{0T})_z \\
\text{(iii)} \qquad \frac{1}{\mu_1}(B_{0I} + B_{0R})_x &= \frac{1}{\mu_2}(B_{0T})_x
\end{aligned}$$

From (ii), since $B_0 = E_0/v$,

$$\begin{aligned}
B_{0I}\sin\theta_I + B_{0R}\sin\theta_R &= B_{0T}\sin\theta_T \\
\sin\theta_I(B_{0I} + B_{0R}) &= B_{0T}\sin\theta_T \qquad (\theta_R = \theta_I) \\
B_{0I} + B_{0R} &= B_{0T}\frac{\sin\theta_T}{\sin\theta_I} \\
\frac{E_{0I}}{v_1} + \frac{E_{0R}}{v_1} &= \frac{E_{0T}}{v_2}\frac{v_2}{v_1} \qquad \text{(Snell's Law)} \\
E_{0I} + E_{0R} &= E_{0T}
\end{aligned}$$

From (iii)

$$\begin{aligned}
\frac{1}{\mu_1}\left(-\frac{E_{0I}}{v_1}\cos\theta_I + \frac{E_{0R}}{v_1}\cos\theta_R\right) &= -\frac{1}{\mu_2 v_2}E_{0T}\cos\theta_T \\
-\frac{\cos\theta_I}{\mu_1 v_1}(E_{0I} - E_{0R}) &= -\frac{\cos\theta_T}{\mu_2 v_2}E_{0T} \\
E_{0I} - E_{0R} &= \frac{\mu_1 v_1}{\mu_2 v_2}\frac{\cos\theta_T}{\cos\theta_I}E_{0T} \\
&= \frac{\mu_1 n_2}{\mu_2 n_1}\frac{\cos\theta_T}{\cos\theta_I}E_{0T} \\
&= \beta\alpha E_{0T}
\end{aligned}$$

So now, substitute $E_{0R} = E_{0T} - E_{0I}$

$$\begin{aligned}
E_{0I} - E_{0T} + E_{0I} &= \alpha\beta E_{0T} \\
E_{0T} &= \frac{2}{1+\alpha\beta}E_{0I}
\end{aligned}$$

This means

$$\begin{aligned} E_{0R} &= (\frac{2}{1+\alpha\beta} - 1)E_{0I} \\ &= \frac{1-\alpha\beta}{1+\alpha\beta}E_{0I} \end{aligned}$$

These are the Fresnel equations:

$$E_{0T} = \frac{2}{1+\alpha\beta}E_{0I} \qquad E_{0R} = \frac{1-\alpha\beta}{1+\alpha\beta}E_{0I}$$

Brewster's angle is the angle of incidence where $E_{0R} = 0$. So we want $1 - \alpha\beta = 0$ or $\alpha\beta = 1$.

$$\begin{aligned} 1 &= \alpha\beta \\ &= \beta\frac{\cos\theta_T}{\cos\theta_I} \\ &= \beta\frac{\sqrt{1-\sin^2\theta_T}}{\cos\theta_I} \\ &= \beta\frac{\sqrt{1-(\frac{n_1}{n_2})^2\sin^2\theta_I}}{\cos\theta_I} \end{aligned}$$

Now if $\mu_1 \approx \mu_2$ then $\frac{n_1}{n_2} \approx \frac{1}{\beta}$, so this condition becomes:

$$\begin{aligned} 1 &= \frac{\sqrt{\beta^2 - \sin^2\theta_I}}{\cos\theta_I} \\ \beta^2 &= \cos^2\theta_I + \sin^2\theta_I = 1 \\ \beta &= 1 \end{aligned}$$

But this will only happen if $n_1 = n_2$.. which means there is no interface. So there is no Brewster's angle for S-polarized light reflecting off an interface.

We want to plot $\frac{E_{0T}}{E_{0I}} = \frac{2}{1+\alpha\beta}$ and $\frac{E_{0R}}{E_{0I}} = \frac{1-\alpha\beta}{1+\alpha\beta}$ as functions of θ_I.

Problem 2. Calculate the reflection coefficient for light at an air-to-silver interface ($\mu_1 = \mu_2 = \mu_0$, $\epsilon = \epsilon_0$, $\sigma = 6\times 10^7(\Omega\text{m})^{-1}$ at $\nu = 4\times 10^{15}$Hz.

$$\begin{aligned} R &= \left(\frac{E_{0R}}{E_{0I}}\right)^2 \\ &= \left|\frac{1-\tilde{\beta}}{1+\tilde{\beta}}\right|^2 \end{aligned}$$

$$\begin{aligned}\tilde{\beta} &= \frac{\mu_1 v_1}{\mu_2 \omega}\tilde{k}_2 \\ &= \frac{c}{\omega}\tilde{k}_2\end{aligned}$$

$$\tilde{k} = k + i\kappa$$

Since silver is a good conductor the real parts and imaginary parts are basically the same.

$$\tilde{k} = k + ik$$

$$k = \sqrt{\frac{\sigma\mu_0\omega}{2}}$$

$$\begin{aligned}\tilde{\beta} &= \frac{c}{\omega}(k + ik) \\ &= \frac{c}{\omega}\sqrt{\frac{\sigma\mu_0\omega}{2}} + i\frac{c}{\omega}\sqrt{\frac{\sigma\mu_0\omega}{2}}\end{aligned}$$

$$\begin{aligned}\frac{c}{\omega}\sqrt{\frac{\sigma\mu_0\omega}{2}} &= c\sqrt{\frac{\sigma\mu_0}{2\omega}} \\ &= 3\times 10^8\sqrt{\frac{6\times 10^7(1.26\times 10^{-6})}{2(2\pi)(4\times 10^{15})}} \\ &\approx 11.63\end{aligned}$$

$$\begin{aligned}R &= \left|\frac{1 - 11.63 - 11.63i}{1 + 11.63 + 11.63i}\right|^2 \\ &= \frac{10.63^2 + 11.63^2}{12.63^2 + 11.63^2} \\ &= 84\%\end{aligned}$$

Problem 3. A 5W beam of light with circular cross section is incident in air upon the plane boundary of a dielectric medium with index of refraction of 5. If the angle of incidence is 60 degrees and the incident wave is parallel polarized, determine the powers contained in the reflected and transmitted beams.

$$\begin{aligned}\alpha &= \frac{\sqrt{1 - (\frac{n_1}{n_2})^2\sin^2\theta_I}}{\cos\theta_I} \\ &= \frac{\sqrt{1 - (\frac{1}{5})^2\sin^2 60}}{\cos 60} \\ &= 0.97/\cos 60 \\ &= 1.94\end{aligned}$$

$$\begin{aligned}\beta &= \frac{\mu_1 n_2}{\mu_2 n_1} \\ &\approx \frac{n_2}{n_1} \\ &= 5\end{aligned}$$

The Reflection and Transmission coefficients are

$$\begin{aligned} R &= \left(\frac{\alpha - \beta}{\alpha + \beta}\right)^2 \\ &= \left(\frac{1.94 - 5}{1.94 + 5}\right)^2 \\ &= 0.194 \end{aligned}$$

$$\begin{aligned} T &= 1 - R \\ &= 0.806 \end{aligned}$$

These are related to the intensities which are related to the powers.

$$\begin{aligned} R + T &= 1 \\ \frac{I_R}{I_I} + \frac{I_T}{I_I} &= 1 \\ RI_I + TI_I &= I_I \end{aligned}$$

The reflected beam has

$$I_R = RI_I = 0.194(5\mathrm{W/m^2}) = 0.97\mathrm{W/m^2}$$

The transmitted beam has

$$I_T = TI_I = 0.806(5) = 4.03\mathrm{W/m^2}$$

Problem 4. The resistivity of glass, silver and copper are 10^{12}, 1.59×10^{-8}, and 1.68×10^{-8} Ωm respectively.

(a) If free charges are embedded in a piece of glass how long would it take for the charge to flow to the surface?

$$\begin{aligned} \tau &= \epsilon/\sigma \\ &= 4.7\epsilon_0\rho \\ &= 4.7(8.85 \times 10^{-12})10^{12} \\ &= 41.6\mathrm{s} \end{aligned}$$

(b) How thick should silver coating be to operate at a frequency of 10^{10}Hz?

The skin depth in a conductor is (assume $\mu = \mu_0$)

$$\begin{aligned} d &= \frac{1}{k} \\ &= \sqrt{\frac{2}{\sigma\omega\mu}} \\ &= \sqrt{\frac{2(1.59 \times 10^{-8})}{2\pi(10^{10})(1.26 \times 10^{-6})}} \\ &= 633\mathrm{nm} \end{aligned}$$

The waves will not penetrate very much beyond 633nm so the thickness should be thinner than that. Very thin?

(c) Find the wavelength and propagation speed in copper for radio waves at 1MHz

$$\begin{aligned}
k &= \sqrt{\frac{\sigma\omega\mu}{2}} \\
&= \sqrt{\frac{(1.68\times10^{-8})2\pi(10^6)(1.26\times10^{-6})}{2}} \\
&= 2.58\times10^{-4}\mathrm{m}^{-1}
\end{aligned}$$

$$\begin{aligned}
\lambda &= \frac{2\pi}{k} \\
&= \frac{2\pi}{2.58\times10^{-4}} \\
&= 2.43\times10^{4}\mathrm{m} \qquad \text{(whaaaat)}
\end{aligned}$$

$$\begin{aligned}
v &= \frac{\omega}{k} \\
&= \frac{2\pi(10^6)}{2.58\times10^{-4}} \\
&= 2.44\times10^{10} m/s \qquad \text{(???)}
\end{aligned}$$

Problem 5. Assume a monochromatic plane wave strikes a thick slab of glass of an aquarium at normal incidence. Find the minimum and maximum transmission coefficients. The water has $n_1 = 4/3$, glass $n_2 = 1.5$, air $n_3 = 1$.

An incident beam goes from the water to the glass. The transmitted amplitude is given by

$$E_{0T} = \left(\frac{2n_1}{n_1+n_2}\right)E_{0I}$$

This transmitted wave then becomes an incident wave on the boundary between the glass and air. So the transmitted wave in the air is

$$\begin{aligned}
E'_{0T} &= \left(\frac{2n_2}{n_2+n_3}\right)E_{0T} \\
&= \left(\frac{2n_2}{n_2+n_3}\right)\left(\frac{2n_1}{n_1+n_2}\right)E_{0I}
\end{aligned}$$

The transmission coefficient from water to air is

$$\begin{aligned}
T &= \frac{\epsilon_3 v_3}{\epsilon_1 v_1}\left(\frac{E'_{0T}}{E_{0I}}\right)^2 \\
&= \frac{n_3}{n_1}\left(\frac{2n_2}{n_2+n_3}\right)^2\left(\frac{2n_1}{n_1+n_2}\right)^2 \\
&= \frac{16n_3n_1n_2^2}{(n_2+n_3)^2(n_1+n_2)^2} \\
&= \frac{16(1)(4/3)(1.5)^2}{(1.5+1)^2(4/3+1.5)^2} \\
&= 0.96
\end{aligned}$$