PHY4311 AUTOCORRELATION ASSIGNMENT

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Problem 1. A short pulse has a Gaussian shape $I(t) = I_0 \exp(-\frac{t^2}{T^2})$

(a) Please write the functional form of the autocorrelation trace. The form is given by

$$\begin{aligned}
G(\tau) &= \int_{-\infty}^{\infty} I(t)I(t-\tau)dt \\
&= I_0^2 \int_{-\infty}^{\infty} \exp\Big(-\frac{t^2}{T^2} - \frac{(t-\tau)^2}{T^2}\Big)dt \\
&= I_0^2 \int_{-\infty}^{\infty} \exp\Big(\frac{-t^2 - t^2 + 2t\tau - \tau^2}{T^2}\Big)dt \\
&= I_0^2 \int_{-\infty}^{\infty} \exp\Big(\frac{-2(t^2 - t\tau + \frac{1}{2}\tau^2)}{T^2}\Big)dt \\
&= I_0^2 \int_{-\infty}^{\infty} \exp\Big(\frac{-2(t^2 - t\tau + \frac{1}{4}\tau^2 + \frac{1}{4}\tau^2)}{T^2}\Big)dt \\
&= I_0^2 \int_{-\infty}^{\infty} \exp\Big(\frac{-2((t - \frac{1}{2}\tau)^2 + \frac{1}{4}\tau^2)}{T^2}\Big)dt \\
&= I_0^2 \int_{-\infty}^{\infty} \exp\Big(\frac{-2(t - \frac{1}{2}\tau)^2 - \frac{1}{2}\tau^2}{T^2}\Big)dt \\
&= I_0^2 \int_{-\infty}^{\infty} \exp\Big(-\frac{2}{T^2}(t-\frac{1}{2}\tau)^2\Big)\exp\Big(-\frac{1}{2}\frac{\tau^2}{T^2}\Big)dt \\
&= I_0^2 \exp\Big(-\frac{1}{2}\frac{\tau^2}{T^2}\Big)\int_{-\infty}^{\infty} \exp\Big(-\frac{2}{T^2}(t-\frac{1}{2}\tau)^2\Big)dt
\end{aligned}$$

To do the integral note that its a bell shape and shifting it doesn't change the area. So

$$\int_{-\infty}^{\infty} \exp\Big(-\frac{2}{T^2}(t-\frac{1}{2}\tau)^2\Big)dt = \int_{-\infty}^{\infty} \exp\Big(-\frac{2}{T^2}t^2\Big)dt = T\sqrt{\frac{\pi}{2}}$$

Since $\int_{-\infty}^{\infty} \exp(-ax^2)dx = \sqrt{\pi/a}$. Substituting this back in

$$\begin{aligned}
G(\tau) &= I_0^2 \exp\Big(-\frac{1}{2}\frac{\tau^2}{T^2}\Big)(T\sqrt{\frac{\pi}{2}}) \\
G(\tau) &= \sqrt{\frac{\pi}{2}} I_0^2 T \exp\Big(-\frac{1}{2}\frac{\tau^2}{T^2}\Big)
\end{aligned}$$

(b) What is the FWHM of the correlation trace?

The peak is at $\tau_0 = 0$. Since $G(\tau_0) = \frac{1}{\pi\delta\tau_0}$ the FWHM is

$$2\delta\tau_0 = \frac{2}{\pi G(0)} = \frac{2}{\pi}\sqrt{\frac{2}{\pi}}\frac{1}{I_0^2 T} = \Big(\frac{2}{\pi}\Big)^{3/2}\frac{1}{I_0^2 T}$$