PHY4311 ASSIGNMENT 8

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Problem 1. Mode Locking

(a) Please show that each pulse in a mode locked train has an intensity that is N times larger than the sum of intensities of the individual modes.

A mode locked train made up of N individual modes of intensity X_0 is given by

$$X(t) = \sum_n X_n(t) = X_0 \frac{\sin(N\Delta\omega t/2)}{\sin(\Delta\omega t/2)} \sin(\omega_0 t + \phi_0)$$

The amplitude of the pulse is (when $\Delta\omega t/2$ is small)

$$X_0 \frac{\sin(N\Delta\omega t/2)}{\sin(\Delta\omega t/2)} \approx X_0 \frac{N\Delta\omega t/2}{\Delta\omega t/2} = NX_0$$

The intensity of the pulse is

$$I_{pulse} = N^2 X_0^2$$

The n-th mode is given by (assuming they all share the same amplitude and phase)

$$X_n(t) = X_0 \sin(\omega_0 t + \Delta\omega t + \phi_0)$$

The maximal intensity of each mode is

$$I_{mode} = X_0^2$$

Since there are N modes, the sum of the intensities of the individual modes is just

$$I_{modes} = I_0 + I_1 + ... = NI_{mode} = NX_0^2$$

So the intensity of the pulse is N times larger than the sum of the intensities of the individual modes.

$$I_{pulse} = NI_{modes}$$

(b) Please show that the energy of a mode-locked train is equal to the sum of the energies of the individual modes.

The energy density E of a mode is

$$E = \frac{1}{2}\epsilon_0 X_n^2 = \frac{1}{2}\epsilon_0 X_0^2$$

Since there are N modes each one carrying the same energy density, the sum of the energies of the individual modes is

$$E_{modes} = \epsilon_0 \frac{N}{2} X_0^2$$

The energy density of the mode locked train is

$$E_{train} = \frac{1}{2}\epsilon_0 X^2 = \epsilon_0 \frac{N^2}{2} X_0^2$$

Different by a factor of N.

Problem 2. A CO_2 laser operating at one atmosphere has a central wavelength of $\lambda = 10.6$ microns and a collion broadened gain linewidth for each ro-vibrational (rotational-vibrational transition) line. Assume that the FWHM gain linewidth of a ro-vibrational line is 10^9Hz and the cavity length is $L = 40$cm.

(a) What is the mode number for the central wavelength?

$$\begin{aligned} k &= \frac{m\pi}{L} \\ \frac{2\pi}{\lambda} &= \frac{m\pi}{L} \\ m &= \frac{2L}{\lambda} = \frac{2(0.4)}{10.6 \times 10^{-6}} \\ m &\doteq 75472 \end{aligned}$$

(b) How many modes contribute to the mode locked pulse if all modes within FWHM gain contribute?

First we need the frequency spacing of the modes. Then we divide the FWHM gain bandwith by the frequency spacing to get the number of modes that would 'fit' in the gain linewidth.

Since in a cavity $n\lambda = 2L$, we know $\nu = n\frac{c}{2L}$, so the frequency spacing is

$$\begin{aligned} \Delta\nu &= \frac{c}{2L} = \frac{3 \times 10^8}{2(0.4)} \\ &= 375 \times 10^6\text{Hz} \end{aligned}$$

The number of modes that contribute is

$$\frac{10^9}{\Delta\nu} = \frac{10^9}{375 \times 10^6} = 2$$

Only two modes.

(c) What is the minimum duration of a pulse in the mode locked pulse train?

The duration of the pulse is given by

$$\frac{1}{N\Delta\nu} = \frac{1}{2(375 \times 10^6)} = 1.3 \times 10^{-9}s$$

(d) What is the time delay between pulses?

The time delay is just

$$\frac{1}{\Delta\nu} = \frac{1}{375 \times 10^6} = 2.7 \times 10^{-9}s$$

Problem 3. If we increase the pressure to 10 atmospheres, then the ro-vibrational lines collisionally broaden enough that they overlap. This allows a much greater bandwidth. Assume that the collisionally broadened FWHM bandwidth is 300 GHz, how short could the individual mode locked pulses be?

The number of modes that contribute would now be

$$\frac{300 \times 10^9}{375 \times 10^6} = 800$$

So the duration of the pulse would be

$$\frac{1}{N\Delta\nu} = \frac{1}{800(375 \times 10^6)} = 3.3 \times 10^{-12} s$$

About a thousand times shorter.