

PHY4311 LASERS ASSIGNMENT 12

MOHAMMED CHAMMA 6379153
MARCH 29 2015

Problem 1. Given a resonator with one plane mirror ($R_1 = \infty$) and one curved mirror ($R_2 = 4\text{m}$), with a separation of $L = 1\text{m}$, is it stable?

$$\begin{aligned} g_1 &= 1 - \frac{L}{R_1} = 1 - 0 = 1 \\ g_2 &= 1 - \frac{L}{R_2} = 1 - \frac{1}{4} = \frac{3}{4} \\ 0 \leq g_1 g_2 &= 1\left(\frac{3}{4}\right) = \frac{3}{4} \leq 1 \end{aligned}$$

Since $0 \leq g_1 g_2 \leq 1$, the resonator is stable.

Write the matrix for one complete round trip of a ray in a cavity consisting of a plane mirror ($R_1 = \infty$) and an $R = 3\text{m}$ mirror separated by $L = 1\text{m}$.

The matrix is

$$\begin{aligned} & \begin{bmatrix} 1 & 0 \\ -\frac{2}{R_1} & 1 \end{bmatrix} \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{2}{R_2} & 1 \end{bmatrix} \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{2}{3} & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{2}{3} & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -\frac{2}{3} & -\frac{2}{3} + 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -\frac{2}{3} & \frac{1}{3} \end{bmatrix} \\ &= \begin{bmatrix} 1 - 2/3 & 1 + 1/3 \\ -\frac{2}{3} & \frac{1}{3} \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{3} & \frac{4}{3} \\ -\frac{2}{3} & \frac{1}{3} \end{bmatrix} \end{aligned}$$

Is the cavity stable?

$$\begin{aligned} g_1 &= 1 - \frac{L}{R_1} = 1 - 0 = 1 \\ g_2 &= 1 - \frac{L}{R_2} = 1 - \frac{1}{3} = \frac{2}{3} \\ g_1 g_2 &= \frac{2}{3} \end{aligned}$$

Since $0 \leq \frac{2}{3} \leq 1$ the cavity is stable.

If $L = 2\text{m}$ is the cavity stable? $g_1 g_2 = (1)(\frac{1}{3}) = \frac{1}{3}$. Yes, it's still stable.