a transverse force and acquires a transverse velocity component upon entry into the wiggler. The resulting trajectory is helical and describes a bulk streaming along the axis of symmetry as well as a transverse circular rotation that lags 180° behind the phase of the wiggler field. The magnitude of the transverse wiggler velocity, denoted by v_w, is proportional to the product of the wiggler amplitude and period. This relationship may be expressed in the form

$$\frac{v_w}{c} \cong 0.934\frac{B_w\lambda_w}{\gamma_b}, \tag{1.3}$$

where the wiggler period is expressed in units of centimetres, B_w denotes the wiggler amplitude in tesla, and $\gamma_b = 1 + E_b/m_ec^2$ denotes the relativistic time-dilation factor associated with the total kinetic energy E_b of the electron beam (where m_e denotes the rest mass of the electron, and m_ec^2 denotes the electron rest energy).

Since the motion is circular, both axial and transverse velocities have a constant magnitude. This is important because the resonant interaction depends upon the axial velocity of the beam. In addition, since the wiggler induces a constant-magnitude transverse velocity, the relation between the total electron energy and the streaming energy can be expressed in terms of the time-dilation factors in the form

$$\gamma_z \cong \frac{\gamma_b}{\surd(1 + 0.872\,B_w^2\lambda_w^2)}. \tag{1.4}$$

As a result, the resonant wavelength depends upon the total beam energy, and the wiggler amplitude and period through

$$\lambda \cong (1 + 0.872\,B_w^2\lambda_w^2)\frac{\lambda_w}{2\gamma_b^2}. \tag{1.5}$$

It is the interaction between the transverse wiggler-induced velocity and the transverse magnetic field of an electromagnetic wave that induces a force normal to both in the axial direction. This is the ponderomotive force. The transverse velocity and the radiation magnetic field are directed at right angles to each other and undergo a simple rotation about the axis of symmetry. A resonant wave must be circularly polarized with a polarization vector that is normal to both the transverse velocity and the wiggler field and which rotates in synchronism with the electrons. This synchronism is illustrated in Fig. 1.2, and is maintained by the aforementioned resonance condition.

In order to understand the energy transfer, we return to the surfer analogy and consider a group of surfers attempting to catch a series of waves. In the attempt to match velocities with the waves, some will catch a wave ahead of the crest and slide forward while others will catch a wave behind the crest and slide backward. As a result, clumps of surfers will collect in the troughs of the waves. Those surfers which slide forward ahead of the wave are accelerated and gain energy at the expense of the wave, while those that slide backward are decelerated and lose energy to the wave. The wave grows if more surfers are decelerated than

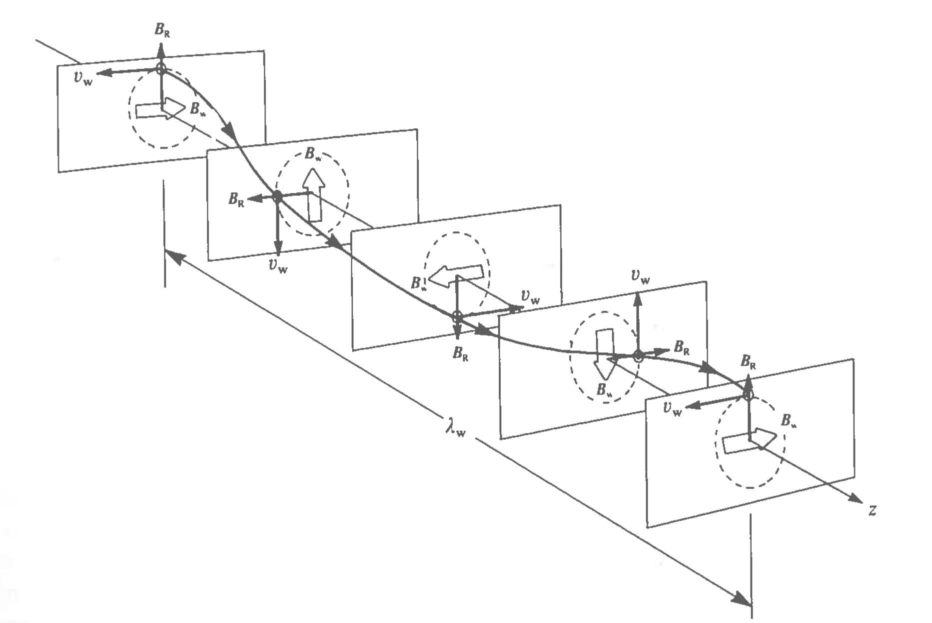

Fig. 1.2 The electron trajectory in a helical wiggler includes bulk streaming parallel to the axis of symmetry as well as a helical gyration. The vector relationships between the wiggler field $\boldsymbol{B}_w$, the transverse velocity $\mathbf{v}_w$, and the radiation field $\boldsymbol{B}_R$ of a resonant wave are shown in the figure projected on to planes transverse to the symmetry axis at intervals of one quarter of a wiggler period. This projection is circular, and the transverse velocity is directed opposite to that of the wiggler. A resonant wave must be circularly polarized with a polarization vector that is normal to both the transverse velocity and the wiggler field and which rotates in synchronism with the electrons. The electrons then experience a slowly varying wave amplitude. The transverse velocity and the radiation field are directed at right angles to each other and undergo a simple rotation. The interaction between the transverse velocity and the radiation field induces a force in the direction normal to both which coincides with the symmetry axis.

accelerated, and there is a net transfer of energy to the wave. The free-electron laser operates by an analogous process. Electrons in near resonance with the ponderomotive wave lose energy to the wave if their velocity is slightly greater than the phase velocity of the wave, and gain energy at the expense of the wave in the opposite case. As a result, wave amplification occurs if the wave lags behind the electron beam.

This process in a free-electron laser is described by a nonlinear pendulum equation. The **ponderomotive phase** $\psi(= (k + k_w)z - \omega t)$ is a measure of the position of an electron in both space and time with respect to the ponderomotive wave. The ponderomotive phase satisfies the circular pendulum equation

$$\frac{d^2}{dz^2}\psi = -K^2 \sin\psi, \tag{1.6}$$