PHY4362
ASSIGNMENT 6 AND 7

MOHAMMED CHAMMA - 6379153

Problem. 1. Draw a block diagram representing the important steps in obtaining the angular momentum and parity selection for the E1 radiation interacting with the two level quantum system.

I think I accidentally just did it for Beta decay and now I'm out of time

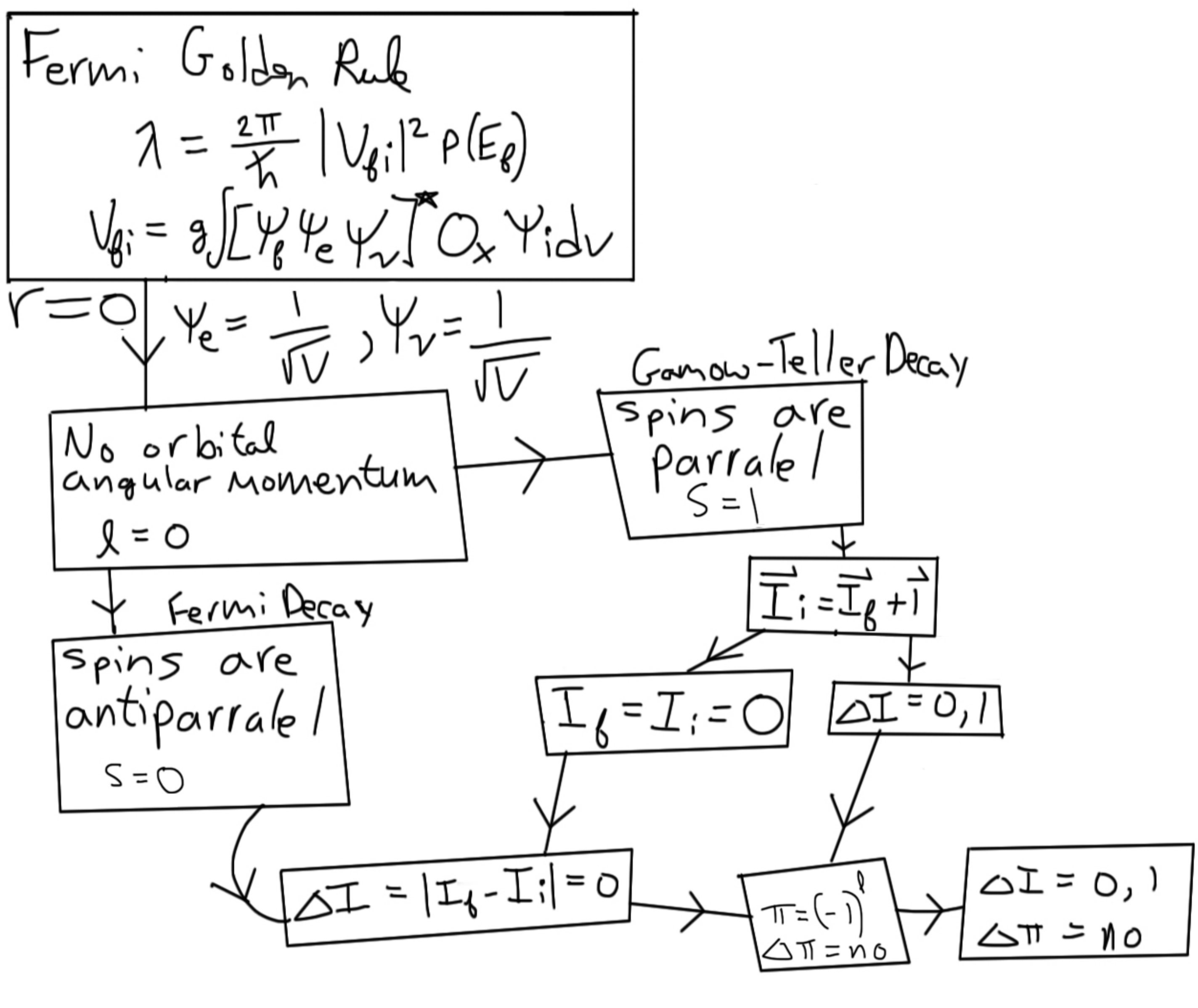

Figure 0.1

Problem. 2. Multipole radiation

a) For the $\frac{9}{2}^- \to \frac{7}{2}^+$ γ transition give all permitted multi-pole radiations and indicate which multi-pole might be the most intense in the emitted radiation.

There is a parity change. To find what L can be we do:

$$\begin{aligned}|J_i - J_f| &\leq L \leq J_i + J_f \\ |\frac{9}{2} - \frac{1}{2} - \frac{7}{2} - \frac{1}{2}| &\leq L \leq \frac{9}{2} - \frac{1}{2} + \frac{7}{2} + \frac{1}{2} \\ 0 &\leq L \leq 8\end{aligned}$$

So L can be 1, 2, 3, 4, 5, 6, 7, 8.
The allowed transitions are E1, M2, E3, M4, E5, M6, E7, M8

b) The natural linewidth would be about $\Gamma = \frac{\hbar}{\tau} = \frac{1.05\times10^{-34}\text{m}^2\text{kg/s}}{10^{-17}\text{s}} = 1.05 \times 10^{-17}\text{m}^2\text{kg/s}^2$

Problem. 3. Calculate the recoil energy for the transitions from the first excited statte to the ground state for the following nuclides (energy given in parentheses) I'm assuming the given energy is the energy of the gamma ray.

The recoil energy is given by $E_R = \frac{E_\gamma^2}{2Mc^2}$. $c^2 = 931.5\text{MeV/u}$

a) ^{15}O ($E_\gamma = 5.183$MeV) $E_R = \frac{()^2}{(931.5)} =$

$$M = 15.003065\text{u} \implies E_R = \frac{(5.183)^2}{2(15.003065)(931.5)} = 9.61 \times 10^{-4}\text{MeV}$$

b) ^{19}O ($E_\gamma = 0.0960$MeV)

$$M = 19.003577\text{u} \implies E_R = \frac{(0.0960)^2}{2(19.003577)(931.5)} = 2.61 \times 10^{-7}\text{MeV}$$

c) ^{57}Fe ($E_\gamma = 0.0144$MeV)

$$M = 56.935396\text{u} \implies E_R = \frac{(0.0144)^2}{2(56.935396)(931.5)} = 1.96 \times 10^{-9}\text{MeV}$$

d) ^{70}Ge ($E_\gamma = 1.0396$MeV)

$$M = 69.924250\text{u} \implies E_R = \frac{(1.0396)^2}{2(69.924250)(931.5)} = 8.3 \times 10^{-6}\text{MeV}$$

e) ^{227}Th ($E_\gamma = 0.0093$MeV)

$$M = 227.0277\text{u} \implies E_R = \frac{(0.0093)^2}{2(227.0277)(931.5)} = 2.05 \times 10^{-10}\text{MeV}$$

Problem. 4. From these transitions compute the natural linewidth Γ, the Doppler width Δ at room temperature T =293K, the Doppler width at liquid helium temperature T =4K and the nuclear recoil energy.

We'll use the following equations:

$$\Gamma = \frac{\hbar}{\tau}, \qquad \Delta = 2\sqrt{\ln 2}E_\gamma\sqrt{\frac{2kT}{Mc^2}}, \qquad E_R = \frac{E_\gamma^2}{2Mc^2}$$

$k = 8.62 \times 10^{-11}\text{MeV/K}$.

a) $E_\gamma = 73\text{keV} = 73 \times 10^{-3}\text{MeV}$, $\tau = 6.3\text{ns}$, $M_{^{193}Ir} = 192.962917\text{u}$

$$\begin{aligned}
\Gamma &= \frac{\hbar}{\tau} = \frac{1.05 \times 10^{-34}}{6.3 \times 10^{-9}} = 1.67 \times 10^{-26} \\
\Delta_{room} &= 2\sqrt{\ln 2}E_\gamma\sqrt{\frac{2kT}{Mc^2}} = 2\sqrt{\ln 2}(73 \times 10^{-3})\sqrt{\frac{2(8.62 \times 10^{-11})(293)}{192.962917(931.5)}} = 6.44 \times 10^{-8}\text{MeV} \\
\Delta_{cold} &= 2\sqrt{\ln 2}(73 \times 10^{-3})\sqrt{\frac{2(8.62 \times 10^{-11})(4)}{192.962917(931.5)}} = 7.53 \times 10^{-9}\text{MeV} \\
E_R &= \frac{E_\gamma^2}{2Mc^2} = \frac{(73 \times 10^{-3})^2}{2(192.962917)(931.5)} = 1.48 \times 10^{-8}\text{MeV}
\end{aligned}$$

b) Same process but with $E_\gamma = 14.4\text{keV} = 14.4 \times 10^{-3}\text{MeV}$, $\tau = 98\text{ns}$, $M_{^{57}Fe} = 56.935396\text{u}$

$$\begin{aligned}
\Gamma &= 1.07 \times 10^{-27}\text{m}^2\text{kg/s}^2 \\
\Delta_{room} &= 2.34 \times 10^{-8}\text{MeV} \\
\Delta_{cold} &= 2.73 \times 10^{-9}\text{MeV} \\
E_R &= 1.95 \times 10^{-9}\text{MeV}
\end{aligned}$$

c) $E_\gamma = 6.2\text{keV} = 6.2 \times 10^{-3}\text{MeV}$, $\tau = 6.6\mu\text{s}$, $M_{^{181}Ta} = 180.947992\text{u}$

$$\begin{aligned}
\Gamma &= 1.59 \times 10^{-29}\text{m}^2\text{kg/s}^2 \\
\Delta_{room} &= 5.65 \times 10^{-9}\text{MeV} \\
\Delta_{cold} &= 6.6 \times 10^{-10}\text{MeV} \\
E_R &= 1.14 \times 10^{-10}\text{MeV}
\end{aligned}$$

d) $E_\gamma = 23.9\text{keV} = 23.9 \times 10^{-3}\text{MeV}$, $\tau = 17.8\text{ns}$, $M_{^{119}Sn} = 118.903311\text{u}$

$$\begin{aligned}
\Gamma &= 5.90 \times 10^{-27}\text{m}^2\text{kg/s}^2 \\
\Delta_{room} &= 2.69 \times 10^{-8}\text{MeV} \\
\Delta_{cold} &= 3.14 \times 10^{-9}\text{MeV} \\
E_R &= 2.58 \times 10^{-9}\text{MeV}
\end{aligned}$$

e) $E_\gamma = 95\text{keV} = 95 \times 10^{-3}\text{MeV}$, $\tau = 22\text{ps}$, $M_{^{165}Ho} = 164.930319\text{u}$

$$\begin{aligned}
\Gamma &= 4.77 \times 10^{-24}\text{m}^2\text{kg/s}^2 \\
\Delta_{room} &= 9.07 \times 10^{-8}\text{MeV} \\
\Delta_{cold} &= 1.06 \times 10^{-8}\text{MeV} \\
E_R &= 2.94 \times 10^{-8}\text{MeV}
\end{aligned}$$

Problem. 5. Graph the spectral profile of two doppler broadened lines L1 and L2 in ^{19}O centered at 91keV separated by 0.1eV. Take the peak intensity of L1 to be twice the peak intensity of L2.

The energy distribution will look like this

$$f(E_\gamma) = \exp\Big(- \Big(\frac{mc^2}{2kT}\Big)\Big(1 - \frac{E_\gamma'}{E_\gamma}\Big)^2\Big)$$

For ^{19}O we take $m = 19.003577$u. The constants are $c^2 = 931.5$MeV/u, and $k = 8.62 \times 10^{-11}$MeV/K. For L1 we'll use $E_\gamma' = 91 - 0.0001 = 90.9999$MeV and for L2 we'll use $E_\gamma' = 91.0001$MeV. To make the peak intensity of L1 twice that of L2 we'll just plot $2f(E_\gamma)$.

a) $T = 293K$
b) $T = 1000K$
c) $T = 4K$

Problem. 6. Standard Fission Reactor

i) What parameters contribute to the four factor formula and what is its physical meaning?

The four factor formula is $k_\infty = \eta f p \epsilon$ and represents the criticality of a nuclear chain reaction in an infinite medium. The four factors are called

$$\begin{aligned} \eta &: \text{reproduction factor} \\ f &: \text{thermal utilization factor} \\ p &: \text{resonance escape probability} \\ \epsilon &: \text{fast fission factor} \end{aligned}$$

η represents the # of fission neutrons produced per absorption in the fuel. f is the probability that an absorbed neutron is absorbed in the fuel material. p is the fraction of fission neutrons that slow down to thermal energies without being absorbed. ϵ is the ratio of the total number of fission neutrons to the number of thermal fission neutrons (?).

ii) If $k > 1$, the reaction is supercritical and will grow exponentially. If $k < 1$ it is subcritical and will decay exponentially. If $k = 1$ is it critical and the neutron population remains constant.

iii) The size of the graphite rods is determined by the diffusion length L_d which is the distance thermal neutrons travels on average before absorption and the slowing distance L_s which is the distance a fast neutron travels before slowing down. The size is also determined by k_∞ and the complete reproduction factor k which takes into account the fraction of fast and thermal neutrons that leave the system ℓ_f and ℓ_t.

Problem. 7. Fission bombs

i) Explain the difference between the first plutonium and uranium bombs.

The first uranium bomb was a gun type bomb where a plug of uranium-235 was fired into a pellet of uranium-235, causing the mass to go supercritical. The first plutonium bomb was an implosion bomb, where a sphere of plutonium is compressed to criticality by chemical explosions surrounding it.

ii) What is the role of a U238 tamper in the plutonium bomb design?

The U238 tamper surrounds the plutonium mass and reflects neutrons back into the core and provides a little extra neutrons to the reaction.

iii) Which Japanese city was spared the conventional carpet bombing to be a test bed for the destructive power of the nuclear device?

Nagasaki.

Problem. 8. Fusion bombs

i) What is the "sloika design"?

It refers to the Russian 'layered' design where fusion and fission fuel are layered

ii) What are neutron bombs and salted bombs?

Neutron bombs are meant to primarily release a lethal pulse of neutron radiation as opposed to explosive destruction. This is meant to clear an area of people by killing them with radiation while largely preserving buildings and structures. The fallout and the explosion are relatively small compared to other nuclear weapons and the primary killer is the neutron radiation

Salted bombs are nuclear weapons that maximize nuclear fallout and are used to make an area uninhabitable by "salting" it with dangerous radioactive waste.

iii) What percentage of the destructive power of a typical 10MT nuclear device comes from actual fusion?

Less than half. Most of the power comes from the extra fission of Uranium that the fusion causes.

Problem. 9. Controlled Fusion

i) What is the laser output radiation used in NIF, and how is it converted first to UV and then to x-rays?

An infrared laser is amplified to energies where it is the converted to an UV beam. The UV beam strikes the hohlraum's inner walls and are re-emitted as xrays.

ii) What is the difference between the stellatron and tokamak?

The difference between the two is the geometry of the torus. The stellatron manipulates the magnetic fields with external currents while the tokamak manipulates it by driving a current through the plasma.

iii) Explain the meaning and significance of the Lawson criterion.

The Lawson criterion states that for a fusion reactor to reach ignition, the temperature of the plasma heated by the products of the fusion reaction must by high enough to maintain the temperature of the plasma.

Problem. 10. Stellar nucleo-synthesis

a) Draw the evolution diagram for a star of the size of our Sun from its birth to the final stages of its existence.

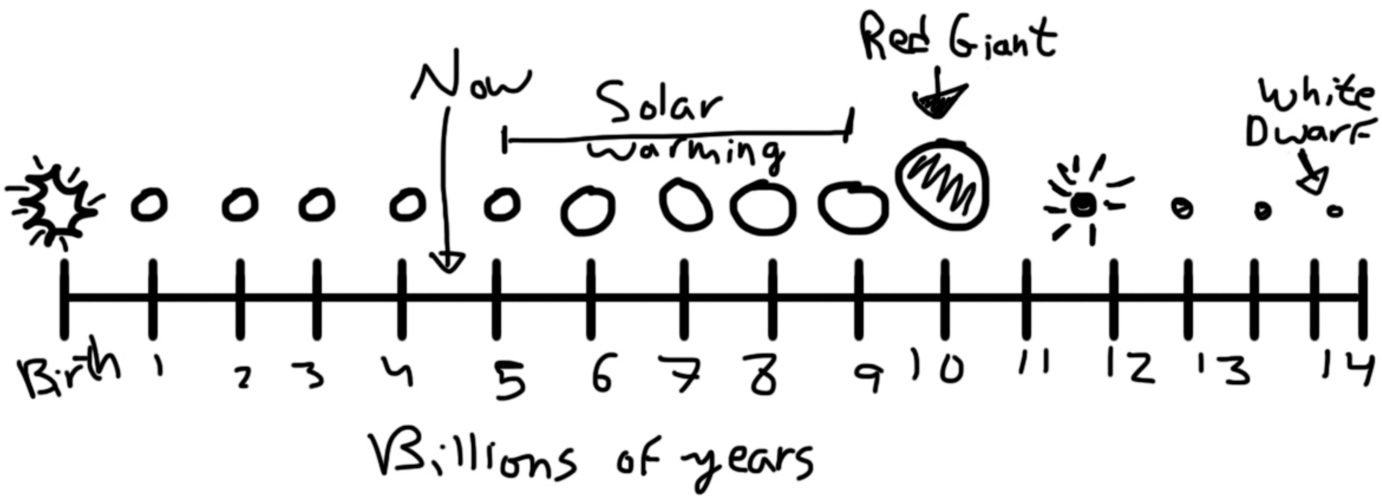

FIGURE 0.2

b) What is the source of the forces stabilizing the size of the very massive star at various stages of its existence.

As a normal star hydrogen fusion provides an outward pressure which is balanced by the inward gravitational force. During the red giant phase the star expands because in addition to fusion in the core, there is fusion in the shells. As a white dwarf the star has collected too much mass, collapses on itself, and is held 'up' by electron degeneracy pressure caused by the Pauli exclusion principle. The star resists further compression because all the lower energy electron states are filled.

c) Explain the conditions necessary for the processes leading to the production of elements heavier than iron to take place.

To make elements heavier than iron you need a red giant to go supernova. In the explosion the neutron densities are theoretically high enough to form the heavier elements.

Problem. 11. Describe the idea of the Pound and Rebka experimental measurement of gravitational redshift (1960).

A photon falling moving in a gravitational field will have its frequency shifted (blueshifted if towards the center and redshifted if away). Atoms will emit photons of a very specific frequency and can only absorb photons of a very specific frequency since the energy levels are discrete.

Take two radioactive cobalt atoms. These atoms emit high energy gamma photons and have sharp spectral lines. One of these atoms will emit a photon which can be absorbed by the other. In a gravitational field, this photon is slightly shifted, and, since the energy levels are discrete, the 'absorber' cannot absorb the photon.

If the emitter is on the ground, photons emitted upward will be redshifted. If the absorber moves towards the source, the photon is blueshifted in the absorber's reference frame, by the special relativistic doppler effect. These two effects can cancel out the shift, and the photon can be absorbed. Measuring the speed necessary for this to happen lets you calculate the gravitational frequency shift, validating a prediction of general relativity.

Atoms that emit photons will experience recoil to conserve momentum. Since the frequency shift is so tiny in the tower that Pound and Rebka did their experiment in, this recoil causes a doppler shift which to counteract would require a shift many orders of magnitude larger than the gravitational shift.

By embedding the cobalt atoms in a lattice of iron atoms, the cobalt emits photons virtually recoil-free, since the lattice as a whole absorbs the recoil. This is called the Mossbauer effect. Taking all this into account Pound and Rebka were able to find a measurement that agreed within 10% of the prediction, and in subsequent experiments the accuracy was increased to better than 1%.