PHY4362
ASSIGNMENT 3
MOHAMMED CHAMMA 6379153

Problem 1. Give the expected shell-model spin and parity assignment for the ground states of ^{7}Li, ^{11}B, ^{15}C and ^{17}F.

- ^{7}Li: Z = 3, so N = 4. The unpaired proton determines the spin-parity assignment. The configuration of the protons are 2 in the $1s_{1/2}$ level, and 1 in the $1p_{3/2}$ level, or simply $(1s_{1/2})^2(1p_{3/2})^1$. The p sublevel corresponds to $l = 1$. The parity is $= (-1)^l = (-1)^1 = -1$. The spin is of the uncoupled proton, 3/2. The spin-parity assignment for ^{7}Li is $(3/2)^-$.
- ^{11}B: Z = 5, so N = 6. The unpaired proton determines the spin-parity assignment. The configuration of the protons is $(1s_{1/2})^2(1p_{3/2})^3$. The spin-parity assignment is $(3/2)^-$.
- ^{15}C: Z = 6, so N = 9. The unpaired neutron determines the spin-parity assignment. The configuration of the neutrons is $(1s_{1/2})^2(1p_{3/2})^4(1p_{1/2})^2(1d_{5/2})^1$. The spin-parity assignment is $(5/2)^+$ since $l = d = 2$.
- ^{17}F: Z = 9, so N = 8. The unpaired proton determines the spin-parity assignment. The configuration of the protons is $(1s_{1/2})^2(1p_{3/2})^4(1p_{1/2})^2(1d_{5/2})^1$. The spin-parity assignment is $(5/2)^+$.

Problem 2. Compute the mass defects of ^{32}S, ^{20}F, and ^{238}U.
The mass defect is calculated as $\triangle m = (m - A)c^2$

- ^{32}S:

  $m = 31.972071$, $A = 32$

  $\triangle m = (31.972071 - 32)c^2 = -0.027929c^2 = -0.027929 * 931.5 = -26.02\,MeV$
- ^{20}F:

  $m = 19.999981$, $A = 20$

  $\triangle m = (19.999981 - 20)c^2 = -0.000019c^2 = -0.0177\,MeV$
- ^{238}U:

  $m = 238.050785$, $A = 238$

  $\triangle m = (238.050785 - 238)c^2 = 0.050785c^2 = 47.3\,MeV$

Problem 3. Prove that the electric quadrupole moment of a uniformly charged ellipsoid of semi-major axis b and semi-minor axis a is given by $eQ = \frac{2}{5}Ze(b^2 - a^2)$

First, we find the charge density $\rho(r)$ for the ellipsoid, which has volume $V = \frac{4}{3}\pi a^2 b$. Since it is uniformly charged,

$$\rho(r) = \frac{Q}{V} = \frac{3Ze}{4\pi a^2 b} = constant$$

Let's map the x, y, z coordinates of the ellipse to a unit sphere: $(x, y, z) = (au, av, bw)$ so that

$$u = x/a$$
$$v = y/a$$
$$w = z/b$$

The Jacobian of this transformation is $a \cdot a \cdot b = a^2 b$.
Now

$$\begin{aligned}
eQ = & \int \psi^* \rho(r)(3z^2 - r^2)\psi dV \\
= & \frac{3Ze}{4\pi a^2 b}\int \psi^*(3z^2 - x^2 - y^2 - z^2)\psi dV \\
= & \frac{3Ze}{4\pi a^2 b}\int \psi^*(2z^2 - x^2 - y^2)\psi dV \\
= & \frac{3Ze}{4\pi a^2 b}a^2 b\int \psi^*(2b^2w^2 - a^2u^2 - a^2v^2)\psi dV \qquad (change\, coordinates) \\
= & \frac{3Ze}{4\pi}(2b^2\int \psi^* w^2 \psi dV - a^2\int \psi^* u^2 \psi dV - a^2 \int \psi^* v^2 \psi dV) \\
= & \frac{3Ze}{4\pi}(2b^2 < w^2 > -a^2 < u^2 > -a^2 < v^2 >) \qquad (expectation\, values)
\end{aligned}$$

Since u, v, and w are in a sphere, $< u^2 >=< v^2 >=< w^2 >$. So,

$$\begin{aligned}
eQ = & \frac{3Ze}{4\pi}(2b^2 < w^2 > -a^2 < w^2 > -a^2 < w^2 >) \\
= & \frac{3Ze}{4\pi}(2b^2 - 2a^2) < w^2 > \\
= & 2\frac{3Ze}{4\pi}(b^2 - a^2) < w^2 > \\
= & \frac{3Ze}{2\pi}(b^2 - a^2)\int \psi^* w^2 \psi d^3u \qquad (d^3u = r^2 \sin\phi dr d\theta d\phi) \\
= & \frac{3Ze}{2\pi}(b^2 - a^2)\int\int\int \psi^*\psi r^2 \cos^2\phi r^2 \sin\phi dr d\theta d\phi \quad (w = r\cos\phi,\, \phi \in [0, \pi]) \\
= & \frac{3Ze}{2\pi}(b^2 - a^2)\int_0^{2\pi} d\theta \int_0^1 r^4 dr \int_0^{\pi} \cos^2\phi \sin\phi d\phi \\
= & \frac{3Ze}{2\pi}(b^2 - a^2)(2\pi)(\frac{1}{5})(-\frac{1}{3}\cos^3\phi|_0^{\pi}) \\
= & \frac{3Ze(b^2 - a^2)}{5}(-\frac{(-1)}{3} + \frac{1}{3}) \\
= & \frac{2}{5}Ze(b^2 - a^2) \qquad \square
\end{aligned}$$

Problem 4. For the scattering of 10MeV alpha particles from Au nuclei, calculate the total scattering cross section for scattering at angles >1°, >5°, >20°. Calculate the differential scattering cross section for scattering at 1°, 5° and 20°.

$$\sigma = \pi Z^2 \Big(\frac{ke^2}{KE}\Big)^2 \Big(\frac{1+\cos\theta}{1-\cos\theta}\Big)$$

Au nuclei implies $Z = 79$. $k = 9 * 10^9 \frac{Nm^2}{C^2}$. $KE = 10MeV = 1.6 * 10^{-12}J$. $e = 1.6 * 10^{-19}C$.
For angles $>\theta = 1°$,

$$\sigma = \pi(79)^2 \Big(\frac{9 * 10^9 (1.6 * 10^{-19})^2}{1.6 * 10^{-12}}\Big)^2 \Big(\frac{1+cos(1)}{1-cos(1)}\Big) = 5.338 * 10^{-24} m^2$$

Similarly, for angles $> \theta = 5°$,

$$\sigma = \pi(79)^2 \Big(\frac{9 * 10^9 (1.6 * 10^{-19})^2}{1.6 * 10^{-12}}\Big)^2 \Big(\frac{1+cos(5)}{1-cos(5)}\Big) = 2.13 * 10^{-25} m^2$$

and for angles $>\theta = 20°$,

$$\sigma = \pi(79)^2 \Big(\frac{9 * 10^9 (1.6 * 10^{-19})^2}{1.6 * 10^{-12}}\Big)^2 \Big(\frac{1+cos(20)}{1-cos(20)}\Big) = 1.308 * 10^{-26} m^2$$

Now, the differential scattering cross-section is given by

$$\frac{d\sigma}{d\Omega} = \Big(kzZe^2\Big)^2 \Big(\frac{1}{4T_\alpha}\Big)^2 \frac{1}{\sin^4\big(\frac{\theta}{2}\big)}$$

For alpha particles, z=2. For $\theta = 1°$,

$$\frac{d\sigma}{d\Omega} = \Big(9 * 10^9 (2)(79)(1.6 * 10^{-19})^2\Big)^2 \Big(\frac{1}{4(1.6 * 10^{-12})}\Big)^2 \frac{1}{\sin^4\big(\frac{1°}{2}\big)} = 5.58 * 10^{-21} m^2/sr$$

For $\theta = 5°$,

$$\frac{d\sigma}{d\Omega} = \Big(9 * 10^9 (2)(79)(1.6 * 10^{-19})^2\Big)^2 \Big(\frac{1}{4(1.6 * 10^{-12})}\Big)^2 \frac{1}{\sin^4\big(\frac{5°}{2}\big)} = 8.94 * 10^{-24} m^2/sr$$

For $\theta = 20°$,

$$\frac{d\sigma}{d\Omega} = \Big(9 * 10^9 (2)(79)(1.6 * 10^{-19})^2\Big)^2 \Big(\frac{1}{4(1.6 * 10^{-12})}\Big)^2 \frac{1}{\sin^4\big(\frac{20°}{2}\big)} = 3.56 * 10^{-26} m^2/sr$$

Problem 5. List and briefly explain the experimental methods used in establishing the nuclear charge radius.

The nuclear charge radius is a property of the nucleus.

- First experiment that probed the radius of the nucleus was the Geiger-Marsden experiment initiated by Rutherford
- Alpha particles fired at atoms would be scattered by the nucleus of the atom. Geiger and Marsden used gold foil.
- They related the energy of the fired alpha particle to the distance of closest approach. This gives an upper bound on the radius of the nucleus (assuming the alpha particle doesn't penetrate the nucleus).
- For gold, they found the upper bound to be $3.2 * 10^{-14} m$.

Problem 6. The spin j^p and excitation energy E of the ground state and a sequence of excited states of the nuclei ^{170}Hf are given in a table

j^p	0^+	2^+	4^+	6^+	8^+
E(eV)	0	100	321	641	1041

A.

Calculate the moment of inertia of the nucleus for each of these excited states.
From Krane (p. 144)

$$E(0^+) = 0 \implies I = 0$$

$$E(2^+) = 6(\hbar^2/2I) \implies I = \frac{6\hbar^2}{2E(2^+)} = \frac{6(6.58 * 10^{-15})^2}{2(100)} = 1.30 * 10^{-26} eV \cdot s^2$$

$$E(4^+) = 20(\hbar^2/2I) \implies I = \frac{20\hbar^2}{2E(4^+)} = \frac{20(6.58 * 10^{-15})^2}{2(321)} = 1.39 * 10^{-25} eV \cdot s^2$$

$$E(6^+) = 42(\hbar^2/2I) \implies I = \frac{42\hbar^2}{2E(6^+)} = \frac{42(6.58 * 10^{-15})^2}{2(641)} = 5.83 * 10^{-25} eV \cdot s^2$$

$$E(8^+) = 72(\hbar^2/2I) \implies I = \frac{72\hbar^2}{2E(8^+)} = \frac{72(6.58 * 10^{-15})^2}{2(1041)} = 1.62 * 10^{-24} eV \cdot s^2$$

B.

Compare your results with the moment of inertia of the nucleus treated as the rigidly rotating sphere.

The moment of inertia of a rigid rotating sphere is $I = \frac{2}{5}mr^2$ where m is the mass and r is the radius of the sphere.

The mass of ^{170}Hf is 169.94 amu or $m = 169.94u * \frac{1.66*10^{-27}kg}{1u} = 2.82 * 10^{-25}kg$
We can find the atomic radius using the empirical radius formula

$$r = r_0 A^{1/3} = 1.2 * 10^{-15} * 170^{1/3} = 6.65 * 10^{-15} m$$

So we find that the moment of inertia of the rigidly rotating sphere approximating the ^{170}Hf nucleus is

$$I = \frac{2}{5}mr^2 = \frac{2}{5}(2.82 * 10^{-25})(6.65 * 10^{-15})^2 = 5.0 * 10^{-54} kg \cdot m^2$$

$$I = 5.0 * 10^{-54} kg \cdot m^2 = 3.125 * 10^{-35} eV \cdot s^2$$

The moment of inertia of the rigid sphere is much smaller.

Problem 7. Calculate the ground state magnetic moment of ^{7}Li, ^{39}K and ^{45}Sc using equation

$$\mu_J = \gamma_J J\hbar = g_J \mu_N = g_J J \mu_B \text{ where } g_J = \left(g_l \pm \frac{g_s - g_l}{2l+1}\right)$$

Where g_l and g_s are orbital and intrinsic g factors for the odd nucleon concerned, l is orbital angular momentum and the upper sign is taken for $J = l + \frac{1}{2}$. The spin values are 3/2, 3/2 and 7/2 respectively. $\mu_B = 5.79 * 10^{-5} \frac{eV}{T}$

^{7}Li

$$J = 3/2\, and\, J = l + 1/2 \implies l = J - 1/2 = \frac{3}{2} - \frac{1}{2} = 1$$

Since $Z = 3$ and $N = 4$, the odd nucleon is the proton, so

$$g_s = 5.59,\ g_l = 1$$

Now

$$g_j = \left(g_l + \frac{g_s - g_l}{2l+1}\right) = \left(1 + \frac{5.59 - 1}{2(1)+1}\right) = 2.53$$

So

$$\mu_J = g_j J \mu_B = 2.53(\frac{3}{2})(5.79 * 10^{-5}) = 2.20 * 10^{-4}\frac{eV}{T}$$

^{39}K

$$J = 3/2\, and\, J = l + 1/2 \implies l = J - 1/2 = \frac{3}{2} - \frac{1}{2} = 1$$

Since $Z = 19$ and $N = 20$, the odd nucleon is the proton, so

$$g_s = 5.59,\ g_l = 1$$

Now

$$g_j = \left(g_l + \frac{g_s - g_l}{2l+1}\right) = \left(1 + \frac{5.59 - 1}{2(1)+1}\right) = 2.53$$

So

$$\mu_J = g_j J \mu_B = 2.53(\frac{3}{2})(5.79 * 10^{-5}) = 2.20 * 10^{-4}\frac{eV}{T}$$

^{45}Sc

$$J = 7/2\, and\, J = l + 1/2 \implies l = J - 1/2 = \frac{7}{2} - \frac{1}{2} = 3$$

Since $Z = 21$ and $N = 24$, the odd nucleon is the proton, so

$$g_s = 5.59,\ g_l = 1$$

Now

$$g_j = \left(g_l + \frac{g_s - g_l}{2l+1}\right) = \left(1 + \frac{5.59 - 1}{2(3)+1}\right) = 1.66$$

So

$$\mu_J = g_j J \mu_B = 1.66(\frac{7}{2})(5.79 * 10^{-5}) = 3.36 * 10^{-4}\frac{eV}{T}$$