PHY4311
ASSIGNMENT 3

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Problem 1. A laser has a 60cm long gain medium and lossless mirrors with reflectivity of r_1 =100% and r_2 =90%

a) What is the threshold gain per unit length?

At the beginning the light of frequency ν. enters the medium from the left with intensity I_0. After passing through the medium of length $z = 60cm$ the light has an intensity $I_0 e^{gz}$ where g is the gain coefficient of the medium. After reflecting off the right mirror the intensity drops and is $r_2 I_0 e^{gz}$. The light then passes through the medium again and the intensity once it leaves the medium is now $r_2 I_0 e^{gz} e^{gz}$. After reflecting off the mirror on the left, the round trip is completed and the light now has intensity $r_1 r_2 I_0 e^{2gz}$. For what gain g do we have that the intensity is unchanged?

$$\begin{aligned} I_0 &= r_1 r_2 I_0 e^{2gz} \\ g &= -\frac{1}{2z}\ln(r_1 r_2) \\ &= -\frac{1}{2(0.6m)}\ln(1.0(0.9)) = 0.0878\text{m}^{-1} \end{aligned}$$

b) If the laser has normal incidence windows, each surface reflecting 4% of the light that is incident on it per pass, what is the threshold gain?

The light goes from Start →Window →Window → Medium →Window →Window →Right mirror →Window →Window →Medium →Window →Window →Left mirror.

So the light's intensity through each step, using $t = 1 - 0.4 = 0.96$ as the transmission coefficient through each window surface,

$$I_0 \to tI_0 \to t^2 I_0 \to t^2 I_0 e^{gz} \to t^3 I_0 e^{gz} \to t^4 I_0 e^{gz} \to r_2 t^4 I_0 e^{gz}$$

Now on the way back

$$r_2 t^4 I_0 e^{gz} \to r_2 t^5 I_0 e^{gz} \to r_2 t^6 I_0 e^{gz} \to r_2 t^6 I_0 e^{gz} e^{gz} \to r_2 t^8 I_0 e^{2gz} \to r_1 r_2 t^8 I_0 e^{2gz}$$

So we want

$$\begin{aligned} I_0 &= r_1 r_2 t^8 I_0 e^{2gz} \\ r_1 r_2 t^8 e^{2gz} &= 1 \\ 2gz &= -\ln(r_1 r_2 t^8) \\ g &= -\frac{1}{2z}\ln(r_1 r_2 t^8) \\ &= -\frac{1}{2(0.6m)}\ln(1.0(0.9)(0.96)^8) \\ &= 0.36\text{m}^{-1} \end{aligned}$$

What this means is that to get a laser to work with these silly windows in the way, we would need a medium whose threshold gain needs to be much higher so the light does not lose intensity.

Problem 2. A typical gain cross section of an atom or molecule usually ranges from 10^{-18} to 10^{-20}cm^2.

a) Using 10^{-18}cm^2, calculate the maximum gain coefficient if the inversion is equal to the number density of air at STP.

The number density of air at STP (according to Wikipedia) is $0.02504 \times 10^{21}\text{cm}^{-3}$. Assuming the inversion is the number density, $\Delta N = 0.02504 \times 10^{21}\text{cm}^{-3}$. The gain coefficient would be

$$g = \sigma \Delta N = 10^{-18}\text{cm}^2(0.02504 \times 10^{21}\text{cm}^{-3}) = 25.04\text{cm}^{-1}$$

b) Same thing with a solid. Let's use diamond, whose number density (and so it's inversion in this case) is $\Delta N = 176.2 \times 10^{21}\text{cm}^{-3}$. So

$$g = \sigma \Delta N = 10^{-18}\text{cm}^2(176.2 \times 10^{21}\text{cm}^{-3}) = 176200\text{cm}^{-1}$$