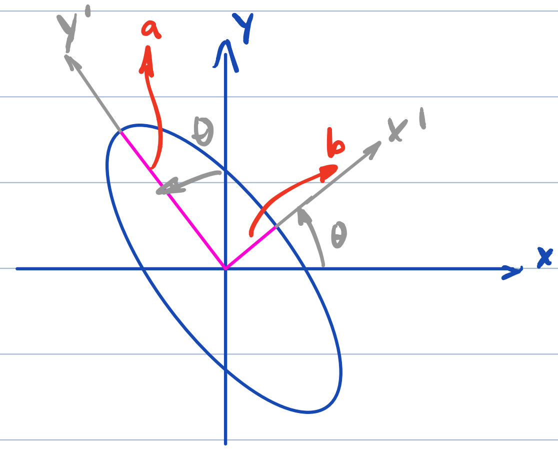
Determination of t_w

The Gaussian function is



$$\left(\frac{x'}{b}\right)^2 + \left(\frac{y'}{a}\right)^2 = 1$$

while we have

$$x' = x\cos\theta + y\sin\theta$$

$$y' = -x\sin\theta + y\cos\theta$$

The Gaussian equation then becomes

$$\frac{1}{b^2}(x\cos\theta + y\sin\theta)^2 + \frac{1}{a^2}(-x\sin\theta + y\cos\theta)^2$$

$$= \frac{1}{b^2}\left(x^2\cos^2\theta + 2xy\sin\theta\cos\theta + y^2\sin^2\theta\right) + \frac{1}{a^2}\left(x^2\sin^2\theta - 2xy\sin\theta\cos\theta + y^2\cos^2\theta\right)$$

$$= x^2\left(\frac{\cos^2\theta}{b^2} + \frac{\sin^2\theta}{a^2}\right) + y^2\left(\frac{\sin^2\theta}{b^2} + \frac{\cos^2\theta}{a^2}\right) + 2xy\sin\theta\cos\theta\left(\frac{1}{b^2} - \frac{1}{a^2}\right)$$

$$= 1$$

If we now set $y=0$, we find

$$x = \frac{\pm ab}{\sqrt{b^2\sin^2\theta + a^2\cos^2\theta}}$$ → set a & b to $\sigma_{y'}$ & $\sigma_{x'}$.

We also need to evaluate the uncertainties on the drift & the burst duration

1. Drift $\rightarrow$ $m = \cot g\,\theta = \dfrac{\cos\theta}{\sin\theta}$

$$\delta m = \frac{\delta\cos\theta}{\sin\theta} + \cos\theta\,\delta\left(\frac{1}{\sin\theta}\right) = -\delta\theta\,\frac{\sin\theta}{\sin\theta} - \cos\theta\,\frac{\cos\theta\,\delta\theta}{\sin^2\theta}$$

$$= -\delta\theta\left(1+\frac{\cos^2\theta}{\sin^2\theta}\right) = -\delta\theta\left(\frac{1}{\sin^2\theta}\right)$$

We then have

$$\boxed{\sigma_m = \frac{\sigma_\theta}{\sin^2\theta}} \quad \text{or} \quad \sigma_m = \sigma_\theta\left(1+m^2\right)$$

2. Burst duration $\rightarrow$ $x_w = \dfrac{ab}{\sqrt{b^2\sin^2\theta + a^2\cos^2\theta}}$

$$\delta x_w = \frac{(\delta a)\,b + a(\delta b)}{\sqrt{b^2\sin^2\theta + a^2\cos^2\theta}} - \frac{1}{2}\,\frac{ab\left[2b\,\delta b\sin^2\theta + 2a\,\delta a\cos^2\theta + 2\,\delta\theta\sin\theta\cos\theta\,(b^2-a^2)\right]}{\left(b^2\sin^2\theta + a^2\cos^2\theta\right)^{3/2}}$$

$$= \left(\frac{\delta a}{a} + \frac{\delta b}{b}\right)x_w - x_w\,\frac{\left[b\,\delta b\sin^2\theta + a\,\delta a\cos^2\theta + \delta\theta\sin\theta\cos\theta\,(b^2-a^2)\right]}{\left(b^2\sin^2\theta + a^2\cos^2\theta\right)}$$

$$= x_w\left[\delta a\left(\frac{1}{a} - \frac{x_w^2\cos^2\theta}{ab^2}\right) + \delta b\left(\frac{1}{b} - \frac{x_w^2\sin^2\theta}{a^2b}\right) + \delta\theta\,\sin\theta\cos\theta\,x_w^2\left(\frac{1}{b^2} - \frac{1}{a^2}\right)\right]$$

$$= x_w\left[\frac{\delta a}{a}\left(1 - \frac{x_w^2\cos^2\theta}{b^2}\right) + \frac{\delta b}{b}\left(1 - \frac{x_w^2\sin^2\theta}{a^2}\right) + \delta\theta\,x_w^2\sin\theta\cos\theta\left(\frac{1}{b^2} - \frac{1}{a^2}\right)\right]$$

We then have (assuming a, b & θ to be uncorrelated)

$$\boxed{\sigma_{x_W} = x_W\left[\frac{\sigma_a^2}{a^2}\left(1-\frac{x_W^2\cos^2\theta}{b^2}\right)^2+\frac{\sigma_b^2}{b^2}\left(1-\frac{x_W^2\sin^2\theta}{a^2}\right)^2+\sigma_\theta^2 x_W^4\sin^2\theta\cos^2\theta\left(\frac{1}{b^2}-\frac{1}{a^2}\right)^2\right]^{1/2}}$$