

Evidence of a shared spectro-temporal law between sources of repeating fast radio bursts

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ABSTRACT

We study the spectro-temporal characteristics of two repeating fast radio bursts (FRBs), namely, FRB 180916.J0158+65 and FRB 180814.J0422+73, and combine the results with those from our earlier analysis on FRB 121102. The relationship between the frequency drift rate, or slope, of individual sub-bursts and their temporal duration is investigated. We consider a broad sample of possible dispersion measure (DM) values for each source to understand the range of valid sub-burst slope and duration measurements for all bursts and to constrain our results. We find good agreement with an inverse scaling law between the two parameters previously predicted using a simple dynamical relativistic model. The remarkably similar behaviour observed in all sources provides strong evidence that a single and common underlying physical phenomenon is responsible for the emission of signals from these three FRBs, despite their associations with different types of host galaxies at various redshifts. It also opens up the possibility that this sub-burst slope law may be a universal property among repeating FRBs, or at least for a significant subclass among them. Our results also suggest that the sub-burst slope law is a proper criterion for dedispersing waterfalls of repeating FRBs, leading to a DM rooted on a simple physical model resting on the relativistic nature of FRBs.

Key words: radiation: dynamics – relativistic processes – radiation mechanisms: non-thermal

1 INTRODUCTION

Fast radio bursts (FRBs) are short duration (\sim millisecond) bursts of energy at radio wavelengths exhibiting large brightness temperatures ($T_B > 10^{32}$ K; Lorimer et al. 2007; Petroff et al. 2019), suggesting that these signals originate from non-thermal objects through some coherent emission mechanism. Still, the origin and underlying physical mechanism of FRBs remain unknown in spite of the large number of proposed models (Platts et al. 2019). FRB signals also undergo a high level of dispersion as they propagate from the source to the observer, a phenomenon quantified through the dispersion measure (DM). This dispersion results from the wavelength dependence of the refractive index of ionized matter in astronomical media through which radiation travels at varying speeds as a function of frequency. While a first Galactic FRB was recently reported by the CHIME/FRB Collaboration and the STARE2 team toward the Galactic magnetar SGR 1935+2154 (The CHIME/FRB

Collaboration et al. 2020; Bochenek et al. 2020), the DM values measured for most FRBs suggest that these signals must emanate from extragalactic sources.

Reported FRBs fall into two groups: one-off events and repeaters. While one-off events form the majority of detections, most of our knowledge about FRBs is based on the study of repeaters. At the time of writing, two repeaters (FRB 121102 and FRB 180916.J0158+65, also respectively known as FRB 20121102A and FRB 20180916B) show periodic behaviours, prompting continued follow-up observations (CHIME/FRB et al. 2020; Rajwade et al. 2020). Importantly, the study of dynamic spectra of repeaters reveals interesting patterns. Among these are a downward drift in the central frequency of consecutive sub-bursts with increasing arrival time within an event (the so-called “sad trombone” effect), and an average reduction in the temporal duration of individual sub-bursts with increasing frequency (Gajjar et al. 2018; Hessels et al. 2019; CHIME/FRB et al. 2019; Josephy et al. 2019).

Several models have been proposed to explain these spectro-temporal characteristics. Some models link these characteristics to

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the intrinsic radiation mechanism of FRBs (Wang et al. 2019; Beloborodov 2020; Metzger et al. 2019) or propagation effects (e.g., plasma lensing in Cordes et al. 2017 or scintillation in Simard & Ravi 2020), while others argue that a combination of both factors can play a part (Hessels et al. 2019). Recently, the detection of the first Galactic FRB (The CHIME/FRB Collaboration et al. 2020; Bochenek et al. 2020) has posed new challenges for existing theoretical models. For example, one sequence of sub-bursts detected toward this source reveals an upward central frequency drift with increasing arrival time (a “happy trombone” effect; see Burst 6 of Hilmarsson et al. 2021 and FRB 190611 in Day et al. 2020 for other examples). A few models have anticipated such a possibility for the spectra of FRBs (Simard & Ravi 2020; Rajabi et al. 2020; Beniamini & Kumar 2020). In particular, Rajabi et al. (2020) proposed a simple dynamical relativistic model where a descending or an ascending central frequency drift for a sequence of sub-bursts can be explained based on the intrinsic properties of the corresponding FRB source (as discussed in Section 1.1 and Appendix B). But more importantly, their model also predicts that a steeper frequency drift should be present within individual sub-bursts (henceforth the “sub-burst slope”¹) where the slope of the FRB signal as displayed in a waterfall (i.e., the signal intensity as a function of frequency and time) obeys a simple law scaling inversely with the temporal duration of the sub-burst. Rajabi et al. (2020) further provided evidence for this sub-burst slope behaviour for FRB 121102 and showed that data taken over a wide range of frequencies for this repeater follow the same law, i.e., the aforementioned inverse scaling of the sub-bursts slope with their corresponding temporal duration. They then argued that this finding implies that the underlying physical process responsible for the signals detected in FRB 121102 is intrinsically narrow-band in nature, while relativistic motions within the source are required to explain the wide observed bandwidths. In this paper, we examine data from two additional repeaters, FRB 180814.J0422+73 (CHIME/FRB et al. 2019) and FRB 180916.J0158+65 (CHIME/FRB Collaboration et al. 2019; CHIME/FRB et al. 2020), and show that this form of law is closely shared between these three FRBs originating from host galaxies at different redshifts. We also consider the effect of the chosen DM on the measurements of the sub-burst slope and the temporal duration in order to understand the robustness of this relationship between the sources. This significant finding reveals new insights on the underlying physical mechanism at the source of FRB signals and helps refine modelling efforts.

1.1 The triggered dynamical model of Rajabi et al. (2020)

Rajabi et al. (2020) introduced a simple dynamical model where a triggering source (e.g., a pulsar or magnetar; see Houde et al. 2019) is located directly behind an FRB source as seen by an observer. An FRB source may contain one or many sub-regions moving towards (or away from) the observer, potentially at relativistic speeds,

¹ We denote an FRB event or burst as containing one lone or many pulses of radiation, as is observed for the so-called sad trombone effect, for example. A single pulse in an event containing several pulses will be defined as a sub-burst. Rajabi et al. 2020 used the term “sub-burst drift” to describe the observed signal delay t_D as a function of the measured frequency ν_{obs} within an individual sub-burst (as in equation (1)). However, since this term is also sometimes found in the literature to denote delays between separate sub-bursts (as in the sad trombone effect), we will instead opt for “sub-burst slope” to describe the internal drift within an individual sub-burst to avoid any possible confusion.

and from which individual sub-burst signals are emanating. Such a scenario is appropriate for situations where the emitted signal is highly collimated, such as is the case for a radiation process based on Dicke’s superradiance which our proposed FRB model is ultimately inspired by (Rajabi & Houde 2016a,b; Rajabi & Houde 2017; Mathews 2017; Houde et al. 2018; Houde et al. 2019; Rajabi et al. 2019; Rajabi & Houde 2020). Under such conditions it can be shown that the slope of a single sub-burst signal (for repeaters an event can contain several sub-bursts) obeys the following relation

$$\frac{1}{\nu_{\text{obs}}} \frac{d\nu_{\text{obs}}}{dt_D} = -\frac{A}{t_w}, \quad (1)$$

where ν_{obs} , t_w and t_D are the frequency, the temporal duration of the sub-burst and the delay before its appearance (in relation to the arrival of the trigger) as measured by the observer. The systemic parameter $A \equiv \tau'_w/\tau'_D$ with τ'_w and τ'_D the corresponding sub-burst proper temporal duration and delay in the reference frame of the corresponding FRB sub-region, respectively.

Following the model of Rajabi et al. (2020), the temporal duration of an FRB sub-burst in the observer rest frame is given by

$$t_w = \tau'_w \frac{\nu_0}{\nu_{\text{obs}}}, \quad (2)$$

where ν_0 is frequency of emission in the FRB rest frame. Equation (2) clearly predicts an inverse relationship between the observed FRB temporal width and frequency, which had previously been noticed and studied. For example, a verification of this effect can be found in Figure 7(b) of Gajjar et al. (2018) for the case of FRB 121102². Although the burst temporal duration exhibits a fair amount of scattering at a given frequency (which could also be inherent to τ'_w in equation (2)), the predicted behaviour is consistent with the observations.

Rajabi et al. (2020) also derived the following equation for the relative drift in the observed central frequency of a sequence of sub-bursts with increasing arrival time

$$\frac{\Delta\nu_{\text{obs}}}{\Delta t_D} = \frac{\nu_{\text{obs}}}{\nu_0} \frac{d\nu_{\text{obs}}}{d\tau'_D}, \quad (3)$$

where the term on the left-hand side is for the relative central frequency drift and τ'_D is the proper temporal delay between the arrival of the trigger and the emission of the ensuing sub-burst in the FRB rest frame. The derivative $d\nu_{\text{obs}}/d\tau'_D$ is a physical parameter characterizing the environment of the FRB source, which determines whether the sequence of sub-bursts has the appearance of a “sad” ($d\nu_{\text{obs}}/d\tau'_D < 0$) or “happy trombone” ($d\nu_{\text{obs}}/d\tau'_D > 0$; see Rajabi et al. 2020 for more details). Equation (3) predicts that the central frequency drift should scale linearly with ν_{obs} , which has previously been verified for FRB 121102 over a wide range of frequencies. This can be asserted, for example, from Figure 3 (top panels) of Hessels et al. (2019). This observed dependency could not be realized if ν_0 changed significantly in equation (3). For example, a change of 50% in ν_0 would markedly affect the appearance of the figure.

We also note that within the context of our triggered model, individual sub-bursts belonging to a single FRB event all results from the same background trigger signal. Their sequence of appearance in time, as seen by the observer, will vary depending on

² As noted in Rajabi et al. (2020), the temporal narrowing effect with observed frequency is likely more pronounced than seen in Figure 7b of Gajjar et al. (2018) as the authors do not measure the duration of individual sub-bursts t_w but rather that of the whole event. The true values for t_w are therefore likely to be lower and the sub-burst slope more pronounced in several instances.

the physical properties of the medium where individual sub-bursts emanate from (which will affect the delay time in the corresponding rest frame τ'_D) and its velocity β (and therefore frequency ν_{obs} relative to the observer). Although we expect sub-bursts belonging to a single FRB event to be clustered in time, it is also possible that sub-bursts belonging to different events be observed relatively closely in time.

2 BURST ANALYSIS

Although equation (1) was tested and verified for FRB 121102 in Rajabi et al. (2020) using previously published data covering more than a decade in frequency (Michilli et al. 2018; Gajjar et al. 2018; Josephy et al. 2019), it was not known at the time whether it applies equally well to other repeating FRBs. We therefore retrieved and analyzed previously published data for two other sources discovered by the CHIME/FRB Collaboration (Fonseca et al. 2020), namely FRB 180916.J0158+65 (CHIME/FRB et al. 2020) and FRB 180814.J0422+73 (CHIME/FRB et al. 2019). These data are all contained within the CHIME/FRB spectral band (approximately 400–800 MHz) and the corresponding dynamic spectra were analyzed using the two-dimensional autocorrelation technique introduced in Hessels et al. (2019), resulting in estimates for the sub-burst slope ($d\nu_{\text{obs}}/dt_D$) and temporal duration (t_w). See Appendix A for more details. These data sources were chosen purely due to their ease of accessibility and the support available. Ultimately we aim to extend this analysis to as many sources and bursts as possible.

2.1 The effect of the Dispersion Measure (DM)

Since the measurement of any drift rate (or almost any other spectro-temporal feature) is strongly dependant on the DM that is used to dedisperse a waterfall, and since the DM of a source can potentially vary from burst to burst as well as with time, we studied the variation of our slope and temporal duration measurements for each sub-burst at different choices of DM. Dedispersion can be performed by optimizing either the signal-to-noise (S/N) or a structure parameter and can result in different values found for the DM depending on the burst (e.g. Fig 1 of Gajjar et al. 2018). In particular, an algorithm seeking to choose a DM by maximizing S/N might stack the individual sub-bursts of a complex FRB event and yield a DM value that is higher than a structure optimizing algorithm. For bursts with components that are not clearly resolved it becomes ambiguous which algorithm is most accurate and the precision in the DMs determined burst to burst can be much narrower than the variations in the DM observed overall for a source (CHIME/FRB et al. 2020). It therefore becomes difficult to uncouple FRB characteristics from the nature of the medium in order to study relationships between spectro-temporal features as we hope to do. One option is to use the DM found on a burst by burst basis. However, doing this can become a complicated process of verifying that the DM algorithm choice is appropriate, which will often be ambiguous for smeared bursts where it is not clear if it consists of multiple components or not. Without a detailed understanding of the emission mechanism, the medium, the source, and the resulting DM distribution as a function of time, it is in fact much simpler and more conservative to choose a DM range as wide as possible based on the history of DMs found for the source. We shall see that despite the significant uncertainties this choice entails, the data still point to the existence of an inverse trend between the sub-burst slope and temporal duration for the three sources considered here.

Source	Data source	DM Range (pc/cm ³)
FRB 121102	Michilli et al. 2018	554.1–565.3
FRB 121102	Gajjar et al. 2018	555–570 (555–583)
FRB 180916.J0158+65	CHIME/FRB et al. 2020	346.82–349.82
FRB 180814.J0422+73	CHIME/FRB et al. 2019	188.7–190.0

Table 1. The range of DMs used to determine the range of possible values of each sub-burst slope and duration. These are chosen to be as wide as possible while still obtaining reasonable sub-burst slope measurements. In general, the published history of DMs found for a source (all bursts considered) determines the range used, with some DMs on the higher end excluded due to resulting positive sub-burst slopes or distortion. The DM range in parentheses is used specially for Burst 11D from FRB 121102 in Gajjar et al. 2018 due to its high S/N optimized DM. See the text for more details.

Table 1 shows the DM ranges chosen for each source and dataset. We aim to consider as broad a range of DMs as possible while still obtaining reasonable sub-burst slope measurements. For the data used from Michilli et al. 2018 DM variations are estimated by those authors to be $\lesssim 1\%$ of 559.7 pc/cm³, and we therefore consider a range of 554.1–565.3 pc/cm³. For the data from Gajjar et al. 2018, due to availability, we use the sub-bursts in Burst 11A and Burst 11D. A structure optimized DM for 11A is found at 565 pc/cm³, and their Figure 1 indicates that DMs between 555–570 are also close to optimal, so we adopt this range. For Burst 11D, due to a lack of structure we consider a range of 555–583 to be closer to its S/N optimized DM, however higher DMs are excluded as the sub-burst slopes start to become positive (which are not physical according to our model and in general usually indicate too aggressive of a dedispersion, as described in Section 2.2). For data from CHIME/FRB et al. 2020 on FRB 180916.J0158+65 a precise DM of 348.82 ± 0.05 pc/cm³ is found for one of the bursts, but burst-to-burst the DM can range from 348.7–350.2. We therefore choose a mid-point of about 348.82 pc/cm³ and adopt a range of 346.82–349.82 pc/cm³. The lower value for the start of the range is chosen to push the limit of acceptable DMs while still obtaining reasonable sub-burst slope measurements. We stay away from the higher end of the observed range due to the sub-burst distortion and positive slopes observed for most cases at that high of a DM. Finally, for data from CHIME/FRB et al. 2019 on FRB 180814.J0422+73, due to the structure present in the bursts, we extend the full range of structure optimized DMs found (188.9–190 pc/cm³) to 188.7–190 pc/cm³. We ignore the higher S/N optimized DMs due to the component stacking and distortion observed when dedispersing to those DMs.

For each source, we generate a grid of DMs over the range chosen and dedisperse all bursts to each DM before performing an autocorrelation analysis. The grid spacing varies from $\Delta\text{DM} \approx 0.1\text{--}2$ pc/cm³ depending on the source, yielding approximately 10–20 trial DMs in each case. For FRB events with multiple components like Burst 11A from Gajjar et al. 2018 for FRB 121102, the components are separated manually by finding valleys in the corresponding time series of the data. When necessary these components are padded with a background sample of the waterfall so that there is a wide enough temporal extent to properly dedisperse the burst. Some bursts are not clearly resolved, but wherever there is indication that the slope suddenly changes mid-burst a manual attempt is made to separate the components.

The autocorrelation analysis (see Appendix A) is then performed for all dedispersed waterfalls to obtain sub-burst slope and

temporal duration measurements for every burst at each DM (Hessels et al. 2019). We use these data to determine the range of possible values for each measurement. Examples of a waterfall for every sub-burst used in this analysis with their corresponding autocorrelation are shown in Figures 6 – 10 at the end of the paper, displayed for one of the trial DMs. The range found for each of these measurements is much larger than the parameter uncertainty resulting from the underlying two-dimensional Gaussian fit of the autocorrelation function used to evaluate them. Since the true underlying DM distribution for each source appears to be narrower than the DM range we have used (considering the distribution so far implied by published DMs and knowing that the distribution can change with time), the range of values found this way must be larger than the range implied by the true uncertainties for each measurement. We therefore treat the range of values found by this analysis as upper-limit estimates of the real measurement uncertainties.

2.2 Measurement exclusions and fitting

With the measurements for each sub-burst at all trial DMs found, there remain measurements that are unconstrained and/or non-physical that need to be discarded before fitting. As previously mentioned we discard any positive sub-burst slope measurements which are non-physical under our model, as well as measurements where the value and/or uncertainties approaches infinity, as is the case for sub-bursts that become near vertical or circular in their autocorrelation.

The result of this exclusion process is that out of a total of 41 sub-bursts analysed, we retain all the measurements made for 28 sub-bursts over the entirety of the DM ranges specified in Table 1. For the remaining 13 sub-bursts the measurements excluded were taken at the higher end of the DM range, since they yielded positive sub-burst slopes. For all but one of the sub-bursts treated this way the DM range is slightly further restricted. The exception being one burst from FRB 180814.J0422+73, where the DM range is limited from $188.7 - 190.0 \text{ pc/cm}^3$ to $188.8 - 188.9 \text{ pc/cm}^3$. We specifically identify (i.e., circle) these sub-bursts when displayed in Figures 1 and 5 below to indicate the smaller range of DMs used.

Using this set of sub-burst slope and temporal duration measurements we find a fit to equation (1) at each DM and compute the reduced- χ^2 to select a representative DM for each source (i.e., the DM with the reduced- χ^2 closest to unity). The representative DMs found are 558.8 and 568.3 pc/cm^3 for FRB 121102 for the data from Michilli et al. 2018 and Gajjar et al. 2018, respectively, 348.82 pc/cm^3 for FRB 180916.J0158+65 and 188.8 pc/cm^3 for FRB 180814.J0422+73.

3 RESULTS AND DISCUSSION

We show in Figure 1 the results of our analysis, where the sub-burst slope (normalized to the frequency of observation ν_{obs}) is plotted against the temporal width t_w for the three FRBs. Normalizing the sub-burst slope has the advantage of allowing us to combine the different sources on the same graph irrespective of the frequency of observation, shifts due to the dynamical Doppler effect or cosmological redshift. Furthermore, we note that equation (1) is also insensitive to temporal scaling transformations. For example, interstellar scintillation, which brings a temporal broadening scaling inversely with the fourth power of the frequency, will have no effect on our analysis. The only consequence being a shift of data points along the specific law characterized by the parameter A in equation

(1). The points displayed in Figure 1 are the measurements of each sub-burst obtained at the representative DM described at the end of Section 2.2, and the capped lines represent the range of possible measurements over the DM range considered.

Examination of Figure 1 reveals that the inverse relationship between the two parameters is clearly seen for all sources on the graph for values ranging over two orders of magnitude for both the normalized sub-burst slope and the temporal duration. Also shown in the figure are fits for the predicted function A/t_w (see equation (1)) for the three sources at their representative DMs (see end of Section 2.2), with $A = 0.078 \pm 0.006$, 0.082 ± 0.006 and 0.076 ± 0.013 for FRB 121102, FRB 180916.J0158+65 and FRB 180814.J0422+73, respectively. The shaded regions for each source represent the range of fits found when considering all DMs in the adopted range. The corresponding range of fit parameters are found to be $A = 0.042 - 0.355$ for FRB 121102, $A = 0.030 - 0.153$ for FRB 180916.J0158+65 and $A = 0.071 - 0.152$ for FRB 180814.J0422+73. These regions overlap significantly, but leave open the possibility of unique and distinct fits between the three sources.

A few important consequences are to be noted from the results presented in Figure 1. First, and most importantly, the inverse relationship between the sub-burst slope and temporal duration is observed to be independent of the selection of the DM. That is, there is an unmistakable tendency for narrower sub-bursts at higher slopes, and vice-versa, irrespective of the DM chosen. Even with the ranges of measurements seen, Figure 1 strongly suggests each source obeys a form of equation (1), where we see a clear decrease in the magnitude of slopes with increasing sub-burst durations.

Second, we note the possibility that not only is the inverse relationship between the sub-burst slope and temporal duration verified for the three sources, but they do so with similar values for A in equation (1) at their representative DMs. The different fits to this systemic parameter are similar given their uncertainties, and it is difficult to visually distinguish between the corresponding curves. This closeness between the values obtained for A is rather remarkable and suggests the existence of a single and common underlying physical phenomenon responsible for the emission of FRB signals in the three sources. This is significant because these FRBs are associated with different types of host galaxies at various redshifts. More precisely, FRB 121102 is localized to a low-metallicity irregular dwarf galaxy at a redshift $z = 0.193$ (Tendulkar et al. 2017), while the redshift of FRB 180814.J0422+73 is estimated to be $z \leq 0.1$ (CHIME/FRB et al. 2019). Furthermore, the candidates for the host galaxy of FRB 180814.J0422+73 are not consistent with those harboring long gamma-ray bursts (LGRBs) or superluminous supernovae (SLSNe), unlike the host galaxy of FRB 121102 (Li et al. 2019). As for FRB 180916.J0158+65, it is precisely localized to a star-forming region in a massive spiral galaxy at a redshift $z = 0.0337$ (Marcote et al. 2020). This source is the closest known extragalactic FRB, whose host galaxy does not show signatures of a strong magnetic field nor a radio counterpart as reported for FRB 121102. The similarities in the values for A between the three sources also suggests that the sub-burst slope law is a suitable criterion for dedispersing waterfalls of repeating FRBs. The resulting choice of DM would have the advantage of being rooted on a simple physical model resting on the relativistic nature of FRBs, instead of more arbitrary choices based on the S/N or other structure criteria.

As was discussed in Rajabi et al. (2020), the three predictions made by their simple dynamical model (i.e., the narrowing of sub-

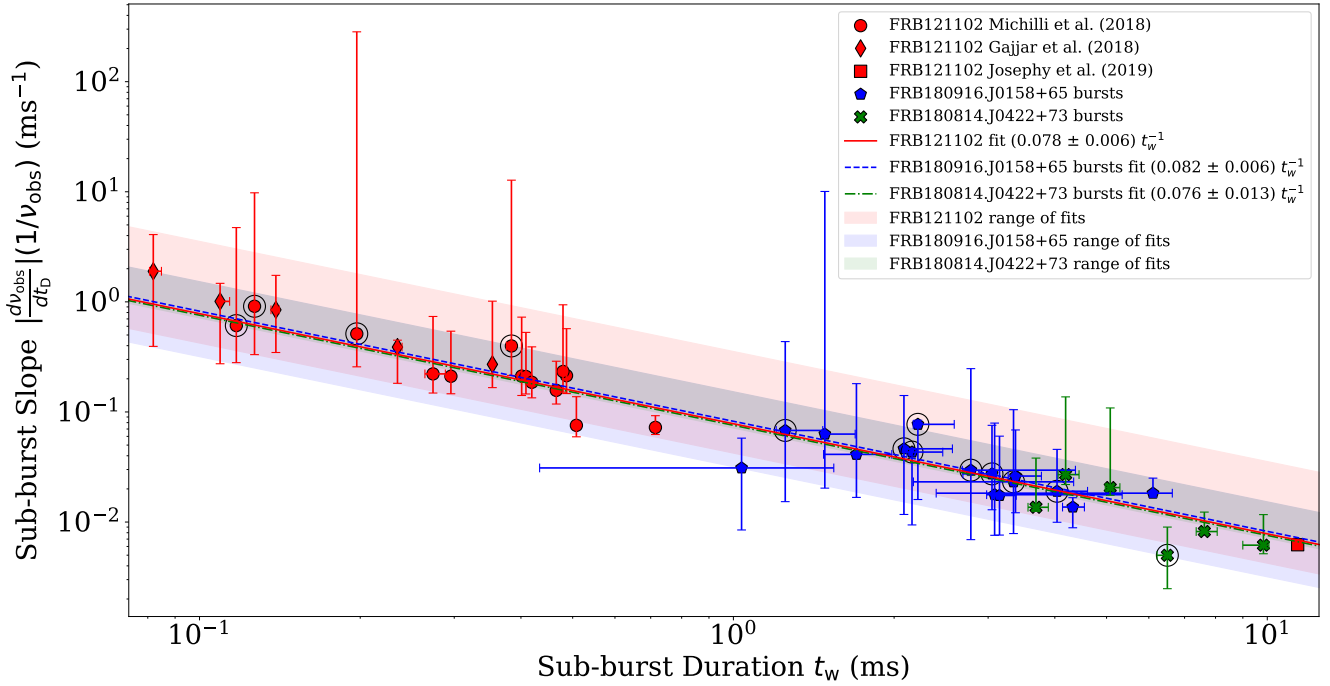


Figure 1. A plot of $|dv_{\text{obs}}/dt_D| (1/\nu_{\text{obs}})$ vs. t_w for bursts from FRB 121102 (red circles, diamonds and square; [Gajjar et al. 2018](#); [Michilli et al. 2018](#); [Josephy et al. 2019](#)), FRB 180916.J0158+65 (blue pentagons; [CHIME/FRB et al. 2020](#)) and FRB 180814.J0422+73 (green crosses; [CHIME/FRB et al. 2019](#)). The sub-burst slope dv_{obs}/dt_D and duration t_w were obtained using the two-dimensional autocorrelation technique of [Hessels et al. \(2019\)](#), while the center frequency ν_{obs} was estimated from the corresponding dynamic spectra. Each burst was dedispersed to a grid of trial DMs over the range specified by Table 1 and the measurements were repeated. The one point from [Josephy et al. 2019](#) was not part of the same analysis and is shown for reference. The red, blue and green lines are for fits of the function A/t_w on the FRB 121102, FRB 180916.J0158+65 and FRB 180814.J0422+73 data, respectively, at the DM within the range of trial DMs for which the reduced- χ^2 of the fit was closest to unity, and are difficult to distinguish from one another. All points for a given source (except for the [Josephy et al. 2019](#) datum) are of measurements made at the same DM used for the corresponding fit. The capped lines at each point represent the range of possible measurements obtained via the autocorrelation analysis for different DMs over the DM ranges chosen. As discussed in Section 2.1, these are used in lieu of, and are larger than, the difficult to determine true measurement uncertainties. The circled points indicate sub-bursts that required a limited DM range to constrain their measurements (see Section 2.2). The shaded regions represent the range of fits found when using measurements obtained at other DMs in the range. These regions overlap significantly, but indicate the possibility of unique and distinct fits between the three sources within the range of possible DMs chosen.

bursts width t_w with increasing frequency ν_{obs} , the sad or happy trombone effect and the sub-burst slope law discussed here) provide strong evidence that the underlying physical phenomenon is narrow-band in nature. This is because the dependencies on ν_{obs} and the frequency of emission in the FRB rest frames ν_0 for the three predicted relationships are such that it would be difficult to envision how they could be realized through the data if ν_0 was allowed to vary substantially (see Section 3.3 for more details). Although data over a significant range of observed frequency is currently only available for FRB 121102 (and constitutes the basis of the analysis presented in [Rajabi et al. 2020](#)), the fact that FRB 180916.J0158+65 and FRB 180814.J0422+73 follow the same law renders it reasonable to expect that the conclusions reached for FRB 121102 also apply to them.

We can use this information with our model to further characterise the environment of the sources responsible for the detected bursts. Indeed using the extensive data available for FRB 121102 one can estimate, although with limited precision at this point, the maximum Lorentz factor and the rest frame frequency of emission ν_0 . To do so we will assume highly simplified conditions, i.e., that the different FRB reference frames from which the individual sub-bursts emanate either move towards or away from the observers with

the same range of speeds. We will denote by $\beta^+ > 0$ and $\beta^- = -\beta^+$ the maximum velocities (divided by the speed of light) towards and away from the observer, respectively, with corresponding observed frequencies ν_{obs}^\pm . It is then straightforward to show that, under this assumption,

$$\beta^+ = \frac{\nu_{\text{obs}}^+ - \nu_{\text{obs}}^-}{\nu_{\text{obs}}^+ + \nu_{\text{obs}}^-} \quad (4)$$

$$\nu_0^2 = \nu_{\text{obs}}^+ \nu_{\text{obs}}^- \quad (5)$$

Using $\nu_{\text{obs}}^+ \approx 7.5$ GHz and $\nu_{\text{obs}}^- \approx 630$ MHz we find $\beta^+ \approx 0.9$ and $\nu_0 \approx 2.6$ GHz for FRB 121102 (taking into account its known redshift $z = 0.193$ from [Tendulkar et al. 2017](#); see Section 3.3 for more details). Evidently, the accuracy for these estimates is set and limited by the frequency coverage of the existing data and is likely to change as more detections are acquired. For example, confirming the purported detection of signals at 111 MHz from [Fedorova & Rodin \(2019\)](#) would further increase β^+ and bring down ν_0 on the order of 1 GHz. At any rate, these results imply that FRB 121102 is potentially very strongly relativistic.

We also know that the spectral width $\Delta\nu_{\text{obs}}$ associated to individual sub-bursts for FRB 121102 scales as $\Delta\nu_{\text{obs}} \sim 0.16 \nu_{\text{obs}}$ on average (see Figure 6 in [Rajabi et al. 2020](#) or Figure 5 in [Houde](#)

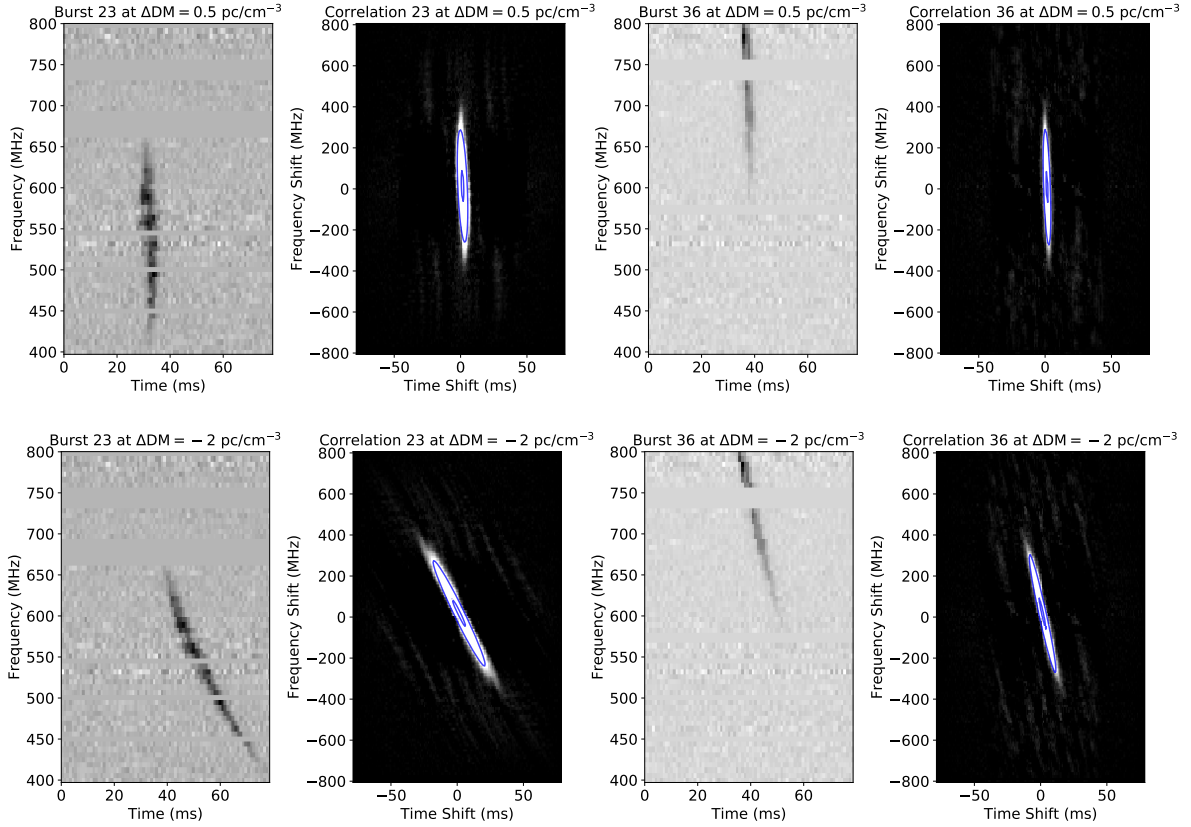


Figure 2. Changes to waterfalls and autocorrelations due to variations in the DM. Sub-bursts 23 (first column) and 36 (third column) for FRB 180916.J0158+65 are shown with their autocorrelation functions (second and fourth columns, respectively) for two offsets $\Delta\text{DM} = 0.5 \text{ pc cm}^{-3}$ (top row) and -2 pc cm^{-3} (bottom row) from the representative value chosen for our analysis (i.e., $\text{DM} = 348.82 \text{ pc cm}^{-3}$). The rotations brought about by the small changes in DM are clearly seen in both the waterfall and autocorrelation functions. See Figure 8 for these bursts dedispersed to their representative DM.

et al. 2019). This spectral extent is the result of motions (through the Doppler effect) within a given FRB rest frame from where a sub-burst centred at ν_{obs} originates. As discussed in Appendix B, the observed spectral width is constrained through

$$2\Delta\beta' \leq \frac{\Delta\nu_{\text{obs}}}{\nu_{\text{obs}}} \leq \frac{2\Delta\beta'}{1 - \Delta\beta'^2}, \quad (6)$$

where the motions in the FRB rest frame are contained within $\pm\Delta\beta'$. We thus find $\Delta\beta' \sim 0.08$ with equation (6) for this source.

We thus have a picture where FRB 121102 and similar sources would consist of systems within which a number of spatially distinct FRB rest frames, whose motions cover a wide range of velocities (some highly relativistic relative to the observer; $|\beta| \lesssim 0.9$ for FRB 121102), are responsible for the emission of individual sub-bursts. In turn, each such rest frame is also host to mildly relativistic motions ($|\Delta\beta'| \lesssim 0.08$ for FRB 121102), which are responsible for the observed wide spectral widths of sub-bursts.

In the following sections we discuss the effect of DM variations on the autocorrelation of waterfalls, as well as the effects of noise and missing data on the measurement of the sub-burst slope and temporal duration. We also further discuss the determination of physical parameters of the source.

3.1 DM variations as a rotation of the autocorrelation function

Following the study of the variation of measurements over ranges of plausible DMs discussed in Sec. 2.1, different DM choices can be modeled as rotations of the autocorrelation function of the burst. As an example, we show in Figure 2 two bursts each at two choices of DM. In a given waterfall we see that the shape of the burst can ‘distort’ due to the ν_{obs}^{-2} dependence on the dispersion, while that of the autocorrelation remains practically the same except for experiencing a rotation. To characterize this further we consider the sub-burst angle parameter (as opposed to the corresponding sub-burst slope derived from said angle) defining the orientation of the fitted ellipsoid’s semi-major axis measured counterclockwise from the positive frequency axis of the autocorrelation function (see Appendix A). The sub-burst angles from FRB 180916.J0158+65 are plotted against the corresponding temporal durations derived from the underlying two-dimensional Gaussian fits in Figure 3. This shows that across different DMs the measured duration varies little while the angle is offset by a constant level from values at other DMs. We can demonstrate this using equations (1) and (A2) to find that the slope angle θ is related to the sub-burst duration through

$$\theta = \arctan \left(\frac{1}{A} \frac{\nu_{\text{res}}}{\nu_{\text{obs}}} \frac{t_{\text{w}}}{t_{\text{res}}} \right), \quad (7)$$

where as before $A \equiv \tau'_{\text{w}}/\tau'_{\text{D}}$, and ν_{res} and t_{res} are the frequency and time resolutions of the waterfall. We also approximated ν_{obs}

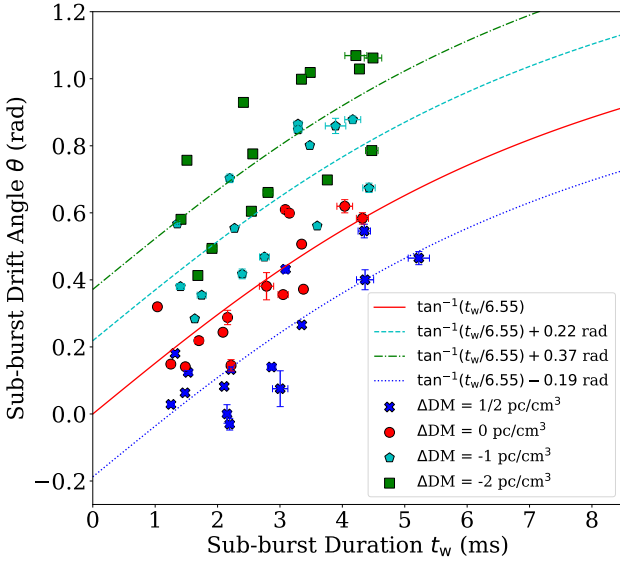


Figure 3. The fit angle θ vs. temporal duration t_w from sub-bursts dedispersed to small variations in the DM for the source FRB 180916.J0158+65. Red circles are sub-bursts at $\Delta\text{DM} = 0$, which corresponds to a DM = 348.82 pc cm⁻³. Blue crosses, cyan pentagons, and green squares are sub-bursts dedispersed to $\Delta\text{DM} = 0.5, -1$, and -2 pc cm⁻³, respectively. Error bars indicate the parameter fitting uncertainty. The red curve is the fit to the red circles and is of the form given in equation (7), derived from the dynamical model described in the main text. Blue, cyan, and green curves are obtained by adding a rotation (i.e., adding an angle) to the $\Delta\text{DM} = 0$ model. As discussed in Section 3.1 this plot demonstrates the rotational effect small variations in the DM can have on the autocorrelation of FRB waterfalls.

to be constant, which is adequate for this purpose. We find that the chosen fit obtained with equation (7) for the sub-bursts at $\Delta\text{DM} = 0$ (i.e., the solid curve in Figure 3) is also satisfactory for angles corresponding to the different ΔDM values when a simple offset angle (i.e. a rotation) is applied. Similar trends appear to hold for the other two sources considered, however it is most clear in the example of FRB 180916.J0158+65.

Considering the sub-burst angle instead of the slope during analysis at first appears to avoid the discontinuity in slope measurements around $\theta = 0$ or π , where its magnitude approaches infinity. The behaviour of slope measurements derived from the parameter angle in the context of autocorrelation noise is discussed in more detail in Pleunis (2020) as well as in the `dftd`³ package.

3.2 Uncertainty due to frequency band masking

In addition to signal noise, the waterfall analyses were complicated by missing frequency bands of data, which would sometimes overlap with the frequency extent of the sub-burst under consideration. In this section we assess the extent of the uncertainty introduced by the missing frequency band data by (1) artificially masking (zero-padding) various trial Gaussian signals of known orientations and characteristic widths, (2) processing them through our pipeline, and (3) comparing the extracted sub-burst slope and duration parameters to the generating parameters.

Consider for example Burst 23 of FRB 180916.J0158+65 (CHIME/FRB et al. 2020) pictured, along with its two-dimensional

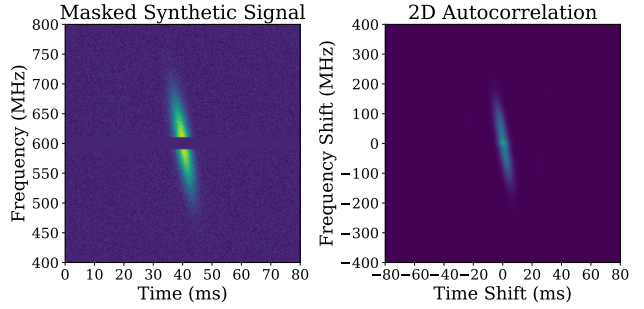


Figure 4. Synthetic Gaussian signal with a masked (zeroed) band (left), and 2D autocorrelation of masked signal (right). The signal shown approximately matches the characteristics of Burst 23 depicted in Figure 8 in each of their temporal widths, their frequency extents, their inclinations, and their total missing frequency bandwidths.

autocorrelation, in Figure 8. Three frequency bands of data are absent from the original data in this burst, and the total missing bandwidth (as a fraction of the frequency extent of the sub-burst) is higher than the fractional bandwidth typically absent from sub-bursts analyzed in the paper.

To estimate the effect of missing frequency channels on our analysis we construct an artificial burst similar to Burst 23 from a Gaussian with a standard deviation along the semi-major axis of $a = 67$ MHz (90 pixels), standard deviation along the semi-minor axis $b = 2.2$ ms (15 pixels), $\theta = 10^\circ$ (inclination from vertical), and with stochastic noise of amplitude 25% that of the Gaussian amplitude. We perform our analysis on a 2D array with dimensions 540×540 pixels, having horizontal and vertical resolutions of 6.75 px/ms and 1.35 px/MHz, respectively. As a first test, we mask a band of width 18.5 MHz (25 pixels) through the center of the burst and pass this zero-padded signal through our pipeline.

The fitting procedure on the 2D autocorrelation returns $a_{\text{fit}} = 102.7$ pixels, $b_{\text{fit}} = 14.8$ pixels, and $\theta_{\text{fit}} = 9.83^\circ$. The process is visualized in Figure 4. For such a small inclination angle, the percentage error in t_w is very close to that of b , and is (in this case) approximately 1%. The corresponding percentage error in the sub-burst slope dv_{obs}/dt_D is 1.7%.

We can generalize this test by shifting the frequency masking band of Figure 4 vertically. Upon doing so, we find that the error is independent of the frequency band's vertical position. The percentage error for the burst duration is found to be $\approx (-1.4 \pm 0.4)\%$, where the $\pm 0.4\%$ uncertainty applies to all band vertical positions tested, while the corresponding error in the angle is $\approx (-1.1 \pm 0.7)\%$.

If we rotate the burst of Figure 4, while retaining the central band mask of 18.5 MHz (25 pixels) on burst centre, we observe a linear enhancement of error with increasing orientation. The effect is, however, a negligible one: for every burst rotation by 10° , the duration error increases by only 0.45%, while the orientation angle error decreases (or increases in magnitude) by only 0.12° . At a 30.0° burst angle, the sub-burst slope error is only 4%.

3.3 The narrow-band nature of the emission process

As discussed in Section 1.1, results from Gajjar et al. (2018) and Hessels et al. (2019) for FRB 121102 point to a rest frame frequency of emission ν_0 that does not change significantly from burst to burst. The results presented here can be shown to also be consistent with a narrow-band emission process by inserting equation (2) into

³ <https://github.com/zpleunis/dftd>

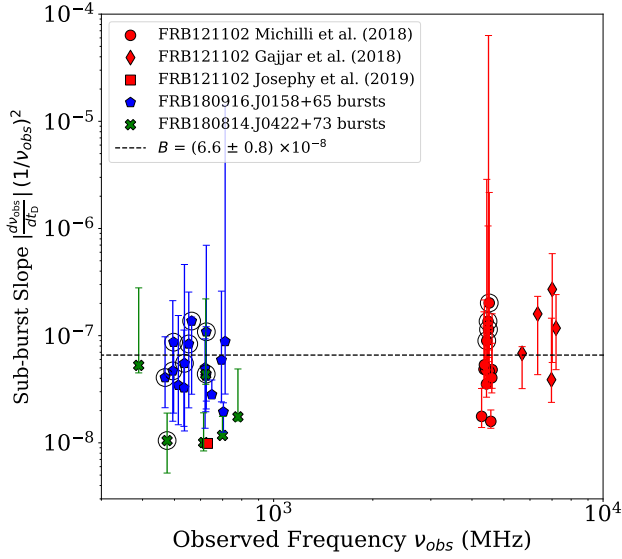


Figure 5. A plot of $|dv_{\text{obs}}/dt_D| (1/\nu_{\text{obs}}^2)$ vs. ν_{obs} for FRB 121102 (red circles, diamonds and square from Gajjar et al. 2018, Michilli et al. 2018 and Josephy et al. 2019, respectively), FRB 180916.J0158+65 (blue pentagons; CHIME/FRB et al. 2020) and FRB 180814.J0422+73 (green crosses; CHIME/FRB et al. 2019). The broken black line is for a fit to a constant B on the combined data for the three sources, with $B \equiv (\tau'_D \nu_0)^{-1} = (6.6 \pm 0.8) \times 10^{-8}$. The capped lines at each point represent the range of possible values due to the range of sub-burst slopes measured at different DMs, as discussed in Section 2.1.

equation (1) to obtain

$$\frac{1}{\nu_{\text{obs}}^2} \frac{d\nu_{\text{obs}}}{dt_D} = -\frac{1}{\nu_0 \tau'_D}, \quad (8)$$

for the sub-burst slope (normalized to ν_{obs}^2), which is then predicted to be independent of ν_{obs} and scale inversely with ν_0 . Figure 5 shows the corresponding plot using the same data as in Figure 1. The broken black line is for a fit to a constant B on the combined data for the three sources, with $B \equiv (\tau'_D \nu_0)^{-1} = (6.6 \pm 0.8) \times 10^{-8}$. While there is some scatter in the data, the result is consistent with the expected lack of dependency on ν_{obs} . Any deviation could easily be accounted for with the uncertainty on the DMs and inherent variations in τ'_D . The combination of this result with the temporal narrowing and sad trombone effects discussed in Section 1.1 for FRB 121102 provides evidence for the narrow-band nature of the emission process.

4 CONCLUSION

We demonstrate a method of studying the sub-burst slope in the context of DM variations from burst to burst and over time by adopting large ranges of possible DMs when measuring spectro-temporal properties of FRBs. This method reveals that even given a wide range of possible DMs for each burst from an FRB source, the slope of an individual sub-burst is inversely proportional to its temporal duration. Furthermore, for the three sources considered in this work, namely FRB 121102, FRB 180916.J0158+65 and FRB 180814.J0422+73, significant overlap between the inverse

trends found is consistent with the three relationships having a nearly identical scaling. That is, the same law can be used to describe sub-bursts from all three sources, though careful analyses over larger data sets at different frequencies would be needed to verify this. Importantly, this result suggests that the sub-burst slope law is a proper criterion for dedispersing waterfalls of repeating FRBs, leading to a DM rooted on a simple physical model resting on the relativistic nature of FRBs.

We believe that the simplest explanation for the existence of this trend is that the emission mechanism of these FRB sources is narrow-band in nature, which would be consistent with our earlier models based on Dicke’s superradiance. Such a mechanism requires a trigger, which leaves room for magnetar-centric models of FRBs. To further study the relationship between the sub-burst slope and temporal duration future analyses of FRBs from all known repeater sources can be performed in the manner presented here. A large sample of sources helps to constrain the uncertainties due to variations in DM, and necessitates convenient and public access to FRB data.

Finally, we note that our discovery of a shared sub-burst slope law among these three sources suggests that this could be a universal property among repeating FRBs or at least a significant subclass of them. If deviations from this relationship exist, then it is likely the sub-burst slope law can serve as a classification tool for FRBs by discriminating sources that follow this law from those that do not. This not only motivates further searches but also provides a new tool to study and categorize FRBs based on their underlying physical mechanism.

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DATA AVAILABILITY

The data pipeline is made available at <https://github.com/mef51/subdriftlaw> and maintained by M.A.C. Aggregate data of the bursts and the code for the figures are also available. Data of the FRB spectra are available either publicly or via the authors of their respective publications. The figures in this paper were prepared using the matplotlib package (Hunter 2007).

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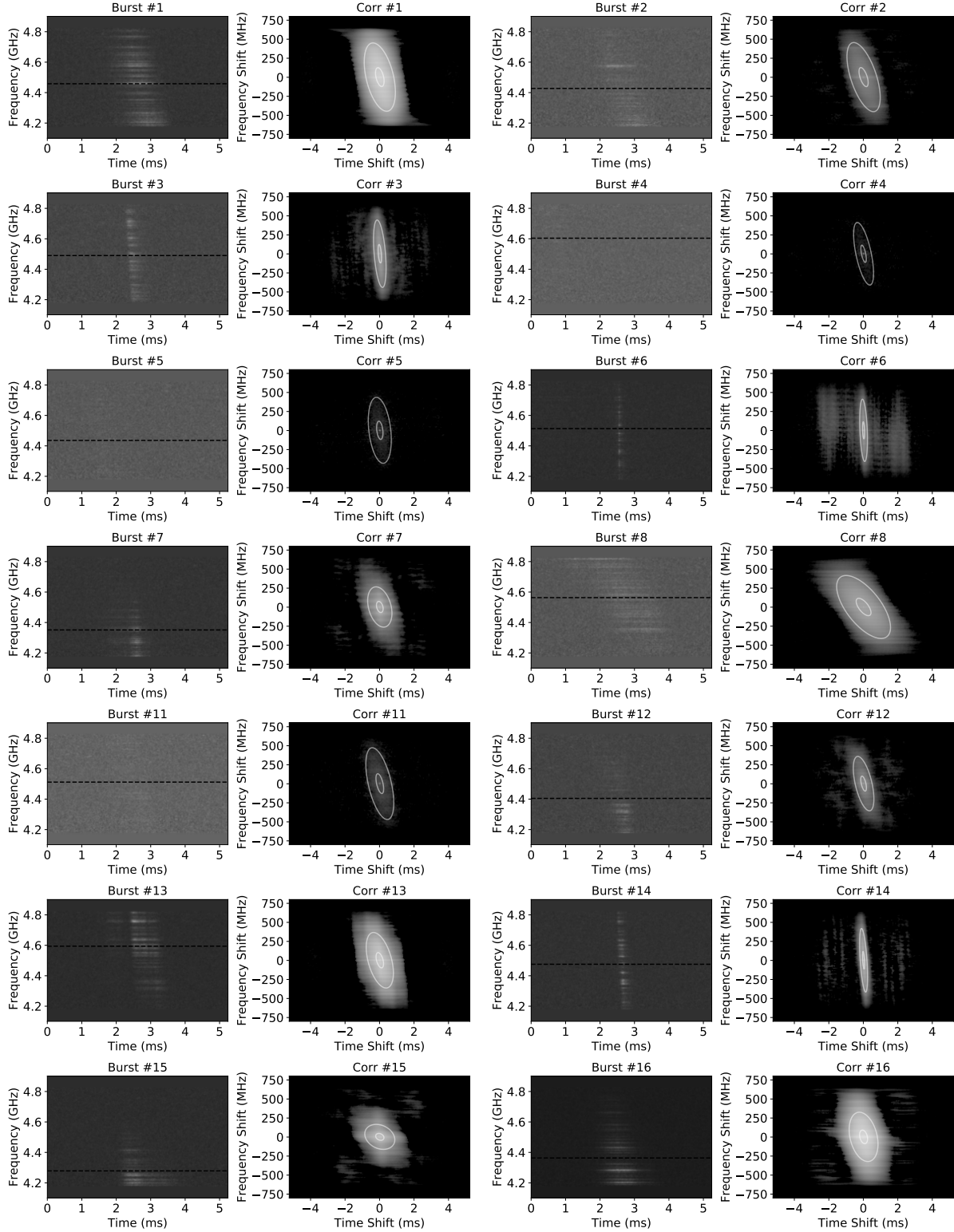


Figure 6. Dynamic spectra (first and third columns) and corresponding autocorrelation functions (second and fourth columns) for FRB 121102 bursts at a frequency of approximately 4–5 GHz from [Michilli et al. \(2018\)](#). The dynamic spectra were dedispersed with a $DM = 559.7 \text{ pc cm}^{-3}$ and the dashed horizontal line in the waterfall denotes the center frequency ν_{obs} used for the analysis. The autocorrelation functions are modelled with a 2D Gaussian ellipsoid whose one- and two-standard deviation levels are shown using the white contours.

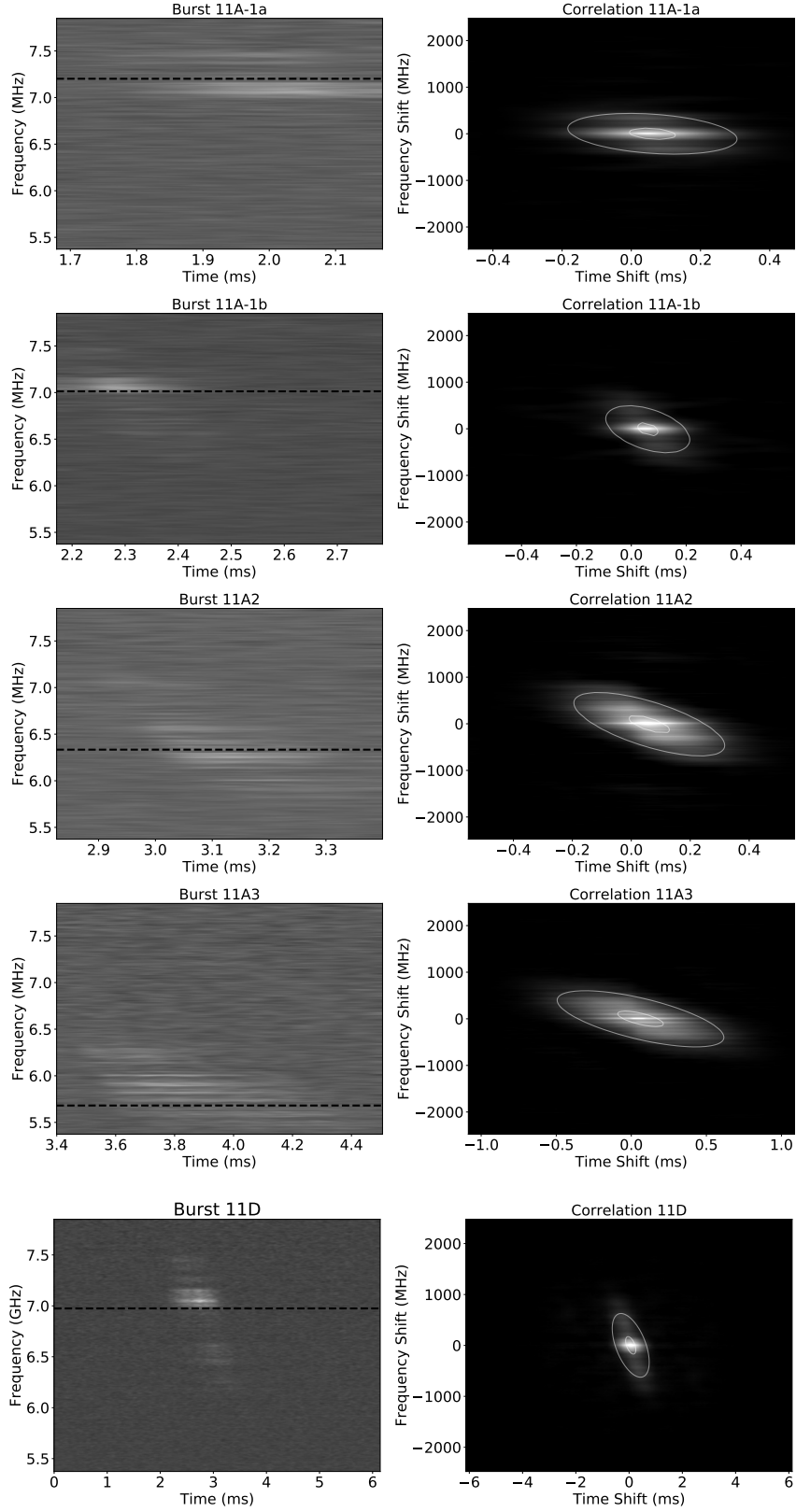


Figure 7. Same as Figure 6 but for the FRB 121102 data at approximately 5–8 GHz published in [Gajjar et al. \(2018\)](#) and dedispersed with a $DM = 565 \text{ pc cm}^{-3}$. The top four sub-bursts are taken from one event, i.e., Burst 11A. Note that the time axes for the autocorrelation functions do not all share the same range, which distorts their relative appearance.

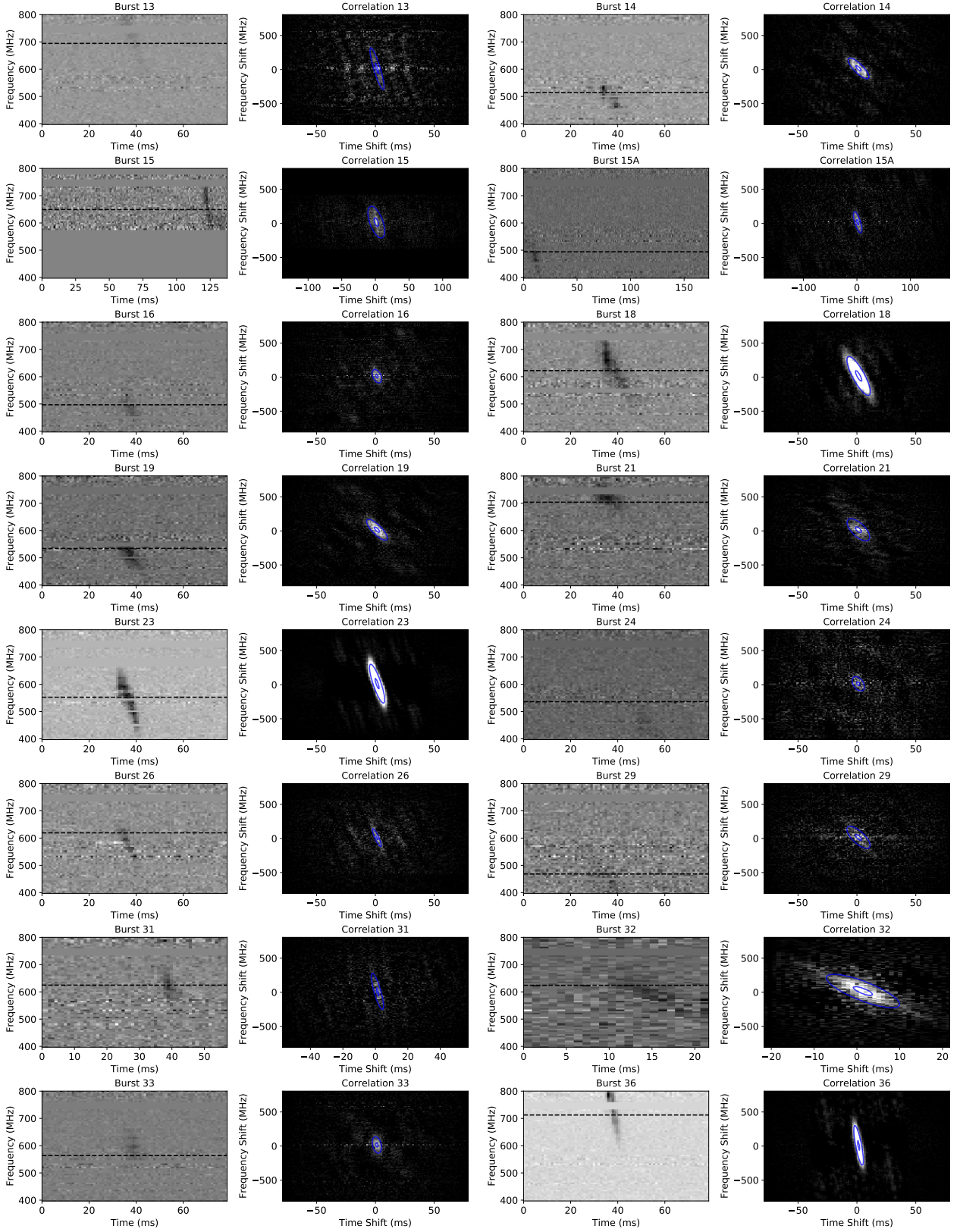


Figure 8. Same as Figure 6 but for FRB 180916.J0158+65 taken from CHIME/FRB et al. (2020). These data were dedispersed with a $DM = 348.82 \text{ pc cm}^{-3}$. Note that the time axes for the autocorrelation functions do not all share the same range, which distorts their relative appearance.

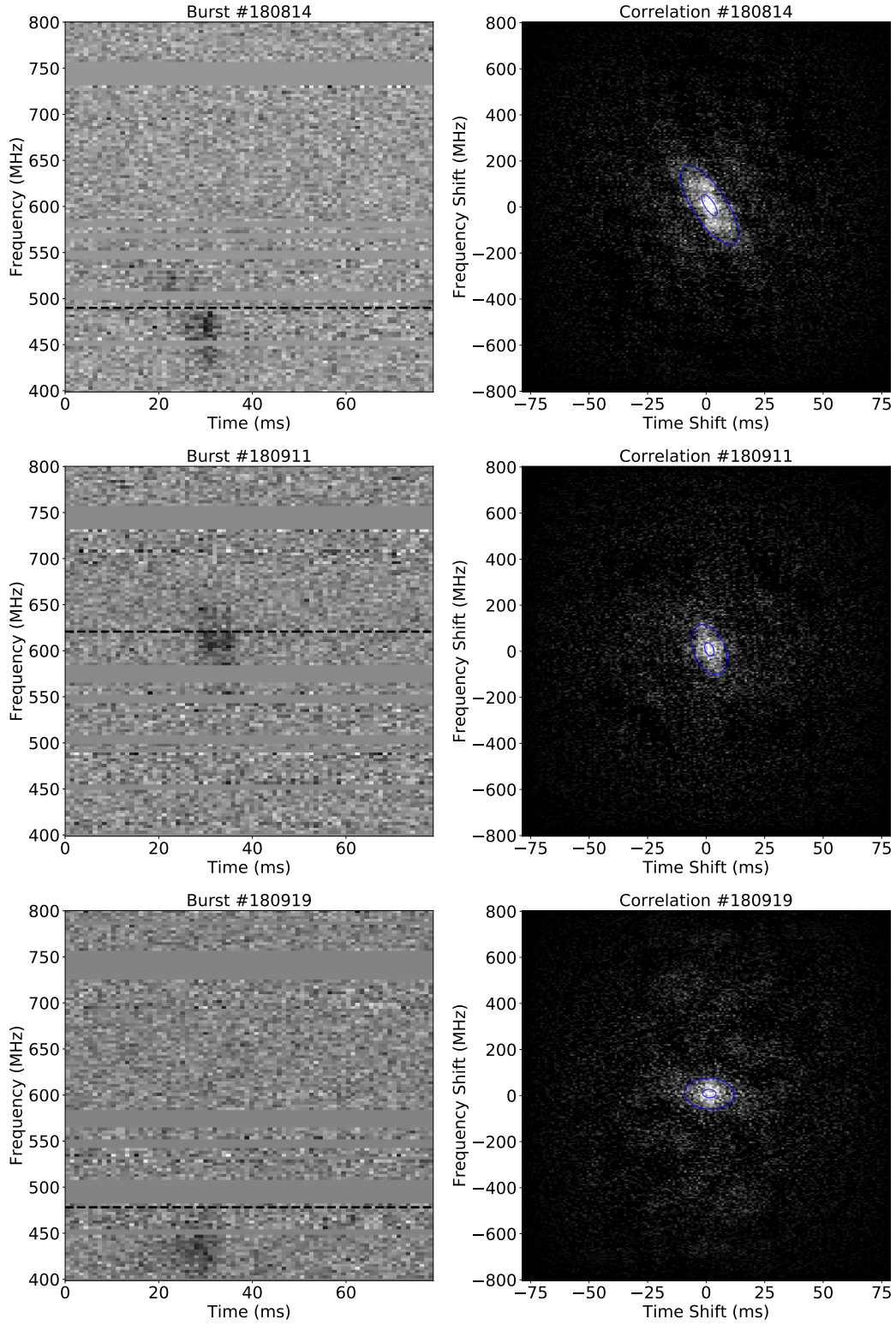


Figure 9. Same as Figure 6 but for FRB 180814.J0422+73 taken from [CHIME/FRB et al. \(2019\)](#). These data were dedispersed with a $DM = 188.9 \text{ pc cm}^{-3}$.

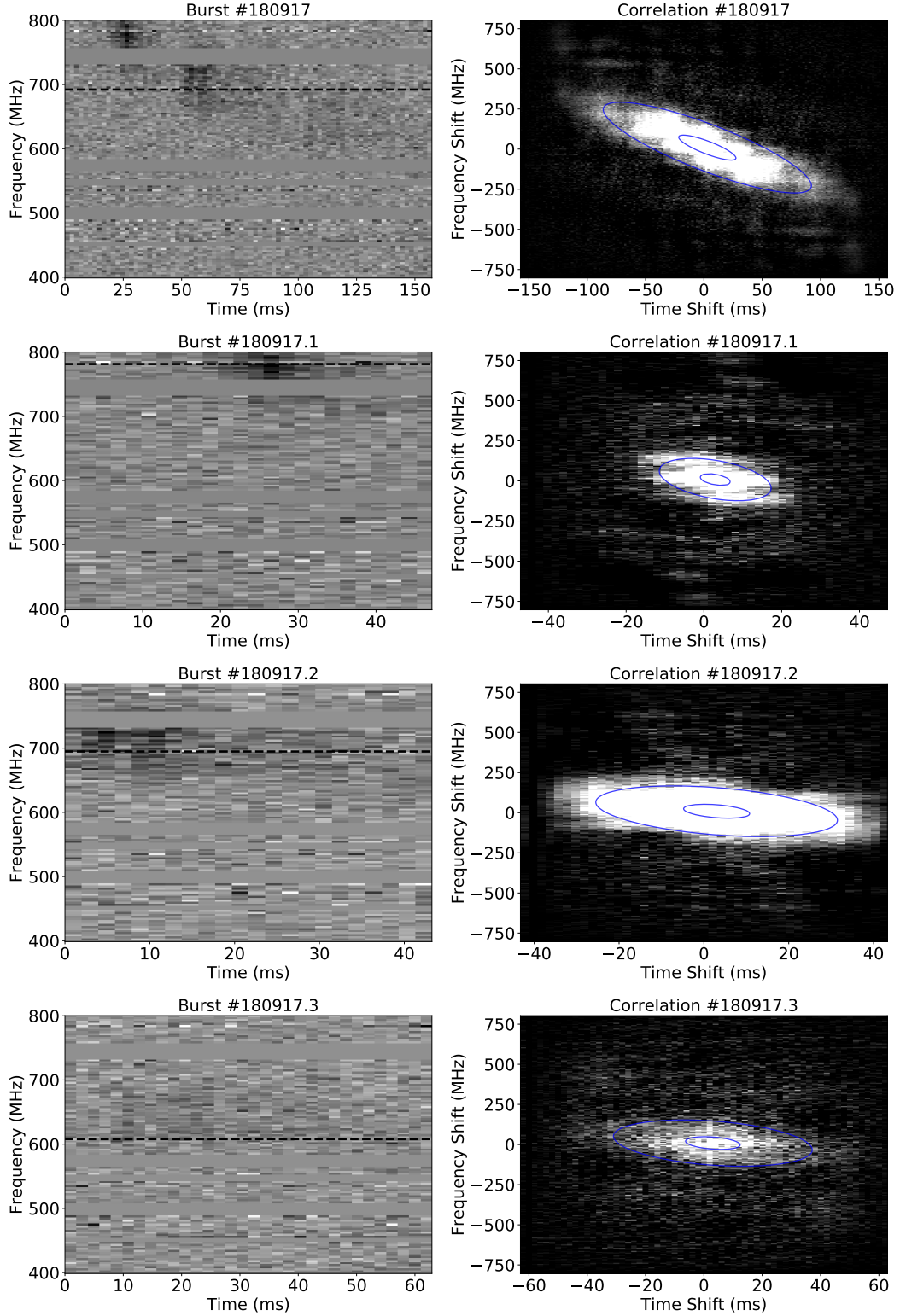


Figure 10. Same as Figure 9 but for Burst #180917 of FRB 180814.J0422+73 taken from CHIME/FRB et al. (2019). The whole event is shown on the top row (not used for Figure 1 of main text), while its three separate sub-bursts are detailed in the bottom three (all used for Figure 1 of main text). Note that the time axes for the autocorrelation functions do not all share the same range, which distorts their relative appearance.

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APPENDIX A: AUTOCORRELATION ANALYSIS

We discuss here the process of preparing and obtaining measurements from the dynamic spectra of bursts, based on the autocorrelation technique described in [Hessels et al. \(2019\)](#).

As mentioned in Section 2.1 when a waterfall consists of a train of multiple sub-bursts we separate the components and measure the slope and temporal duration of each sub-burst separately. The dynamic spectra of every sub-burst used in this analysis with its autocorrelation is shown in Figures 6 – 10.

The pipeline that every sub-burst undergoes is written in Python and consists of computing the autocorrelation of the signal, fitting a two-dimensional (2D) Gaussian to the resulting autocorrelation function, and a calculation of the physical quantities of interest from the fit parameters: namely the sub-burst slope

and temporal duration. The autocorrelation of the waterfall measures the self-similarity of the sub-burst in frequency-time space and for FRBs can be approximated by an ellipsoid with an intensity that follows a 2D Gaussian ([Hessels et al. 2019](#)). Before computing the autocorrelation and depending on the source and/or burst, some noise removal is performed. For the bursts from FRB 121102 and FRB 180916.J0158+65 this is done by subtracting from the entire spectrum a background signal obtained from a time-average of twenty or so samples taken prior to the burst. For FRB 180814.J0422+73, due to the raw format the bursts are provided in, a noise mask was acquired through correspondence with members of the CHIME/FRB Collaboration and the channels are normalized by the standard deviation of the intensity. Missing or blocked out frequency channels in dynamic spectra (e.g., because of radio frequency interference (RFI)) are zeroed out before performing the autocorrelation.

The computation of the autocorrelation function is facilitated and sped up by using a Fast Fourier Transform (FFT) of the waterfall, which is then squared and inverted (through an FFT) back to the frequency-time domain ([Press et al. 2007](#)). The autocorrelation function is then modelled with the following functional form for a rotated 2D Gaussian

$$G(x, y) = C \exp \left\{ -\frac{1}{2} \left[x^2 \left(\frac{\cos^2 \theta}{b^2} + \frac{\sin^2 \theta}{a^2} \right) + 2xy \sin \theta \cos \theta \left(\frac{1}{b^2} - \frac{1}{a^2} \right) + y^2 \left(\frac{\sin^2 \theta}{b^2} + \frac{\cos^2 \theta}{a^2} \right) \right] \right\}, \quad (\text{A1})$$

with the free parameters C , a , b , and θ for, respectively, the amplitude, the semi-major and semi-minor axes (i.e., the standard deviations) of the ellipsoid, and the sub-burst angle for the orientation of the semi-major axis measured counterclockwise from the positive y -axis. More precisely, the x - (i.e., for the time lag) and y -axes (i.e., for the frequency lag) are respectively oriented horizontally and vertically on the autocorrelation plots shown in Figures 6–10. To find these parameters we use the `scipy.optimize.curve_fit` package, which performs a non-linear least squares fit. The package also returns a covariance matrix, which is used to calculate the uncertainty of the fitted parameters. These uncertainties are then scaled by the square-root of the reduced- χ^2 computed from the residual between the autocorrelation function and its Gaussian fit. We note again that the uncertainty calculated this way does not capture nearly the entire error budget which depends more significantly on the error in the DM (discussed in Section 2.1) as well the parts of the burst spectra that have been masked out and the shape of its autocorrelation.

Using the angle θ , the sub-burst slope is calculated via

$$\frac{d\nu_{\text{obs}}}{dt_{\text{D}}} = -\frac{\nu_{\text{res}}}{t_{\text{res}}} \cot \theta, \quad (\text{A2})$$

where ν_{res} and t_{res} are the frequency and time resolutions of the waterfall. We obtain the sub-burst duration from the fit parameters through

$$t_{\text{w}} = t_{\text{res}} \frac{ab}{\sqrt{b^2 \sin^2 \theta + a^2 \cos^2 \theta}}. \quad (\text{A3})$$

These expressions are also used to propagate the fit parameter uncertainties to the values of $d\nu_{\text{obs}}/dt_{\text{D}}$ and t_{w} . These uncertainties are used to confirm the assertion that DM variations are the largest source of error, as stated in Section 2.1.

The observation frequency ν_{obs} of each sub-burst is estimated

via an intensity-weighted average of the spectrum over the whole time range. While this decreases the accuracy of the estimate as opposed to using just the on-pulse region, we find it has little bearing on the result. To fit equation (1) we used the `scipy.odr.RealData` package, which uses orthogonal distance regression and the uncertainties on the data to find a fit.

APPENDIX B: DETERMINATION OF β^+ , ν_0 AND $\Delta\beta'$

The equations presented in this section apply to cases where the source of radiation travels directly toward or away from the observer.

For the determination of the maximum speed of an FRB rest frame toward the observer $\beta^+ > 0$ and ν_0 , the frequency of emission within it, we can generally set $\beta^- = -a\beta^+$ with $a \geq 0$ for the greatest (i.e., most negative) speed away from the observer. Using the relativistic Doppler shift formula (Rybicki & Lightman 1979) for the corresponding frequencies in the observer's rest frame

$$\nu_{\text{obs}}^{\pm} = \nu_0 \sqrt{\frac{1 + \beta^{\pm}}{1 - \beta^{\pm}}}, \quad (\text{B1})$$

we find that

$$\beta^+ = \left(\frac{1+a}{2a} \right) \left(\frac{\nu_{\text{obs}}^{+2} + \nu_{\text{obs}}^{-2}}{\nu_{\text{obs}}^{+2} - \nu_{\text{obs}}^{-2}} \right) \times \left[1 - \sqrt{1 - \frac{4a}{(1+a)^2} \left(\frac{\nu_{\text{obs}}^{+2} - \nu_{\text{obs}}^{-2}}{\nu_{\text{obs}}^{+2} + \nu_{\text{obs}}^{-2}} \right)^2} \right] \quad (\text{B2})$$

$$\nu_0^2 = \nu_{\text{obs}}^+ \nu_{\text{obs}}^- \sqrt{\frac{1 - (1-a)\beta^+ - a\beta^{+2}}{1 + (1-a)\beta^+ - a\beta^{+2}}}. \quad (\text{B3})$$

The discussion in Section 3 where the FRB rest frames span the range $\pm\beta^+$ corresponds to the case $a = 1$, which reduces equations (B2)-(B3) to equations (4)-(5) of the main text.

For the determination of $\Delta\beta'$, we start by considering that for a signal initially observed at frequency ν_{obs} a velocity change $\Delta\beta$ in the observer's rest frame will be accompanied by a change $\delta\nu_{\text{obs}}$ in frequency given by

$$\frac{\delta\nu_{\text{obs}}}{\nu_{\text{obs}}} = \frac{\Delta\beta}{1 - \beta^2}, \quad (\text{B4})$$

where β is the initial velocity relative to the observer. Using the special relativistic velocity addition law (Rybicki & Lightman 1979) we can relate the velocity changes in the observer and FRB rest frames through

$$\Delta\beta = \Delta\beta' \left(\frac{1 - \beta^2}{1 + \beta\Delta\beta'} \right), \quad (\text{B5})$$

with $\Delta\beta'$ the corresponding velocity change in the FRB frame.

Allowing for the motions within the FRB rest frame to span the range $\pm\Delta\beta'$ (with $\Delta\beta' \geq 0$; for simplicity, we assume a symmetric velocity range about zero), while using equations (B1) (to express β as a function of ν_{obs} and ν_0) and (B4)-(B5), we find the following relation for the total observed bandwidth covered by the corresponding signals

$$\frac{\Delta\nu_{\text{obs}}}{\nu_{\text{obs}}} = 2\Delta\beta' \left[1 - \Delta\beta'^2 \left(\frac{\nu_{\text{obs}}^2 - \nu_0^2}{\nu_{\text{obs}}^2 + \nu_0^2} \right)^2 \right]^{-1}. \quad (\text{B6})$$

Equation (6) follows from this relation, which reaches a maximum value when $\nu_{\text{obs}} = 0$ or $\nu_{\text{obs}} \gg \nu_0$. While equation (B6) shows

little variations whenever $\Delta\beta' \ll 1$, it could, in principle, be used to evaluate the FRB rest frame frequency ν_0 independently of equation (B3) since it reaches a minimum of $2\Delta\beta'$ at $\nu_{\text{obs}} = \nu_0$. However, the effect is probably too small (on the order of 1% for FRB 121102) to be measurable given the scatter inherent to FRB data.

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