MORE ON DE-DISPERSION

From Kulkarni (2020), the following equation defines the DM

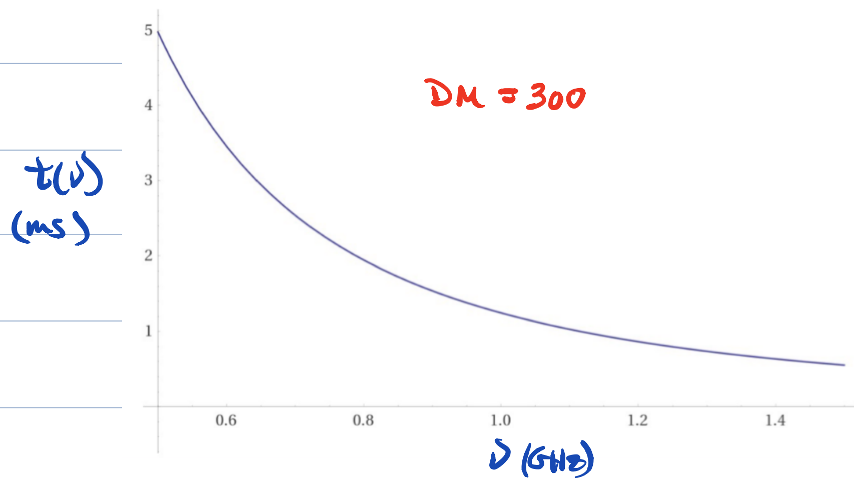
$$DM = K \nu^2 \left[t(\nu) - t(\infty) \right] \rightarrow \nu \text{ is in GHz}$$

where $K = 241.03 \text{ GHz}^{-2} \text{ cm}^{-3} \text{ pc s}^{-1}$; $t(\nu)$ is the time delay at ν due to dispersion. We note that $t(\infty) = L/c$ with L the path length of propagation. For simplicity we will ignore $t(\infty)$ since we are interested in time delay differences between pairs of distinct frequencies. We then write

$$t(\nu) = \frac{a\,DM}{\nu^2}$$

with $a = K^{-1} = 4.15 \times 10^{-3} \text{ GHz cm}^3 \text{pc}^{-1} \text{ s}$

Let's consider an example of $t(\nu)$ vs. ν for $DM = 300 \text{ pc/cm}^3$; $0.5 \text{ GHz} \le \nu \le 1.5 \text{ GHz}$



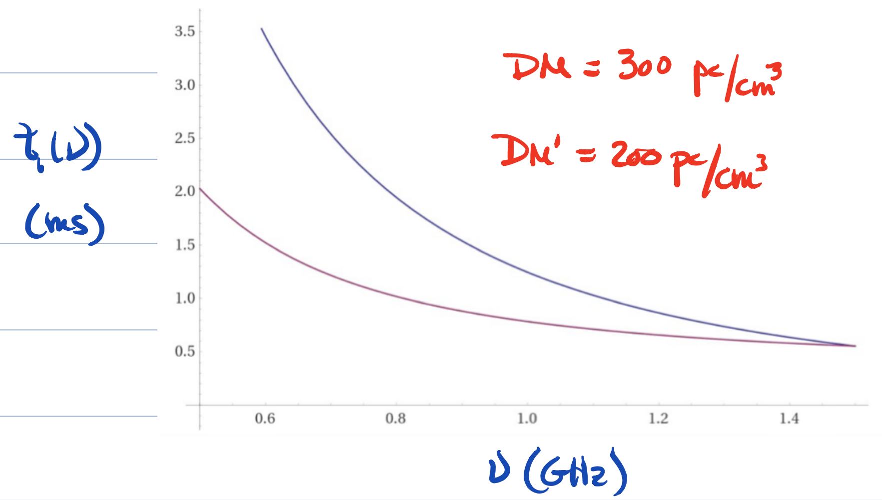
We now use the highest frequency ($\nu = 1.5$ GHz) as the anchor point for a de-dispersion assuming a $DM' = 200$ pc/cm^{-3}. The prime notation is to emphasise that this DM is purposely chosen to be wrong to better see the effects of the de-dispersion.

The relative time delay at frequency $\nu < 1.5$ GHz is defined to be positive with

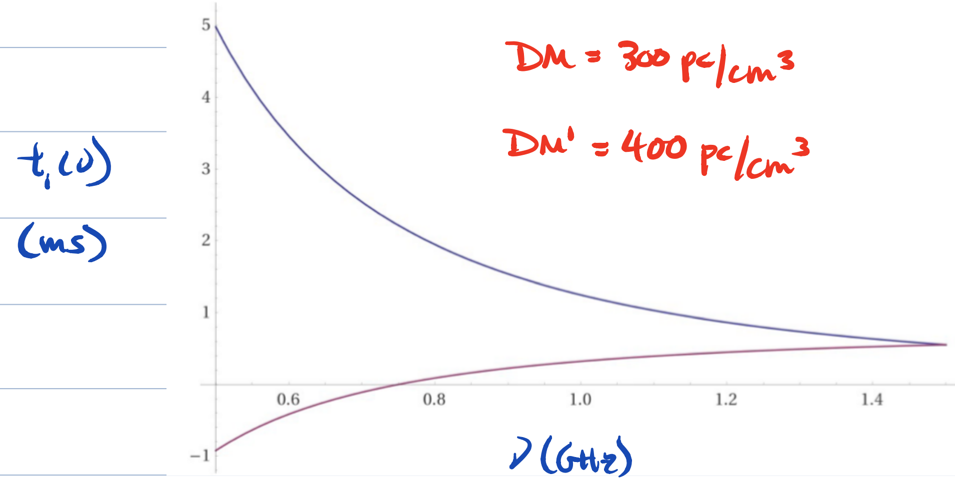
$$\Delta t = a\,DM' \left[\frac{1}{\nu^2} - \frac{1}{(1.5)^2} \right] > 0$$

After de-dispersion we find the red curve below; the blue curve is the same dispersed curve as above. The red de-dispersed curve corresponds to

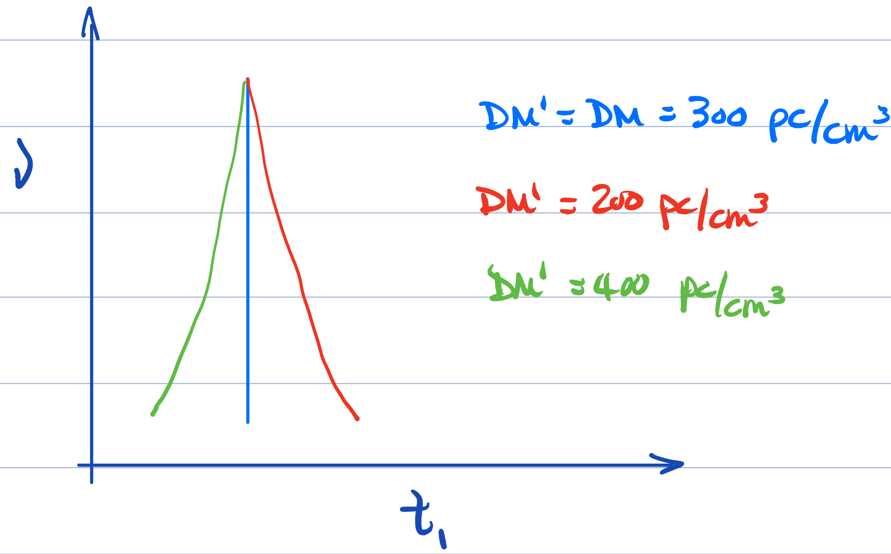
$$t_1(\nu) = t(\nu) - \Delta t(\nu)$$
$$= \frac{a\,DM}{\nu^2} \left[1 - \frac{DM'}{DM} \left(1 - \left(\frac{\nu}{1.5} \right)^2 \right) \right]$$



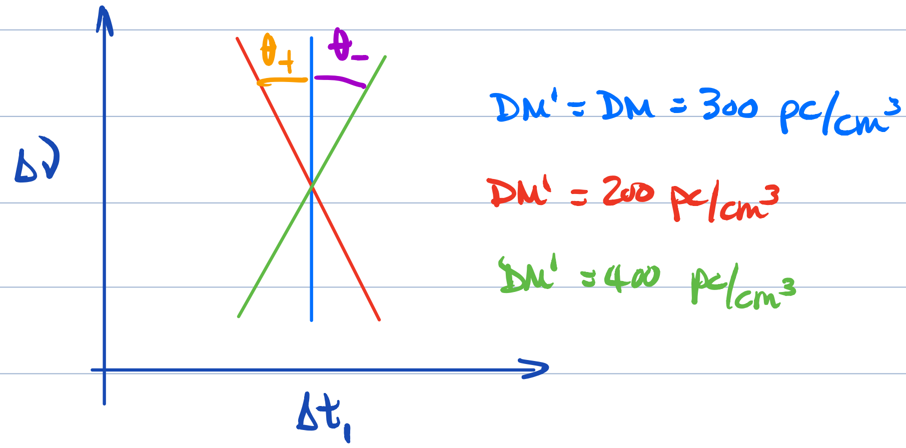
Alternatively, we can set $DM' = 400\ pc/cm^3$ to find



If we were to plot these two graphs as ν vs. $t(\nu)$, as usual for dynamic spectra, we would see



The corresponding autocorrelation functions would be

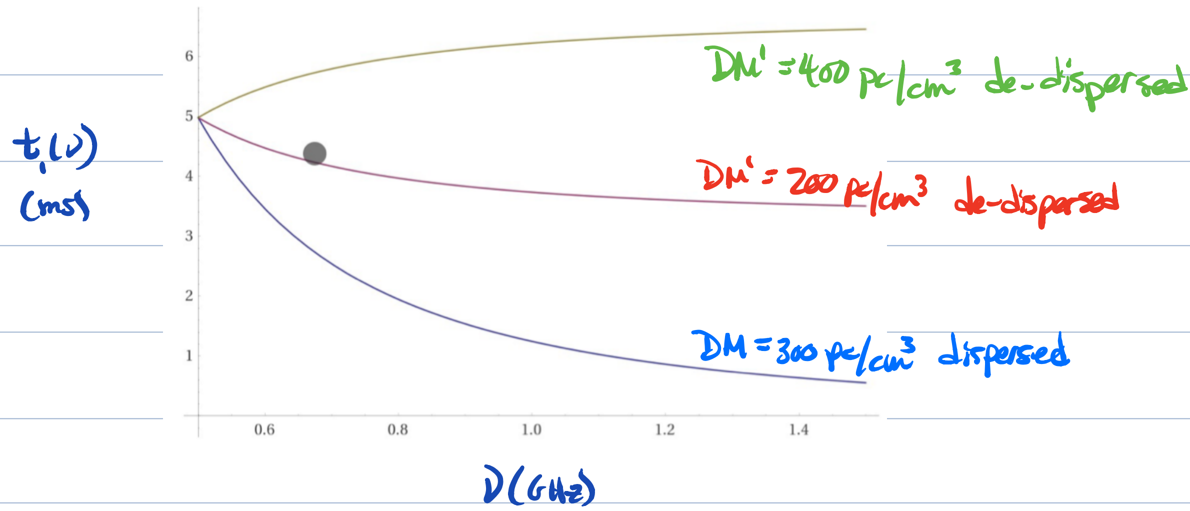


We therefore see that the cases

$DM' < DM \rightarrow$ brings a rotation of increasing angle $\theta+$ relative to correct de-dispersion $DM' = DM$. That is, the drift becomes more negative.

$DM' > DM \rightarrow$ brings a rotation of decreasing angle $\theta-$ relative to correct de-dispersion $DM' = DM$. That is, the drift becomes more positive.

Let us now take the lowest frequency ($\nu = 0.5$ GHz) as the anchor point for de-dispersion. The plot below has the true $DM = 400\ pc/cm^3$ in blue, $DM' = 200\ pc/cm^3$ in red & $DM' = 400\ pc/cm^3$ in green.



In this case we used

$$\Delta t = a\,DM' \left[\frac{1}{(0.5)^2} - \frac{1}{\nu^2} \right] > 0$$

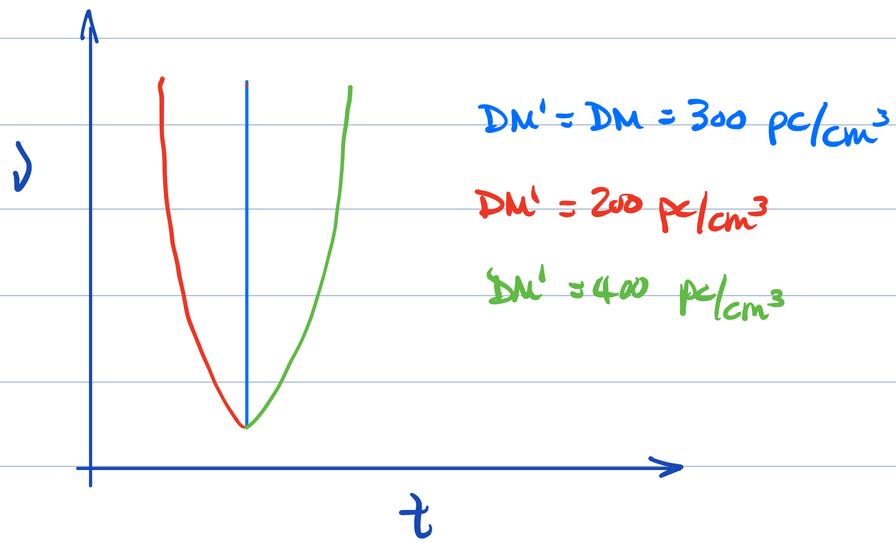
⋮

$$t_1(\nu) = t(\nu) + \Delta t(\nu)$$

$$= \frac{a\,DM}{\nu^2}\left[1 + \frac{DM'}{DM}\left(\left(\frac{\nu}{0.5}\right)^2 - 1\right)\right]$$

Please the '+' sign in the above equation.

If we plot this as a dynamic spectrum with ν vs. t we then have



Comparing with the corresponding dynamic spectrum when the upper frequency $\nu = 1.5$ GHz was the anchor, we find that the shapes of the de-dispersed curves are unchanged. Likewise, the autocorrelation will be the same as before.