ASTRONOMY 9602
COMPUTER PROJECT #2

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Shockfronts

The shockfronts are obtained by plotting the following solution to the Kompaneets model (eq. A8):

$$r(z, y) = 2H \arccos\left[\frac{1}{2} e^{z/2H}\left(1 - \frac{y^2}{4H^2} + e^{-z/H}\right)\right]$$

The function $r(z, y)$ gives one half of the shockfront, so we need to plot $-r(z, y)$ as well to obtain the full shape. Figure 1 shows the plot of the shockfronts for different values of y. An animation of the expanding bubble can be found at https://github.com/mef51/superbubbles/blob/master/blast.gif

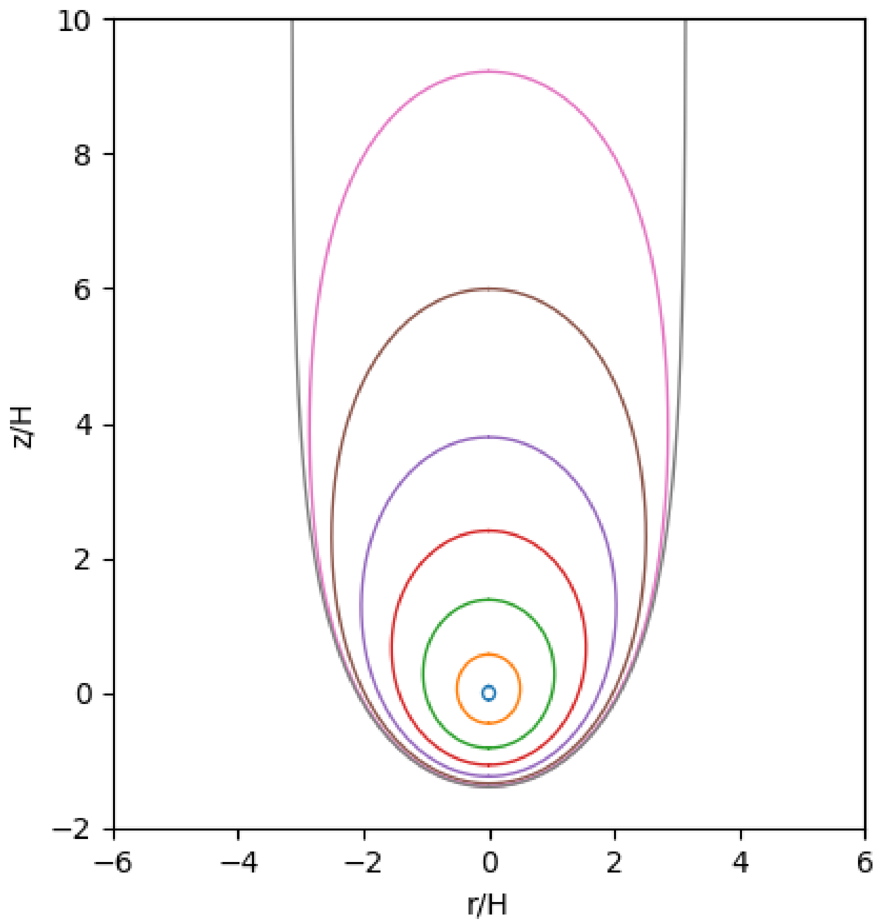

Figure 1: Shockfronts with $y = 0.1, 0.5, 1, 1.4, 1.7, 1.9, 1.98, 2.0$, where 0.1 is the small initial circle and 2.0 is the blown out bubble

Integrating

The integration is performed by first differencing. That is, we approximate all derivatives of a function f as

$$\frac{df}{dt} \approx \frac{f_{n+1} - f_n}{\Delta t}$$

and we get better precision on our result by choosing a smaller and smaller value of Δt (at the expense of computation time). The values we are interesting in obtaining are $\tilde{y}(t)$ and $\tilde{E}_{th}(t)$. We will also need $\Omega(t)$. We find the dimensionless values $\tilde{y}$ and $\tilde{E}$ by simply setting the physical constants H, ρ_0, $L_0 = 1$ in the equations. We use a value of $\gamma = 5/3$. We also use an initial value of $P = 1$.

The integration can begin once we have initial values for $\tilde{y}$, $\tilde{E}_{th}$ and $\tilde{\Omega}$. They are calculated as

$$\begin{aligned}
\tilde{y}_0 &= 0.01 \qquad \text{(small spherical shockfront)} \\
\Omega_0 &= \pi \int_{z_2}^{z_1} r^2(z, \tilde{y}_0) dz \\
\tilde{E}_0 &= \frac{P}{\gamma - 1} \Omega_0
\end{aligned}$$

To calculate Ω_0, z_1 and z_2 (the top and bottom of the shockfront, respectively) are first calculated according to eq. (A9) using the value of $\tilde{y}_0$ before performing the integration:

$$z_{1,2} = -2H \ln\left(1 \mp \frac{y}{2H}\right)$$

Then $\tilde{E}_0$ is calculated by rearranging (A2). With the initial values, the next values $\tilde{y}_1$, $\tilde{y}_2$,... and so on are calculated from the appropriate derivatives/equations. The relevant equations are

$$\begin{aligned}
\frac{dy}{dt} &= \sqrt{\frac{\gamma^2 - 1}{2} \frac{E_{th}}{\rho_0 \Omega}} \quad (A6) \\
\Omega &= \pi \int_{z_2}^{z_1} r^2(z, y) dz \quad (A3) \\
\frac{dE}{dt} &= L_0 - (\gamma - 1) \frac{E_{th}}{\Omega} \frac{d\Omega}{dt} \quad (A7)
\end{aligned}$$

where $P = (\gamma - 1)\frac{E_{th}}{\Omega}$ is substituted into (A7) since P is not, in fact, constant.

Converting these with the first differencing approach, each iteration is calculated with:

$$\begin{aligned}
\tilde{y}_{n+1} &= \sqrt{\frac{\gamma^2 - 1}{2} \frac{\tilde{E}_0}{\rho_0 \Omega_0}} \Delta t + \tilde{y}_n \\
\Omega_{n+1} &= \pi \int_{z_2}^{z_1} r^2(z, y_n) dz \\
\tilde{E}_{n+1} &= L_0 \Delta t - (\gamma - 1) \tilde{E}_n \frac{\Omega_{n+1} - \Omega_n}{\Omega_n}
\end{aligned}$$

where again, z_1 and z_2 are recalculated each iteration with the most recent value of $\tilde{y}_n$.

For my integration I chose $\Delta t = 0.0001$ (since I got strange results with larger dt) and integrate over $t \in [0.005, 10]$. Again, the solutions are dimensionless because I set all physical constants to 1. The code takes about 20 seconds to run on a thinkpad with 8gigs of RAM and an intel i5 quadcore.

The results of the integration are shown in Figure 2, and they are identical to Figure 10 of Basu et al. (1999)

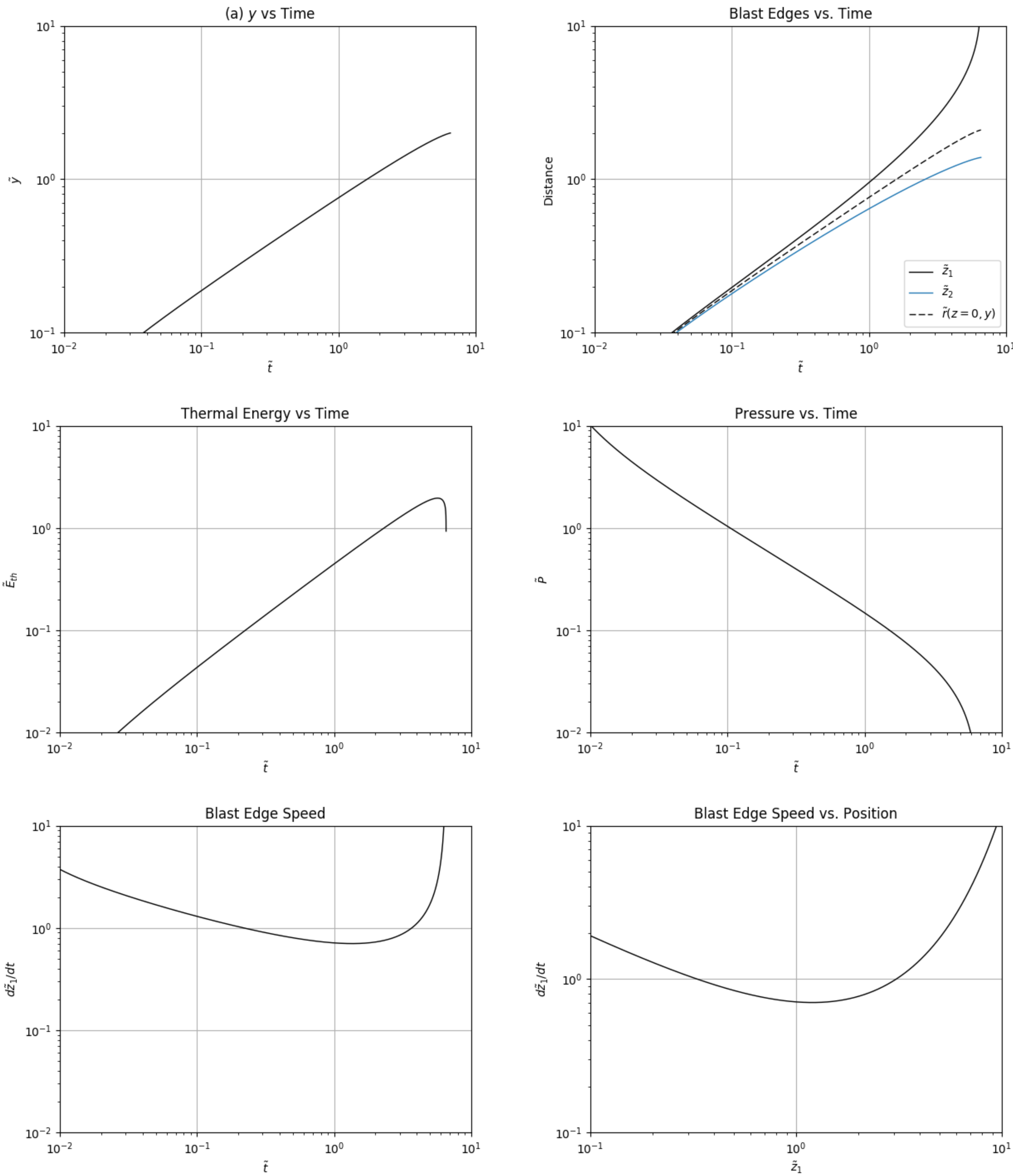

Figure 2: Time evolution of the shockfront, mimicking Fig. 10 of Basu et al. 1999

Conclusion

We've plotted various characteristics of an expanding bubble from the Kompaneets model as applied to superbubbles in the Galaxy and we see from Fig. 3 (Basu et al. 1999) that though this model originated in understanding the expansion of shockfronts from nuclear blasts, it works fairly well when applied to windblown bubbles seen in the galaxy (like W4), and can give us valuable physical information about observed bubbles, like scale heights and timescales, despite there being no 'explosion' or supernova to initiate the bubble.

Code

All code is available at https://github.com/mef51/superbubbles .

The main file that performs the integration and plots the results is superbubbles.py.