Introduction to Differential Equations
Assignment # 12

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P1.

(a)

$$x' = \begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix} x$$

$$\det(A - rI) = \begin{bmatrix} 3-r & -2 \\ 2 & -2-r \end{bmatrix} = r^2 - r - 2 = 0$$

$$r_1 = -1,\ r_2 = 2$$

$$\det(A - r_1 I) = \begin{bmatrix} 4 & -2 \\ 2 & -1 \end{bmatrix} \xi_1 = 0$$

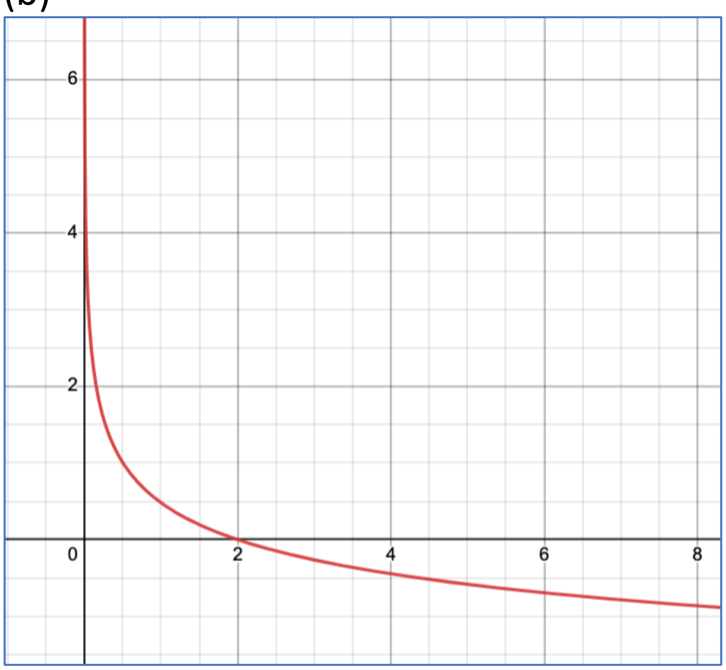
$$\xi_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\det(A - r_2 I) = \begin{bmatrix} 1 & -2 \\ 2 & -4 \end{bmatrix} \xi_2 = 0$$

$$\xi_2 = \begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix}$$

$$x(t) = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix} e^{-2t}$$

(b)



P2.

(a)

$$x' = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} x$$

$$\det(A - rI) = \begin{bmatrix} -2 - r & 1 \\ 1 & -2 - r \end{bmatrix} = r^2 + 4r + 3 = 0$$

$$r_1 = -3,\ r_2 = -1$$

$$\det(A - r_1 I) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \xi_1 = 0$$

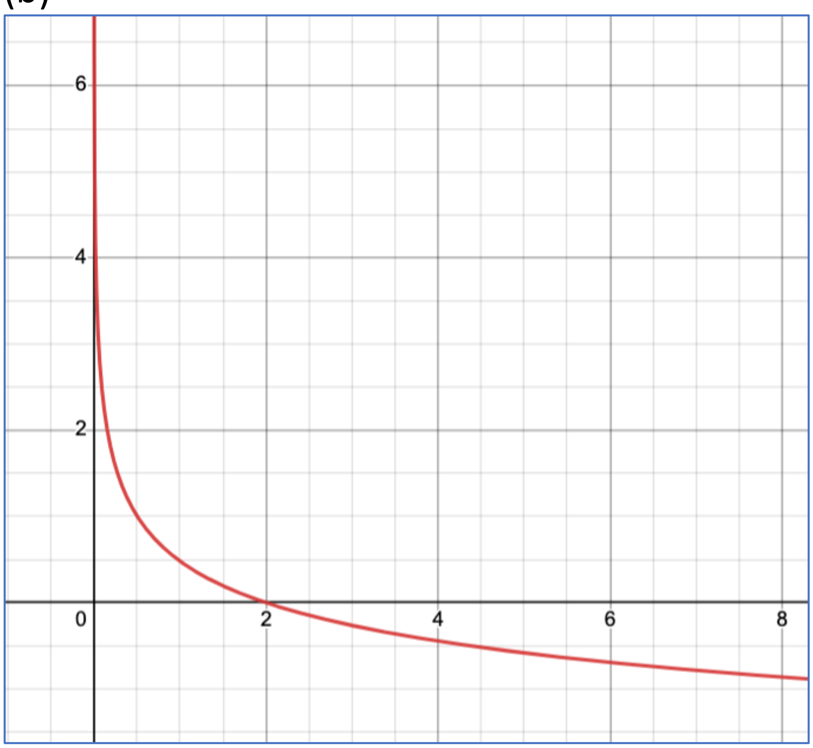
$$\xi_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\det(A - r_2 I) = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \xi_2 = 0$$

$$\xi_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$x(t) = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-3t} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-t}$$

(b)



P3.

$$x' = \begin{bmatrix} 4 & -3 \\ 8 & -6 \end{bmatrix} x$$

$$\det(A - rI) = \begin{bmatrix} 4 - r & -3 \\ 8 & -6 - r \end{bmatrix} = r^2 + 2r = 0$$

$$r_1 = 0,\ r_2 = -2$$

$$\det(A - r_1 I) = \begin{bmatrix} 4 & -3 \\ 8 & -6 \end{bmatrix} \xi_1 = 0$$

$$\xi_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$\det(A - r_2 I) = \begin{bmatrix} 6 & -3 \\ 8 & -4 \end{bmatrix} \xi_2 = 0$$

$$\xi_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$x(t) = c_1 \begin{bmatrix} 3 \\ 4 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-2t}$$

(b)

P4.

$$x' = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix} x$$

$$\det(A - rI) = \begin{bmatrix} 3-r & 2 & 4 \\ 2 & 0-r & 2 \\ 4 & 2 & 3-r \end{bmatrix} = r^2 + 2r = 0$$

$$r_1 = 0,\ r_2 = -2$$

P5.

$$x' = \begin{bmatrix} \frac{1}{2} & 0 \\ 1 & -\frac{1}{2} \end{bmatrix} x,\ x(0) = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\det(A - rI) = \begin{bmatrix} \frac{1}{2} - r & 0 \\ 1 & -\frac{1}{2} - r \end{bmatrix} = r^2 - \frac{1}{4} = \left(r + \frac{1}{2}\right)\left(r - \frac{1}{2}\right) = 0$$

$$r_1 = \frac{1}{2},\ r_2 = -\frac{1}{2}$$

$$\det(A - r_1 I) = \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix} \xi_1 = 0$$

$$\xi_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\det(A - r_2 I) = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \xi_2 = 0$$

$$\xi_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x(t) = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{\frac{t}{2}} + c_2 \begin{bmatrix} 0 \\ 0 \end{bmatrix} e^{-\frac{t}{2}}$$

$$c_1 + c_2 = 3$$

$$c_1 + c_2 = 5$$