Notes for lecture 3

1. Date: April 25th. This lecture is based on Sections 6 and 4 of Chapter 2 of the main textbook (see Chapter2.6.pdf and Chapter2.4.pdf file)
2. Section 2.6 deals with the so called exact equations. This concept is more difficult than that of separable equations but not that much
   a. In understanding exact equations, it is worthwhile to refresh your knowledge of calculus, and specifically the chain rule of computing the derivatives of complex functions. Once you remember it, the rest is conceptually easy.
   b. It is also good to remember partial differentiation of a function of two variables. Note that in the textbook and lecture slides partial derivatives have s special notation. Namely, for a function of two variables $f(x,y)$ the partial derivatives are denoted as $f_x(x,y) = \partial f(x,y)/\partial x$ and $f_y(x,y) = \partial f(x,y)/\partial y$.
   c. In solving practical problems, we first check the condition for ODE to be exact. If this condition is satisfied, then the solution requires two integration and is conducted in two steps (see lecture slides)
   d. Again, when dealing with nonlinear equations we often end up with the implicit form of solution $\psi(x,y) = c$, where a is arbitrary constant.
   e. Finding the integration factor is a kind of art (no systematic techniques). In this connection, all you requested to know is that non-exact equation can become exact after multiplication by an integrating factor.
3. Section 2.4 introduces the theorem of uniqueness and existence of solution.
   a. This theorem is an important part of mathematical culture. In a nutshell, when some conditions are not met there may be no solutions (on some interval of x) or there maybe two or more solutions passing through the same point (x,y). It does not happen if the coefficients of differential equations are smooth enough. But it may happen when the coefficients are not smooth (not continuous) functions.
4. Explanation of the content is accompanied by examples. In addition, you can look at the sample problems (see SampleProblems3.pdf file)
5. Additional Internet resources
   a. As before, Wolfram Alpha for computing integrals https://www.wolframalpha.com/examples/mathematics/calculus-and-analysis/ and plotting graphs (not required but highly recommended) https://www.wolframalpha.com/examples/mathematics/plotting-and-graphics/
   b. A short video from Khan Academy on exact equations https://www.khanacademy.org/math/differential-equations/first-order-differential-equations#exact-equations can also be recommended
6. The deadline for submitting homework, Assignment 3 (refer to Assignment3.pdf) is May 2, 13:00.