Notes for lecture 5

1. Date: May 9th. This lecture is based on Sections 3.3 and 3.4 of Chapter 3 of the main textbook (see Chapter3.3.pdf and Chapter3.4.pdf file)
2. Section 3.3 (see Chapter3.3.pdf and Lecture5.pdf) consider linear differential equation of 2nd order (with constant coefficients). The key points are as follows.
    a. When the roots of characteristic equation are complex numbers $r_{1,2} = \lambda \pm \mu i$, the two independent solutions are $e^{\lambda t}\cos \mu t$ and $e^{\lambda t}\sin \mu t$, and the general solution is constructed as a linear combination of these two solutions.
    b. The structure of the two independent solutions is derived with the use of Euler's formula for complex numbers, and the fact that they are independent is tested by checking the Wronskian.
3. Section 3.4 (see Chapter3.4.pdf and Lecture5.pdf) deals with equation having one (but repeated, since we have 2nd order equation) root.
    a. The first solution $y_1(t)$ is established straightforwardly (corresponds to the repeated root). To find 2nd solution, a techniques conceptually similar to the variation of constants (which we studied before) is used. Specifically, the 2nd solution is searched in the form $y_2(t) = \nu(t) y_1(t)$, where $\nu(t)$ is at first is unknown but, later on, its structure of is established from the given differential equation. Again, the fact that the two solutions are independent is tested by checking the Wronskian.
    b. The part Section 3.4 related to reduction of order is optional since it deals equations where the coefficients are not constant but functions of t. The main idea here is that if you happen to know one of the solution, $y_1(t)$, the second one can be established as $y_2(t) = \nu(t) y_1(t)$, and finding $\nu(t)$ requires solving a differential equation of 1st (not 2nd) order. Hence the name of the technique.
4. Explanation of the content is accompanied by examples. In addition, you can look at sample problems (see SampleProblems5.pdf file).
5. To brush up your knowledge of complex numbers, see file ComplexNumbers.pdf.
6. Additional Internet resources
    a. Short videos from Khan Academy on linear homogeneous equations https://www.khanacademy.org/math/differential-equations/second-order-differential-equations#complex-roots-characteristic-equation are recommended.
    b. Complex numbers https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:complex and also https://en.wikipedia.org/wiki/Complex_number
7. The deadline for submitting homework, Assignment 5 (refer to Assignment5.pdf) is May 16, 13:00. Solutions to this assignment (refer to Assignment5_sol.pdf) will be uploaded to Resource Section on May 16 after the class.