Introduction to Differential Equations
Assignment # 9

Tian Xiaoyang
26001904581

P1.

$f(t) = t^n e^{at}$

$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$

$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} t^n e^{at}\, dt$

$= \int_0^\infty t^n e^{(a-s)t}\, dt$

$= t^n e^{(a-s)t} - []$

P2.

P3.

$F(s) = \frac{2s+1}{s^2-2s-2} = \frac{2s+1}{(s^2-2s+1)-2-1} = \frac{2s+1}{(s-1)^2-3} = \frac{2s+1}{(s-1)^2-\sqrt{3}^2}$

$= \mathcal{L}^{-1}\left\{\frac{2[(s-1)+1]+1}{(s-1)^2-\sqrt{3}^2}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{2(s-1)+2+1}{(s-1)^2-\sqrt{3}^2}\right\}$

$= \mathcal{L}^{-1}\left\{2\frac{(s-1)}{(s-1)^2-\sqrt{3}^2}\right\} + \mathcal{L}^{-1}\left\{2\frac{\frac{3}{2}}{(s-1)^2-\sqrt{3}^2} \cdot \frac{\sqrt{3}}{\sqrt{3}}\right\}$

$= \mathcal{L}^{-1}\left\{2\frac{(s-1)}{(s-1)^2-\sqrt{3}^2}\right\} + \mathcal{L}^{-1}\left\{\frac{2}{\sqrt{3}}\frac{\frac{3}{2}\sqrt{3}}{(s-1)^2-\sqrt{3}^2}\right\}$

$= 2e^{1t}\cos -\sqrt{3}t + \frac{2}{\sqrt{3}}e^{1t}\sin -\frac{3}{2}\sqrt{3}t$

P4.

$F(s) = \frac{8s^2-4s+12}{s(s^2+4)}$

$\mathcal{L}^{-1}\left\{\frac{8s^2-4s+12}{s(s^2+4)}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{8s^2-4s+12}{s(s^2+2^2)}\right\}$

P5.

$y'' - 2y' + 4y = 0$

$\mathcal{L}\{y''\} - 2\mathcal{L}\{y'\} + 4\mathcal{L}\{y\} = 0$

$\mathcal{L}\{y''\} = s\mathcal{L}\{y'\} - y'(0)$

$= s\big(s\mathcal{L}\{y\} - y(0)\big) - y'(0)$

$2\mathcal{L}\{y'\} = 2(s\mathcal{L}\{y\} - y(0))$

$\mathcal{L}\{y''\} + 2\mathcal{L}\{y'\} + 4\mathcal{L}\{y\}$

$= s\big(s\mathcal{L}\{y\} - y(0)\big) - y'(0) - 2\big(s\mathcal{L}\{y\} + y(0)\big) + 4\mathcal{L}\{y\}$

$= \big(s^2\mathcal{L}\{y\} - sy(0)\big) - y'(0) - 2\big(s\mathcal{L}\{y\} + y(0)\big) + 4\mathcal{L}\{y\}$

$= s^2\mathcal{L}\{y\} - 2s - 0 - 2s\mathcal{L}\{y\} + 4 + 4\mathcal{L}\{y\}$

$= \mathcal{L}\{y\}(s^2 - 2s + 4) - 2s + 4$

$\mathcal{L}\{y\}(s^2 - 2s + 4) - 2s + 4 = 0$

$\mathcal{L}\{y\}(s^2 - 2s + 4) = 2s - 4$

$\mathcal{L}\{y\} = \frac{2s-4}{s^2-2s+4}$

$= \frac{2s-4}{s^2-2s+1+4-1}$

$= \frac{2s-4}{(s-1)^2+3}$

$= \frac{2s-4}{(s-1)^2+\sqrt{3}^2}$

$= \frac{2(s-1)-2}{(s-1)^2+\sqrt{3}^2}$

$= 2\frac{(s-1)}{(s-1)^2+\sqrt{3}^2} - 2\frac{1}{(s-1)^2+\sqrt{3}^2} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= 2\frac{(s-1)}{(s-1)^2+\sqrt{3}^2} - \frac{2}{\sqrt{3}}\frac{\sqrt{3}}{(s-1)^2+\sqrt{3}^2}$

$= 2e^t \cos\sqrt{3}t - \frac{2}{\sqrt{3}}e^t \sin\sqrt{3}t$

$y = 2e^t \cos\sqrt{3}t - \frac{2}{\sqrt{3}}e^t \sin\sqrt{3}t$

P6.

$y'' - 2y' + 2y = e^{-t}$

$\mathcal{L}\{y''\} - 2\mathcal{L}\{y'\} + 2\mathcal{L}\{y\} = e^{-t}$

$\mathcal{L}\{y''\} = sy(s) - sy(0) - y'(0)$

$\mathcal{L}\{y'\} = 2sy(s) - 2y(0)$

$\mathcal{L}\{y\} = 2y(s)$

$\mathcal{L}\{y''\} - 2\mathcal{L}\{y'\} + 2\mathcal{L}\{y\} = e^{-t}$

$sy(s) - sy(0) - y'(0) - 2sy(s) + 2y(0) + 2y(s) = e^{-t}$

$sy(s) - 1 - 2sy(s) + 2y(s) = e^{-t}$

$sy(s) - 2sy(s) + 2y(s) = e^{-t} + 1$

$y(s)(s + 2s + 2) = e^{-t} + 1$

$y(s) = \frac{e^{-t}+1}{s+2s+2}$