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P1.
$$y''' + 6y'' + 12y' - 8y = 0$$
$$r^3 - 6r^2 + 12r - 8 = 0$$
$$(r - 2)(r - 2)(r - 2) = 0$$
$$r_1 = 2, r_2 = 2, r_3 = 2$$
$$y = c_1 e^{2t} + tc_2 e^{2t} + t^2 c_3 e^{2t}$$

P2.
$$16y^{(4)} + 24y'' + 9y = 0$$
$$16r^4 + 24r^2 + 9 = 0$$
$$(4r^2 + 3)(4r^2 + 3) = 0$$
$$4r^2 = -3$$
$$r^2 = -\frac{3}{4}$$
$$r = \pm\sqrt{\frac{3}{4}}i$$
$$y = c_1 \cos\sqrt{\frac{3}{4}}t + c_2 \sin\sqrt{\frac{3}{4}}t + c_3 \cos\sqrt{\frac{3}{4}}t + c_4 \sin\sqrt{\frac{3}{4}}t$$

P3.
$$y^{(6)} + y = 0$$
$$r^6 + 1 = 0$$
$$(r^2 + 1)(r^4 - r^2 + 1) = 0$$
$$(r + i)(r - i)(r + i)^2(r - i)^2 = 0$$
$$r = -i, r = i, r = -i, r = i, r = -i, r = i$$

$$y = c_1 \cos t + c_2 \sin t + tc_3 \cos t + t^2 c_4 \cos t + tc_5 \sin t + t^2 c_6 \sin t$$

P4.

$$y^{(4)} + 2y^{(2)} + y = 0$$
$$r^4 + 2r^2 + 1 = 0$$
$$(r^2 + 1)(r^2 + 1) = 0$$
$$(r + i)(r - i)(r + i)(r - i)$$
$$y = c_1 \cos t + c_2 \sin t + c_3 t \cos t + c_4 t \sin t$$

P5.

$$y''' + 12y'' + 36y' = 0$$
$$r^3 + 12r^2 + 36r = 0$$
$$r(r + 6)(r + 6) = 0$$
$$r = 0, r = -6$$
$$y = c_1 + c_2 e^{-6t} + t c_3 e^{-6t}$$
$$y' = -6c_2 e^{-6t} + c_3 e^{-6t} - 6c_3 e^{-6t} t$$
$$y'' = 36c_2 e^{-6t} - 6c_3 e^{-6t} + 36c_3 e^{-6t} t - 6c_3 e^{-6t}$$

$$y(0) = 0, y'(0) = 1, y''(0) = 6$$

$$y = c_1 + c_2 = 0$$
$$y' = -6c_2 + c_3 = 1$$
$$y'' = 36c_2 - 6c_3 - 6c_3 = 6$$
$$= 6c_2 - 2c_3 = 1$$
$$-c_3 = 2, c_3 = -2$$
$$c_2 = -\frac{1}{2}$$
$$c_1 = \frac{1}{2}$$
$$y = \frac{1}{2} - \frac{1}{2} e^{-6t} - 2t e^{-6t}$$

P6.

$$y = c_1 + c_2 e^{2t} \cos 5t + c_3 e^{2t} \sin 5t$$

$$r = 0, r = 2 + 5i, r = 2 - 5i$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = 2 \pm 5i$$

$$if\ a = 1$$

$$-\frac{b}{2} = 2, b = -4$$

$$5i = \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$10i = \sqrt{b^2 - 4ac}$$

$$-100 = 16 - 4c$$

$$c = 29$$

$$r^3 - 4r^2 + 29r = 0$$

$$y''' - 4y'' + 29y' = 0$$