Introduction to Differential Equations
Assignment # 3

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P1.

$y' + p(t)y = g(t), y(t_0) = y_0$

$(t-3)y' + (\ln t)y = 2t\,, y(1) = 2$

$y' + \frac{(\ln t)y}{(t-3)} = \frac{2t}{(t-3)}, y(1) = 2$

$p(t)$ and $g(t)$ will not be continuous at $t = 3$

$p(t)$ and $g(t)$ will be continuous at $-\infty < t < 3, 3 < t < \infty$

$y(1) = 2, t = 1$

$-\infty < t < 3$ is the interval with certain solution

P2.

$(4 - t^2)y' + 2ty = 3t^2, y(-3) = 1$

$y' + \frac{2ty}{(4-t^2)} = \frac{3t^2}{(4-t^2)}$

$p(t)$ and $g(t)$ will not be continuous at $t = \pm 2$

$p(t)$ and $g(t)$ will not be continuous at $-\infty < t < -2,\ -2 < t < 2, 2 < t < \infty$

$y(-3) = 1, t = -3$

$-3 < t < -2$

P3.

For an equation to be exact $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$M = 3x^2 - 2xy + 2$

$N = 6y^2 - x^2 + 3$

$\frac{\partial M}{\partial y} = 2x$

$\frac{\partial N}{\partial x} = 2x$

$\frac{\partial M}{\partial y} = 2x = \frac{\partial N}{\partial x}$

$\int Md_x = x^3 - yx^2 + 2x + c(y)$
$\int Nd_y = 2y^3 - x^2y + 3y + c(x)$
$x^3 + 2y^3 - x^2y + 3y + 2x + 3y = c$

P4.
$(ye^{xy}\cos 2x - 2e^{xy}\sin 2x + 2x) + (xe^{xy}\cos 2x - 3)\, y'$
$M = ye^{xy}\cos 2x - 2e^{xy}\sin 2x + 2x$
$N = xe^{xy}\cos 2x - 3$
$\frac{\partial M}{\partial y} = e^{xy}\cos 2x - 2e^{xy}\sin 2x$
$\frac{\partial N}{\partial x} = xe^{xy}\cos 2x$
$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

P5.
$y' + p(t)y = g(t), y(t_0) = y_0$
$(2x - y) + (2y - x)y' = 0, y(1) = 3$
$y'(2y - x) = -(2x - y)$
$y' = \frac{y-2x}{2y-x}$
$(2y - x)dy = (y - 2x)dx$
$\int(2y - x)dy = \int(y - 2x)dx$
$y^2 - xy = yx - x^2 + c$
$y^2 = 2xy - x^2 + c$
$3^2 = 2 \times 1 \times 3 - 1^2 + c$
$c = 4$
$y^2 = 2xy - x^2 + 4$

$p(t)$ and $g(t)$ will not be continuous at $y = 2x$