Introduction to Differential Equations
Assignment # 13

Date Given: July 4, 2022
Date Due: July 11, 2022

P1. (2 points)

(a) Express the general solution of the system of equations

$$\boldsymbol{x}' = \begin{bmatrix} 1 & 2 \\ -5 & -1 \end{bmatrix} \boldsymbol{x}$$

in terms of real-valued functions

(b) Draw a direction field, sketch a few of the trajectories, and describe the behavior of the solutions as $\to \infty$.

P2. (1 point) Express the general solution of the system of equations

$$\boldsymbol{x}' = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \boldsymbol{x}$$

in terms of real-valued functions.

P3. (2 points)

(a) Find the solution of the initial value problem

$$\boldsymbol{x}' = \begin{bmatrix} 6 & -1 \\ 5 & 4 \end{bmatrix} \boldsymbol{x}, \qquad \boldsymbol{x}(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

(b) Describe the behavior of the solution as $t \to \infty$.

P4. (3 points) The system of differential equations is given as

$$\boldsymbol{x}' = \begin{bmatrix} 2 & -5 \\ \alpha & -2 \end{bmatrix} \boldsymbol{x},$$

where α is a constant parameter.

(a) Determine the eigenvalues in terms of α

(b) Find the critical value of α where the qualitative nature of the phase portrait for the system changes.

(c) Draw a phase portrait for a value of α slightly below, and for another value slightly above, each critical value.

P5. (2 points)

(a) Find a fundamental matrix for the system of equations.

$$\boldsymbol{x}' = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} \boldsymbol{x}.$$

(b) Find the fundamental matrix $\boldsymbol{\Phi}(t)$ satisfying $\boldsymbol{\Phi}(0) = \boldsymbol{I}$.

P6. (2 point)

(a) Find a fundamental matrix for the system of equations.

$$\boldsymbol{x}' = \begin{bmatrix} 1 & -1 \\ 5 & -3 \end{bmatrix} \boldsymbol{x}.$$

(b) Find the fundamental matrix $\boldsymbol{\Phi}(t)$ satisfying $\boldsymbol{\Phi}(0) = \boldsymbol{I}$.