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P1.

$y''' - 6y'' = 3 - \cos t$

$y''' - 6y'' = 0$

$r^3 - 6r^2 = 0$

$r - 6 = 0$

$r = 6$

$y = c_1 e^{6t} + c_2 e^{6t}$

$y_p(x) = A \cos t + B \sin t$

$y'_p(x) = -A \sin t + B \cos t$

$y''_p(x) = -A \cos t - B \sin t$

$y'''_p(x) = A \sin t - B \cos t$

$A \sin t - B \cos t - 6(-A \cos t - B \sin t) = 3 - \cos t$

$A \sin t - B \cos t + 6 A \cos t + 6B \sin t = 3 - \cos t$

$(A + 6B) \sin t + (-B + 6A) \cos t = 3 - \cos t$

$-B + 6A = -1, (A + 6B) \sin t = 3$

P2.

$y''' - 3y'' + 3y' - y = t - 4e^t$

$y''' - 3y'' + 3y' - y = 0$

$r^3 - 3r^2 + 3r = 0$

$r^2 - 3r + 3 = 0$

$r = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{3 \pm \sqrt{3^2 - 4 \times 1 \times 3}}{2 \times 1} = \dfrac{3}{2} \pm \sqrt{3}i$

$r_1 = \dfrac{3}{2} + \sqrt{3}i, r_2 = \dfrac{3}{2} - \sqrt{3}i$

$$y = c_1 \cos\frac{3}{2} + \sqrt{3}t + c_2 \sin\frac{3}{2} - \sqrt{3}t + c_3 \cos\frac{3}{2} + \sqrt{3}t + c_4 \sin\frac{3}{2} - \sqrt{3}t$$

$y_p(x) = t - 4e^t$
$y'_p(x) = 1 - 4e^t$
$y''_p(x) = -4e^t$
$y'''_p(x) = -4e^t$

$-4e^t - 3(-4e^t) + 3(1 - 4e^t) - t - 4e^t = t - 4e^t$
$-4e^t + 3 = 0$
$-4e^t = -3$
$e^t = \frac{3}{4}$
$t = -0.2877$

$$y = c_1 \cos(\frac{3}{2} + \sqrt{3}t) + c_2 \sin(\frac{3}{2} - \sqrt{3}t) + c_3 \cos(\frac{3}{2} + \sqrt{3}t) +$$
$$c_4 \sin(\frac{3}{2} - \sqrt{3}t) - 0.2877 - 4e^{-0.2877}$$

P3.
$y^{(4)} - y'' = 4t + 2te^{-t}$
$y^{(4)} - y'' = 0$
$(r^4 - r^2) = 0$
$(r^2 + r)(r^2 - r) = 0$
$r^2 - r = 0$
$r = \pm 1$
$y = c_1 e^t + c_2 e^t$

$y_p(x) = 4t + 2te^{-t}$
$y'_p(x) = 4 + 2te^{-t} + 2e^{-t}$
$y''_p(x) = -2te^{-t} + 2e^{-t} - 2e^{-t}$
$y'''_p(x) = 2te^{-t} - 2e^{-t} - 2e^{-t} + 2e^{-t}$
$y^{(4)}_p(x) = -2te^{-t} - 2e^{-t} + 2e^{-t} + 2e^{-t} - 2e^{-t}$

$$y^{(4)} - y'' = 4t + 2te^{-t}$$
$$-2te^{-t} - 2e^{-t} + 2e^{-t} + 2e^{-t} - 2e^{-t} - 2te^{-t} + 2e^{-t} - 2e^{-t} =$$
$$4t + 2te^{-t}$$
$$4te^{-t} = 4t + 2te^{-t}$$
$$2te^{-t} = 4t$$
$$e^{-t} = 2$$
$$t = -0.6931$$
$$y = -2.7726 - 1.3863e^{0.6931}$$

$$y = c_1 e^t + c_2 e^t + -2.7726 - 1.3863e^{0.6931}$$

P4.
$$y''' + 4y' = t, y(0) = 0, y'(0) = 0, y''(1) = 0$$
$$y''' + 4y' = 0$$
$$r^3 + 4r = 0$$
$$r^2 + 4 = 0$$
$$r^2 = -4$$
$$r = \pm 2i$$
$$y = c_1 \cos t + c_1 \sin t + c_3 \cos t + c_4 \sin t$$

$$y_p(x) = t$$
$$y'_p(x) =$$
$$y''_p(x) =$$
$$y'''_p(x) =$$