Introduction to Differential Equations
Assignment # 5

Tian Xiaoyang
26001904581

P1.

$y = c_1 e - 3t\, cos\, 4t + c_2 e - 3t\, sin\, 4t$

$y = e^{-3t}(c_1 cos\, 4t + c_2 sin\, 4t)$

$y_1 = e^{-3t}(c_1 cos\, 4t + i sin\, 4t)$

$y_2 = e^{-3t}(c_1 \cos 4t + i sin\, 4t)$

$r_1 = -3 + 4i$

$r_1 = -3 - 4i$

$r_1 = -3 \pm 4i$

$-3 = -\frac{b}{2a}$

$4i = \frac{\sqrt{b^2-4ac}}{a}$

$a = 1, b = 6$

$4i = \frac{\sqrt{36-4ac}}{2}$

$-16 = 36 - 4c$

$c = 25$

$y'' + 3y' + 6.25y = 0$

P2.

(1)

$a = 9, b = 0, c = 4$

$r = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$r = \pm 3i$

$y = c_1 cos 3i + c_2 sin 3i$

(2)

$a = 1, b = -4, c = 5$

$r = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
$r = 2 \pm i$
$y = e^{2t}(c_1 cost + c_2 isint)$

(3)
$a = 2, b = 2, c = 1$
$r = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
$r = -\frac{1}{2} \pm \frac{1}{2}i$
$y = e^{-\frac{1}{2}t}(c_1 cost + c_2 sint)$

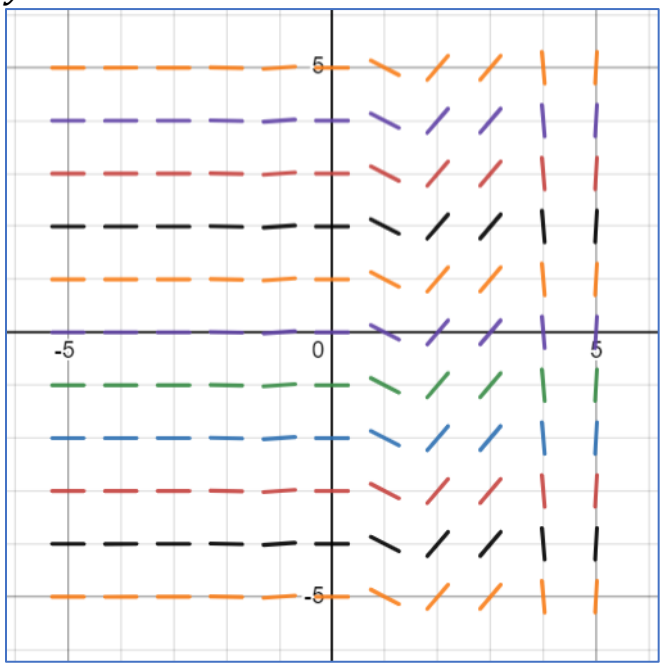
P3.
$a = 1, b = -2, c = 5$
$r = 1 \pm 2i$
$y = e^{t}(c_1 cos2t + c_2 sin2t)$

$t = \frac{\pi}{2}$
$0 = e^{\frac{\pi}{2}}(c_1 cos2\frac{\pi}{2} + c_2 sin2\frac{\pi}{2})$
$0 = e^{\frac{\pi}{2}}(c_1 cos\pi + c_2 sin\pi)$
$0 = -c_1 + 0$
$c_1 = 0$

$y = e^{t}c_1 cos2t + e^{t}c_2 sin2t$
$y = e^{t}c_2 sin2t$
$y' = e^{t}c_2 sin2t + e^{t}2c_2 cos2t$
$2 = e^{\frac{\pi}{2}}c_2 sin2\frac{\pi}{2} + e^{\frac{\pi}{2}}2c_2 cos2\frac{\pi}{2}$
$2 = e^{\frac{\pi}{2}}c_2 sin\pi + e^{\frac{\pi}{2}}2c_2 cos\pi$
$2 = 0 - e^{\frac{\pi}{2}}2c_2$
$2 = -2e^{\frac{\pi}{2}}c_2$

$-1 = e^{\frac{\pi}{2}}c_2$
$-0.208 = c_2$

$y = -0.208e^t sin2t$



P4.
(a)
$t^2y'' + 4ty' + 2y = 0, t > 0$
$\mathrm{x} = \ln t$
$\alpha = 4, \beta = 2$
$y = \frac{d^2y}{dx^2} + (\alpha - 1)\frac{dy}{dx} + \beta y = 0$
$= \frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y$
$r^2 + 3r + 2 = 0$
$r_1 = -1, r_2 = -2$
$y(x) = c_1e^{-x} + c_2e^{-2x}$
$y = c_1e^{-\ln t} + c_2e^{-2\ln t}$
$y = c_1t^{-1} + c_2t^{-2}$

(b)

$t^2y'' + 2ty' + 0.25y = 0, t > 0$

$\mathrm{x} = \ln t$

$\alpha = 2, \beta = 0.25$

$y = \frac{d^2y}{dx^2} + (\alpha - 1)\frac{dy}{dx} + \beta y = 0$

$= \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 0.25y$

$r^2 + 2r + \frac{1}{4} = 0$

$r_1 = -\frac{2+\sqrt{3}}{2}, r_2 = -\frac{2-\sqrt{3}}{2}$

$y(x) = c_1 e^{-\frac{2+\sqrt{3}}{2}x} + c_2 e^{-\frac{2-\sqrt{3}}{2}x}$

$y = c_1 e^{-\frac{2+\sqrt{3}}{2}\ln t} + c_2 e^{-\frac{2-\sqrt{3}}{2}\ln t}$

$y = c_1 t^{-\frac{2+\sqrt{3}}{2}} + c_2 t^{-\frac{2-\sqrt{3}}{2}}$

P5.

$a = 25, b = -20, c = 4$

$r = \frac{-(-20)\pm\sqrt{(-20)^2-4\times4\times25}}{2\times25}$

$r = \frac{20\pm0}{50} = \frac{2}{5}$

$y = e^{-\frac{1}{2}t}(c_1 cost + c_2 sint)$

P6.

$a = 1, b = -6, c = 9$

$r = \frac{-(-6)\pm\sqrt{(-6)^2-4\times1\times9}}{2}$

$r = \frac{6\pm0}{2}$

$r = 3$

$y = e^{3t}(c_1 cost + c_2 sint)$

$$0 = e^0(c_1 cos0 + c_2 sin0)$$
$$0 = c_1 + 0$$
$$0 = c_1$$

$$y' = 3e^{3t}c_2 sint + 3e^{3t}c_2 cost$$
$$2 = 3e^0 c_2 sin0 + 3e^0 c_2 cos0$$
$$2 = 3c_2$$
$$c_2 = \frac{2}{3}$$

$$y = e^{3t}\frac{2}{3}sint$$