Introduction to Differential Equations
Assignment # 4

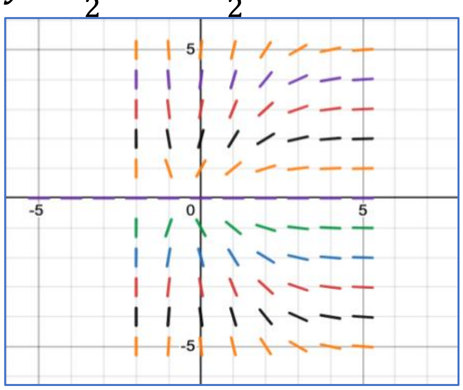
Tian Xiaoyang
26001904581

P1.

$y'' + 5y' = 0$
$ay'' + by' + cy = 0$
General solution
$ar^2e^{rx} + bre^{rx} + ce^{rx} = 0$
$ar^2 + br + c = 0$
$1r^2 + 5r = 0$
$r = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
$r = 0, r = -5$
$y = c_1e^{r_1x} + c_2e^{r_2x}$
$y = c_1e^{0\cdot x} + c_2e^{-5x}$
$y = c_1 + c_2e^{-5x}$

P2.

$y'' + 4y' + 3y = 0, y(0) = 2, y'(0) = -1$
$1r^2 + 4r + 3 = 0$
$(r+1)(r+3) = 0$
$r = -1, r = -3$
$y = c_1e^{-x} + c_2e^{-3x}$
$\left.\begin{matrix} c_1+c_2=2 \\ -c_1+-3c_2=-1 \end{matrix}\right\} \Rightarrow c_1 = \frac{5}{2}, c_2 = -\frac{1}{2}$
$y = \frac{5}{2}e^{-x} - \frac{1}{2}e^{-3x}$



When y>0, for the same y value, as t increases, the slope of the graph decreases.
When y<0, for the same y value, as t increases, the slope of the graph increases.
When y=0, slope is equal to 0

P3.

$y = c_1e^{2t} + c_2e^{-3t}$
$r_1 = 2, r_2 = -3$
$(r-2)(r+3) = 0$
$r^2 + r - 6 = 0$
$y'' + y' - 6y = 0$

P4.

$y'' + (3-\alpha)y' - 2(\alpha - 1)y = 0$

P5.

$y_1(x) = x, y_2(x) = xe^x$
$x^2y'' - x(x+2)y' + (x+2)y = 0, x > 0$
$y = y_1(x) = x$
$y' = 1$
$y'' = 0$
$x^2y'' - x(x+2)y' + (x+2)y = x^2 \cdot 0 - x(x+2) \cdot 1 + (x+2) \cdot x$
$= x^2 + 2x - x^2 - 2x$
$= 0$
$y_1(x) = x \ is \ a \ solution$

$y = y_2(x) = xe^x$
$y' = xe^x + e^x$
$y'' = xe^x + e^x + e^x$
$x^2y'' - x(x+2)y' + (x+2)y$
$= x^2 \cdot (xe^x + e^x + e^x) - x(x+2) \cdot (xe^x + e^x) + (x+2) \cdot xe^x$
$= (x^3e^x + x^2e^x + x^2e^x) - (x^3e^x + 2x^2e^x + x^2e^x + 2xe^x) + x^2e^x +$
$2xe^x$
$= 0$
$y_2(x) = xe^x \quad is \ a \ solution$

$$W(y_1, y_2) = \begin{vmatrix} x & xe^x \\ 1 & xe^x + e^x \end{vmatrix}$$
$$= x(xe^x + e^x) - xe^x$$
$$= x^2e^x - xe^x - xe^x$$
$$= x^2e^x \neq 0$$

$y_1(x) = x, y_2(x) = xe^x$ are fundamental set of solutions

P6.

$$ty'' + 2y' + te^ty = 0$$
$$p(t) = \frac{2}{t}, q(t) = e^t$$
$$W[y_1y_2](t) = ce^{-\int p(t)dt}$$
$$= ce^{-\int \frac{2}{t}dt}$$
$$= ce^{-2\ln t}$$

$$W(y_1, y_2) = 2, t = 1$$
$$2 = c \cdot e^{-2ln1}$$
$$c = 2$$

$$W(y_1, y_2) = 2, t = 5$$
$$W(y_1, y_2) = 2 \times 5^2$$
$$= 50$$