

Introduction to Differential Equations
Assignment # 11

Date Given: June 20, 2022

Date Due: June 27, 2022

- P1.** (1 point) Find the Laplace transform of $f(t) = \int_0^t \sin(t - \tau) \cos \tau \, d\tau$.
- P2.** (1 point) By using the convolution theorem, find (express in terms of a convolution integral) the inverse Laplace transform of $F(s) = \frac{1}{(s+1)^2(s^2+4)}$.
- P3.** (2 points) Express in terms of a convolution integral the solution of the following initial value problem: $y'' + 2y' + 2y = \sin \alpha t$; $y(0) = 0, y'(0) = 0$.
- P4.** (2 points) Express in terms of a convolution integral the solution of the following initial value problem: $y'' + 4y' + 4y = g(t)$; $y(0) = 2, y'(0) = -3$.
- P5.** (1 point) Transform the differential equation $u^{(4)} - u = 0$ with initial conditions $u(0) = 0, u'(0) = 1, u''(0) = 2, u'''(0) = 3$, into a system of first order equations corresponding to this initial value problem.
- P6.** (3 points)
- (a) Transform the system $x_1' = -0.5x_1 + 2x_2, x_2' = -2x_1 - 0.5x_2$ into a single equation of second order.
 - (b) Find x_1 and x_2 that also satisfy the initial conditions $x_1(0) = -2, x_2(0) = 2$.
 - (c) Sketch the graph of the solution in the x_1x_2 -plane.