Introduction to Differential Equations
Assignment # 10

Date Given: June 13, 2022
Date Due: June 20, 2022

P1. (1 point) Find the Laplace transform of

$$f(t) = \begin{cases} 0, & \text{if } t < 1 \\ t^2 - 2t + 2, & \text{if } t \geq 1 \end{cases}$$

P2. (1 point) Find the inverse Laplace transform of $F(s) = \dfrac{e^{-2s}}{s^2 + s - 2}$.

P3. (2 points) Find the solution of the following initial value problem

$$y'' + y = g(t), \quad y(0) = 0, y'(0) = 1, \quad g(t) = \begin{cases} t/2, & \text{if } 0 \leq t < 6 \\ 3, & \text{if } t \geq 6 \end{cases}$$

Draw the graphs of the forcing function and of the solution.

P4. (2 points) Find the solution of the following initial value problem

$$y'' + y = u_\pi(t) - u_{3\pi}(t), \quad y(0) = 0, y'(0) = 0.$$

Draw the graphs of the forcing function and of the solution.

P5. (2 points) Find the solution of the following initial value problem $y'' + 2y' + 2y = \delta(t - \pi)$; $y(0) = 1, y'(0) = 0$. Draw a graph of the solution.

P6. (2 points) Find the solution of the following initial value problem $y'' + 3y' + 2y = \delta(t - 5) + u_{10}(t)$; $y(0) = 0, y'(0) = 1/2$. Draw a graph of the solution.