Introduction to Differential Equations
Assignment # 10

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P1.

$$f\begin{cases} 0, & if\ t<1 \\ t^2-2t+2, & if\ t\geq 1 \end{cases}$$

$$f(t)=0+u_1(t)\,t^2-2t+2$$
$$=u_1(t)\,(t^2-2t+2)-1$$
$$\mathcal{L}\{f(t)\}=e^{-s}(\frac{2!}{s^3}-\frac{2}{s^2}+\frac{3}{s})$$

P2.

$$\frac{e^{-2s}}{s^2+2-2}$$
$$=\frac{e^{-2s}}{(s-2)(s+1)}$$

$$\frac{a}{s+2}+\frac{b}{s-1}$$
$$a(s-1)+b(s+2)=1$$
$$as-a+bs+2b=1$$
$$as+bs=0$$
$$a=-b$$
$$3b=1$$
$$b=\frac{1}{3}$$
$$a=-\frac{1}{3}$$
$$\frac{1}{3(s+2)}+\frac{1}{3(s-1)}$$
$$=-\frac{1}{3}e^{-2t}+\frac{1}{3}e^{t}$$

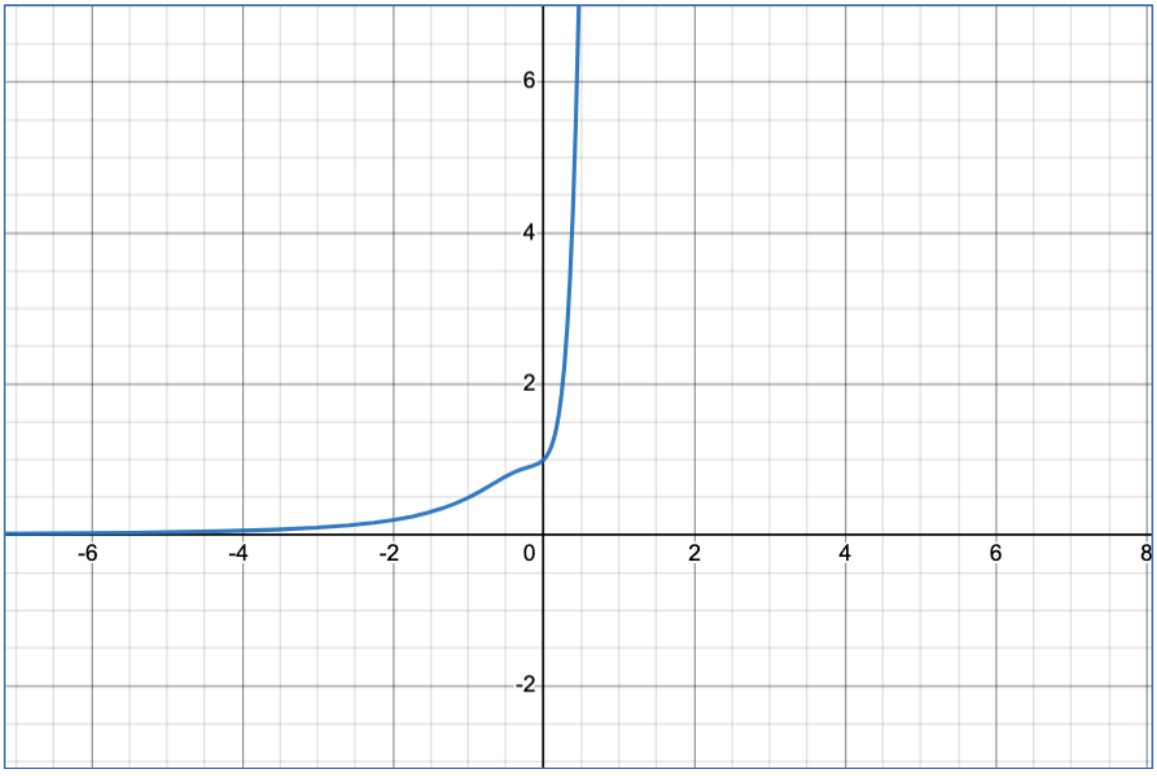
$$\mathcal{L}^{-1}\{G(s)\}=-\frac{1}{3}e^{-2t}+\frac{1}{3}e^{t}$$

$$\mathcal{L}^{-1}\{e^{-2s}G(s)\} = \frac{1}{3}e^{-2(t-2)} + \frac{1}{3}e^{t-2}$$

P3.

$$y'' + y = g(t)$$
$$y(0) = 0, y'(0) = 1$$
$$g(t) = \begin{cases} \frac{t}{2}, & if\ 0 \le t < 6 \\ 3, & if\ t \ge 6 \end{cases}$$

$$s^2y(s) + sy(0) - y'(0) + y(s) = \mathcal{L}^{-1}g(t)$$
$$s^2y(s) + y(s) = \frac{e^{-6s}}{s}$$
$$s^2y(s) + y(s) = \frac{e^{-6s}}{s} + 1$$
$$y(s) = \frac{s}{e^{-6s}(s^2+1)} + \frac{1}{s^2+1}$$



P4.

$y'' + y = u_\pi(t) - u_{3\pi}(t),\ y(0) = 0, y'(0) = 0$

$\mathcal{L}\{g(t)\} = \mathcal{L}\{u_\pi(t) - u_{3\pi}(t)\}$

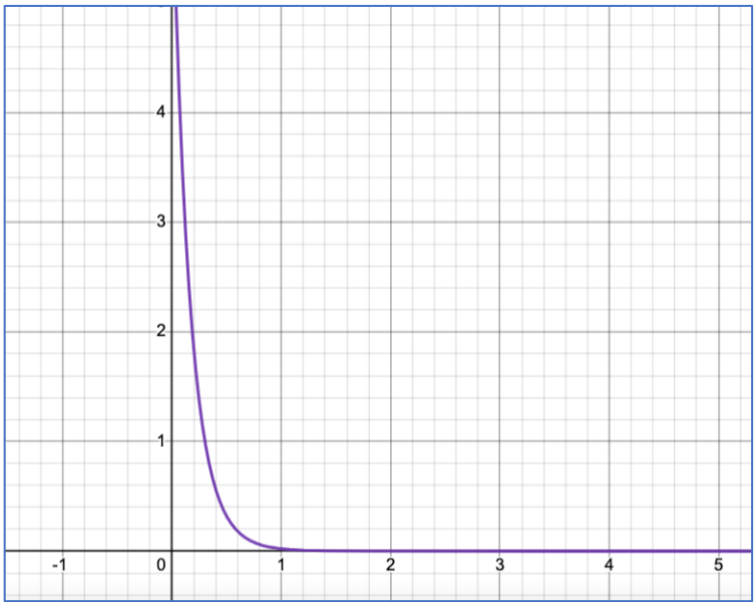
$= \frac{e^{-\pi s} - e^{-3\pi s}}{s}$

$y'' + y = s^2 y(s) + sy(0) - y'(0) + y(s)$

s

$s^2 y(s) + y(s) = \frac{e^{-\pi s} - e^{-3\pi s}}{s}$

$y(s) = \frac{e^{-\pi s} - e^{-3\pi s}}{s(s^2+1)}$



P5.

$y'' + 2y' + 2y = \delta(t - \pi), y(0) = 1, y'(0) = 0$

$y'' + 2y' + 2y = s^2 Y(s) - sy(0) - y'(0) + 2\big(sY(s) - y(0)\big) + 2Y(s)$

$\delta(t - \pi) = e^{-\pi s}$

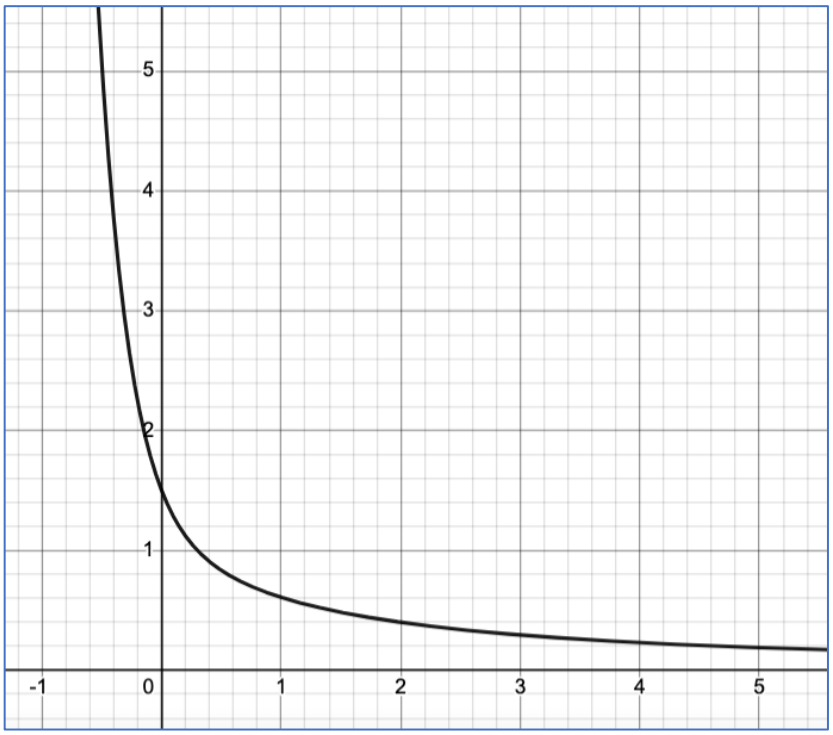
$s^2 Y(s) - sy(0) - y'(0) + 2\big(sY(s) - y(0)\big) + 2Y(s) = e^{-\pi s}$

$s^2 Y(s) - s + 2sY(s) - 2 + 2Y(s) = e^{-\pi s}$

$s^2 Y(s) + 2sY(s) + 2Y(s) = e^{-\pi s} + s + 2$

$Y(s)(s^2 + 2s + 2) = e^{-\pi s} + s + 2$

$$Y(s) = \frac{e^{-\pi s}+s+2}{s^2+2s+2}$$
$$Y(s) = \frac{e^{-\pi s}}{s^2+2s+2} + \frac{s}{s^2+2s+2} + \frac{2}{s^2+2s+2}$$



P6.

$$y'' + 3y' + 2y = \delta(t-5) + u_{10}(t), y(0) = 0, y'(0) = \frac{1}{2}$$
$$y'' + 3y' + 2y = s^2Y(s) - sy(0) - y'(0) + 3\big(sY(s) - y(0)\big) + 2Y(s)$$
$$= s^2Y(s) - \frac{1}{2} + 3sY(s) - 3y(0) + 2Y(s)$$
$$= s^2Y(s) - \frac{1}{2} + 3sY(s) + 2Y(s)$$

$$\delta(t-5) = e^{-5s}$$
$$u_{10}(t) = \frac{e^{-10s}}{s}$$

$$s^2Y(s) - \frac{1}{2} + 3sY(s) + 2Y(s) = e^{-5s} + \frac{e^{-10s}}{s}$$

$$Y(s)(s^2 + 3s + 2) = e^{-5s} + \frac{e^{-10s}}{s} + \frac{1}{2}$$

$$Y(s) = \frac{e^{-5s}}{s^2+3s+2} + \frac{e^{-10s}}{s(s^2+3s+2)} + \frac{1}{2(s^2+3s+2)}$$