Introduction to Differential Equations
Assignment # 1

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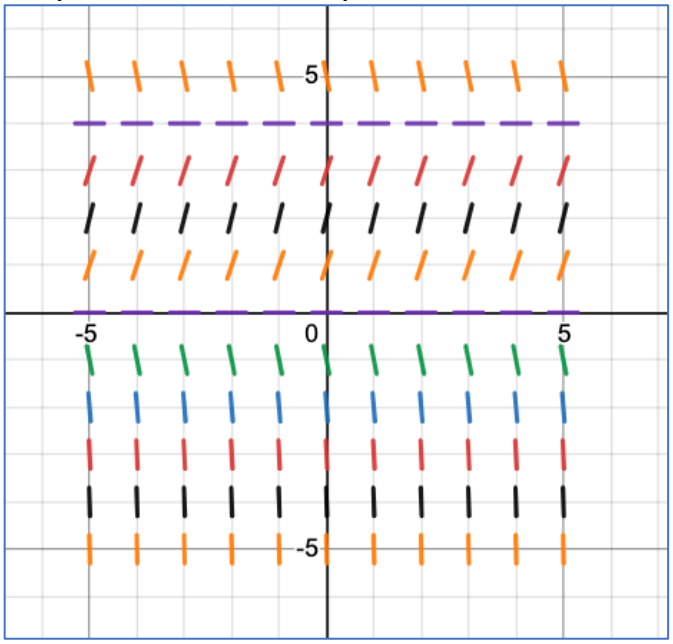
P1.

$y' = y(4 - y)$

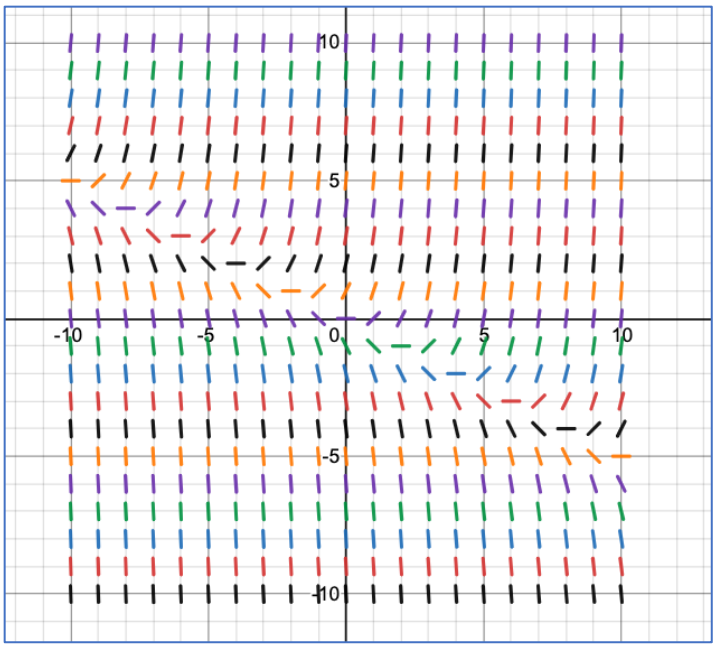
Slope field of the equation



P2.

$y' = t + 2y$

Slope field of the equation



From the differential equation and the slope equation, the solutions have decency on both values t and y.

When $y > 0$, as t increases in value, the slope is positive and goes steeper.

When $y < 0$, as t increases in value, the slope is negative and goes steeper.

When $y = -\frac{t}{2}$, the slope is 0.

P3.

$y' = ay - b, y(0) = y_0$

$y = \frac{b}{a} + ce^{at},\ c = y - \frac{b}{a}$

$y = \frac{b}{a} + (y_0 - \frac{b}{a})e^{at}$

$\frac{dy}{dt} = y - 5, y(0) = y_0$

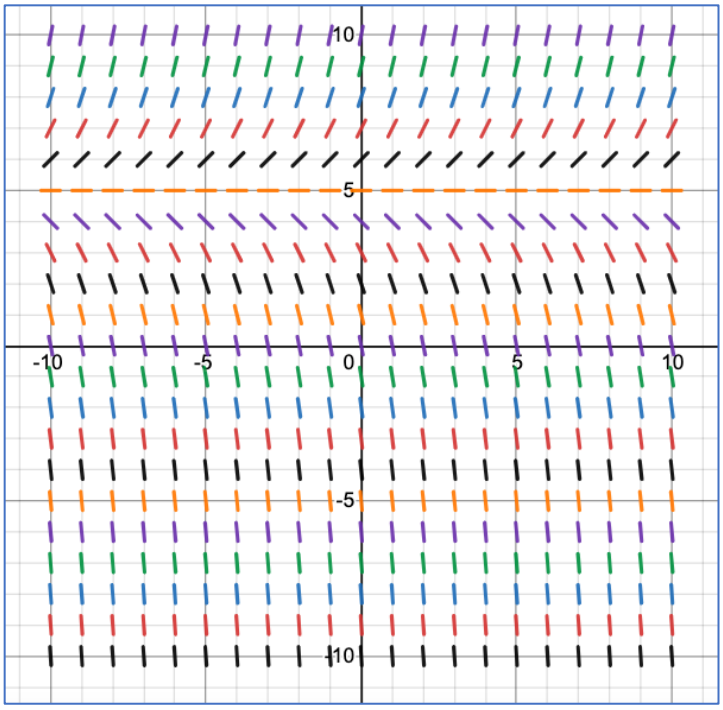
$a = 1, b = 5$

$y = \frac{5}{1} + (y_0 - \frac{5}{1})e^{1 \cdot 0}$

$= 5 + (y_0 - 5)$

$\frac{dy}{dt} = y - 5$ has no dependency on t, the solutions to the differential equation only depend on values of y.

The slope field of the equation



$t = 0,\ y = 0$
$\frac{dy}{dt} = 0 - 5 = -5$
$y = 5 + (0 - 5)$
$= 5 - 5$
$= 0$

$t = 0,\ y = 1$
$\frac{dy}{dt} = 1 - 5 = -4$
$y = 5 + (1 - 5)$
$= 5 - 4$
$= 1$

$t = 0,\ y = 2$

$\frac{dy}{dt} = 2 - 5 = -3$
$y = 5 + (2 - 5)$
$= 5 - 3$
$= 2$

$t = 0,\ y = -1$
$\frac{dy}{dt} = -1 - 5 = -6$
$y = 5 + (-1 - 5)$
$= 5 - 6$
$= -1$

$t = 0,\ y = -2$
$\frac{dy}{dt} = -2 - 5 = -7$
$y = 5 + (-2 - 5)$
$= 5 - 7$
$= -2$

P4.
(a)
$(1 + y^2)\frac{d^2y}{dt^2} + \frac{dy}{dt} + y = e^t$
Highest derivative is $\frac{d^2y}{dt^2}$, therefore, this is a second order differential equation.
(b)
$\frac{dy}{dt} + ty = 0$
Highest derivative is $\frac{dy}{dt}$, therefore, this is a first order differential equation.

(c)

$$\frac{d^3y}{dt^3} + t\frac{dy}{dt} + ycos^2t = t^3$$

Highest derivative is $\frac{d^3y}{dt^3}$, therefore, this is a third order differential equation.