Introduction to Differential Equations
Assignment # 9

Date Given: June 6, 2022
Date Due: June 13, 2022

P1. (1 points) Use integration by parts to find the Laplace transform of $f(t) = t^n e^{at}$, where a is a real constant, and n is a non-negative integer.

P2. (1 point) Find the Laplace transform of

$$f(t) = \begin{cases} t, & \text{if } 0 \leq t < 1 \\ 0, & \text{if } 1 \leq t < \infty \end{cases}$$

P3. (1 point) Find the inverse Laplace transform of $F(s) = \dfrac{2s + 1}{s^2 - 2s - 2}$

P4. (1 point) Find the inverse Laplace transform of $F(s) = \dfrac{8s^2 - 4s + 12}{s(s^2 + 4)}$

P5. (2 points) Use the Laplace transform to solve the following initial value problem: $y'' - 2y' + 4y = 0$, $y(0) = 2, y'(0) = 0$.

P6. (2 points) Use the Laplace transform to solve the following initial value problem: $y'' - 2y' + 2y = e^{-t}$, $y(0) = 0, y'(0) = 1$.

P7. (2 points) Find the Laplace transform $Y(s) = \mathcal{L}[y]$ of the solution of the following initial value problem

$$y'' + 4y = \begin{cases} t, & \text{if } 0 \leq t < 1 \\ 1, & \text{if } 1 \leq t < \infty \end{cases}$$

$y(0) = 0, y'(0) = 0$.