Introduction to Differential Equations
Assignment # 2

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P1.

$y(1) = 1/2, t > 0$

$ty' + 2y = t2 - t + 1$

General solution

$\mu(t)\frac{dy}{dt} + p(t)\mu(t)y = \mu(t)g(t)$

$\mu(t) = e^{\int p(t)dt}$

$y' + \frac{2}{t}y = 2 - 1 + \frac{1}{t}$

$\mu(t) = e^{\int \frac{2}{t}dt} = e^{2ln|t^2|} = t^2$

$t^2y' + 2ty = 2t^2 - t^2 + t$

$t^2y' = 2t^2 - t^2 + t$

$t^2y = t^2 + \frac{2}{3}t^3 - \frac{1}{3}t^3 + \frac{1}{2}t^2 + c$

$y = \frac{3}{2} - \frac{1}{3}t + \frac{c}{t^2}$

$\frac{1}{2} = \frac{3}{2} - \frac{1}{3} + c$

$-\frac{2}{3} = c$

$y = \frac{3}{2} - \frac{1}{3}t - \frac{2}{3t^2}$

P2.

$y(\pi/2) = 1, t > 0$

$ty' + 2y = 2\, sin\, t$

$\mu(t) = e^{\int p(t)dt}$

$ty' + 2y = 2\, sin\, t$

$y' + \frac{2}{t}y = \frac{2\, sin\, t}{t}$

$\mu(t) = e^{\int \frac{2}{t} dt} = e^{2ln|t^2|} = t^2$
$t^2 y' + 2ty = 2t\ sin\ t$
$t^2 y = 2t\ cos\ t + 2sin\ t + c$
$y = \frac{2t\ cos\ t}{t^2} + \frac{2sin\ t}{t^2} + \frac{c}{t^2}$
$1 = \frac{2\frac{\pi}{2} cos \frac{\pi}{2}}{\frac{\pi^2}{2}} + \frac{2sin\ \frac{\pi}{2}}{\frac{\pi^2}{2}} + \frac{c}{\frac{\pi^2}{2}}$
$1 = 1.295 + \frac{c}{\frac{\pi^2}{2}}$
$-0.295 = \frac{c}{\frac{\pi^2}{2}}$
$c = 0.7279$

P4.
$y' + y^2 sinx = 0$
$\frac{dy}{dt} = -y^2 sinx$
$-\frac{1}{y^2} dy = \sin x\, dt$
$\int \left(-\frac{1}{y^2}\right) dy = \int \sin x\, dx$
$-\frac{1}{y} = \cos x + c$
$y = -\frac{1}{\cos x + c}$

P5.
$\frac{dy}{dx} = \frac{x^2}{1+y^2}$
$(1 + y^2)dy = x^2 dx$
$\int (1 + y^2)dy = \int x^2 dx$
$y + \frac{1}{3} y^3 = \frac{1}{3} x^3 + c$

P6.

(a)

$y' = 2x/(1+2y), y(2) = 0$

$\frac{dy}{dx} = \frac{2x}{1+2y}$

$(1+2y)dy = 2xdx$

$\int(1+2y)dy = \int 2xdx$

$y^2 = x^2 - y$

$y = 1 \pm \sqrt{x^2 - y}$

(b)