DSP Midterm Exam

Monday, May 24, 2021

Question 1: T or F, Multiple Choice, and Short Answer
(Estimated time: 5-10 minutes)

1. **T** or **F**: The time index n for DT signals usually has units of seconds.

2. **T** or **F**: DT signals are typically plotted as a series of lollipops.

3. **T** or **F**: A DT frequency of π is considered a high frequency.

4. **T** or **F**: A DT frequency of 2π is considered a low frequency.

5. **T** or **F**: The DTFT of a DT signal is periodic with a period of 2π.

6. The DTFT of a rectangle is a:

   (a) Sine wave
   (b) **Periodic sinc**
   (c) Rectangle
   (d) None of the above

7. The DTFT of a single impulse is:

   (a) A single impulse
   (b) **A constant**
   (c) Infinite
   (d) None of the above

8. A DT signal is given as follows:

$$x(n) = \big(3\delta(n) - 5\delta(n-1) + 3\delta(n-2)\big)u(n+1)$$

   The value of $x(n)$ at $n = 3$ is:

   (a) 0
   (b) 0.25
   (c) 0.5
   (d) 1
   (e) None of the above

$$x(3) = \big(3\delta(3) - 5\delta(2) + 3\delta(1)\big)u(4)$$
$$= (0 - 0 + 0)1 = 0$$

9. A DT signal $x(n)$ is given as follows:

$$x(n) = 101$$

The length of $x(n)$ is:

(a) 0

(b) 1

(c) 101

$\boxed{\textbf{(d) } \infty}$

(e) None of the above

10. A DT signal $x(n)$ is given as follows:

$$x(n) = \text{rect}_5(n)$$

We want to create a new signal $y(n)$ that is identical to $x(n)$ but is centered about $n = 0$ (i.e., we want a rectangle that is symmetric about time zero). To create $y(n)$, we should use:

(a) $y(n) = x(2n)$

$\boxed{\textbf{(b) } \boldsymbol{y(n) = x(n+2)}}$

(c) $y(n) = x(n-2)$

(d) $y(n) = x(n+2) + x(n-2)$

(e) None of the above

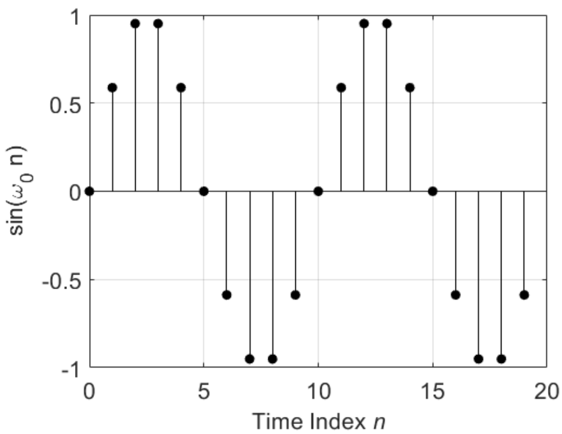
The normal length-5 rectangle starts at $n = 0$ and goes to $n = 4$:

$$\text{rect}_5(n) = \delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3) + \delta(n-4)$$

We want the rectangle to start at time $n = -2$ and go to time $n = 2$, so we need to shift to the left by two samples via $n \to n+2$:

$$\text{rect}_5(n+2) = \delta(n+2) + \delta(n+1) + \delta(n) + \delta(n-1) + \delta(n-2)$$

11. Two cycles of a DT sine wave $\sin(\omega_0 n)$ are shown in the following plot:



The value of ω_0 is:

(a) 1/5

$\boxed{\textbf{(b) } \boldsymbol{\pi/5}}$

(c) 5

(d) 5π

(e) None of the above

Looking at the signal, 1 cycle contains 10 samples.
But, 1 cycle = 2π radians, thus in 2π radians there are 10 samples:

$$\omega_0 = \frac{2\pi}{10} = \frac{\pi}{5}$$

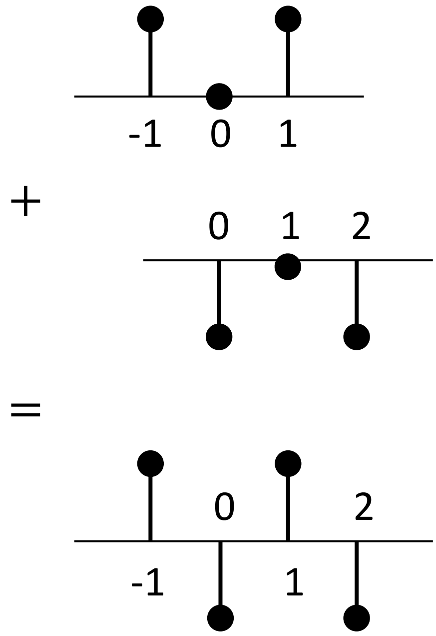
12. Two DT signals, $x_1(n)$ and $x_2(n)$, are given as follows:

$$x_1(n) = \delta(n+1) + \delta(n-1)$$

$$x_2(n) = \delta(n) - \delta(n-1)$$

In the space provided below, sketch the convolution of these two signals. You must show your work to receive credit.

(Note: All lollipops have height 1 or -1)



$1x_1(n-0)$
This is $x_1(n)$ scaled by 1 and shifted by 0 (i.e., simply $x_1(n)$) due to the first term in $x_2(n)$.

$-1x_1(n-1)$
This is $x_1(n)$ scaled by -1 and shifted by -1 due to the second term in $x_2(n)$.

13. A DT signal $x(n)$ is given as follows:

$$x(n) = \delta(n+2) + 2\delta(n) + \delta(n-2)$$

Which of the following is the DTFT $X(e^{j\omega})$ of this signal?

(a) $\boxed{e^{j2\omega} + 2 + e^{-j2\omega}}$

(b) $2e^{j\omega} + 2 + 2e^{-j\omega}$

(c) $\delta(2\omega) + 2\delta(\omega) + \delta(-2\omega)$

(d) $2\delta(\omega) + \delta(2\omega) - 2\delta(\omega)$

(e) None of the above

Use the table of transform pairs:

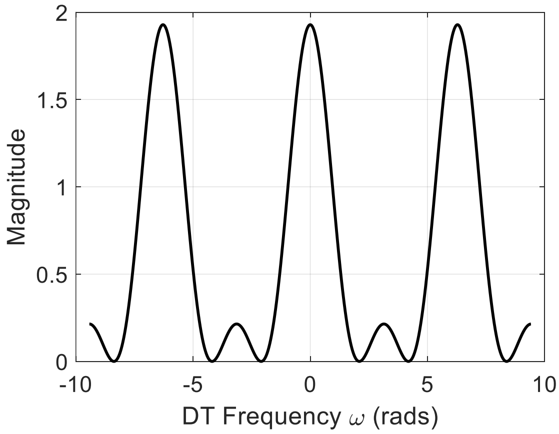
$$\delta(n) \leftrightarrow 1$$

$$\delta(n-n_0) \leftrightarrow e^{-j\omega n_0}$$

14. In simple terms, describe the key difference between the DTFT of $\mathrm{rect}_3(n)$ and the DTFT of $\mathrm{rect}_3(n+1)$:

Both have the same magnitude spectrum, but their phase spectra differ.
Specifically, the phase spectrum of the first rectangle is $-\omega$ (with jumps of $\pm\pi$ when the magnitude spectrum is negative).
The phase spectrum of the second rectangle is zero (with jumps of $\pm\pi$ when the magnitude spectrum is negative).

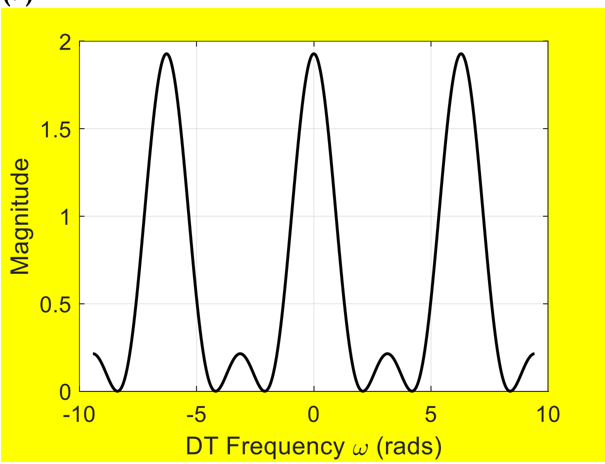
15. A DT signal $x(n)$ has a magnitude spectrum as shown below (showing the range -3π to $+3\pi$):



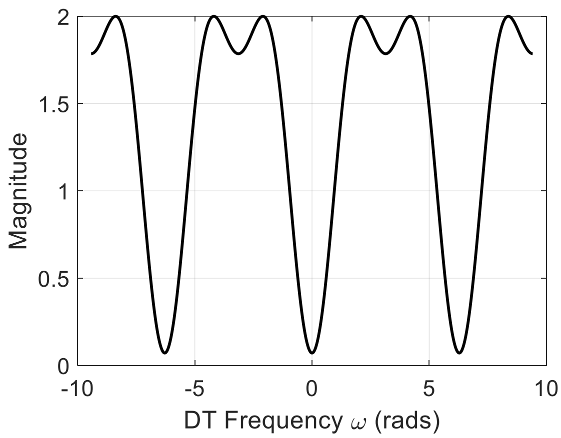
The magnitude spectrum of $x(n-2)$ would look like:

A shift in the time domain causes a phase change in the frequency domain, thus the magnitude spectrum is the same.

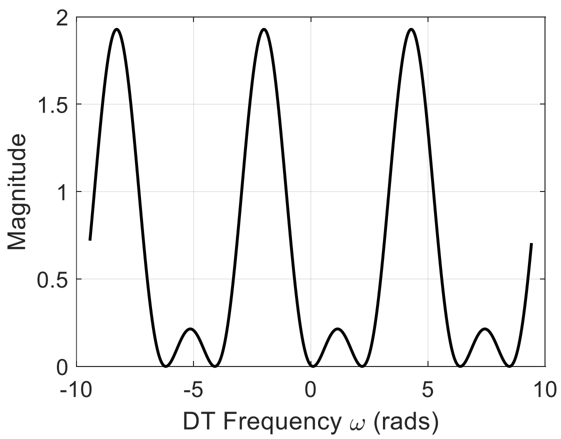
(a)



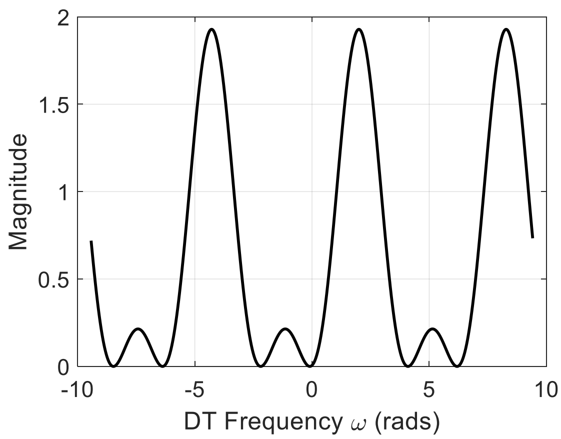
(b)



(c)

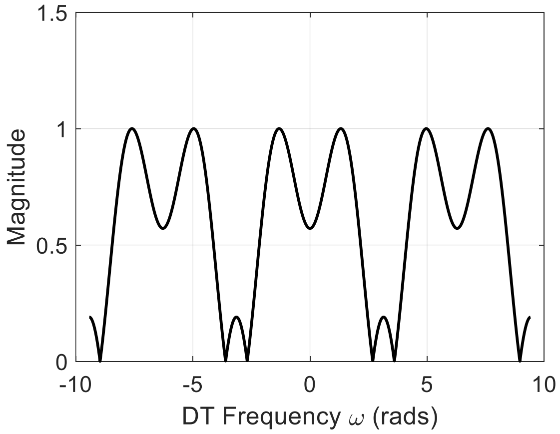


(d)



(e) None of the above

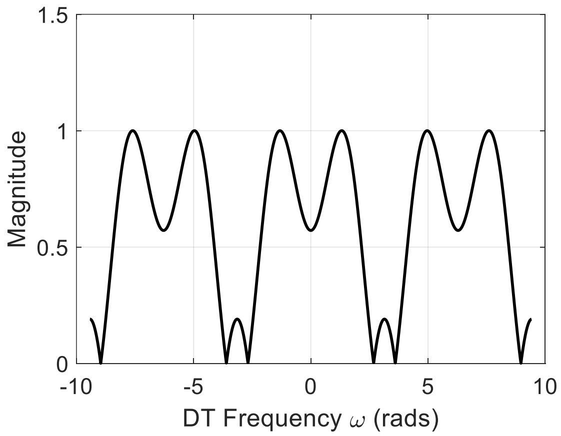
16. A DT signal $x(n)$ has a magnitude spectrum as shown below (showing the range -3π to $+3\pi$):



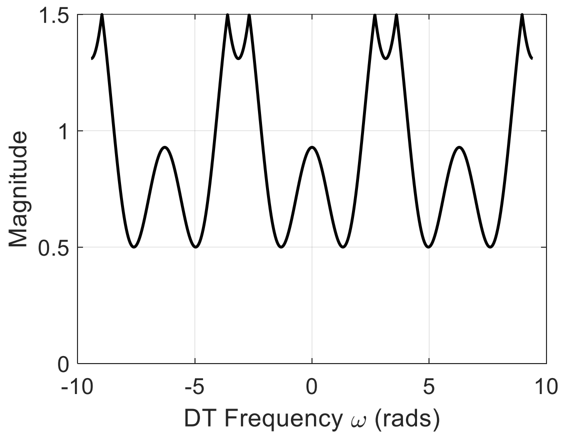
Multiplying by a scalar in the time domain does the same multiplication in the frequency domain ($-1.5\ \times$) but the negative will not show in the magnitude spectrum.

The magnitude spectrum of $-1.5x(n)$ would look like:

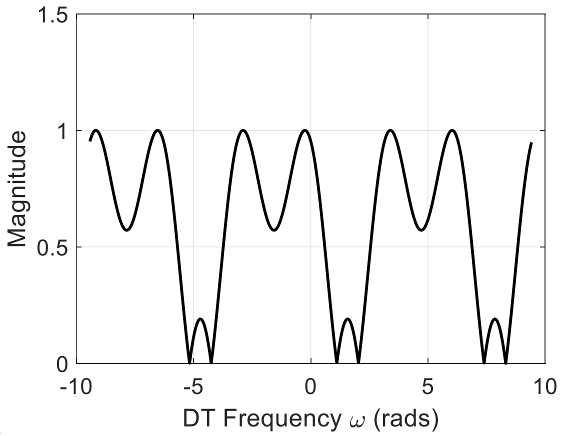
(a)



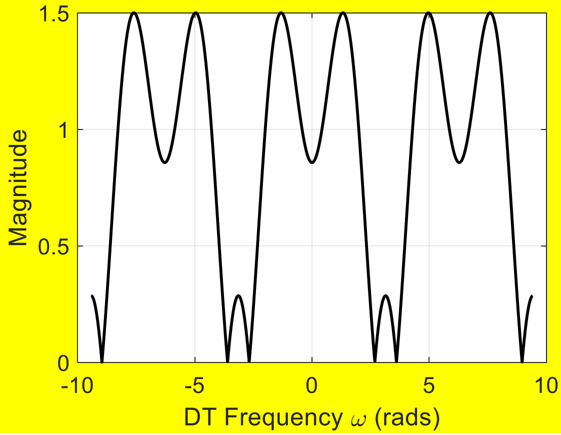
(b)



(c)

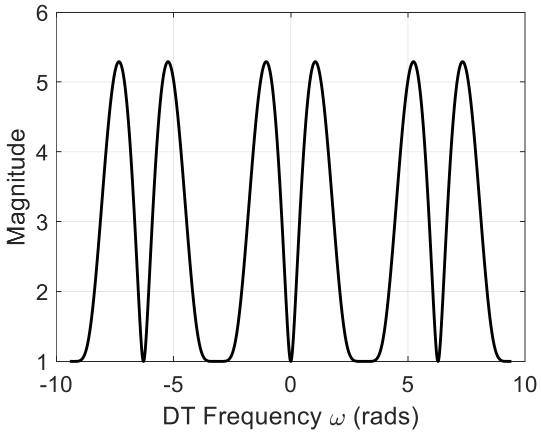


(d)



(e) None of the above

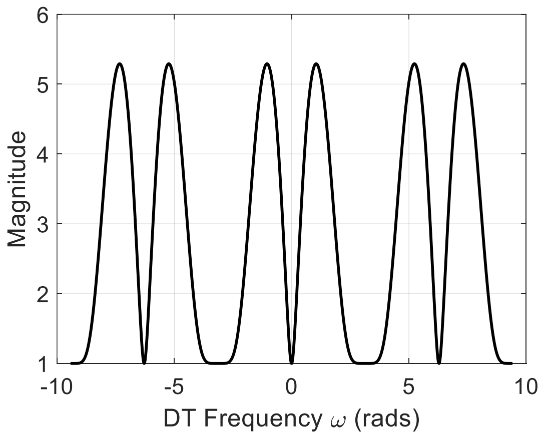
17. A DT signal $x(n)$ has a magnitude spectrum as shown below (showing the range -3π to $+3\pi$):



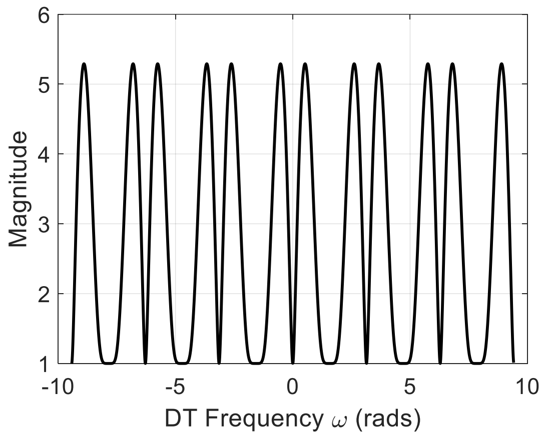
Multiplying by $\cos(\pi n)$ in the time domain will horizontally shift the spectrum by π.

The magnitude spectrum of $\cos(\pi n)\,x(n)$ would look like:

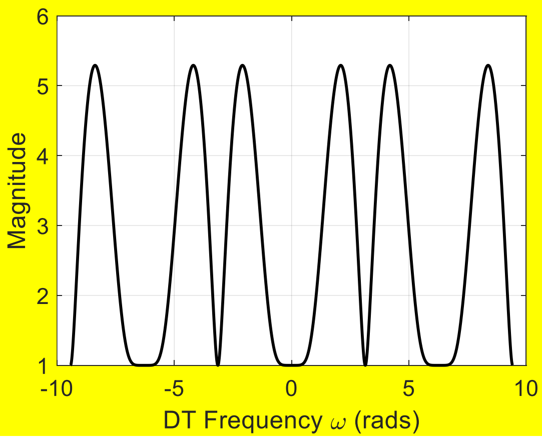
(a)



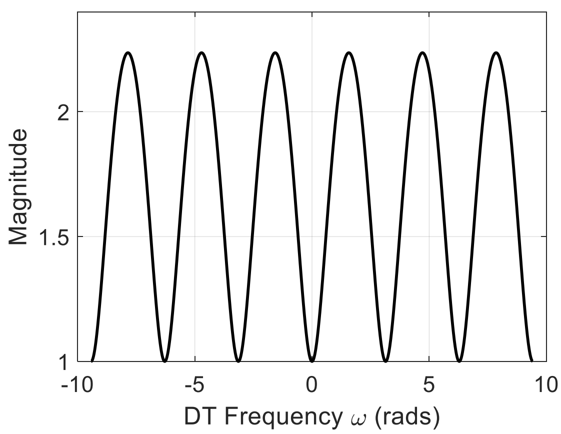
(b)



(c)



(d)



(e) None of the above