Digital Signal Processing

Spring Semester 2022

Welcome to DSP!

Damon M. Chandler <chandler@fc.ritsumei.ac.jp>
Visual Information Engineering Laboratory
Ritsumeikan University

Outline

1. Course overview
2. Today’s learning objectives
3. Lecture 1 technical content

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Grading Scheme

- In-class activities: 25%
- Midterm exam: 25% (Week 7)
- Final exam: 25% (Week 15)
- Final project: 25% (due after Finals week)

Outline

1. Course overview
2. Today's learning objectives
3. Lecture 1 technical content

Today's learning objectives

From **today's lecture**, you should **be able to**…

- Define the term "discrete-time signal"
- Write a DT signal using mathematical notation
- Draw a signal as a plot
- Write and use common DT signals
- Compute very basic signal properties

Outline

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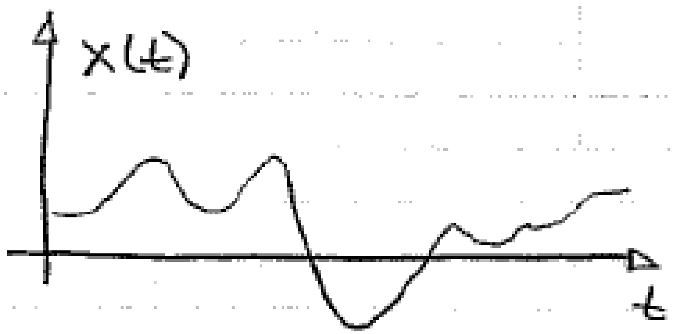
What is a signal?

Notation: A continuous-time signal $x(t)$ is a continuous-valued function of a continuous independent (time) variable



dependent variable

$$x(t)$$

independent variable

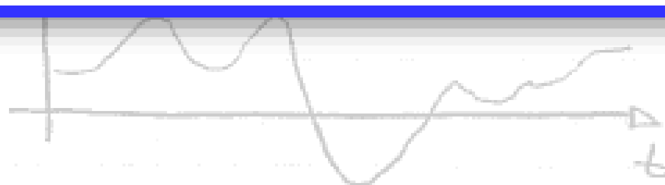
$x(t_0) \in \mathbb{R}$ or $\mathbb{C}$

$t \in \mathbb{R}$

What is a signal?

Notation: A continuous-time signal $x(t)$ is a continuous-valued function of a

DSP folks refer to "continuous time" via the acronym "CT."



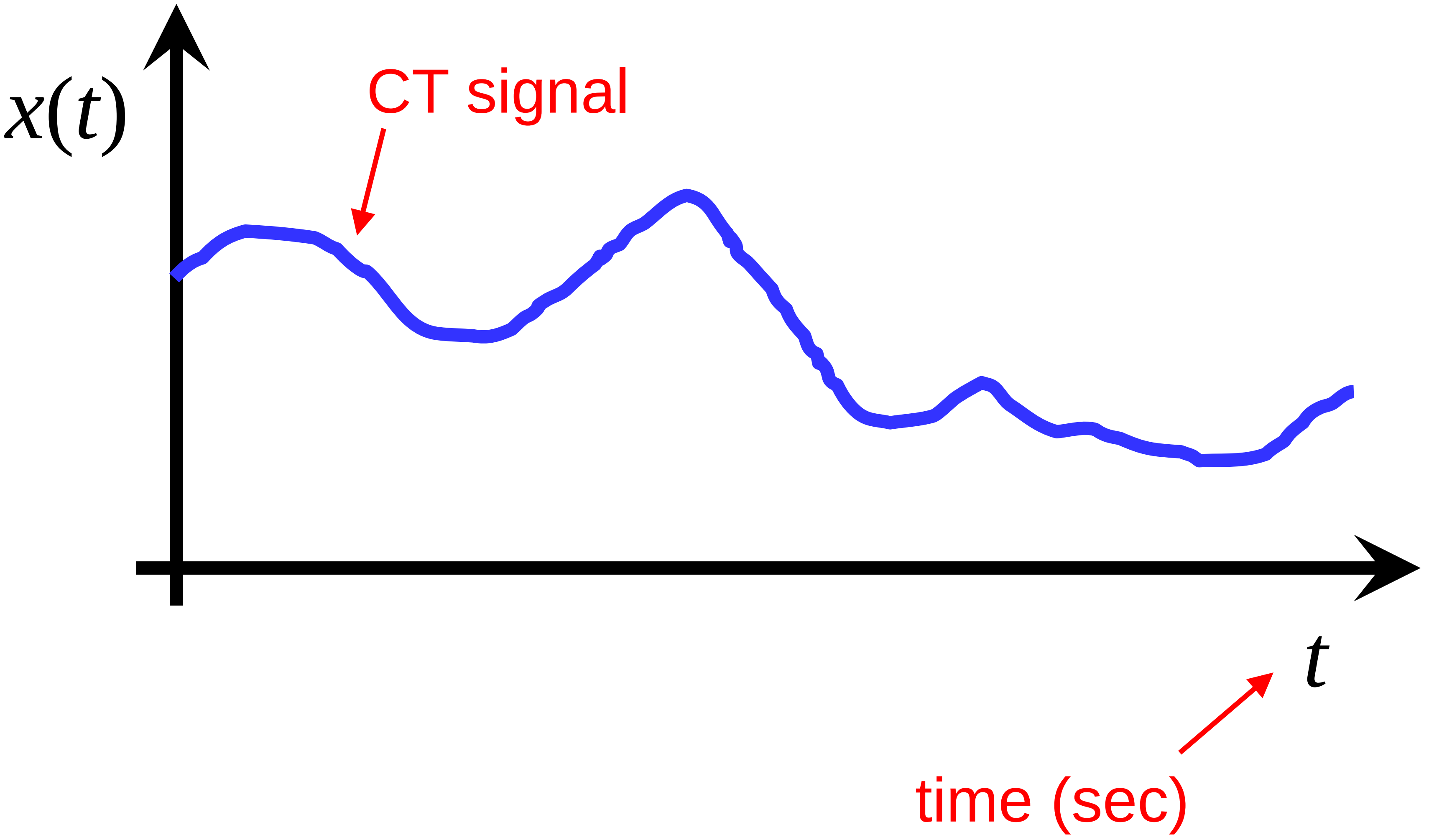
independent variable

$x(t_0) \in \mathbb{R}$ or $\mathbb{C}$

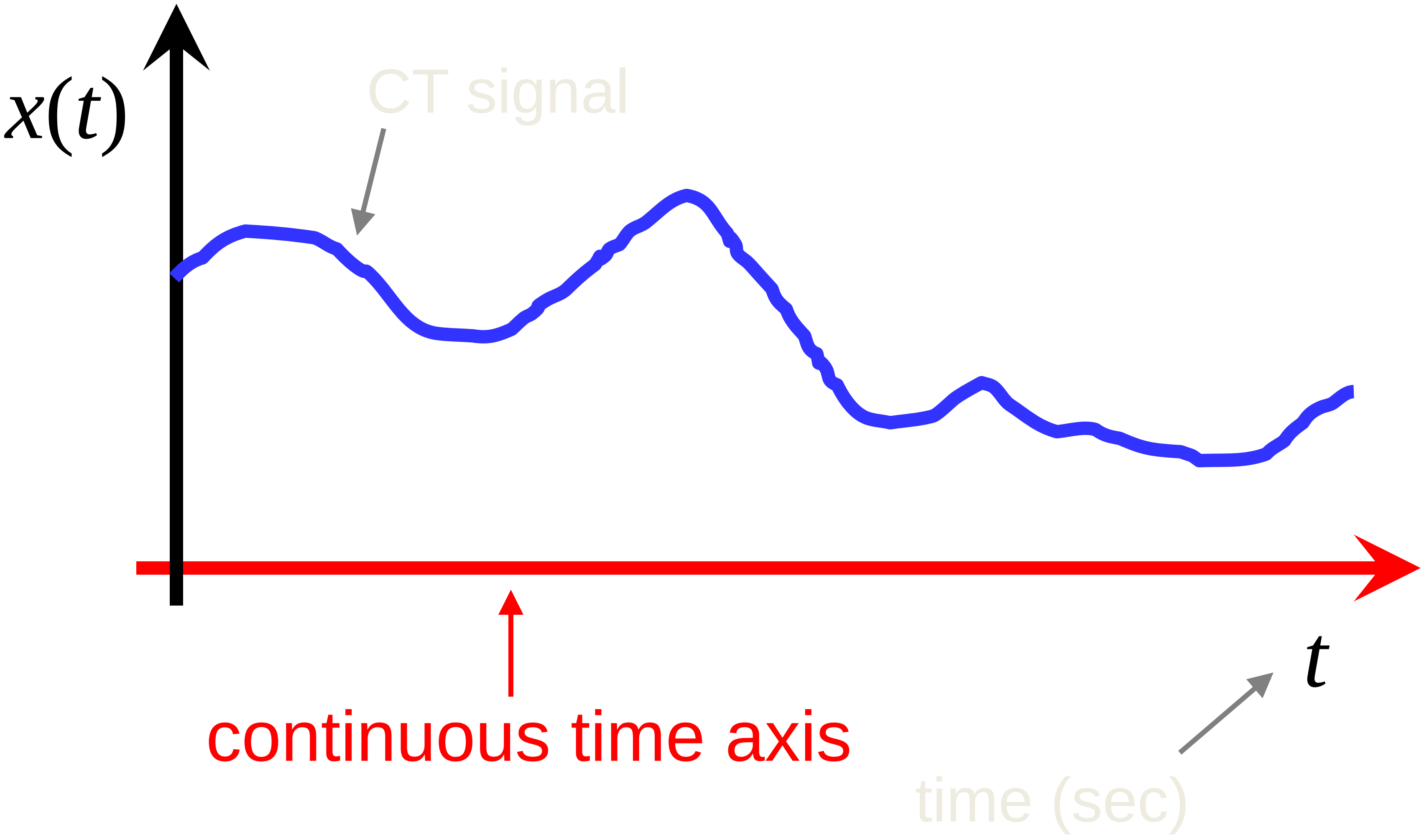
$t \in \mathbb{R}$

https://www.neumann.com/homestudio/en/how-to-connect-a-microphone-to-your-computer

CT signals vs. DT signals



CT signals vs. DT signals



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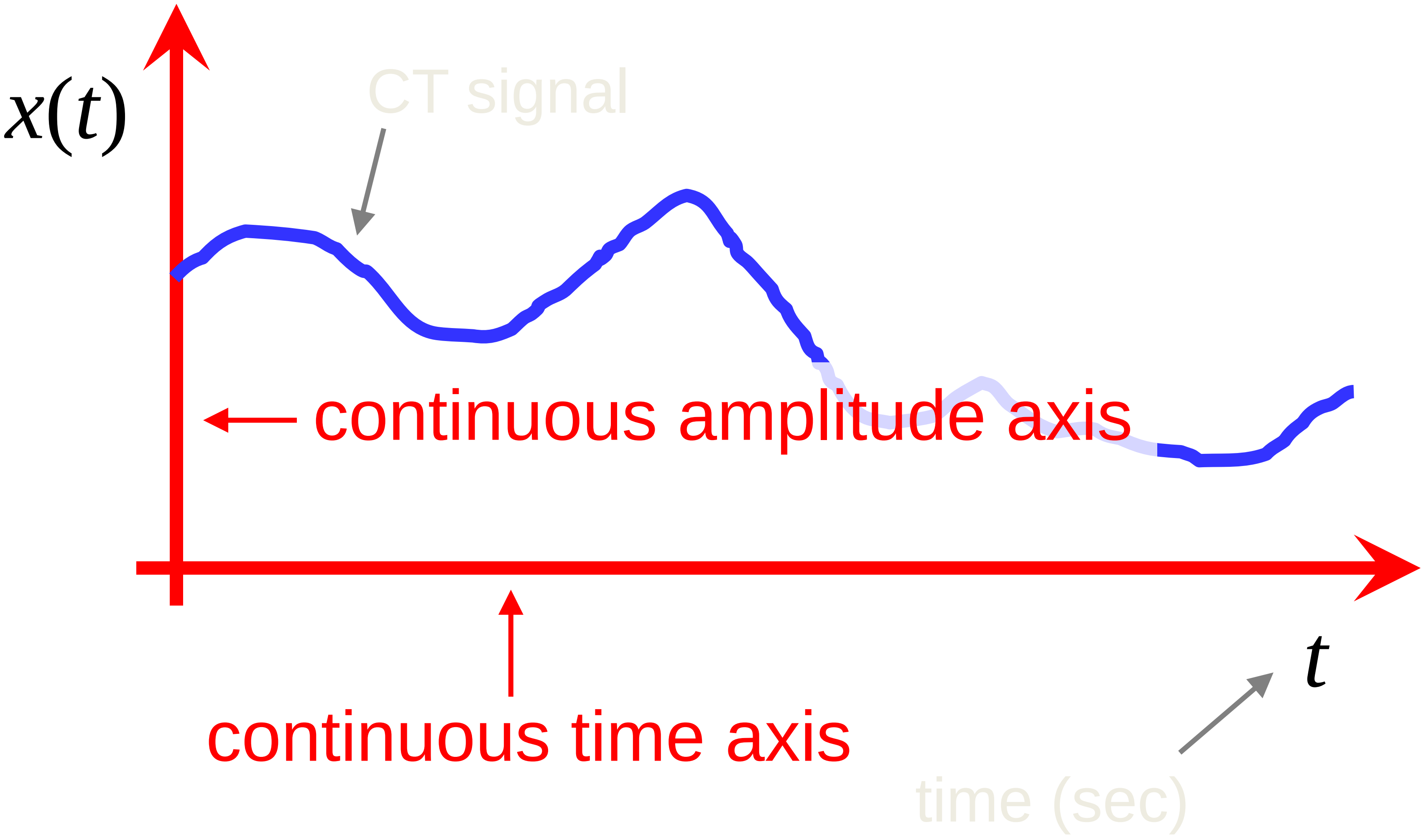

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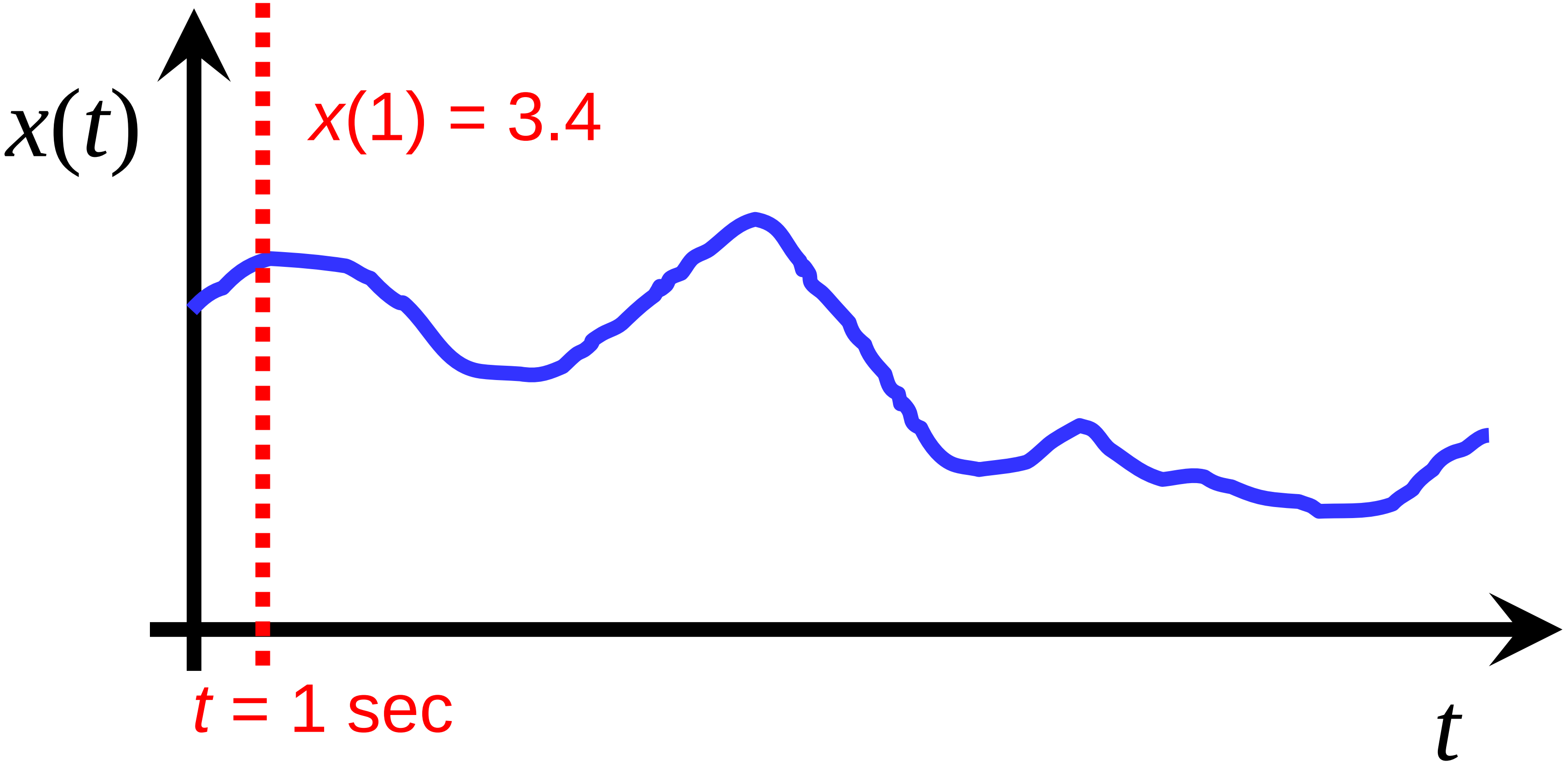

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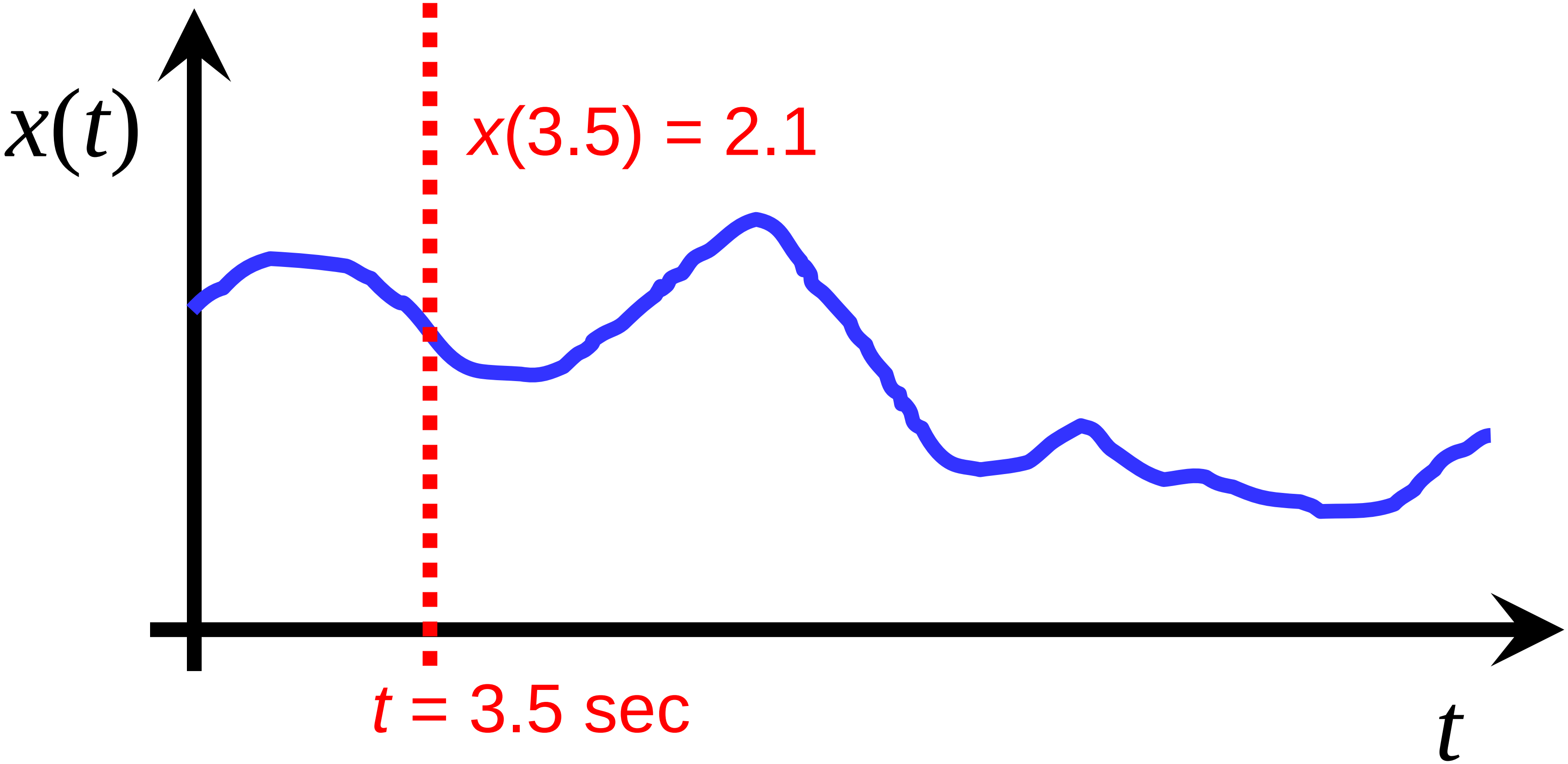

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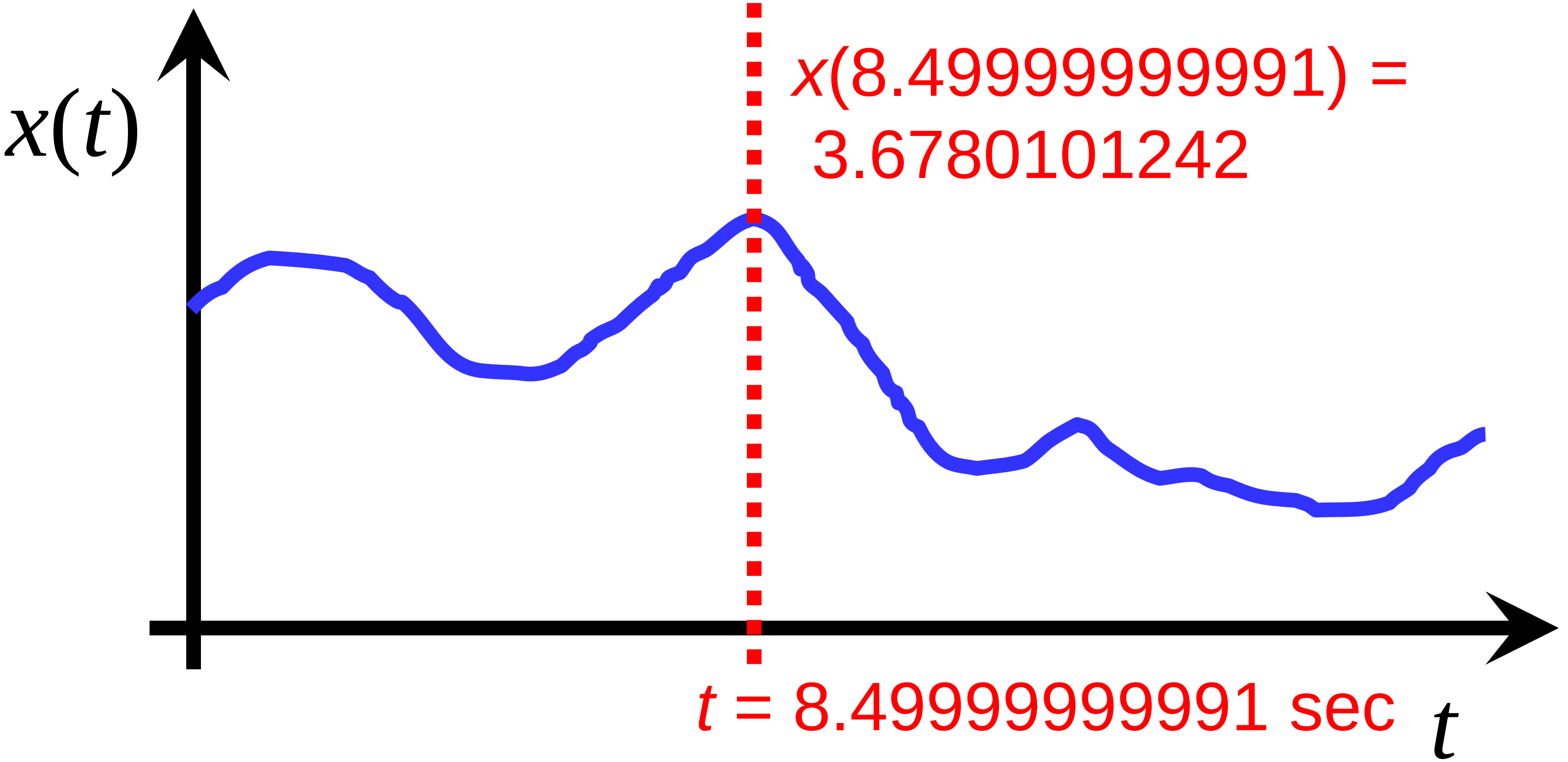
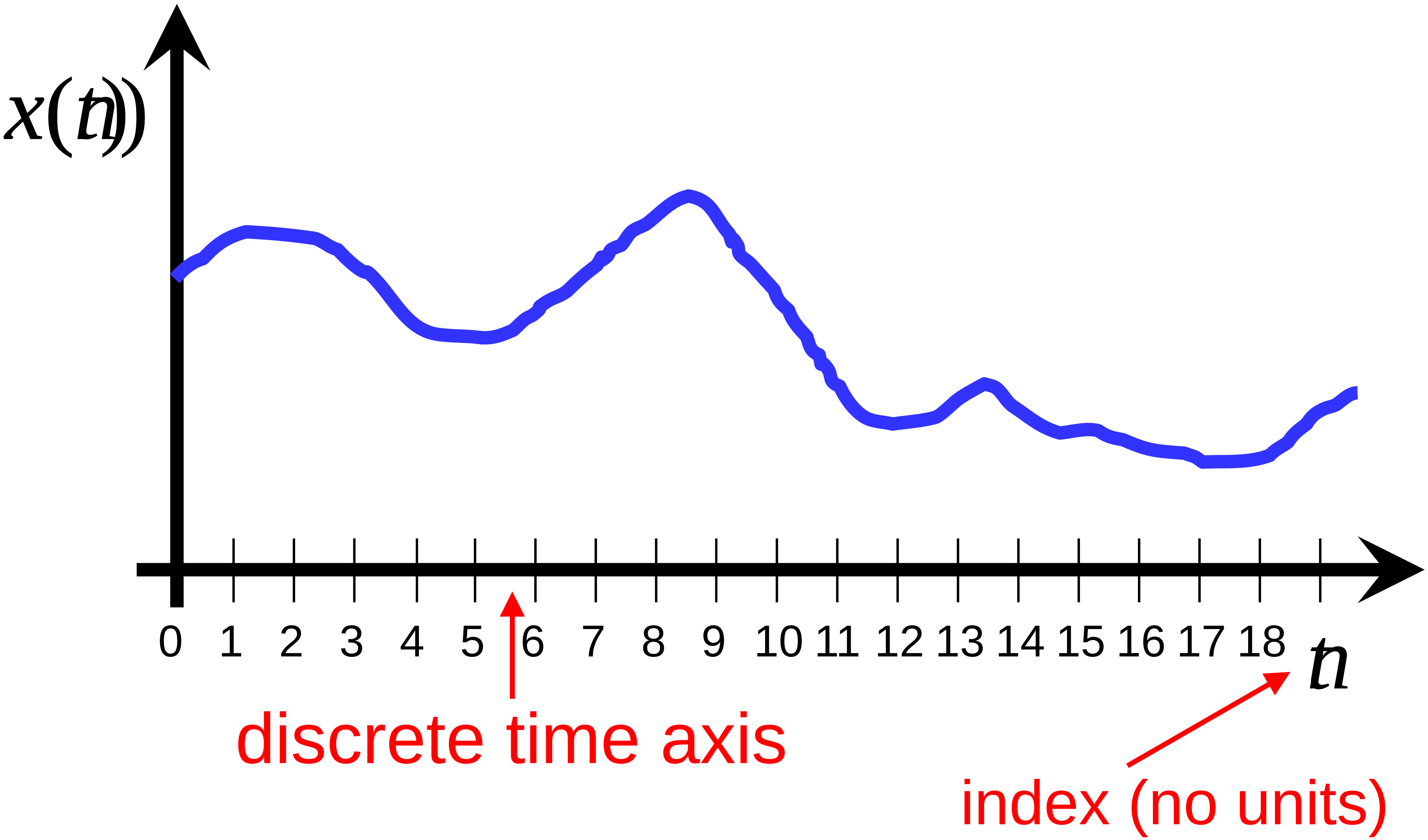


CT signals vs. DT signals



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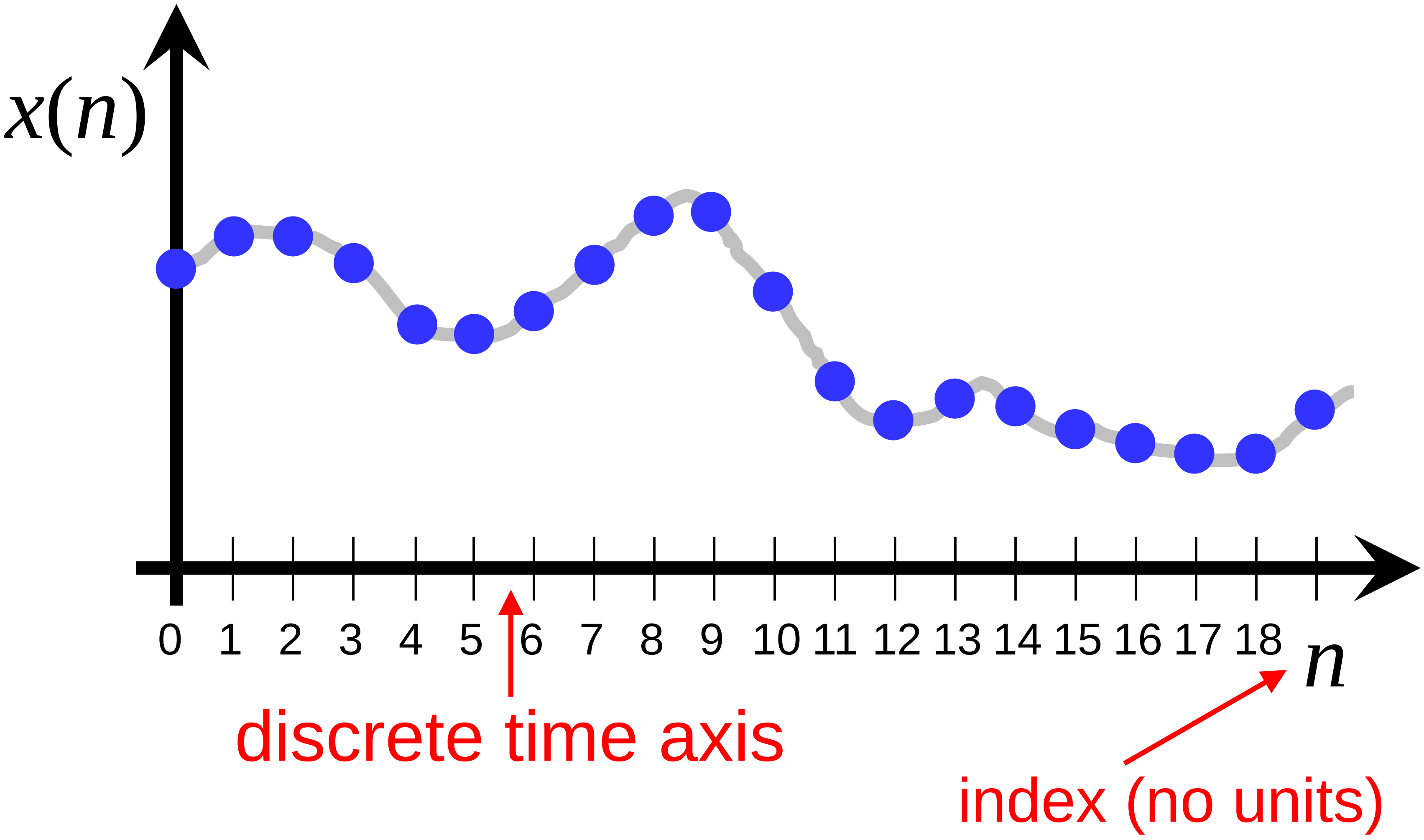
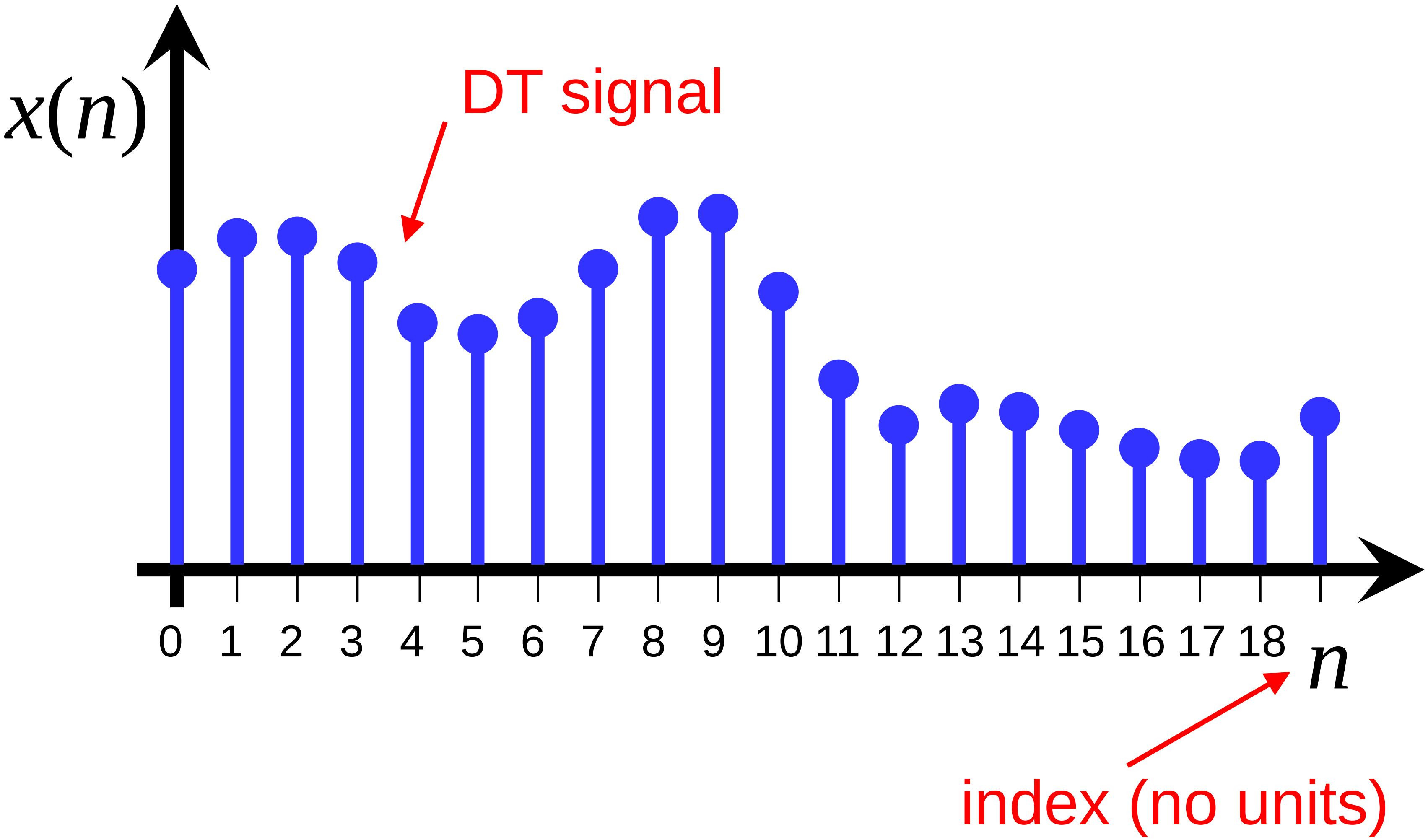
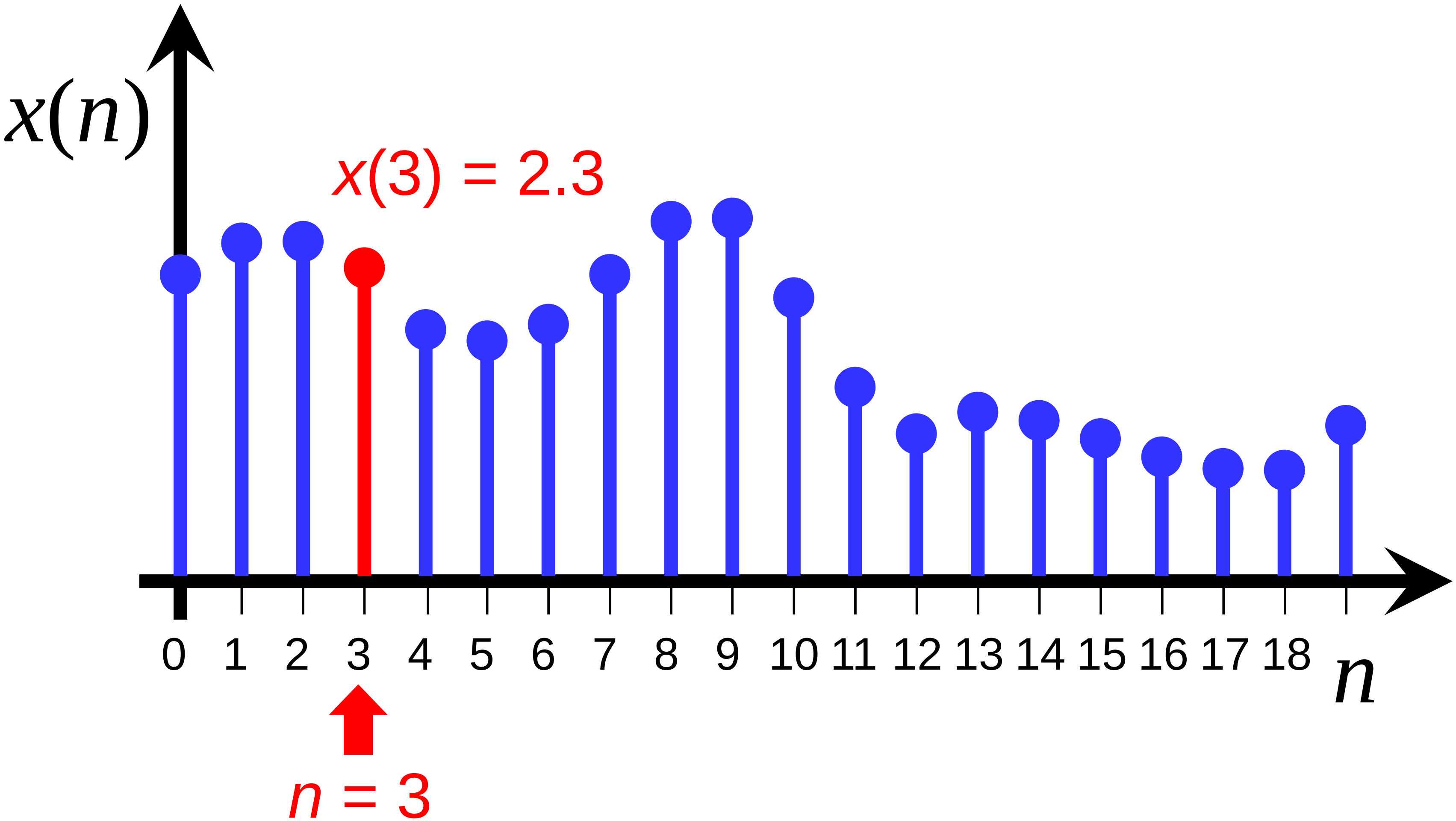


CT signals vs. DT signals



CT signals vs. DT signals



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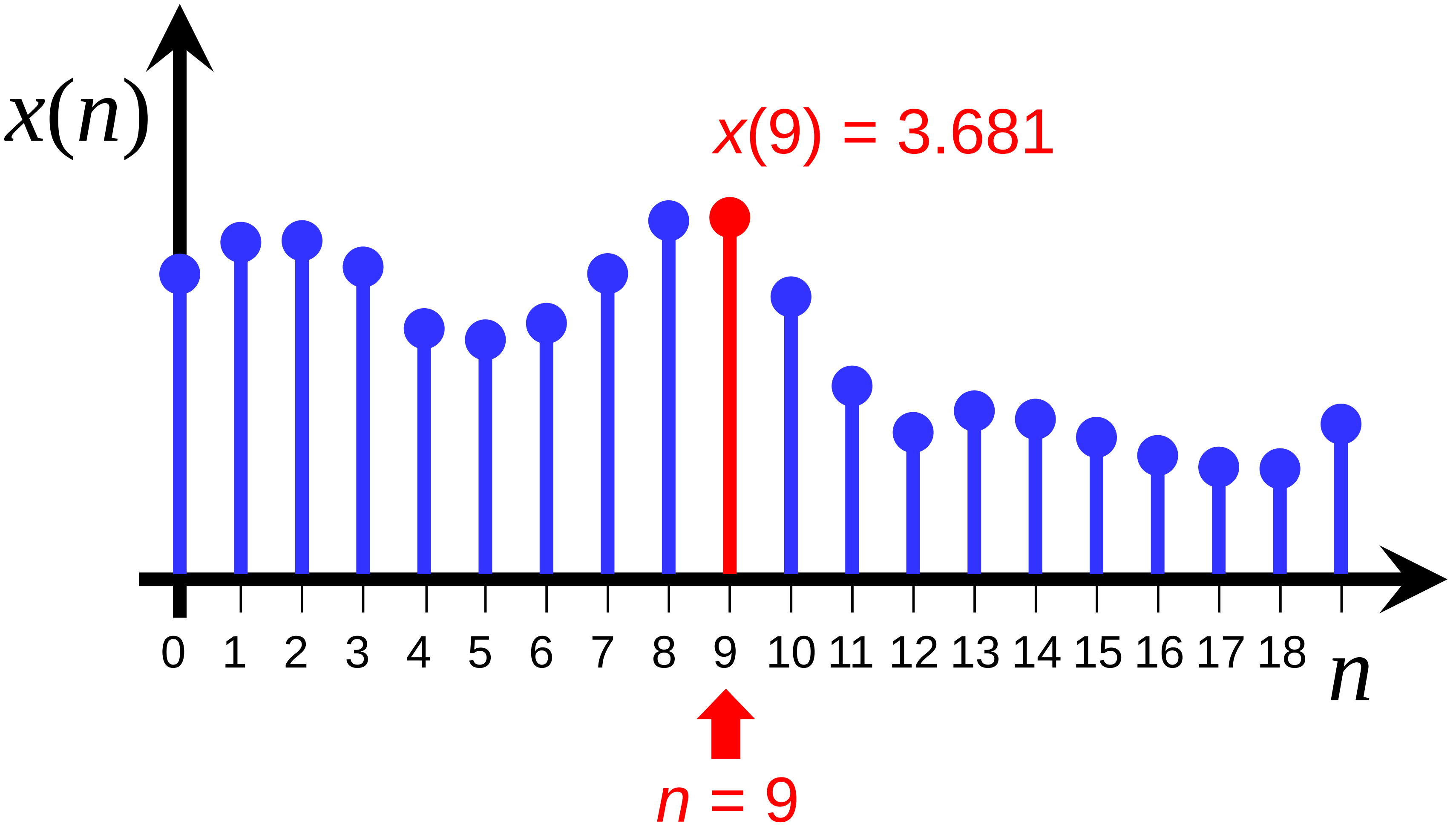
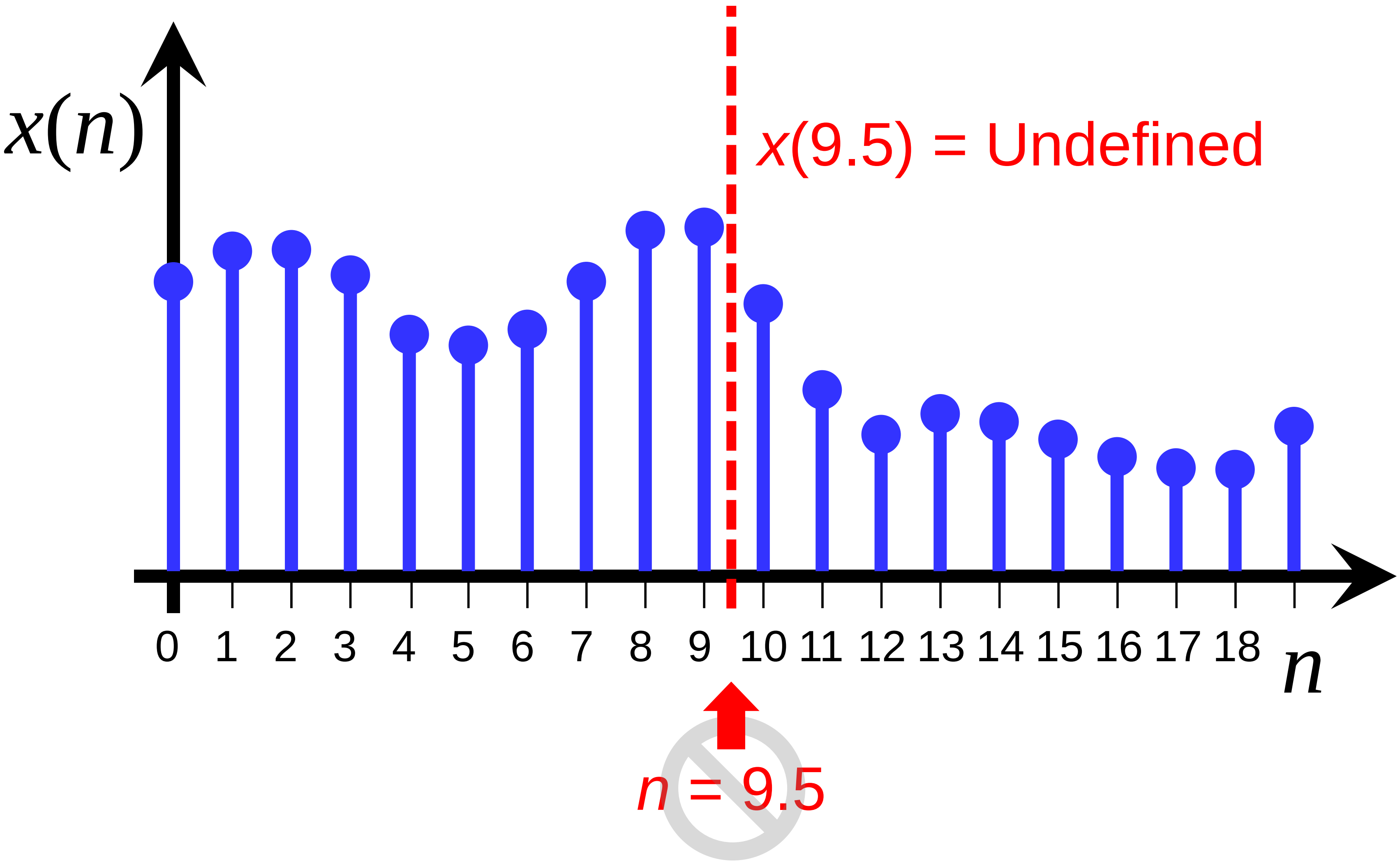
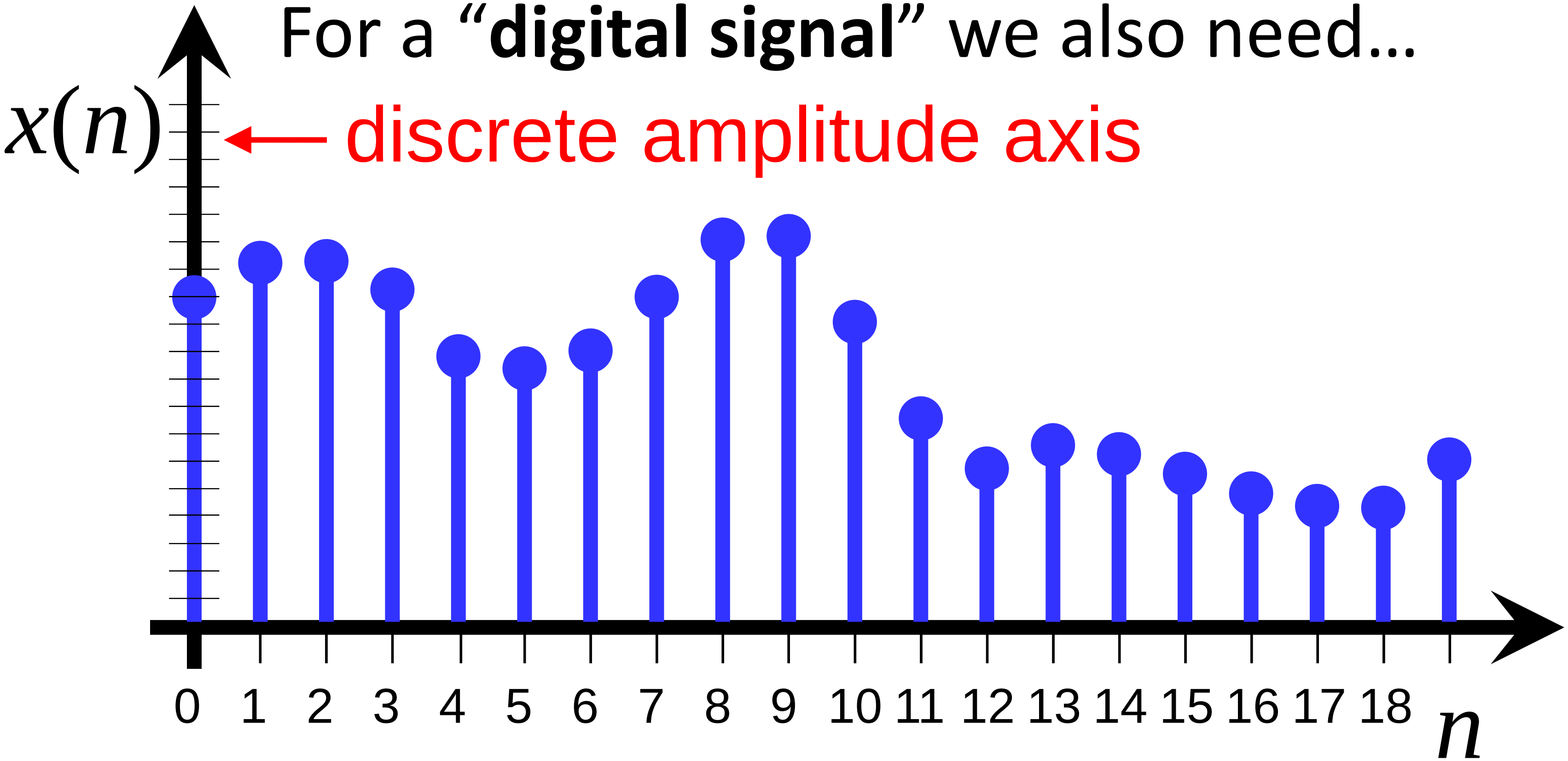


CT signals vs. DT signals



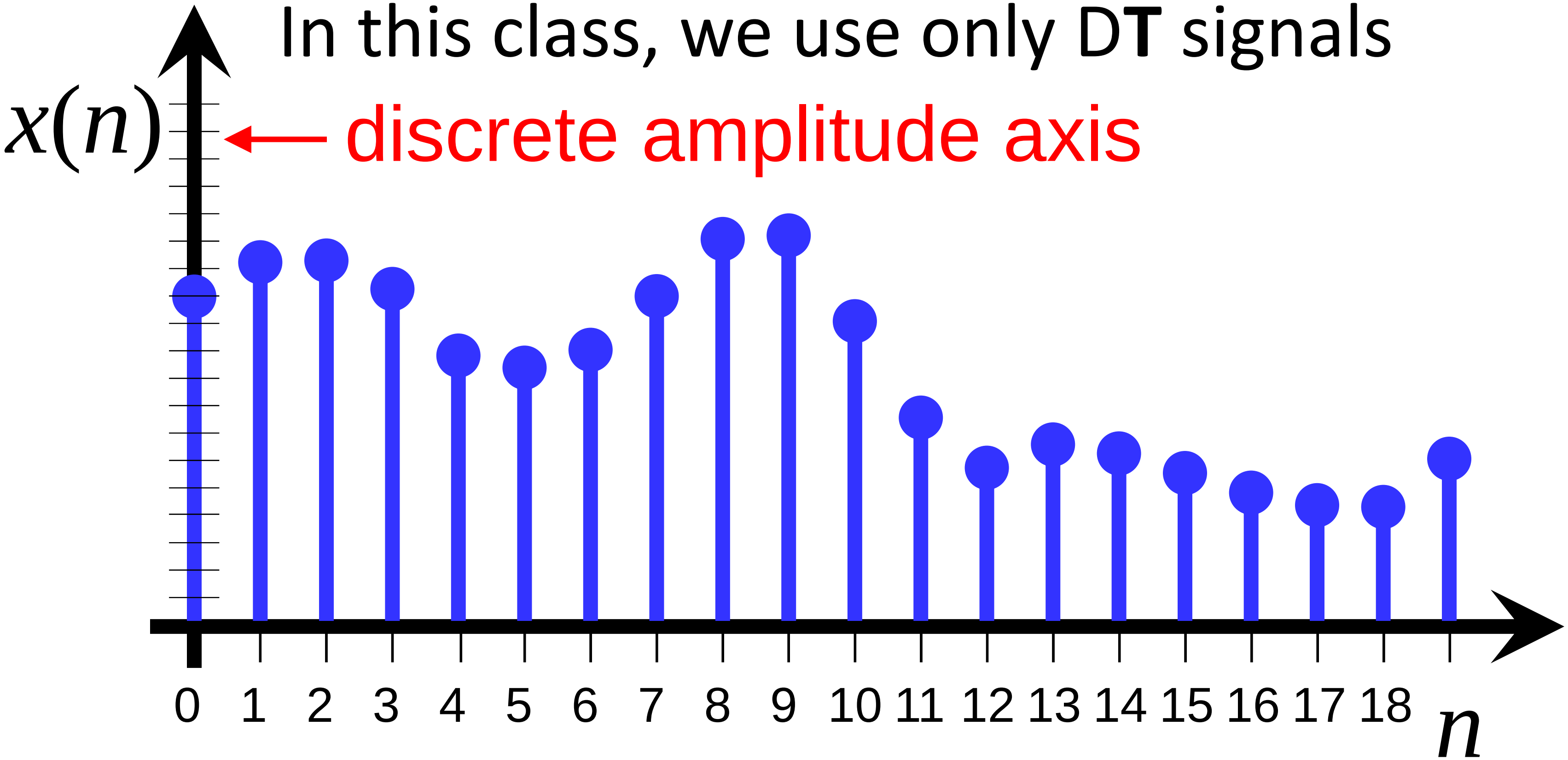
CT signals vs. DT signals

For a "**digital signal**" we also need…



CT signals vs. DT signals

In this class, we use only D**T** signals



How to write DT signals

Defn: A DT signal is a sequence of (possibly complex) numbers

— A function defined on the positive and negative integers

dependent variable

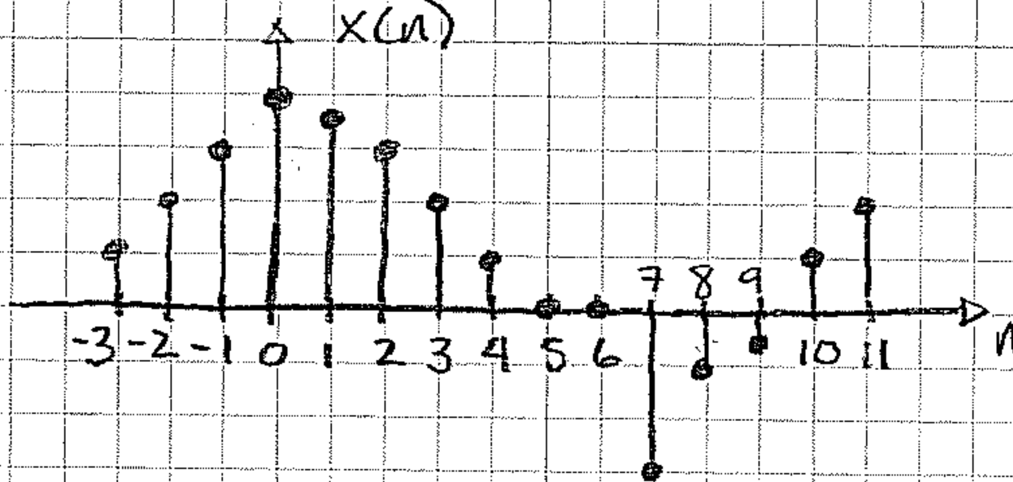
$x(n)$

independent variable

- time index $n \in \mathbb{Z}$

- the signal value or sample $x(n_0) \in \mathbb{R}$ or $\mathbb{C}$

How to write DT signals



We plot DT signals as lollipops

Important Note: $x(n)$ is defined only for integer values of n

$x(-1) = 4.1$, $x(0) = 3$, $x(1) = 2.4$, $x(2) = 10$, etc.

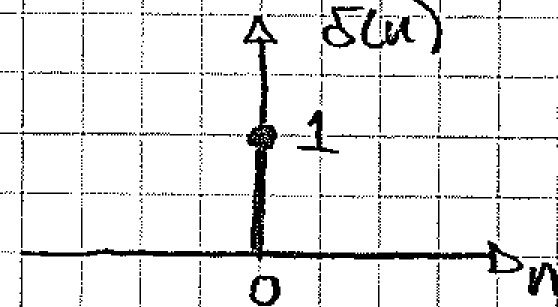
Undefined: $x(0.2) = \phi$, $x(-10.1) = \phi$, $x(\frac{1}{2}) = \phi$

Most important DT signal

The DSP alphabet contains only one letter: δ

- Unit impulse (unit sample)

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & \text{else} \end{cases}$$



(a.k.a. Kronecker delta)

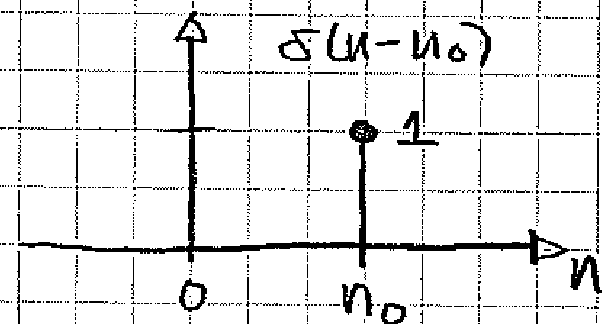
Most important DT signal

But, the δ can be shifted in time

– Time-shifted unit impulse

$$\delta(n - n_0) = \begin{cases} 1, & n = n_0 \\ 0, & \text{else} \end{cases}$$

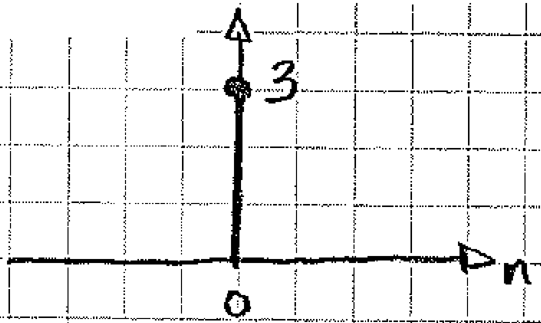
integer



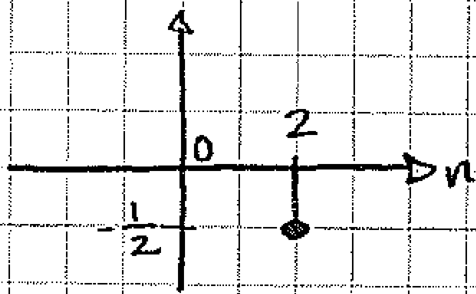
Most important DT signal

And, the δ can be scaled in amplitude

$$3\delta(n) = \begin{cases} 3, & n=0 \\ 0, & \text{else} \end{cases}$$



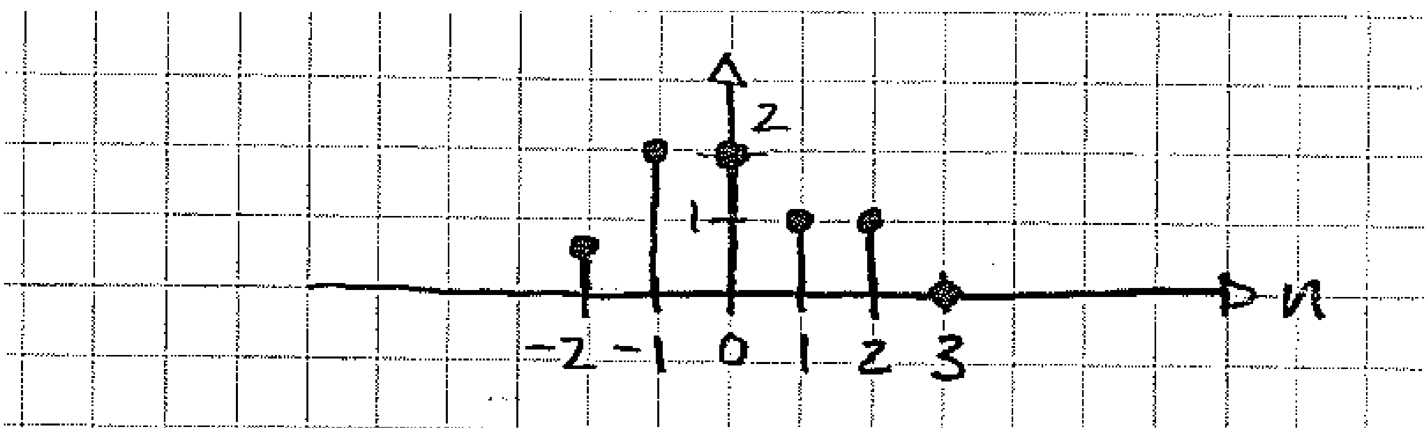
$$-\frac{1}{2}\delta(n-2) = \begin{cases} -\frac{1}{2}, & n=2 \\ 0, & \text{else} \end{cases}$$



Ways of writing DT signals

1. As a weighted sum of time-shifted impulses

$$x(n) = \tfrac{1}{2}\delta(n+2) + 2\delta(n+1) + 2\delta(n) + \delta(n-1) + \delta(n-2)$$



Ways of writing DT signals

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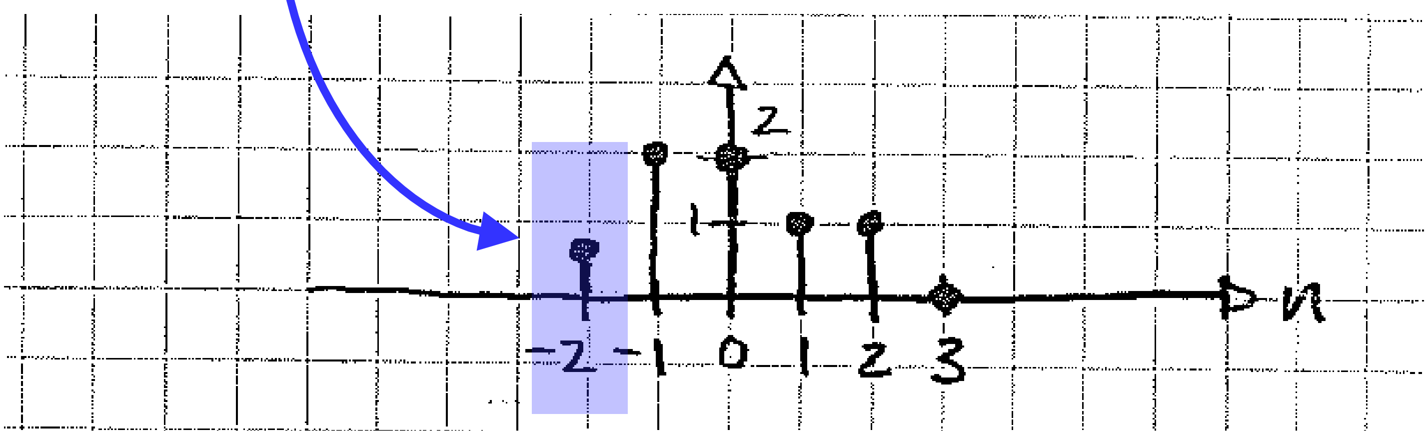

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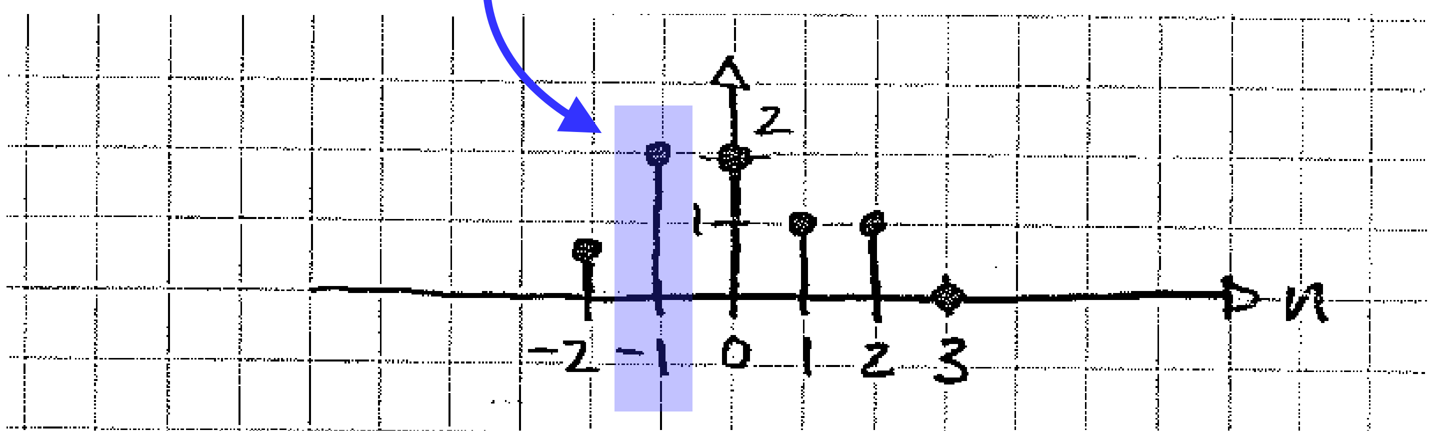

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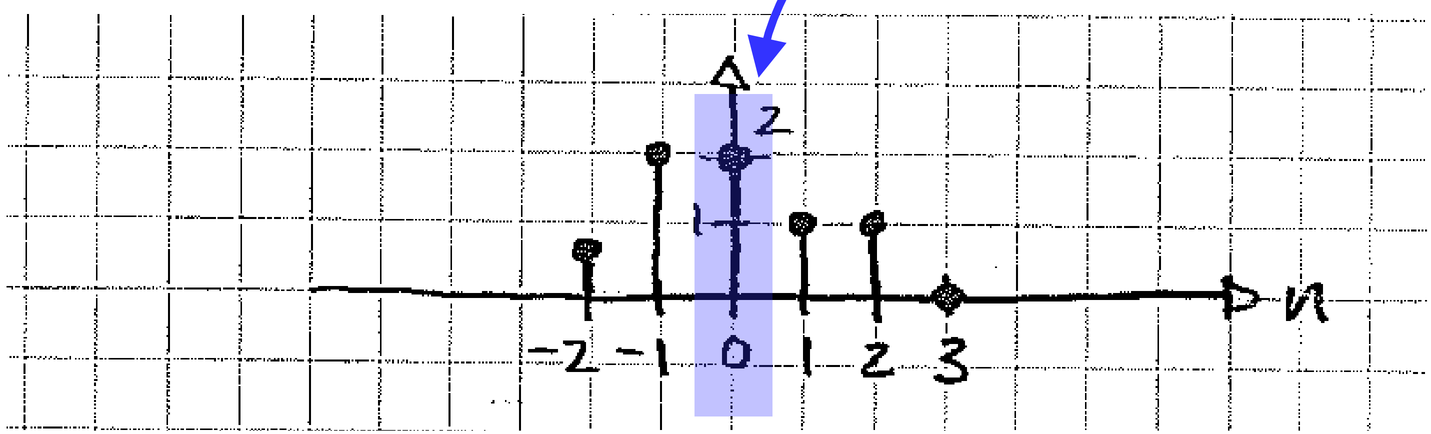

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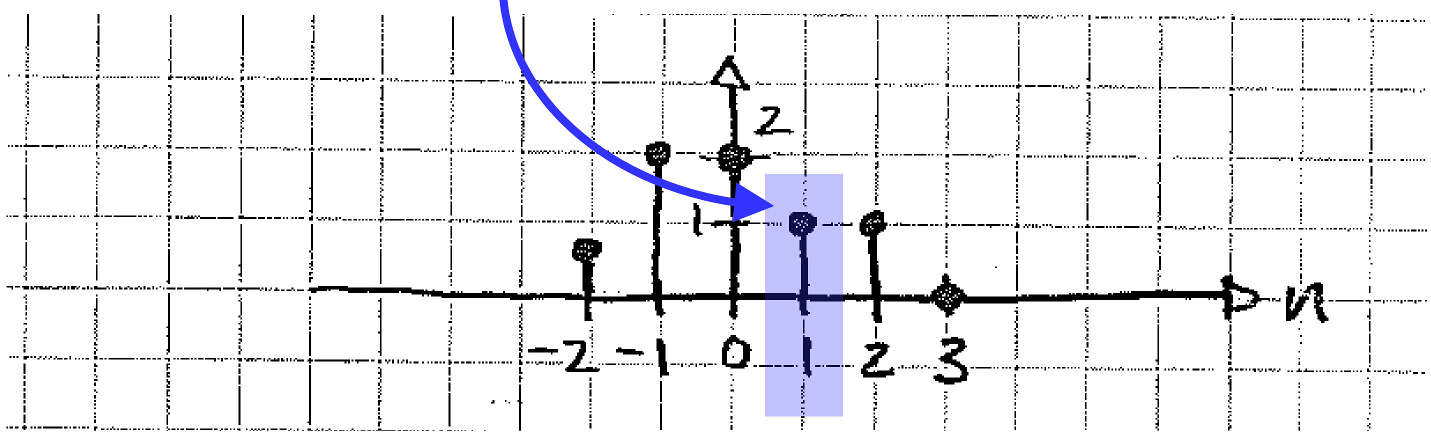

Ways of writing DT signals

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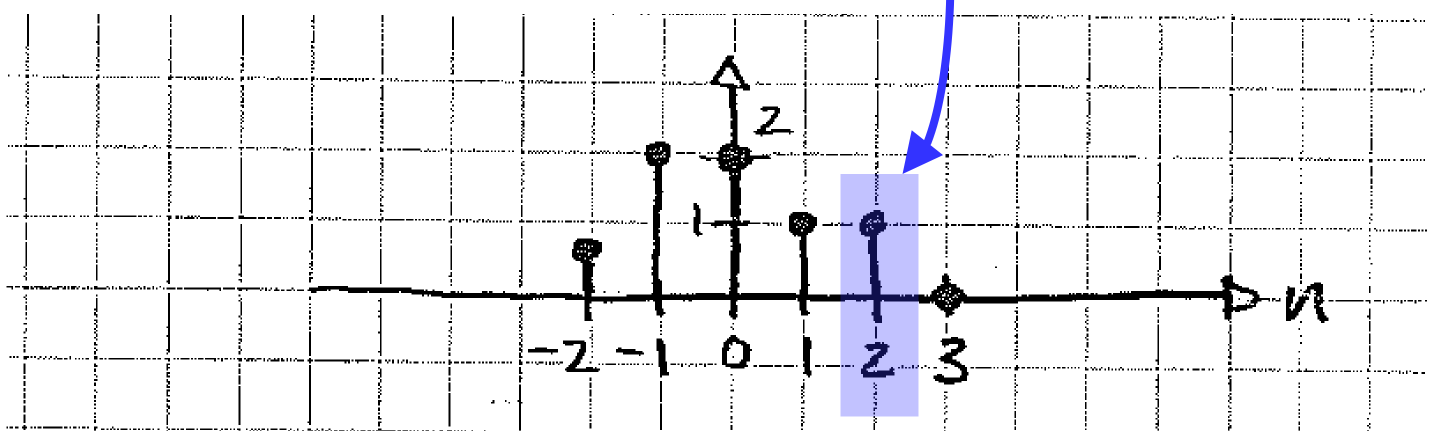


Ways of writing DT signals

2. As a closed-form expression

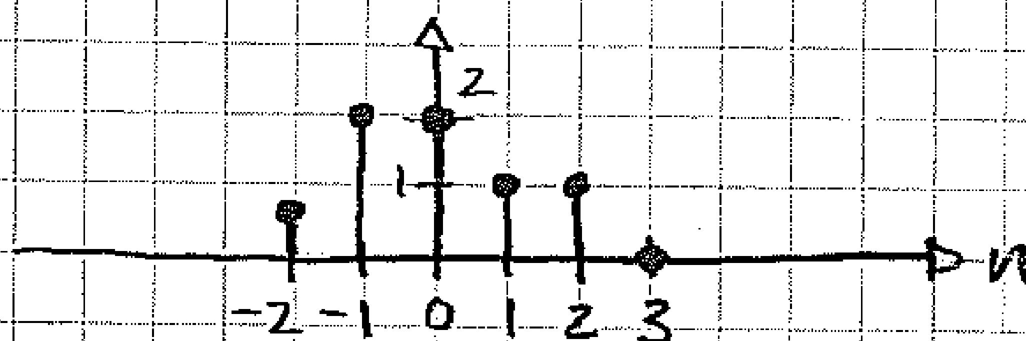
e.g., $x(n) = (3.6)^n \cos(n)$

$x(n) = n^3$

Ways of writing DT signals

3. As a sequence of #s

$$x(n) = \left\{ \tfrac{1}{2},\ 2,\ \underset{\uparrow}{2},\ 1,\ 1,\ 0 \right\}$$



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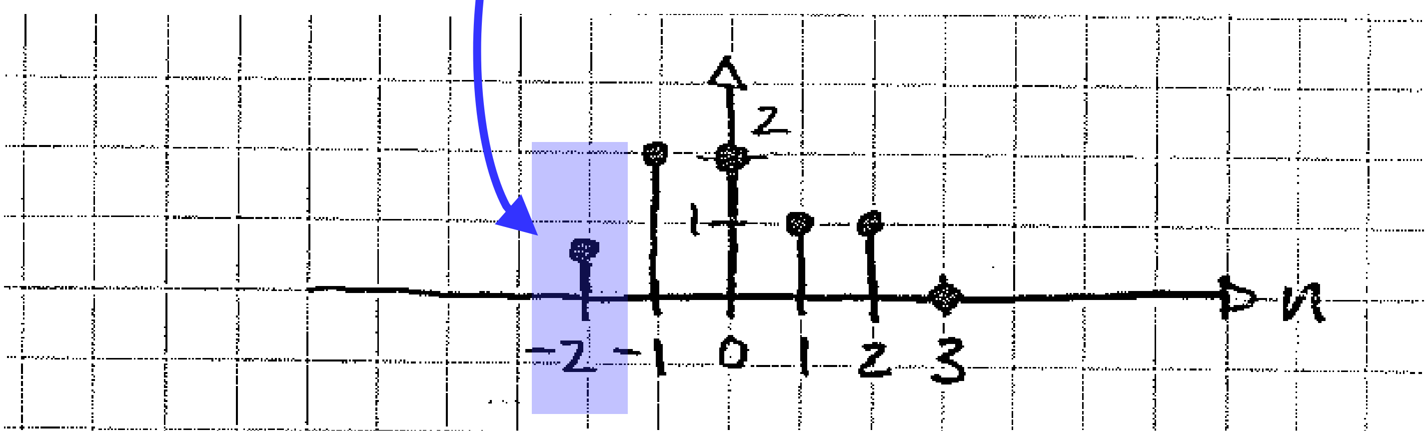

Ways of writing DT signals

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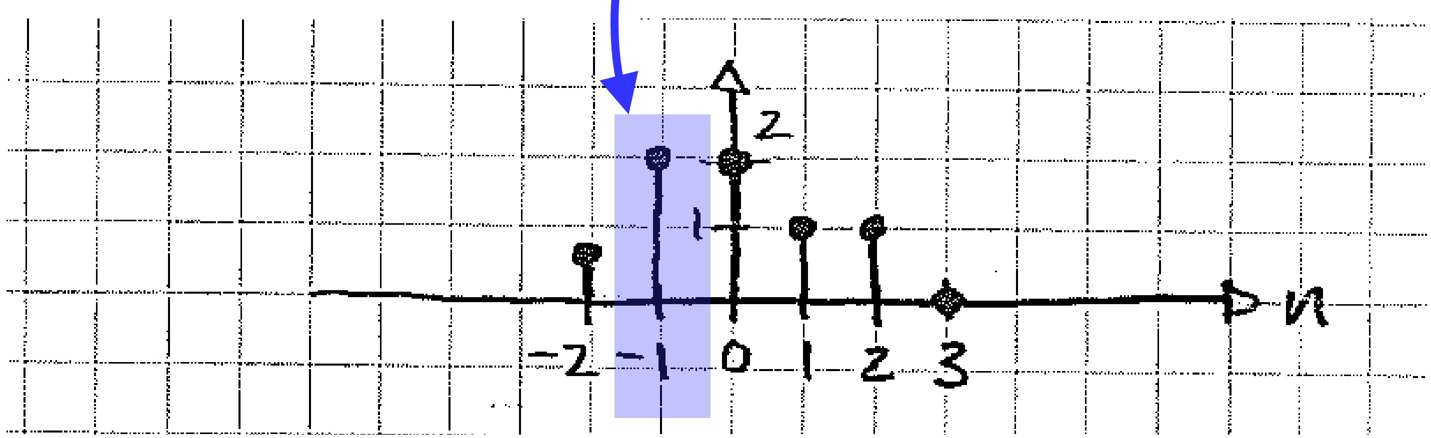

Ways of writing DT signals

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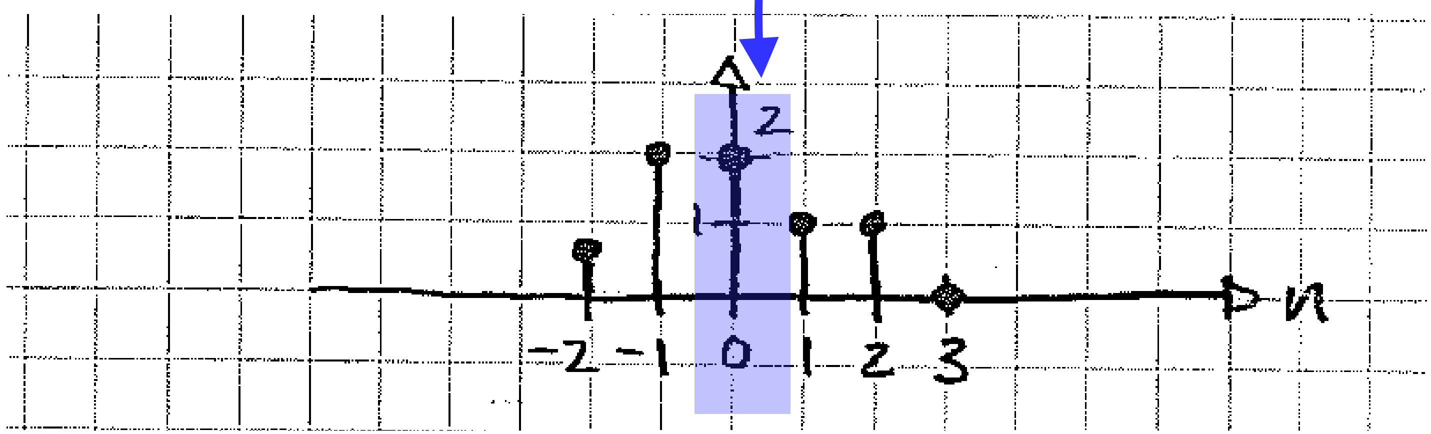

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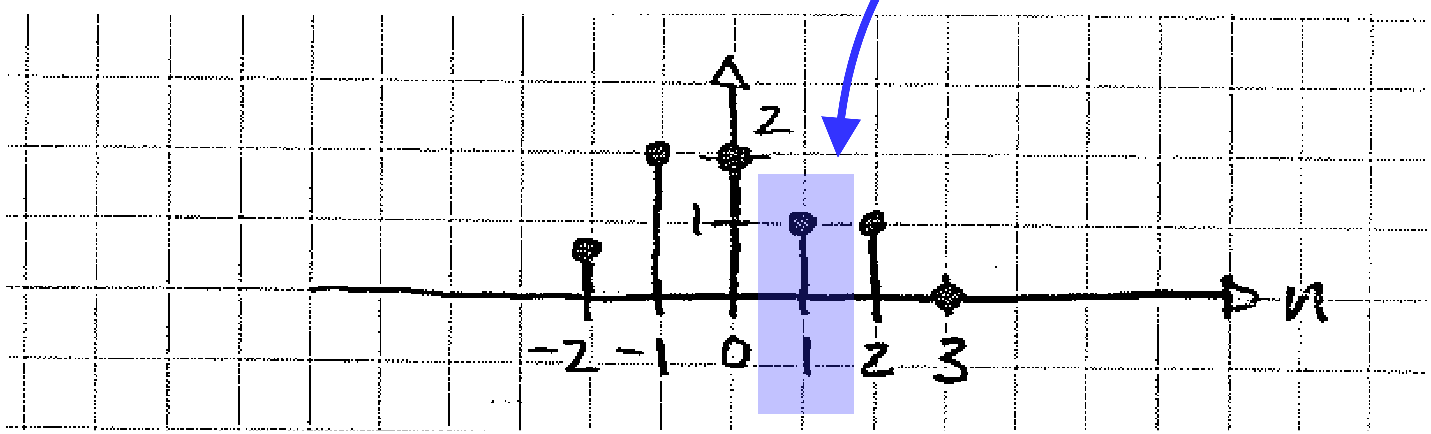

Ways of writing DT signals

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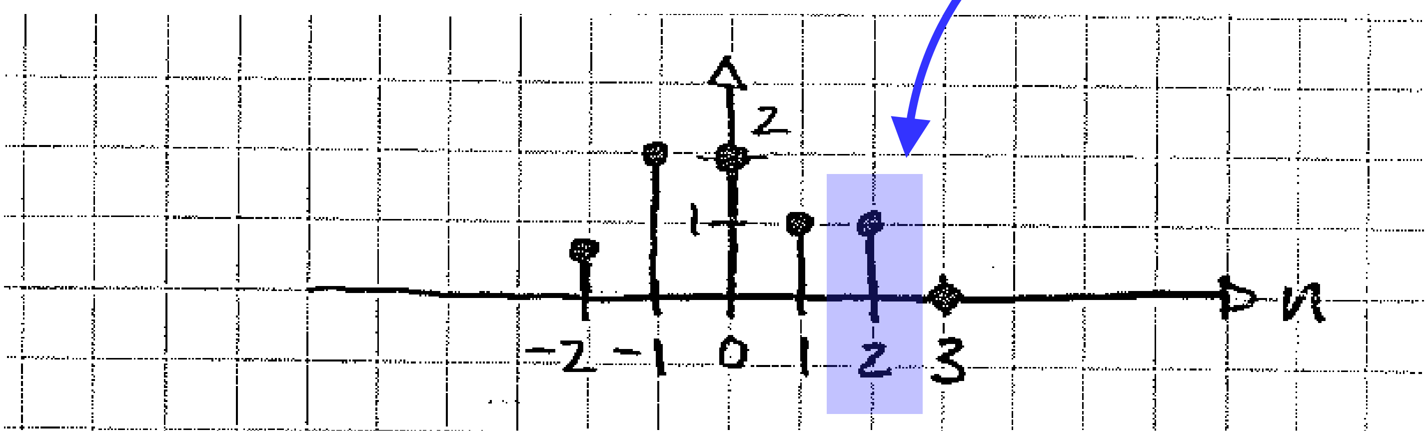

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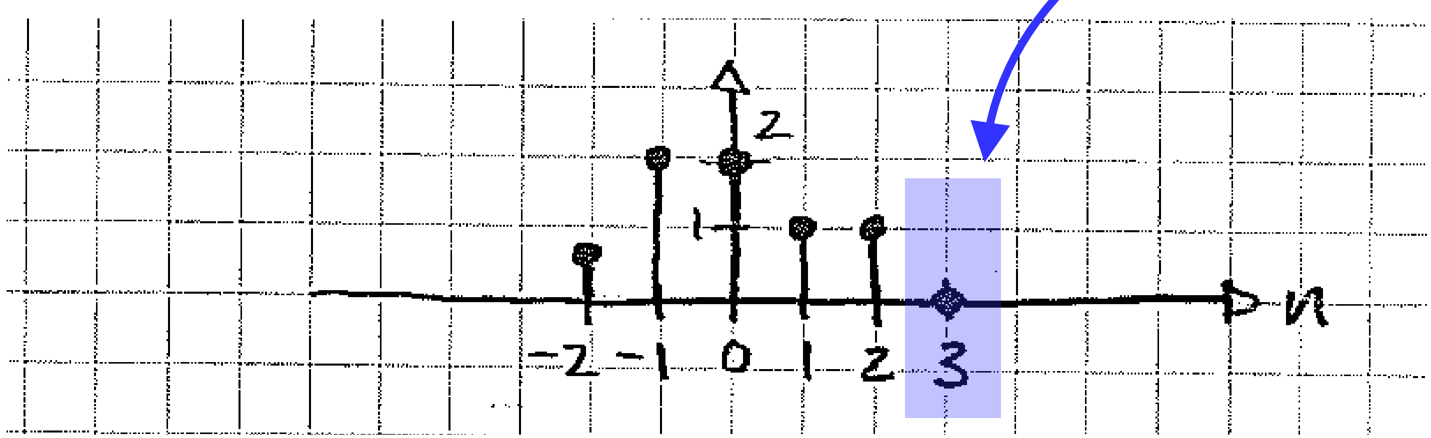
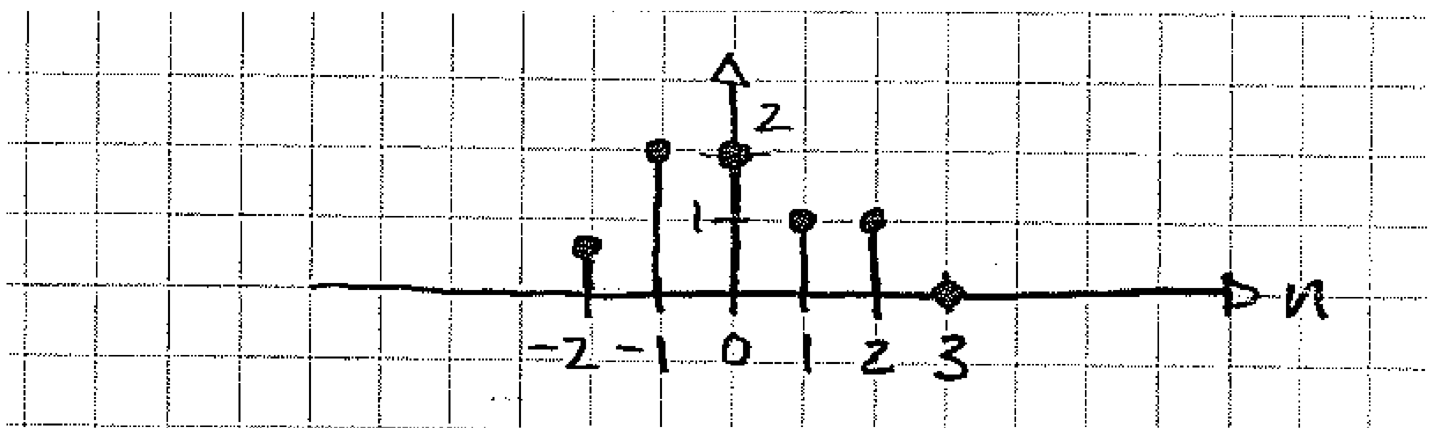


Ways of writing DT signals

4. Brute force

$x(-2) = \frac{1}{2}$, $x(-1) = 2$, $x(0) = 2$,

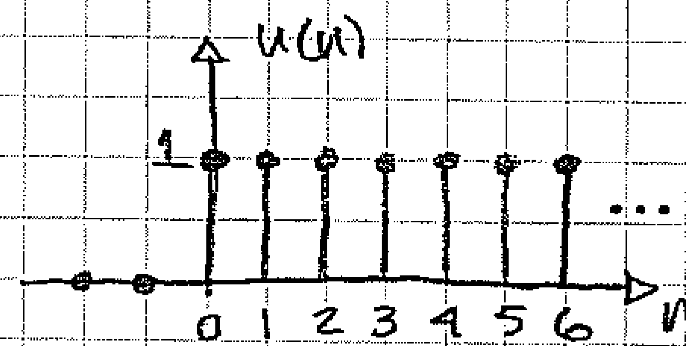
$x(1) = 1$, $x(2) = 1$, $x(3) = 0$



Common DT Signals

- <u>Unit Step function</u>

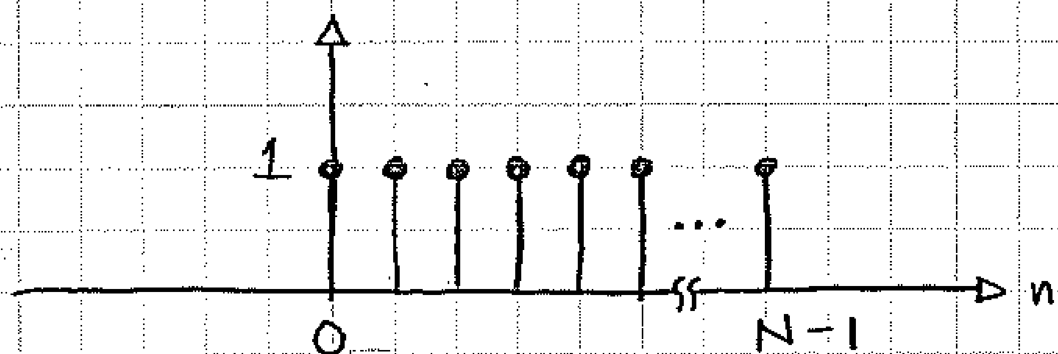
$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & \text{else} \end{cases}$$



$$u(n) = \sum_{m=0}^{\infty} \delta(n-m) = \delta(n) + \delta(n-1) + \delta(n-2) + \dots$$

DT rectangles

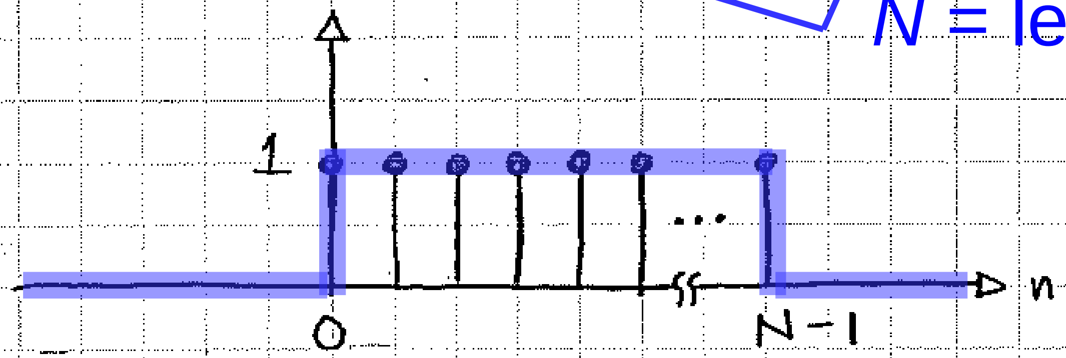
$$\mathrm{rect}_N(n) = \begin{cases} 1, & 0 \le n < N \\ 0, & \text{else} \end{cases}$$



DT rectangles

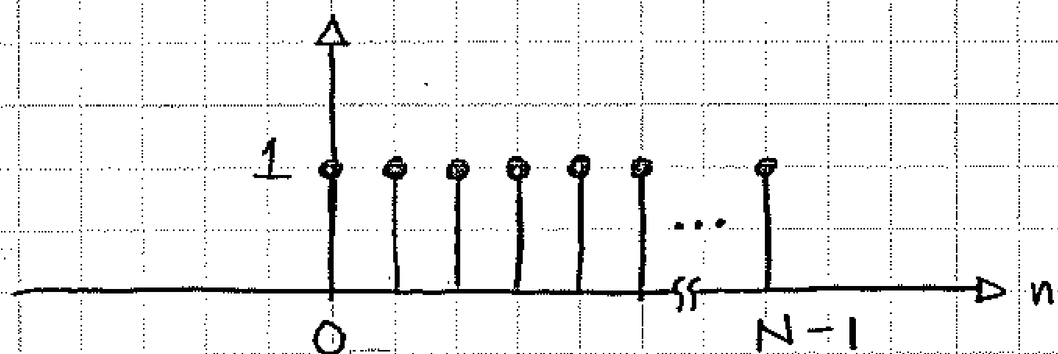
$$\mathrm{rect}_N(n) = \begin{cases} 1, & 0 \le n < N \\ 0, & \text{else} \end{cases}$$

N = length



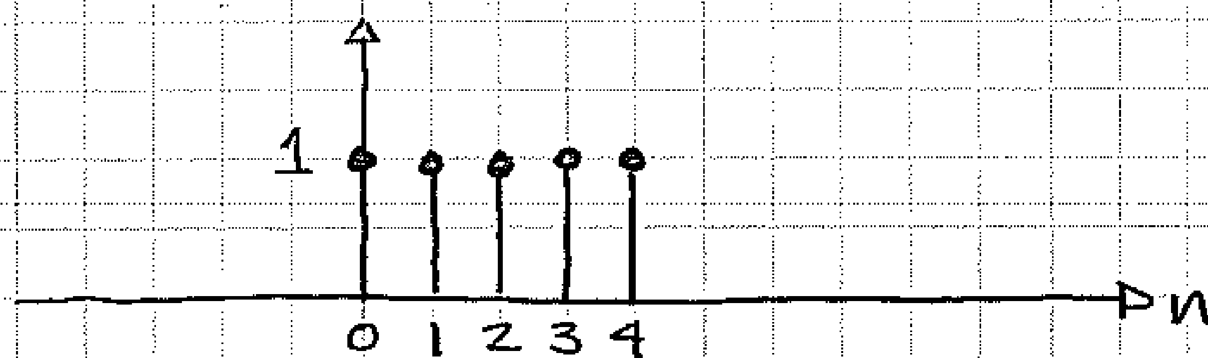
DT rectangles

$$\mathrm{rect}_N(n) = \begin{cases} 1, & 0 \le n < N \\ 0, & \text{else} \end{cases} = \sum_{k=0}^{N-1} \delta(n-k)$$



DT rectangles

<u>Ex:</u> $\text{rect}_5(n) = \begin{cases} 1, & 0 \le n < 5 \\ 0, & \text{else} \end{cases}$



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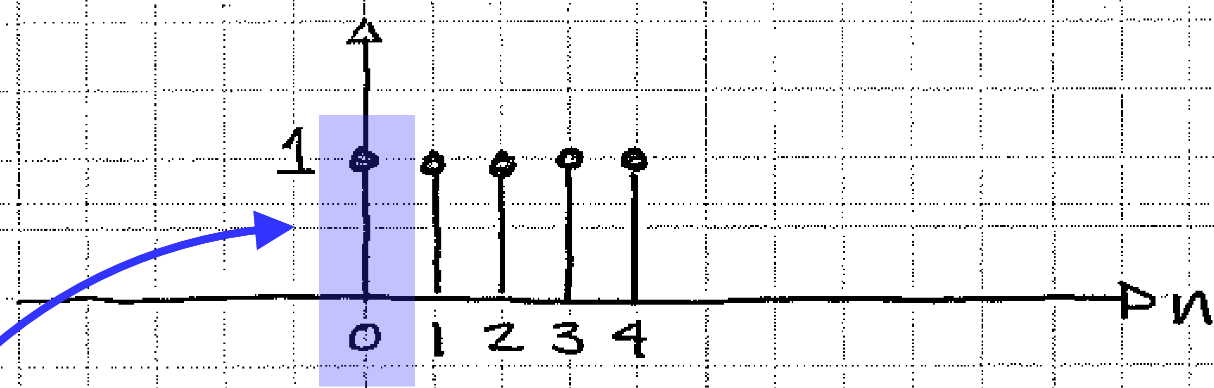

$$\mathrm{rect}_5(n) = \sum_{k=0}^{4} \delta(n-k)$$

$$= \boxed{\delta(n)} + \delta(n-1) + \delta(n-2) + \delta(n-3) + \delta(n-4)$$

DT rectangles

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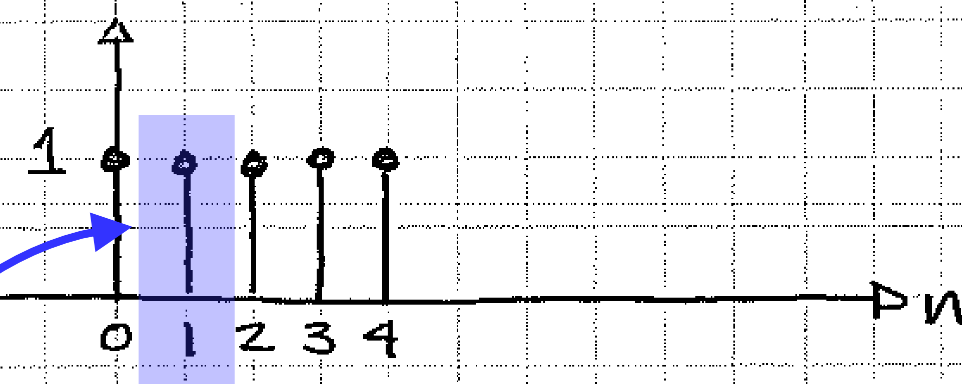

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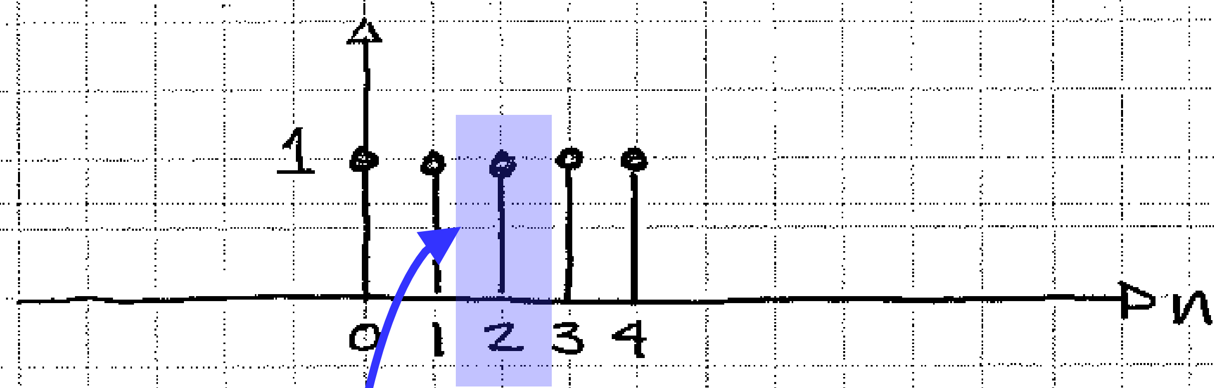

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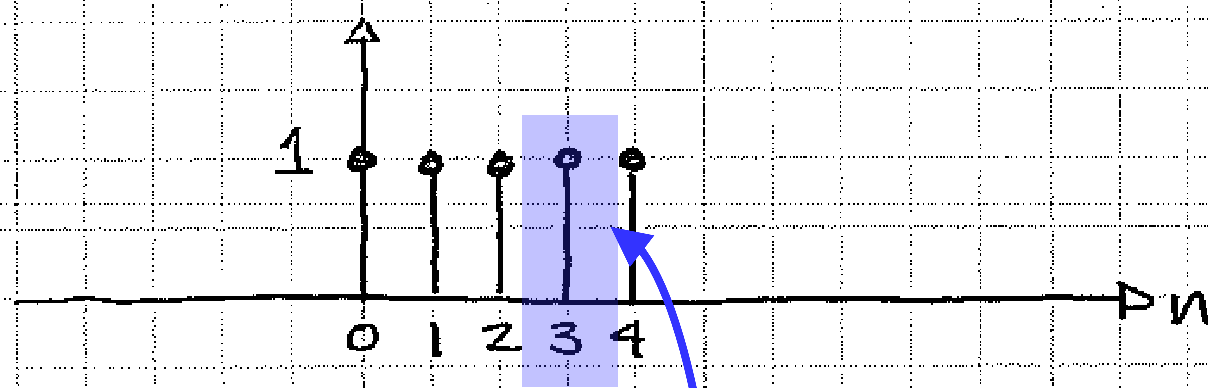

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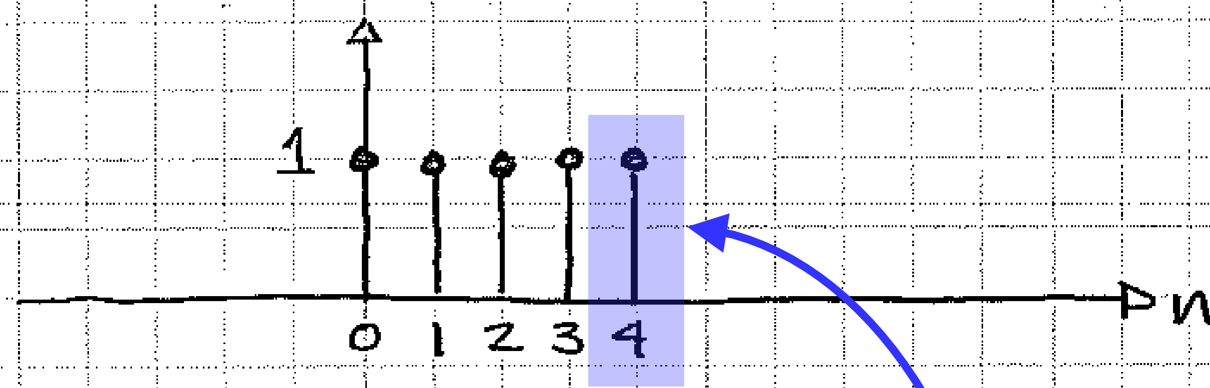

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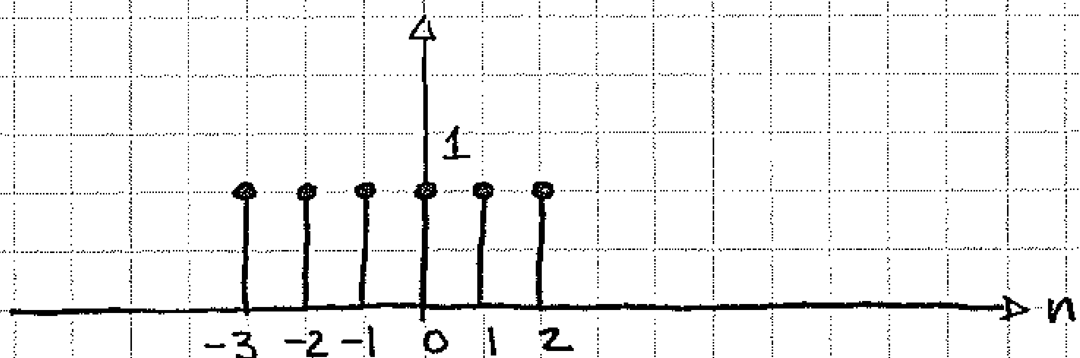
$$= \delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3) + \boxed{\delta(n-4)}$$

DT rectangles

Ex: $\mathrm{rect}_6(n+3) = \begin{cases} 1, & 0 \le n+3 < 6 \rightarrow -3 \le n < 3 \\ 0, & \text{else} \end{cases}$



$$\mathrm{rect}_6(n+3) = \sum_{k=-3}^{2} \delta(n-k)$$

$$= \delta(n+3) + \delta(n+2) + \delta(n+1) + \delta(n) + \delta(n-1) + \delta(n-2)$$

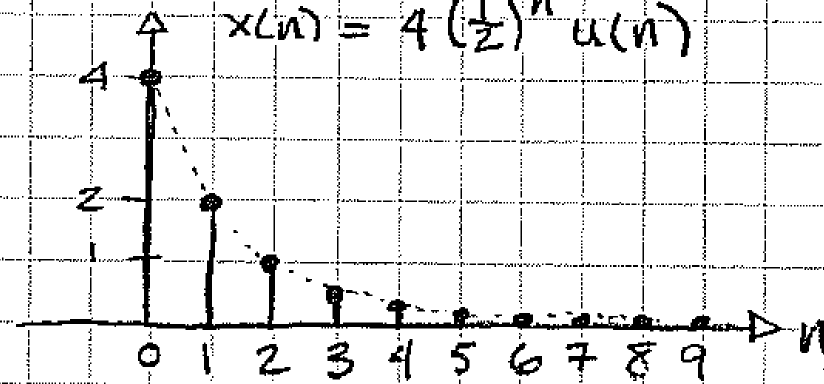
Common DT Signals

- Real exponential

$$x(n) = \alpha^n$$

$$\alpha \in \mathbb{R}$$

$x(n) = 4\left(\frac{1}{2}\right)^n u(n)$



Common DT Signals

- Complex exponential

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$$x(n) = \alpha^n$$

$$\alpha \in \mathbb{C}$$

$$\alpha = re^{j\omega}, \quad \text{where} \quad r = |\alpha| \quad \text{and} \quad \omega = \measuredangle \alpha$$

$$\hookrightarrow x(n) = (re^{j\omega})^n$$

$$= r^n e^{j\omega n}$$

Special case ($r = 1$): $x(n) = e^{j\omega n}$

$$= \cos(\omega n) + j\sin(\omega n)$$

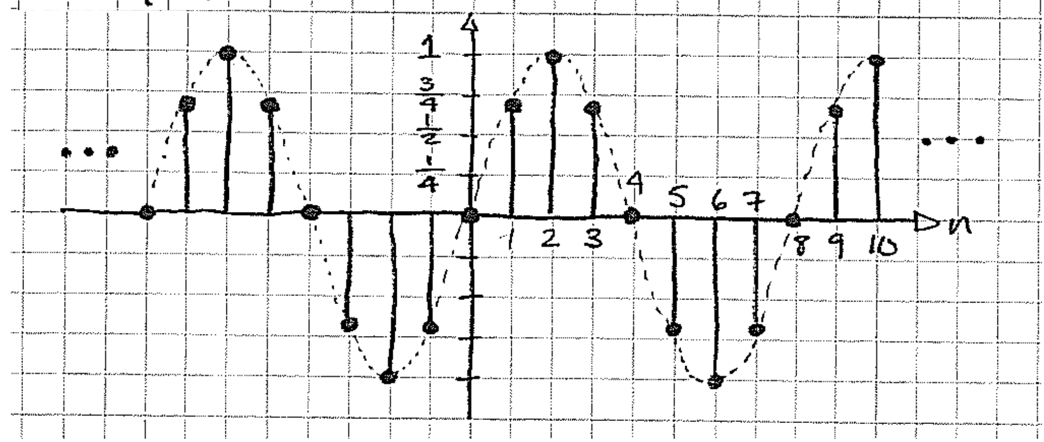
Common DT Signals

- Sine and cosine

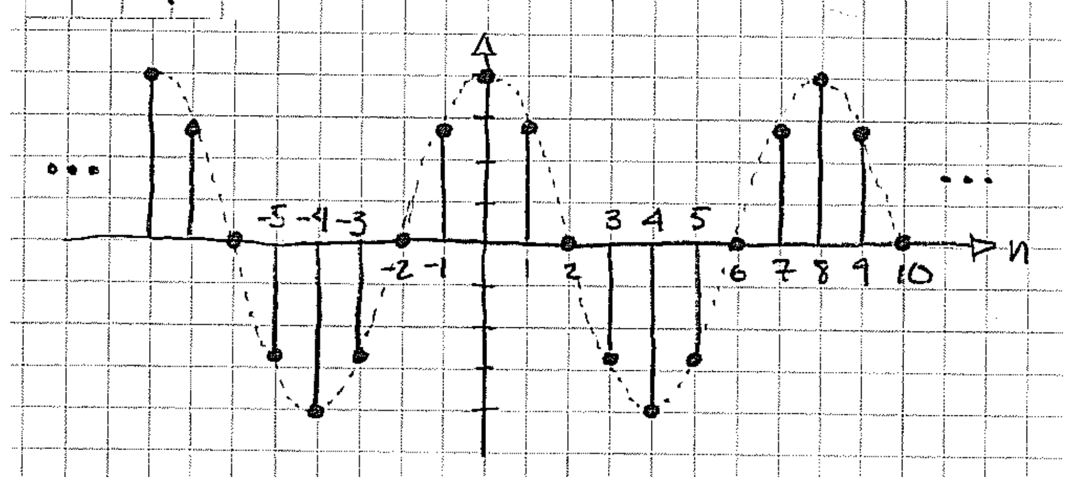
$$x(n) = \sin(\omega n)$$

$$x(n) = \cos(\omega n)$$

$\sin(\frac{\pi}{4} n)$

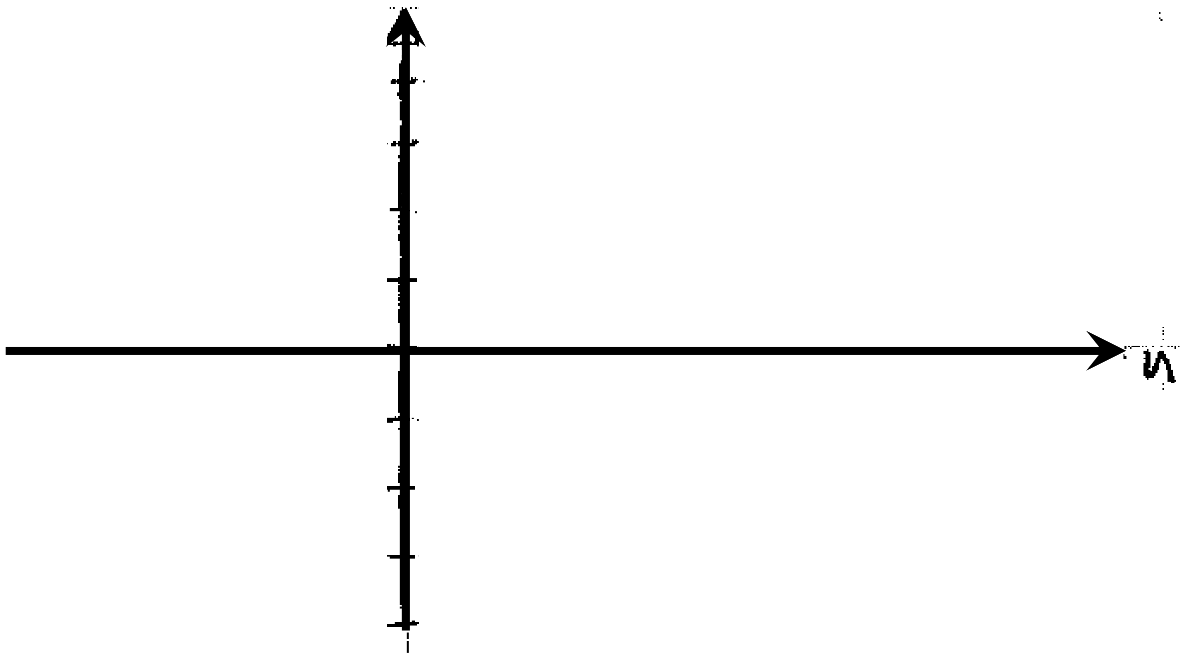


$\cos(\frac{\pi}{4} n)$



DT frequency

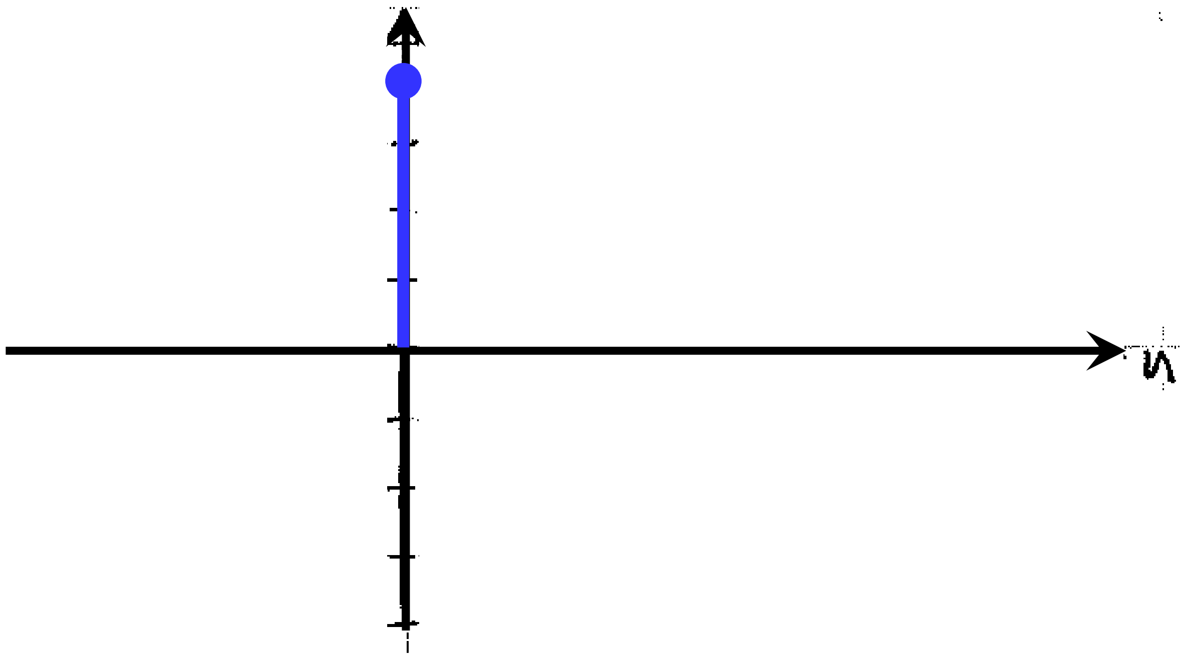
$$x(n) = \cos(\tfrac{\pi}{4}n)$$



n	$\cos(\frac{\pi}{4}n)$
0	
1	
2	
3	
4	
5	
6	
7	

DT frequency

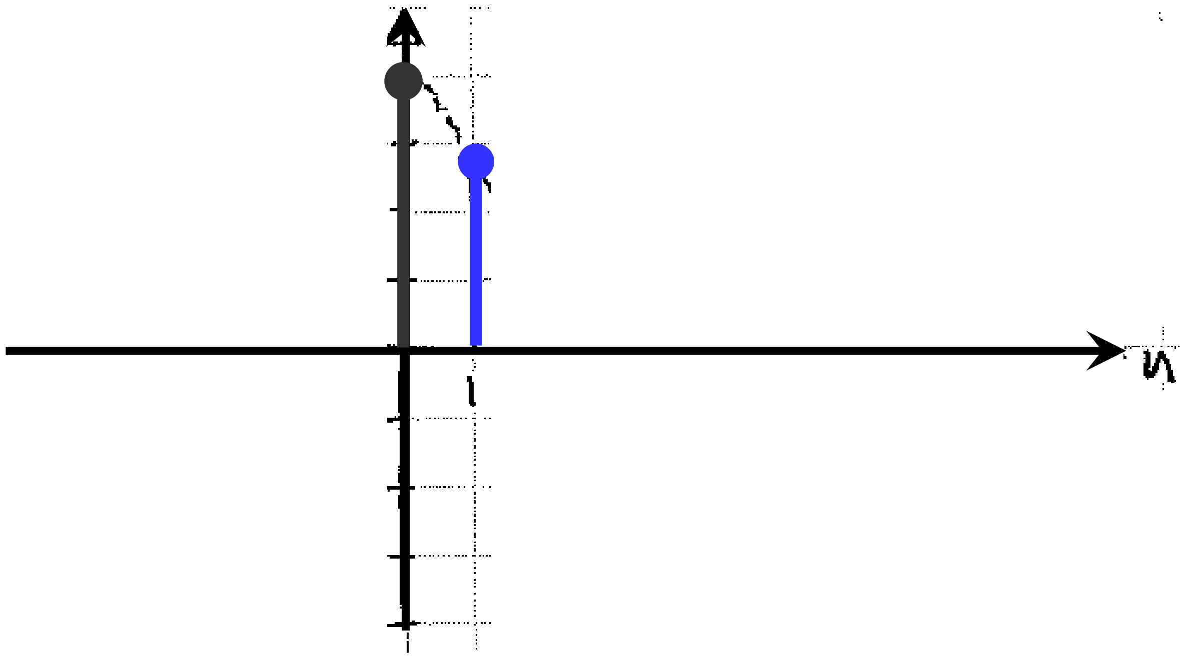
$$x(n) = \cos(\tfrac{\pi}{4} n)$$



n	$\cos(\frac{\pi}{4} n)$
0	1
1	
2	
3	
4	
5	
6	
7	

DT frequency

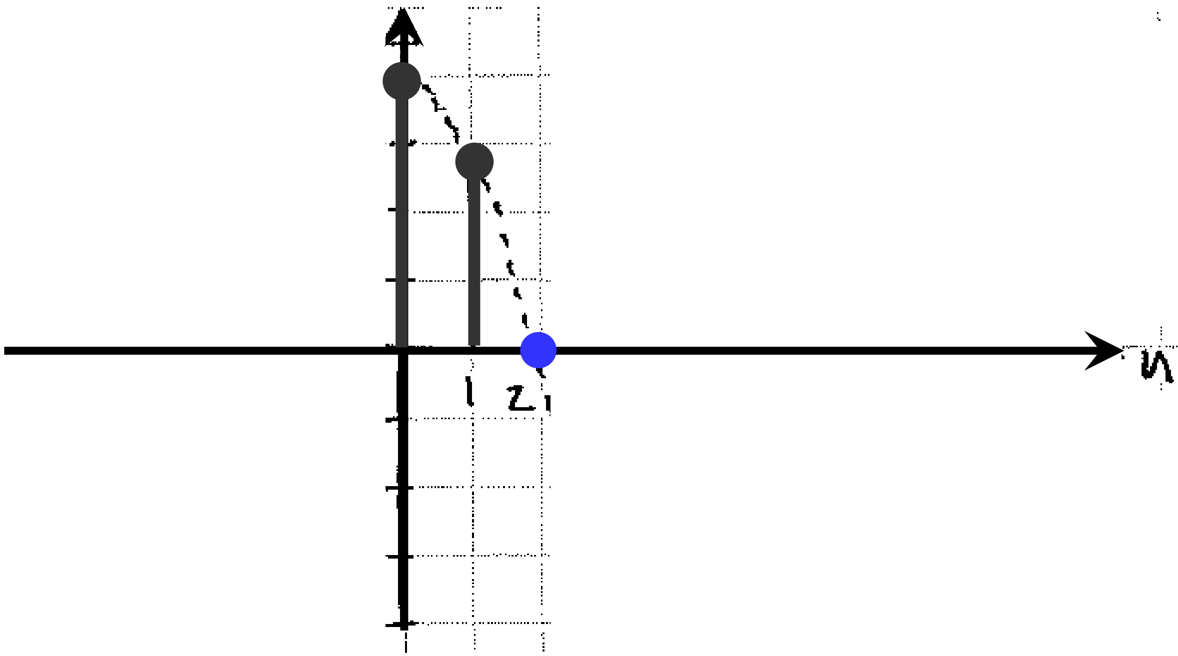
$$x(n) = \cos(\tfrac{\pi}{4} n)$$



n	$\cos(\frac{\pi}{4} n)$
0	1
1	$\sqrt{2}/2$
2	
3	
4	
5	
6	
7	

DT frequency

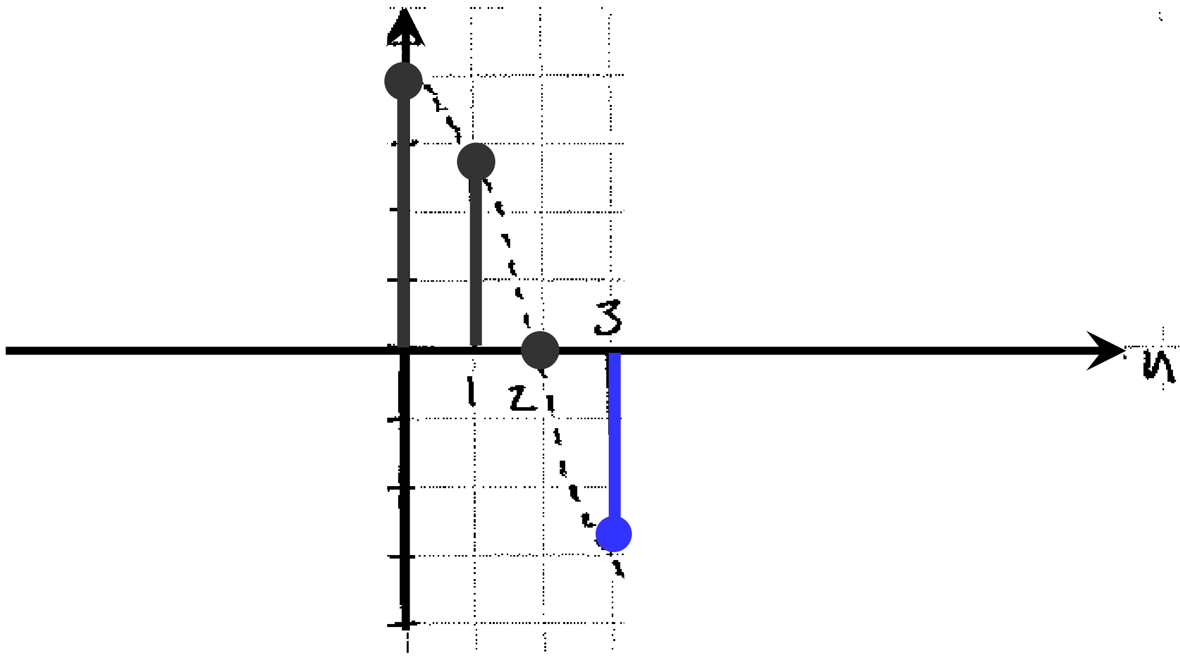
$$x(n) = \cos(\tfrac{\pi}{4} n)$$



n	$\cos(\frac{\pi}{4} n)$
0	1
1	$\sqrt{2}/2$
2	0
3	
4	
5	
6	
7	

DT frequency

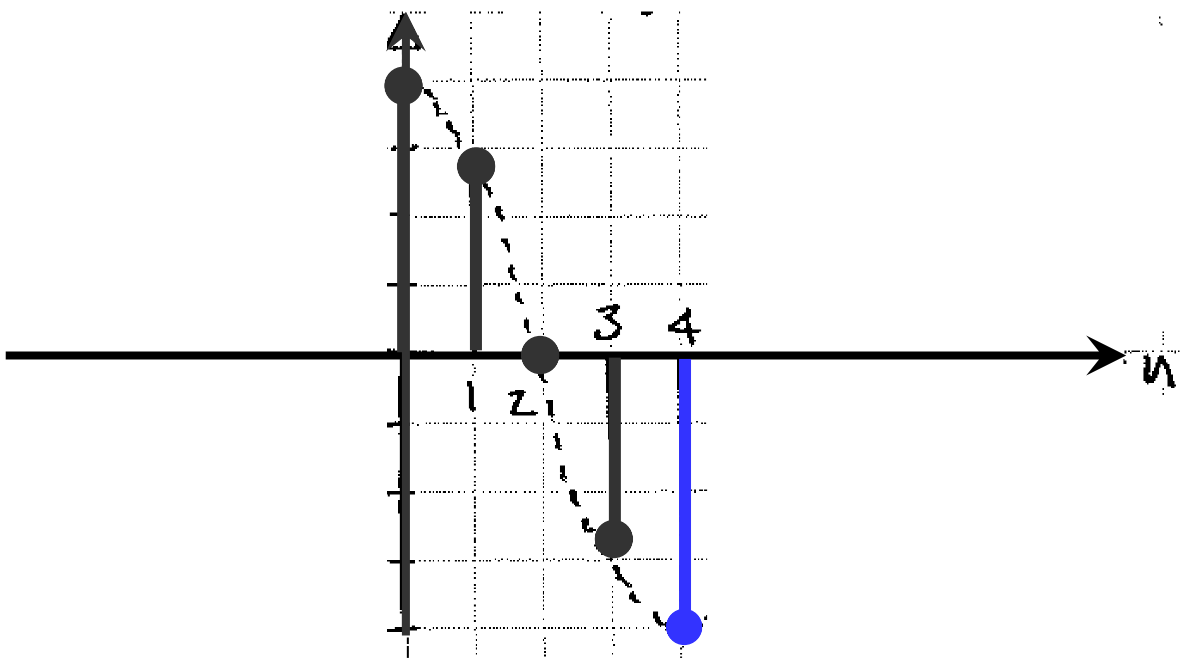
$$x(n) = \cos\left(\tfrac{\pi}{4}\, n\right)$$



n	$\cos(\frac{\pi}{4} n)$
0	1
1	$\sqrt{2}/2$
2	0
3	$-\sqrt{2}/2$
4	
5	
6	
7	

DT frequency

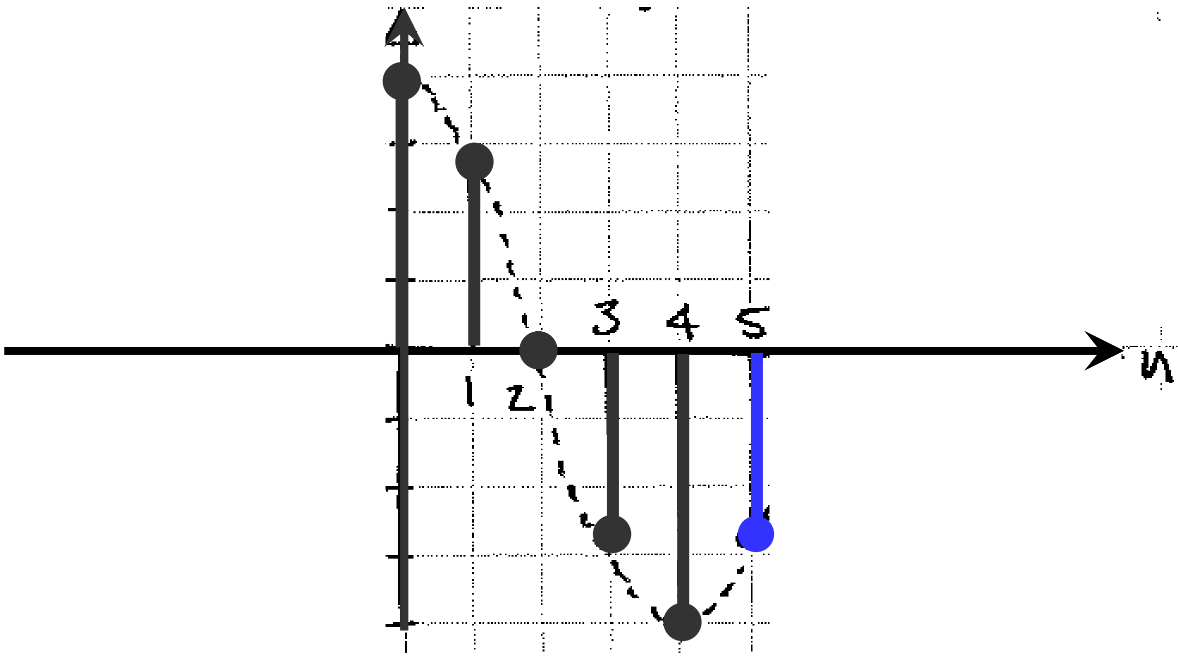
$$x(n) = \cos(\tfrac{\pi}{4} n)$$



n	$\cos(\frac{\pi}{4} n)$
0	1
1	$\sqrt{2}/2$
2	0
3	$-\sqrt{2}/2$
4	-1
5	
6	
7	

DT frequency

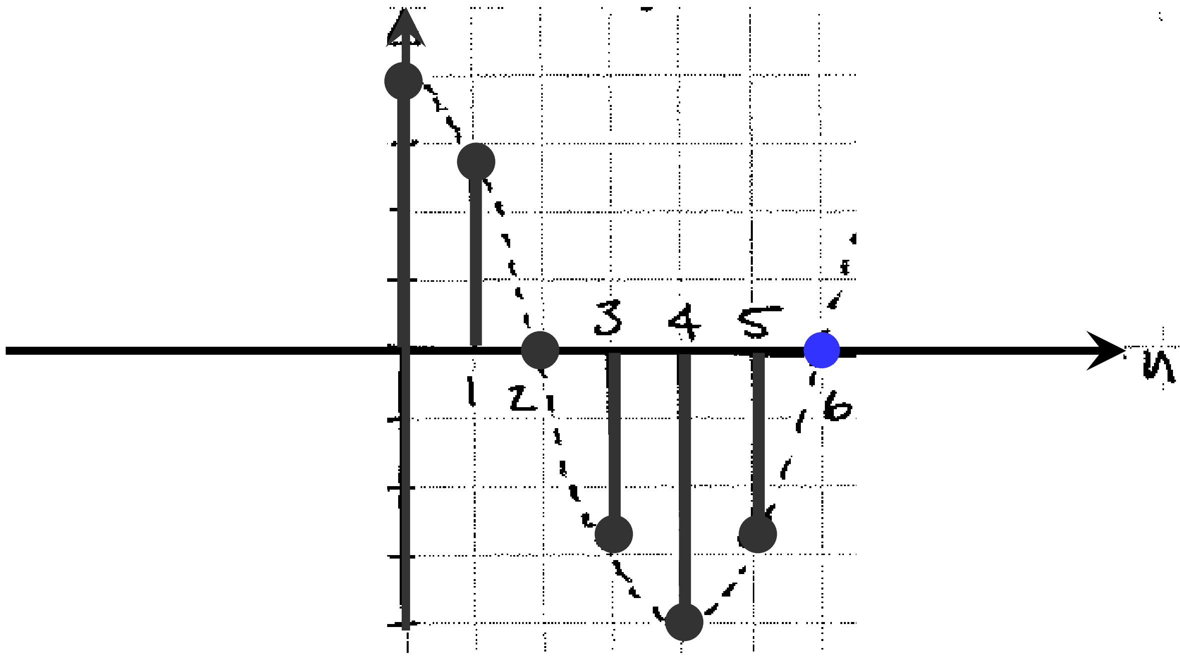
$$x(n) = \cos(\tfrac{\pi}{4} n)$$



n	$\cos(\frac{\pi}{4}n)$
0	1
1	$\sqrt{2}/2$
2	0
3	$-\sqrt{2}/2$
4	-1
5	$-\sqrt{2}/2$
6	
7	

DT frequency

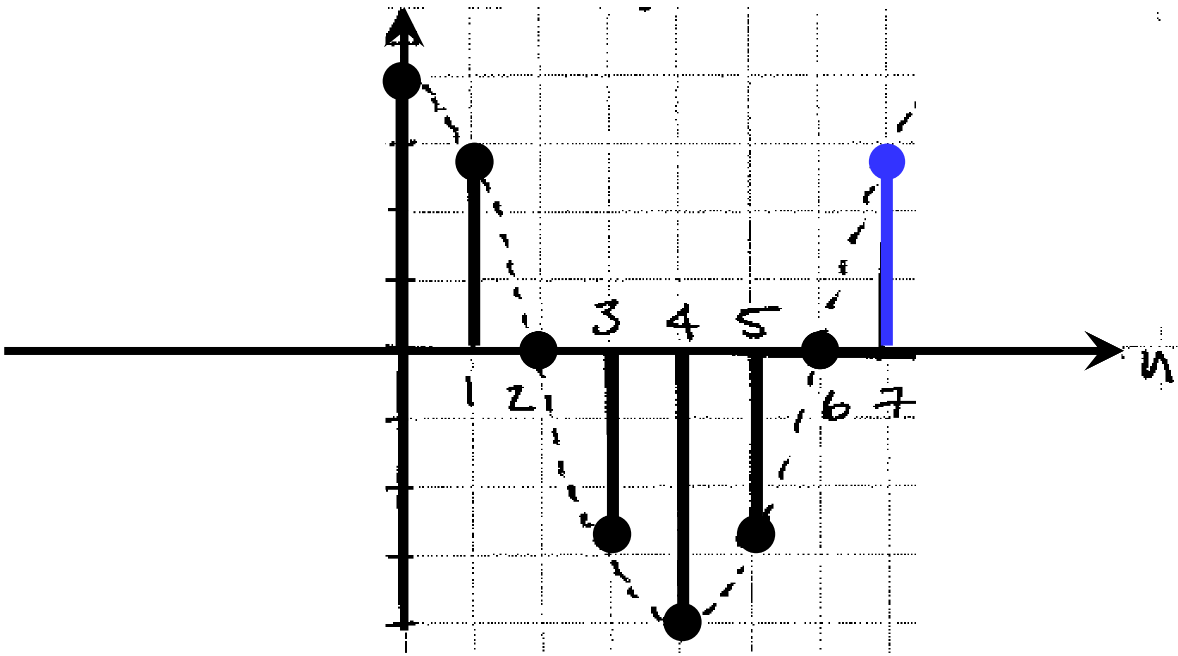
$$x(n) = \cos(\tfrac{\pi}{4} n)$$



n	$\cos(\frac{\pi}{4} n)$
0	1
1	$\sqrt{2}/2$
2	0
3	$-\sqrt{2}/2$
4	-1
5	$-\sqrt{2}/2$
6	0
7	

DT frequency

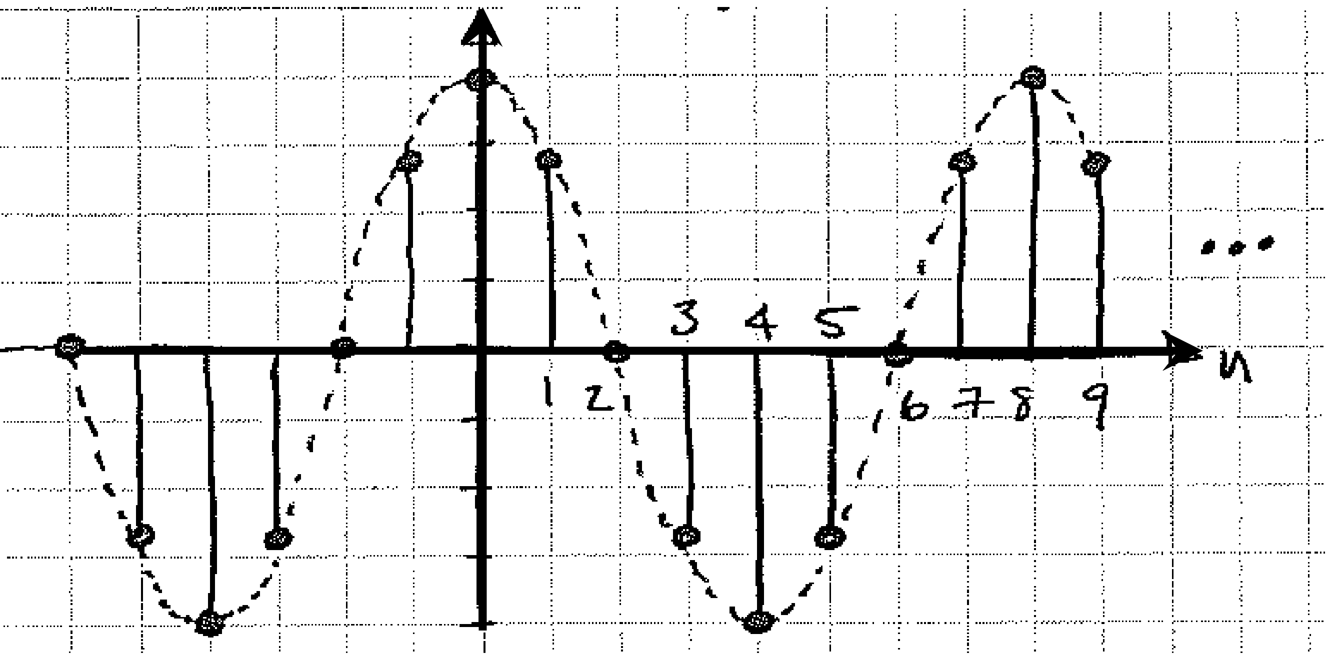
$$x(n) = \cos(\tfrac{\pi}{4} n)$$



n	$\cos(\frac{\pi}{4}n)$
0	1
1	$\sqrt{2}/2$
2	0
3	$-\sqrt{2}/2$
4	-1
5	$-\sqrt{2}/2$
6	0
7	$\sqrt{2}/2$

DT frequency

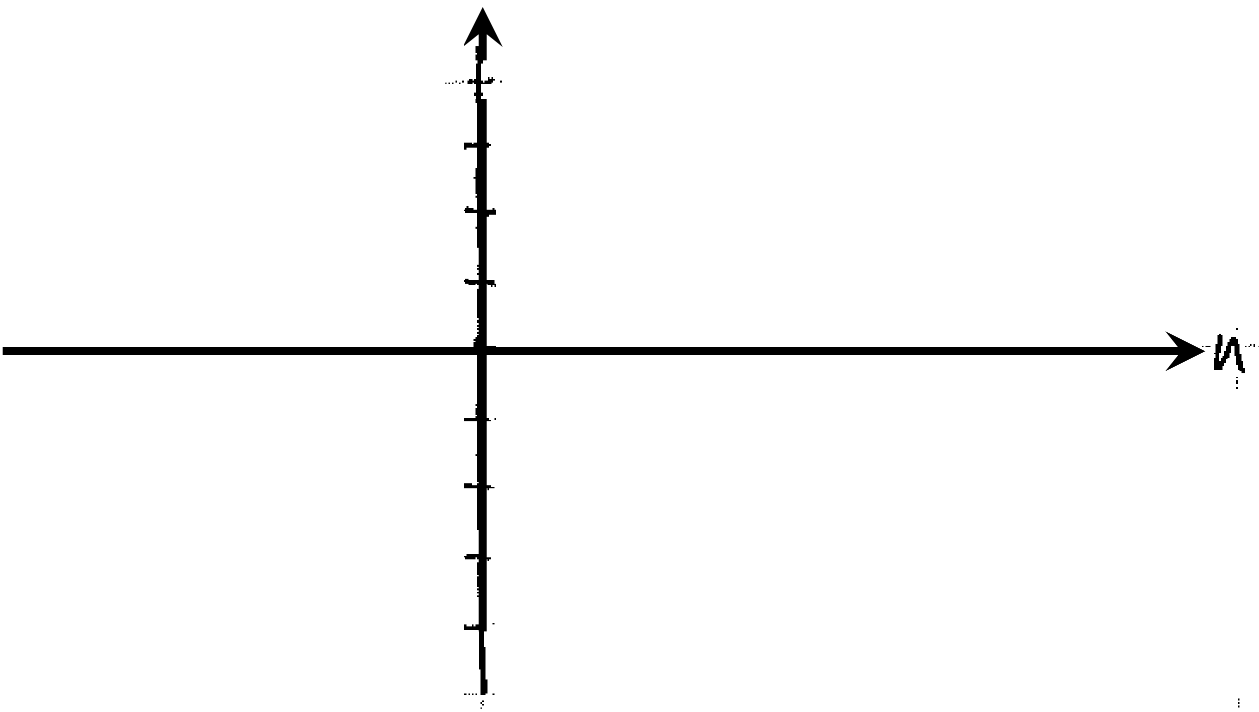
$$x(n) = \cos(\tfrac{\pi}{4} n)$$



n	$\cos(\frac{\pi}{4} n)$
0	1
1	$\sqrt{2}/2$
2	0
3	$-\sqrt{2}/2$
4	-1
5	$-\sqrt{2}/2$
6	0
7	$\sqrt{2}/2$

DT frequency

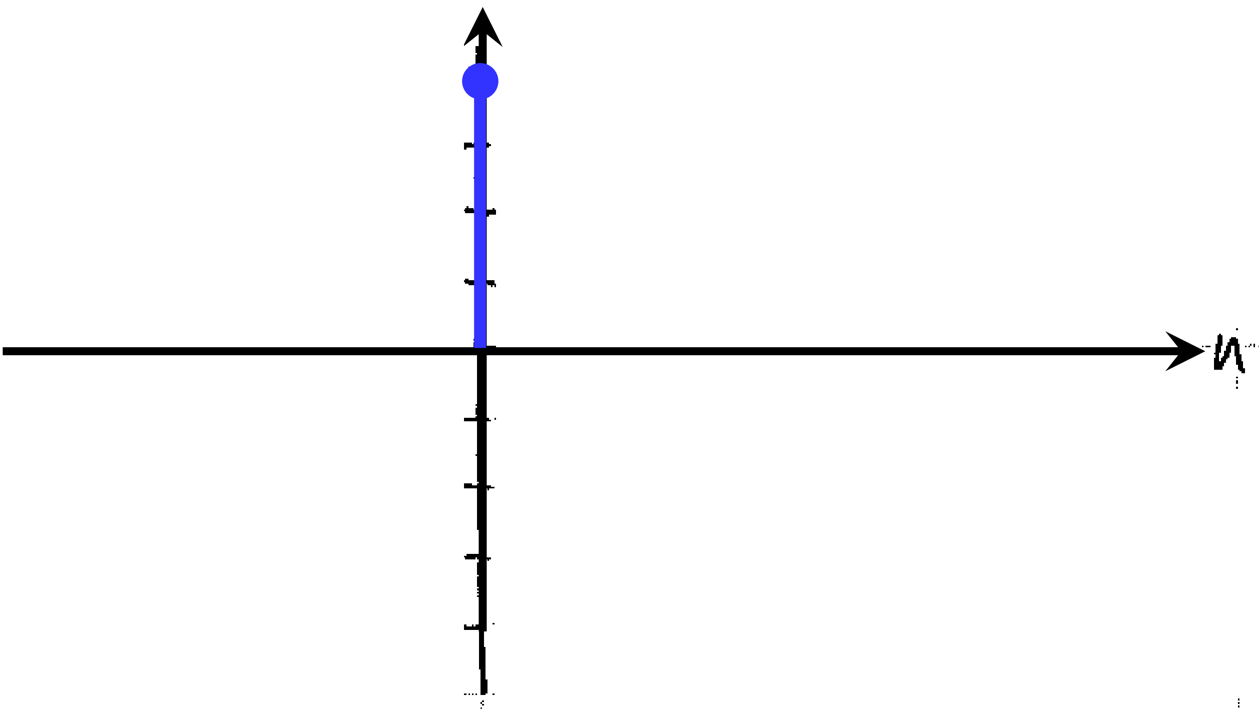
$$x(n) = \cos(\tfrac{\pi}{2} n)$$



n	$\cos(\frac{\pi}{2} n)$
0	
1	
2	
3	
4	
5	
6	
7	

DT frequency

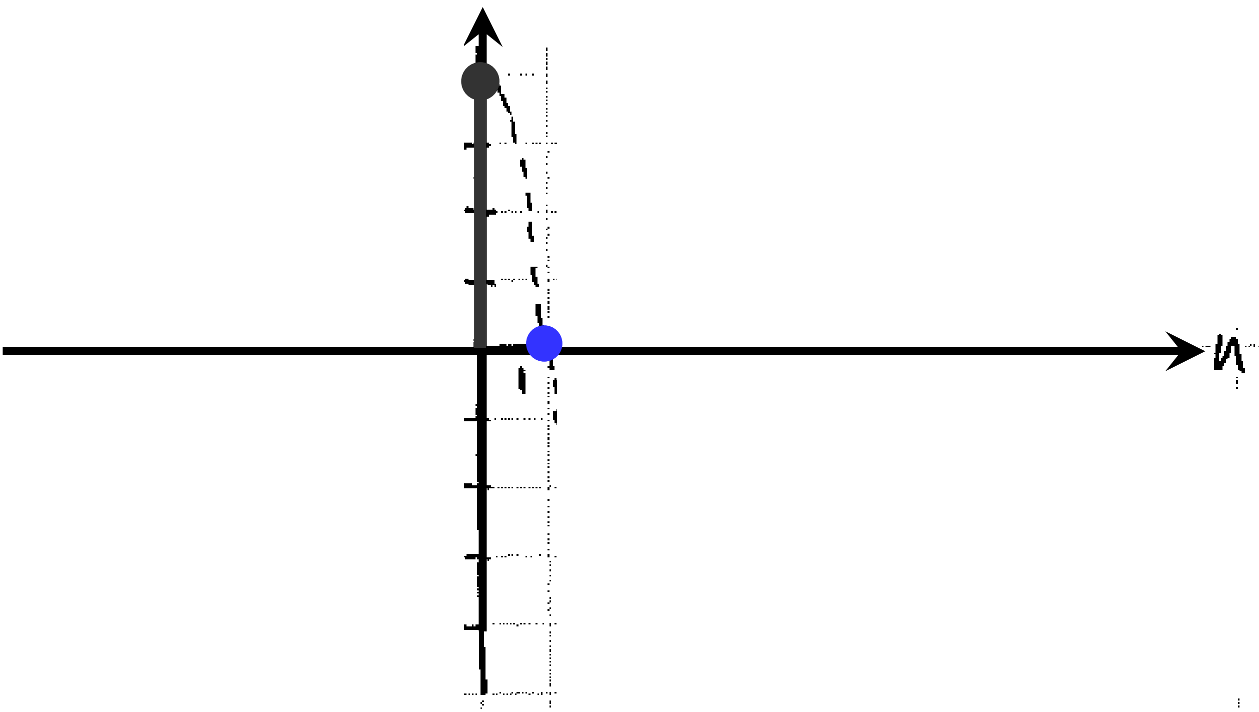
$$x(n) = \cos(\tfrac{\pi}{2} n)$$



n	$\cos(\frac{\pi}{2} n)$
0	1
1	
2	
3	
4	
5	
6	
7	

DT frequency

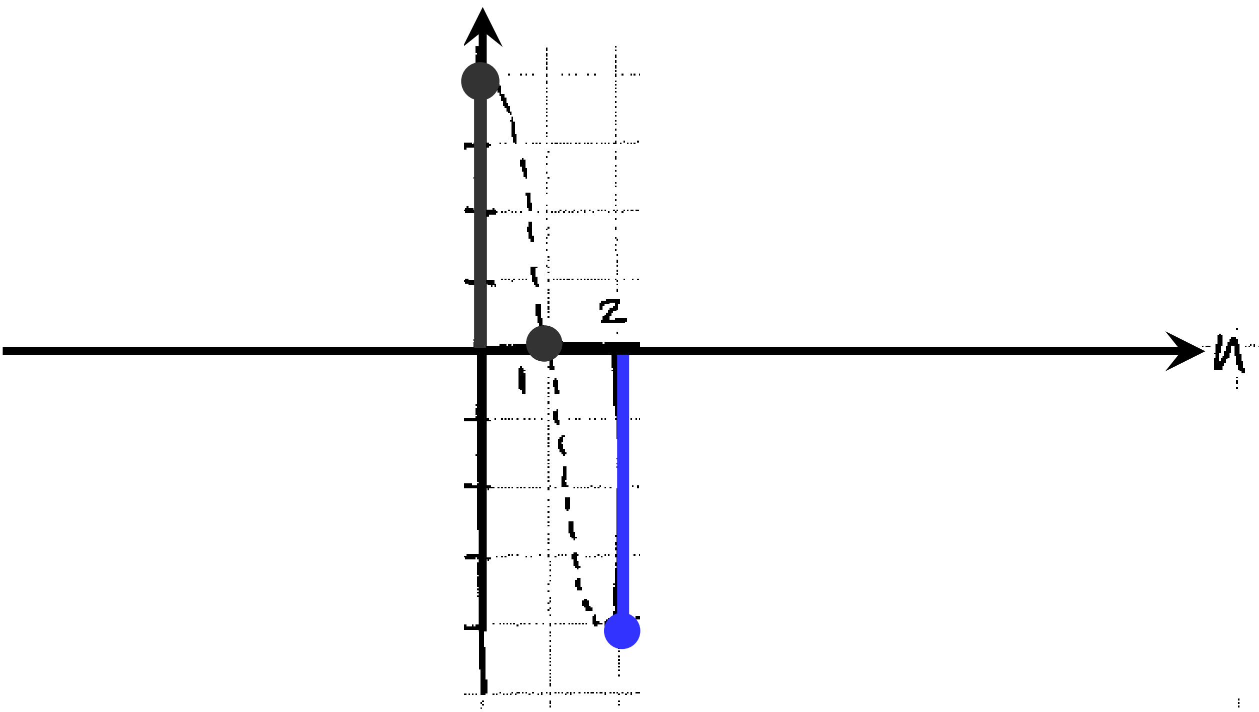
$$x(n) = \cos(\tfrac{\pi}{2} n)$$



n	$\cos(\frac{\pi}{2} n)$
0	1
1	0
2	
3	
4	
5	
6	
7	

DT frequency

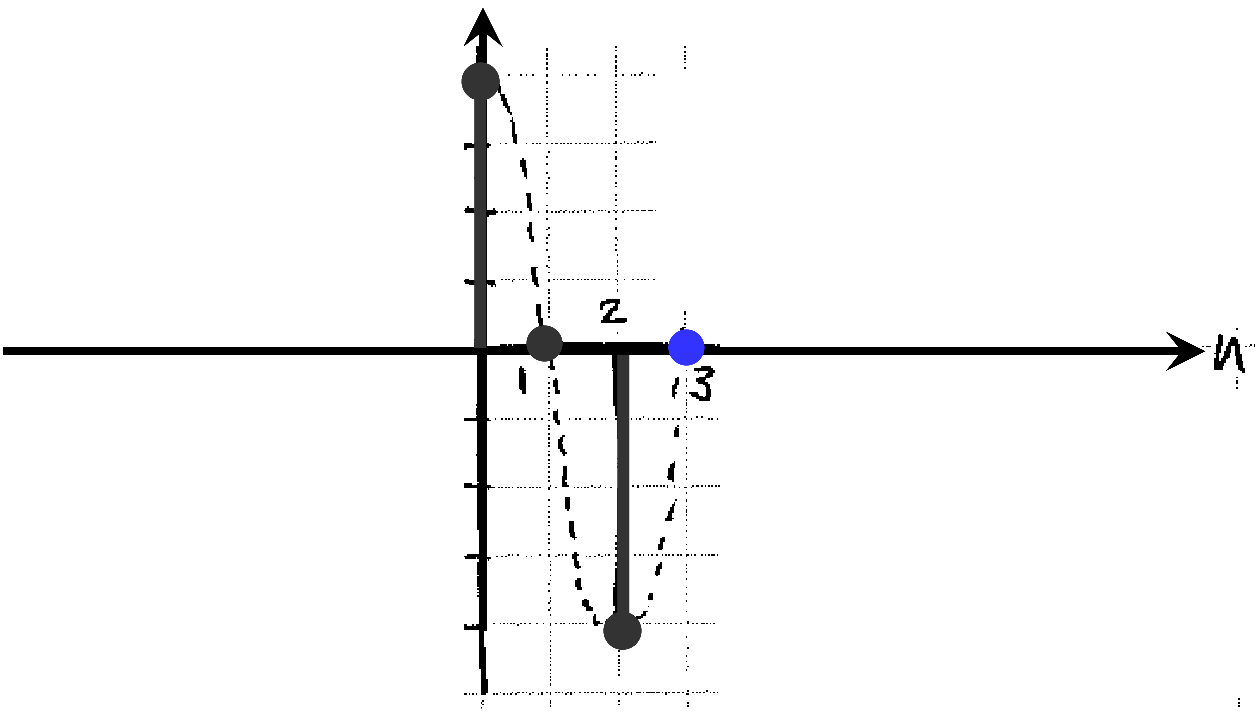
$$x(n) = \cos(\tfrac{\pi}{2} n)$$



n	$\cos(\frac{\pi}{2} n)$
0	1
1	0
2	-1
3	
4	
5	
6	
7	

DT frequency

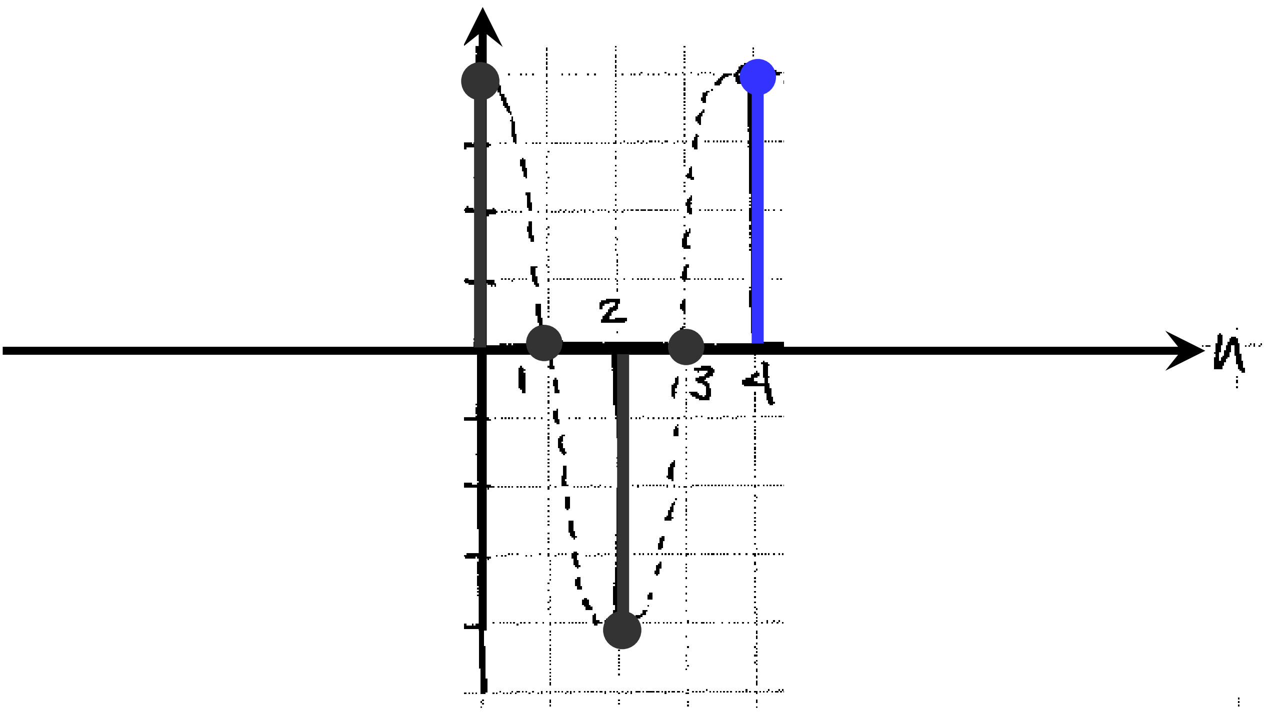
$$x(n) = \cos(\tfrac{\pi}{2} n)$$



n	$\cos(\frac{\pi}{2} n)$
0	1
1	0
2	-1
3	0
4	
5	
6	
7	

DT frequency

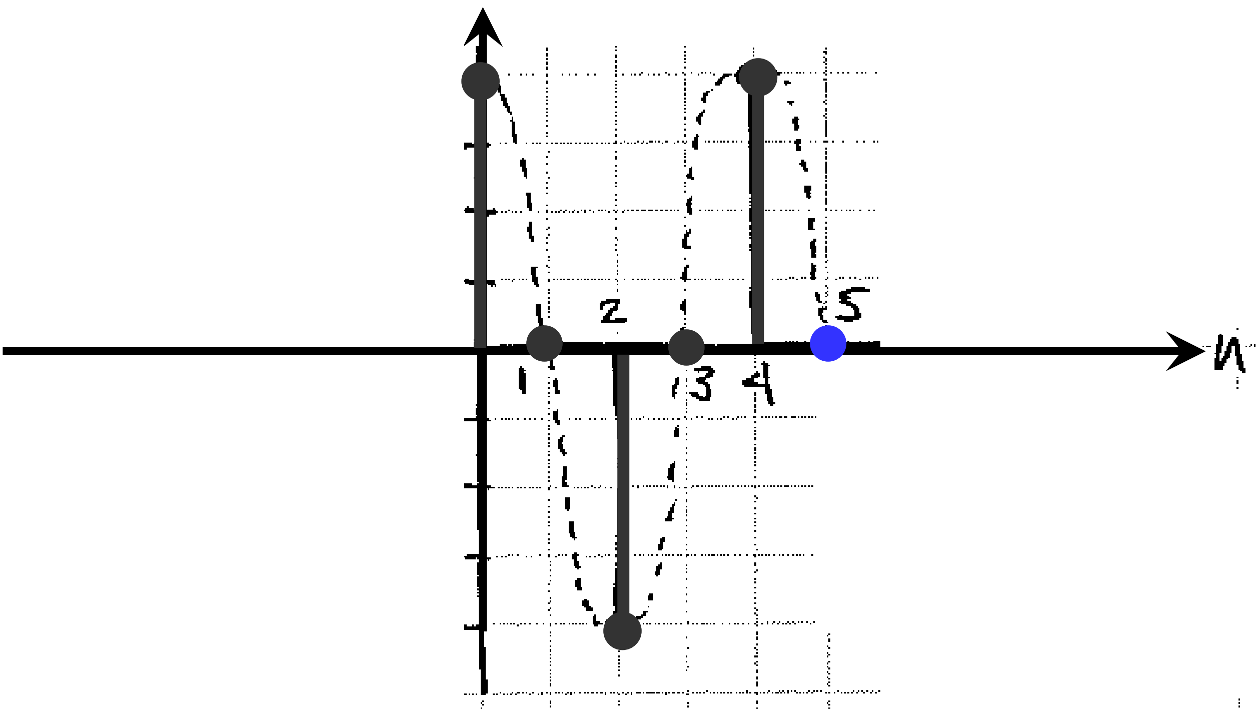
$$x(n) = \cos(\tfrac{\pi}{2} n)$$



n	$\cos(\frac{\pi}{2} n)$
0	1
1	0
2	-1
3	0
4	1
5	
6	
7	

DT frequency

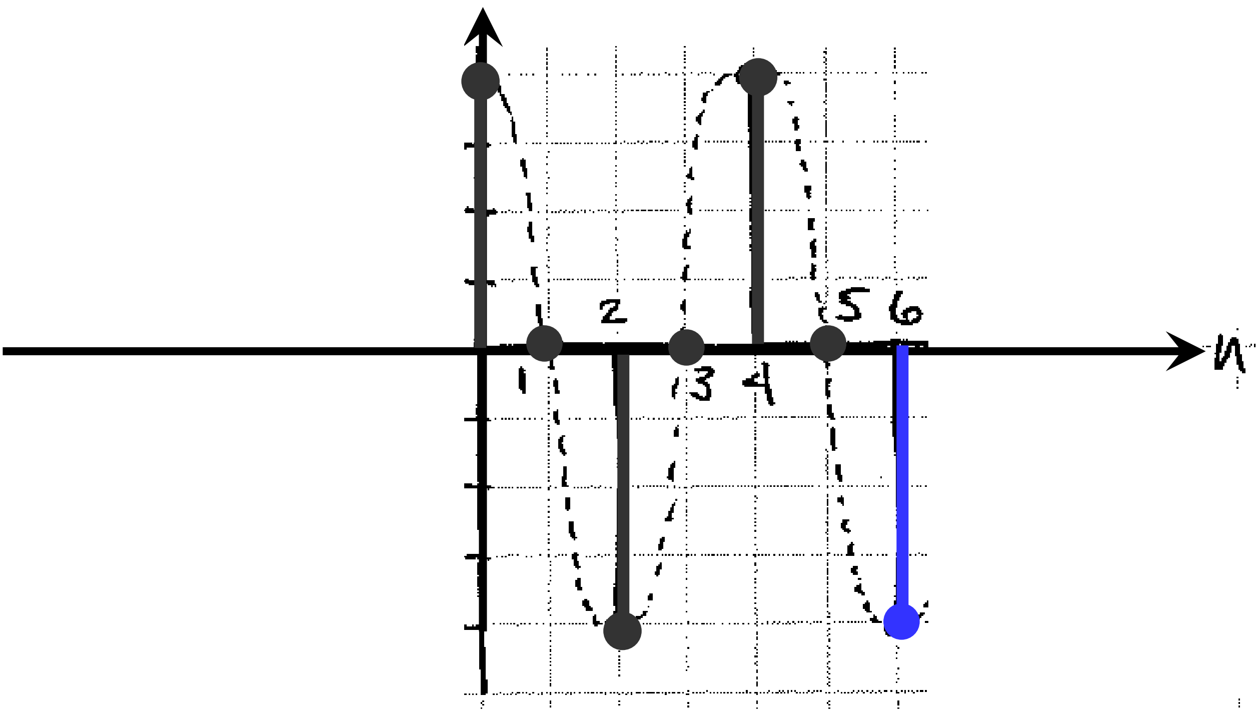
$$x(n) = \cos(\tfrac{\pi}{2} n)$$



n	$\cos(\frac{\pi}{2} n)$
0	1
1	0
2	−1
3	0
4	1
5	0
6	
7	

DT frequency

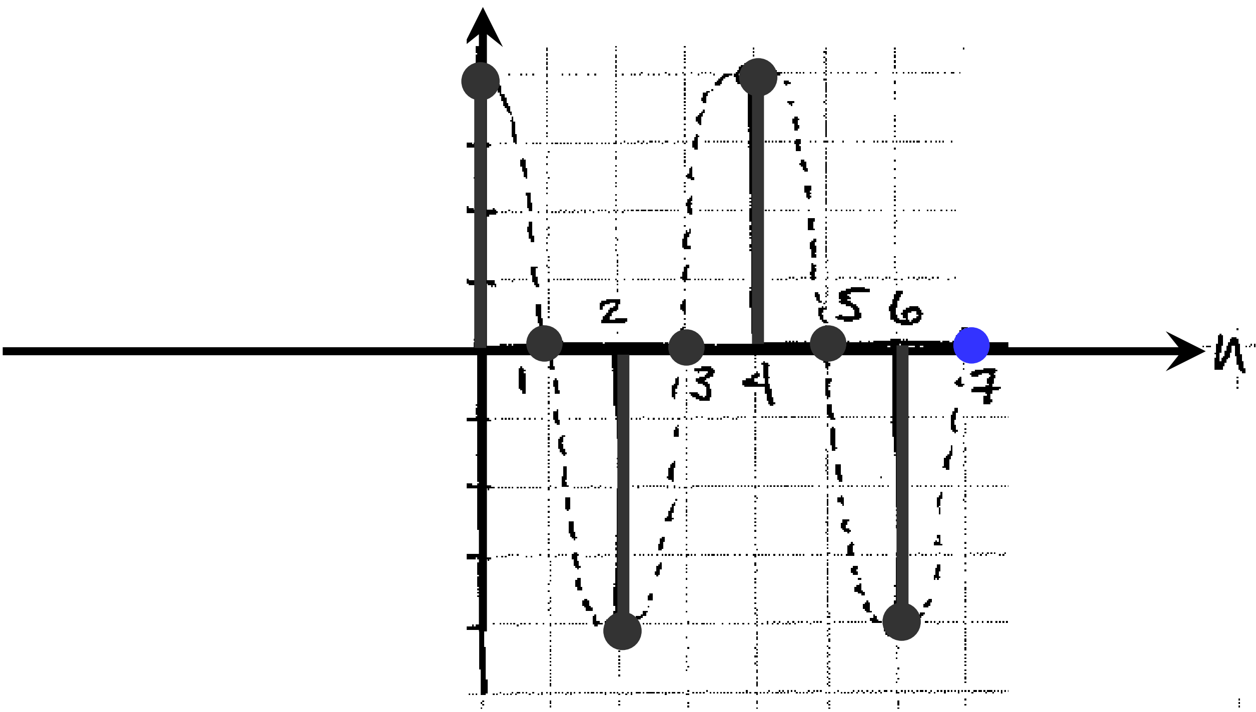
$$x(n) = \cos(\tfrac{\pi}{2} n)$$



n	$\cos(\frac{\pi}{2} n)$
0	1
1	0
2	−1
3	0
4	1
5	0
6	−1
7	

DT frequency

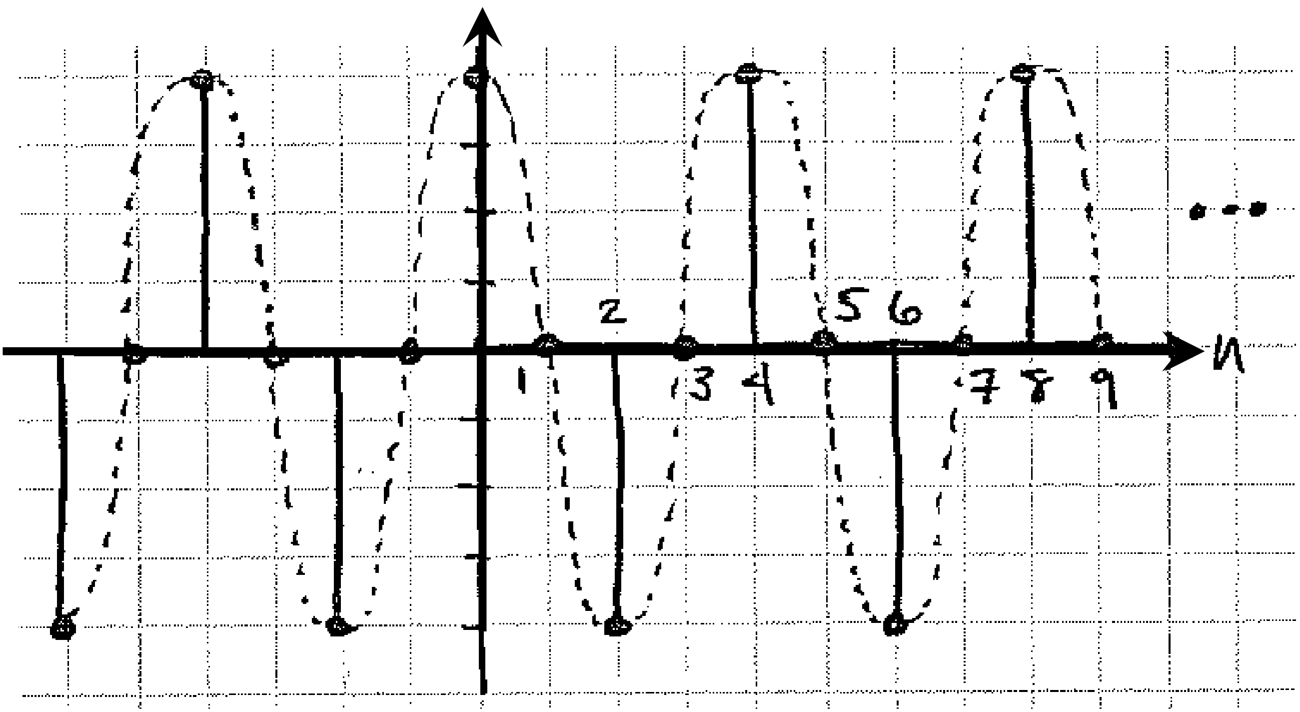
$$x(n) = \cos(\tfrac{\pi}{2} n)$$



n	$\cos(\frac{\pi}{2} n)$
0	1
1	0
2	-1
3	0
4	1
5	0
6	-1
7	0

DT frequency

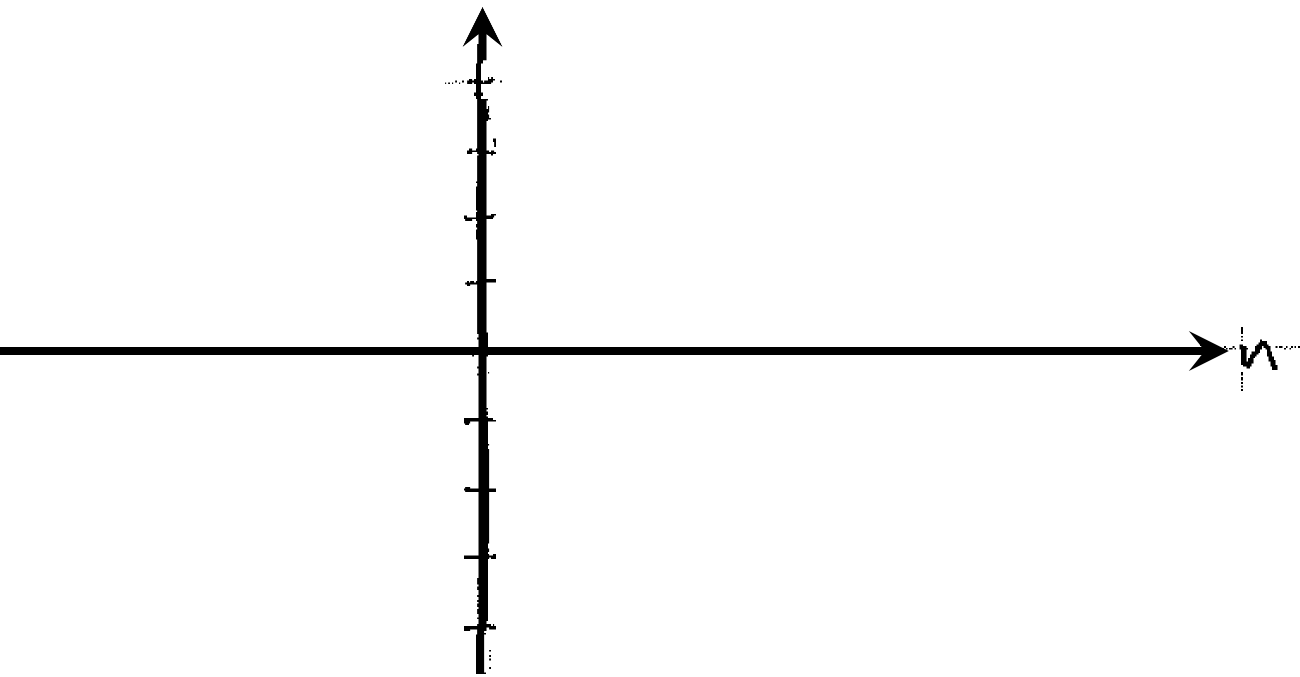
$$x(n) = \cos(\tfrac{\pi}{2} n)$$



n	$\cos(\frac{\pi}{2} n)$
0	1
1	0
2	−1
3	0
4	1
5	0
6	−1
7	0

DT frequency

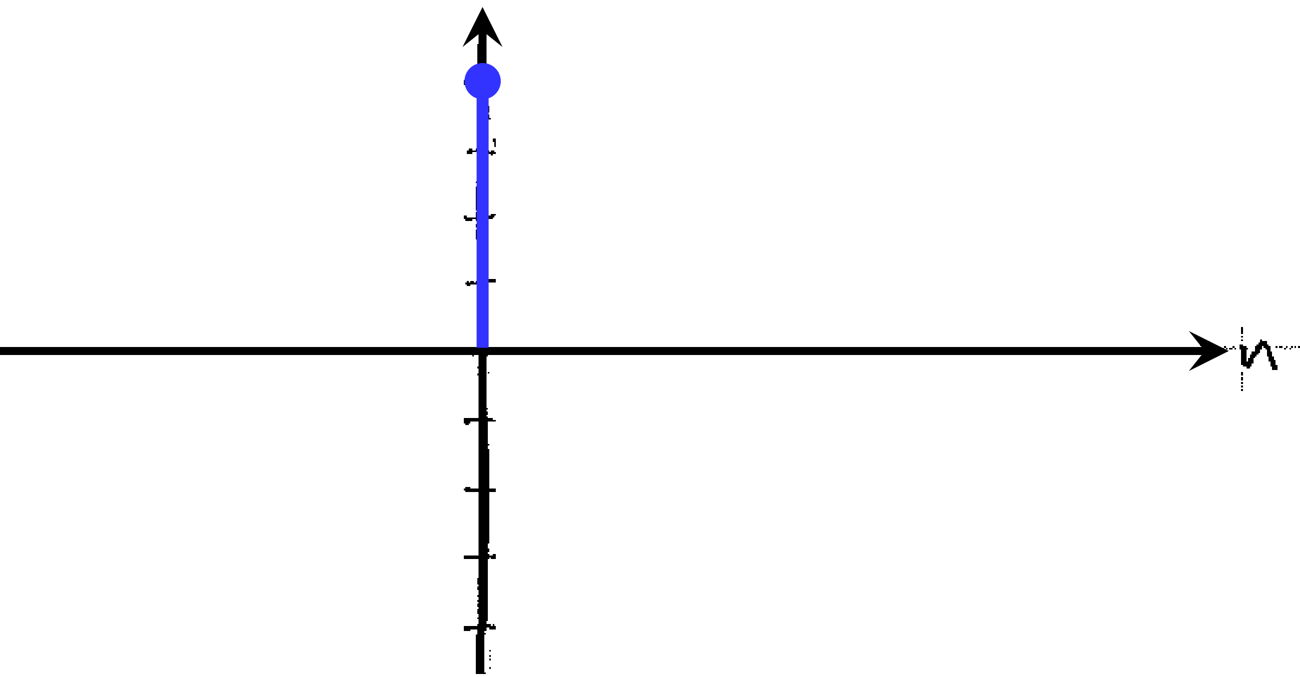
$$x(n) = \cos(\pi n)$$



n	cos(πn)
0	
1	
2	
3	
4	
5	
6	
7	

DT frequency

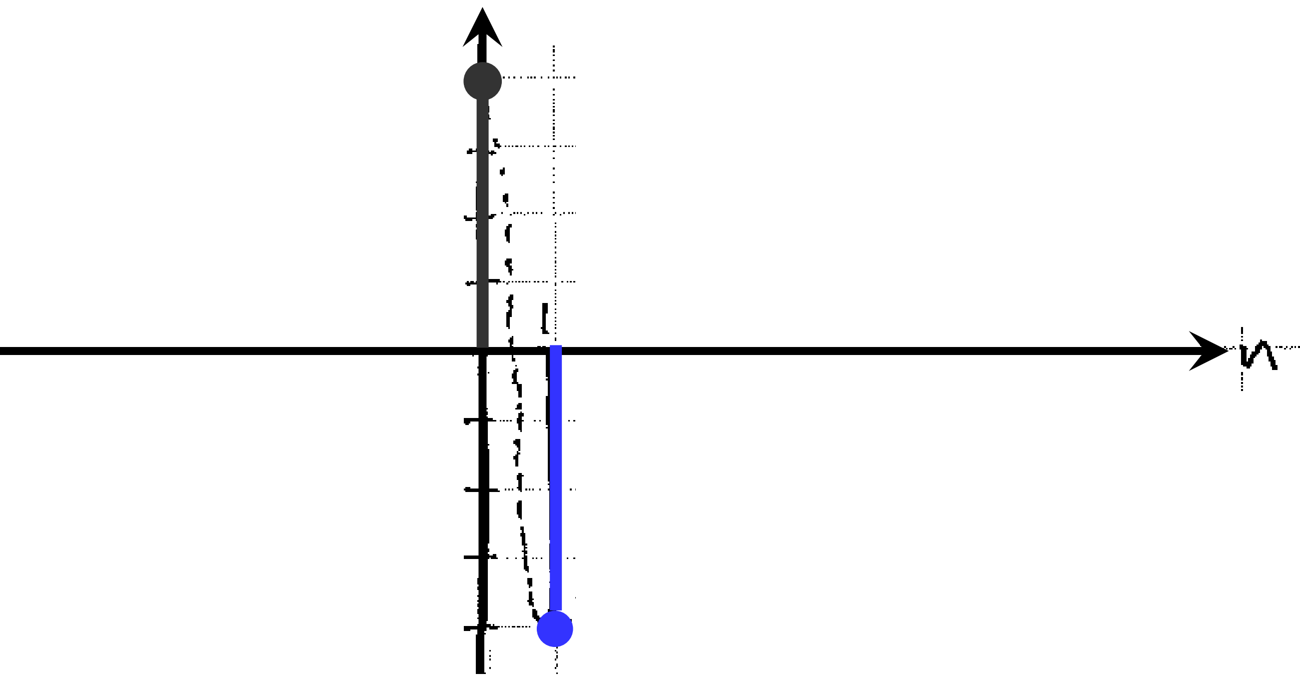
$$x(n) = \cos(\pi n)$$



n	$\cos(\pi n)$
0	1
1	
2	
3	
4	
5	
6	
7	

DT frequency

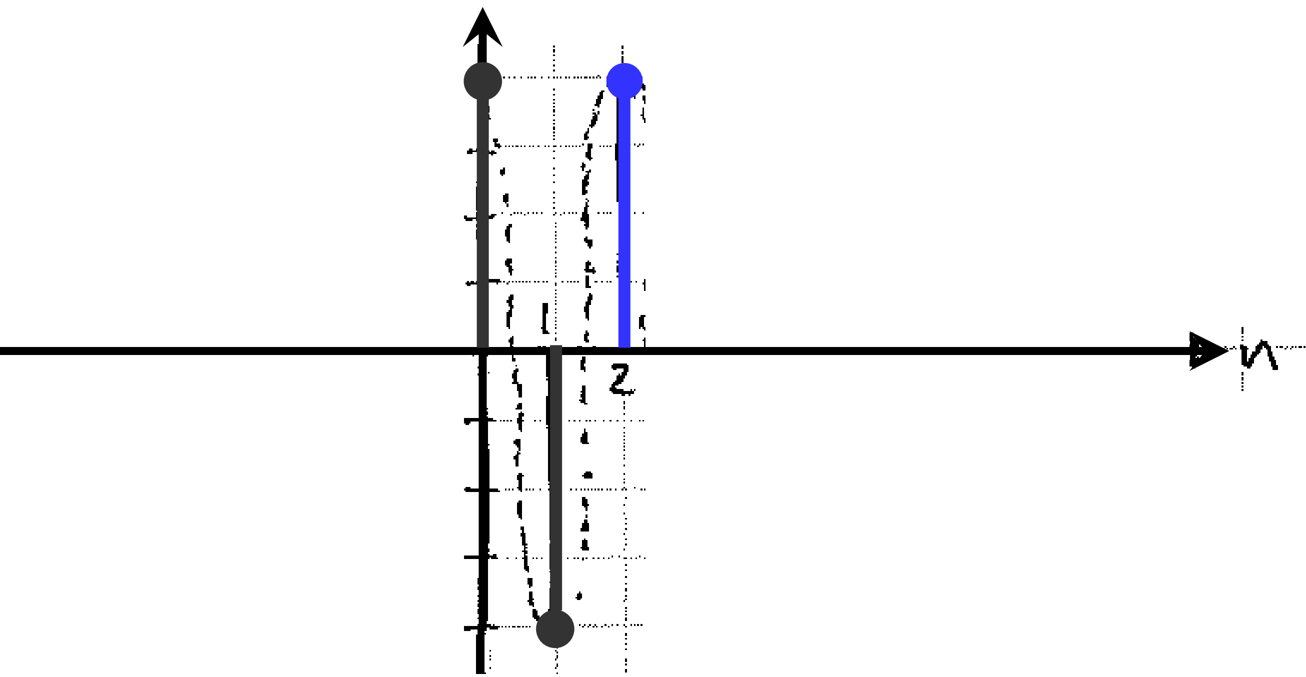
$$x(n) = \cos(\pi n)$$



n	$\cos(\pi n)$
0	1
1	−1
2	
3	
4	
5	
6	
7	

DT frequency

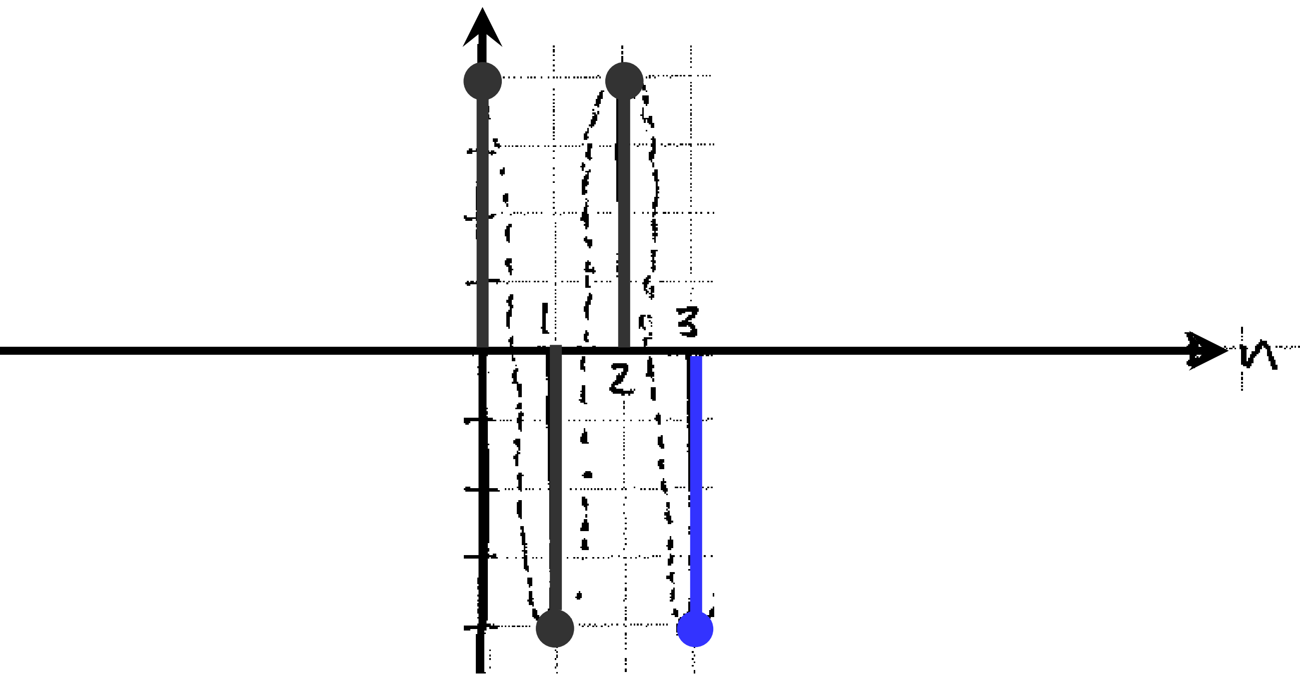
$$x(n) = \cos(\pi n)$$



n	$\cos(\pi n)$
0	1
1	-1
2	1
3	
4	
5	
6	
7	

DT frequency

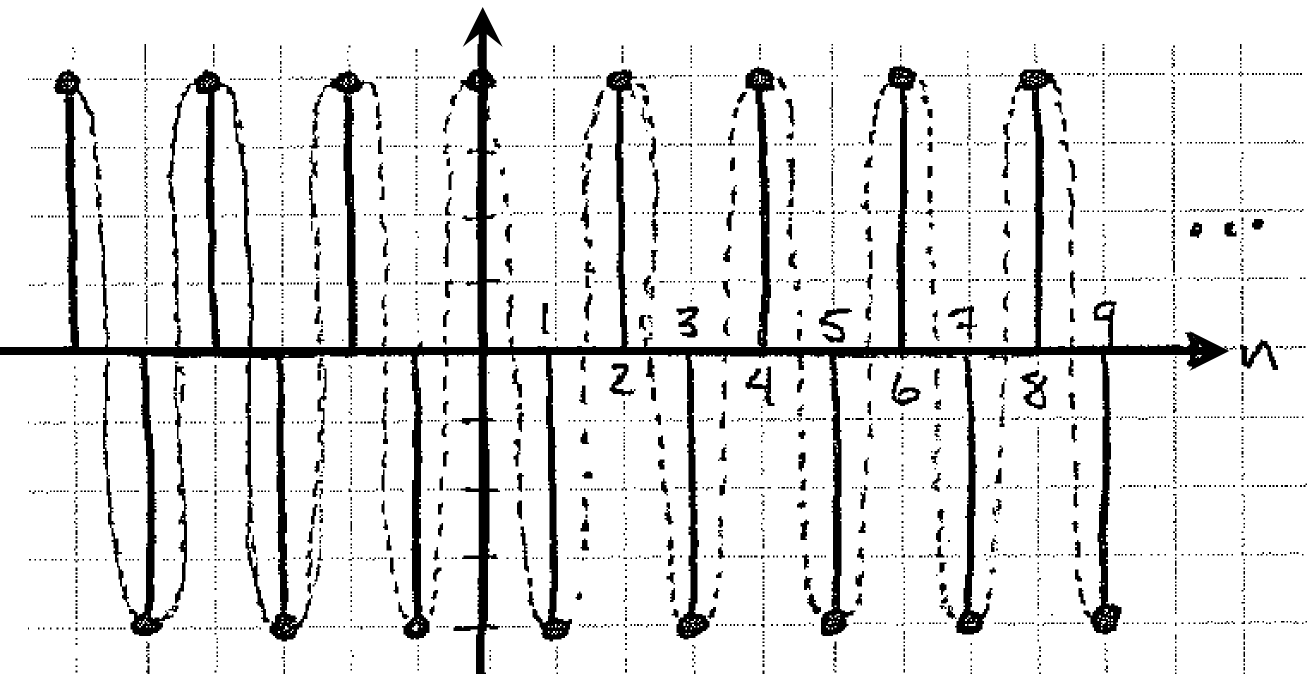
$$x(n) = \cos(\pi n)$$



n	$\cos(\pi n)$
0	1
1	−1
2	1
3	−1
4	
5	
6	
7	

DT frequency

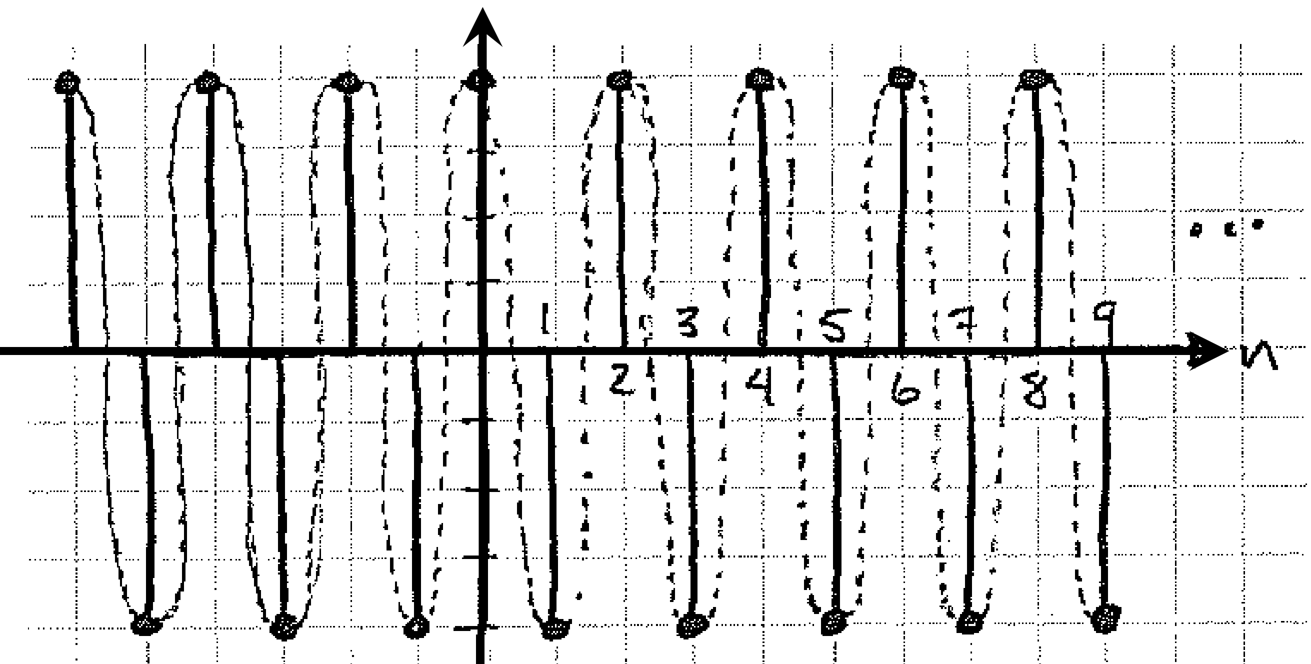
$$x(n) = \cos(\pi n)$$



n	$\cos(\pi n)$
0	1
1	-1
2	1
3	-1
4	1
5	-1
6	1
7	-1

DT frequency

$$x(n) = \cos(\pi n)$$



n	$\cos(\pi n)$
0	1
1	−1
2	1
3	−1
4	1
5	−1
6	1

For DT signals, the maximum frequency occurs @ $\omega = \pi$