Digital Signal Processing

Spring Semester 2022

Digital Systems, Part 1

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Last time's learning objectives

- Explain how the Discrete/Fast Fourier Transform (DFT/FFT) differs from the DTFT
  - ✓ The DFT is the DTFT computed at discrete values of ω
  - ✓ The FFT is a fast algorithm for computing the DFT
- Compute the DFT/FFT on paper
  - ✓ Compute the DTFT, then replace ω with k (where $k = 0, 1, 2, \ldots\ N-1$; N=DFT length)
- Compute the DFT/FFT in Matlab (and interpret the results)
  - ✓ Use the **`fft()`** function

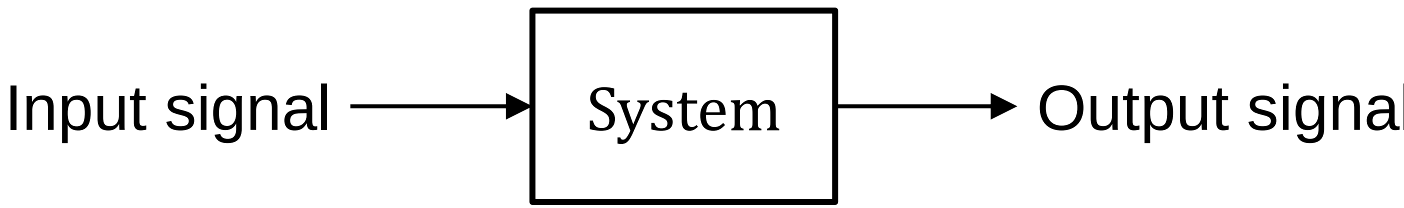
Today's learning objectives

From **today's lecture**, you should **be able to**…

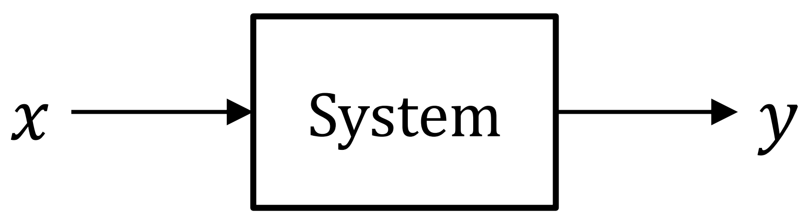
- Explain the terms "digital system" and "digital filter"
- List the ways to characterize a filter
- ~~List the ways to apply a filter~~

Digital systems and filters

In signal processing, a **system** takes an input signal and yields an ouput signal:



By convention, the **input** is denoted by $\boldsymbol{x}$ and the **output** is denoted by $\boldsymbol{y}$:



When both the input and the output are DT signals, the system can be called a "digital system."

Digital systems and filters

A digital **filter** is a digital **system** designed to:

- Remove unwanted frequencies (e.g., noise)
- Attenuate (reduce) certain frequencies
- Amplify (increase) certain frequencies

In this class, we will focus on $\overbrace{\textbf{LTI}}^{\text{Linear and Time-Invariant}}$ DT systems.

$$x(n) \longrightarrow \boxed{\text{LTI filter}} \longrightarrow y(n)$$

Example: Length-3 moving average filter

Input/output equation (called a **difference equation**):

$$y(n) = \frac{1}{3}\big(x(n+1) + x(n) + x(n-1)\big)$$

Length-3 MA filter: Common Questions

- What's the filter's impulse response?
- Is the filter memoryless?
- Is the filter causal?
- What type of filter is it?
  - Is it a lowpass filter? A highpass filter? A bandpass filter?
  - What frequencies does it attenuate?
- Does the filter induce phase distortion?

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Impulse response of LTI systems

LTI systems (filters) can be characterized by how they change an impulse:

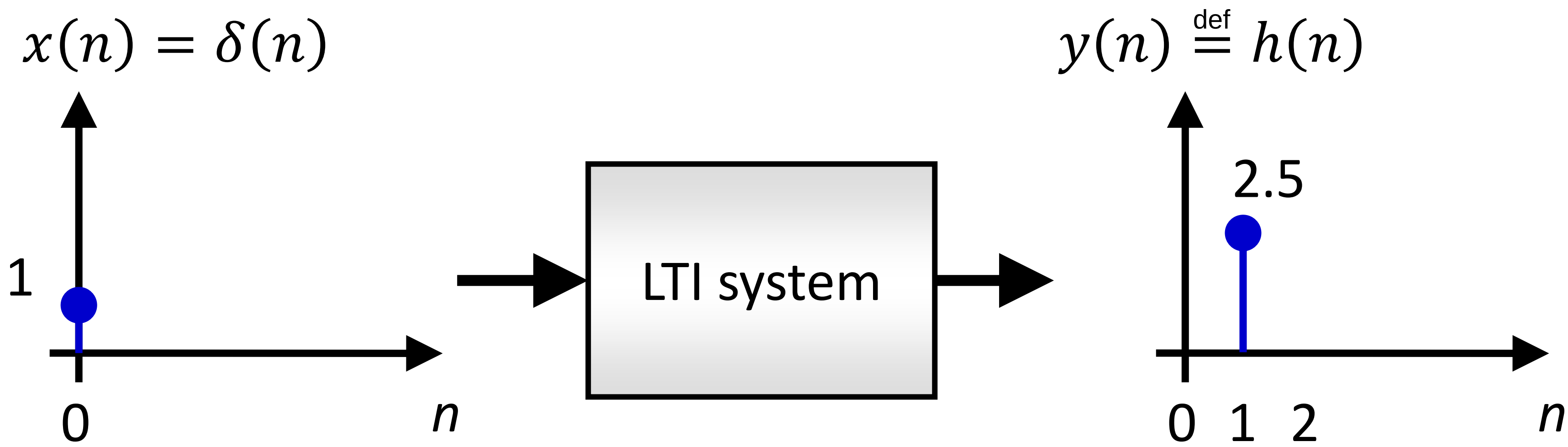
$$x(n) = \delta(n) \longrightarrow \boxed{\text{LTI system}} \longrightarrow y(n) = h(n)$$

impulse input

response to impulse

We call $h(n)$ the "impulse response" of the system.

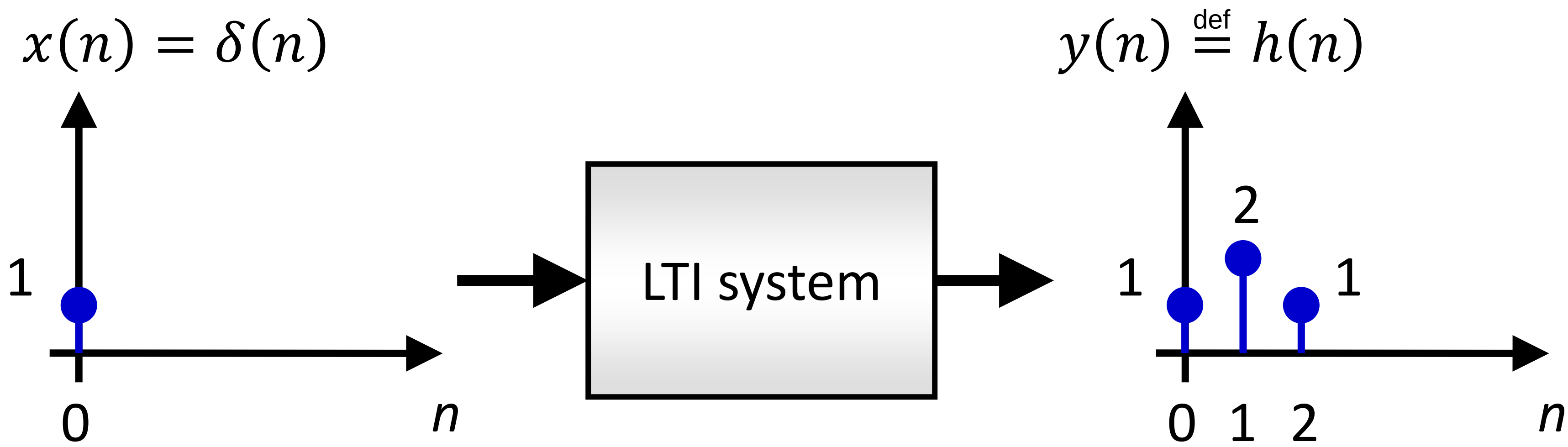
Example of getting impulse response



Thus, the system's **impulse response** is…

$$\boxed{h(n) = 2.5\delta(n-1)}$$

Example of getting impulse response



Thus, the system's **impulse response** is…

$$\boxed{h(n) = \delta(n) + 2\delta(n-1) + \delta(n-2)}$$

Impulse response of MA filter

Input/output equation (called a **difference equation**):

$$y(n) = \frac{1}{3}\big(x(n+1) + x(n) + x(n-1)\big)$$

The system's **impulse response** is…

$$\boxed{h(n) = \frac{1}{3}\big(\delta(n+1) + \delta(n) + \delta(n-1)\big)}$$

Just replace $x(n)$ with $\delta(n)$ in the I/O equation!

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More properties of DT systems

<u>Memoryless</u>: A system is memoryless iff the output at time n depends only on the input at time n.

Length-3 MA filter's **difference equation**:

$$y(n) = \frac{1}{3}\big(x(n+1) + x(n) + x(n-1)\big)$$

→ **Not memoryless** (i.e., requires storing previous/future samples in order to compute current output)

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More properties of DT systems

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More properties of DT systems

Memoryless: A system is memoryless iff the output at time n depends only on the input at time n.

Causal: A system is causal iff the output at time n depends only on current and past inputs (i.e., inputs at time instants $\leq n$)

Note: Memoryless $\Rightarrow$ causal

Note: Real-time systems must be causal

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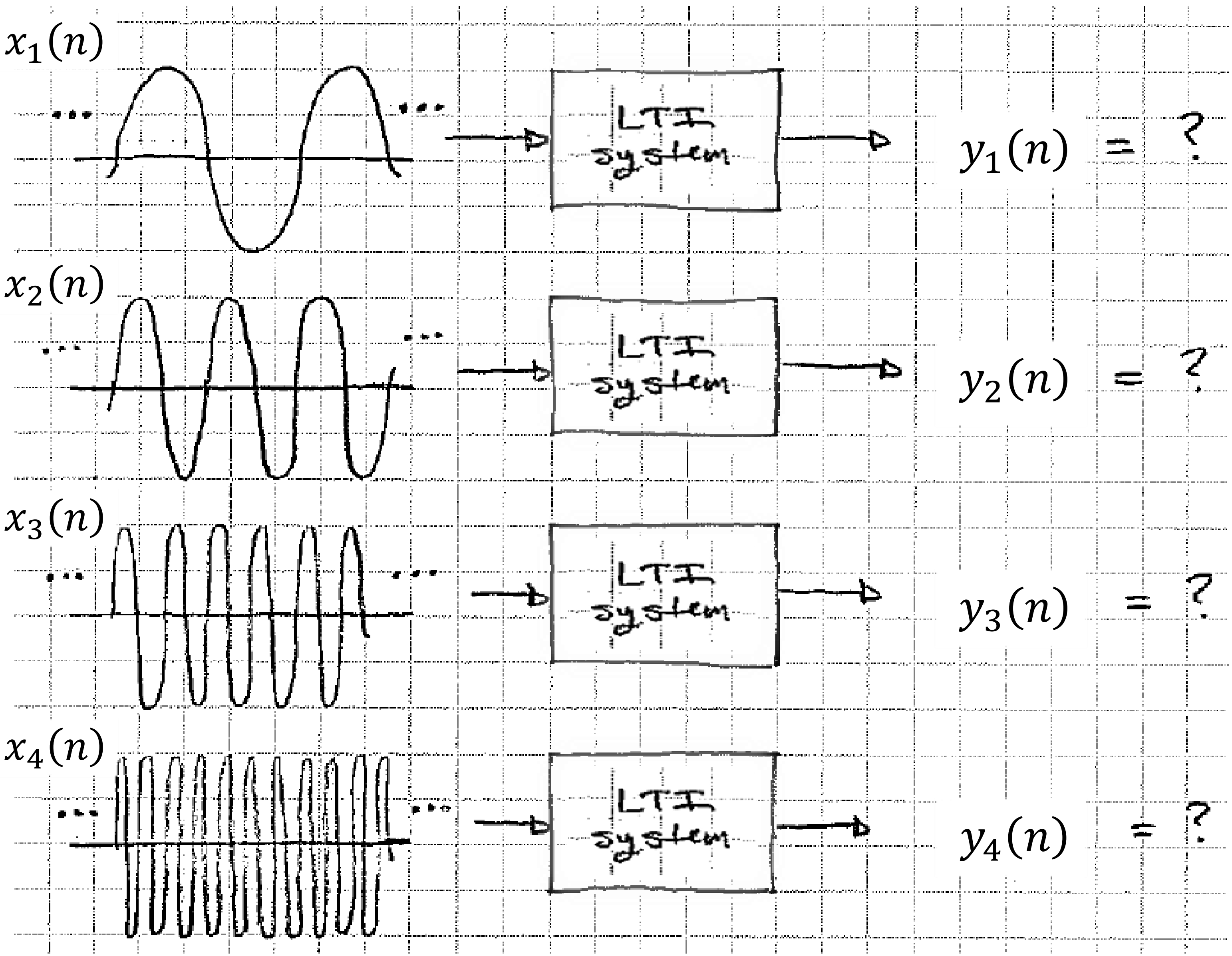
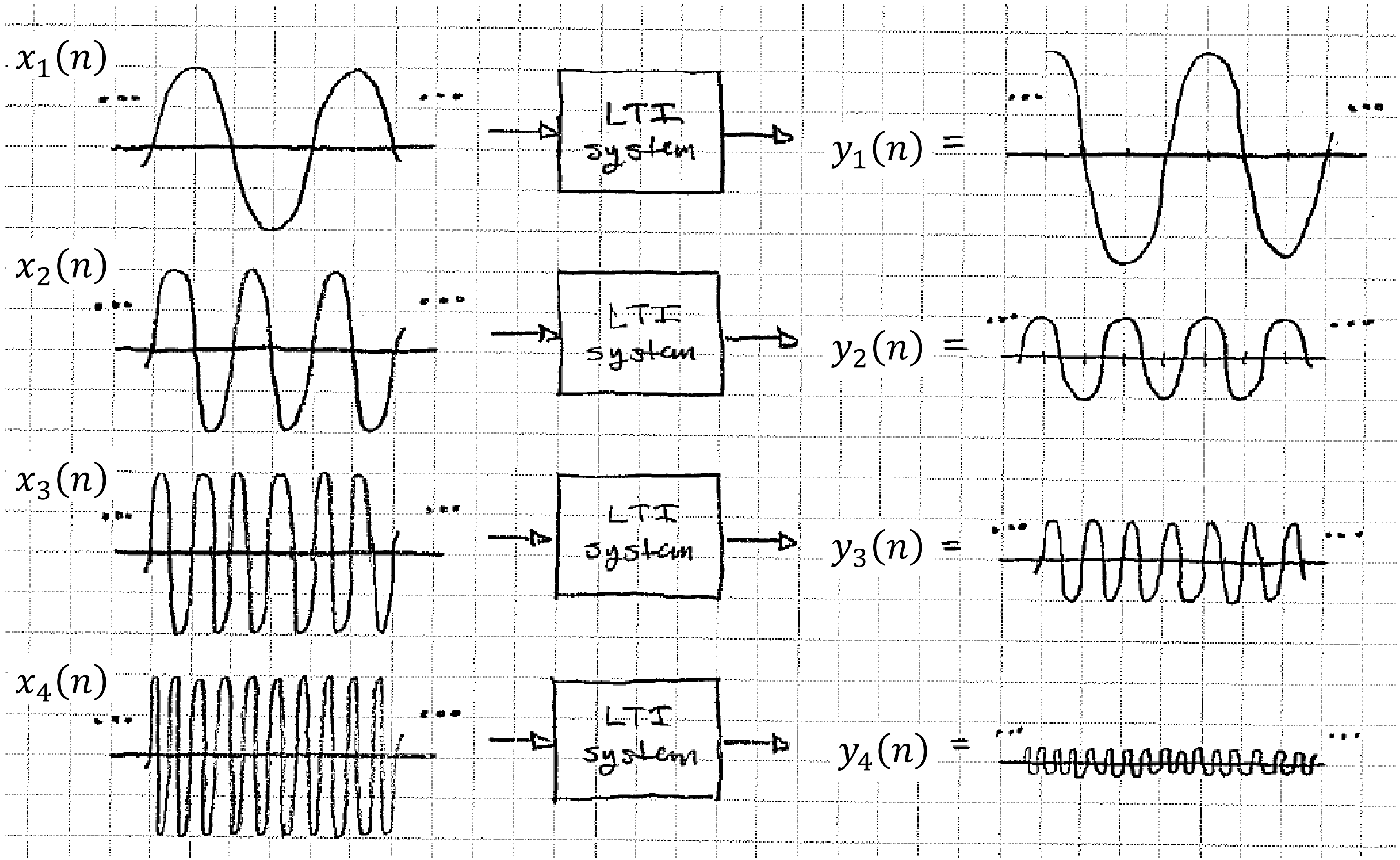
Frequency response



Frequency response



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Examples of linear-phase filters (no phase distortion)

(Trick: They have **symmetric impulse responses**)