Digital Signal Processing

Spring Semester 2022

Digital Systems, Part 3

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Last time's learning objectives

- Use convolution to filter a signal
- Characterize a filter in terms of its frequency response

Today’s learning objectives

From **today’s lecture**, you should **be able to**…

- Use windowing to design a filter
- Apply a filter in Matlab

How do you process a signal?

- How is a filter characterized?
  - Two ways:
    1. By examining how the filter changes an impulse
       - Called the filter's "***impulse response***"
    2. By examining how the filter changes sines/cosines of various frequencies
       - Called the filter's "***frequency response***"
- How is a filter applied?
  - Two ways:
    1. Convolution in the time domain
    2. Multiplication in the frequency domain

Summary

- **Impulse response:** $h(n)$
  - How the system responds to an impulse
- **Frequency response:** $H\left(e^{j\omega}\right)$
  - How the system responds to various frequencies ($\sin(\omega n)$, $\cos(\omega n)$, $e^{j\omega n}$)
  - $H\left(e^{j\omega}\right) = DTFT[\, h(n)\,]$
  - **Magnitude of $\boldsymbol{H\left(e^{j\omega_0}\right)}$** = how system will change amplitude at frequency ω_0
  - **Phase of $\boldsymbol{H\left(e^{j\omega_0}\right)}$** = how system will change phase (i.e., time-shift) at frequency ω_0

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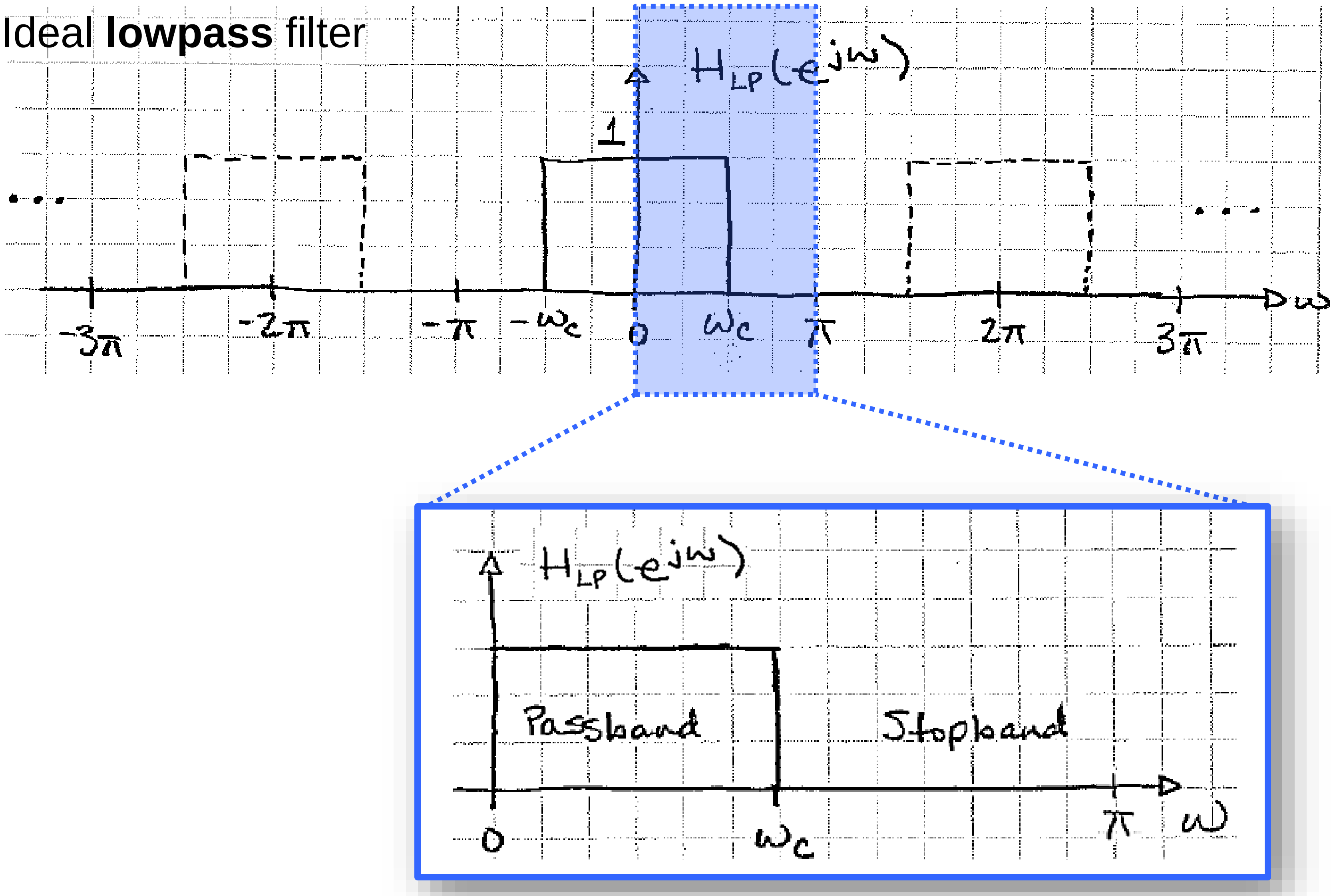
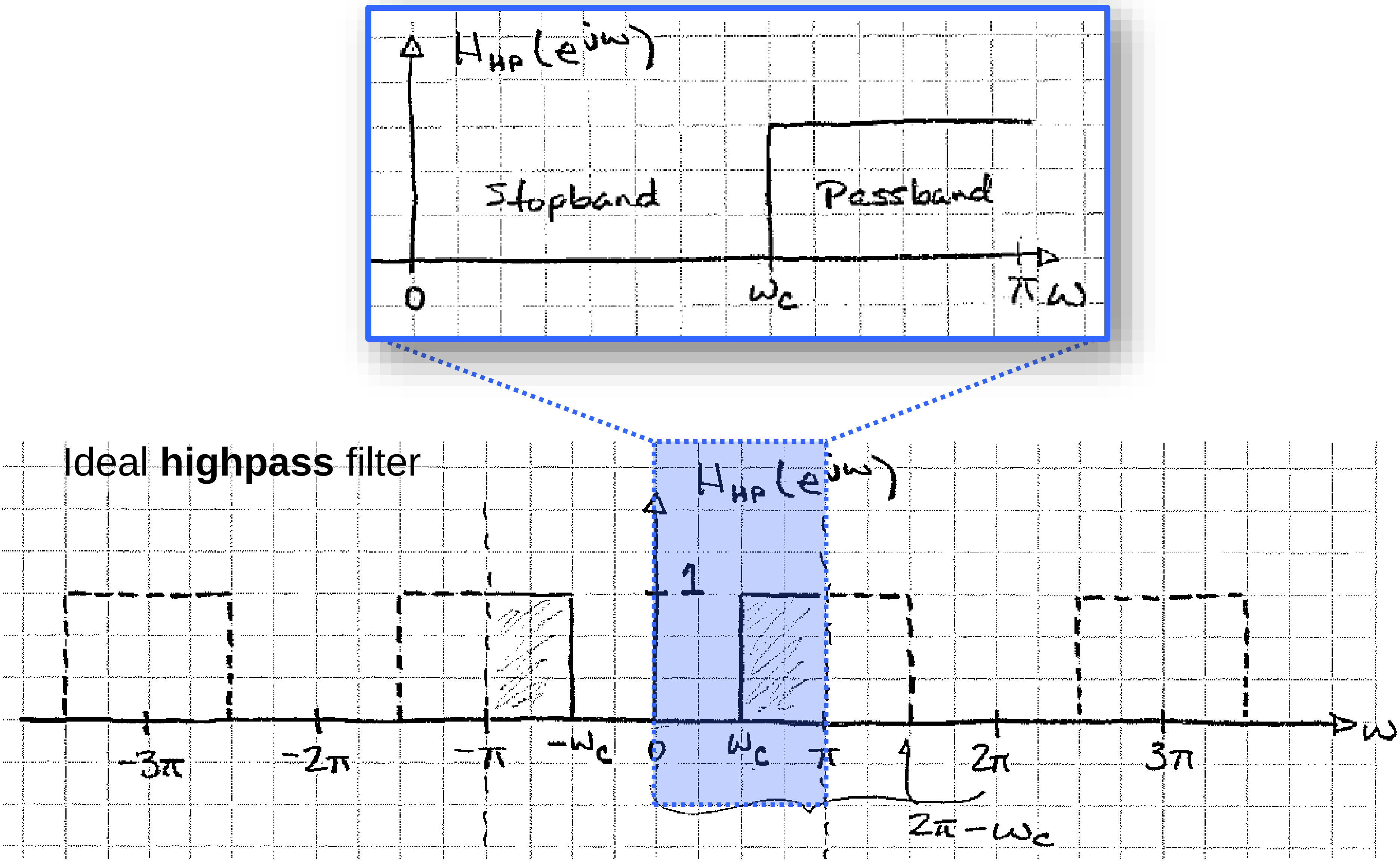
A first look at DT lowpass and higpass filters

Ideal lowpass and highpass filters

Ideal **lowpass** filter

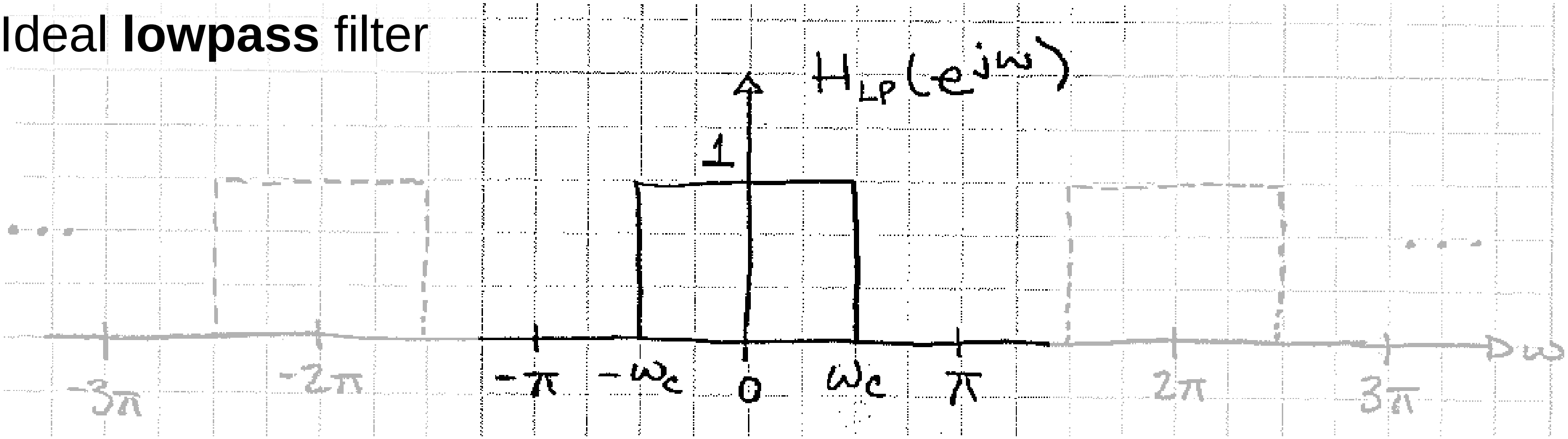


Ideal lowpass and highpass filters

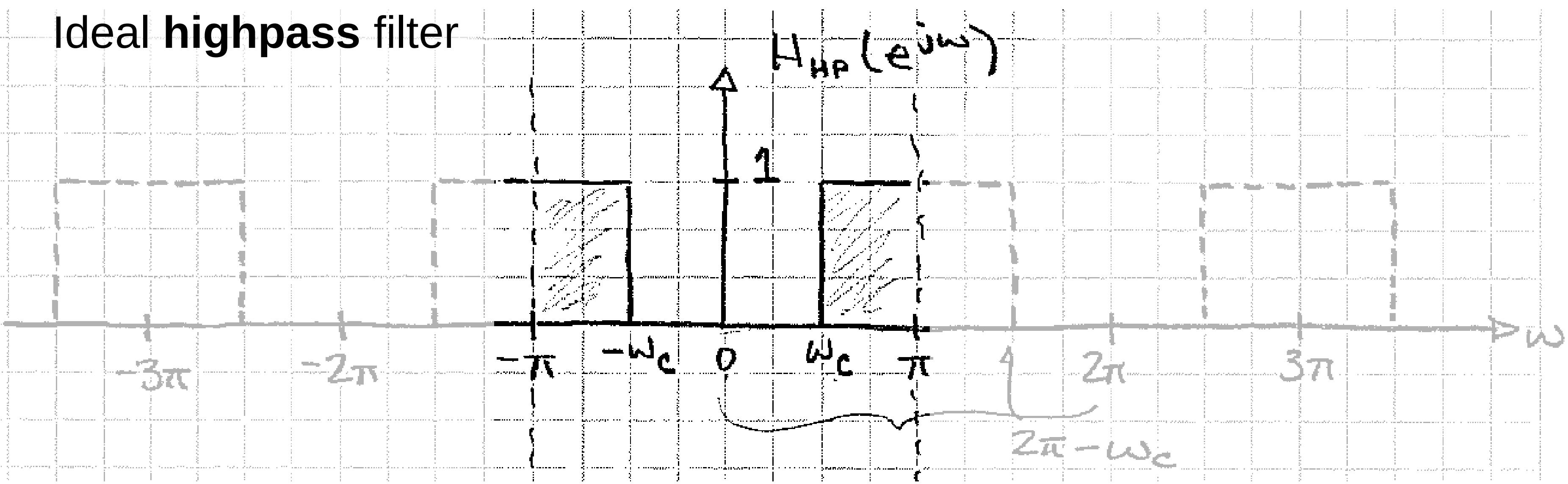


Ideal lowpass and highpass filters

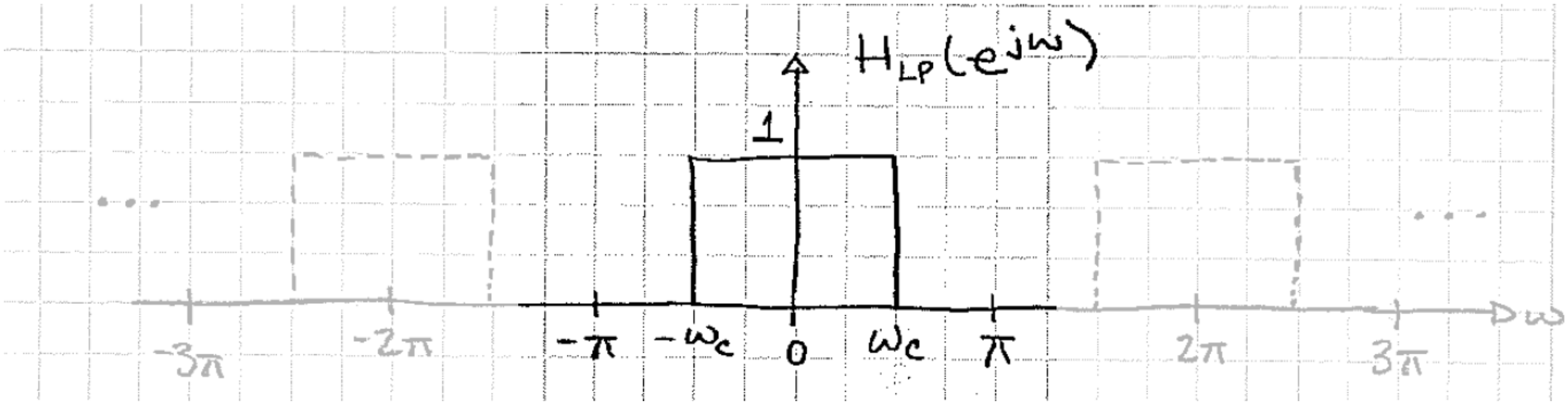
Ideal **lowpass** filter

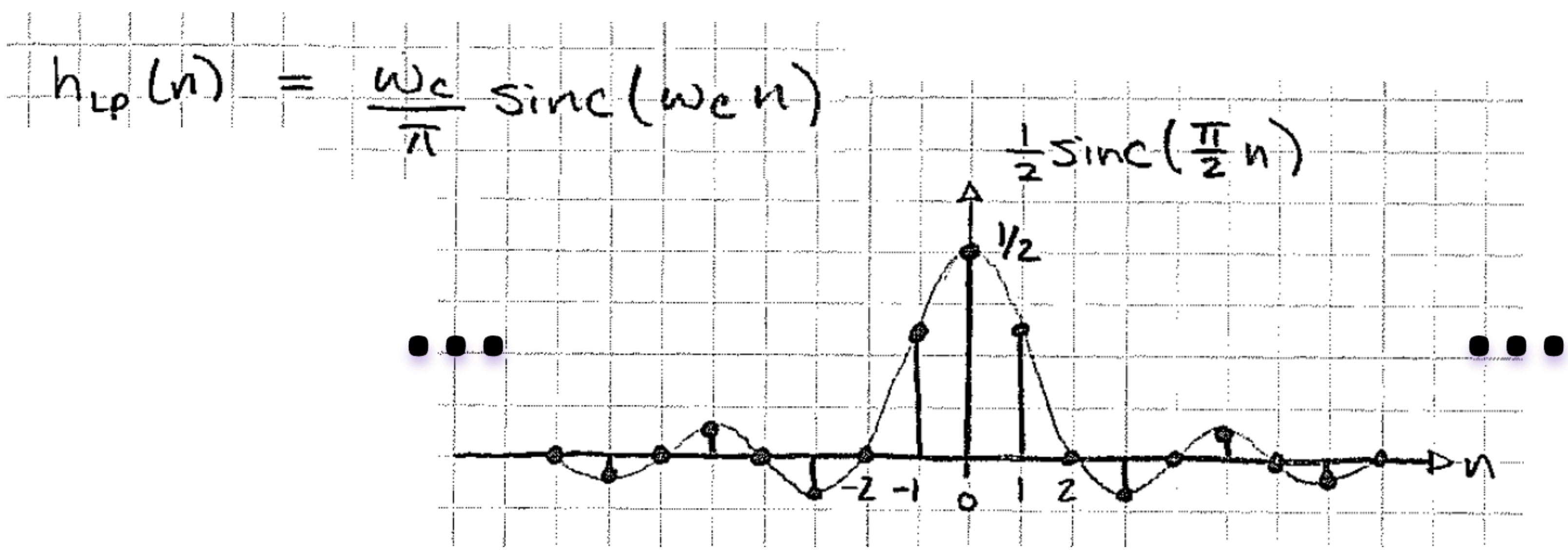


Ideal **highpass** filter

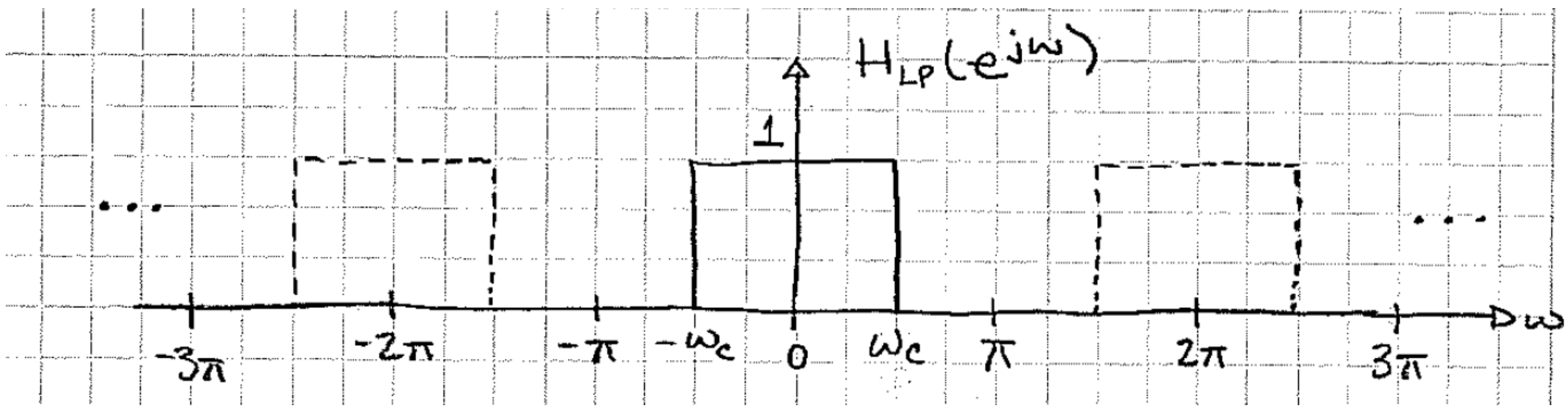


Ideal lowpass filter

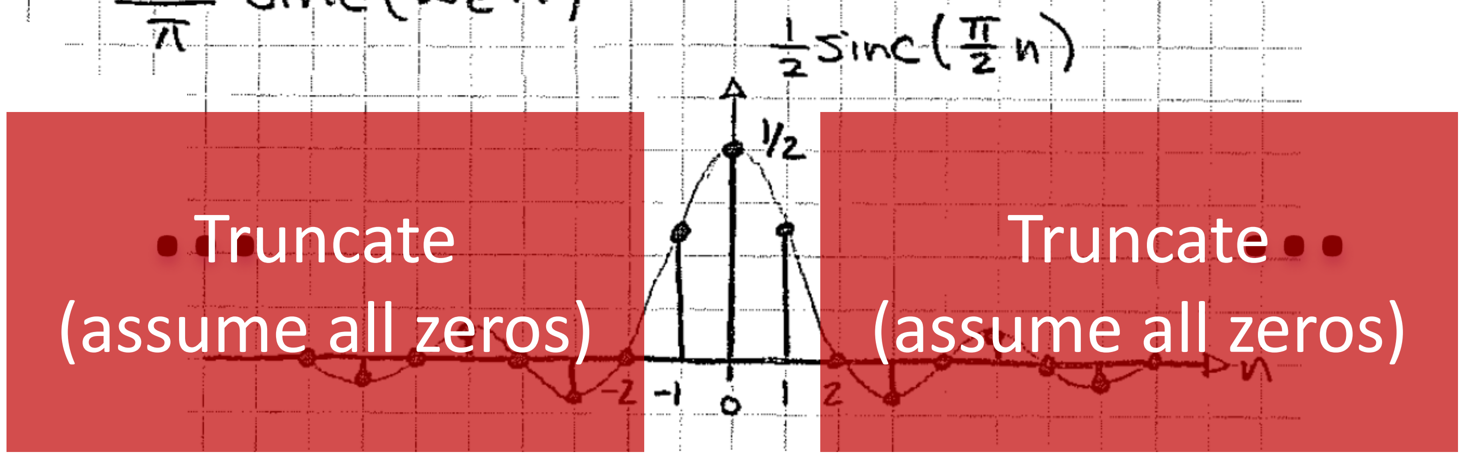




Ideal lowpass filter



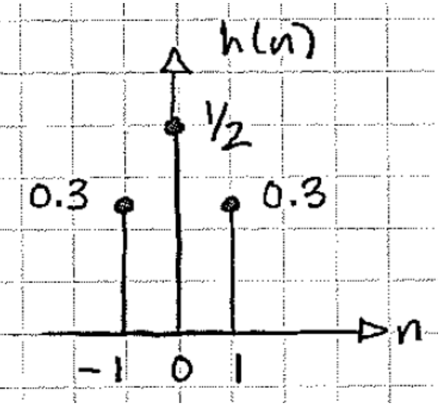
$$h_{LP}(n) = \frac{\omega_c}{\pi}\,\mathrm{sinc}(\omega_c n)$$



Example: Truncated sinc filter

$$h(n) = h_{LP}(n)\,\mathrm{rect}_3(n+1)$$

$$= \tfrac{1}{2}\,\mathrm{sinc}\!\left(\tfrac{\pi}{2} n\right) \mathrm{rect}_3(n+1)$$



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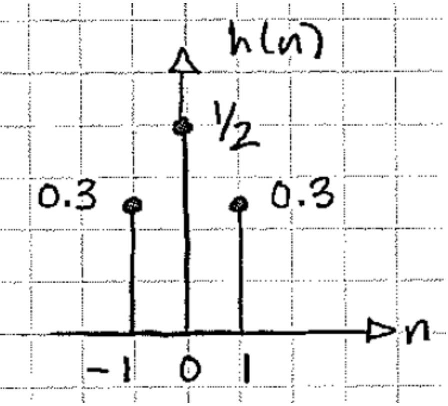

$$h(n) = 0.3\,\delta(n+1) + 0.5\,\delta(n) + 0.3\,\delta(n-1)$$

Impulse response

Example: Truncated sinc filter

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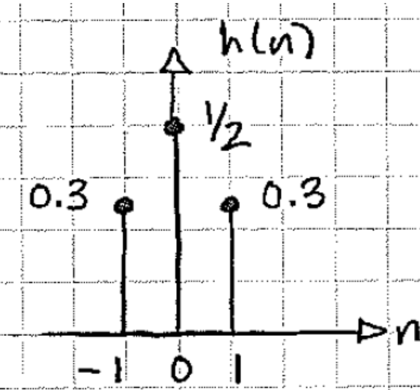


$$h(n) = 0.3\,\delta(n+1) + 0.5\,\delta(n) + 0.3\,\delta(n-1)$$

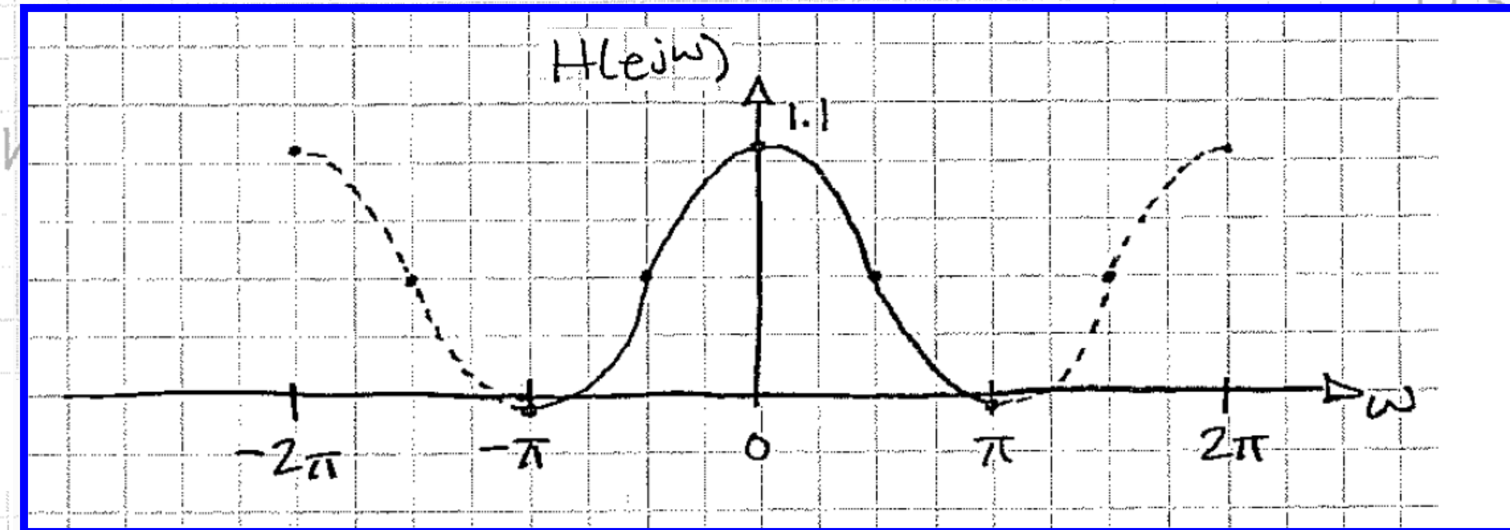
Impulse response

$$H(e^{j\omega}) = 0.3\,e^{j\omega} + 0.5 + 0.3\,e^{-j\omega}$$

$$= 0.5 + 0.6\cos(\omega)$$

Frequency response

Example: Truncated sinc filter



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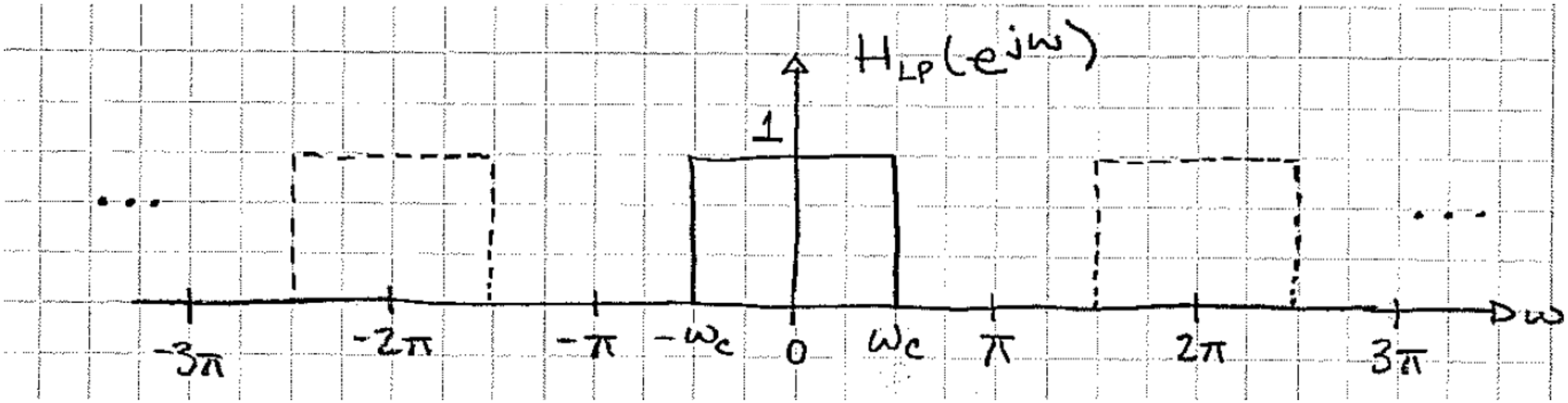
Impulse response

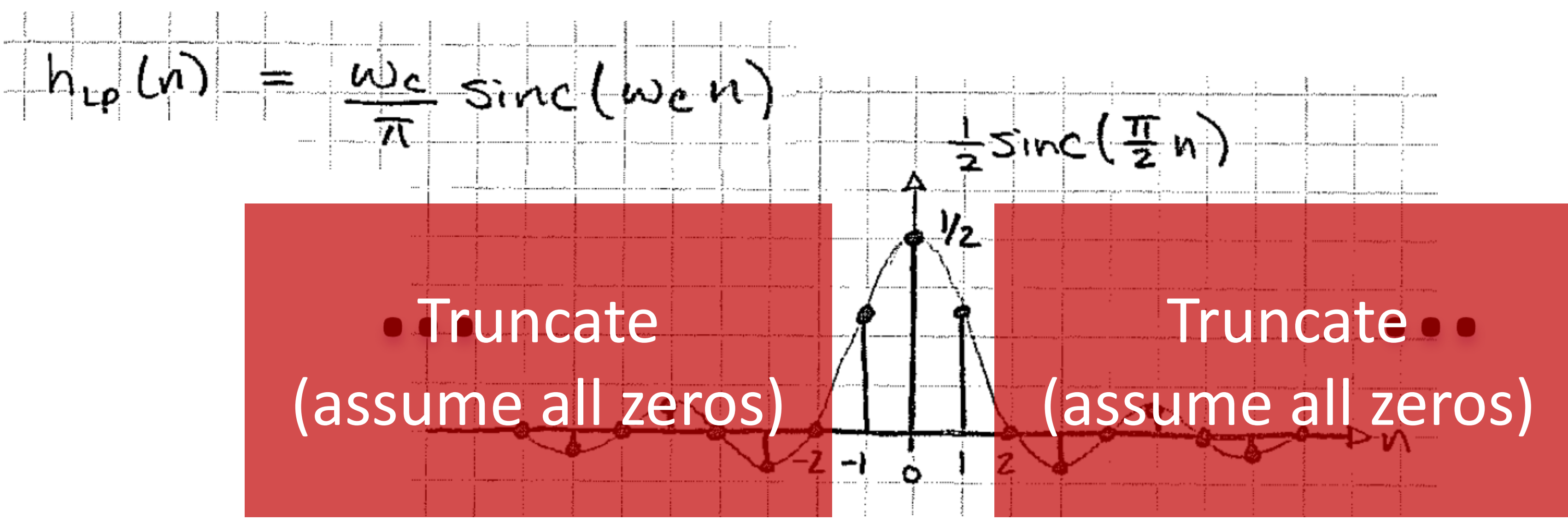
$$H(e^{j\omega}) = 0.3\,e^{j\omega} + 0.5 + 0.3\,e^{-j\omega}$$

$$= \boxed{0.5 + 0.6\cos(\omega)}$$

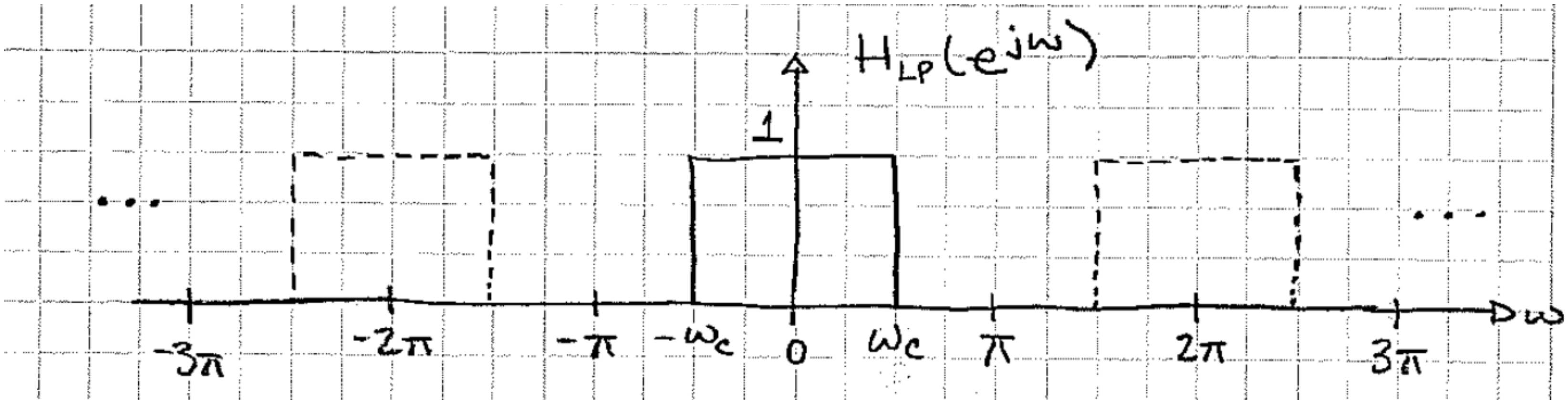
Frequency response

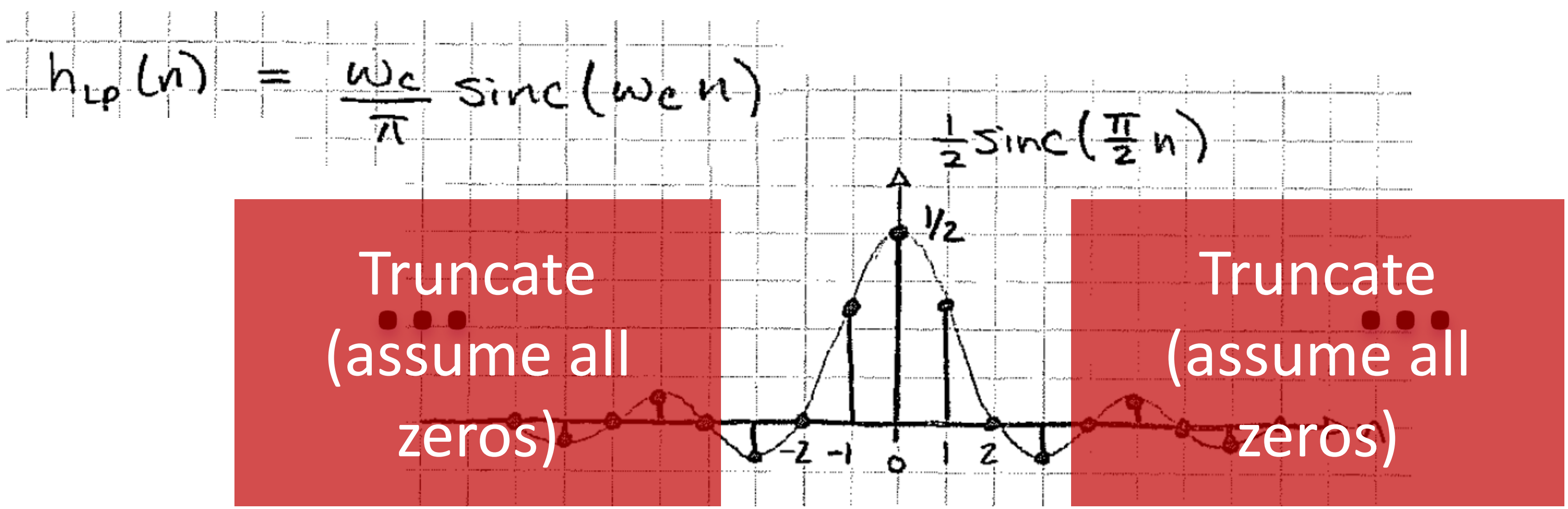
Ideal lowpass filter





Ideal lowpass filter

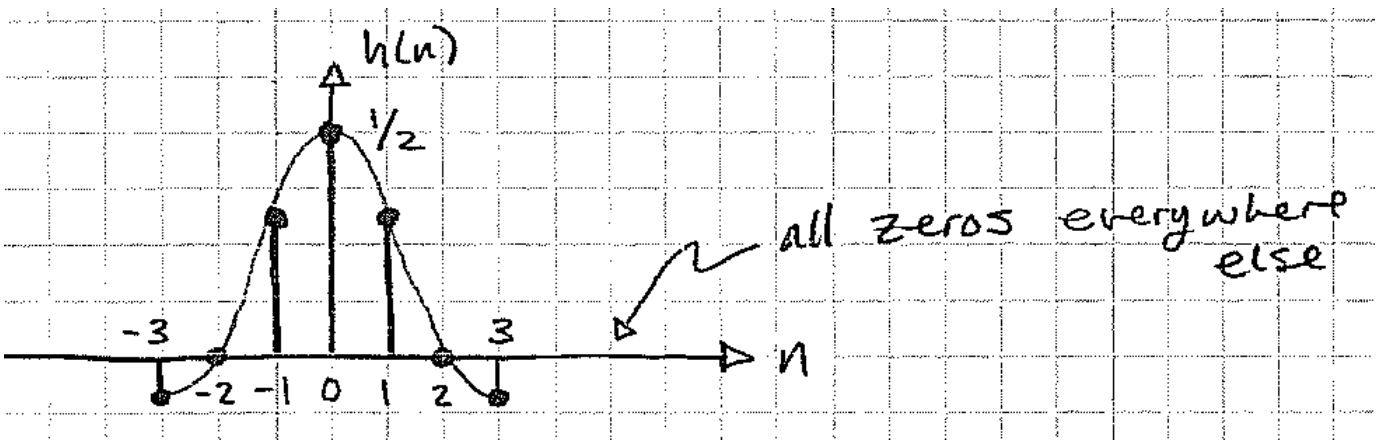




Example: Longer truncated sinc filter

$$h(n) = h_{LP}(n)\, \mathrm{rect}_7(n+3)$$

$$= \tfrac{1}{2}\, \mathrm{sinc}\left(\tfrac{\pi}{2} n\right) \mathrm{rect}_7(n+3)$$



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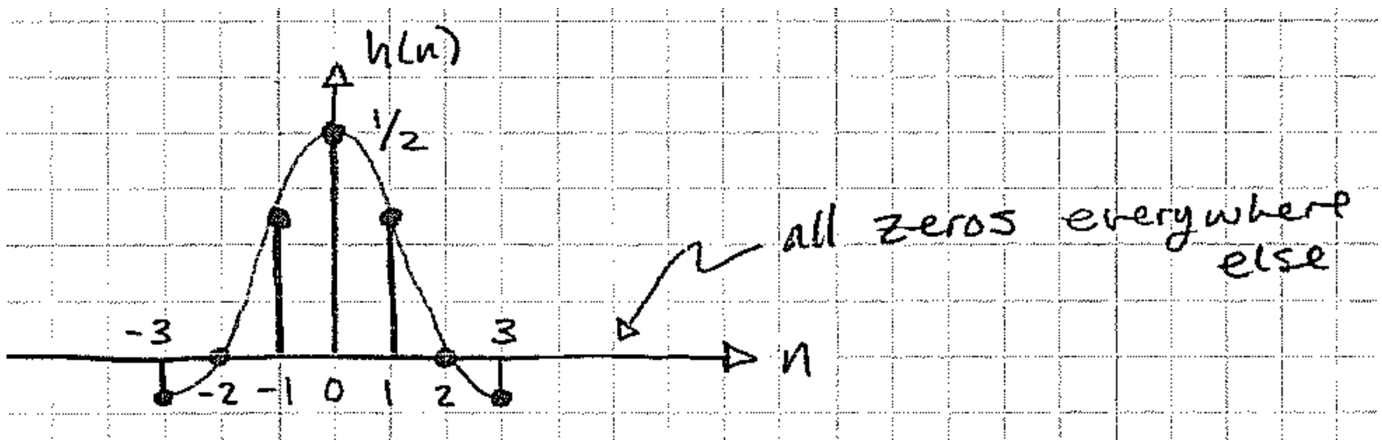
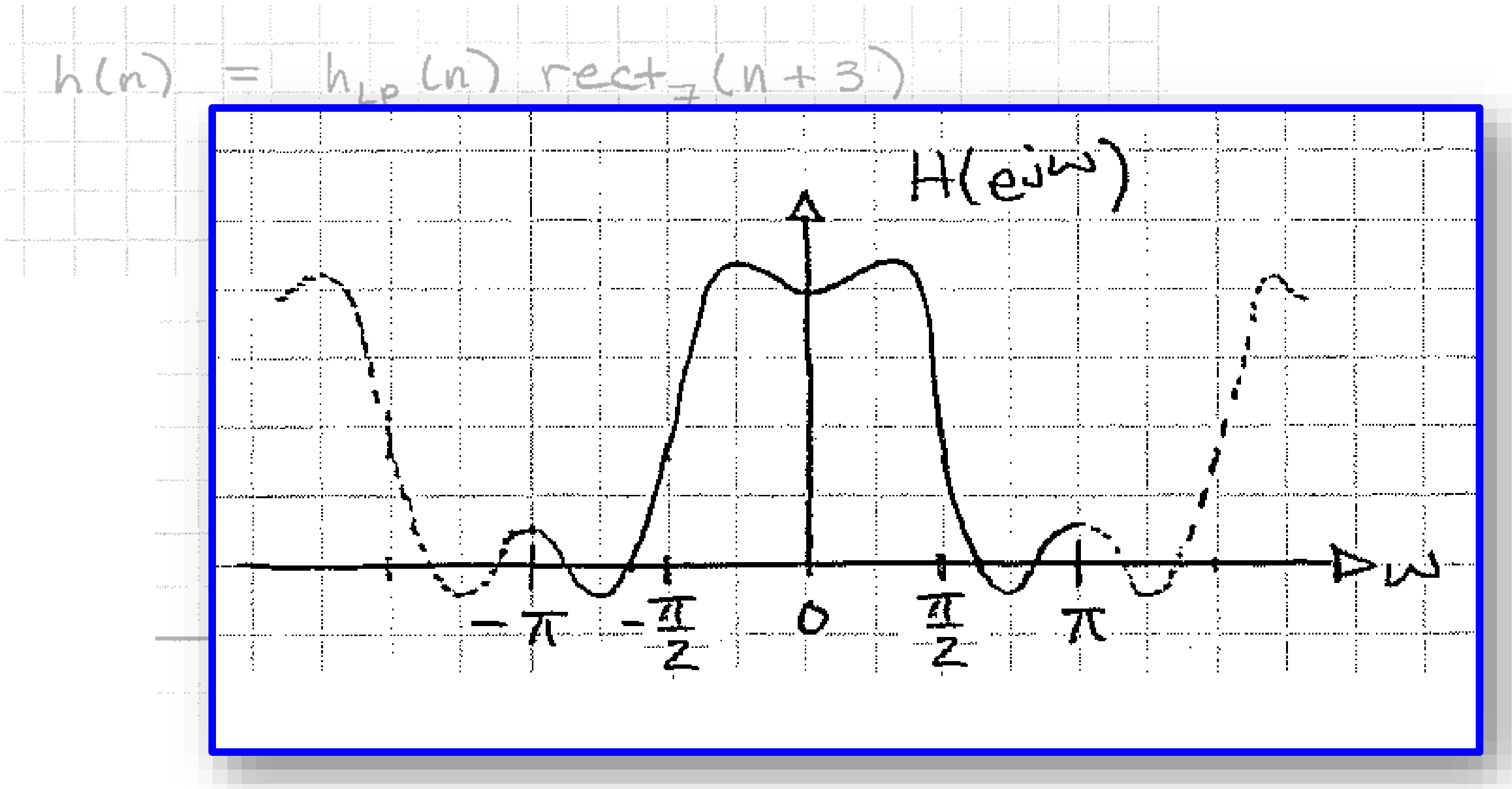


$$H(e^{j\omega}) = 0.5 + 0.6\cos(\omega) - 0.2\cos(3\omega)$$

Example: Longer truncated sinc filter



$$H(e^{j\omega}) = \boxed{0.5 + 0.6\cos(\omega) - 0.2\cos(3\omega)}$$

Example: Longer truncated sinc filter

$$h(n) = \frac{\omega_c}{\pi} \operatorname{sinc}(\omega_c n)\, \mathrm{rect}_7(n+3)$$

$$H(e^{j\omega}) = \frac{1}{2\pi} H_{LP}(e^{j\omega}) \circledast \frac{\sin(\frac{3}{2}\omega)}{\sin(\frac{1}{2}\omega)}$$

Example: Longer truncated sinc filter

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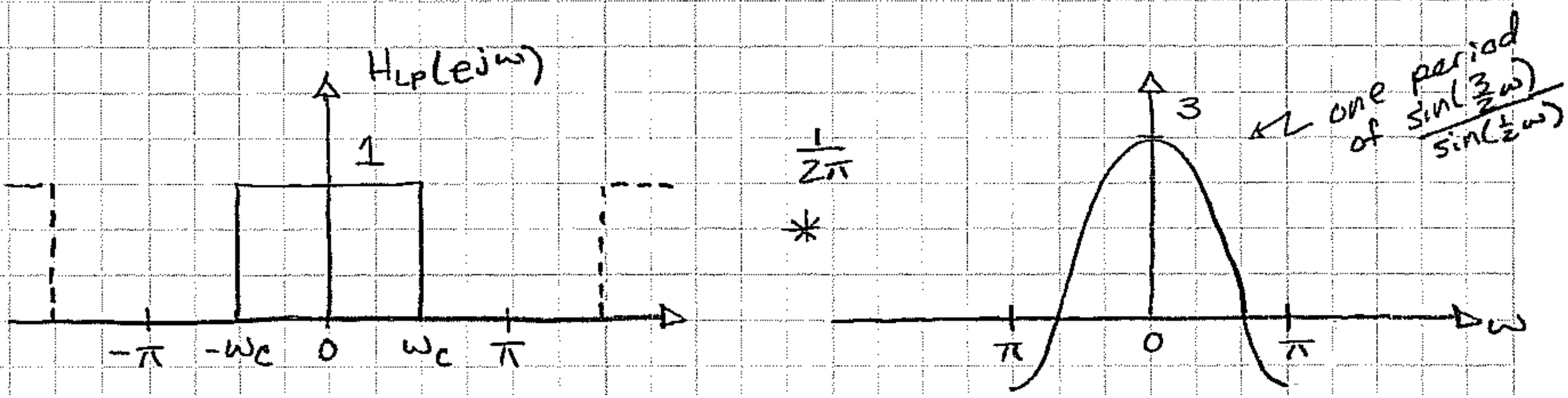

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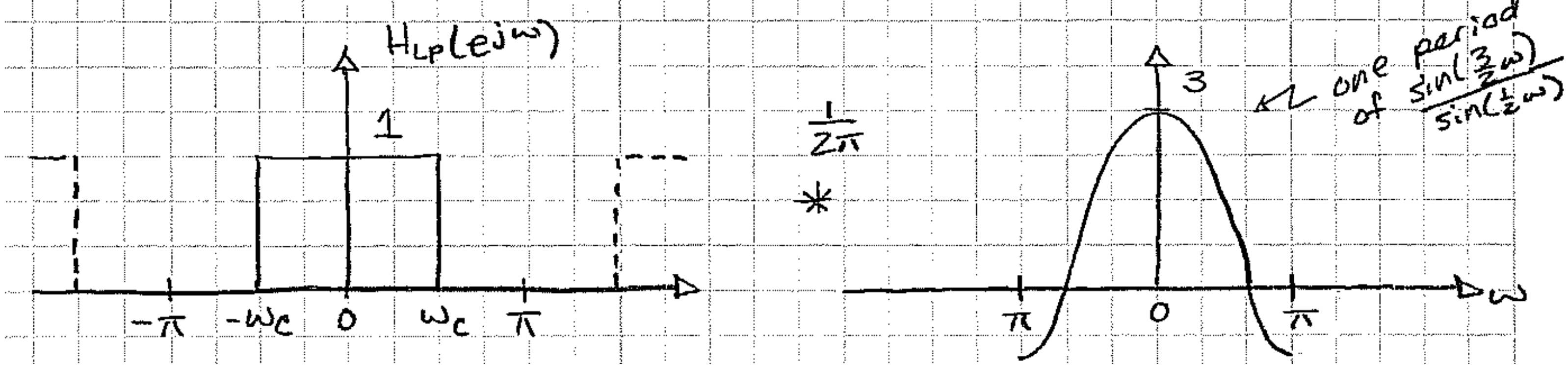
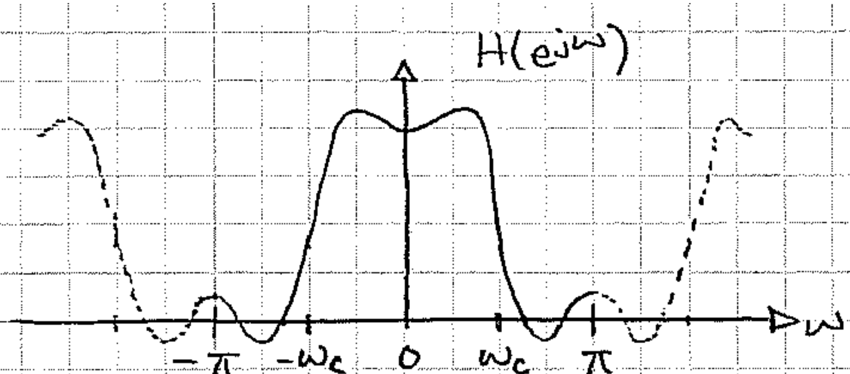
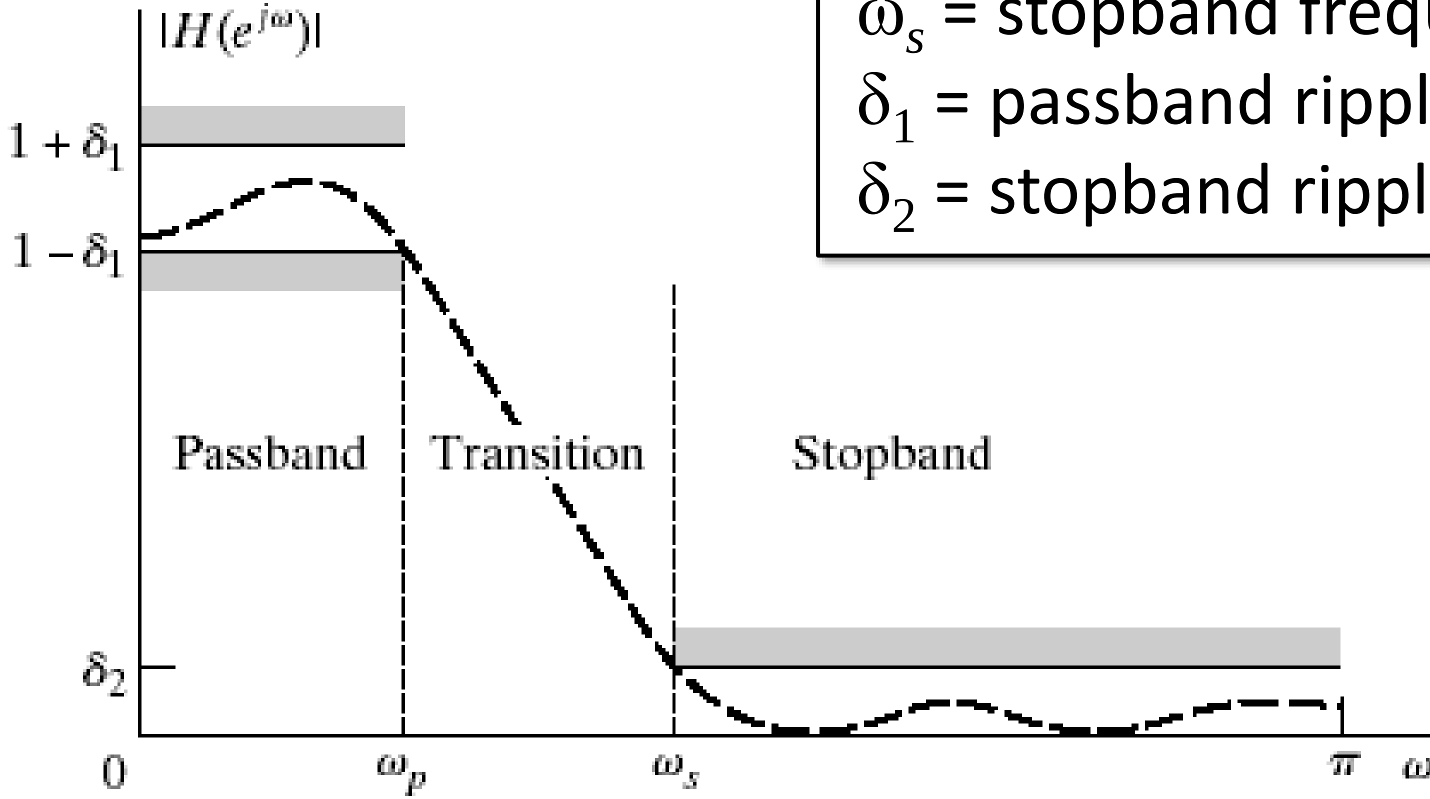




Non-ideal behavior

ω_p = passband frequency
ω_s = stopband frequency
δ_1 = passband ripple
δ_2 = stopband ripple



Techniques for FIR Filter Design

1. **Windowing Method**

   Idea: Window an ideal filter's impulse response

2. Frequency Sampling Method

   Idea: Take samples of an ideal filter's DTFT and then perform an inverse DFT

3. Computer-Based Optimization

   Idea: Iteratively adjust the filter coefficients to achieve an "optimal" frequency response

General Windowing Approach

- Goal: Design an FIR filter $h(n) \leftrightarrow H(e^{j\omega})$ with M+1 coefficients that **approximates** a **desired** freq. response

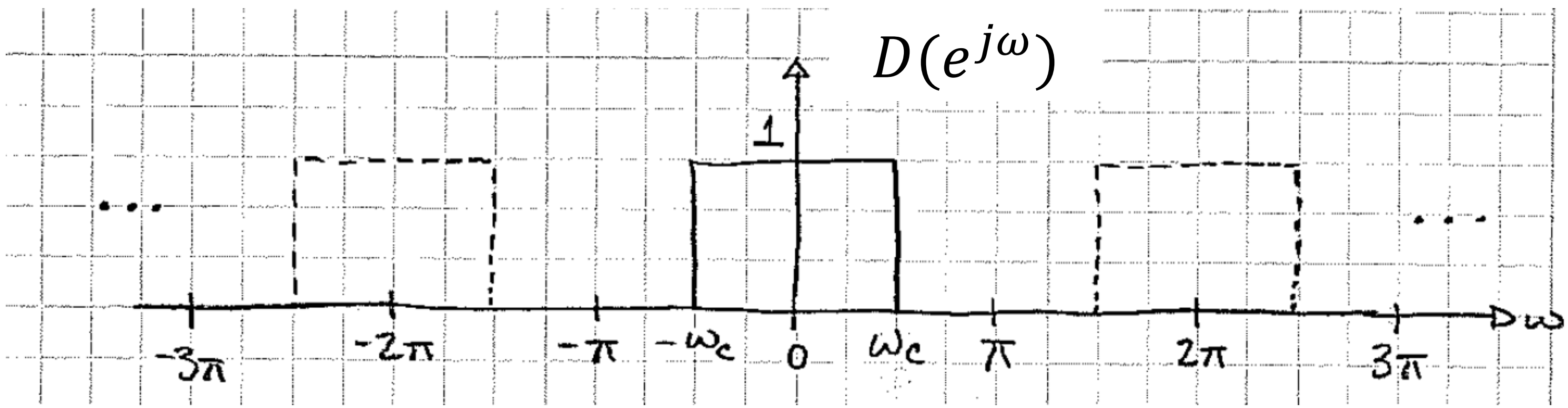
$$D(e^{j\omega}) = |D(e^{j\omega})| \angle D(e^{j\omega})$$

- $d(n)$ can be directly computed by

$$d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} D(e^{j\omega}) e^{j\omega n} d\omega$$

which typically has infinite duration, or is longer than M+1

Ideal lowpass filter



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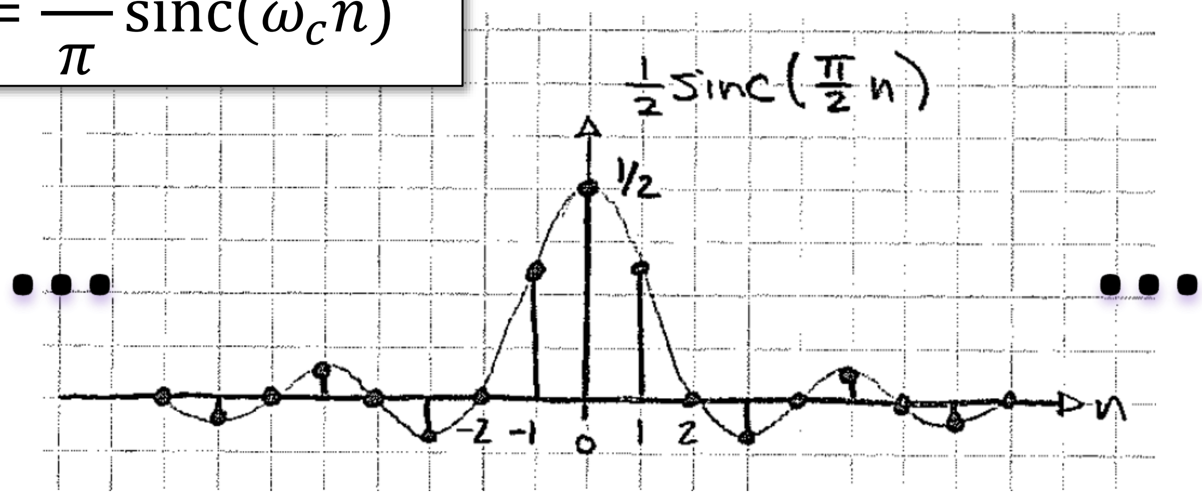

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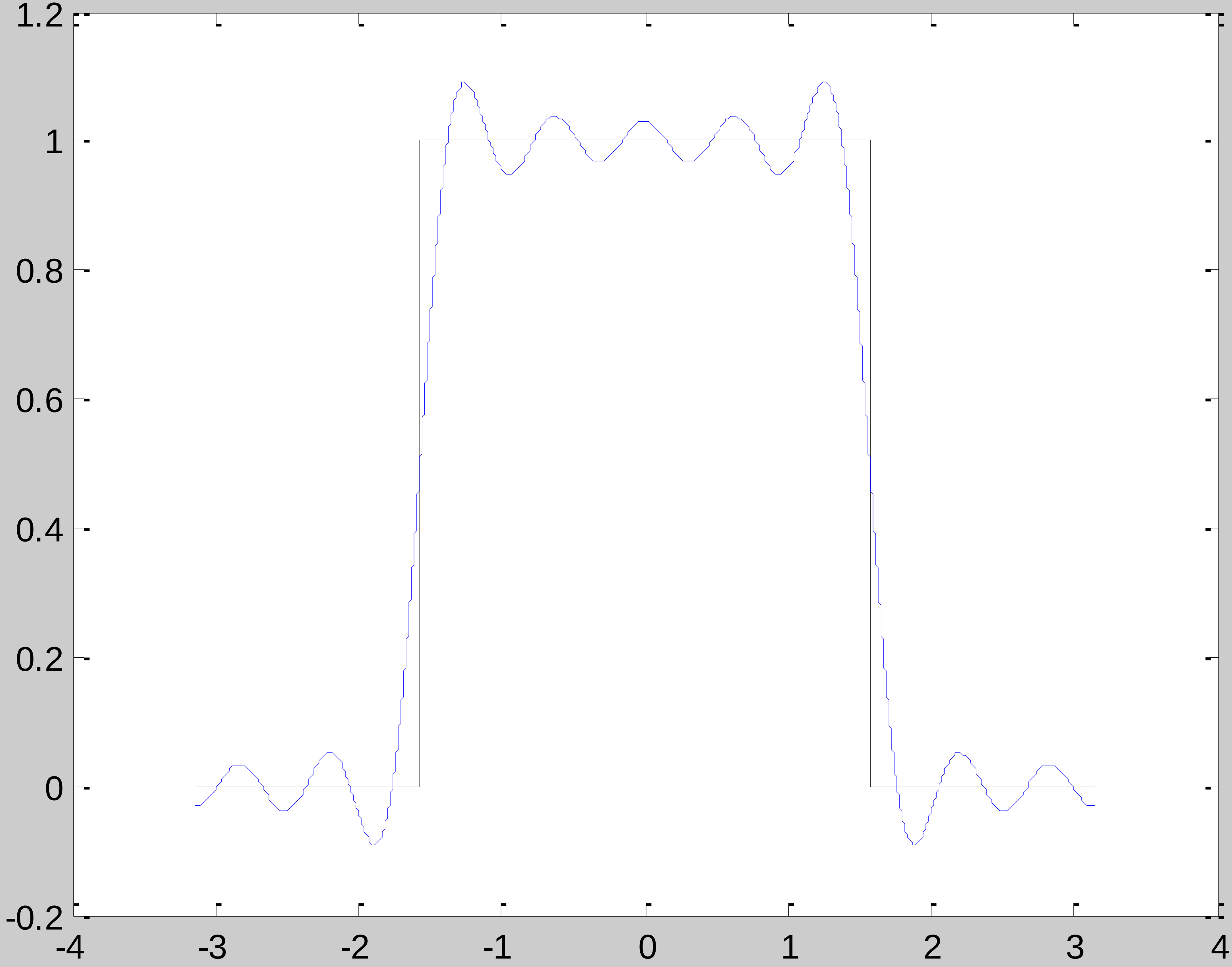
- The windowing approach simply limit the filter length to M+1 by applying a window to $d(n)$.
- Typical window types:

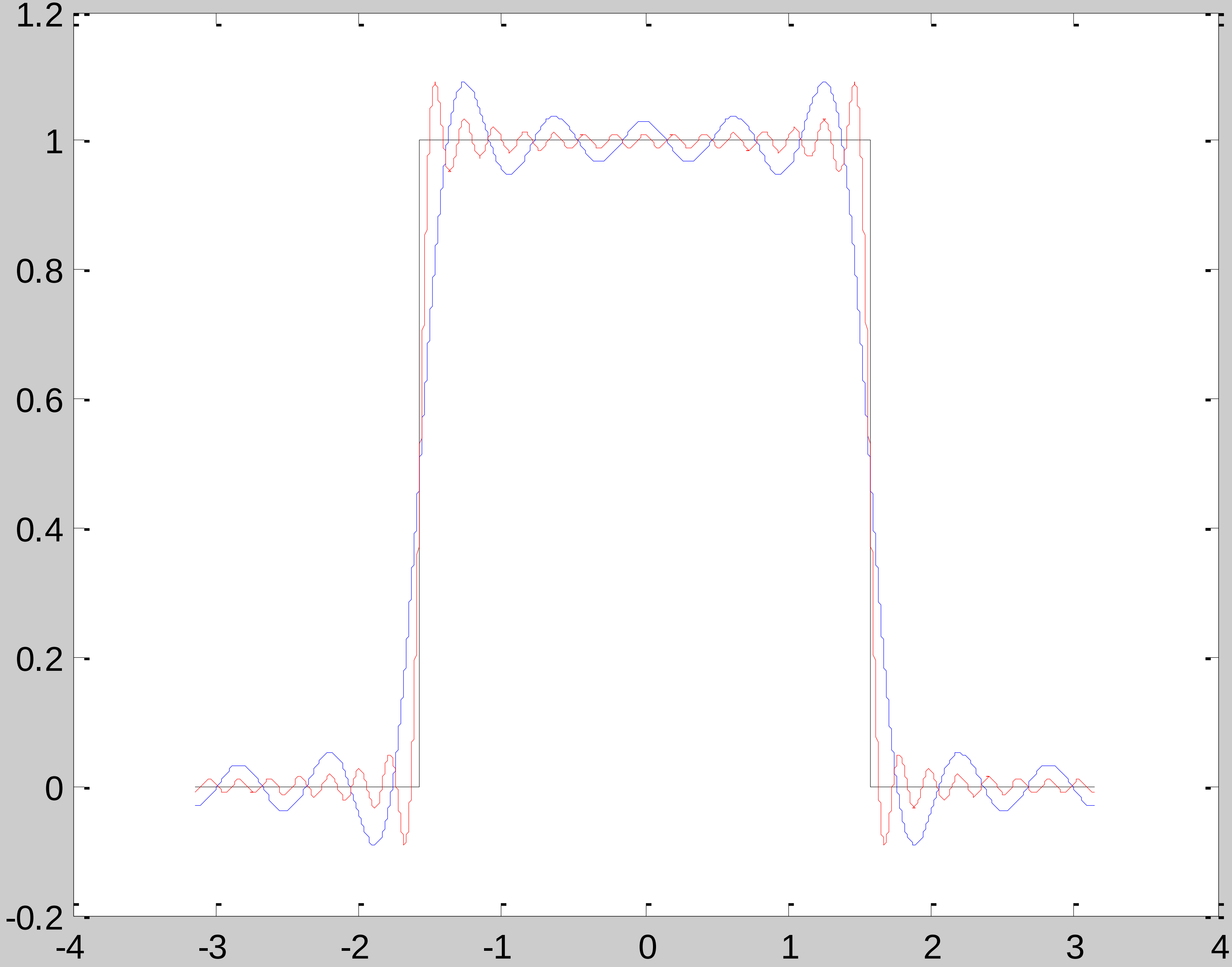
(1) Rectangular (2) Triangular (3) Hamming
(4) Hanning (5) Blackman (6) Kaiser

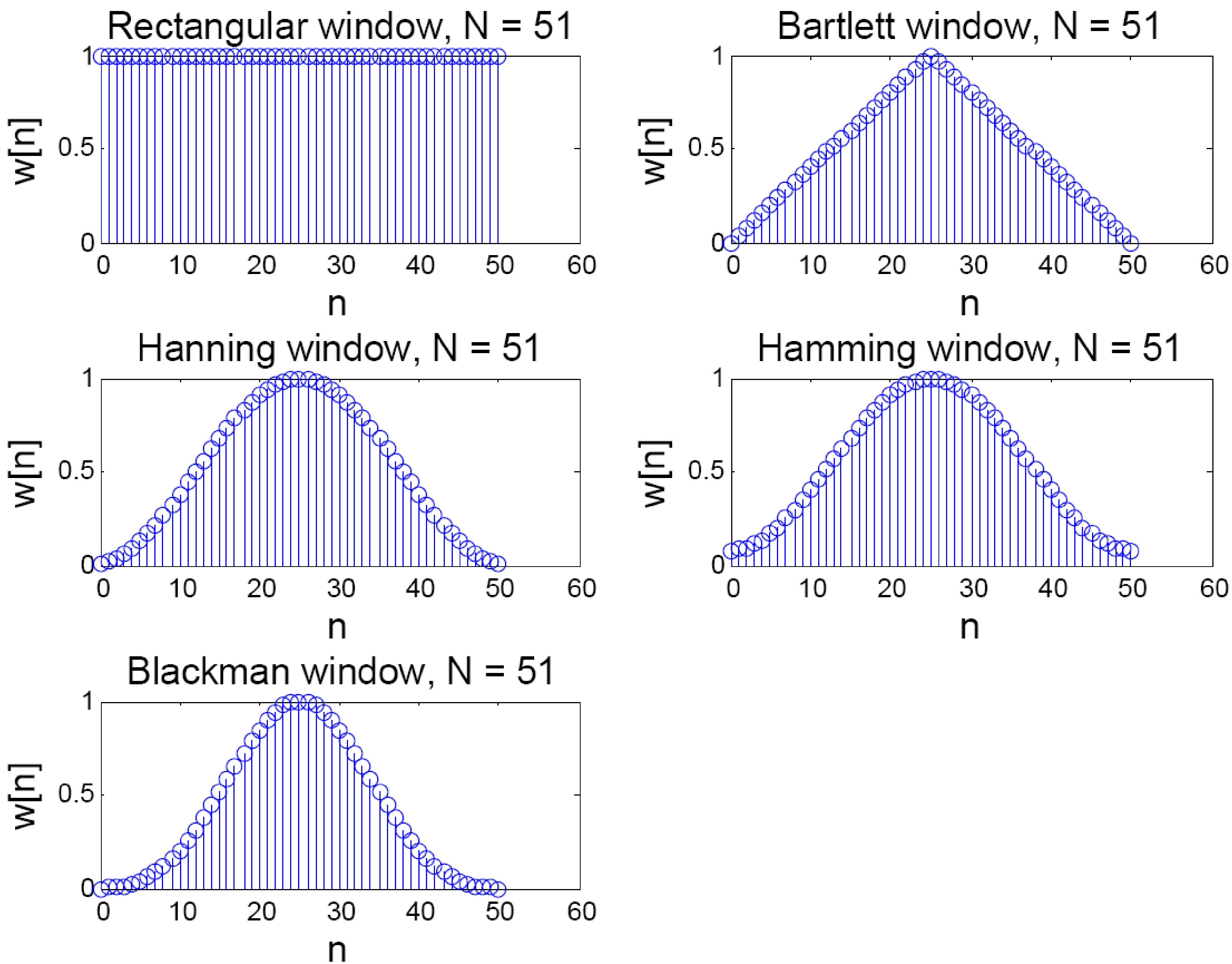
Truncated sinc filter

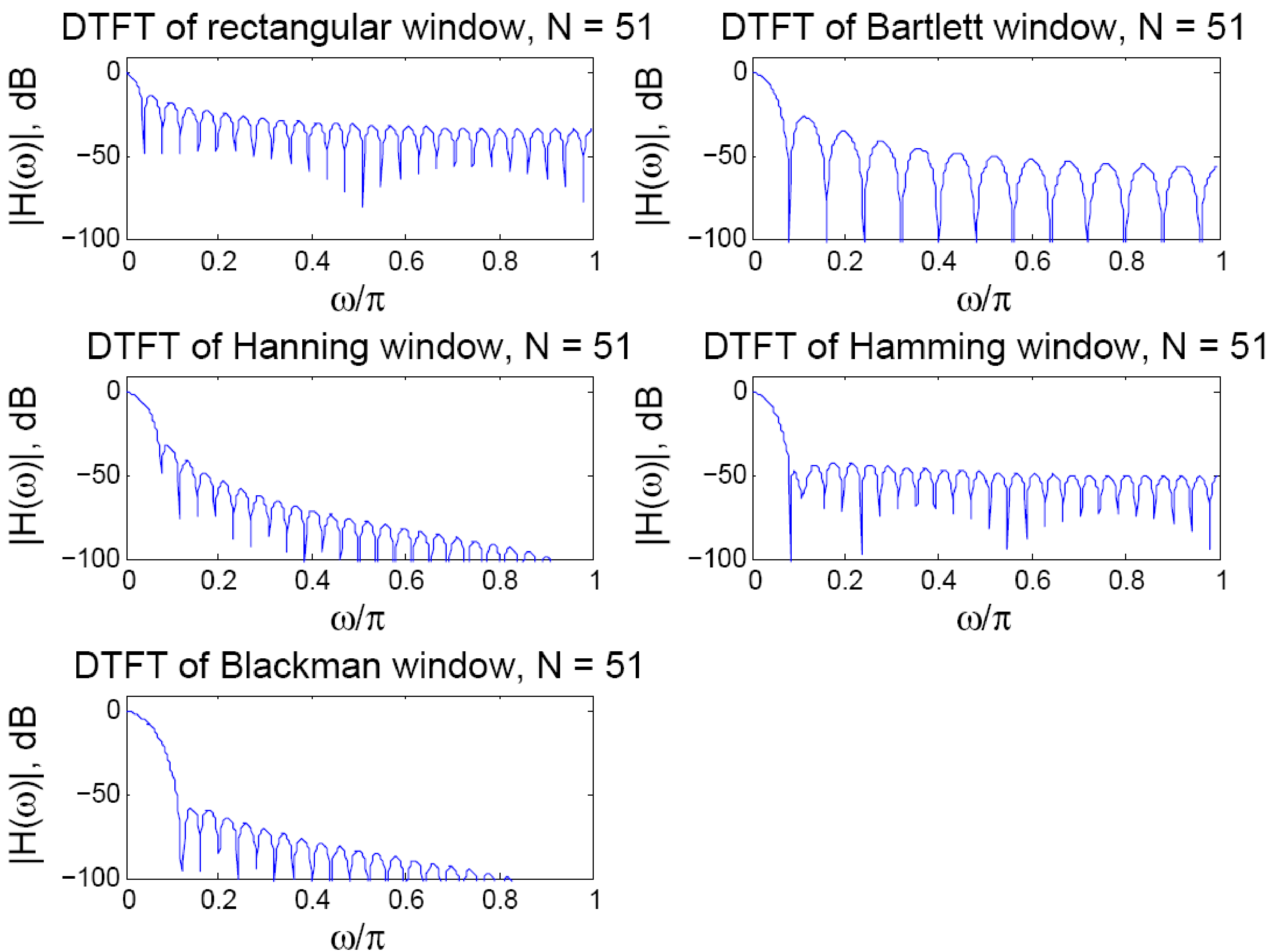
```
// create the windowed sinc filter
N = 10;
n = -N:N; // length 21
w_c = 0.5*%pi;
h = w_c/%pi * sinc(w_c*n);

// plot the filter's frequency response
w = linspace(-%pi, %pi, 1000);
H = dsp_dtft(h, n, w);
scf(1); plot(w/%pi, H);
scf(2); plot(w/%pi, 20*log10(H));
```


1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-4
-3
-2
-1
0
1
2
3
4


1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-4
-3
-2
-1
0
1
2
3
4


Rectangular window, N = 51
Bartlett window, N = 51
Hanning window, N = 51
Hamming window, N = 51
Blackman window, N = 51
w[n]
n
0
0.5
1
0
10
20
30
40
50
60


DTFT of rectangular window, N = 51
DTFT of Bartlett window, N = 51
DTFT of Hanning window, N = 51
DTFT of Hamming window, N = 51
DTFT of Blackman window, N = 51
|H(ω)|, dB
ω/π
0
−50
−100
0
0.2
0.4
0.6
0.8
1

Comparison of Windows

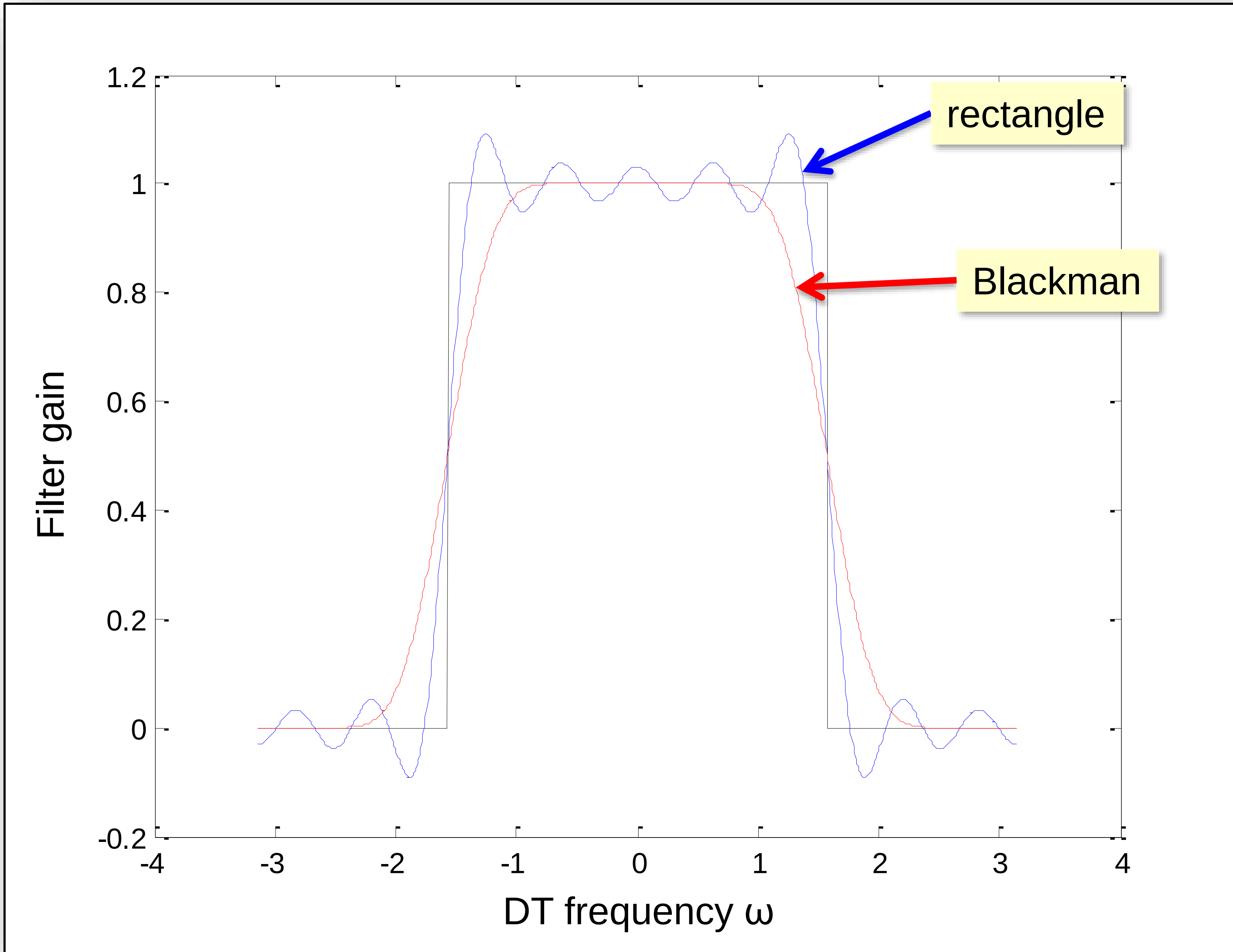
Window	Transition width	Stopband attenutation
Rectangle	$4\pi / (M+1)$	-21 dB
Hanning	$8\pi / M$	-44 dB
Hamming	$8\pi / M$	-53 dB
Blackman	$12\pi / M$	-74 dB

Observations of Windowing Approach

1. The **transition width** is determined by the **length** of the window

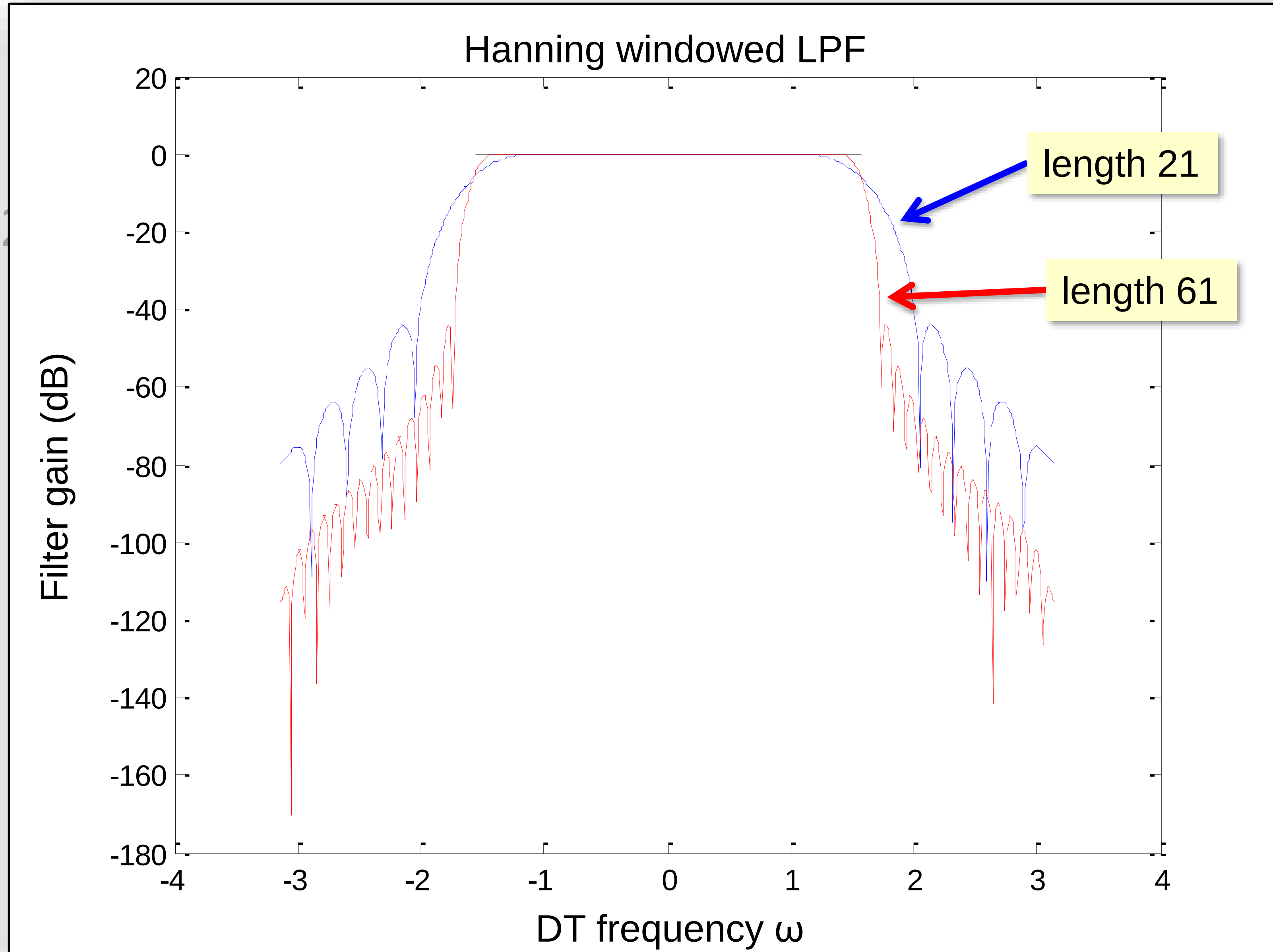
   - Approximately equal to the MLW of the window’s magnitude spectrum
   - Rectangle has the narrowest MLW of $4\pi/(M+1)$
   - Blackman window has a MLW of $12\pi/M$

1. of

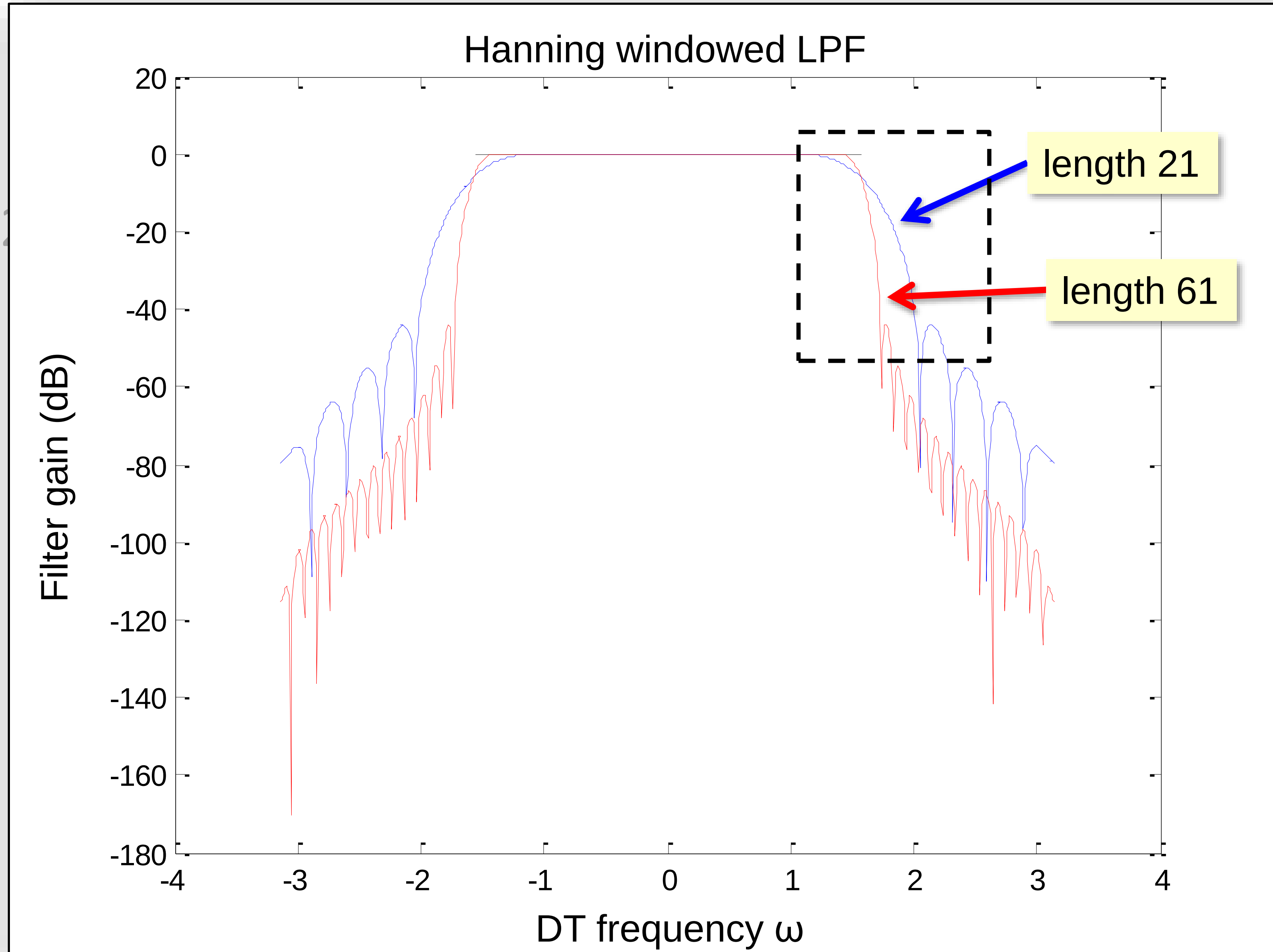

rectangle
Blackman
Filter gain
DT frequency ω
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-4
-3
-2
-1
0
1
2
3
4

Observations of Windowing Approach

2. The **stopband attenuation** is mainly determined by the **shape** of the window
   - Largely insensitive to the length of the window

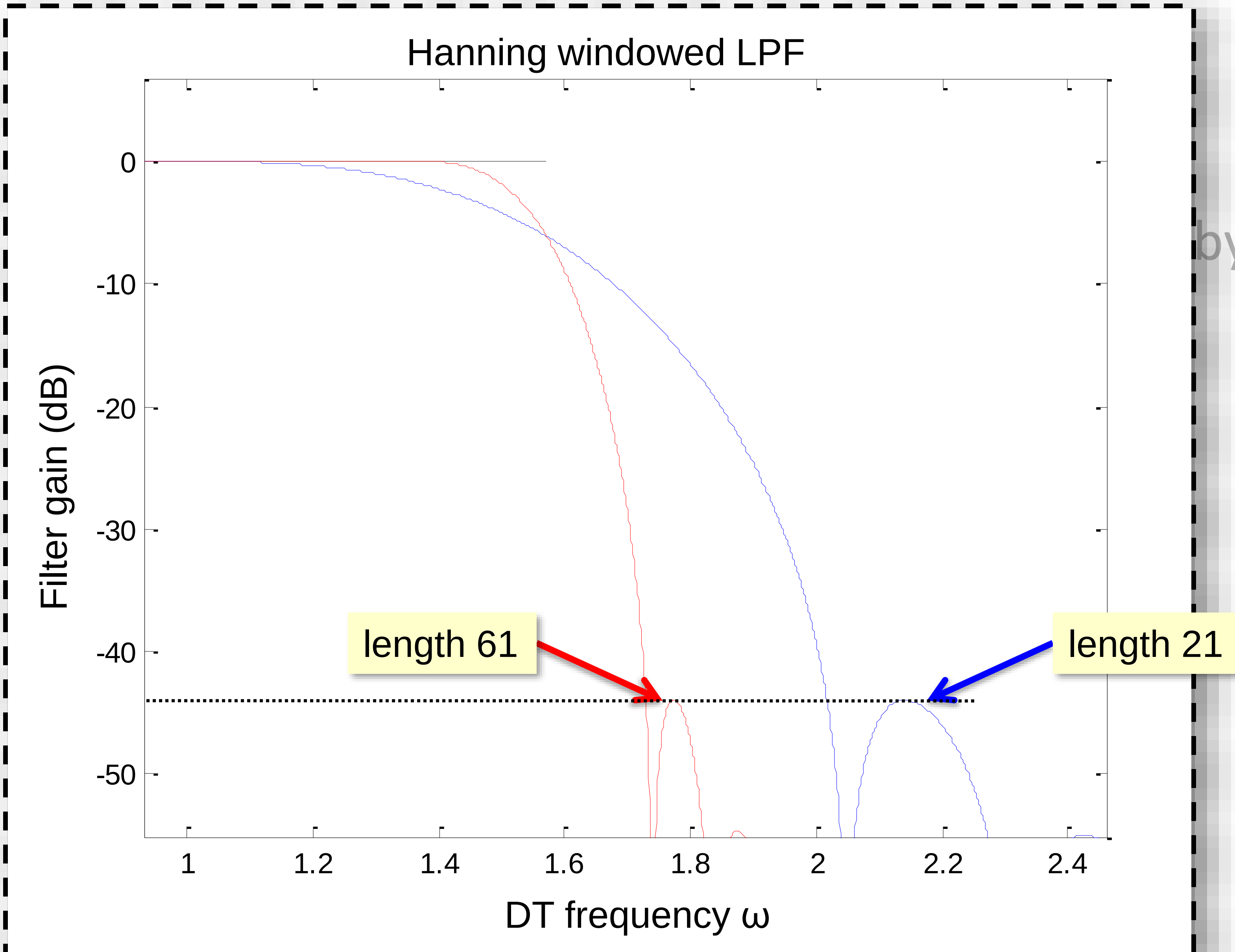


Hanning windowed LPF
Filter gain (dB)
20
0
-20
-40
-60
-80
-100
-120
-140
-160
-180
-4
-3
-2
-1
0
1
2
3
4
DT frequency ω
length 21
length 61



Hanning windowed LPF
length 21
length 61
Filter gain (dB)
DT frequency ω
20
0
-20
-40
-60
-80
-100
-120
-140
-160
-180
-4
-3
-2
-1
0
1
2
3
4

2.



Hanning windowed LPF
Filter gain (dB)
0
-10
-20
-30
-40
-50
1
1.2
1.4
1.6
1.8
2
2.2
2.4
DT frequency ω
length 61
length 21

by

Observations of Windowing Approach

2. The **stopband attenuation** is mainly determined by the **shape** of the window

    - Relatively insensitive to the length of the window
    - **Rectangle** has a (first-sidelobe) stopband attenuation of approximately **-21 dB**
    - **Blackman** window has a (first-sidelobe) stopband attenuation of approximately **-74 dB**