Physics
Quiz # 3

Date Given: April 21, 2022
Date Due: April 28, 2022

Q1. (1 point) The time derivative of a vector is a:

(a) Tensor (multi-dimensional array of numbers).
(b) Either a scalar or a vector.
(c) Scalar.
(d) Vector.

Answer:
(d) The time derivative of a vector is itself a vector having both a magnitude and a direction.

Q2. (1 point) The magnitude of the velocity vector is called the:

(a) Acceleration and it is a vector.
(b) Acceleration and it is a scalar.
(c) Speed and it is a scalar.
(d) Speed and it is a vector.

Answer:
(c) The magnitude of the velocity vector is called the speed and it is a scalar.

Q3. (2 points) For a particle moving in 3D along a helix curve given as $\boldsymbol{p}(t) = \alpha\cos\omega t\boldsymbol{i} + \alpha\sin\omega t\boldsymbol{j} + \gamma t\boldsymbol{k}$, where α, γ, ω are some constants:

(a) Show that the speed of the particle is constant.
(b) Find out the angle between the velocity and acceleration vectors.

Answer:
By direct differentiation one obtains

(a) $\boldsymbol{v} = -\alpha\omega\sin\omega t\boldsymbol{i} + \alpha\omega\cos\omega t\boldsymbol{j} + \gamma\boldsymbol{k}$. Therefore $v = |\boldsymbol{v}| = \sqrt{\alpha^2\omega^2 + \gamma^2} = \text{const.}$
(b) $\boldsymbol{a} = -\alpha\omega^2\cos\omega t\boldsymbol{i} - \alpha\omega^2\sin\omega t\boldsymbol{j}$. Thus $\boldsymbol{v}\cdot\boldsymbol{a} = 0$, and therefore vectors $\boldsymbol{v}$ and $\boldsymbol{a}$ are perpendicular.

Q4. (2 points) A particle which moves in two-dimensional motion has coordinates given in millimeters by $x = t^2 - 4t + 20$ and $y = 3\sin 2t$, where the time t is in seconds. Determine the magnitude of the velocity vector and the angle between the velocity and acceleration vectors at time $t = 3$s.

Answer: Here we have $x = t^2 - 4t + 20$, $v_x = \mathrm{d}x/\mathrm{d}t = 2t - 4$, $a_x = \mathrm{d}v_x/\mathrm{d}t = 2\text{mm/s}^2$, and $y = 3\sin 2t$, $v_y = \mathrm{d}y/\mathrm{d}t = 6\cos 2t$, $a_y = \mathrm{d}v_y/\mathrm{d}t = -12\sin 2t\text{mm/s}^2$.

(a) For $t = 3$s we have $v_x = 2$mm/s, $v_y = 5.76$mm/s, Therefore, $v = |\boldsymbol{v}| = \sqrt{v_x^2 + v_y^2} \approx 6.10$mm/s,
(b) For $t = 3$s we have $a_x = 2$mm/s, $a_y = 3.35$mm/s. For $\boldsymbol{v} = 2\boldsymbol{i} + 5.76\boldsymbol{j}$, and $\boldsymbol{a} = 2\boldsymbol{i} + 3.35\boldsymbol{j}$ one gets and $\theta = \arccos\frac{\boldsymbol{v}\cdot\boldsymbol{a}}{|\boldsymbol{v}|\,|\boldsymbol{a}|} \approx 11.67^{\circ}$.

Q5. (1 point) The path followed by a bouncing ping-pong ball is a:

(a) Hyperbola.
(b) Circle.
(c) Straight line.

(d) Parabola.

Answer:
(d) A stroboscopic photograph of a bouncing pig-pong ball shows that the ball follows a parabolic path. This is also easy to derive mathematically.

Q6. (3 points) The ball is kicked from point A with the initial velocity $v_A = 10\text{m/s}$. Determine a) the maximum height h it reaches, and b) the range x_C, and c) the speed when the ball strikes the ground.

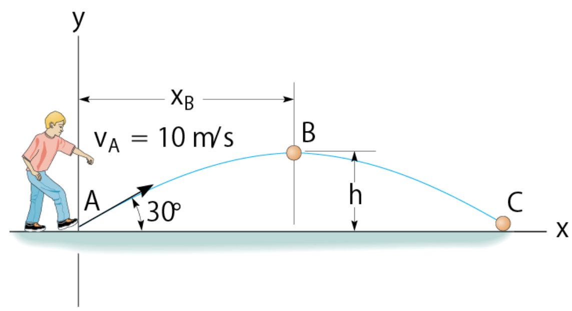

Figure 1: Illustration to Q6.

Answer:

(a) From $v_{By}^2 = v_{Ay}^2 + 2a_y(y_B - y_A)$, where $v_{By} = 0$, $v_{Ay} = v_A \sin 30^o = 5\text{m/s}$, $a_y = -9.81\text{m/s}^2$. Therefore $h = y_B - y_A = 1.27\text{m}$.

(b) From $y_C = y_A + v_{Ay}t_{AC} + \frac{1}{2}a_y t_{AC}^2$, where $y_C = y_A = 0$, we find $5 - \frac{1}{2}8.81t_{AC}^2 = 0$ and therefore $t_{AC} = 1.0194\text{s}$. Therefore $x_C = x_A + v_{Ax}t_{AC}$, where $x_A = 0$ and $v_{Ax} = v_A \cos 30^o = 8.660\text{m/s}$. Thus, $x_C = 8.83\text{m}$.

(c) Since $v_{Cy} = v_{Ay} + a_y t_{AC} = -5\text{m/s}$, and $v_{Cx} = v_{Ax} = 8.660\text{m/s}$, we have $v_C = \sqrt{v_{Cx}^2 + v_{Cy}^2} = 10\text{m/s}$