Physics
Quiz # 5

Date Given: May 12, 2022
Date Due: May 19, 2022

Q1. (1 point) In the case of uniform circular motion the magnitude of the acceleration is constant.

(a) True

(b) False

Answer:

(a) In the case of uniform circular motion the tangential component of acceleration is zero, and the normal component of the acceleration is calculated as the square of the velocity of the particle divided by the radius of the circle.

Q2. (1 point) In the case of uniform circular motion the velocity and acceleration vectors are

(a) parallel

(b) perpendicular

(c) neither from the above

Answer:

(b) In the case of uniform circular motion the tangential component of acceleration is zero, and the normal component of the velocity is zero too. Hence the velocity and acceleration vectors are perpendicular.

Q3. (2 points) Find an equation in polar coordinates that has the same graph as the given equation in rectangular coordinates.

(a) $(x^2 + y^2 - 2ax)^2 = 4a^2(x^2 + y^2)$

(b) $\sqrt{(x^2 + y^2)^3} = 2axy$

Answer:

(a) $r = 2a(1 + \cos\theta)$

(b) $r = a \sin 2\theta$

Q4. (2 points) The car has a speed of 10m/s. Determine the angular velocity $\dot{\theta}$ of the radial line OA at this instant.

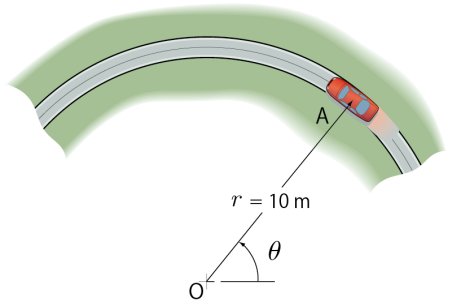

Figure 1: Illustration to Q4.

Answer: The speed $v = \sqrt{\dot{r}^2 + (r\dot{\theta})^2}$. Since r is constant, $\dot{r} = 0$. Therefore $v^2 = r^2\dot{\theta}^2$ and

$$\dot{\theta} = 1\text{rad/s}.$$

Q5. (2 points) A particle is moving along a circular path having a radius of 1m such that its position as a function of time is given by $\theta = \cos 2t$, where θ is in radians and t is in seconds. Determine the magnitude of the acceleration of the particle when $\theta = \frac{1}{2}$rad.

Answer: The the magnitude of the acceleration is $a = \sqrt{(\ddot{r} - r\dot{\theta}^2)^2 + (r\ddot{\theta} + 2\dot{r}\dot{\theta})^2}$. Since r is constant, $\dot{r} = 0$ and $\ddot{r} = 0$. Next, for $\theta = 1/2$ we have $\cos 2t = 1/2$ and therefore $2t = \pi/3$, that is $t = \pi/6$. Since, $\dot{\theta}(t) = -2\sin 2t$ and $\ddot{\theta}(t) = -4\cos 2t$, for $2t = \pi/3$ we have $\dot{\theta} = -\sqrt{3}$ and $\ddot{\theta} = -2$. Finally

$$a = \sqrt{r^2\dot{\theta}^4 + r^2\ddot{\theta}^2} = r\sqrt{\dot{\theta}^4 + \ddot{\theta}^2} = \sqrt{13} \approx 3.605\text{m/s}.$$