Physics
Quiz # 11

Date Given: June 23, 2022
Date Due: June 30, 2022

Q1. (1 point) In a conservative (potential) force field the work done against the force

(a) Is independent of the particular path followed in reaching the new position.
(b) Depends on the path the particle followed in reaching the new position.
(c) Is independent of the position of the particle.
(d) Is independent of the velocity of the particle.

Answer:
$\boxed{\text{(a)}}$ In a conservative force field the work done against the force is independent of the particular path

Q2. (1 point) In a conservative force field the work done against the force along a closed path

(a) Is equal to the area of the geometric figure bounded by the path.
(b) Is equal to the value of the potential function at the start point.
(c) Is equal to the value of the potential function at the end point.
(d) Is zero.

Answer:
$\boxed{\text{(d)}}$ In a conservative force field the work done against the force along a closed path is zero.

Q3. (1 point) A force $\boldsymbol{F} = F_x(x,y,z)\boldsymbol{i} + F_y(x,y,z)\boldsymbol{j} + F_z(x,y,z)\boldsymbol{k}$ is conservative (potential) if

(a) $\frac{\partial F_z}{\partial y} = \frac{\partial F_y}{\partial z}$ and $\frac{\partial F_x}{\partial z} = \frac{\partial F_z}{\partial x}$ and $\frac{\partial F_y}{\partial x} = \frac{\partial F_x}{\partial y}$
(b) $\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} = 0$
(c) $\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} = 1$
(d) $\frac{\partial F_x}{\partial y} = \frac{\partial F_x}{\partial z}$ and $\frac{\partial F_y}{\partial x} = \frac{\partial F_y}{\partial z}$ and $\frac{\partial F_z}{\partial x} = \frac{\partial F_z}{\partial y}$

Answer: $\boxed{\text{(a)}}$ A force $\boldsymbol{F} = F_x(x,y,z)\boldsymbol{i} + F_y(x,y,z)\boldsymbol{j} + F_z(x,y,z)\boldsymbol{k}$ is conservative (potential) if $\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\boldsymbol{i} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\boldsymbol{j} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\boldsymbol{k} = \boldsymbol{0}$.

Q4. (2 points) Compute the work done by the force $\boldsymbol{F} = (2x+y)\boldsymbol{i} + (x+z^2)\boldsymbol{j} + (2yz+1)\boldsymbol{k}$, given as a function of position with $F_x = (2x+y)$, $F_y = (x+z^2)$, $F_z = (2yz+1)$, along a path consisting of straight line segments from $(0,0,0)$ to $(1,1,1)$ to $(1,1,0)$ to $(0,0,0)$.

Answer: Here we have $\frac{\partial F_z}{\partial y} = 2z$, $\frac{\partial F_y}{\partial z} = 2z$, $\frac{\partial F_x}{\partial z} = 0$, $\frac{\partial F_z}{\partial x} = 0$, $\frac{\partial F_y}{\partial x} = 1$, $\frac{\partial F_x}{\partial y} = 1$. The force is potential because

$$\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) = 0, \quad \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right) = 0, \quad \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right) = 0.$$

Since $\boldsymbol{F}$ is potential and the path from $(0,0,0)$ to $(1,1,1)$ to $(1,1,0)$ to $(0,0,0)$ is closed,
$\boxed{\text{the work done by } \boldsymbol{F} \text{ along this path is } 0.}$

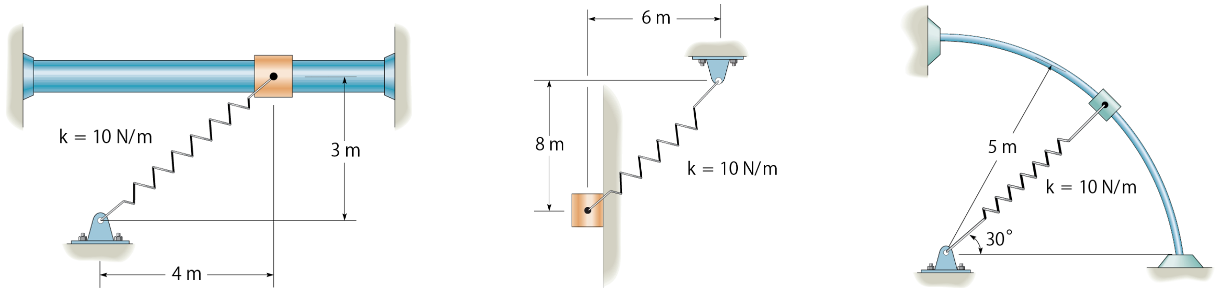

Figure 1: Illustration to Question 5.

Q5. (3 points) Determine the potential energy in the spring shown in Figure 1 (a, b, and c). The spring has an unstretched length of 4 m.

Answer:

(a) $V = \frac{1}{2}10\text{N/m}(5\text{m} - 4\text{m})^2 = \boxed{5 \text{ J.}}$

(b) $V = \frac{1}{2}10\text{N/m}(10\text{m} - 4\text{m})^2 = \boxed{180 \text{ J.}}$

(c) $V = \frac{1}{2}10\text{N/m}(5\text{m} - 4\text{m})^2 = \boxed{5 \text{ J.}}$

Q6. (2 points) The bead of mass m can slide in the vertical plane on the smooth ring of radius R. The spring of stiffness k is attached to the bead as shown in Figure 2. At the start position A the spring is unstretched. The bead is released from rest at A and slides down the ring. For given $m = 10$kg and $R = 1$m, define the stiffness k so that the bead stops at position B (reaches B with zero velocity).

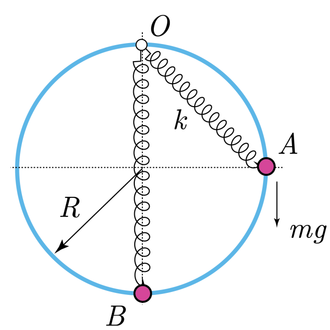

Figure 2: Illustration to Problem 6.

Answer:

- All the active forces in this problem are conservative (potential), and hence the total energy is conserved. Therefore we can write

$$T_A + V_A = T_B + V_B.$$

At point A the system is at rest and therefore $T_A = 0$. The potential energy has two sources, the gravity force and the elastic force of the spring, that is $V_A = V_{A,g} + V_{A,e}$. Let us set the reference frame (datum) at point A, with the vertical axis pointing from A to B. Then the potential energy due to gravity $V_{A,g} = 0$. Since at A the spring is unstretched, $l_0 = l_A = R\sqrt{2}$, and the potential energy due to elasticity of the spring $V_{A,e} = \frac{1}{2}k(l_A - l_0)^2 = 0$.

- At point B we have $T_B = \frac{1}{2}mv_B^2 = 0$. The potential energy $V_B = V_{B,g} + V_{B,e}$. The potential energy due to gravity $V_{B,g} = -mgR$, where $g = 9.81\,\text{m/s}^2$. The potential energy due to elasticity of the spring $V_{B,e} = \frac{1}{2}k(l_B - l_0)^2$, where $l_B = 2R$ is the length of the spring at state B.
- Now, from the energy conservation equation at A and B, we obtain

$$0 = \frac{1}{2}k(l_B - l_0)^2 - mgR \quad\Longrightarrow\quad 0 = \frac{1}{2}kR^2\left(2-\sqrt{2}\right)^2 - mgR \quad\Longrightarrow$$

$$\boxed{k = \frac{2mg}{R\left(2-\sqrt{2}\right)^2} \approx 571.769\,\text{N/m}}$$