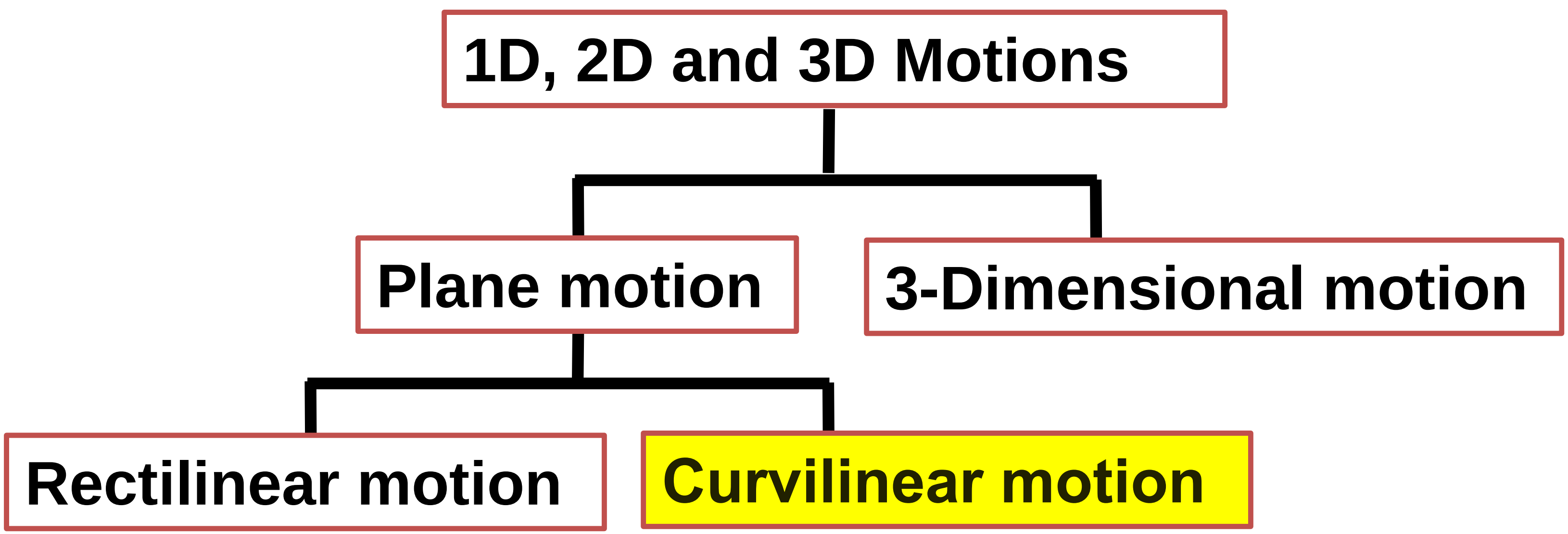
Physics

Lecture 3
Kinematics of Curvilinear Motion: Rectangular Coordinates

Description of Motion

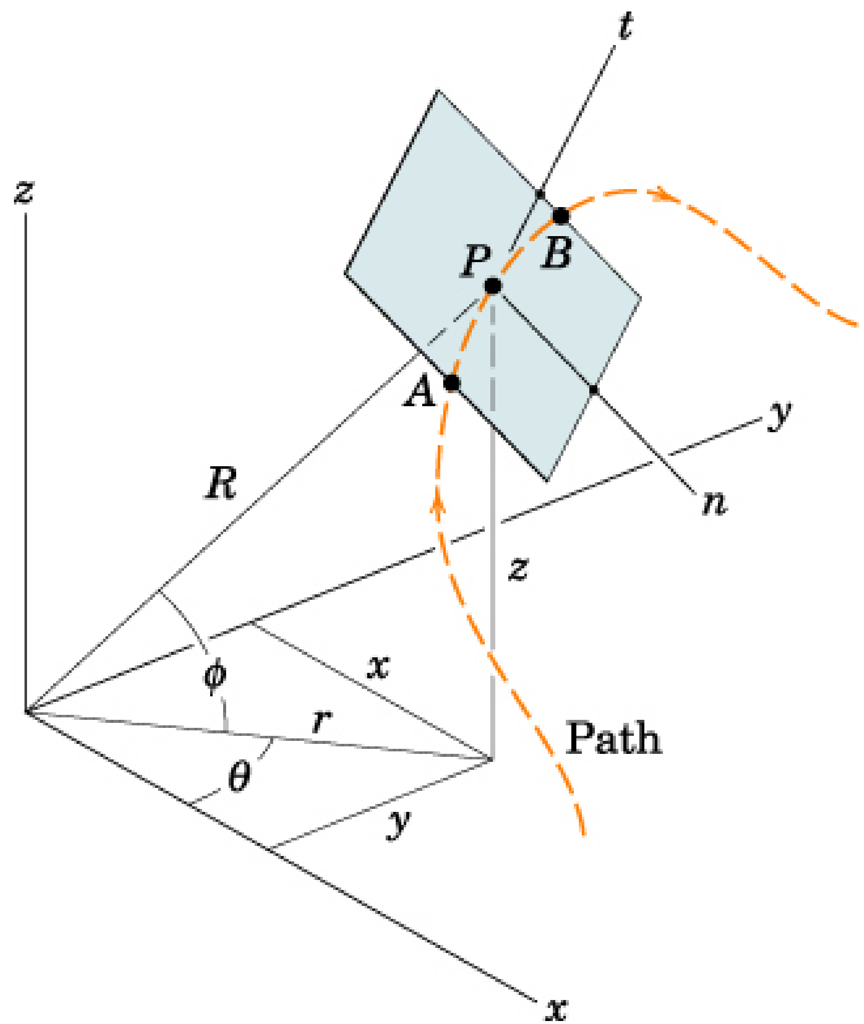


Contents

- Position, velocity and acceleration in two/three dimensions – rectangular coordinates

- Motion of projectile

Different Coordinates To Measure Positions



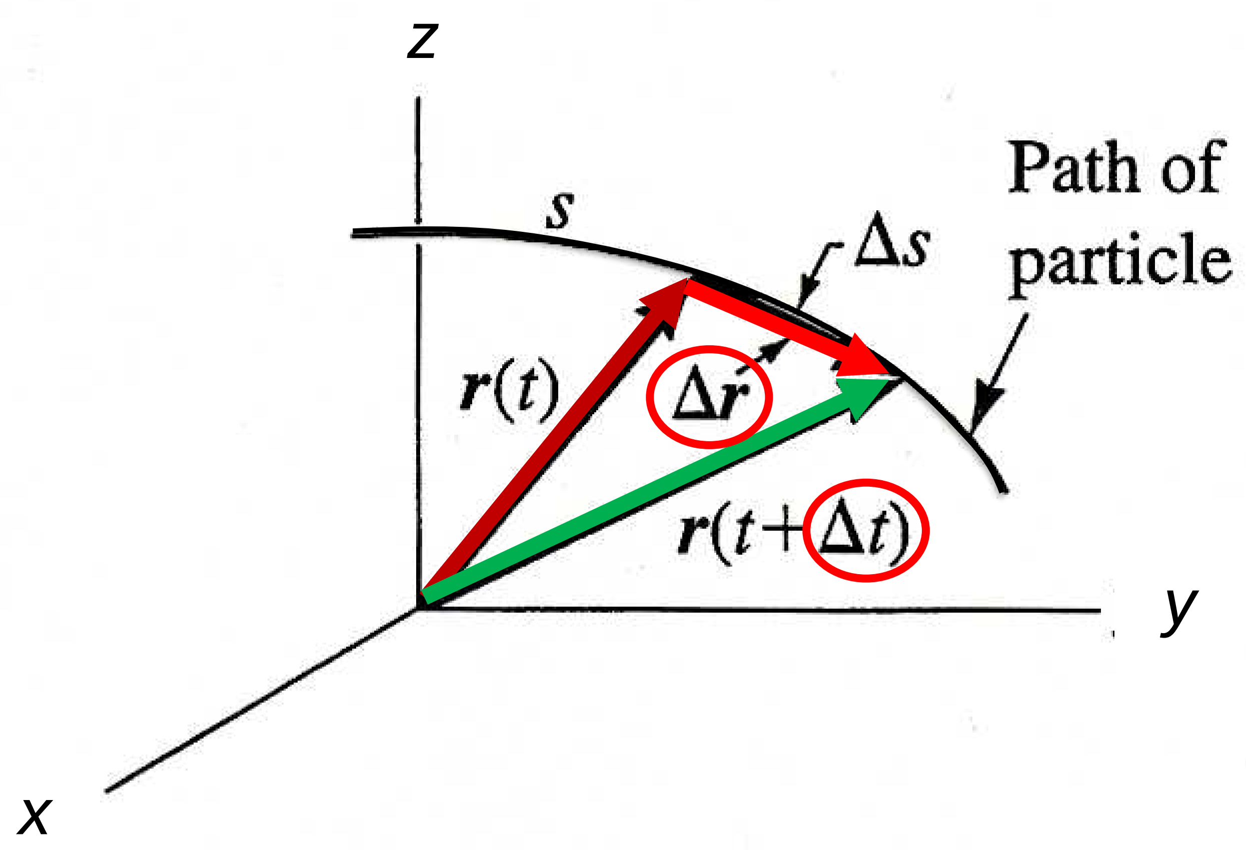
Rectangular (x, y, z)

Cylindrical (r, θ, z)

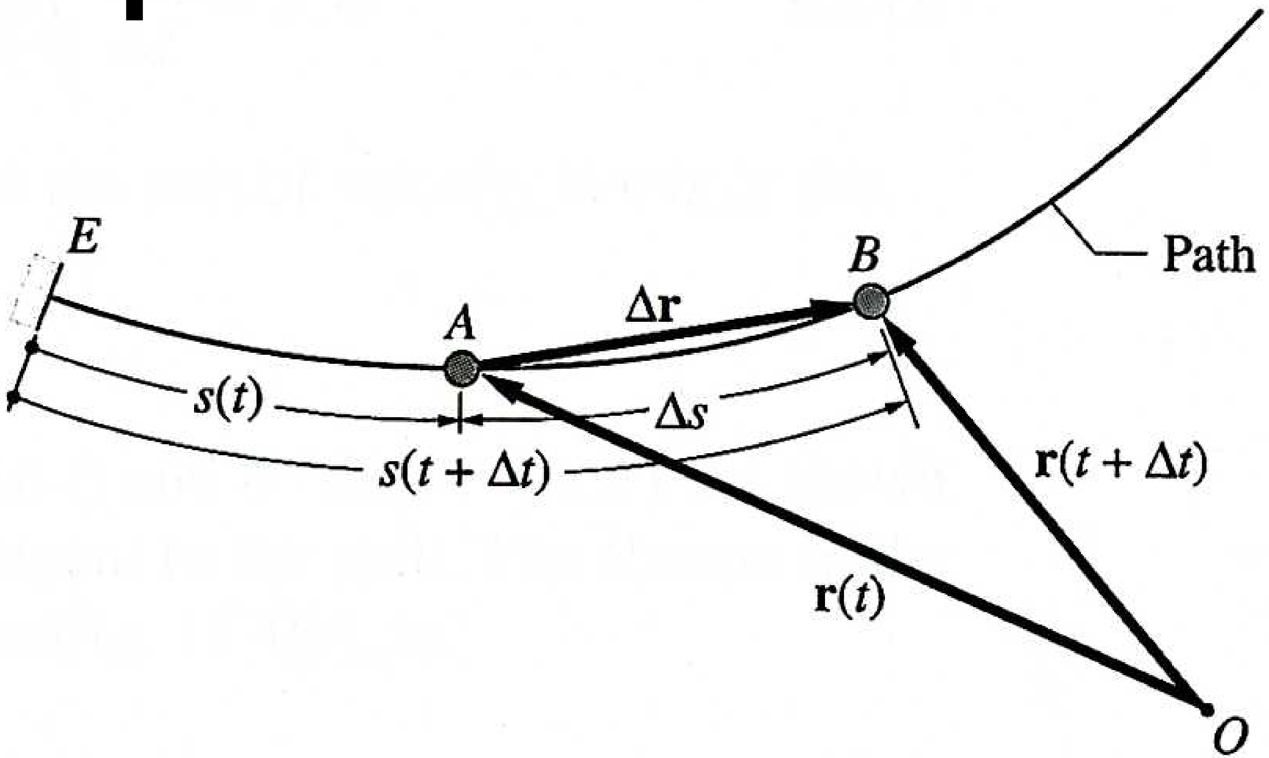
Spherical (R, θ, ϕ)

Position Vector of a Particle

Change of position vector



Displacement of a Particle



Displacement vector $\mathbf{r}(t)$

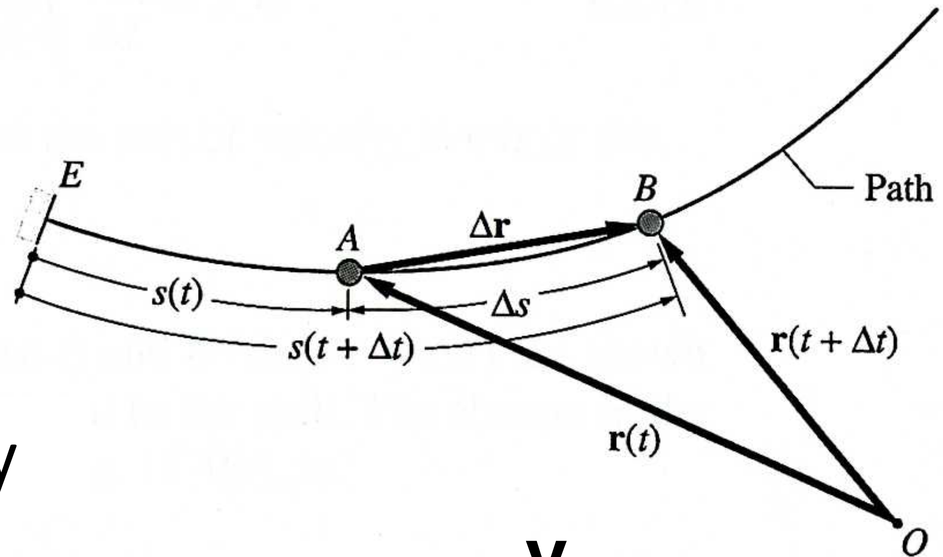
Path Coordinate $s(t)$

Change in Path Length $\Delta s = s(t+\Delta t) - s(t)$

Velocity of a Particle

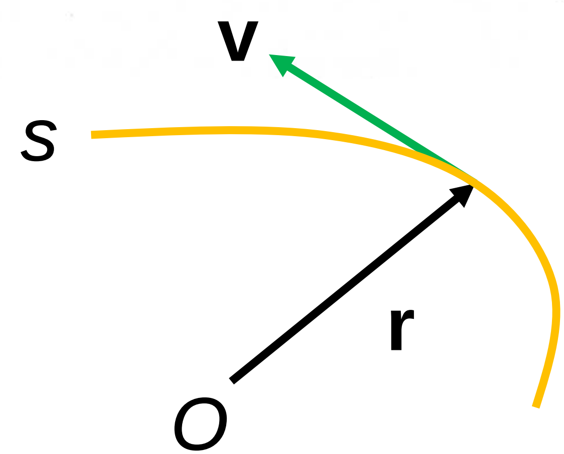
Average Velocity

$$\mathbf{v}_{average} = \frac{\Delta \mathbf{r}}{\Delta t}$$



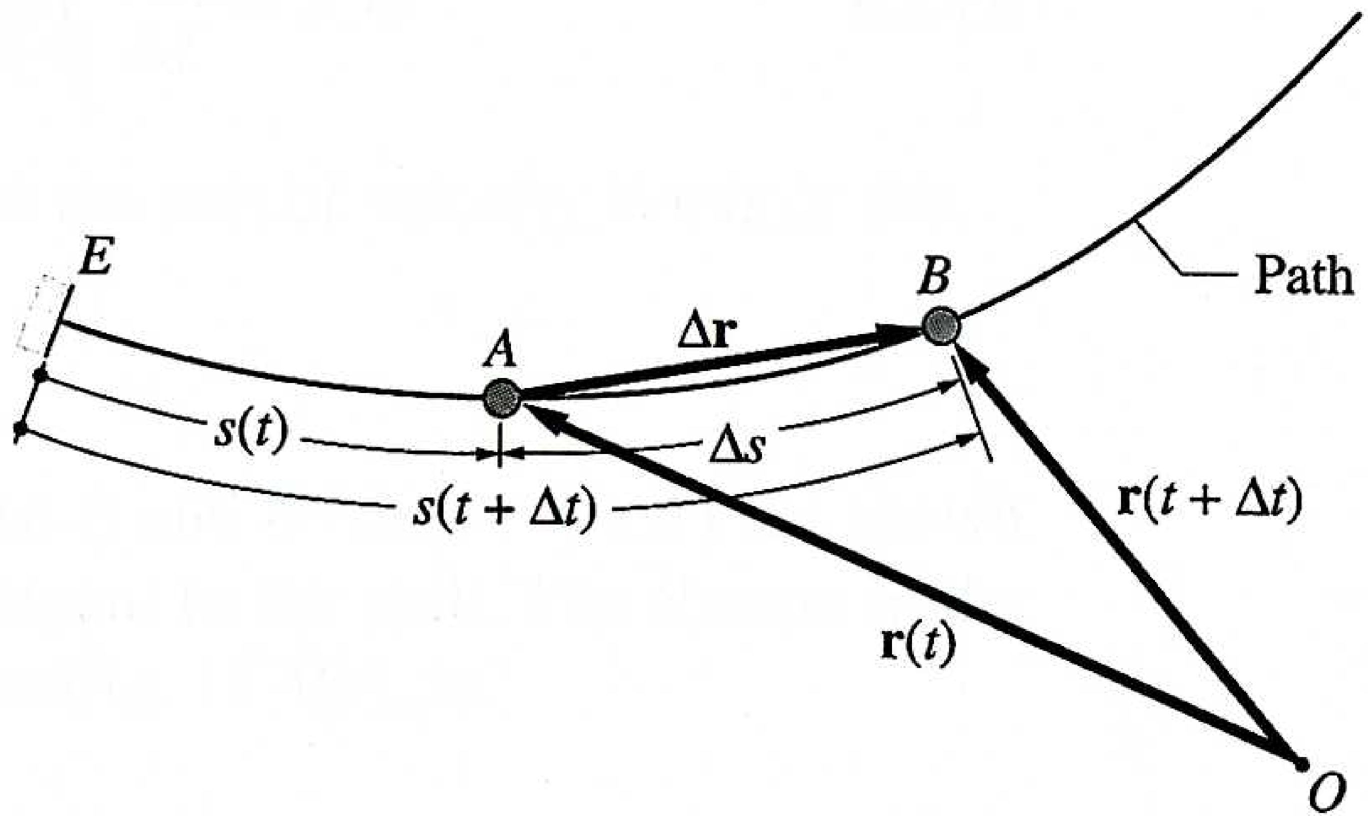
Instantaneous Velocity

$$\mathbf{v}(t) = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\mathbf{r}(t+\Delta t) - \mathbf{r}(t)}{\Delta t}$$



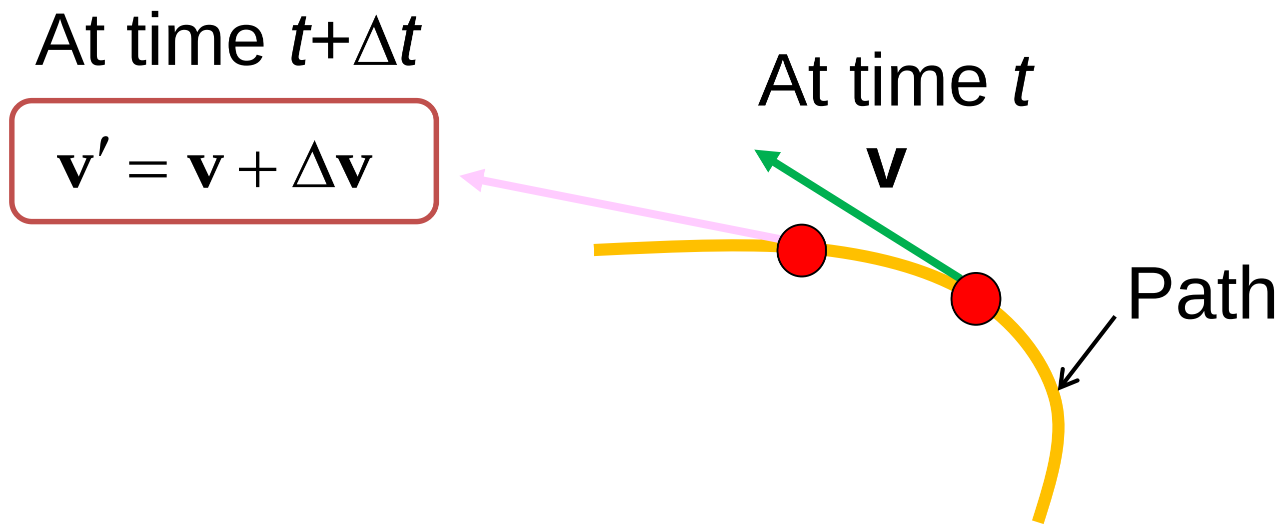
Direction of Velocity

Velocity and Speed of a Particle



Speed = Magnitude of Velocity
(the length of vector $\mathbf{v}(t)$)

Acceleration of a Particle



Average Acceleration

$$\mathbf{a}_{average} = \frac{\Delta\mathbf{v}}{\Delta t}$$

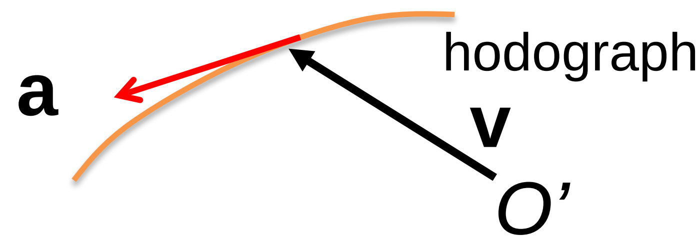
Instantaneous Acceleration

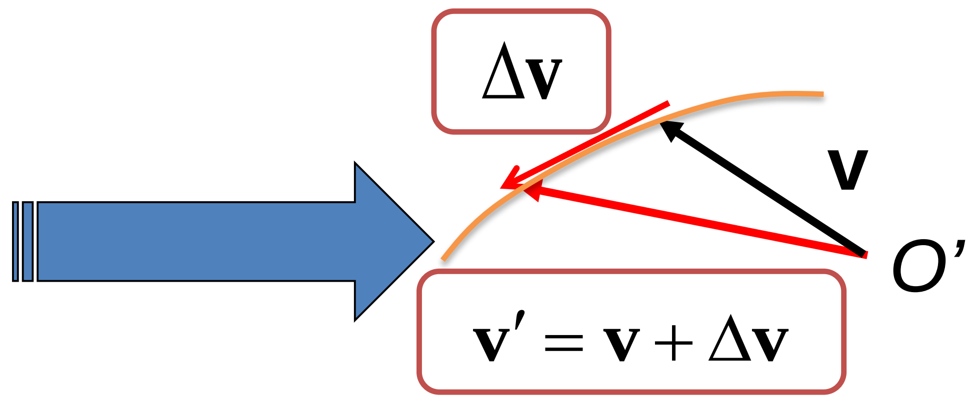
$$\mathbf{a}(t) = \lim_{\Delta t \to 0} \frac{\Delta\mathbf{v}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\mathbf{v}(t+\Delta t) - \mathbf{v}(t)}{\Delta t}$$

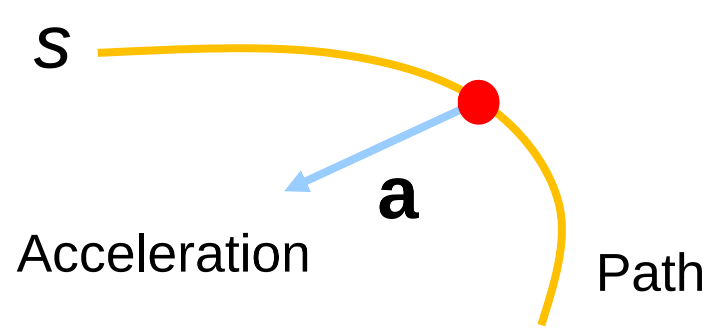
Hodographs of r and v

Direction of acceleration:

tangent to hodograph of v







but not necessarily tangent
to hodograph of r

Position, Velocity and Acceleration

Position vector

$$\boxed{\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}}$$

Time derivative of position vector

$$\boxed{\frac{d\mathbf{r}}{dt} = \frac{dx}{dt}\mathbf{i} + \frac{dy}{dt}\mathbf{j} + \frac{dy}{dt}\mathbf{k}}$$

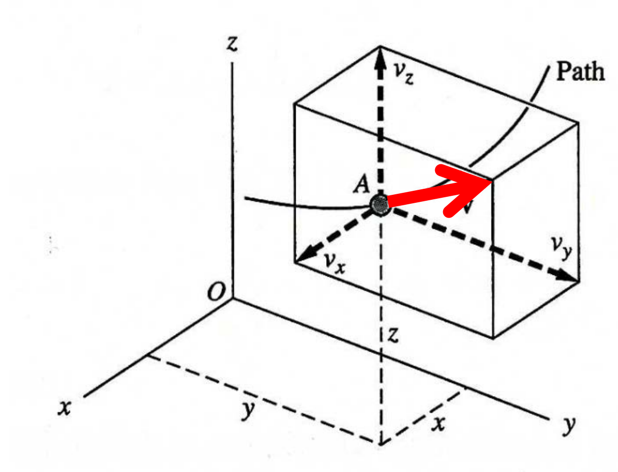
Time derivative of velocity vector

$$\boxed{\frac{d\mathbf{v}}{dt} = \frac{d^2x}{dt^2}\mathbf{i} + \frac{d^2y}{dt^2}\mathbf{j} + \frac{d^2y}{dt^2}\mathbf{k}}$$

Position, Velocity and Acceleration

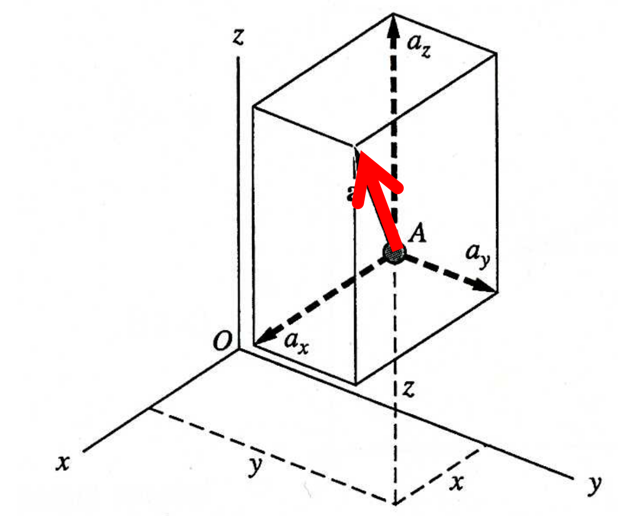
Velocity

$$\frac{d\mathbf{r}}{dt} = \mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}$$



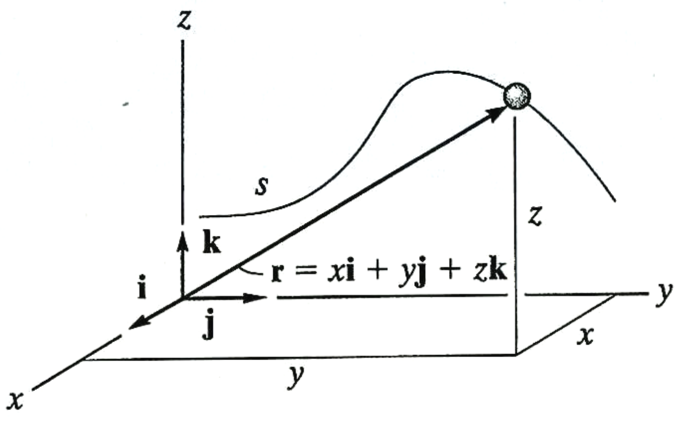
Acceleration

$$\frac{d\mathbf{v}}{dt} = \mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$$

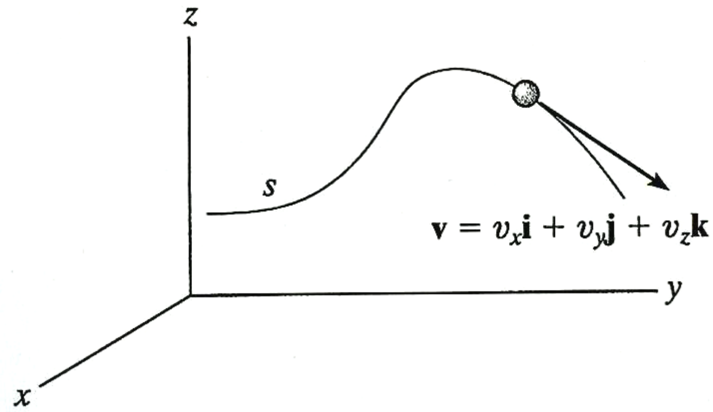


Magnitude and Direction of **r**, **v** and **a**

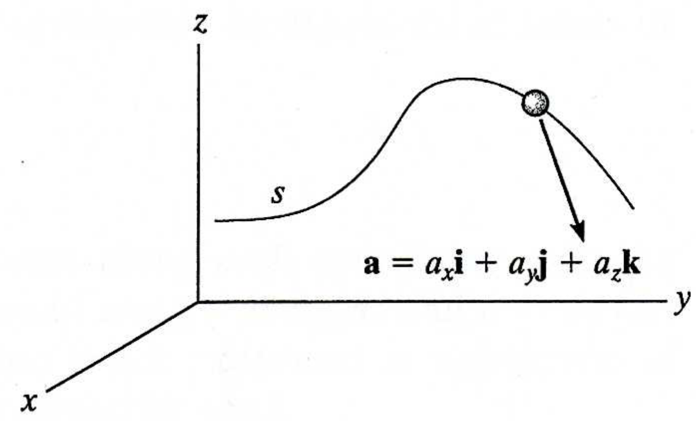
Magnitude and direction of **r**



Magnitude and direction of **v**



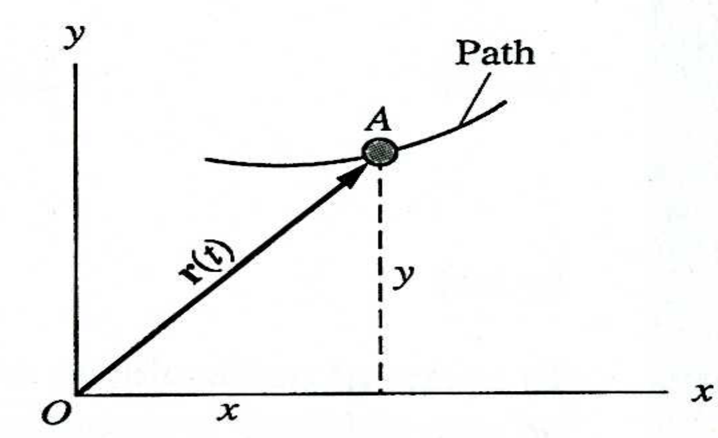
Magnitude and direction of **a**



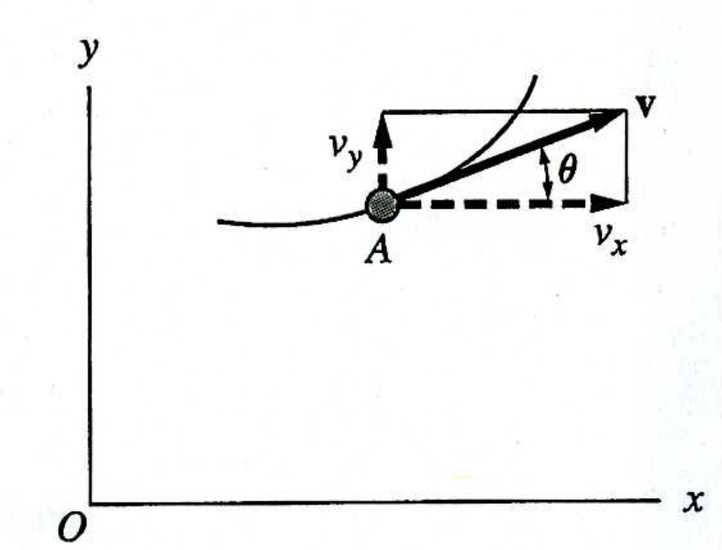
Plane Motion

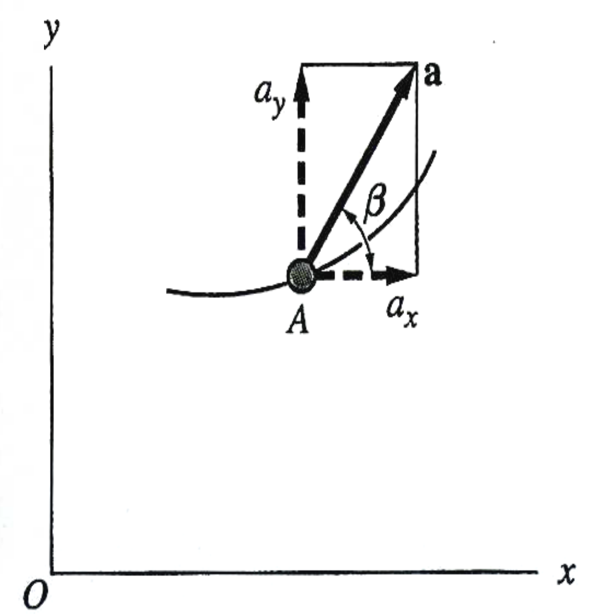
All movement occurs in or can be represented as occurring in a single plane

Position



Velocity





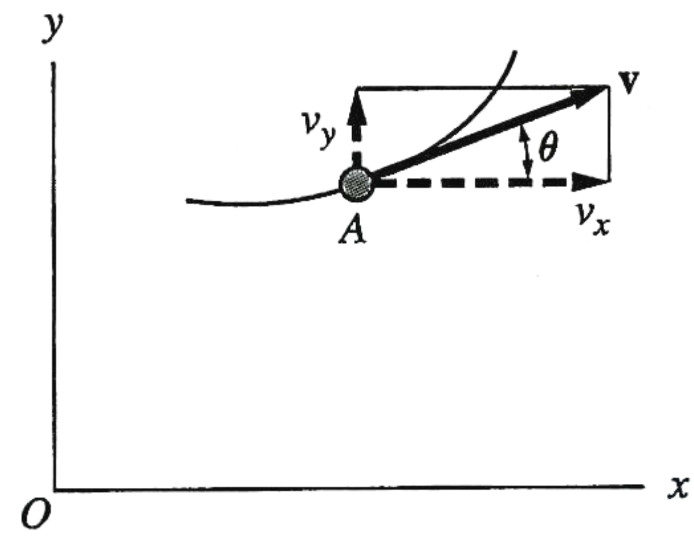
Acceleration

Direction of Velocity and Acceleration

Rectangular Components of Velocity

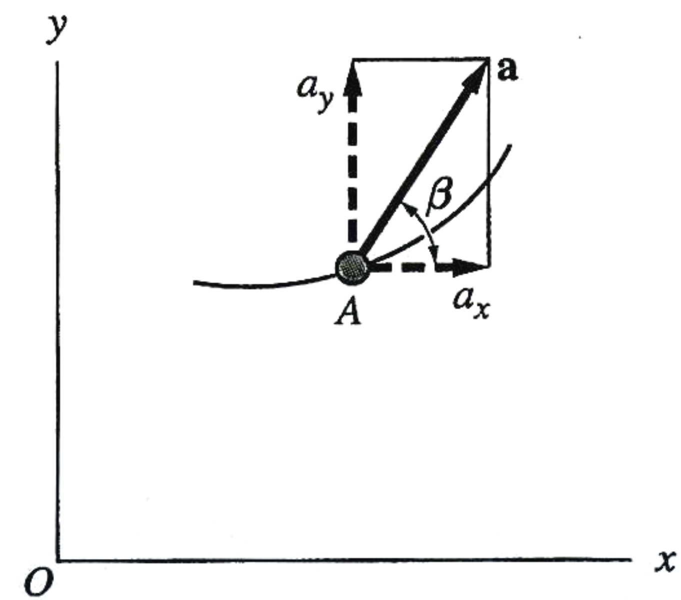
- **Velocity is tangent to the path**

$$\tan\theta = \frac{dy}{dx}$$



Rectangular Components of Acceleration

- **Acceleration is NOT necessarily tangent to the path**



Contents

- Concept of position, velocity and acceleration in two/three dimensions – rectangular coordinates

- **Motion of projectile**

Constant Accelerated Motion

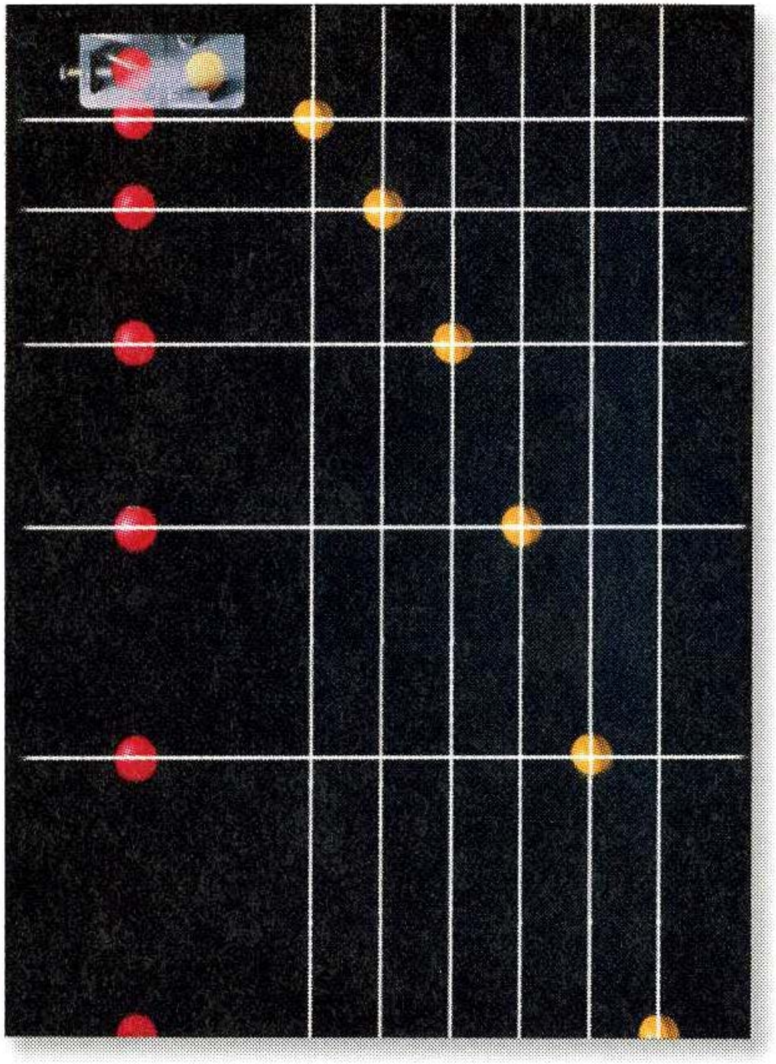
When a body falls freely towards the earth and if we assume no air resistance

Typical Example

Motion of a projectile

Elevation

Horizontal distance

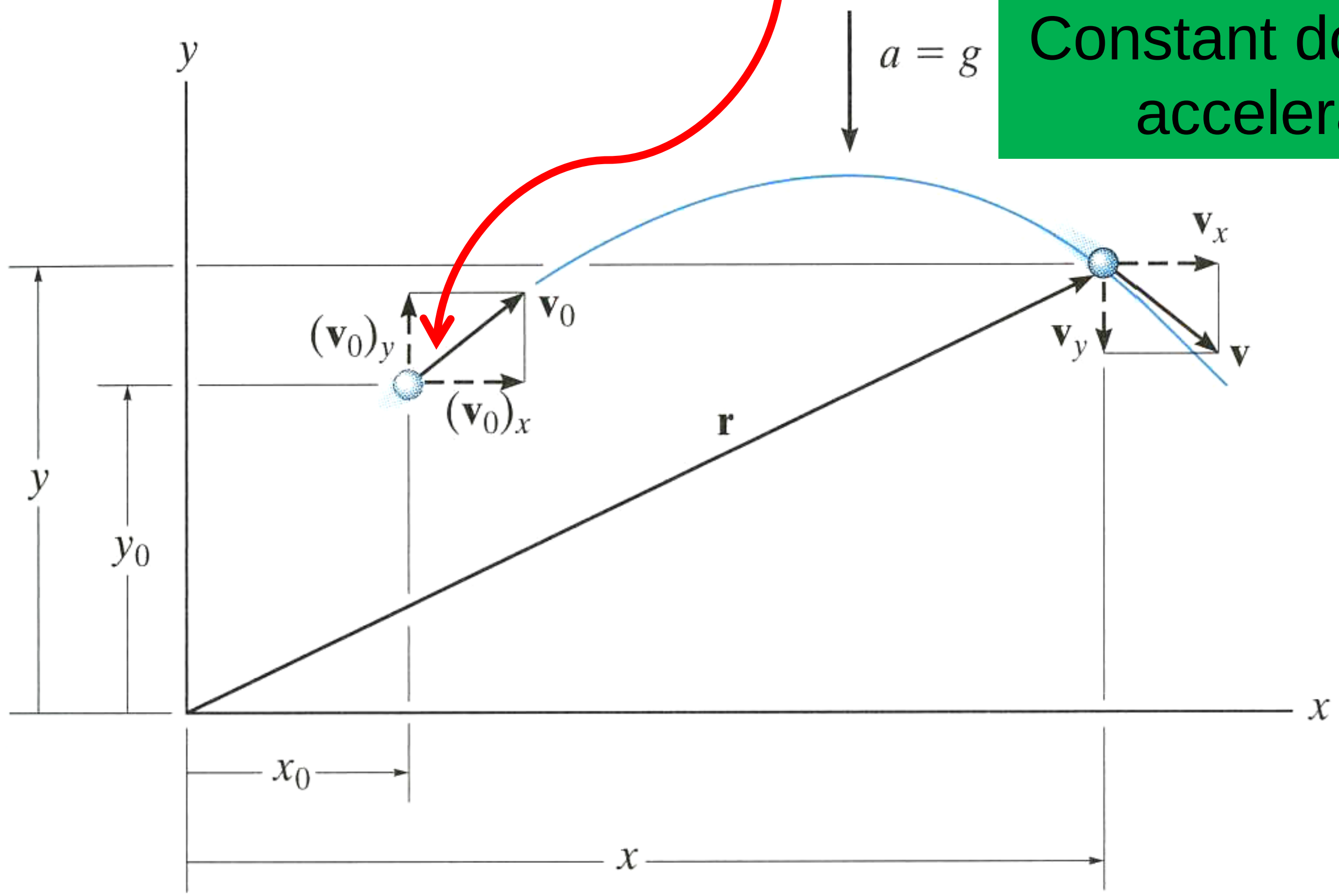


Projectile Motion

Initial conditions

At point (x_0, y_0)
Initial velocity = v_0

Constant downward acceleration



Solution for Projectile Motion

Horizontal motion

$a_x = 0$

Apply equations for constant acceleration

$$v_x(t) = v_{0x} \qquad x(t) = x_{0x} + v_{0x}\,t$$

Vertical motion

$a_y = -g$

Apply equations for constant acceleration

$$v_y(t) = v_{0y} - g\,t \qquad y(t) = y_0 + v_{0y}\,t - \frac{1}{2}g\,t^2$$

Projectile Motion