Physics

Lecture 13
Angular Impulse & Momentum; Generalization to system of points

Today's Contents

- <u>Angular impulse and momentum</u>
- Principle of angular impulse and momentum
- Generalization to systems of points
  - <u>Linear impulse and momentum</u>

Work Energy Method Vs. Impulse Momentum Method

- Work is a scalar quantity that is associated with a force and a change in the position of the point of application
- Impulse is a vector quantity associated with a force and a time interval.
- Kinetic energy is a scalar quantity associated with a mass and its speed at an instant of time
- Momentum is a vector quantity associated with a mass and its velocity vector at an instant of time
- The work-energy principle is a scalar relationship, whereas the impulse-momentum principle is a vector relationship

Review: Principle of Conservation of Linear Momentum

If the Impulse acting on a particle is zero during a given time interval, the momentum of the particle will be conserved during that interval

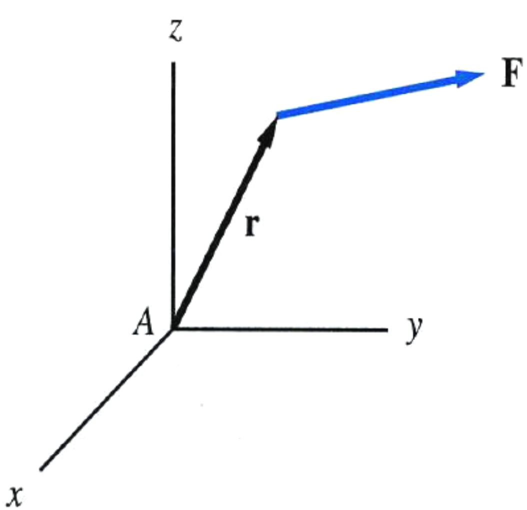
- **If there is no resultant force, the momentum will be conserved.**
- **Impulse can be zero, even if the force is not zero.**
- **It is possible that one or two components of impulse will be conserved even though the total momentum itself is not conserved.**

Angular Impulse

The angular impulse of a force F about point A

$$\left(\boldsymbol{H}_A\right)_{1-2} = \int_{t_1}^{t_2} \mathbf{r} \times \mathbf{F}\,dt = \int_{t_1}^{t_2} \mathbf{M}_A\,dt$$

Unit is N·m·s

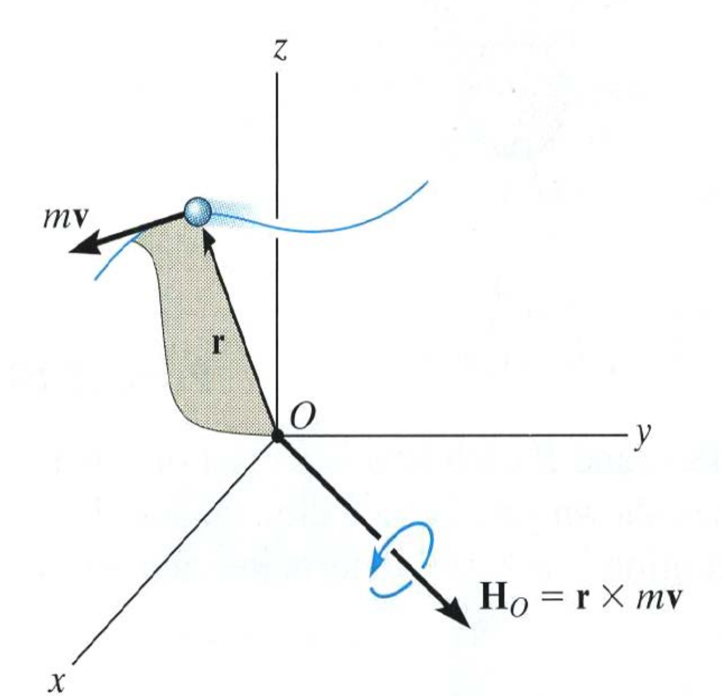


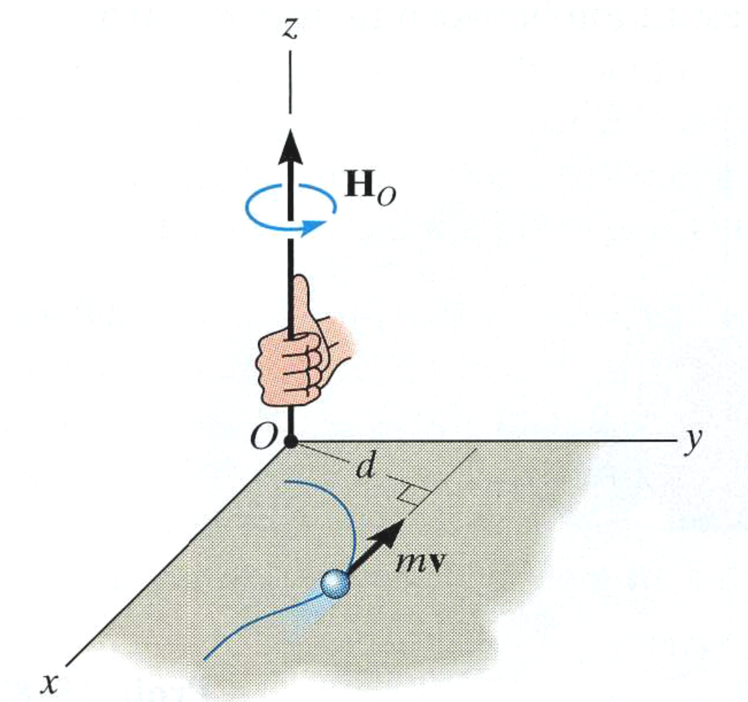
If the direction and magnitude of M_A are constant

Angular Momentum

The angular momentum of a particle about point O = Moment of the particle's linear momentum about O

Unit is kg·m²/s





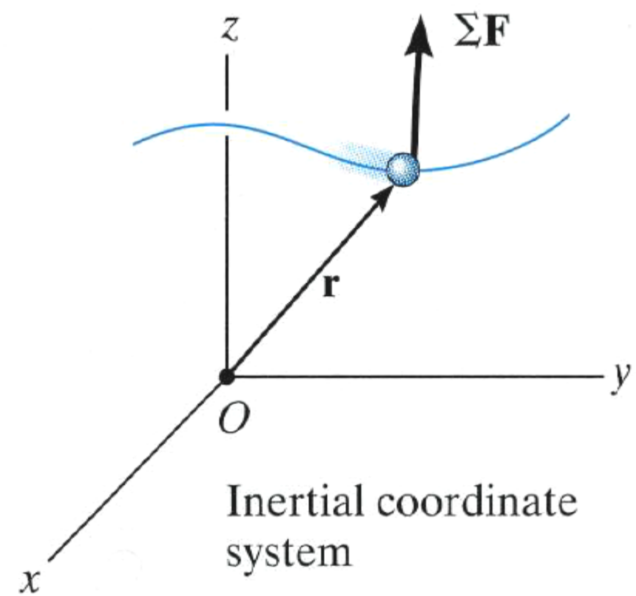
Vector formulation

$$\mathbf{H}_0 = \mathbf{r} \times m\mathbf{v}$$

Scalar formulation

$$(\mathbf{H}_0)_z = (d)(mv)$$

Relation Between Moment of a Force and Angular Momentum



The resultant moment about point O of all the forces acting on the particle = Time rate of change of particle’s angular momentum about point O

Principle of Angular Impulse and Momentum

$$\boxed{(H_0)_1 + \int_{t_1}^{t_2} \sum \mathbf{M}_0 dt = (H_0)_2}$$

- This is a vector equation where changes in direction as well as magnitude may occur during the interval of integration

$\boxed{\text{Conservation of angular momentum}}$

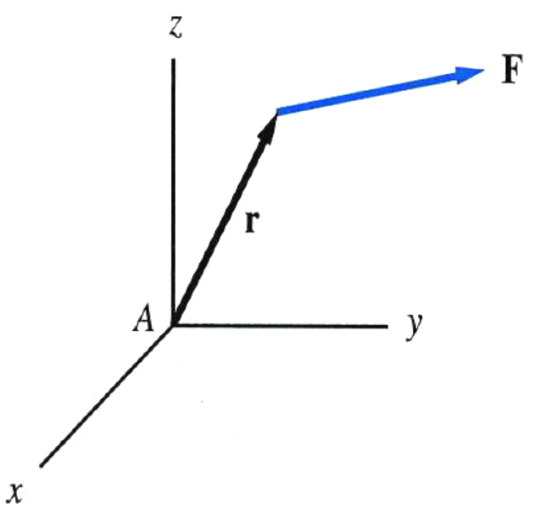
Central Force

When a particle moves under the influence of a force directed toward a fixed center of attraction, the motion is called *central-force motion*.

Simple example of center force: spring force

Central Force

The moment of central force about the origin is zero. Therefore, angular momentum is conserved in central force motion because a central force does not produce a moment about the point about which the particle is moving.

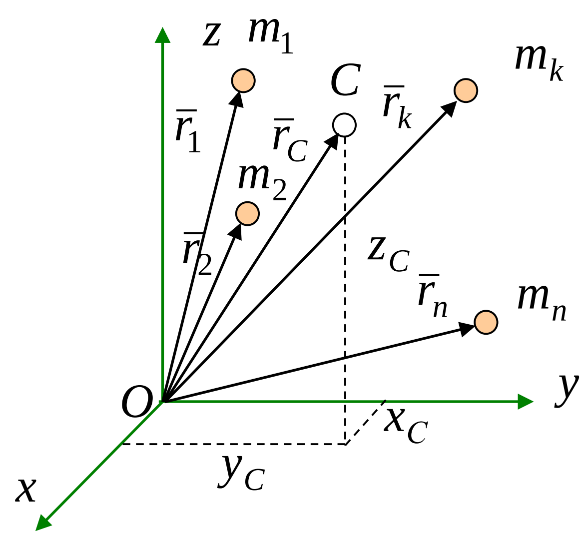


$$\text{if} \quad \boldsymbol{F} = \alpha \boldsymbol{r}$$

$$\boldsymbol{M}_A = \boldsymbol{r} \times \boldsymbol{F} = \boldsymbol{r} \times \alpha \boldsymbol{r} = 0$$

Hence, angular momentum $\boldsymbol{H}_A$ stays constant (is conserved)

Center of mass (systems of points)



Given a system of points (defined by vectors $\overline{r}_1, \overline{r}_2, ..., \overline{r}_n$) with masses $m_1, m_2, ..., m_n$, define the center of mass

$$\overline{r}_C = \frac{\sum m_k \overline{r}_k}{M} \quad \Leftrightarrow \quad x_C = \frac{\sum m_k x_k}{M}, \quad y_C = \frac{\sum m_k y_k}{M}, \quad z_C = \frac{\sum m_k z_k}{M},$$

where $M = \sum m_k$ is the total mass of the system

Motion of the Center of Mass

Forces applied to the points are divided to external F_k and internal f_k. Write motion equations

$$m\ddot{r}_k = F_k + f_k, \quad i = 1, n$$

and summate them

$$\sum m\ddot{r}_k = \sum F_k + \sum f_k$$

since $\sum f_k = 0$ **by the Newton 3rd law, and** $\sum m\ddot{r}_k = M\,\ddot{r}_C$ **we have**

$$\boxed{M\,\ddot{r}_C = \sum F_k}$$

center of mass moves as a particle under action of all external forces

Linear momentum

If $G_k = m_k \dot{\bar{r}}_k$ **is the linear momentum for point** k **, then**

$$G = \sum G_k = \sum m_k \dot{\bar{r}}_k = \frac{\mathrm{d}}{\mathrm{d}t}\left(M\,\bar{r}_C\right) = M\,\bar{v}_C$$

is the total linear momentum. Then

$$\frac{\mathrm{d}G}{\mathrm{d}t} = \sum F_k$$

<u>Conservation of linear momentum</u>: if the sum of all external forces is zero, then

$$G = M\,\bar{v}_C = \mathrm{const}$$

that is, velocity of the center of mass stays constant