Physics

Lecture 8
Kinetics: rectangular coordinates

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Types of Problems in Dynamics

First type

In which the acceleration of the particle is either specified or can be determined directly from known kinematic conditions

Use Newton's second law directly

$$\mathbf{F} = m\mathbf{a}$$

Second type

The forces acting on the particle are specified, and we must determine the resulting motion

Approaches differs for Second Type

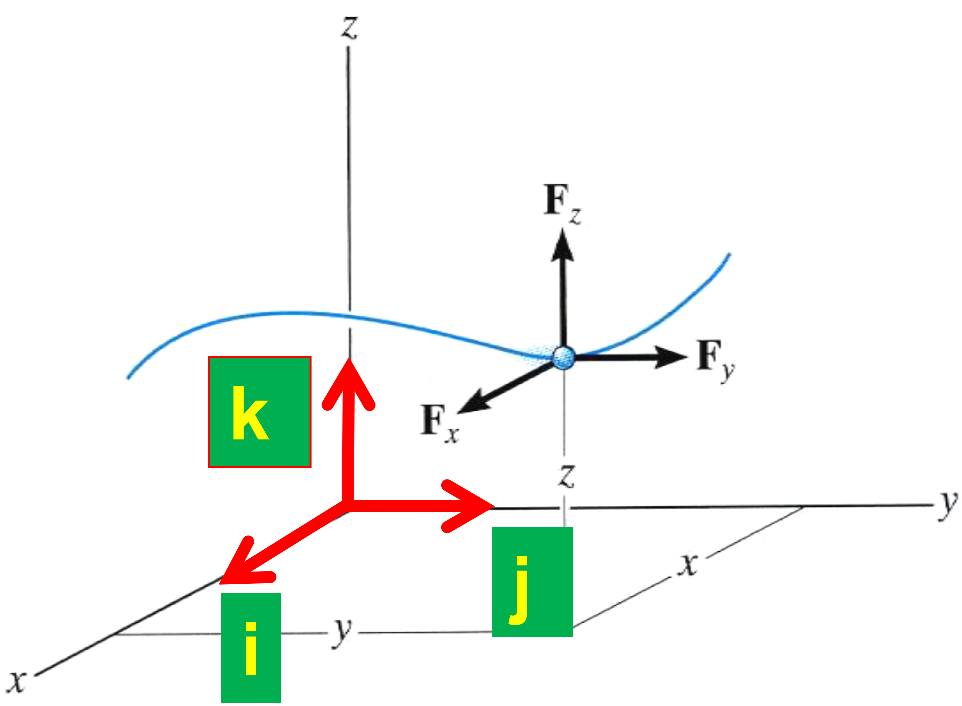
Forces are constant

Forces are functions of time, position, or velocity

Equations of Motion: Rectangular Coordinates

$$\sum \mathbf{F} = m\mathbf{a} = m\ddot{\mathbf{r}}$$

$$\sum F_x \mathbf{i} + \sum F_y \mathbf{j} + \sum F_z \mathbf{k} = m(a_x \mathbf{i} + a_y \mathbf{j} + a_z \mathbf{k})$$



$$\sum F_x = ma_x$$

$$\sum F_y = ma_y$$

$$\sum F_z = ma_z$$

Important points

- The equation of motion is based on experimental evidence and is valid only when applied within an inertial frame of reference.
- The equation of motion states that the unbalanced force on a particle causes it to accelerate.
- An inertial frame of reference does not rotate, rather its axes either translate with constant velocity or are at rest.
- Mass is a property of matter that provides a quantitative measure of its resistance to a change in velocity. It is an absolute quantity and so it does not change from one location to another.
- Weight is a force that is caused by the earth’s gravitation. It is not absolute; rather it depends on the altitude of the mass from the earth’s surface.

Procedures for Analysis

Step 1: Construct free body diagram

- Select the inertial coordinate system.
- Once the coordinates are established, draw the particle's free body diagram. Drawing this diagram a graphical representation that accounts for all the forces $\boldsymbol{F}$ which act on the particle, and thereby makes it possible to resolve these forces into their x, y, z components.
- The direction and sense of the particle's acceleration should also be established. If the sense is unknown, for mathematical convenience assume that the sense of each acceleration component acts in the same direction as its positive inertial coordinate axis.
- The acceleration may be represented as the $m\boldsymbol{a}$ vector on the kinetic diagram.
- Identify the unknowns in the problem.

Procedures for Analysis

Step 2: Construct equations of motion

- If the forces can be resolved directly from the free-body diagram, apply the equations of motion in their scalar component form.
- If the geometry of the problem appears complicated, which often occurs in three dimensions, Cartesian vector analysis can be used for the solution.
- Friction. If a moving particle contacts a rough surface, it may be necessary to use the frictional equation, which relates the frictional and normal forces (F_f and N) acting at the surface of contact by using the coefficient of kinetic friction, i.e., $F_f = \mu N$. Remember that F_f always acts on the free-body diagram such that it opposes the motion of the particle relative to the surface it contacts.

Procedures for Analysis

Step 2: Construct equations of motion

- Spring. If the particle is connected to an elastic spring having negligible mass, the spring force F_s can be related to the deformation of the spring by the equation $F_s = k s$. Here k is the spring's stiffness measured as a force per unit length, and s is the stretch or compression defined as the difference between the deformed length L and the undeformed length L_0, i.e., $s = L - L_0$.

Procedures for Analysis

Step 3: Use kinematics to find solution

- If the velocity or position of the particle is to be found, it will be necessary to apply the necessary kinematic equations once the particle's acceleration is determined from $\boldsymbol{F} = m\,\boldsymbol{a}$.
- If acceleration is a function of time, use $a = \mathrm{d}v / \mathrm{d}t$ and $v = \mathrm{d}s / \mathrm{d}t$ which, when integrated, yield the particle's velocity and position, respectively.
- If acceleration is constant, use $v = v_0 + a\,t,\;\; s = s_0 + v\,t + a\,t^2/2,\;\; v^2 = v_0^2 + 2a\,(s - s_0)$ to determine the velocity or position of the particle.
- If the solution for an unknown vector component yields a negative scalar, it indicates that the component acts in the direction opposite to that which was assumed.