Physics for Computer Science

Lecture 2
Kinematics of Particles

Contents

Introduction and Classification

Particle dynamics

Kinematics

Study of motion without reference to the forces which cause motion

Geometry of motion

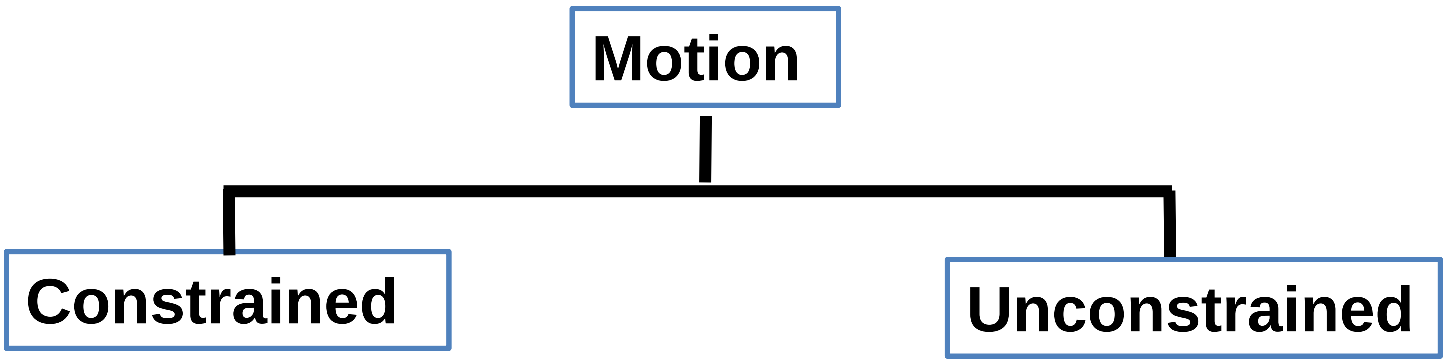
Kinetics

Study of relationship between the forces acting on the body and the resulting motion

Application of Kinematics

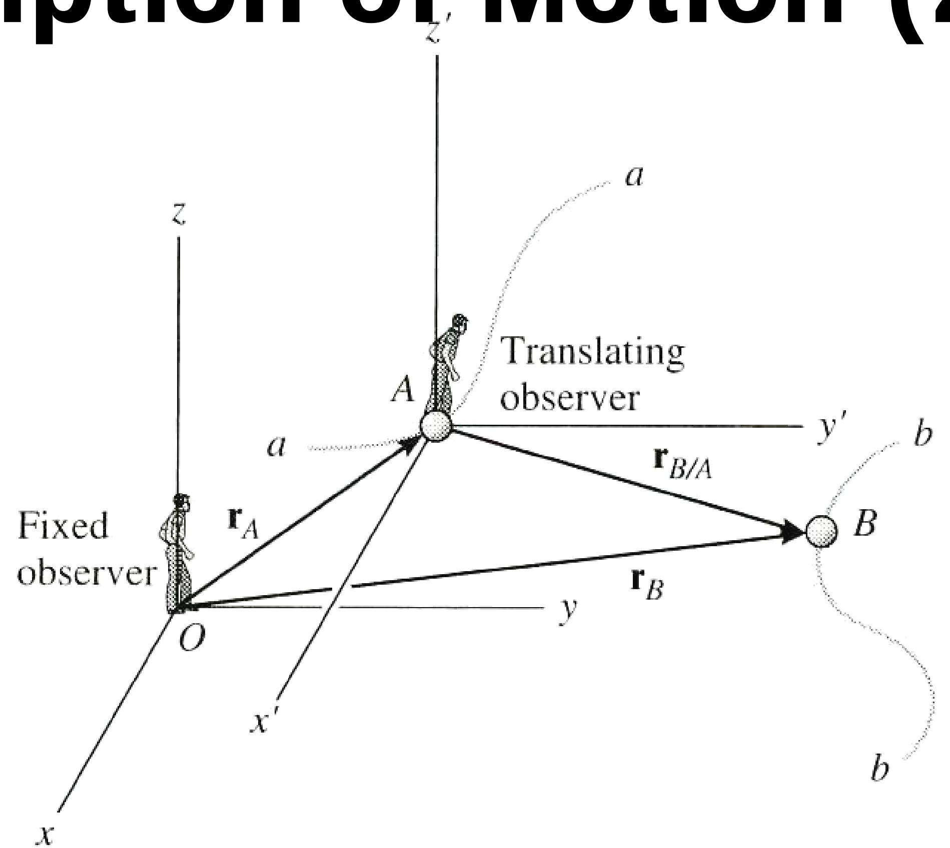
- **Animation (and also design of machine linkages and joints)**
- **Calculation of trajectories for robots, human arms, legs…**

Description of Motion (1)



- Unconstrained: motion in the free space
- Constrained: motion (of a particle) is restricted to a subspace of the free space

Description of Motion (2)

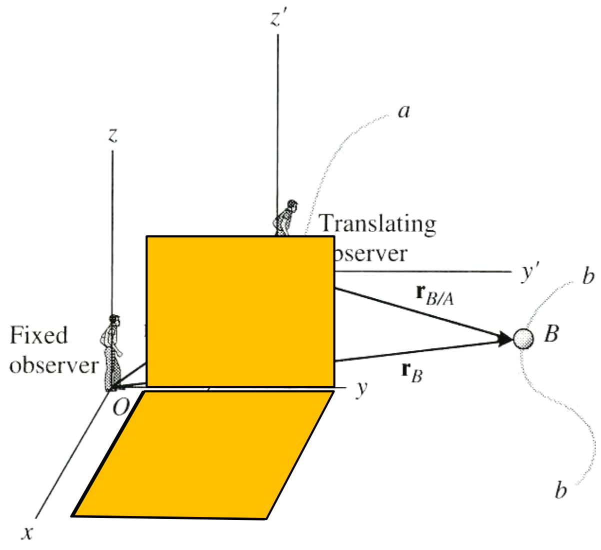


Motion (measured with respect to what)

Absolute motion (measured w.r.t fixed observer

Relative motion (measured w.r.t to moving observer)

Description of Motion (3)



1D, 2D and 3D Motions

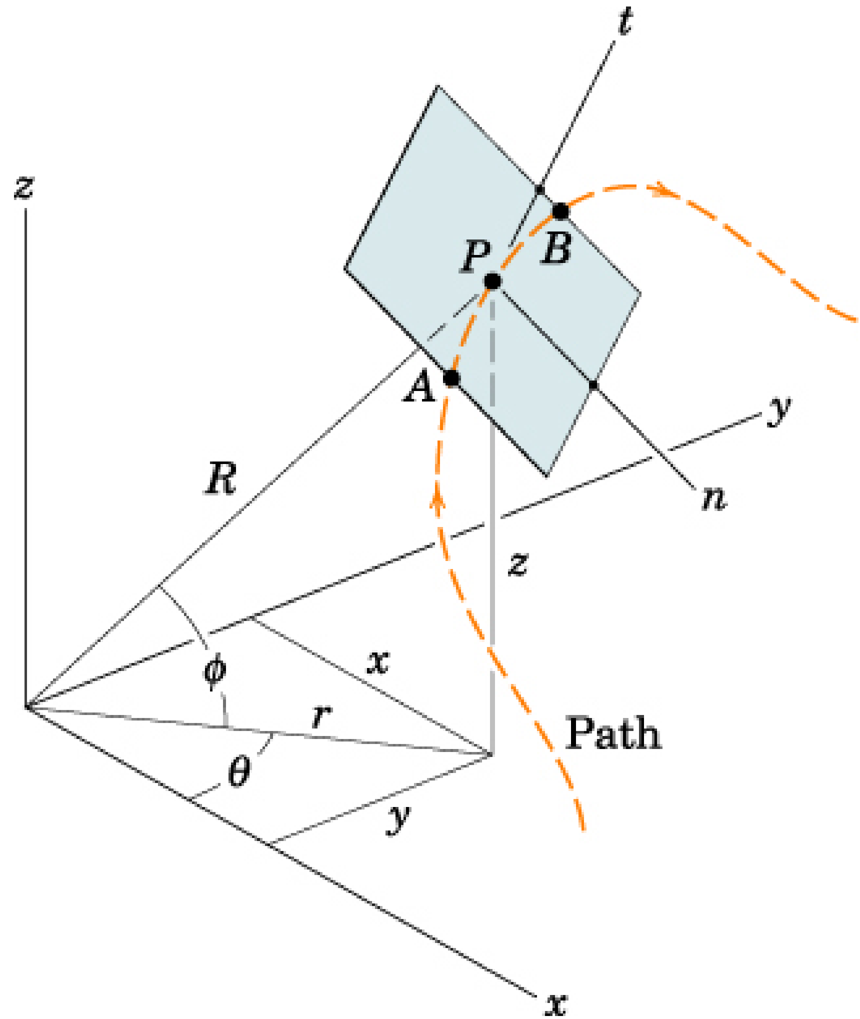
Plane motion

3-Dimensional Motion

Rectilinear motion

Curvilinear motion

Description of Motion (4)
Types of Coordinates



Rectangular (x, y, z)

Cylindrical (r, θ, z)

Spherical (R, θ, ϕ)

Contents

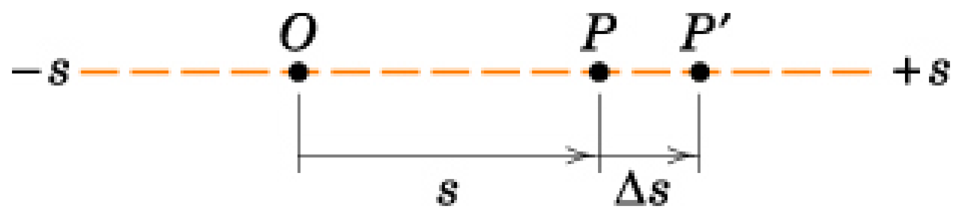
- Introduction, classification, and description of motion of a particle

- **Calculation of position, velocity and acceleration in rectilinear motion**

Curvilinear vs. Rectilinear Motion

Rectilinear (one-dimensional) Motion

If the motion of a particle is along a straight line, the motion is said to be rectilinear.



Position $s(t)$

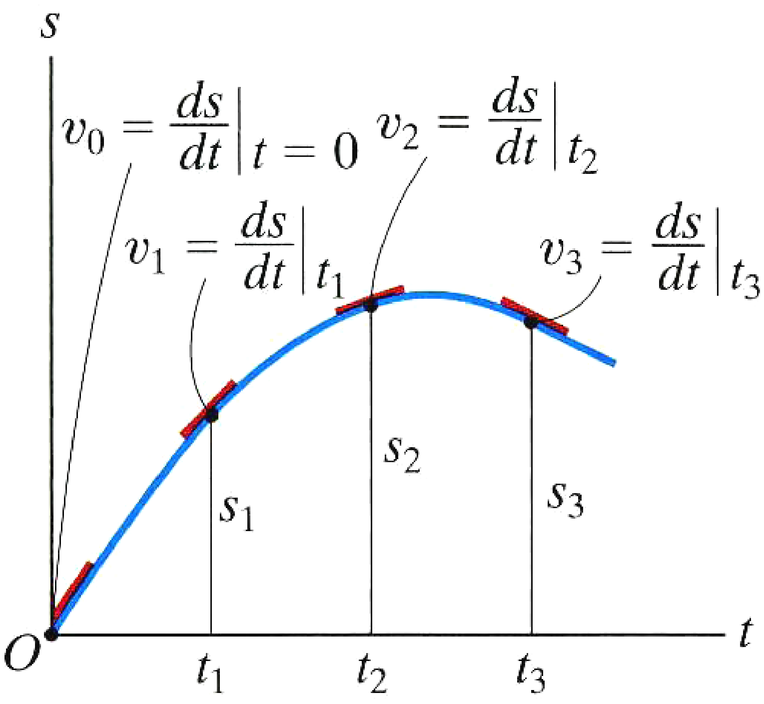
Velocity $v = \dfrac{ds}{dt} = \dot{s}(t)$

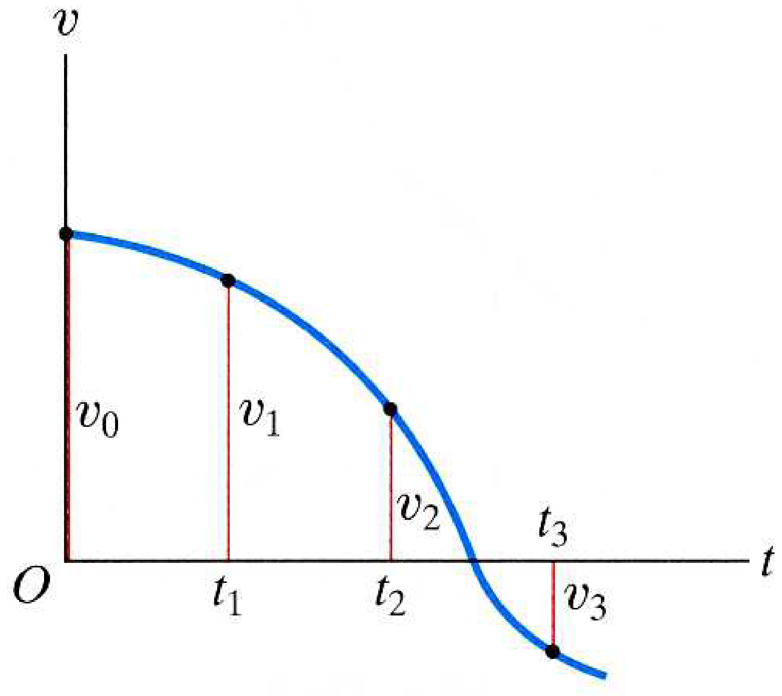
Acceleration $a = \dfrac{dv}{dt} = \ddot{s}(t)$

from $dt = \dfrac{ds}{v} = \dfrac{dv}{a}$ one gets $v\,dv = a\,ds$

Graphical Interpretation (*s-t* vs *v-t*)

Given the s-*t* graph,
Construct the *v-t graph*



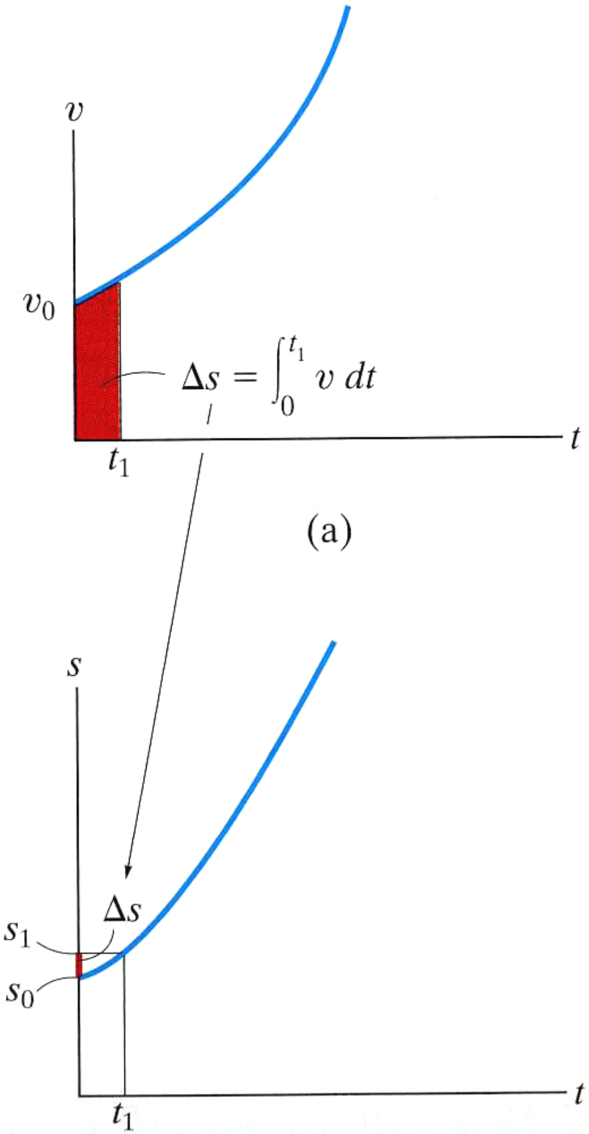


Graphical Interpretation (*v-t* vs *s-t*)

Given the v-*t* graph,
Construct the *s-t graph*

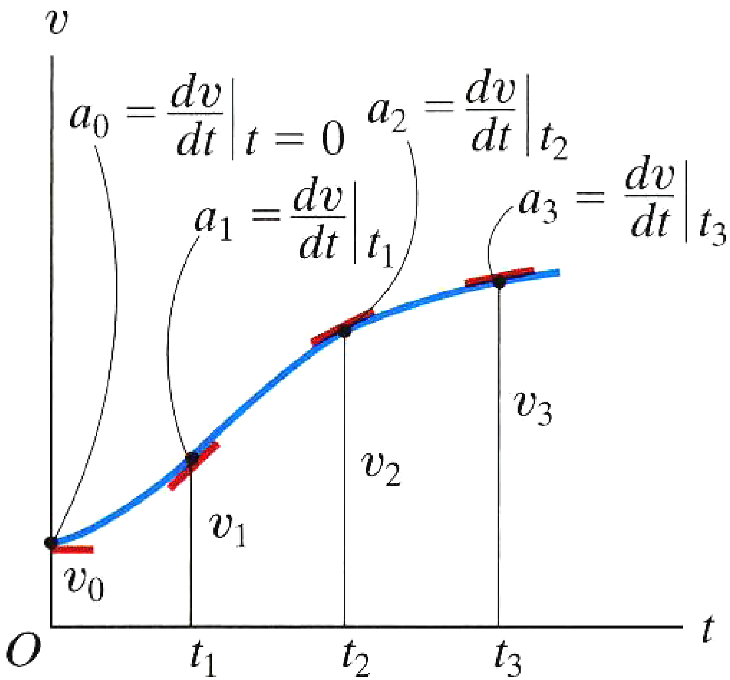
$$\int_{s_1}^{s_2} ds = \int_{t_1}^{t_2} v dt \quad \Rightarrow$$

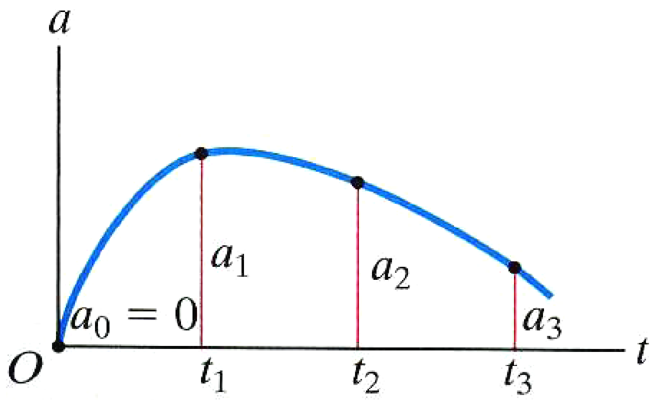
$$s_2 - s_1 = \text{area under } v - t \text{ curve}$$



Graphical Interpretation (*v-t* vs *a-t*)

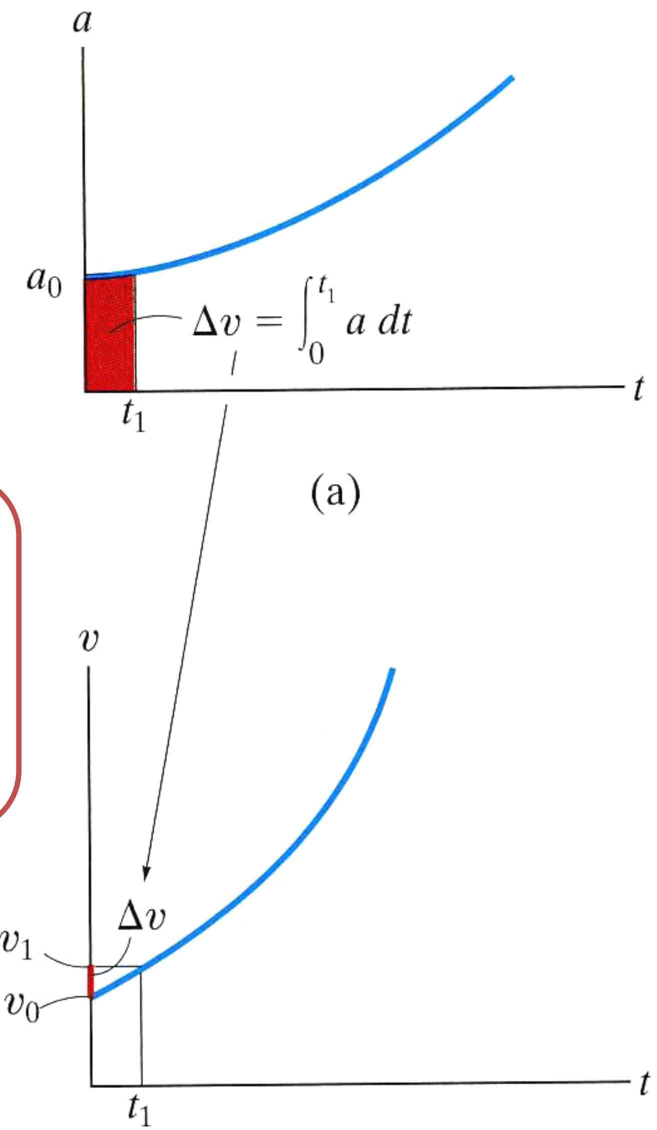
Given the v-*t* graph,
Construct the *a-t graph*





Graphical Interpretation (*a-t* vs *v-t*)

Given the *a-t* graph,
Construct the *v-t graph*

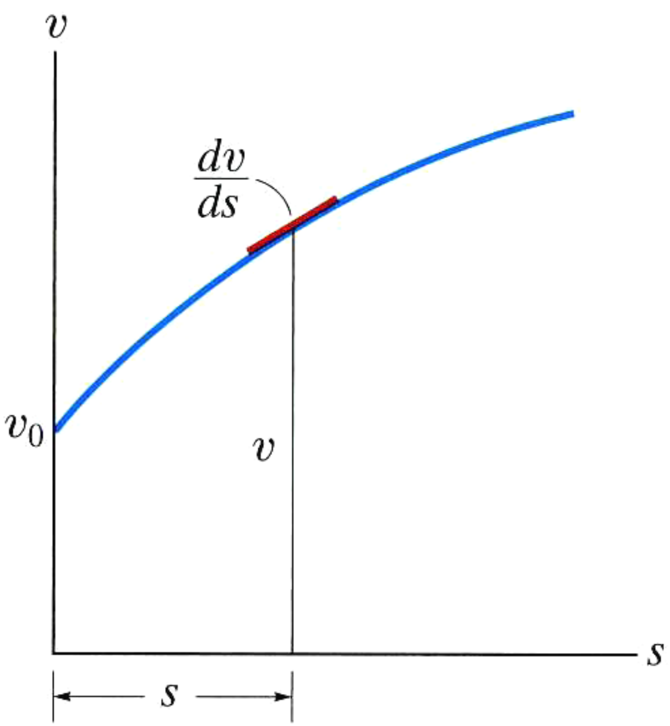


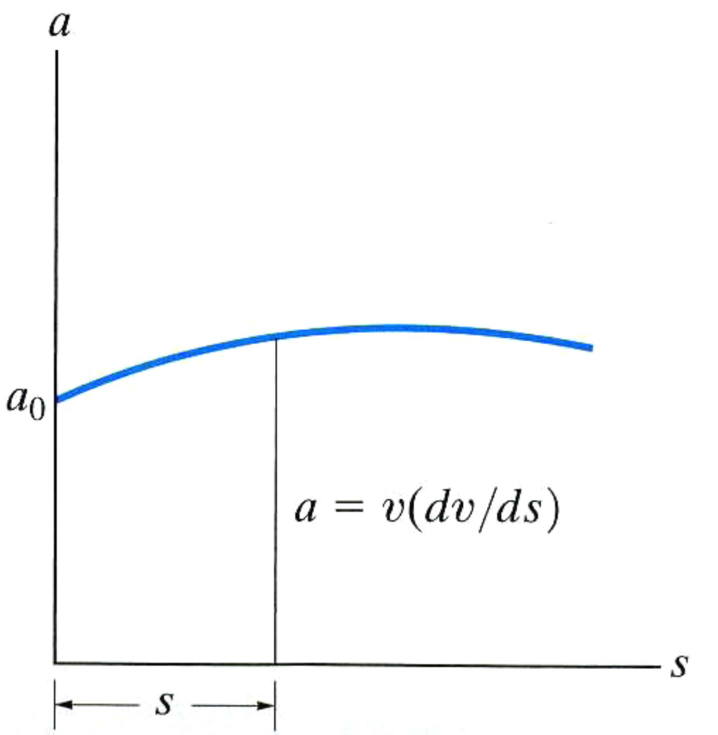
$$\int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a\,dt \quad \Rightarrow$$

$$v_2 - v_1 = \text{area under } a - t \text{ curve}$$

Graphical Interpretation (*a-s* vs *v-s*)

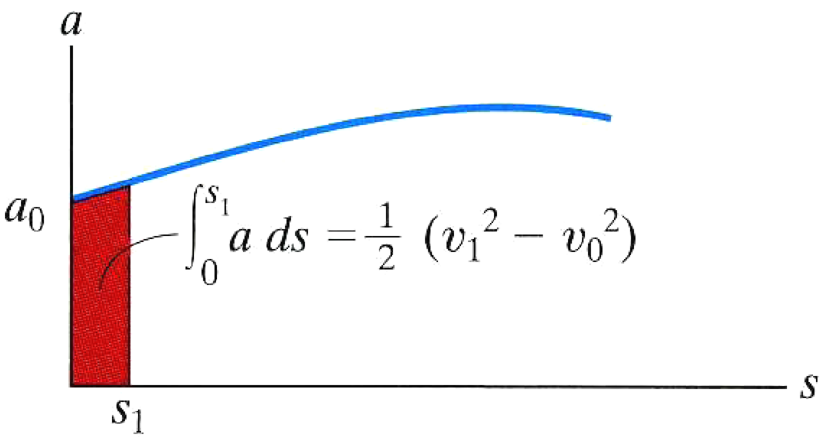
Given the *v-s* graph,
Construct the *a-s graph*

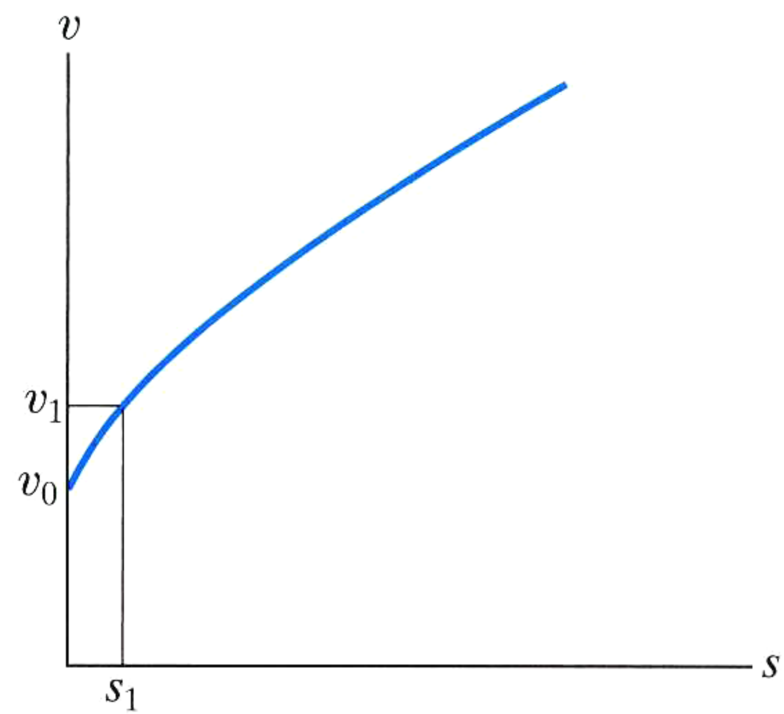




Graphical Interpretation (*a-s* vs *v-s*)

Given the *a-s* graph,
Construct the *v-s graph*





$$\int_{v_1}^{v_2} v\,dv = \int_{s_1}^{s_2} a\,ds \qquad \Rightarrow$$

$$\frac{v_2^2 - v_1^2}{2} = \text{area under } a - s \text{ curve}$$

Constant Acceleration

Initial conditions At $t = 0$, $s = s_0$ and $v = v_0$ Express

- Velocity as a function of time

$$\boxed{\int_{v_0}^{v} dv = a\int_{0}^{t} dt \quad \Rightarrow \quad v = v_0 + at}$$

- Velocity as a function of position

$$\boxed{\int_{v_0}^{v} v\,dv = a\int_{s_0}^{s} ds \quad \Rightarrow \quad v^2 = v_0^2 + 2a(s - s_0)}$$

Constant Acceleration

Initial conditions At $t = 0$, $s = s_0$ and $v = v_0$

- Express position as a function of time

$$\int_{s_0}^{s} dv = \int_{0}^{t} (v_0 + at)dt \quad \Rightarrow \quad s = s_0 + v_0 t + at^2 / 2$$

Variable Acceleration

Initial conditions

At $t = 0$, $s = s_0$ and $v = v_0$

- Acceleration can be given as a function of time, $a = f(t)$

- Acceleration can be given as a function of position, $a = f(s)$

- Acceleration can be given as a function of velocity, $a = f(v)$

Kinematics of Hybrid Motion

Hybrid motion:

When a particle's position, velocity and acceleration CANNOT be described by a single continuous mathematical function along the entire path

A series of functions will be required to specify the motion at different interval of time

Given a graph of motion relating to any two of the variables, subsequent graphs relating to two other variables can be constructed