Physics
Quiz # 4

Date Given: April 28, 2022
Date Due: May 12, 2022

Q1. (4 points) Solve the following problems.

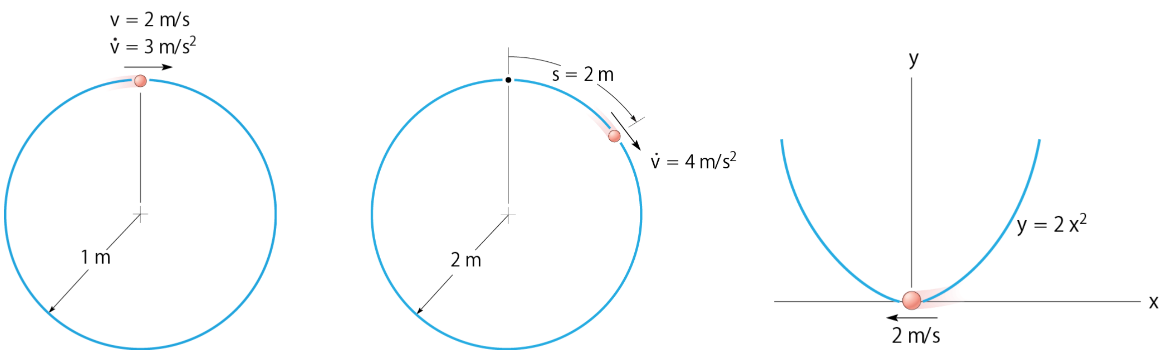

Figure 1: (a)-left, (b)-middle, (c)-right

(a) (1 point) Determine the magnitude of acceleration at the instant shown in Figure 1(a).

(b) (2 point) Determine the speed and the normal component of acceleration at $s = 2$m (see Figure 1(b)). At $s = 0$, $v = 0$.

(c) (1 points) Determine the acceleration at the instant shown in Figure 1(c). The particle has a constant speed of 2m/s.

Answer:

(a) $a_t = \dot{v} = 3\text{m/s}^2$; $a_n = v^2/\rho = 2^2/1 = 4\text{m/s}^2$. Thus, $a = \sqrt{a_t^2 + a_n^2} = \sqrt{3^2 + 4^2} = 5\text{m/s}^2$.

(b) $a_t = \dot{v} = 4\text{m/s}^2$; Next, since $v\mathrm{d}v = a_t \mathrm{d}s$, we have $\int_0^v v\mathrm{d}v = \int_0^s a_t \mathrm{d}s \Longrightarrow \left[\frac{v^2}{2}\right]_0^v = a_t \left[s\right]_0^s \Longrightarrow \frac{v^2}{2} = a_t s \Longrightarrow v^2 = 2a_t s = 2 \times 4 \times 2$ and therefore $v = 4\text{m/s}$. Thus, $a_n = v^2/\rho = 4^2/2 = 8\text{m/s}^2$

(c) Here $a_t = 0$ and $v = 2$. The radius of curvature $\rho(x) = \frac{(1+(\mathrm{d}y/\mathrm{d}x)^2)^{3/2}}{|\mathrm{d}^2y/\mathrm{d}x^2|} = \frac{(1+(4x)^2)^{3/2}}{4}$, and for $x = 0$, $\rho = \frac{1}{4}$m. Then $a_n = v^2/\rho = 16\text{m/s}^2$, and $a = \sqrt{a_t^2 + a_n^2} = \sqrt{16^2} = 16\text{m/s}^2$.

Q2. (2 points) Determine the normal and tangential component of acceleration at $s = 0$ if $v = (4s+1)\text{m/s}$ (see Figure 2).

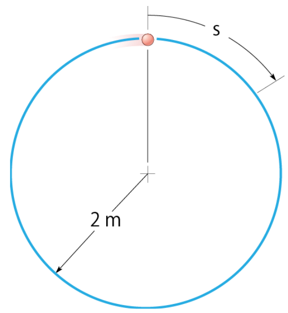

Figure 2: Illustration to Question 2.

Answer: Here $v = (4s + 1)$ and therefore $\mathrm{d}v = 4\mathrm{d}s$. Next, from $a_t\mathrm{d}s = v\mathrm{d}v$ we have $a_t\mathrm{d}s = (4s + 1)4\mathrm{d}s$ and $a_t = (4s + 1)4$. Therefore at $s = 0$ we have $a_t = 4\text{m/s}^2$. Next, from $a_n = v^2/\rho$ we get $a_n = (4s + 1)^2/2$. Therefore at $s = 0$ we have $a_n = 0.5\text{m/s}^2$.

Q3. (2 points) Determine the acceleration at $s = 2\text{m}$ if $\dot{v} = (2s)\text{m/s}^2$ where s is in meters (see Figure 3). At $s = 0$, $v = 1\text{m/s}$.

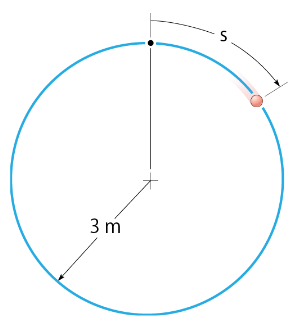

Figure 3: Illustration to Question 3.

Answer: Since $v\mathrm{d}v = a_t\mathrm{d}s$ and $a_t = \dot{v} = 2s$, we have $\int_1^v v\mathrm{d}v = \int_0^s a_t\mathrm{d}s = \int_0^s 2s\mathrm{d}s$ and therefore $\frac{1}{2}(v^2 - 1) = s^2$ and $v = \sqrt{1 + 2s^2}$. Thus, for $s = 2$ we have $v = 3\text{m/s}$. Next, $a_t = \dot{v} = 2s$ and for $s = 2$ we have $a_t = 4\text{m/s}^2$. Then, from $a_n = v^2/\rho$ we get $a_n = (1 + 2s^2)^2/3$, and for $s = 2$ we have $a_n = 3\text{m/s}^2$. Finally, $a = \sqrt{a_t^2 + a_n^2} = 5\text{m/s}^2$.

Q4. (2 points) Determine the acceleration when $t = 1\text{s}$ if $v = (4t^2 + 2)\text{m/s}$ where t is in seconds (see Figure 4).

Answer: Here $a_t = \dot{v} = 8t$, and for $t = 1$ we have $a_t = 8\text{m/s}^2$. Next, $a_n = v^2/\rho = (4t + 2)^2/6$, and for $t = 1$ we have $a_n = 6\text{m/s}^2$. Finally, $a = \sqrt{a_t^2 + a_n^2} = 10\text{m/s}^2$.

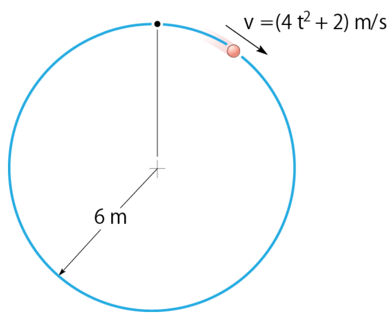

Figure 4: Illustration to Question 4.