Physics
Quiz # 10

Date Given: June 16, 2022
Date Due: June 23, 2022

Q1. (2 points) Determine the work of the force when it displaces 2 m as shown in Figure 1 (a and b).

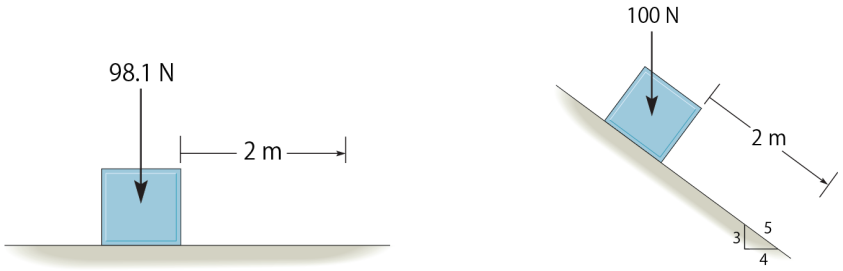

Figure 1: Illustration to Question 1.

Answer:

(a) $U = 0$ J.

(b) $U = 100\text{N}\left(\frac{3}{5}\ 2\text{m}\right) = 120$ J.

Q2. (2 points) Determine the work of the force when it displaces 2 m as shown in Figure 2 (a and b).

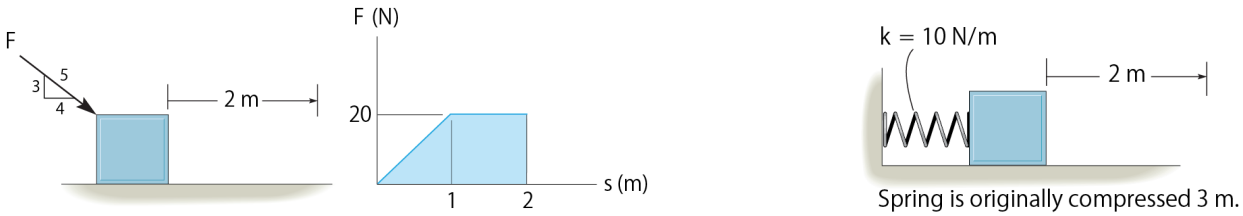

Figure 2: Illustration to Question 2.

Answer:

(a) $U = \frac{4}{5}(\text{Area}) = \frac{4}{5}\left(\frac{1}{2}1 \times 20 + 1 \times 20\right) = 24$ J.

(b) $U = \frac{1}{2}10\text{N/m}\left((3\text{m})^2 - (1\text{m})^2\right) = 40$ J.

Q3. (2 points) Determine the kinetic energy of the 10-kg block shown in Figure 3 (a and b).

Answer:

(a) $T = \frac{1}{2}10\text{kg}(2\text{m/s})^2 = 20$ J.

(b) $T = \frac{1}{2}10\text{kg}(6\text{m/s})^2 = 180$ J.

Figure 3: Illustration to Question 3.

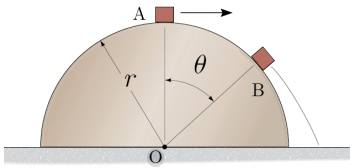

Figure 4: Illustration to Problem 4.

Q4. (2 points) A small box of mass m is given a speed of v_0 at the top of the smooth[1] half cylinder. Determine the angle θ at which the box leaves the cylinder if the initial speed $v_0 = \sqrt{\alpha g r}$, where g is the gravitational acceleration, r is the radius of the cylinder, and $\alpha = 1/4$.

Answer:

(a) *Principle of Work and Energy:* By referring to the free-body diagram of the block, Figure 5, notice that the normal reaction N does no work, while the gravity force W, displacing the block downward (from A to B) through a distance of $h = r - r\cos\theta$, does positive work mgh. From the work-energy equation, $T_1 + U_{1-2} = T_2$, one gets

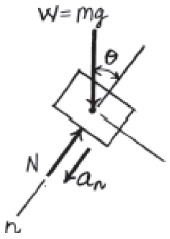

Figure 5: Illustration to Problem 4.

$$\frac{1}{2}m\alpha g r + mg(r - r\cos\theta) = \frac{1}{2}mv^2 \quad \Longrightarrow \quad v^2 = gr(\alpha + 2 - 2\cos\theta)$$

(b) *Equations of Motion:* in the normal direction we have $ma_n = \sum F_n$, where $a_n = v^2/r$ and the total force in the normal direction is $\sum F_n = -N + mg\cos\theta$. Therefore

$$mg(\alpha + 2 - 2\cos\theta) = -N + mg\cos\theta \quad \Longrightarrow \quad N = mg(3\cos\theta - \alpha - 2)$$

The block leaves the track when $N = 0$, that is when $3\cos\theta - \alpha - 2 = 0$, and therefore $\cos\theta = \frac{\alpha+2}{3}$. For $\alpha = 1/4$ we get $\cos\theta = 3/4$

$$\cos\theta = 3/4 \quad \Longrightarrow \quad \theta \approx 0.722734 \text{ rad} \approx 41.41^\circ.$$

[1] This implies that friction is negligible.