Physics

Lecture 10
Introduction to Work

Review: General Approaches to Problems in Kinetics

Approaches in Kinetics

Force-Mass-Acceleration

Work & Energy

Impulse & Momentum

Unbalanced forces acting on a particle

Integration of forces w.r.t. displacement

Procedures for Analysis to Solve Problems in Kinetics

Step 1

Construct free body diagram

Step 2

Construct equations of motion

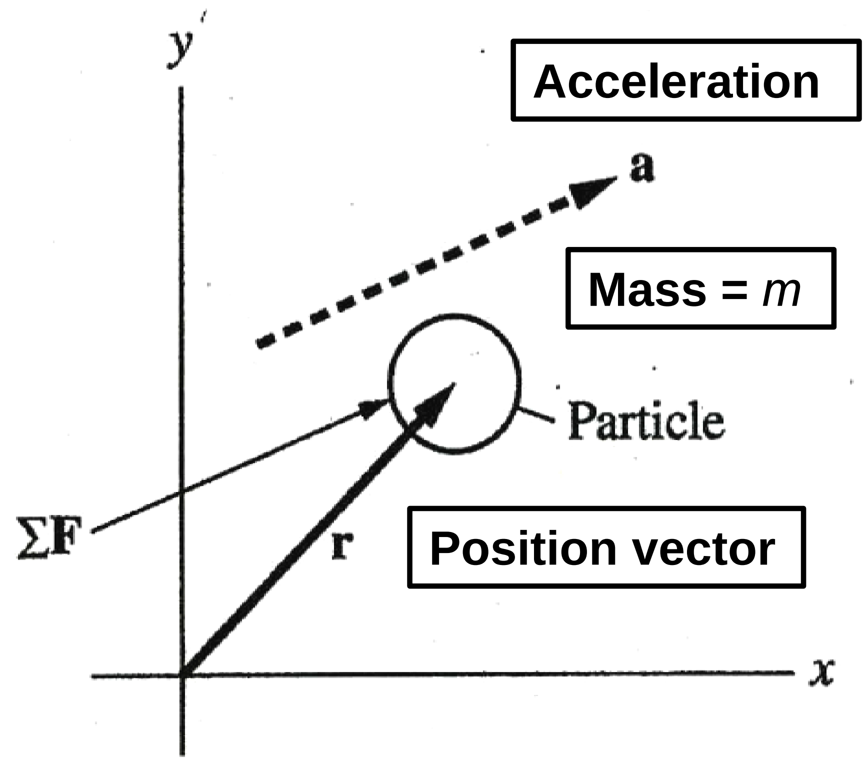
Step 3

Use kinematics to find solution

Equation of Motion

$$\sum \mathbf{F} = m\mathbf{a} = m\ddot{\mathbf{r}}$$

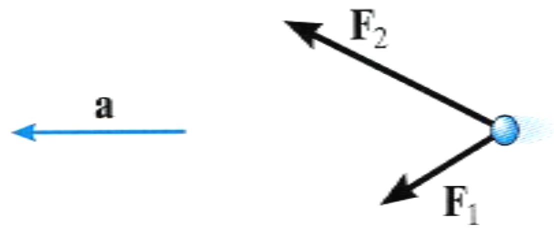
$$\sum \mathbf{F} - m\mathbf{a} = 0$$



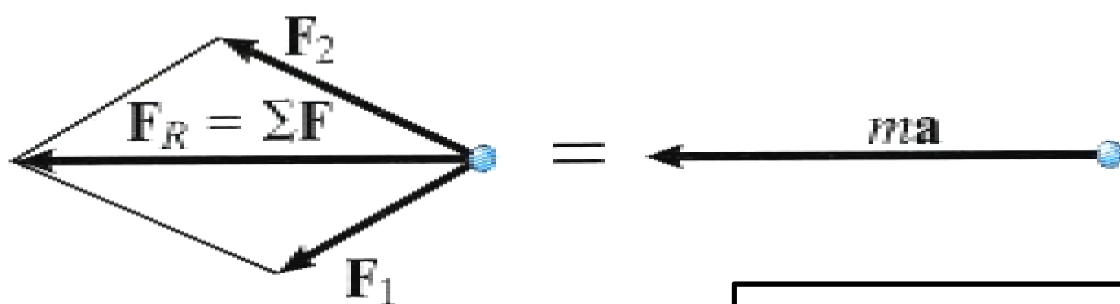
D'Alembert's Principle

An imaginary force that is collinear with Σ**F** but oppositely sensed and of magnitude m**a** would, if applied to a particle, cause it to be in equilibrium

Free Body Diagram (FBD) & Kinetic Diagram (KD)



$$\mathbf{F}_R = \sum \mathbf{F} = m\mathbf{a}$$



Kinetic Diagram

Free Body Diagram (FBD)

The diagram of an isolated body with the representation of all external forces acting on it is called a Free Body Diagram

Today's Contents

- Definition of work
- Calculation of work
- Types of work
- Principle of work and energy
- Power and efficiency

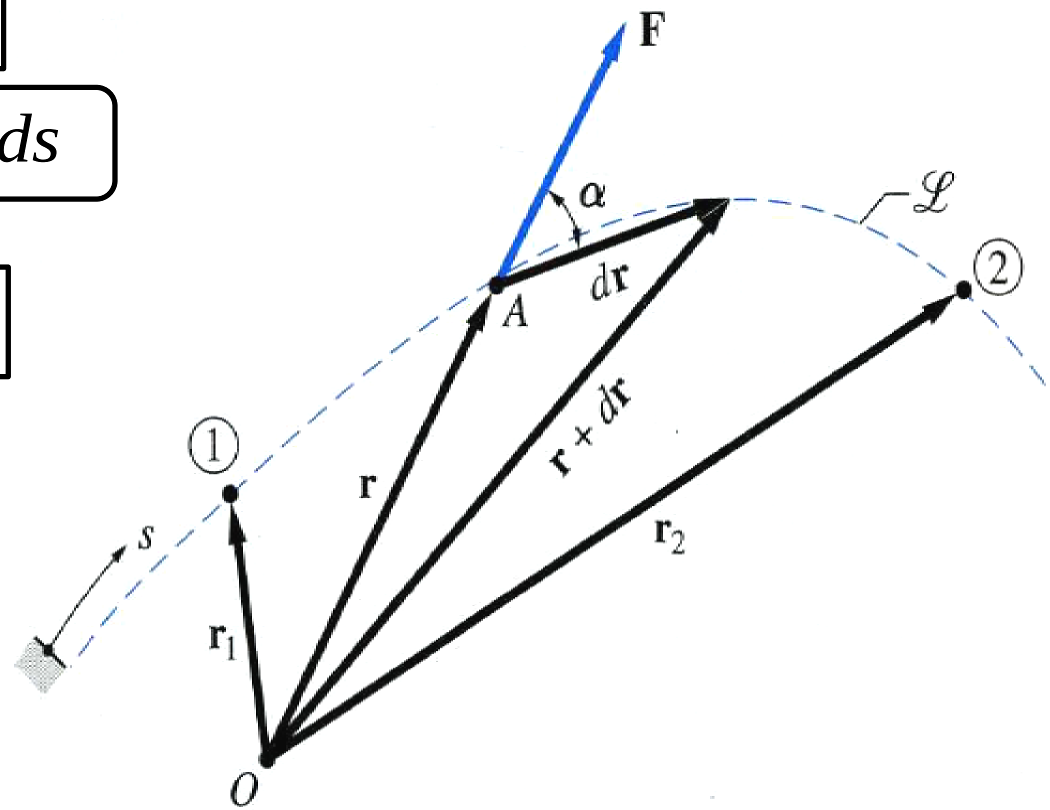
Work of a Force

Differential work

$$dU = \mathbf{F} \bullet d\mathbf{r} = F \cos\alpha \, ds$$

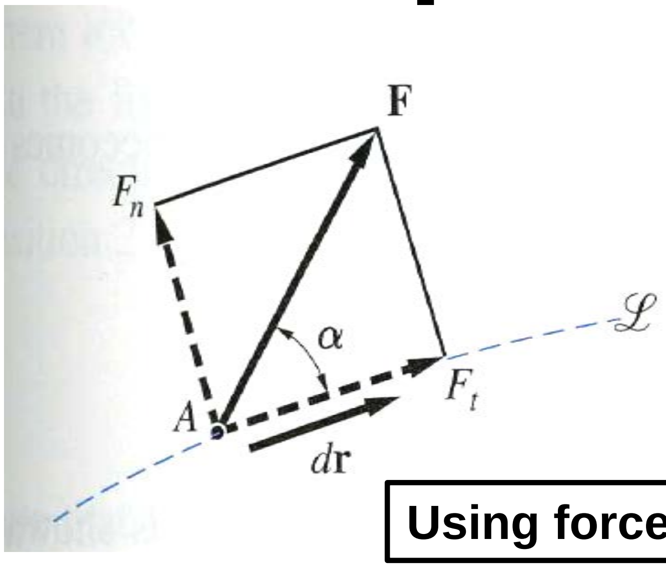
Work done by F

$$U_{1-2} = \int_{r_1}^{r_2} \mathbf{F} \bullet d\mathbf{r}$$



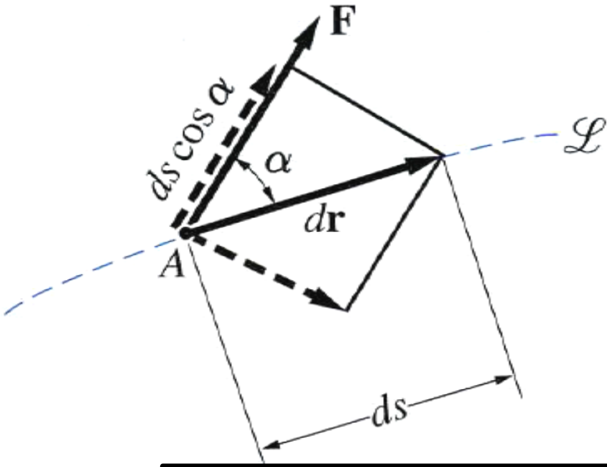
Unit of work in SI system is joule (J): 1 J = 1 N-m

Expressions for Work(1)



Using force

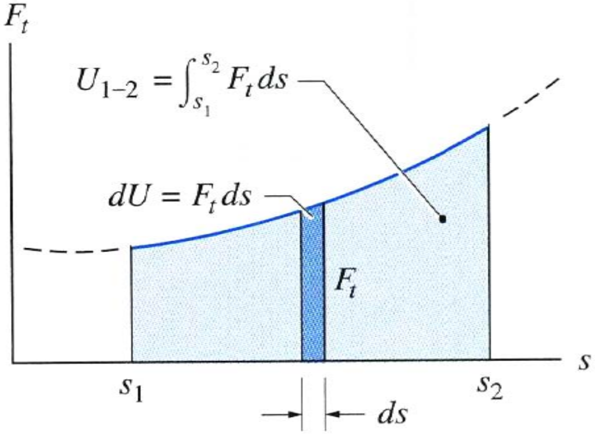
Work done by F



Using displacement

Incremental work

Geometric interpretation



Incremental work

Expressions for Work(2)

(1) Incremental work using force

$$\boxed{dU = F_t\ ds}$$

$$\boxed{= (\text{Working component of } \mathbf{F}) \times (\text{Magnitude of } \mathrm{d}\mathbf{r})}$$

(2) Incremental work using displacement

$$\boxed{dU = F \cos\alpha\, ds}$$

(3) Expression for work in rectangular coordinates

$$U_{1-2} = \int_{\boldsymbol{r}_1}^{\boldsymbol{r}_2} \boldsymbol{F} \bullet d\boldsymbol{r} = \int_{s_1}^{s_2} F \cos\alpha\, ds$$

Types of Work Encountered

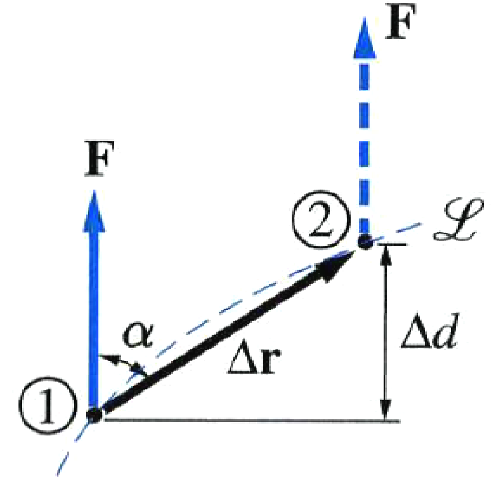
- Work of a variable force
- Work of a constant force
- Work of a central force
    - Work of a spring force
    - Work of a weight

Work of a Constant Force

Work done by F

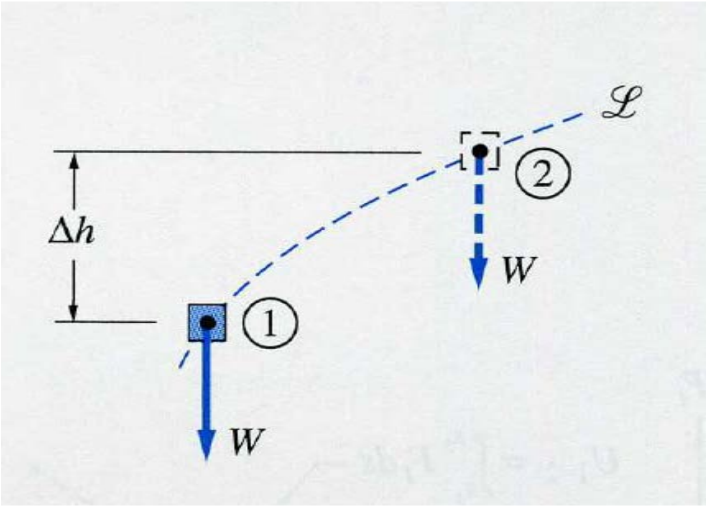
$$U_{1-2} = \mathbf{F} \bullet \int_{r_1}^{r_2} d\mathbf{r} = \mathbf{F} \bullet \Delta\mathbf{r}$$

$$U_{1-2} = F\Delta d$$



Work done by weight W

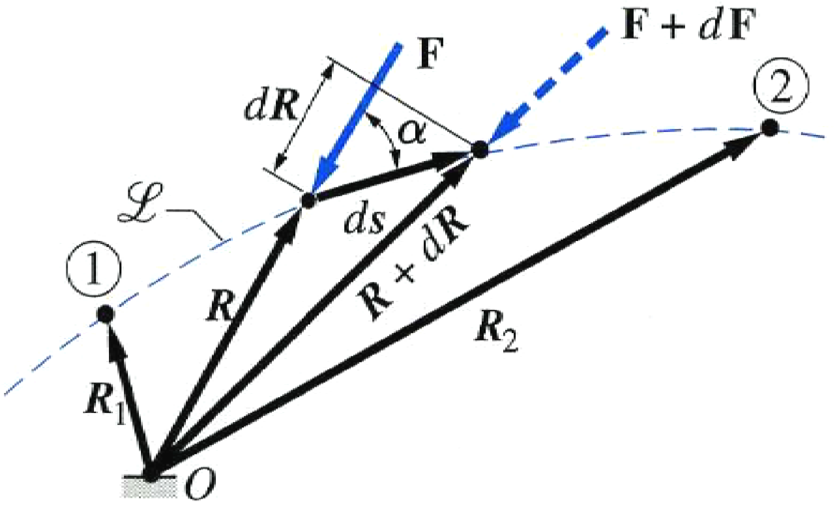
$$U_{1-2} = -W\Delta h$$



Work of a Central Force

Characteristics of central force

- Always directed toward a fixed point
- Magnitude is a function of the distance between the fixed point and the point of application



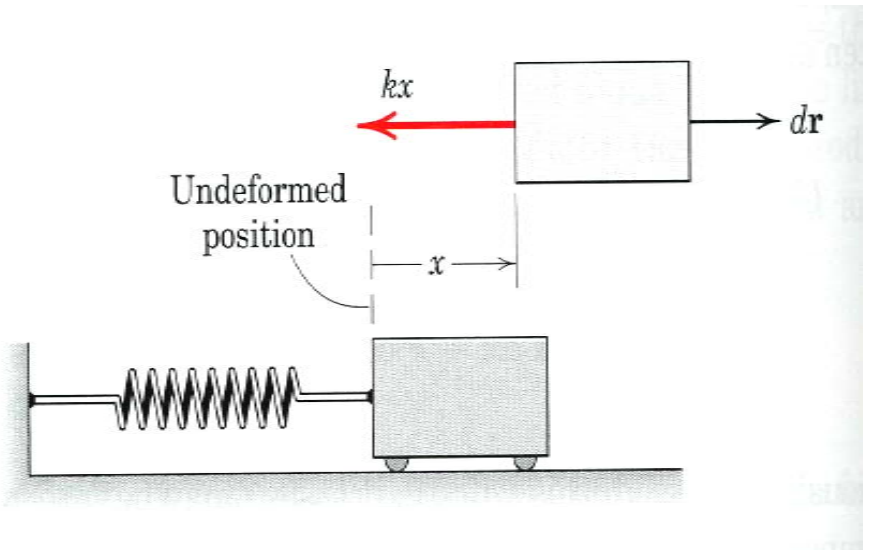
Examples of central force

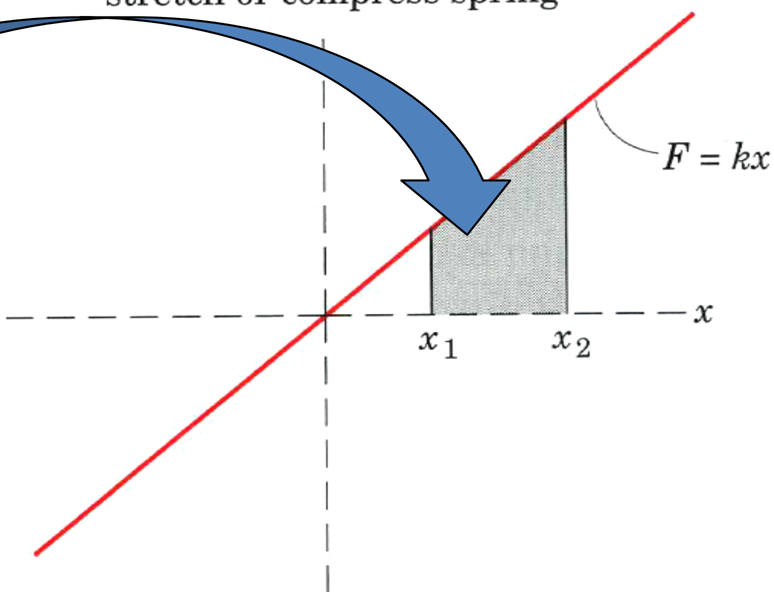
- Force exerted by a spring
- Gravitational attraction

Incremental work

Work of a Spring Force

Work done





Valid for both elongation (positive x) as well as contraction (negative x)

Work of a Gravitational Force(1)

Two types

- **Near the surface (g = constant)**
- **Away from the surface (g NOT constant) use Newton's law of Gravitation**

$$F = G\frac{m_1 m_2}{r^2}$$

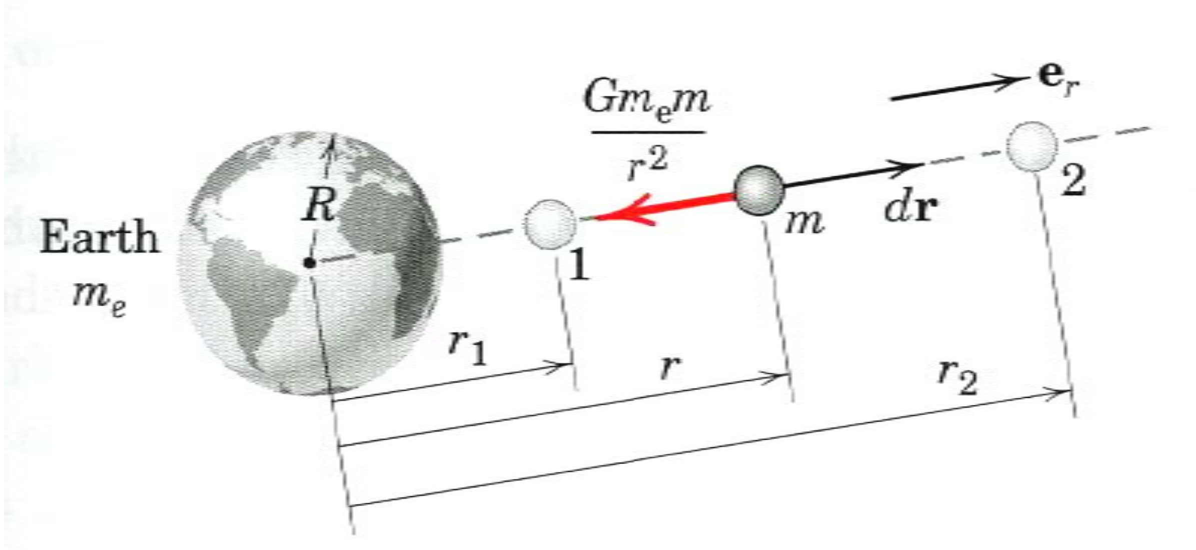
***F*: Mutual force of attraction between particles**

***G*: Constant of Gravitation**

m_1, m_2: masses of the two particles

***r*: Distance between the centers of the particles**

Work of a Gravitational Force (2)



Work done

$$U_{1-2} = -Gm_em\int_{r_1}^{r_2}\frac{dr}{r^2} = Gm_em\left(\frac{1}{r_2}-\frac{1}{r_1}\right) = mgR^2\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$$

***R*: Radius of the earth**

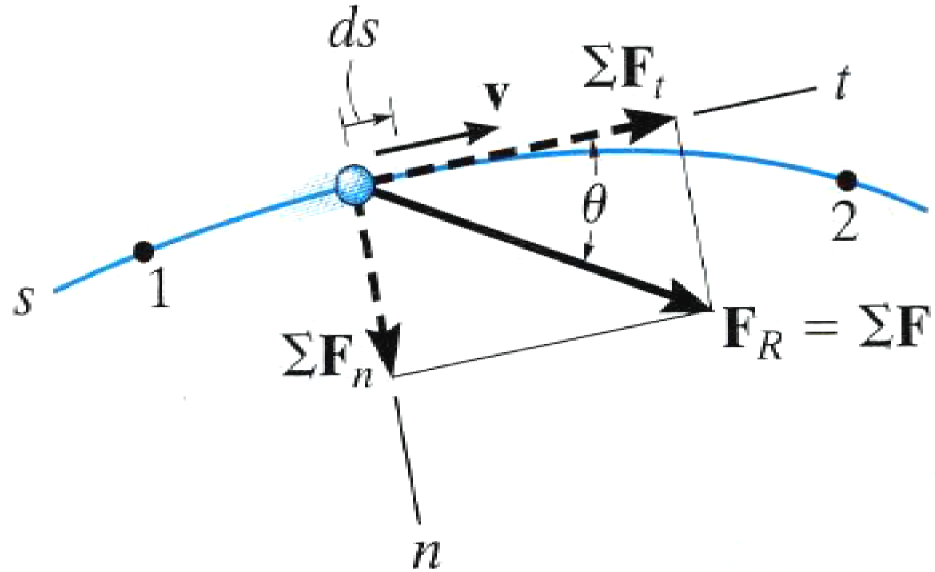
***g*: Acceleration due to gravity**

Principles of Work and Energy

Principle

$$\sum U_{1-2} = \frac{1}{2}m(v_2^2 - v_1^2)$$

$$= T_2 - T_1$$



Advantages of Work Energy Method over FMA Method

- Sometimes work can be calculated without integration
- Only forces that DO work need be considered
- Determine speed as a function of position of the particle

Few Important Points Related to Work Energy Method

- Work of a force is a scalar quantity (positive, negative or zero) that is associated with the change in the position of the point of application

- Kinetic energy is a scalar quantity (always positive) associated with the speed of a particle at a given instant of time.

Procedures for Analysis

- **Establish the inertial coordinate system and draw a free body diagram of the particle in order to account for all the forces that DO WORK on the particle**

- **The kinetic energy is always positive, since it involves the speed squared**

- **Work is positive when the force component is in the same sense of direction as its displacement, otherwise it is negative**

Power and Efficiency

Power (P) **The rate at which work is done**

$$P = \frac{dU}{dt} = \frac{F \bullet dr}{dt} = F \bullet \frac{dr}{dt} = F \bullet V$$

Efficiency (η)

$$\eta = \frac{Output\ power}{Input\ power} \times 100\%$$