Physics

Lecture 9
Kinetics: curvilinear coordinates

Contents

- Types of problems in dynamics
- Equations of motion in x-y-z coordinates
- Equations of motion in path coordinates
- Equations of motion in cylindrical (polar) coordinates

Equations of Motion: Path Coordinates

$$\sum \mathbf{F} = m\mathbf{a} = m\ddot{\mathbf{r}}$$

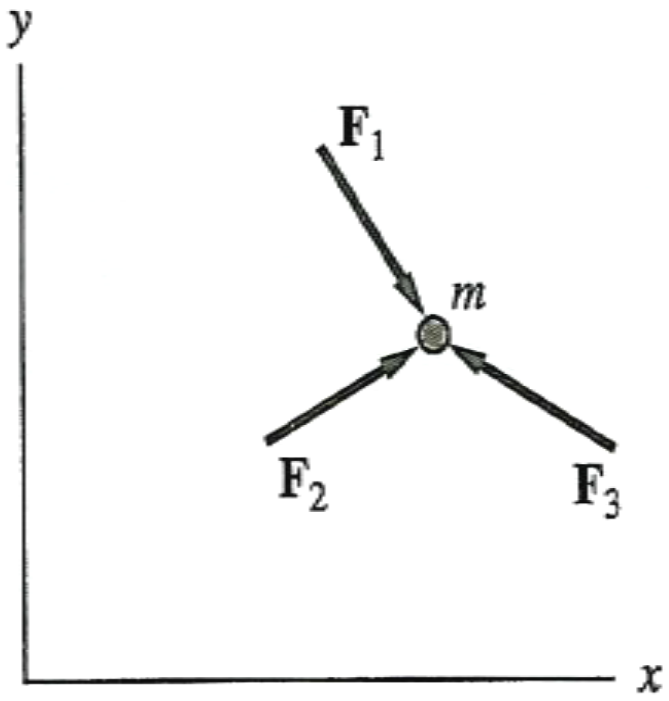
$$\sum F_n \mathbf{e}_n + \sum F_t \mathbf{e}_t = m\mathbf{a}_n + m\mathbf{a}_t$$

$$a_t = \dot{v} = v\frac{dv}{ds}$$

$$a_n = \frac{v^2}{\rho}$$

$$\sum F_n = ma_n$$

$$\sum F_t = ma_t$$



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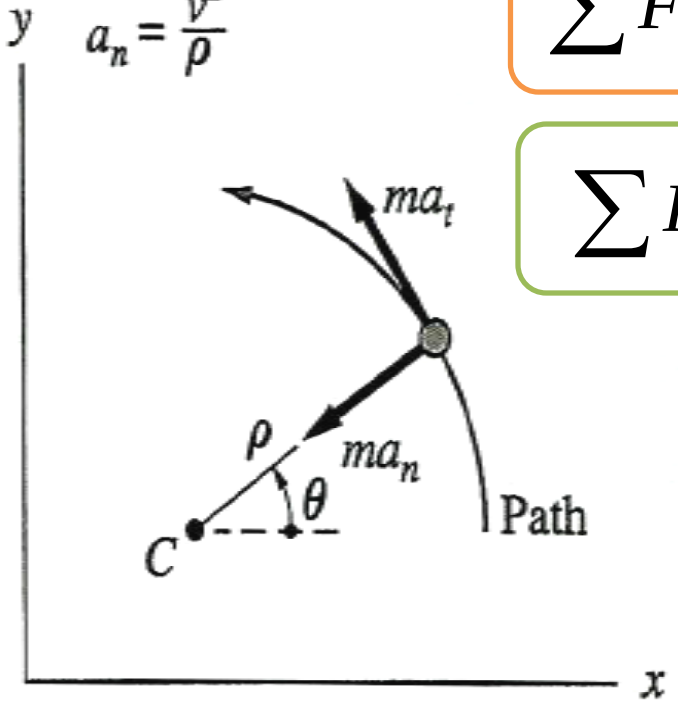


Free Body Diagram (FBD)

Kinetic Diagram (KD)

Procedures for Analysis

Step 1: Construct free body diagram

- Establish the *t, n* coordinate system at the particle and draw the particle's free body diagram and kinetic diagram
- The particle's normal acceleration always acts in the positive *n* directions
- If the particle's tangential acceleration is unknown, assume it acts in the positive *t* directions

Step 2: Construct equations of motion

Step 3: Use kinematics to find solution

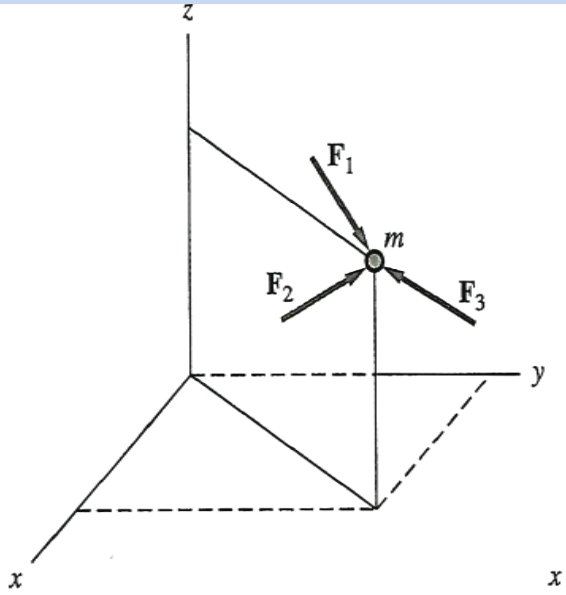
- Formulate the tangential and normal components of acceleration
- Find the radius of curvature ρ, if the path is defined as y = f(x)

Equations of Motion: Cylindrical Coordinates

$$\sum \mathbf{F} = m\mathbf{a} = m\ddot{\mathbf{R}}$$

$$\sum F_R \mathbf{e}_R + \sum F_\theta \mathbf{e}_\theta + \sum F_z \mathbf{e}_z = ma_R \mathbf{e}_R + ma_\theta \mathbf{e}_\theta + ma_z \mathbf{e}_z$$

Free Body Diagram (FBD)



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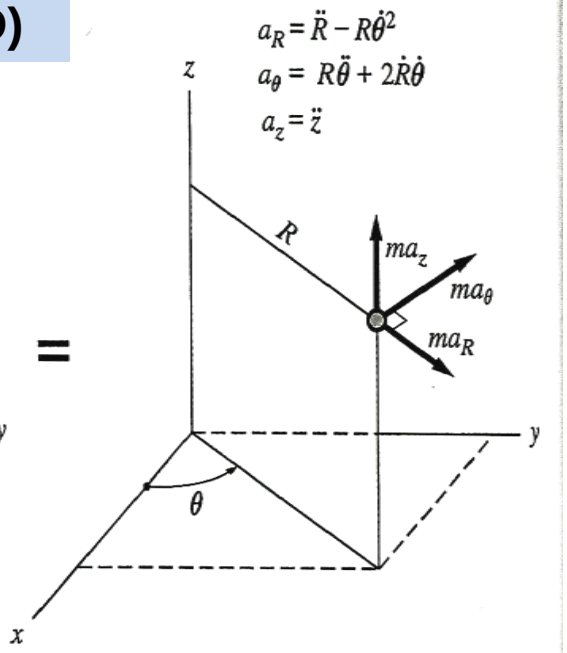

Kinetic Diagram (KD)

$$\sum F_R = ma_R$$

$$\sum F_\theta = ma_\theta$$

$$\sum F_z = ma_z$$

Procedures for Analysis

Step 1: Construct free body diagram

- Establish the R, θ, z inertial coordinate system and draw the particle's free body diagram and kinetic diagram
- Assume a_R, a_θ, a_z act in the positive directions of R, θ, z if they are unknown
- Identify all the unknown in the problem

Step 2: Construct equations of motion

Step 3: Use kinematics to find solution

- Determine R and the time derivative of R, θ, z, and then evaluate the acceleration components
- If any of the acceleration components is computed as a negative quantity, it indicates it acts in its negative coordinate direction