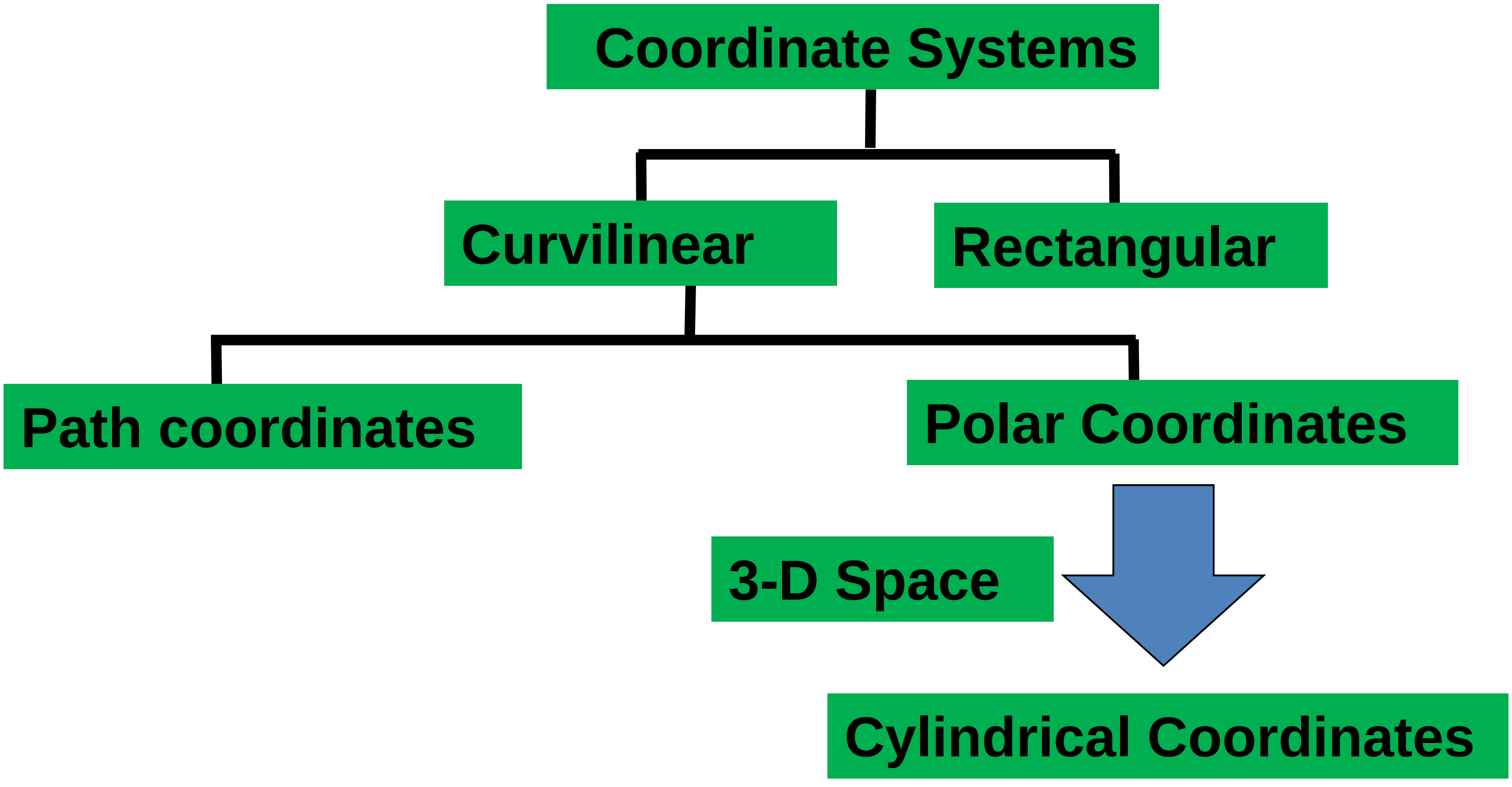
Physics

Lecture 5
Kinematics of Curvilinear Motion
(Polar Coordinates)

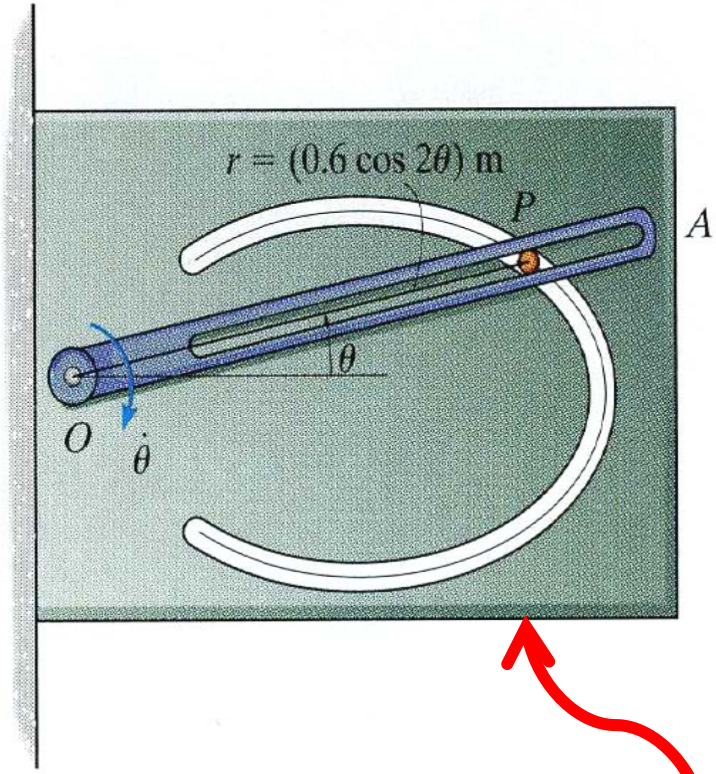
Contents

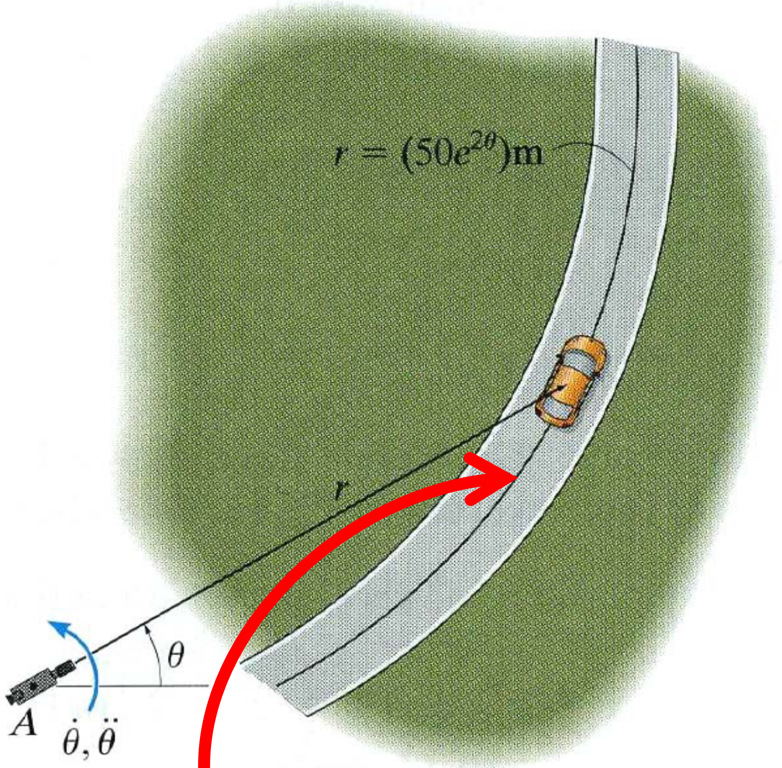
- Polar coordinate system
- Base vectors of polar coordinate system
- Velocity and acceleration in polar coordinates
- Circular motion using polar coordinates

Types of Coordinate Systems



When do we use polar coordinates



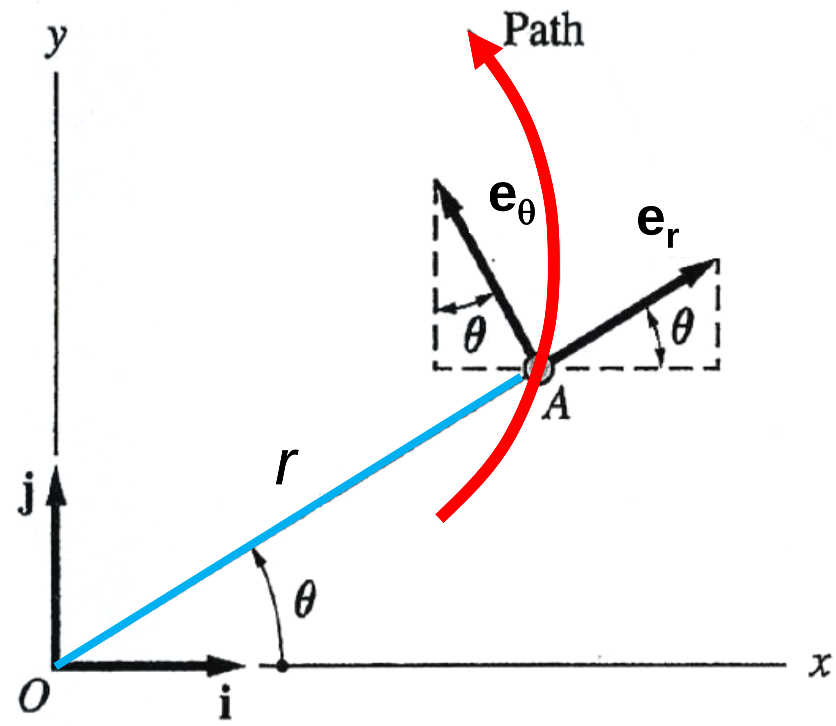


- When a motion is constrained through the control of a radial distance and an angular position
- Or when an unconstrained motion is observed by measuring the radial distance and angular position

Polar Coordinates

Characteristics

- Location is specified by a radial coordinate *r* from a fixed origin and a transverse coordinate θ, which is the counterclockwise angle between a fixed reference line and the *r* axis

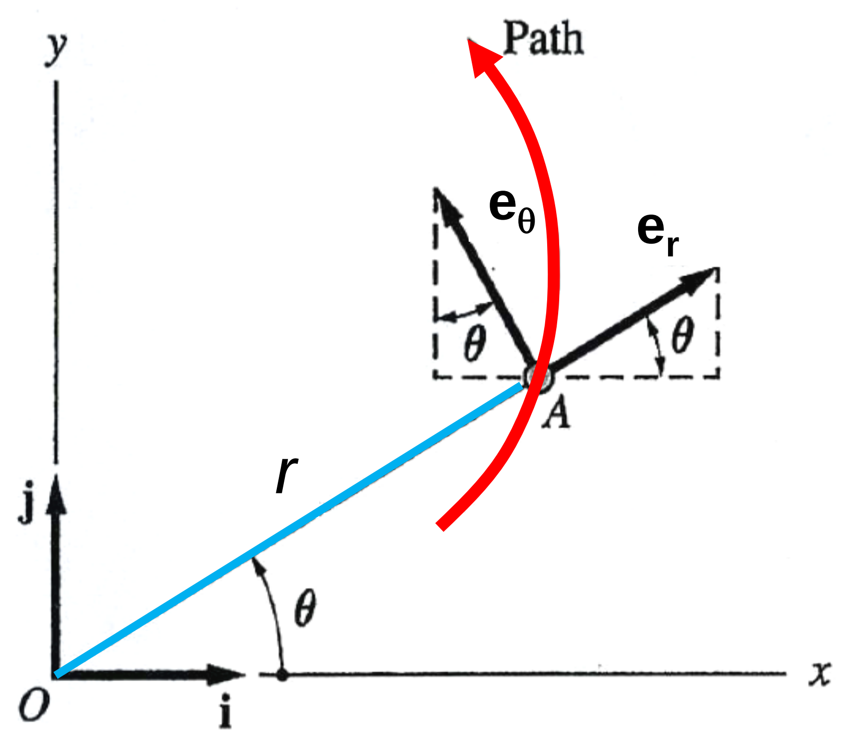


- The positive directions of *r* and θ coordinates are defined by base vectors $\mathbf{e}_r$ and $\mathbf{e}_\theta$

- $\mathbf{e}_r$ is in the direction of increasing *r* when θ is held fixed, and $\mathbf{e}_\theta$ is in a direction of increasing θ when *r* is held fixed

Base Vectors in Polar Coordinates

- Mutually perpendicular of unit magnitude
- Directions of $\mathbf{e}_r$ and $\mathbf{e}_\theta$ are NOT fixed
- Directions depend on the location of the particle



What is the difference between $\mathbf{e}_n$ and $\mathbf{e}_t$ (Base Vectors in path coordinates) and $\mathbf{e}_r$ and $\mathbf{e}_\theta$

Path Coordinates Vs. Polar Coordinates

- Path coordinates depend on the path, and the direction of motion of the particle, whereas polar coordinates are determined solely by the position of the particle

- In both the coordinate systems, the base vectors possess nonzero derivatives, even though their magnitudes are constant (equal to unity)

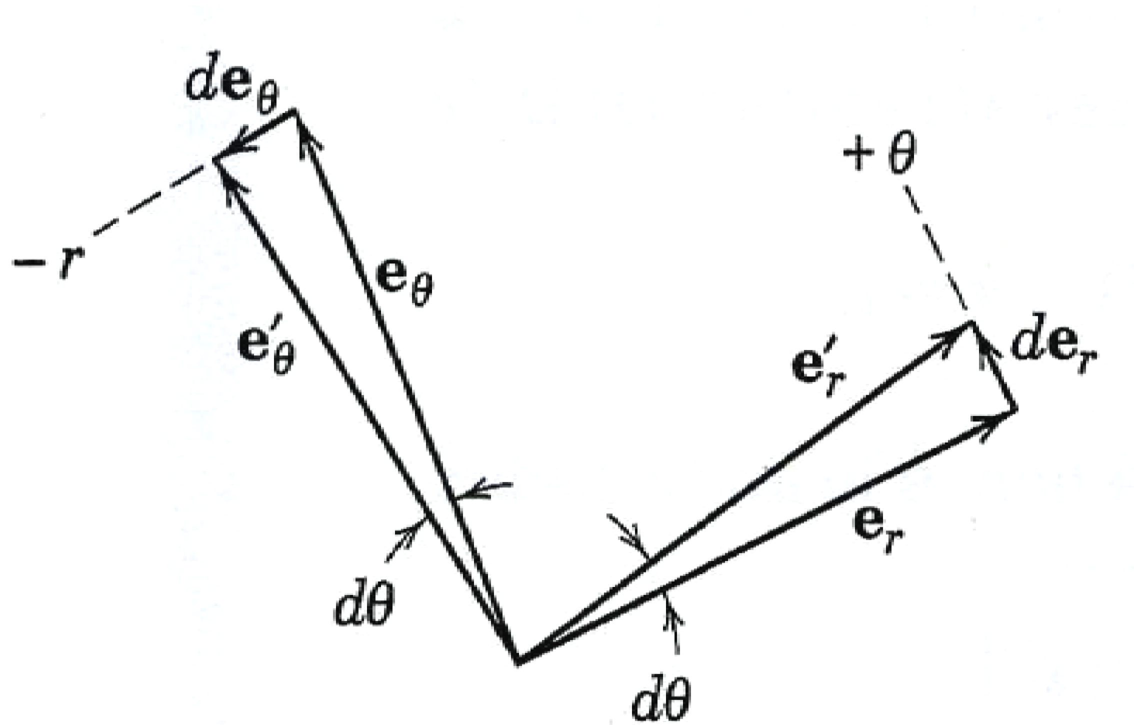
Derivative of the Base Vectors

$$\boxed{\dot{\mathbf{e}}_r = \dot{\theta}\,\mathbf{e}_\theta}$$

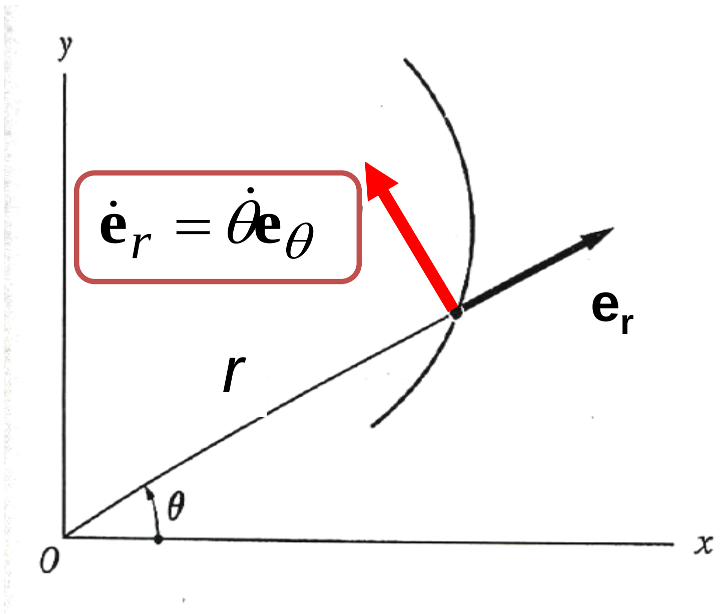
$$\boxed{\dot{\mathbf{e}}_\theta = -\dot{\theta}\,\mathbf{e}_r}$$

$\boxed{\dot{\theta}}$ angular velocity

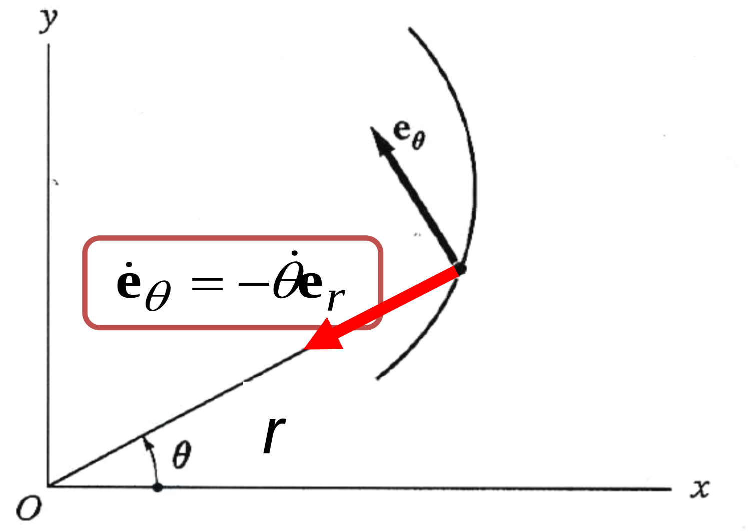
During time Δt



Characteristics of Derivatives of the Base Vectors



Direction of $\mathbf{e}_r$



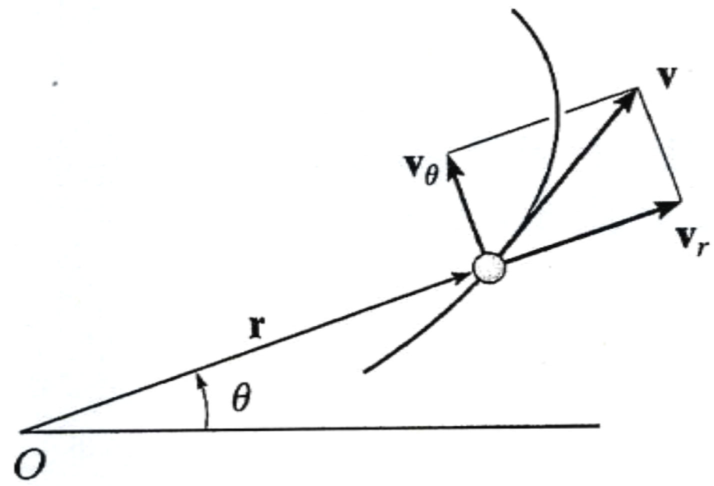
Direction of $\mathbf{e}_\theta$

Velocity in Polar Coordinates

$$\mathbf{v} = v_r \mathbf{e}_r + v_\theta \mathbf{e}_\theta$$

$$\boxed{v_r = \dot{r}} \qquad \boxed{v_\theta = r\dot{\theta}}$$

$\boxed{\dot{\theta} =}$ Angular velocity



Velocity has two components:
radial (v_r) and transverse (v_θ)

The two components of velocity are mutually perpendicular

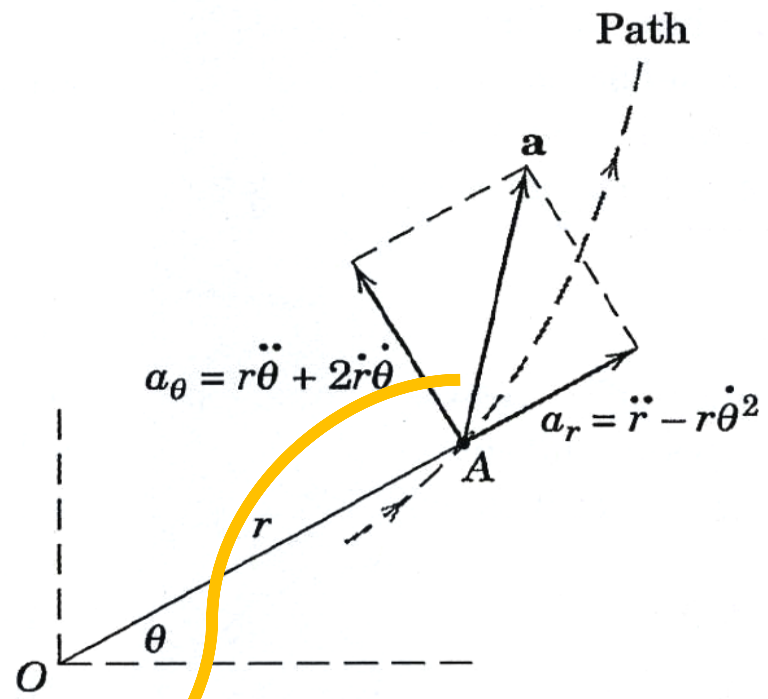
Acceleration in Polar Coordinates

$$\boxed{\mathbf{a} = a_r \mathbf{e}_r + a_\theta \mathbf{e}_\theta}$$

$$\boxed{a_r = \ddot{r} - r\dot{\theta}^2}$$

$$\boxed{a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}}$$

$\boxed{\ddot{\theta}}$ angular acceleration



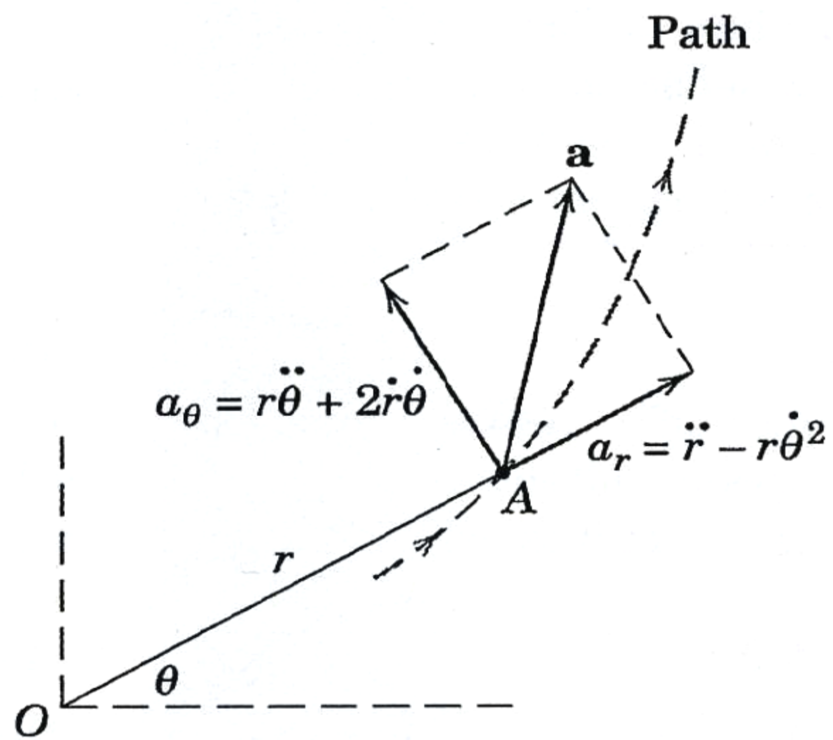
What is the magnitude & direction of acceleration?

$$\boxed{a = |\boldsymbol{a}| = \sqrt{a_r^2 + a_\theta^2}}$$

Do not Confuse

$$\boxed{a_r \neq \dot{v}_r}$$

$$\boxed{a_\theta \neq \dot{v}_\theta}$$



Circular Motion using Polar Coordinates

constant radius r

$$v_r = 0$$

$$v_\theta = r\dot{\theta}$$

$$a_r = -r\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta}$$