3 Kinetics of Particles

CHAPTER OUTLINE

3/1 Introduction

According to Newton's second law, a particle will accelerate when it is subjected to unbalanced forces. Kinetics is the study of the relations between unbalanced forces and the resulting changes in motion. In Chapter 3 we will study the kinetics of particles. This topic requires that we combine our knowledge of the properties of forces, which we developed in statics, and the kinematics of particle motion just covered in Chapter 2. With the aid of Newton's second law, we can combine these two topics and solve engineering problems involving force, mass, and motion.

The three general approaches to the solution of kinetics problems are: (A) direct application of Newton's second law (called the force-mass-acceleration method), (B) use of work and energy principles, and (C) solution by impulse and momentum methods. Each approach has its special characteristics and advantages, and Chapter 3 is subdivided into Sections A, B, and C, according to these three methods of solution. In addition, a fourth section, Section D, treats special applications and combinations of the three basic approaches. Before proceeding, you should review carefully the definitions and concepts of Chapter 1, because they are fundamental to the developments which follow.

SECTION A FORCE, MASS, AND ACCELERATION

3/2 Newton's Second Law

The basic relation between force and acceleration is found in Newton's second law, Eq. 1/1, the verification of which is entirely experimental. We now describe the fundamental meaning of this law by considering an ideal experiment in which force and acceleration are assumed to be measured without error. We subject a mass particle to the action of a single force $\mathbf{F}_1$, and we measure the acceleration $\mathbf{a}_1$ of the particle in the primary inertial system.* The ratio F_1/a_1 of the magnitudes of the force and the acceleration will be some number C_1 whose value depends on the units used for measurement of force and acceleration. We then repeat the experiment by subjecting the same particle to a different force $\mathbf{F}_2$ and measuring the corresponding acceleration $\mathbf{a}_2$. The ratio F_2/a_2 of the magnitudes will again produce a number C_2. The experiment is repeated as many times as desired.

We draw two important conclusions from the results of these experiments. First, the ratios of applied force to corresponding acceleration all equal the *same* number, provided the units used for measurement are not changed in the experiments. Thus,

$$\frac{F_1}{a_1} = \frac{F_2}{a_2} = \cdots = \frac{F}{a} = C, \qquad \text{a constant}$$

We conclude that the constant C is a measure of some invariable property of the particle. This property is the *inertia* of the particle, which is its *resistance to rate of change of velocity*. For a particle of high inertia (large C), the acceleration will be small for a given force F. On the other hand, if the inertia is small, the acceleration will be large. The mass m is used as a quantitative measure of inertia, and therefore, we may write the expression $C = km$, where k is a constant introduced

*The primary inertial system or astronomical frame of reference is an imaginary set of reference axes which are assumed to have no translation or rotation in space. See Art. 1/2, Chapter 1.

to account for the units used. Thus, we may express the relation obtained from the experiments as

$$F = kma \tag{3/1}$$

where F is the magnitude of the resultant force acting on the particle of mass m, and a is the magnitude of the resulting acceleration of the particle.

The second conclusion we draw from this ideal experiment is that the acceleration is always in the direction of the applied force. Thus, Eq. 3/1 becomes a *vector* relation and may be written

$$\mathbf{F} = km\mathbf{a} \tag{3/2}$$

Although an actual experiment cannot be performed in the ideal manner described, the same conclusions have been drawn from countless accurately performed experiments. One of the most accurate checks is given by the precise prediction of the motions of planets based on Eq. 3/2.

Inertial System

Although the results of the ideal experiment are obtained for measurements made relative to the "fixed" primary inertial system, they are equally valid for measurements made with respect to any nonrotating reference system which translates with a constant velocity with respect to the primary system. From our study of relative motion in Art. 2/8, we know that the acceleration measured in a system translating with no acceleration is the same as that measured in the primary system. Thus, Newton's second law holds equally well in a nonaccelerating system, so that we may define an *inertial system* as any system in which Eq. 3/2 is valid.

If the ideal experiment described were performed on the surface of the earth and all measurements were made relative to a reference system attached to the earth, the measured results would show a slight discrepancy from those predicted by Eq. 3/2, because the measured acceleration would not be the correct absolute acceleration. The discrepancy would disappear when we introduced the correction due to the acceleration components of the earth. These corrections are negligible for most engineering problems which involve the motions of structures and machines on the surface of the earth. In such cases, the accelerations measured with respect to reference axes attached to the surface of the earth may be treated as "absolute," and Eq. 3/2 may be applied with negligible error to experiments made on the surface of the earth.*

*As an example of the magnitude of the error introduced by neglect of the motion of the earth, consider a particle which is allowed to fall from rest (relative to earth) at a height h above the ground. We can show that the rotation of the earth gives rise to an eastward acceleration (Coriolis acceleration) relative to the earth and, neglecting air resistance, that the particle falls to the ground a distance

$$x = \frac{2}{3}\omega\sqrt{\frac{2h^3}{g}}\cos\gamma$$

east of the point on the ground directly under that from which it was dropped. The angular velocity of the earth is $\omega = 0.729(10^{-4})$ rad/s, and the latitude, north or south, is γ. At a latitude of 45° and from a height of 200 m, this eastward deflection would be $x = 43.9$ mm.

An increasing number of problems occur, particularly in the fields of rocket and spacecraft design, where the acceleration components of the earth are of primary concern. For this work it is essential that the fundamental basis of Newton's second law be thoroughly understood and that the appropriate absolute acceleration components be employed.

Before 1905 the laws of Newtonian mechanics had been verified by innumerable physical experiments and were considered the final description of the motion of bodies. The concept of *time*, considered an absolute quantity in the Newtonian theory, received a basically different interpretation in the theory of relativity announced by Einstein in 1905. The new concept called for a complete reformulation of the accepted laws of mechanics. The theory of relativity was subjected to early ridicule, but has been verified by experiment and is now universally accepted by scientists. Although the difference between the mechanics of Newton and that of Einstein is basic, there is a practical difference in the results given by the two theories only when velocities of the order of the speed of light (300×10^6 m/s) are encountered.* Important problems dealing with atomic and nuclear particles, for example, require calculations based on the theory of relativity.

Systems of Units

It is customary to take k equal to unity in Eq. 3/2, thus putting the relation in the usual form of Newton's second law

$$\boxed{\mathbf{F} = m\mathbf{a}} \qquad \textbf{[1/1]}$$

A system of units for which k is unity is known as a *kinetic* system. Thus, for a kinetic system the units of force, mass, and acceleration are not independent. In SI units, as explained in Art. 1/4, the units of force (newtons, N) are derived by Newton's second law from the base units of mass (kilograms, kg) times acceleration (meters per second squared, m/s^2). Thus, $N = kg \cdot m/s^2$. This system is known as an *absolute* system since the unit for force is dependent on the absolute value of mass.

In U.S. customary units, on the other hand, the units of mass (slugs) are derived from the units of force (pounds force, lb) divided by acceleration (feet per second squared, ft/sec^2). Thus, the mass units are slugs = $lb\text{-}sec^2/ft$. This system is known as a *gravitational* system since mass is derived from force as determined from gravitational attraction.

For measurements made relative to the rotating earth, the relative value of g should be used. The internationally accepted value of g relative to the earth at sea level and at a latitude of 45° is 9.806 65 m/s^2. Except where greater precision is required, the value of 9.81 m/s^2 will be used for g. For measurements relative to a nonrotating earth, the absolute value of g should be used. At a latitude of 45° and at sea level, the absolute value is 9.8236 m/s^2. The sea-level variation in both

*The theory of relativity demonstrates that there is no such thing as a preferred primary inertial system and that measurements of time made in two coordinate systems which have a velocity relative to one another are different. On this basis, for example, the principles of relativity show that a clock carried by the pilot of a spacecraft traveling around the earth in a circular polar orbit of 644 km altitude at a velocity of 27 080 km/h would be slow compared with a clock at the pole by 0.000 001 85 s for each orbit.

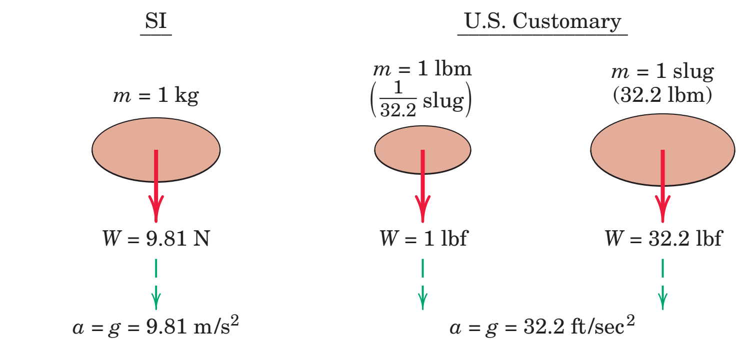
the absolute and relative values of g with latitude is shown in Fig. 1/1 of Art. 1/5.

In the U.S. customary system, the standard value of g relative to the rotating earth at sea level and at a latitude of 45° is 32.1740 ft/sec^2. The corresponding value relative to a nonrotating earth is 32.2230 ft/sec^2.

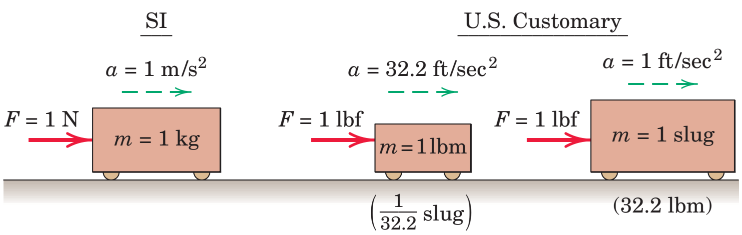
Force and Mass Units

We need to use both SI units and U.S. customary units, so we must have a clear understanding of the correct force and mass units in each system. These units were explained in Art. 1/4, but it will be helpful to illustrate them here using simple numbers before applying Newton's second law. Consider, first, the free-fall experiment as depicted in Fig. 3/1*a* where we release an object from rest near the surface of the earth. We allow it to fall freely under the influence of the force of gravitational attraction W on the body. We call this force the *weight* of the body. In SI units for a mass $m = 1$ kg, the weight is $W = 9.81$ N, and the corresponding downward acceleration a is $g = 9.81$ m/s^2. In U.S. customary units for a mass $m = 1$ lbm (1/32.2 slug), the weight is $W =$ 1 lbf and the resulting gravitational acceleration is $g = 32.2$ ft/sec^2. For a mass $m = 1$ slug (32.2 lbm), the weight is $W = 32.2$ lbf and the acceleration, of course, is also $g = 32.2$ ft/sec^2.

In Fig. 3/1*b* we illustrate the proper units with the simplest example where we accelerate an object of mass m along the horizontal with a force F. In SI units (an absolute system), a force $F = 1$ N causes a mass $m = 1$ kg to accelerate at the rate $a = 1$ m/s^2. Thus, 1 N = 1 kg·m/s^2. In the U.S. customary system (a gravitational system), a force $F = 1$ lbf



(*a*) Gravitational Free-Fall



(*b*) Newton's Second Law

Figure 3/1

causes a mass $m = 1$ lbm (1/32.2 slug) to accelerate at the rate $a = 32.2$ ft/sec^2, whereas a force $F = 1$ lbf causes a mass $m = 1$ slug (32.2 lbm) to accelerate at the rate $a = 1$ ft/sec^2.

We note that in SI units where the mass is expressed in kilograms (kg), the weight W of the body in newtons (N) is given by $W = mg$, where $g = 9.81$ m/s^2. In U.S. customary units, the weight W of a body is expressed in pounds force (lbf), and the mass in slugs (lbf-sec^2/ft) is given by $m = W/g$, where $g = 32.2$ ft/sec^2.

In U.S. customary units, we frequently speak of the weight of a body when we really mean mass. It is entirely proper to specify the mass of a body in pounds (lbm) which must be converted to mass in slugs before substituting into Newton's second law. Unless otherwise stated, the pound (lb) is normally used as the unit of force (lbf).

3/3 Equation of Motion and Solution of Problems

When a particle of mass m is subjected to the action of concurrent forces $\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3, \ldots$ whose vector sum is $\Sigma\mathbf{F}$, Eq. 1/1 becomes

$$\boxed{\Sigma\mathbf{F} = m\mathbf{a}} \tag{3/3}$$

When applying Eq. 3/3 to solve problems, we usually express it in scalar component form with the use of one of the coordinate systems developed in Chapter 2. The choice of an appropriate coordinate system depends on the type of motion involved and is a vital step in the formulation of any problem. Equation 3/3, or any one of the component forms of the force-mass-acceleration equation, is usually called the *equation of motion*. The equation of motion gives the instantaneous value of the acceleration corresponding to the instantaneous values of the forces which are acting.

Two Types of Dynamics Problems

We encounter two types of problems when applying Eq. 3/3. In the first type, the acceleration of the particle is either specified or can be determined directly from known kinematic conditions. We then determine the corresponding forces which act on the particle by direct substitution into Eq. 3/3. This problem is generally quite straightforward.

In the second type of problem, the forces acting on the particle are specified and we must determine the resulting motion. If the forces are constant, the acceleration is also constant and is easily found from Eq. 3/3. When the forces are functions of time, position, or velocity, Eq. 3/3 becomes a differential equation which must be integrated to determine the velocity and displacement.

Problems of this second type are often more formidable, as the integration may be difficult to carry out, particularly when the force is a mixed function of two or more motion variables. In practice, it is frequently necessary to resort to approximate integration techniques, either numerical or graphical, particularly when experimental data are involved. The procedures for a mathematical integration of the acceleration when it is a function of the motion variables were developed in Art. 2/2, and these same procedures apply when the force is a specified function of these same parameters, since force and acceleration differ only by the constant factor of the mass.

Constrained and Unconstrained Motion

There are two physically distinct types of motion, both described by Eq. 3/3. The first type is *unconstrained* motion where the particle is free of mechanical guides and follows a path determined by its initial motion and by the forces which are applied to it from external sources. An airplane or rocket in flight and an electron moving in a charged field are examples of unconstrained motion.

The second type is *constrained* motion where the path of the particle is partially or totally determined by restraining guides. An ice-hockey puck is partially constrained to move in the horizontal plane by the surface of the ice. A train moving along its track and a collar sliding along a fixed shaft are examples of more fully constrained motion. Some of the forces acting on a particle during constrained motion may be applied from outside sources, and others may be the reactions on the particle from the constraining guides. *All forces*, both applied and reactive, which act *on* the particle must be accounted for in applying Eq. 3/3.

The choice of an appropriate coordinate system is frequently indicated by the number and geometry of the constraints. Thus, if a particle is free to move in space, as is the center of mass of the airplane or rocket in free flight, the particle is said to have *three degrees of freedom* since three independent coordinates are required to specify the position of the particle at any instant. All three of the scalar components of the equation of motion would have to be integrated to obtain the space coordinates as a function of time.

If a particle is constrained to move along a surface, as is the hockey puck or a marble sliding on the curved surface of a bowl, only two coordinates are needed to specify its position, and in this case it is said to have *two degrees of freedom*. If a particle is constrained to move along a fixed linear path, as is the collar sliding along a fixed shaft, its position may be specified by the coordinate measured along the shaft. In this case, the particle would have only *one degree of freedom*.

Key Concepts

FREE-BODY DIAGRAM

When applying any of the force-mass-acceleration equations of motion, you must account correctly for *all* forces acting on the particle. The only forces which we may neglect are those whose magnitudes are negligible compared with other forces acting, such as the forces of mutual attraction between two particles compared with their attraction to a celestial body such as the earth. The vector sum $\Sigma\mathbf{F}$ of Eq. 3/3 means the vector sum of *all* forces acting *on* the particle in question. Likewise, the corresponding scalar force summation in any one of the component directions means the sum of the components of *all* forces acting *on* the particle in that particular direction.

The only reliable way to account accurately and consistently for every force is to *isolate* the particle under consideration from *all* contacting and influencing bodies and replace the bodies removed by the forces they exert on the particle isolated. The resulting *free-body diagram* is the means by which every force, known and unknown, which acts on the particle is represented and thus accounted for. Only after this vital step has been completed should you write the appropriate equation or equations of motion.

The free-body diagram serves the same key purpose in dynamics as it does in statics. This purpose is simply to establish a *thoroughly reliable method* for the correct evaluation of the resultant of all actual forces acting on the particle or body in question. In statics this resultant equals zero, whereas in dynamics it is equated to the product of mass and acceleration. When you use the vector form of the equation of motion, remember that it represents several scalar equations and that every equation must be satisfied.

Careful and consistent use of the *free-body method* is the *most important single lesson* to be learned in the study of engineering mechanics. When drawing a free-body diagram, clearly indicate the coordinate axes and their positive directions. When you write the equations of motion, make sure all force summations are consistent with the choice of these positive directions. As an aid to the identification of external forces which act on the body in question, these forces are shown as heavy red vectors in the illustrations in this book. Sample Problems 3/1 through 3/5 in the next article contain five examples of free-body diagrams. You should study these to see how the diagrams are constructed.

In solving problems, you may wonder how to get started and what sequence of steps to follow in arriving at the solution. This difficulty may be minimized by forming the habit of first recognizing some relationship between the desired unknown quantity in the problem and other quantities, known and unknown. Then determine additional relationships between these unknowns and other quantities, known and unknown. Finally, establish the dependence on the original data and develop the procedure for the analysis and computation. A few minutes spent organizing the plan of attack through recognition of the dependence of one quantity on another will be time well spent and will usually prevent groping for the answer with irrelevant calculations.

3/4 Rectilinear Motion

We now apply the concepts discussed in Arts. 3/2 and 3/3 to problems in particle motion, starting with rectilinear motion in this article and treating curvilinear motion in Art. 3/5. In both articles, we will analyze the motions of bodies which can be treated as particles. This simplification is possible as long as we are interested only in the motion of the mass center of the body. In this case we may treat the forces as concurrent through the mass center. We will account for the action of nonconcurrent forces on the motions of bodies when we discuss the kinetics of rigid bodies in Chapter 6.

If we choose the x-direction, for example, as the direction of the rectilinear motion of a particle of mass m, the acceleration in the y- and z-directions will be zero and the scalar components of Eq. 3/3 become

$$\begin{aligned} \Sigma F_x &= ma_x \\ \Sigma F_y &= 0 \\ \Sigma F_z &= 0 \end{aligned} \tag{3/4}$$

For cases where we are not free to choose a coordinate direction along the motion, we would have in the general case all three component equations

$$\begin{aligned} \Sigma F_x &= ma_x \\ \Sigma F_y &= ma_y \\ \Sigma F_z &= ma_z \end{aligned} \tag{3/5}$$

where the acceleration and resultant force are given by

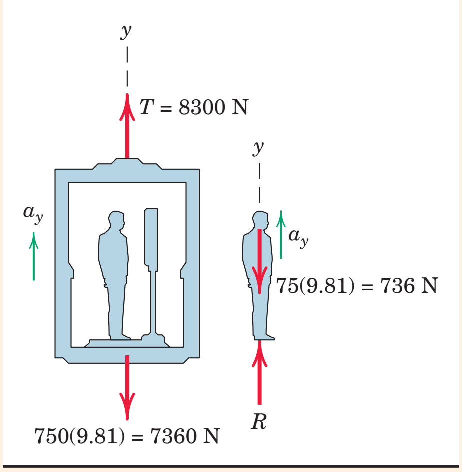
$$\begin{aligned} \mathbf{a} &= a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k} \\ a &= \sqrt{a_x{}^2 + a_y{}^2 + a_z{}^2} \\ \Sigma\mathbf{F} &= \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k} \\ |\Sigma\mathbf{F}| &= \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2} \end{aligned}$$

Koji Sasahara/AP Images

This view of a car-collision test suggests that very large accelerations and accompanying large forces occur throughout the system of the two cars. The crash dummies are also subjected to large forces, primarily by the shoulder-harness/seat-belt restraints.

Sample Problem 3/1

A 75-kg man stands on a spring scale in an elevator. During the first 3 seconds of motion from rest, the tension T in the hoisting cable is 8300 N. Find the reading R of the scale in newtons during this interval and the upward velocity v of the elevator at the end of the 3 seconds. The total mass of the elevator, man, and scale is 750 kg.



Solution. The force registered by the scale and the velocity both depend on the acceleration of the elevator, which is constant during the interval for which the forces are constant. From the free-body diagram of the elevator, scale, and man taken together, the acceleration is found to be

$$[\Sigma F_y = ma_y] \qquad 8300 - 7360 = 750a_y \qquad a_y = 1.257 \text{ m/s}^2$$

The scale reads the downward force exerted on it by the man's feet. The equal and opposite reaction R to this action is shown on the free-body diagram of the man alone together with his weight, and the equation of motion for him gives

① $$[\Sigma F_y = ma_y] \qquad R - 736 = 75(1.257) \qquad R = 830 \text{ N} \qquad \textit{Ans.}$$

The velocity reached at the end of the 3 seconds is

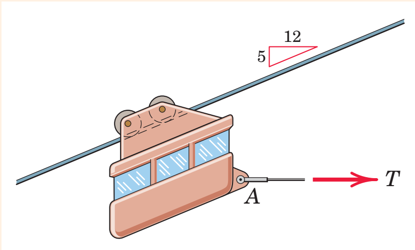
$$\left[\Delta v = \int a\, dt\right] \qquad v - 0 = \int_0^3 1.257\, dt \qquad v = 3.77 \text{ m/s} \qquad \textit{Ans.}$$

Helpful Hint

① If the scale were calibrated in kilograms, it would read 830/9.81 = 84.6 kg which, of course, is not his true mass since the measurement was made in a noninertial (accelerating) system. *Suggestion:* Rework this problem in U.S. customary units.

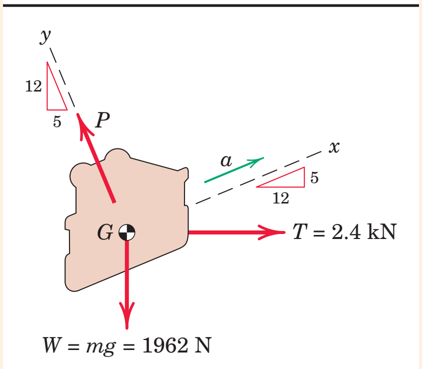
Sample Problem 3/2

A small inspection car with a mass of 200 kg runs along the fixed overhead cable and is controlled by the attached cable at A. Determine the acceleration of the car when the control cable is horizontal and under a tension $T = 2.4$ kN. Also find the total force P exerted by the supporting cable on the wheels.



Solution. The free-body diagram of the car and wheels taken together and treated as a particle discloses the 2.4-kN tension T, the weight $W = mg = 200(9.81) = 1962$ N, and the force P exerted on the wheel assembly by the cable.

The car is in equilibrium in the y-direction since there is no acceleration in this direction. Thus,



$$[\Sigma F_y = 0] \qquad P - 2.4(\tfrac{5}{13}) - 1.962(\tfrac{12}{13}) = 0 \qquad P = 2.73 \text{ kN} \qquad \textit{Ans.}$$

① In the x-direction the equation of motion gives

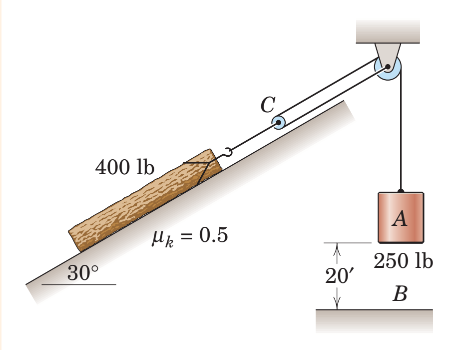
$$[\Sigma F_x = ma_x] \qquad 2400(\tfrac{12}{13}) - 1962(\tfrac{5}{13}) = 200a \qquad a = 7.30 \text{ m/s}^2 \qquad \textit{Ans.}$$

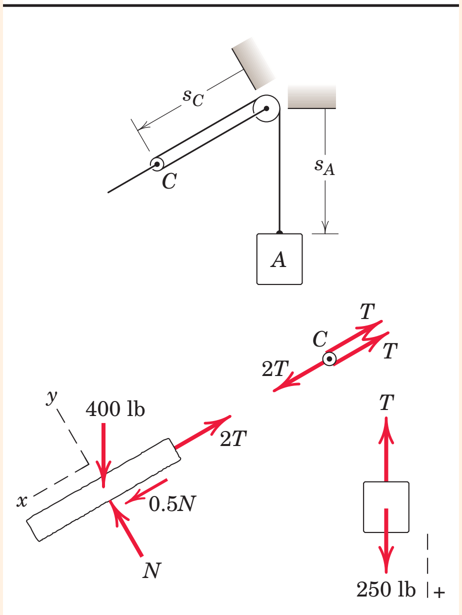
Helpful Hint

① By choosing our coordinate axes along and normal to the direction of the acceleration, we are able to solve the two equations independently. Would this be so if x and y were chosen as horizontal and vertical?

Sample Problem 3/3

The 250-lb concrete block A is released from rest in the position shown and pulls the 400-lb log up the 30° ramp. If the coefficient of kinetic friction between the log and the ramp is 0.5, determine the velocity of the block as it hits the ground at B.





Solution. The motions of the log and the block A are clearly dependent. Although by now it should be evident that the acceleration of the log up the incline is half the downward acceleration of A, we may prove it formally. The constant total length of the cable is $L = 2s_C + s_A + \text{constant}$, where the con-
① stant accounts for the cable portions wrapped around the pulleys. Differentiating twice with respect to time gives $0 = 2\ddot{s}_C + \ddot{s}_A$, or

$$0 = 2a_C + a_A$$

We assume here that the masses of the pulleys are negligible and that they turn with negligible friction. With these assumptions the free-body diagram of the pulley C discloses force and moment equilibrium. Thus, the tension in the cable attached to the log is twice that applied to the block. Note that the accelerations of the log and the center of pulley C are identical.

The free-body diagram of the log shows the friction force $\mu_k N$ for motion up the plane. Equilibrium of the log in the y-direction gives

② $[\Sigma F_y = 0]$ $\quad N - 400 \cos 30° = 0 \qquad N = 346 \text{ lb}$

and its equation of motion in the x-direction gives

$[\Sigma F_x = ma_x]$ $\quad 0.5(346) - 2T + 400 \sin 30° = \dfrac{400}{32.2} a_C$

For the block in the positive downward direction, we have

③ $[+\downarrow \Sigma F = ma]$ $\quad 250 - T = \dfrac{250}{32.2} a_A$

Solving the three equations in a_C, a_A, and T gives us

$$a_A = 5.83 \text{ ft/sec}^2 \qquad a_C = -2.92 \text{ ft/sec}^2 \qquad T = 205 \text{ lb}$$

④ For the 20-ft drop with constant acceleration, the block acquires a velocity

$[v^2 = 2ax]$ $\quad v_A = \sqrt{2(5.83)(20)} = 15.27 \text{ ft/sec}$ *Ans.*

Helpful Hints

① The coordinates used in expressing the final kinematic constraint relationship must be consistent with those used for the kinetic equations of motion.

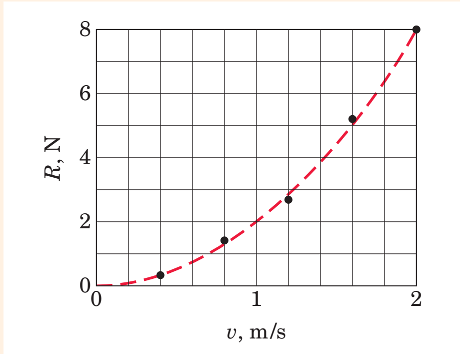
② We can verify that the log will indeed move up the ramp by calculating the force in the cable necessary to initiate motion from the equilibrium condition. This force is $2T = 0.5N + 400 \sin 30° = 373$ lb or $T = 186.5$ lb, which is less than the 250-lb weight of block A. Hence, the log will move up.

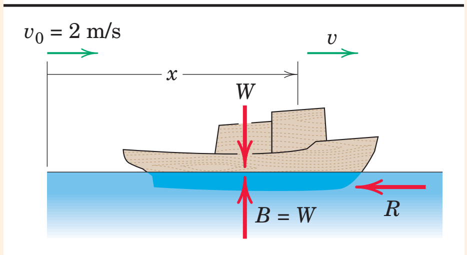
③ Note the serious error in assuming that $T = 250$ lb, in which case, block A would not accelerate.

④ Because the forces on this system remain constant, the resulting accelerations also remain constant.

Sample Problem 3/4

The design model for a new ship has a mass of 10 kg and is tested in an experimental towing tank to determine its resistance to motion through the water at various speeds. The test results are plotted on the accompanying graph, and the resistance R may be closely approximated by the dashed parabolic curve shown. If the model is released when it has a speed of 2 m/s, determine the time t required for it to reduce its speed to 1 m/s and the corresponding travel distance x.





Solution. We approximate the resistance–velocity relation by $R = kv^2$ and find k by substituting $R = 8$ N and $v = 2$ m/s into the equation, which gives $k = 8/2^2 = 2\ \text{N}\cdot\text{s}^2/\text{m}^2$. Thus, $R = 2v^2$.

The only horizontal force on the model is R, so that

① $$[\Sigma F_x = ma_x] \qquad -R = ma_x \qquad \text{or} \qquad -2v^2 = 10\frac{dv}{dt}$$

We separate the variables and integrate to obtain

$$\int_0^t dt = -5\int_2^v \frac{dv}{v^2} \qquad t = 5\left(\frac{1}{v} - \frac{1}{2}\right)\ \text{s}$$

Thus, when $v = v_0/2 = 1$ m/s, the time is $t = 5(\frac{1}{1} - \frac{1}{2}) = 2.5$ s. *Ans.*

The distance traveled during the 2.5 seconds is obtained by integrating $v = dx/dt$. Thus, $v = 10/(5 + 2t)$ so that

② $$\int_0^x dx = \int_0^{2.5} \frac{10}{5+2t}\,dt \qquad x = \frac{10}{2}\ln(5+2t)\Big|_0^{2.5} = 3.47\ \text{m} \qquad \textit{Ans.}$$

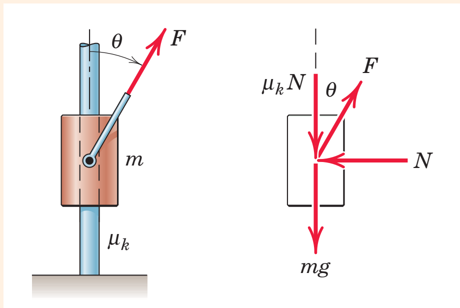
Helpful Hints

① Be careful to observe the minus sign for R.

② *Suggestion:* Express the distance x after release in terms of the velocity v and see if you agree with the resulting relation $x = 5\ln(v_0/v)$.

Sample Problem 3/5

The collar of mass m slides up the vertical shaft under the action of a force F of constant magnitude but variable direction. If $\theta = kt$ where k is a constant and if the collar starts from rest with $\theta = 0$, determine the magnitude F of the force which will result in the collar coming to rest as θ reaches $\pi/2$. The coefficient of kinetic friction between the collar and shaft is μ_k.



Solution. After drawing the free-body diagram, we apply the equation of motion in the y-direction to get

① $$[\Sigma F_y = ma_y] \qquad F\cos\theta - \mu_k N - mg = m\frac{dv}{dt}$$

where equilibrium in the horizontal direction requires $N = F\sin\theta$. Substituting $\theta = kt$ and integrating first between general limits give

$$\int_0^t (F\cos kt - \mu_k F\sin kt - mg)\,dt = m\int_0^v dv$$

which becomes

$$\frac{F}{k}[\sin kt + \mu_k(\cos kt - 1)] - mgt = mv$$

For $\theta = \pi/2$ the time becomes $t = \pi/2k$, and $v = 0$ so that

② $$\frac{F}{k}[1 + \mu_k(0 - 1)] - \frac{mg\pi}{2k} = 0 \qquad \text{and} \qquad F = \frac{mg\pi}{2(1-\mu_k)} \qquad \textit{Ans.}$$

Helpful Hints

① If θ were expressed as a function of the vertical displacement y instead of the time t, the acceleration would become a function of the displacement and we would use $v\,dv = a\,dy$.

② We see that the results do not depend on k, the rate at which the force changes direction.