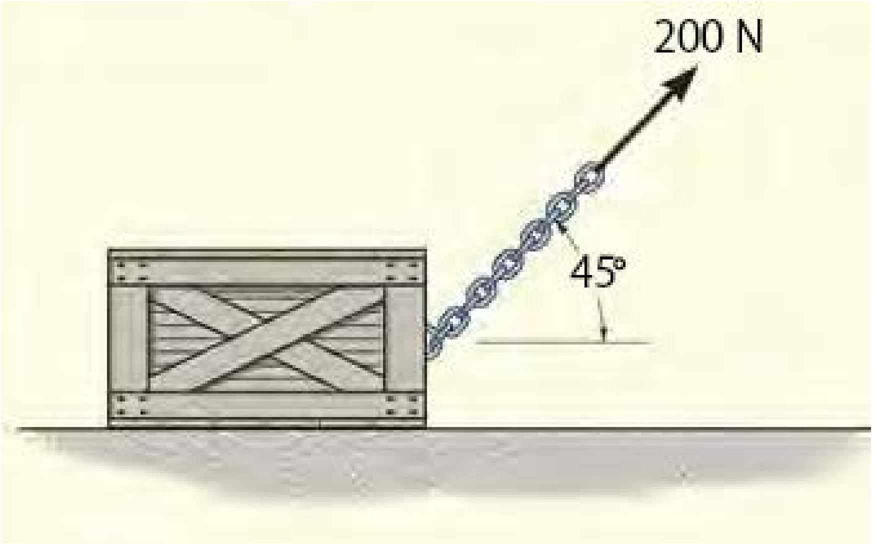
Exercises in Physics

Lecture 12
Impulse & Momentum

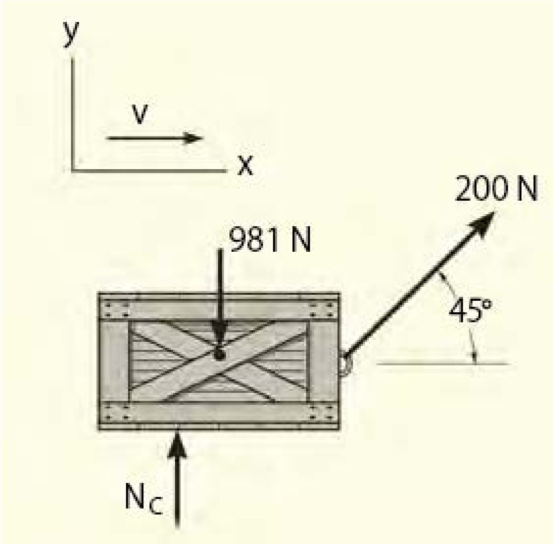
Sample Problem 1

The 100-kg crate is originally at rest on the smooth horizontal surface. If a towing force of 200 N, acting at an angle of 45°, is applied for 10 s, determine the final velocity and the normal force which the surface exerts on the crate during this time interval.

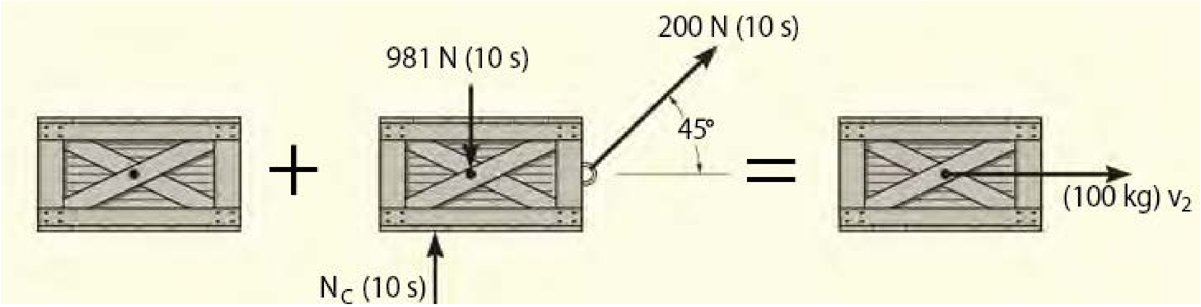


SOLUTION. This problem can be solved using the principle of impulse and momentum since it involves force, velocity, and time.

Solution



Free-Body Diagram. Since all the forces acting are *constant*, the impulses are simply the product of the force magnitude and 10 s, that is. $I = F(t_2 - t_1)$ Note the alternative procedure of drawing the crate's impulse and momentum diagrams is shown below.



Solution

Principle of Impulse and Momentum.

$$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x \mathrm{d}t = m(v_x)_2 \;\Rightarrow$$

$$0 + 200\mathrm{N} \cos 45^{\mathrm{o}} \times 10\mathrm{s} = 100\mathrm{kg}\; v_2 \;\Rightarrow\; v_2 = 14.1 m/s$$

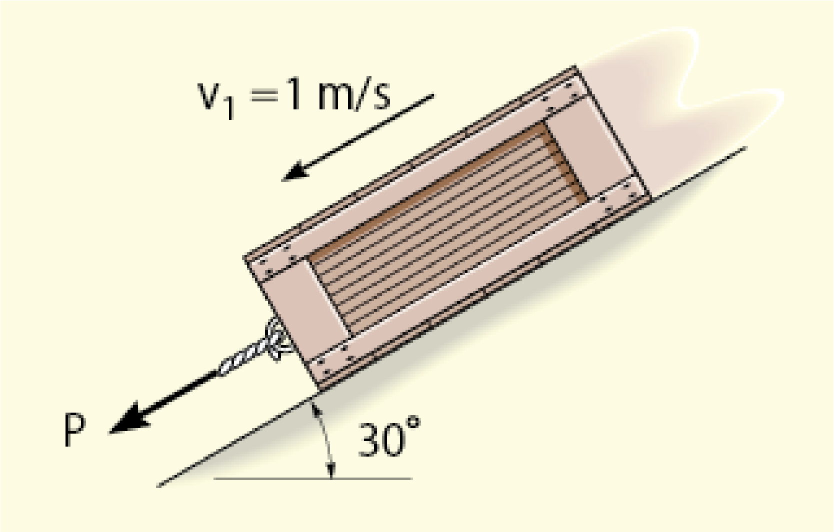
$$m(v_y)_1 + \sum \int_{t_1}^{t_2} F_y \mathrm{d}t = m(v_y)_2 \;\Rightarrow$$

$$0 + N_C \times 10\mathrm{s} - 981\mathrm{N} \times 10\mathrm{s} + 200\mathrm{N} \sin 45^{\mathrm{o}} \times 10\mathrm{s} = 0 \;\Rightarrow\; N_C = 840\mathrm{N}$$

NOTE: Since no motion occurs in the y direction, direct application of the equilibrium equation $\sum F_y = 0$ gives the same result for N_C. Try to solve the problem by first applying $\sum F_x = ma_x$, then $v = v_0 + a_x t$.

Sample Problem 2

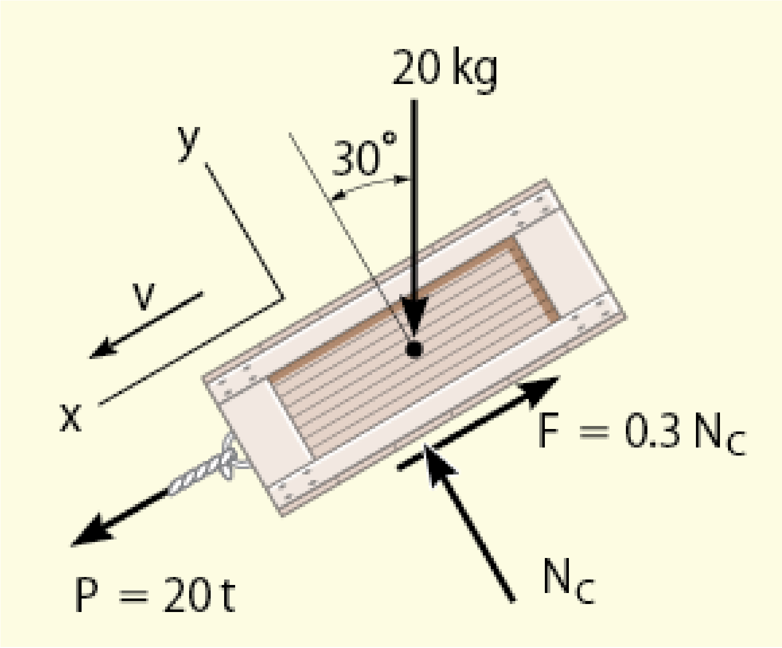
The 20-kg crate is acted upon by a force having a variable magnitude P = (20t) N, where t is in seconds. Determine the crate's velocity 2 s after P has been applied. The initial velocity is $v_1 = 1 \mathrm{m/s}$ down the plane, and the coefficient of kinetic friction between the crate and the plane is $\mu_k = 0.3$.



SOLUTION. This problem can be solved using the principle of impulse and momentum since it involves force, velocity, and time.

Solution

Free-Body Diagram. Since the magnitude of force P = 20t varies with time, the impulse it creates must be determined by integrating over the 2-s time interval.



Solution

Principle of Impulse and Momentum.

$$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x \mathrm{d}t = m(v_x)_2 \;\Rightarrow$$

$$m(v_x)_1 + \int_{t_1}^{t_2} P\mathrm{d}t - \int_{t_1}^{t_2} \mu_k N_C \mathrm{d}t + \int_{t_1}^{t_2} mg\sin 30^{\mathrm{o}} \mathrm{d}t = m(v_x)_2 \;\Rightarrow$$

$$20\times 1 + \int_0^2 20t\mathrm{d}t - 0.3\times N_C \times 2 + 20\times 9.81\sin 30^{\mathrm{o}} \times 2 = 20\; v_2 \;\Rightarrow$$

$$256.2 - 0.6\, N_C = 20 v_2$$

The equation of equilibrium can be applied in the y direction. Why?

$$N_C - 20\times 9.81\cos 30^{\mathrm{o}} = 0$$

Solving,

$$N_C = 169.914\,\mathrm{N}, \quad v_2 = 7.7126\,\mathrm{m/s}$$

Solution

NOTE: We can also solve this problem using the equation of motion.

$$\sum F_x = ma \;\Rightarrow$$

$$20t - \mu_k N_C + mg\sin 30^{\circ} = ma \;\Rightarrow$$

$$a = t - 0.3\times(169.914)/20 + 9.81/2 = t + 2.3563$$

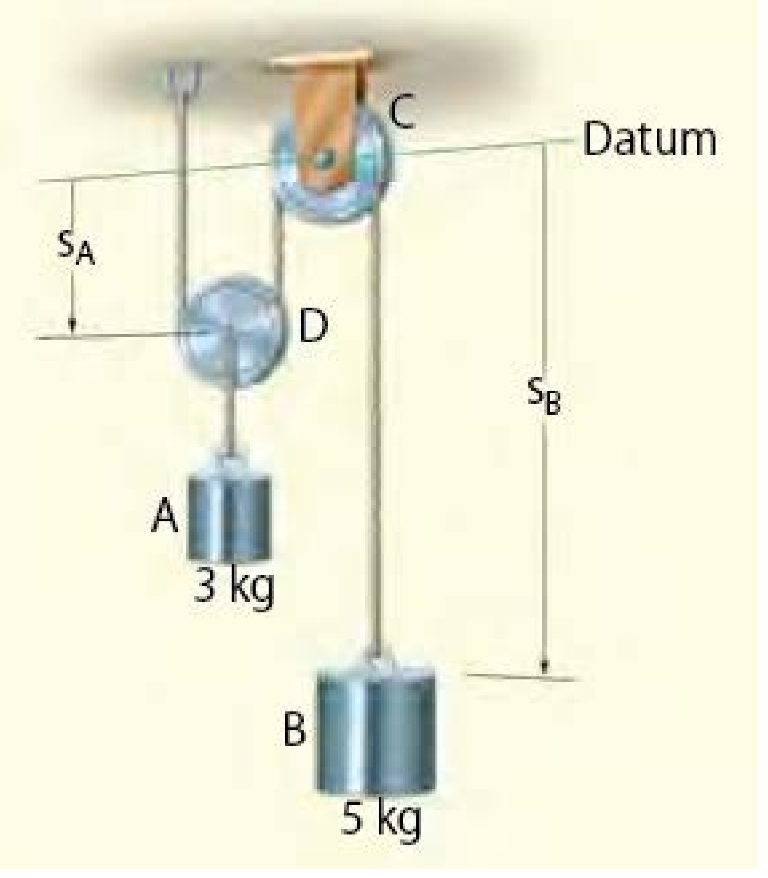
Using kinematics

$$\mathrm{d}v = a\mathrm{d}t \;\Rightarrow \int_1^v \mathrm{d}v = \int_0^2 a\mathrm{d}t = \int_0^2 (t + 2.3563)\mathrm{d}t \;\Rightarrow\; v = 7.7126\,\mathrm{m/s}$$

By comparison, application of the principle of impulse and momentum eliminates the need for using kinematics (a = dv/dt) and thereby yields an easier method for solution.

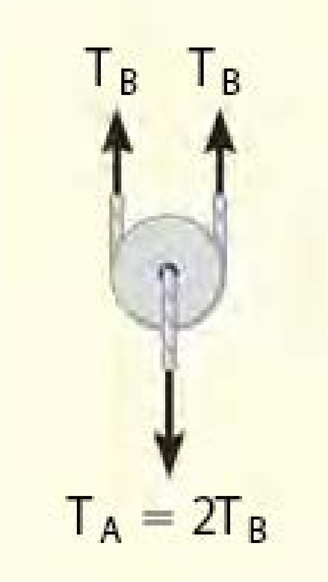
Sample Problem 3

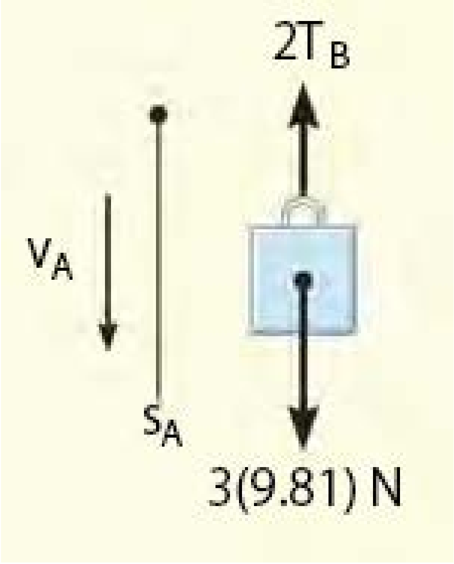
Blocks A and B have a mass of 3 kg and 5 kg, respectively. If the system is released from rest, determine the velocity of block B in 6 s. Neglect the mass of the pulleys and cord.

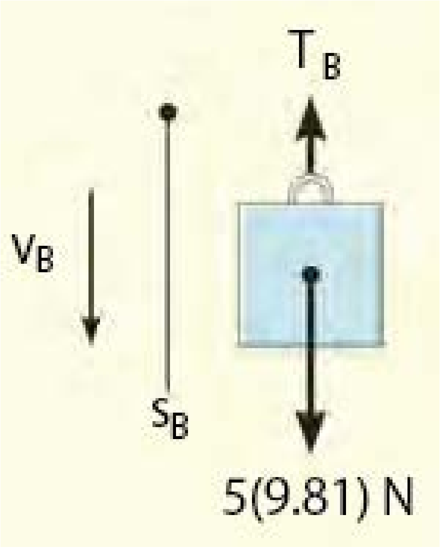


Solution

Free-Body Diagram. Since the weight of each block is constant, the cord tensions will also be constant. Furthermore, since the mass of pulley D is neglected, the cord tension $T_A = 2T_B$. Note that the blocks are both assumed to be moving downward in the positive coordinate directions, s_A and s_B .







Solution

Principle of Impulse and Momentum.

Block A:

$$m(v_A)_1 + \sum \int_{t_1}^{t_2} F_y \mathrm{d}t = m(v_A)_2 \;\Rightarrow$$

$$0 - 2T_B \times 6\,\mathrm{s} + 3 \times 9.81\mathrm{N} \times 6\mathrm{s} = 3\mathrm{kg}\;(v_A)_2$$

Block B:

$$m(v_B)_1 + \sum \int_{t_1}^{t_2} F_y \mathrm{d}t = m(v_B)_2 \;\Rightarrow$$

$$0 - T_B \times 6\,\mathrm{s} + 5 \times 9.81\mathrm{N} \times 6\mathrm{s} = 5\mathrm{kg}\;(v_B)_2$$

Solution

Kinematics. Since the blocks are subjected to dependent motion, the velocity of A can be related to that of B by using the kinematic analysis. A horizontal datum is established through the fixed point at C, and the position coordinates, s_A and s_B, are related to the constant total length l of the vertical segments of the cord by the equation

$$2s_A + s_B = l$$

Taking the time derivative yields.

$$2v_A + v_B = 0$$

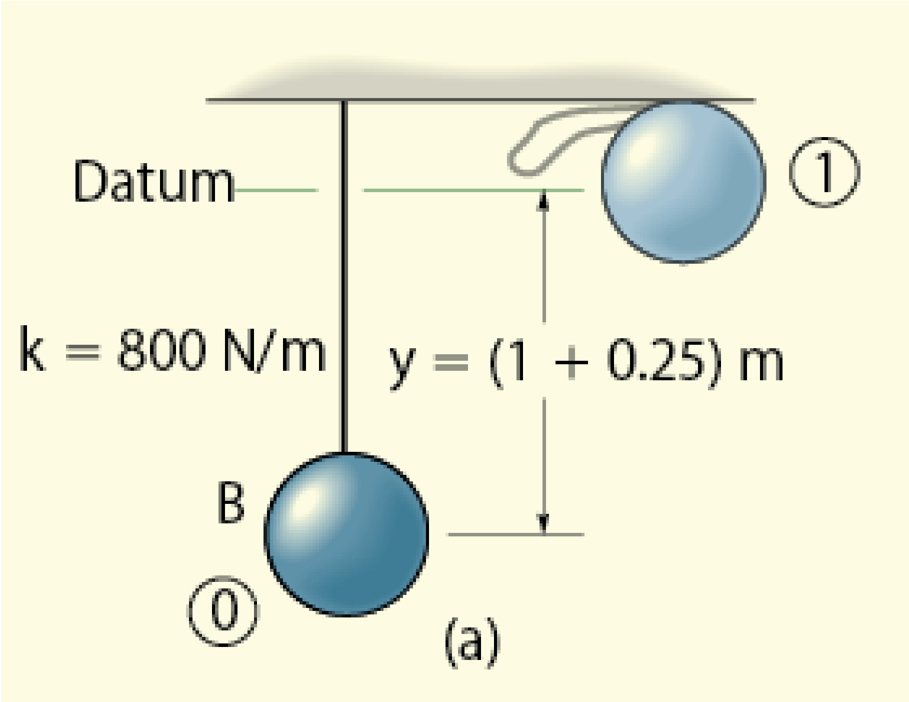
As indicated by the negative sign, when B moves downward A moves upward. Substituting this result into equations and solving them yields

$$(v_B)_2 = 35.8\,\mathrm{m/s}, \qquad T_B = 19.2\,\mathrm{N}$$

NOTE: Realize that the positive (downward) direction for v_A and v_B is consistent in the figures and equations. This is important since we are seeking a simultaneous solution of equations.

Sample Problem 4

Ball B has a mass of 1.5 kg and is suspended from the ceiling by a 1-m-long elastic cord. If the cord is *stretched* downward 0.25 m and the ball is released from rest, determine how far the cord stretches after the ball rebounds from the ceiling. The stiffness of the cord is k = 800 N/m, and the coefficient of restitution between the ball and ceiling is e = 0.8. The ball makes a central impact with the ceiling.



Solution

First we must obtain the velocity of the ball *just before* it strikes the ceiling using energy methods, then consider the impulse and momentum between the ball and ceiling, and finally again use energy methods to determine the stretch in the cord.

Conservation of Energy. With the datum located as shown in the figure, realizing that initially y = y0 = (1 + 0.25) m = 1.25 m, we have

$$T_0 + V_0 = T_1 + V_1 \;\Rightarrow\; \frac{1}{2}m(v_B)_0^2 - mgy_0 + \frac{1}{2}ks^2 = \frac{1}{2}m(v_B)_1^2 + 0 \;\Rightarrow$$

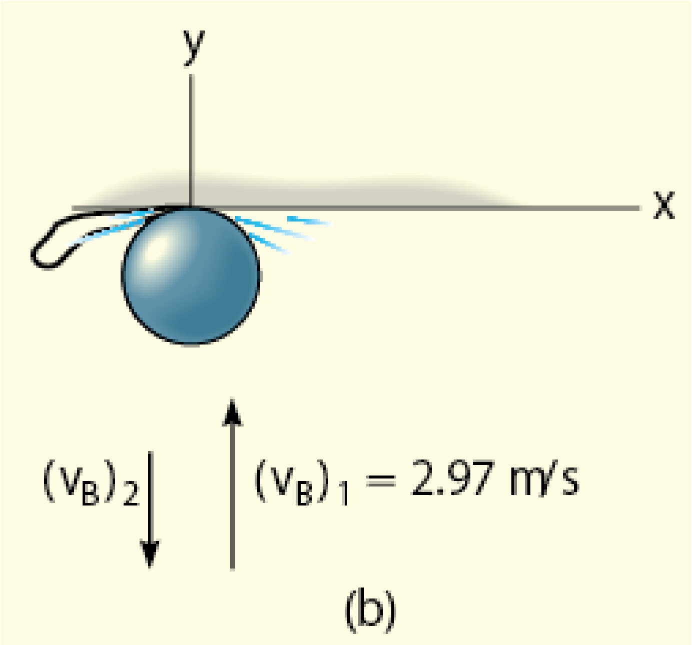
$$0 - 1.5 \times 9.81 \times 1.25 + \frac{1}{2}800 \times (0.25)^2 = \frac{1}{2}1.5 \times (v_B)_1^2 \;\Rightarrow$$

$$v_B = 2.698 \text{ m/s}$$

The interaction of the ball with the ceiling will now be considered using the principles of impact (*the weight of the ball is considered a non-impulsive force*). Since an unknown portion of the mass of the ceiling is involved in the impact, the conservation of momentum for the ball–ceiling system will not be written. The “velocity” of this portion of ceiling is zero since it (or the earth) are assumed to remain at rest both before and after impact.

Solution

Coefficient of Restitution.

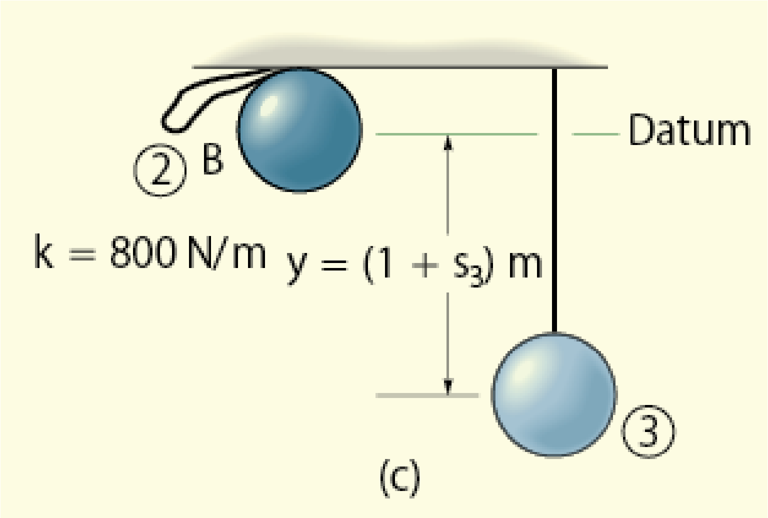


$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1} \quad \Rightarrow \quad 0.8 = \frac{(v_B)_2 - 0}{0 - (v_B)_1} \quad \Rightarrow$$

$$(v_B)_2 = -2.374\text{m / s}$$

Solution

Conservation of Energy.
The maximum stretch s3 in the cord can be determined by again applying the conservation of energy equation to the ball just after collision. Assuming that y = y3 = (1 + s3) m, then



$$T_2 + V_2 = T_3 + V_3 \quad \Rightarrow \frac{1}{2}m(v_B)_2^2 + 0 = \frac{1}{2}m(v_B)_3^2 + mgy_3 + \frac{1}{2}ks_3^2 \Rightarrow$$

$$\frac{1}{2}1.5 \times (2.374)^2 + 0 = 0 - 1.5 \times 9.81 \times (1 + s_3) + \frac{1}{2}800 \times s_3^2 \Rightarrow$$

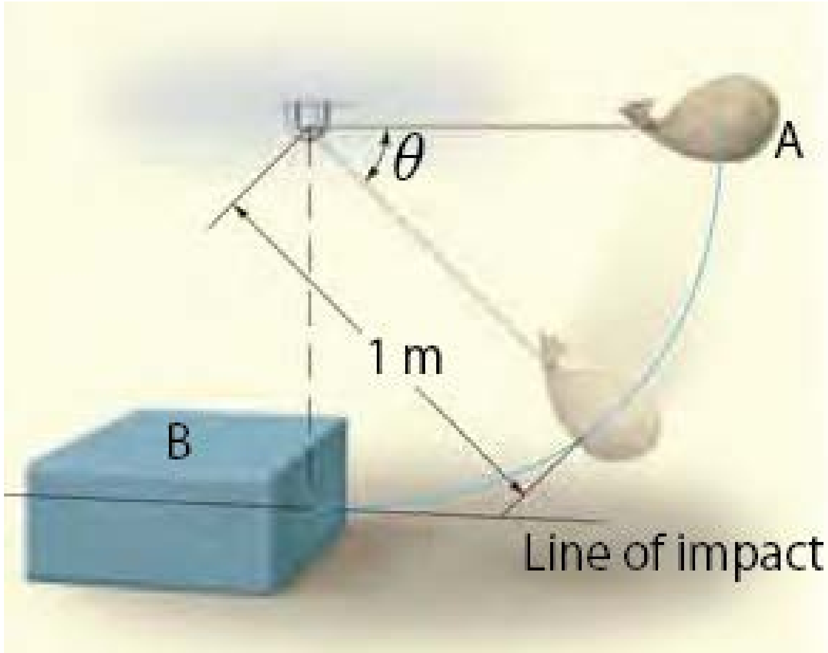
$$400s_3^2 - 14.715s_3 - 18.94 = 0$$

Solving this quadratic equation for the positive root yields

$$s_3 = 0.237\text{m}$$

Sample Problem 5

The bag A, having a weight of 3 kg, is released from rest at the position $\theta = 0^{o}$. After falling to $\theta = 90^{o}$, it strikes an 9-kg box B. If the coefficient of restitution between the bag and box is e = 0.5, determine the velocities of the bag and box just after impact. What is the loss of energy during collision?



Solution

This problem involves central impact. Why? Before analyzing the mechanics of the impact, however, it is first necessary to obtain the velocity of the bag *just before* it strikes the box.

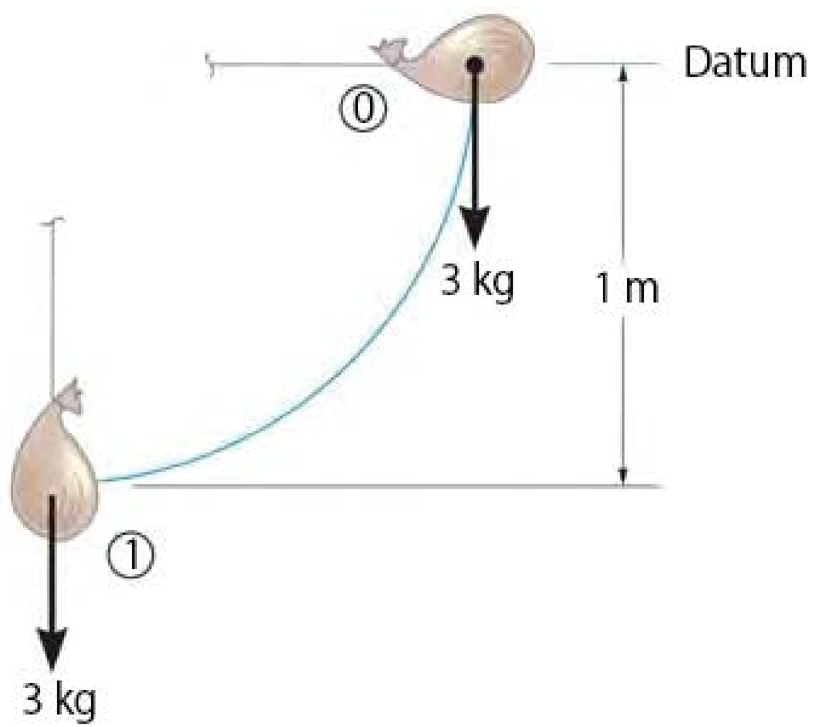
Conservation of Energy. With the datum at $\theta = 0^{o}$, we have

$$T_0 + V_0 = T_1 + V_1 \quad \Rightarrow$$

$$0 + 0 = \frac{1}{2} m_A (v_A)_1^2 + m_A g h_A \Rightarrow$$

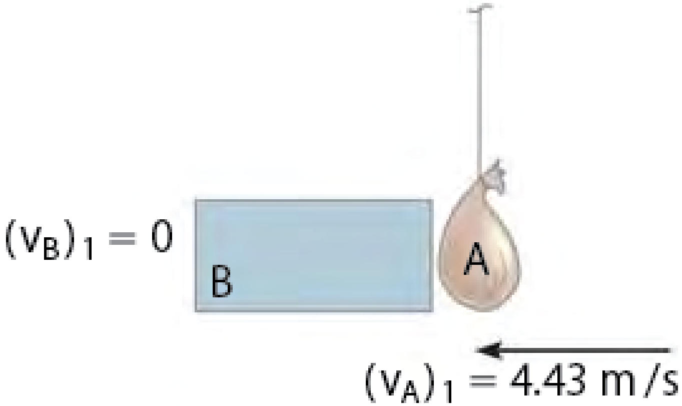
$$(v_A)_1 = \sqrt{2 g h_A} = \sqrt{2 \times 9.81 \times 1} \Rightarrow$$

$$(v_A)_1 = 4.43 \mathrm{m/s}$$



Solution

Conservation of Momentum. After impact we will assume A and B travel to the left. Applying the conservation of momentum to the system, we have



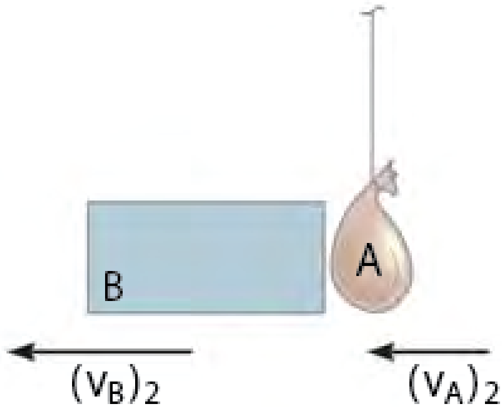
Just before impact

$$m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$\Rightarrow$$

$$3 \times 4.43 = 3(v_A)_2 + 9(v_B)_2$$

$$\Rightarrow (v_A)_2 = 4.43 - 3(v_B)_2$$



Just after impact

Solution

Coefficient of Restitution. Realizing that for separation to occur after collision $(v_B)_2 > (v_A)_2$, we have

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1} \Rightarrow 0.5 = \frac{(v_B)_2 - (v_A)_2}{4.43 - 0} \Rightarrow (v_A)_2 = (v_B)_2 - 2.2147$$

Solving equations with respect to $(v_B)_2, (v_A)_2$, we have

$$(v_A)_2 = -0.55\mathrm{m/s}, \qquad (v_B)_2 = 1.66\mathrm{m/s}$$

Loss of energy. Applying the principle of work and energy to the bag and box just before and just after collision, we have

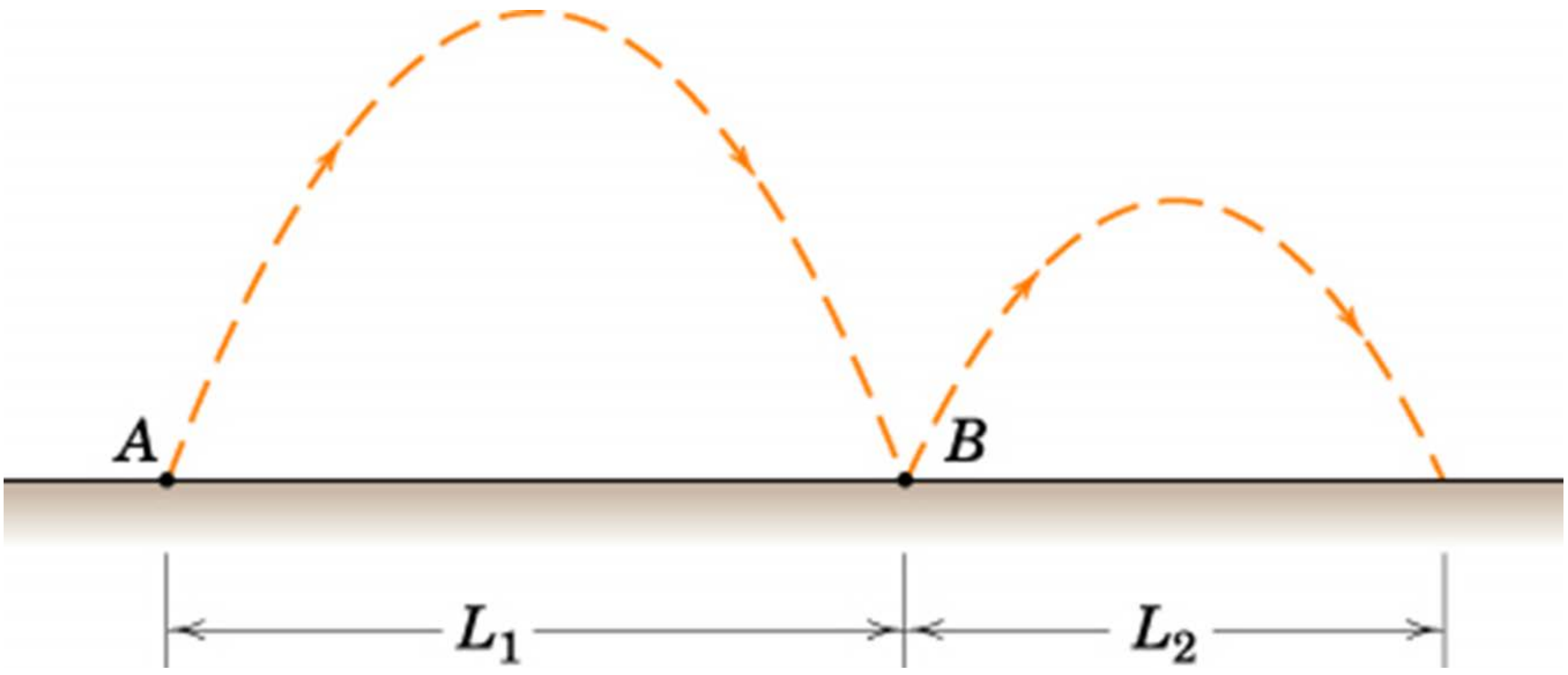
$$\sum U_{1-2} = T_2 - T_1 = \frac{1}{2} m_B (v_B)_2^2 + \frac{1}{2} m_A (v_A)_2^2 - \frac{1}{2} m_A (v_A)_1^2 \Rightarrow$$

$$\sum U_{1-2} = \frac{1}{2} 8 \times (1.66)^2 + \frac{1}{2} 3 \times (-0.55)^2 - \frac{1}{2} 3 \times (4.43)^2 = -17.96\,\mathrm{J}$$

NOTE: The energy loss occurs due to inelastic deformation during the collision.

Sample Problem 6

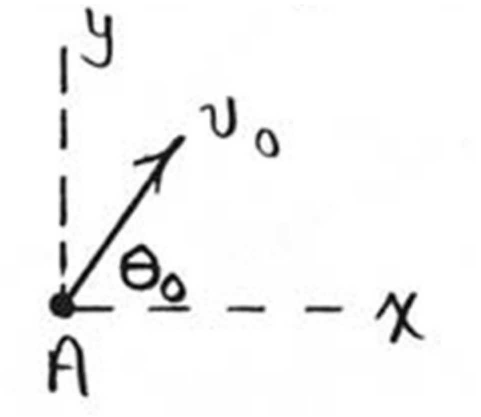
A projectile is launched from point A and has horizontal range L1 as shown. If the coefficient of restitution is e, determine the distance L2.



Solution

Kinematics.

Let the launch condition at point A be speed v_0 and the launch angle θ_0. The range L_1 is then defined (*refer to the projectile motion analysis (Chapter 2) or re-derive it by yourself by setting* $a_x = 0$ *and* $a_y = -g$) as



$$L_1 = \frac{2v_0^2 \sin\theta_0 \cos\theta_0}{g}$$

and the velocity components coming into point B are

$$v_x = v_0 \cos\theta_0, \qquad v_y = -v_0 \sin\theta_0$$

Solution

Coefficient of Restitution.

The velocity components right after impact at point B are

$$v_x = v_0 \cos\theta_0 , \qquad v_y = e\, v_0 \sin\theta_0 ,$$

which results into the range

$$L_2 = \frac{2ev_0^2 \sin\theta_0 \cos\theta_0}{g}$$

Hence,

$$L_2 = e\, L_1$$