Exercises in Physics
Assignment # 3

Date Given: April 21, 2022
Date Due: April 28, 2022

P1. (2 points) A particle which moves with curvilinear motion has coordinates in millimeters which vary with the time t in seconds according to $x = 3t^2 - 4t$ and $y = 4t^2 - t^3/3$. Determine the magnitudes of the velocity $\boldsymbol{v}$ and acceleration $\boldsymbol{a}$ and the angles which these two vectors make with x-axis when $t = 2\text{s}$.

Solution: Here we have $x = 3t^2 - 4t$, $v_x = \mathrm{d}x/\mathrm{d}t = 6t - 4$, $a_x = \mathrm{d}v_x/\mathrm{d}t = 6\text{mm/s}^2$, and $y = 4t^2 - t^3/3$, $v_y = \mathrm{d}y/\mathrm{d}t = 8t - t^2$, $a_y = \mathrm{d}v_y/\mathrm{d}t = 8 - 2t\text{mm/s}^2$.
When $t = 2\text{s}$, one obtains

- $v_x = 8\text{mm/s}$, $v_y = 12\text{mm/s}$, $v = \sqrt{v_x^2 + v_y^2} \approx 14.42\text{mm/s}$, and the angle between $\boldsymbol{v}$ and $\boldsymbol{i}$ is defined as follows: $\cos(\widehat{\boldsymbol{v},\boldsymbol{i}}) = v_x/v = \cos\alpha$, $\cos(\widehat{\boldsymbol{v},\boldsymbol{j}}) = v_y/v = \cos(\frac{\pi}{2} - (\widehat{\boldsymbol{v},\boldsymbol{i}})) = \sin(\widehat{\boldsymbol{v},\boldsymbol{i}}) = \sin\alpha$. Therefore, $\alpha = \arctan v_y/v_x \approx 56.3^\mathrm{o}$
- $a_x = 6\text{mm/s}$, $a_y = 4\text{mm/s}$, $a = \sqrt{a_x^2 + a_y^2} \approx 7.21\text{mm/s}$, and the angle between $\boldsymbol{a}$ and $\boldsymbol{i}$ is defined as follows: $\cos(\widehat{\boldsymbol{a},\boldsymbol{i}}) = a_x/a = \cos\beta$, $\cos(\widehat{\boldsymbol{a},\boldsymbol{j}}) = a_y/a = \cos(\frac{\pi}{2} - (\widehat{\boldsymbol{a},\boldsymbol{i}})) = \sin(\widehat{\boldsymbol{a},\boldsymbol{i}}) = \sin\beta$. Therefore, $\beta = \arctan a_y/a_x \approx 33.7^\mathrm{o}$

P2. (2 points) The position of a point that moves in the xy plane is given by $\boldsymbol{r} = \left(\frac{2}{3}t^3 - \frac{3}{2}t^2\right)\boldsymbol{i} + \frac{t^4}{12}\boldsymbol{j}$, where $\boldsymbol{r}$ is in meters and t is in seconds. Determine the angle between the velocity $\boldsymbol{v}$ and the acceleration $\boldsymbol{a}$ when (a) $t = 2\text{s}$ and (b) $t = 3\text{s}$.

Solution: Here we have $\boldsymbol{r} = \left(\frac{2}{3}t^3 - \frac{3}{2}t^2\right)\boldsymbol{i} + \frac{t^4}{12}\boldsymbol{j}$, $\boldsymbol{v} = \mathrm{d}\boldsymbol{r}/\mathrm{d}t = \left(2t^2 - 3t\right)\boldsymbol{i} + \frac{t^3}{3}\boldsymbol{j}$, $\boldsymbol{a} = \mathrm{d}\boldsymbol{v}/\mathrm{d}t = (4t - 3)\,\boldsymbol{i} + t^2\boldsymbol{j}$,

- For $t = 2\text{s}$ one obtains $\boldsymbol{v} = 2\boldsymbol{i} + \frac{8}{3}\boldsymbol{j}$, $\boldsymbol{a} = 5\boldsymbol{i} + 4\boldsymbol{j}$, and $\theta = \arccos\frac{\boldsymbol{v}\cdot\boldsymbol{a}}{|\boldsymbol{v}|\,|\boldsymbol{a}|} \approx 14.47^\mathrm{o}$.
- For $t = 3\text{s}$ one obtains $\boldsymbol{v} = 9\boldsymbol{i} + 9\boldsymbol{j}$, $\boldsymbol{a} = 9\boldsymbol{i} + 9\boldsymbol{j}$, and $\theta = 0^\mathrm{o}$ because $\boldsymbol{v} \parallel \boldsymbol{a}$.

P3. (2 points) The basketball player likes to release his foul shots at an angle $\theta = 50^\mathrm{o}$ to the horizontal as shown. What initial speed[1] v_0 will cause the ball to pass through the center of the rim?

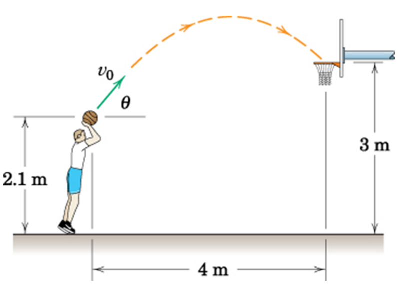

Figure 1: Illustration to Problem 3.

Solution: Use the conventional xy coordinate system with the origin at the release point. Then $x(t) = x_0 + v_{0x}t$ and $y(t) = y_0 + v_{y0}t - \frac{1}{2}gt^2$, where $x_0 = y_0 = 0$, $v_{0x} = v_0\cos\theta$, $v_{0x} = v_0\sin\theta$, and $\theta = 50^\mathrm{o}$.

[1]By the speed v_0 we understand the magnitude of the vector $\boldsymbol{v}_0$ shown in Figure 1.

Let t_f be the moment the ball reaches the hoop. Then $x(t_f) = 4$ and $y(t_f) = 3 - 2.1 = 0.9$. From equation for x coordinate, $4 = v_0 \cos 50^o t_f$, one obtains $t_f = 6.22/v_0$. Substitute this t_f into equation for y coordinate, $0.9 = v_0 \sin 50^o \left(\frac{6.22}{v_0}\right) - \frac{9.81}{2}\left(\frac{6.22}{v_0}\right)^2$. By solving this equation, one obtains $v_0 \approx 7.01$m/s.

P4. (4 points) A projectile is launched from point A with an initial speed[2] $v_0 = 30$m/s. Determine the value of the launch angle α for which the projectile will land at point B.

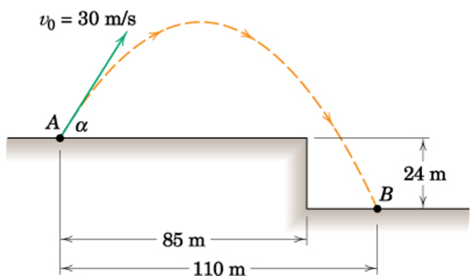

Figure 2: Illustration to Problem 4.

Solution: Use the conventional xy coordinate system with the origin at point A. Then $x(t) = x_0 + v_{0x}t$ and $y(t) = y_0 + v_{y0}t - \frac{1}{2}gt^2$, where $x_0 = y_0 = 0$, $v_{0x} = v_0 \cos\alpha$, $v_{0x} = v_0 \sin\alpha$, and $v_0 = 30$.

Let t_f be the moment the projectile reaches point B. Then $x(t_f) = 110$ and $y(t_f) = -24$. By solving the system

$$\begin{aligned} 110 &= 30(\cos\alpha)t_f \\ -24 &= 30(\sin\alpha)t_f - \frac{9.81}{2}t_f^2 \end{aligned}$$

with respect to α and t_f, one obtains two[3] possible solutions

(a) $t_f \approx 4.26$s, $\alpha \approx 30.5^o$

(b) $t_f \approx 5.39$s, $\alpha \approx 47.2^o$

To discriminate between these two solutions, let us check at what y coordinate the projectile will be when it reach the coordinate $x = 85$. Assume it will reach it some moment of time t_c.

(a) From $x(t_c) = 85 = (30\cos 30.5^o)t_c$ one obtains $t_c \approx 3.29$s. At this moment of time $y(t_c) = 30(\sin 30.5^o)t_c - \frac{9.81}{2}t_c^2 \approx -2.93$m. Therefore, conditions (a) are not possible.

(b) From $x(t_c) = 85 = (30\cos 47.2^o)t_c$ one obtains $t_c \approx 4.17$s. At this moment of time $y(t_c) = 30(\sin 47.2^o)t_c - \frac{9.81}{2}t_c^2 \approx 6.50$m. Therefore, conditions (b) represent positive clearance at the corner

Answer: $\alpha \approx 47.2^o$.

[2]By the speed v_0 we understand the magnitude of the vector $\boldsymbol{v}_0$ shown in Figure 2.

[3]From 1st equation we have $t_f = \frac{110}{30\cos\alpha}$. Substitute it to 2nd equation, $-24 = 30\sin\alpha\frac{110}{30\cos\alpha} - \frac{9.81}{2}\left(\frac{110}{30\cos\alpha}\right)^2 \Longrightarrow -24 = 110\tan\alpha - \frac{9.81}{2}\left(\frac{110^2}{30^2}\right)(1 + \tan^2\alpha)$. Solve this quadratic (with respect to $\tan\alpha$) equation and define two possible solutions for α and t_f.