Exercises in Physics
Assignment # 12

Date Given: June 30, 2022
Date Due: July 7, 2022

P1. (2 points) The ballistic pendulum is a simple device to measure projectile velocity v by observing the maximum angle θ to which the box of sand with embedded projectile swings. Calculate the angle θ if the 60-g projectile is fired horizontally into the suspended 20-kg box of sand with a velocity $v = 600\,\mathrm{m/s}$.

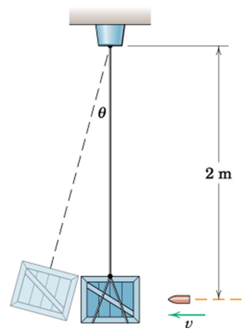

Figure 1: Illustration to Problem 1.

Solution: Let m be the mass of the bullet and M be the combined mass of the bullet and the sand box. Let u be the velocity of the combined bullet-box system right after the impact. As there are no external forces in the horizontal direction the linear momentum (in this direction, before and after the impact) is conserved and we can write

$$mv = Mu, \quad \Longrightarrow \quad u = \frac{m}{M}v = \frac{0.06}{20+0.06}600 \approx 1.795\mathrm{m/s}.$$

After the impact the pendulum moves under action of gravity only, and therefore we can use the energy conservation principle, $T_1 + V_1 = T_2 + V_2$. Here, $T_1 = \frac{1}{2}Mu^2$, $V_1 = 0$, $T_2 = 0$, and $V_2 = Mgh$, where $h = l(1 - \cos\theta)$ and $l = 2\,\mathrm{m}$. Therefore

$$\frac{1}{2}Mu^2 = Mgh = Mgl(1-\cos\theta) \quad \Longrightarrow \quad \cos\theta = 1 - \frac{u^2}{2gl} \quad \Longrightarrow \quad \cos\theta \approx 0.918$$

$$\Longrightarrow \quad \boxed{\theta \approx 0.408407\mathrm{rad} \approx 23.4^{\mathrm{o}}.}$$

P2. (2 points) A tennis player strikes the tennis ball with her racket while the ball is still rising. The ball speed before impact with the racket is $v_1 = 15\,\mathrm{m/s}$ and after impact its speed is $v_2 = 22\,\mathrm{m/s}$, with directions as shown in the figure. If the 60-g ball is in contact with the racket for 0.05s, determine the magnitude of the average force $\boldsymbol{R}$ exerted by the racket on the ball. Find the angle β made by $\boldsymbol{R}$ with the horizontal.

Solution: From the linear impulse-momentum equations we have

$$\begin{aligned} mv_{x_1} + \int_{t_1}^{t_2} \sum F_x \mathrm{d}t = mv_{x_2} \quad &\Longrightarrow \quad mv_{x_1} + R_x(t_2 - t_1) = mv_{x_2}, \\ mv_{y_1} + \int_{t_1}^{t_2} \sum F_y \mathrm{d}t = mv_{y_2} \quad &\Longrightarrow \quad mv_{y_1} + (R_y - mg)(t_2 - t_1) = mv_{y_2}, \end{aligned}$$

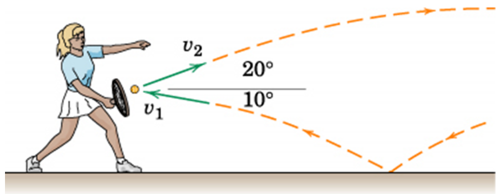

Figure 2: Illustration to Problem 2.

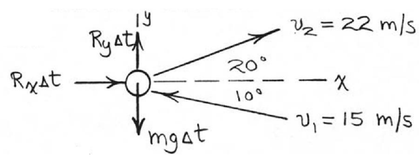

Figure 3: Illustration to Problem 2.

where $v_{x_1} = -v_1 \cos 10^o$, $v_{y_1} = v_1 \sin 10^o$, $v_{x_2} = v_2 \cos 20^o$, $v_{y_2} = v_2 \sin 20^o$. Therefore

$$\begin{aligned} R_x = m\tfrac{v_{x_2}-v_{x_1}}{t_2-t_1} &= 0.06\,\frac{22\cos 20^o + 15\cos 10^o}{0.05} \approx 42.5\,\mathrm{N}, \\ R_y = mg + m\tfrac{v_{y_2}-v_{y_1}}{t_2-t_1} &= 0.06\times 9.81 + 0.06\,\frac{22\sin 20^o - 15\sin 10^o}{0.05} \approx 6.49\,\mathrm{N}, \end{aligned}$$

and

$$\boxed{R = \sqrt{R_x^2 + R_y^2} \approx 43\,\mathrm{N}, \qquad \beta = \arctan\tfrac{R_y}{R_x} \approx 8.68^o.}$$

P3. (1 point) As a check of the basketball before the start of a game, the referee releases the ball from the overhead position shown, and the ball rebounds to about waist level. Determine the coefficient of restitution e and the percentage n of the original energy lost during the impact.

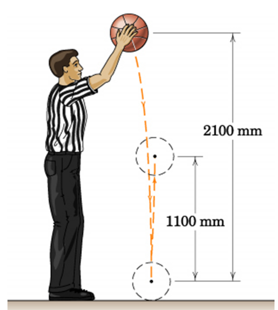

Figure 4: Illustration to Problem 3.

Solution: The velocities of the basketball before (v) and after (v') impact are related to the corresponding heights as

$$v = \sqrt{2gh}, \quad v' = \sqrt{2gh'}.$$

Therefore

$$\boxed{e = \frac{v'}{v} = \sqrt{\frac{h'}{h}} = \sqrt{\frac{1100}{2100}} \approx 0.724,}$$

and

$$\boxed{n = \frac{mgh - mgh'}{mgh}(100\%) = 1 - \frac{h'}{h}(100\%) \approx 47.6\%.}$$

P4. (3 points) A child throws a ball from point A with a speed of $15\,\mathrm{m/s}$. It strikes the wall at point B and then returns exactly to point A. Determine the necessary angle α if the coefficient of restitution in the wall impact is $e = 0.5$.

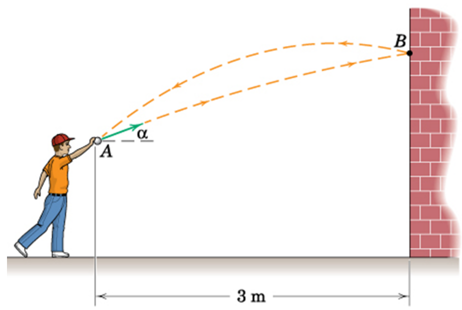

Figure 5: Illustration to Problem 4.

Solution: Set the coordinate system at point A. The launch condition at A are defined by speed $v_0 = 15\,\mathrm{m/s}$ and launch angle α. The components of the initial velocity are $v_{xA} = v_0\cos\alpha$ and $v_{yA} = v_0 \sin\alpha$.

- Motion from A to B. Here, we have

$$v_x(t) = v_{x_A}, \quad x(t) = v_{x_A} t, \qquad v_y(t) = v_{y_A} - gt, \quad y(t) = v_{y_A} t - \frac{1}{2} g t^2.$$

At point B $x_B = 3\,\mathrm{m}$, so we can find the traveling time

$$x_B = v_{x_A} t \quad \Longrightarrow \quad t = \frac{x_B}{v_{x_A}}.$$

Therefore, the components of the velocity just before the impact and the y-coordinate are

$$v_{x_B} = v_{x_A}, \qquad v_{y_B} = v_{y_A} - gt = v_{y_A} - \frac{g}{v_{x_A}} x_B, \qquad y_B = \frac{v_{y_A}}{v_{x_A}} x_B - \frac{1}{2}\frac{g}{v_{x_A}^2} x_B^2. \quad (1)$$

Figure 6: Illustration to Problem 4.

- Impact at B. The components of the after-impact velocity (denoted by prime) are defined as

$$v'_{y_B} = v_{y_B}, \qquad v'_{x_B} = -e v_{x_B}$$

- Motion from B to A. Here, we have new initial velocities (v'_{x_B}, and v'_{y_B}) and therefore

$$\begin{aligned} v'_x(t) &= v'_{x_B} = -e v_{x_B} & x'(t) &= x_B + v'_{x_B} t = x_B - e v_{x_B} t, \\ v'_y(t) &= v'_{y_B} - gt = v_{y_B} - gt, & y'(t) &= y_B + v'_{y_B} t - \tfrac{1}{2} g t^2 = y_B + v_{y_B} t - \tfrac{1}{2} g t^2 \end{aligned}$$

At point A we have $x_A = 0$, so we can find the traveling time

$$x_A = x_B + v'_{x_B} t = x_B - e v_{x_B} t = 0 \quad \Longrightarrow \quad t = \frac{x_B}{e v_{x_B}}.$$

If the ball returns to point A, then

$$y_A = y_B + v_{y_B} t - \frac{1}{2} g t^2 = y_B + v_{y_B} \frac{x_B}{e v_{x_B}} - \frac{1}{2} g \left(\frac{x_B}{e v_{x_B}} \right)^2 = 0. \tag{2}$$

- <u>Numerical analysis.</u> Substitute (1) into (2) and then replace $v_{xA} = v_0 \cos\alpha$, $v_{yA} = v_0 \sin\alpha$. After collecting the terms and using trigonometric identity ($\frac{1}{\cos^2\alpha} = 1 + \tan^2\alpha$), one obtains a quadratic (with respect to $\tan\alpha$) equation:

$$\frac{v_{y_A}}{v_{x_A}} x_B - \frac{1}{2} \frac{g}{v_{x_A}^2} x_B^2 + v_{y_B} \frac{x_B}{e v_{x_B}} - \frac{1}{2} g \left(\frac{x_B}{e v_{x_B}} \right)^2 = \frac{v_{y_A}}{v_{x_A}} x_B - \frac{1}{2} \frac{g}{v_{x_A}^2} x_B^2 + \frac{v_{y_B}}{e v_{x_A}} x_B - \frac{1}{2} g \left(\frac{x_B}{e v_{x_A}} \right)^2 =$$
$$\frac{v_{y_A}}{v_{x_A}} x_B - \frac{1}{2} \frac{g}{v_{x_A}^2} x_B^2 + \frac{v_{y_A}}{e v_{x_A}} x_B - \frac{g}{e v_{x_A}^2} x_B^2 - \frac{1}{2} g \left(\frac{x_B}{e v_{x_A}} \right)^2 = \left(1 + \frac{1}{e}\right) \frac{v_{y_A}}{v_{x_A}} x_B - \frac{1}{2} \frac{g}{v_{x_A}^2} \left(1 + \frac{2}{e} + \frac{1}{e^2}\right) x_B^2 =$$
$$\left(1 + \frac{1}{e}\right) x_B \tan\alpha - \frac{1}{2} \frac{g}{v_0^2} \left(1 + \frac{2}{e} + \frac{1}{e^2}\right) x_B^2 - \frac{1}{2} \frac{g}{v_0^2} \left(1 + \frac{2}{e} + \frac{1}{e^2}\right) x_B^2 \tan^2\alpha = 0.$$

In numbers, we have

$$0.18 g \tan^2\alpha - 9 \tan\alpha + 0.18 g = 0 \quad \Longrightarrow \quad \tan^2\alpha - \frac{50}{g} \tan\alpha + 1 = 0,$$

which admits two solutions:

$$\tan\alpha = \tfrac{1}{2} \tfrac{50}{g} - \tfrac{1}{2} \sqrt{\left(\tfrac{50}{g}\right)^2 - 4} \quad \Longrightarrow \quad \tan\alpha \approx 0.204397 \quad \Longrightarrow \quad \boxed{\alpha \approx 0.20162 \text{ rad} \approx 11.552^{\circ}.}$$
$$\tan\alpha = \tfrac{1}{2} \tfrac{50}{g} + \tfrac{1}{2} \sqrt{\left(\tfrac{50}{g}\right)^2 - 4} \quad \Longrightarrow \quad \tan\alpha \approx 4.89244 \quad \Longrightarrow \quad \boxed{\alpha \approx 1.36918 \text{ rad} \approx 78.448^{\circ}.}$$