Sample Problem 2/5

The curvilinear motion of a particle is defined by $v_x = 50 - 16t$ and $y = 100 - 4t^2$, where v_x is in meters per second, y is in meters, and t is in seconds. It is also known that $x = 0$ when $t = 0$. Plot the path of the particle and determine its velocity and acceleration when the position $y = 0$ is reached.

Solution. The x-coordinate is obtained by integrating the expression for v_x, and the x-component of the acceleration is obtained by differentiating v_x. Thus,

$$\left[\int dx = \int v_x\,dt\right] \qquad \int_0^x dx = \int_0^t (50 - 16t)\,dt \qquad x = 50t - 8t^2 \text{ m}$$

$$[a_x = \dot{v}_x] \qquad a_x = \frac{d}{dt}(50 - 16t) \qquad a_x = -16 \text{ m/s}^2$$

The y-components of velocity and acceleration are

$$[v_y = \dot{y}] \qquad v_y = \frac{d}{dt}(100 - 4t^2) \qquad v_y = -8t \text{ m/s}$$

$$[a_y = \dot{v}_y] \qquad a_y = \frac{d}{dt}(-8t) \qquad a_y = -8 \text{ m/s}^2$$

We now calculate corresponding values of x and y for various values of t and plot x against y to obtain the path as shown.

When $y = 0$, $0 = 100 - 4t^2$, so $t = 5$ s. For this value of the time, we have

$$v_x = 50 - 16(5) = -30 \text{ m/s}$$

$$v_y = -8(5) = -40 \text{ m/s}$$

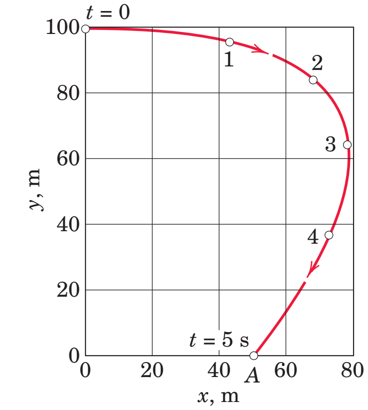
$$v = \sqrt{(-30)^2 + (-40)^2} = 50 \text{ m/s}$$

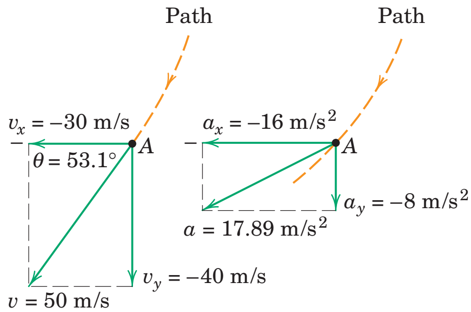
$$a = \sqrt{(-16)^2 + (-8)^2} = 17.89 \text{ m/s}^2$$

The velocity and acceleration components and their resultants are shown on the separate diagrams for point A, where $y = 0$. Thus, for this condition we may write

$$\mathbf{v} = -30\mathbf{i} - 40\mathbf{j} \text{ m/s} \qquad \textit{Ans.}$$

$$\mathbf{a} = -16\mathbf{i} - 8\mathbf{j} \text{ m/s}^2 \qquad \textit{Ans.}$$

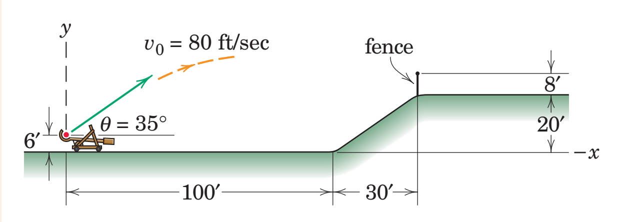




Helpful Hint

We observe that the velocity vector lies along the tangent to the path as it should, but that the acceleration vector is not tangent to the path. Note especially that the acceleration vector has a component that points toward the inside of the curved path. We concluded from our diagram in Fig. 2/5 that it is impossible for the acceleration to have a component that points toward the outside of the curve.

Sample Problem 2/6



A team of engineering students designs a medium-size catapult which launches 8-lb steel spheres. The launch speed is $v_0 = 80$ ft/sec, the launch angle is $\theta = 35°$ above the horizontal, and the launch position is 6 ft above ground level. The students use an athletic field with an adjoining slope topped by an 8-ft fence as shown. Determine:

(a) the time duration t_f of the flight

(b) the x-y coordinates of the point of first impact

(c) the maximum height h above the horizontal field attained by the ball

(d) the velocity (expressed as a vector) with which the projectile strikes the ground (or the fence)

Repeat part (b) for a launch speed of $v_0 = 75$ ft/sec.

Solution. We make the assumptions of constant gravitational acceleration ① and no aerodynamic drag. With the latter assumption, the 8-lb weight of the projectile is irrelevant. Using the given x-y coordinate system, we begin by checking the y-displacement at the horizontal position of the fence.

$$[x = x_0 + (v_x)_0 t] \qquad 100 + 30 = 0 + (80 \cos 35°)t \qquad t = 1.984 \text{ sec}$$

$$[y = y_0 + (v_y)_0 t - \tfrac{1}{2}gt^2] \quad y = 6 + 80 \sin 35°(1.984) - \tfrac{1}{2}(32.2)(1.984)^2 = 33.7 \text{ ft}$$

(a) Because the y-coordinate of the top of the fence is $20 + 8 = 28$ feet, the projectile clears the fence. We now find the flight time by setting $y = 20$ ft:

$$[y = y_0 + (v_y)_0 t - \tfrac{1}{2}gt^2] \quad 20 = 6 + 80 \sin 35°(t_f) - \tfrac{1}{2}(32.2)t_f^2 \quad t_f = 2.50 \text{ sec} \qquad \textit{Ans.}$$

$$[x = x_0 + (v_x)_0 t] \quad x = 0 + 80 \cos 35°(2.50) = 164.0 \text{ ft}$$

(b) Thus the point of first impact is $(x, y) = (164.0, 20)$ ft. *Ans.*

(c) For the maximum height:

② $$[v_y^2 = (v_y)_0^2 - 2g(y - y_0)] \quad 0^2 = (80 \sin 35°)^2 - 2(32.2)(h - 6) \quad h = 38.7 \text{ ft} \qquad \textit{Ans.}$$

(d) For the impact velocity:

$$[v_x = (v_x)_0] \qquad v_x = 80 \cos 35° = 65.5 \text{ ft/sec}$$

$$[v_y = (v_y)_0 - gt] \qquad v_y = 80 \sin 35° - 32.2(2.50) = -34.7 \text{ ft/sec}$$

So the impact velocity is $\mathbf{v} = 65.5\mathbf{i} - 34.7\mathbf{j}$ ft/sec. *Ans.*

If $v_0 = 75$ ft/sec, the time from launch to the fence is found by

$$[x = x_0 + (v_x)_0 t] \qquad 100 + 30 = (75 \cos 35°)t \qquad t = 2.12 \text{ sec}$$

and the corresponding value of y is

$$[y = y_0 + (v_y)_0 t - \tfrac{1}{2}gt^2] \quad y = 6 + 80 \sin 35°(2.12) - \tfrac{1}{2}(32.2)(2.12)^2 = 24.9 \text{ ft}$$

For this launch speed, we see that the projectile hits the fence, and the point of impact is

$$(x, y) = (130, 24.9) \text{ ft} \qquad \textit{Ans.}$$

For lower launch speeds, the projectile could land on the slope or even on the level portion of the athletic field.

Helpful Hints

① Neglecting aerodynamic drag is a poor assumption for projectiles with relatively high initial velocities, large sizes, and low weights. In a vacuum, a baseball thrown with an initial speed of 100 ft/sec at 45° above the horizontal will travel about 311 feet over a horizontal range. In sea-level air, the baseball range is about 200 ft, while a typical beachball under the same conditions will travel about 10 ft.

② As an alternative approach, we could find the time at apex where $v_y = 0$, then use that time in the y-displacement equation. Verify that the trajectory apex occurs over the 100-ft horizontal portion of the athletic field.