Exercises in Physics
Assignment

Date Given: June 2, 2022
Date Due: June 9, 2022

P1. (2 points) Determine the tension P in the cable which will give the 50kg block a steady acceleration of 2m/s^2 up the incline. The kinetic friction coefficient is given as $\mu_k = 0.25$.

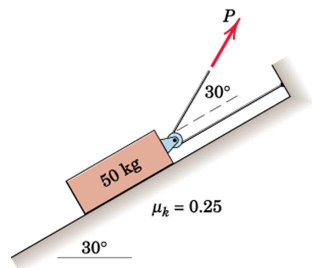

Figure 1: Illustration to Problem 1.

Solution: The forces are sketched in Figure 8. By projecting the Newton vector equation ($m\boldsymbol{a} =$

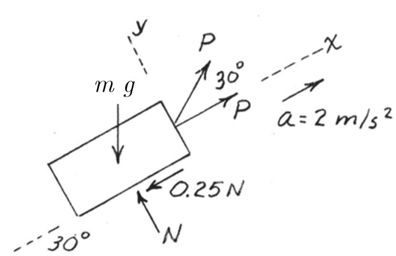

Figure 2: Illustration to Problem 1.

$\boldsymbol{F}$) onto x-axis, one obtains

$$ma_x = P + P\cos 30^{\text{o}} - mg\sin 30^{\text{o}} - \mu_k N,$$

where $m = 50\text{kg}$, $a_x = 2\text{m/s}^2$, $\mu_k = 0.25$, and $g = 9.81\text{m/s}^2$. Projection onto y-axis gives

$$ma_y = 0 = P\sin 30^{\text{o}} + N - mg\cos 30^{\text{o}}.$$

By solving these equations with respect to N and P, we obtain[1] $N \approx 311.415\text{N}$, and $\boxed{P \approx 226.741\text{N}.}$

P2. (3 points) Determine the vertical acceleration of the 30kg cylinder for each of the two cases. Neglect friction and the mass of the pulleys.

Solution: The difference between the two cases shown in Figure 3 is that in the first case the tensile force T is given explicitly ($T = 20g$) while in the second case it is to be found.

[1] Don't confuse here the normal reaction force N (shown in slanted font) the newtons N (not slanted).

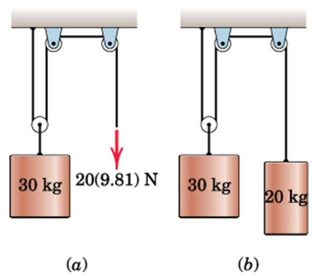

Figure 3: Illustration to Problem 2.

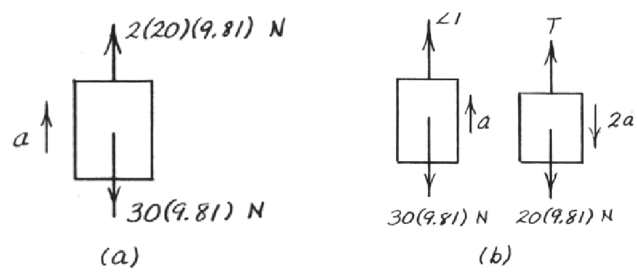

Figure 4: Illustration to Problem 2.

(a) Since the tensile force is known, we have

$$ma = 2T - mg,$$

where $m = 30\text{kg}$, $T = 20g$, and $g = 9.81\text{m/s}^2$. Therefore $\boxed{a = -g + 2T/m = g/3 = 3.27\text{m/s}^2.}$

(b) Here we have

$$m_1 a_1 = 2T - m_1 g, \qquad m_2 a_2 = T - m_2 g,$$

where $m_1 = 30\text{kg}$, $m_2 = 20\text{kg}$, a_1 is the acceleration of the first cylinder, and a_2 is that of the second one. If x_1 and x_2 are the corresponding positions, then by differentiating the constraint equation ($2x_1 + x_2 = \text{const}$) one obtains $2v_1 + v_2 = 0$ and $2a_1 + a_2 = 0$. Therefore $a_2 = -2a_1$, and we have

$$m_1 a_1 = 2T - m_1 g, \qquad 2m_2 a_1 = m_2 g - T.$$

By solving these equations with respect to a_1 and T, one obtains.

$$\boxed{a_1 = \frac{2m_2 - m_1}{m_1 + 4m_2} g = g/11 \approx 0.892\text{m/s}^2}, \qquad T = \frac{3m_1 m_2}{m_1 + 4m_2} g = 180g/11 \approx 160.5\text{N}.$$

P3. (2 points) A force P is applied to the initially stationary cart. Determine the velocity and displacement at time $t = 5\text{s}$ for each of the force histories $P = P_1(t)$ and $P = P_2(t)$. Neglect friction.

Solution: The problem is sketched in Figure 6. By projecting the Newton vector equation ($m\boldsymbol{a} = \boldsymbol{F}$) onto x-axis[2] one obtains

$$ma_x = P(t),$$

where the total mass $m = 10\text{kg}$, the time-depended force P needs to identified from the graph, and the initial conditions $v_x(0) = 0$ and $x(0) = 0$ (the block is at rest at $t = 0$).

[2]Projection onto y-axis gives $N - mg = 0$, but the normal reaction N does not contribute to motion along x-axis. .

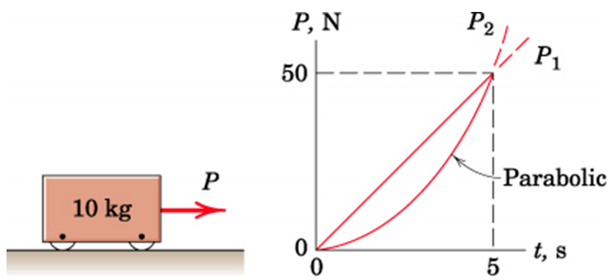

Figure 5: Illustration to Problem 3.

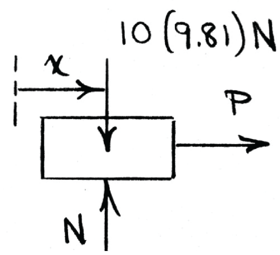

Figure 6: Illustration to Problem 3.

(a) $P = P_1(t) = kt$ and from the graph we find $k = 10$. Therefore

$$a_x = 10t/10 = t \quad \Longrightarrow \quad \mathrm{d}v_x = a_x(t)\mathrm{d}t \quad \Longrightarrow \quad v_x(t) = \int_0^t a_x(t)\mathrm{d}t = \int_0^t t\mathrm{d}t = t^2/2 \quad \text{and}$$

$$\mathrm{d}x/\mathrm{d}t = v_x(t) \quad \Longrightarrow \quad \mathrm{d}x = v_x(t)\mathrm{d}t \quad \Longrightarrow \quad x(t) = \int_0^t v_x(t)\mathrm{d}t = \int_0^t (t^2/2)\mathrm{d}t = t^3/6.$$

At $t = 5$s we have $v_x = 12.5$m/s and $x = 20.8$m.

(b) $P = P_2(t) = kt^2$, and from the graph we find $k = 2$. Therefore

$$a_x = 2t^2/10 = t^2/5 \quad \Longrightarrow \quad \mathrm{d}v_x = a_x(t)\mathrm{d}t \quad \Longrightarrow \quad v_x(t) = \int_0^t a_x(t)\mathrm{d}t = \int_0^t (t^2/5)\mathrm{d}t = t^3/15 \quad \text{and}$$

$$\mathrm{d}x/\mathrm{d}t = v_x(t) \quad \Longrightarrow \quad \mathrm{d}x = v_x(t)\mathrm{d}t \quad \Longrightarrow \quad x(t) = \int_0^t v_x(t)\mathrm{d}t = \int_0^t (t^3/15)\mathrm{d}t = t^4/60.$$

At $t = 5$s we have $\boxed{v_x = 8.33\text{m/s}}$ and $\boxed{x = 10.42\text{m.}}$

P4. (3 points) A small box is deposited by the conveyor belt onto the 30° ramp at A with velocity 0.8m/s. Calculate the distance s on the level surface BC at which the package comes to rest. The coefficient of kinetic friction for the box and supporting surface from a to C is 0.3

Solution: In solving this problem we need consider motion on parts AB and BC and use the fact that velocity at B is the same. The force diagrams for parts BC and AC are sketched in, respectively, left and right parts of Figure 8.

- Part AB. By projecting the Newton vector equation ($m\boldsymbol{a} = \boldsymbol{F}$) onto y-axis, one obtains

$$ma_y = 0 = N - mg\cos 30^{\circ} \quad \Longrightarrow \quad N = mg\cos 30^{\circ},$$

where $m =$ is the mass of the box, and and $g = 9.81\text{m/s}^2$. Next, projection onto x-axis gives

$$ma_x = mg\sin 30^{\circ} - \mu_k N = mg\sin 30^{\circ} - \mu_k mg\cos 30^{\circ} \quad \Longrightarrow \quad a_x = g\sin 30^{\circ} - \mu_k g\cos 30^{\circ} \approx 2.36\text{m/s}^2$$

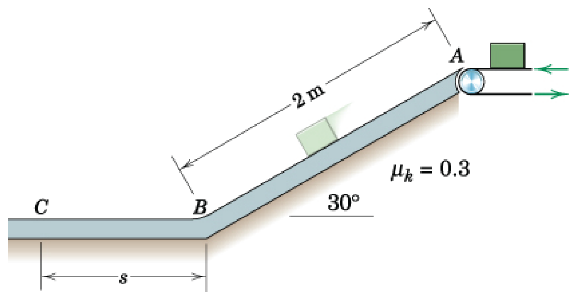

Figure 7: Illustration to Problem 4.

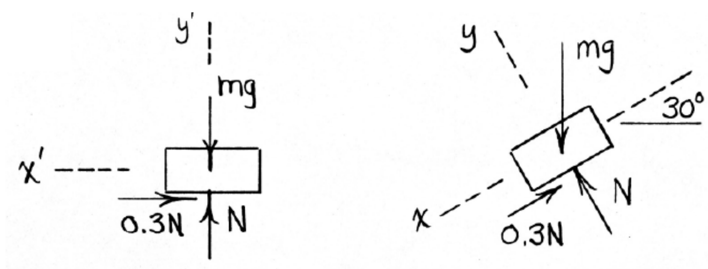

Figure 8: Illustration to Problem 1.

Now, we can compute the velocity at point B. If $v_A = 0.8\text{m/s}$ and the length of the path A is $d = 2\text{m}$, then

$$\frac{v_B^2 - v_A^2}{2} = a_x d \quad \Longrightarrow \quad v_B = 3.17\text{m/s}.$$

- Part BC. By projecting the Newton vector equation ($m\boldsymbol{a} = \boldsymbol{F}$) onto y'-axis, one obtains

$$ma_y = 0 = N - mg \quad \Longrightarrow \quad N = mg.$$

Next, projecting onto x'-axis and taking into account that $\mu_k = 0.3$, gives

$$ma_{x'} = -\mu_k N = -\mu_k mg \Longrightarrow a_{x'} = -\mu_k g = \approx -2.94\text{m/s}^2.$$

Now, since at point C we have $v_C = 0$, we have

$$\frac{v_C^2 - v_B^2}{2} = a_{x'} s \quad \Longrightarrow \quad \boxed{s \approx 1.710\text{m.}}$$