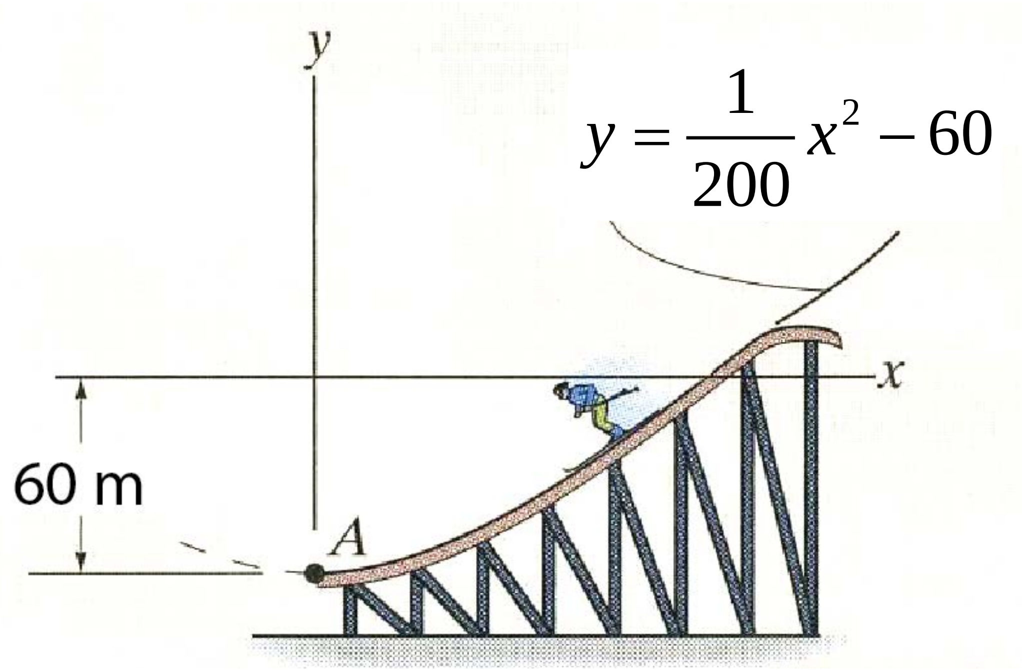
Lecture 9
Kinetics: curvilinear coordinates

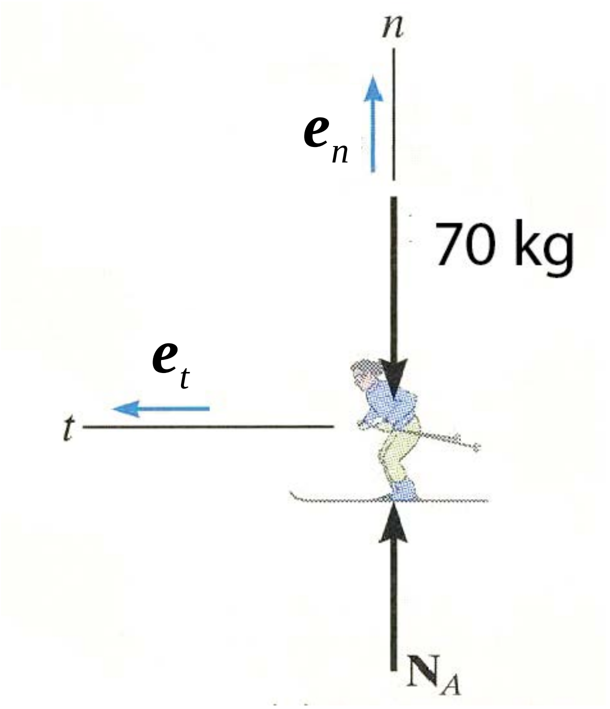
Sample Problem 1

Design of the ski jump requires knowing the type of forces that will be exerted on the skier and her approximate trajectory. If in this case the jump can be approximated by the parabola, determine the normal force on the 70 kg skier the instant she arrives at the end of the jump, point A, where her velocity is $v = 20\,\mathrm{m/s}$. What is the magnitude of her acceleration at this point?



Solution

Free-Body Diagram. Choose the proper coordinate system (in this case path coordinates) and construct the free body diagram



Since $dy/dx = x/100\big|_{x=0} = 0$, the slope at A is horizontal. The free-body diagram of the skier when she is at A is shown in the figure. Since the path is *curved*, there are two components of acceleration, a_n and a_t. Since a_n can be calculated, the unknowns are a_t and N_A.

Solution

Equations of Motion.

$$\sum F_n = ma_n \quad \Rightarrow \quad N_A - mg = ma_n = mv^2 / \rho$$

$$\sum F_t = ma_t \quad \Rightarrow \quad 0 = ma_t \quad \Rightarrow a_t = 0$$

The radius of curvature ρ for the path must be determined at point *A*(0, -60m). Here

$$y = \frac{1}{200} x^2 - 60, \quad dy / dx = \frac{1}{100} x, \quad dy^2 / dx^2 = \frac{1}{100}$$

So that at $x = 0$

$$\rho = \left. \frac{\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2}}{\left| \frac{d^2 y}{dx^2} \right|} \right|_{x=0} = \frac{(1 + 0^2)^{3/2}}{\left| \frac{1}{100} \right|} = 100\,\mathrm{m}$$

Solution

Now, we can compute

$$N_A = mg + mv^2/\rho = 70(9.81 + 20^2/100) \approx 966.7\,\mathrm{N}$$

Kinematics. Since $a_t = 0$ (see motion equations) and

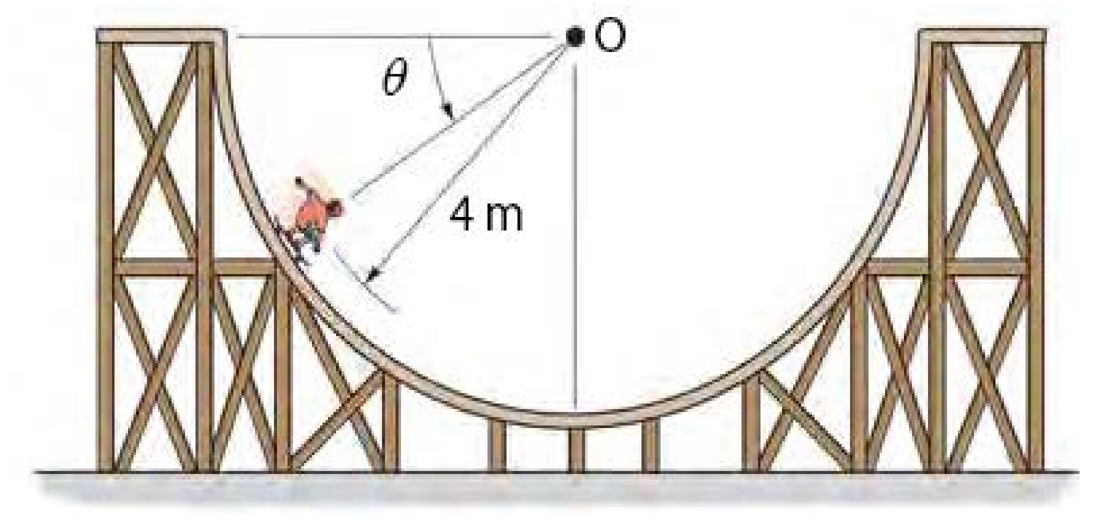
$$a_n = v^2/\rho = (20^2/100) = 4\mathrm{m/s^2}$$

Therefore, the magnitude of the acceleration is

$$a = \sqrt{a_n^2 + a_t^2} = a_n = 4\mathrm{m/s^2}$$

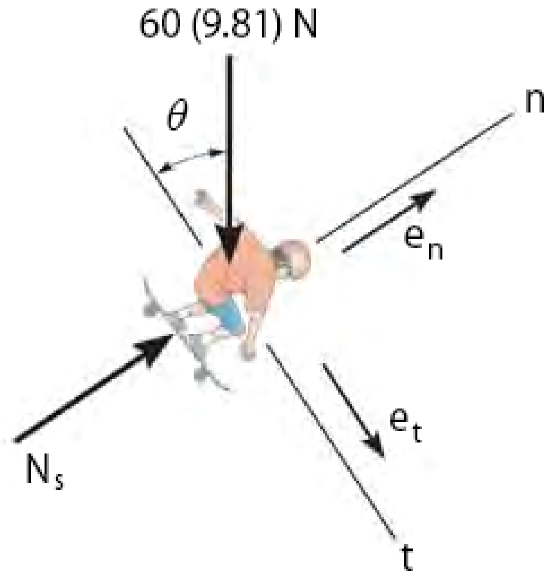
Sample Problem 2

The 60-kg skateboarder coasts down the circular track. If he starts from rest when θ=0, determine the magnitude of the normal reaction the track exerts on him when θ= 60^0. Neglect his size for the calculation.



Solution

Free-Body Diagram. Choose the proper coordinate system (in this case path coordinates). The diagram of the skateboarder when he is at an arbitrary position θ is shown below. At $\theta = 60^{\circ}$ there are three unknowns, N_s, a_t and a_n (or v).



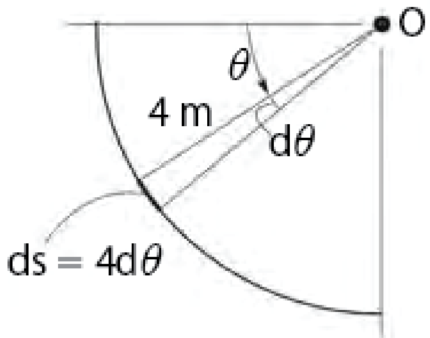
Equations of Motion.

$$\sum F_n = ma_n \;\Rightarrow\; N_s - mg\sin\theta = mv^2/\rho \;\Rightarrow\; N_s = mg\sin\theta + mv^2/\rho$$

$$\sum F_t = ma_t \;\Rightarrow\; mg\cos\theta = ma_t \;\Rightarrow\; a_t = g\cos\theta$$

Solution

Kinematics. Since a_t is expressed in terms of θ , the equation $v\mathrm{d}v = a_t\mathrm{d}s$ must be used to determine the speed of the skateboarder when $\theta = 60^\circ$. Using the geometric relation $s = \theta\rho$, where $\mathrm{d}s = \rho\mathrm{d}\theta = (4\mathrm{m})\mathrm{d}\theta$, and the initial condition $v = 0$ at $\theta = 0$, we have,



$$v\mathrm{d}v = a_t\mathrm{d}s \Rightarrow \int_0^v v\mathrm{d}v = \int_0^{60^\circ} a_t\mathrm{d}s = \int_0^{60^\circ} g\cos\theta(\rho\mathrm{d}\theta) = \rho g\int_0^{60^\circ}\cos\theta\,\mathrm{d}\theta$$

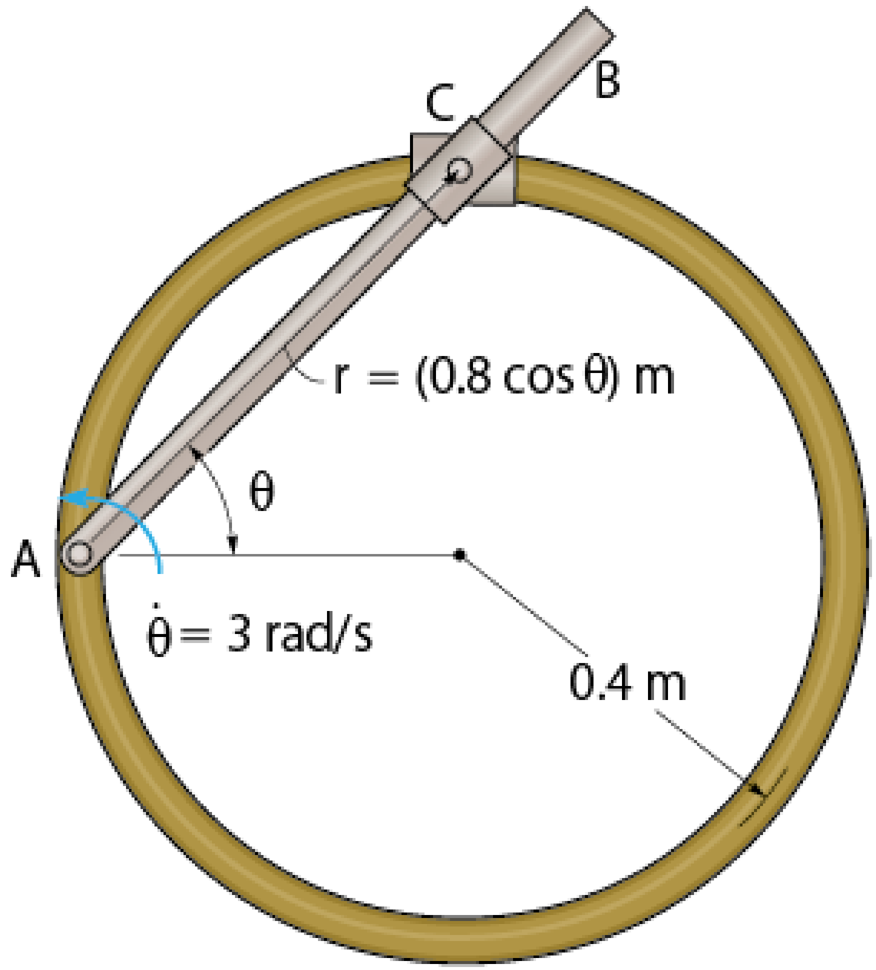
$$\left.\frac{v^2}{2}\right|_o^v = 39.24\sin\theta\Big|_o^{60^\circ} \Rightarrow \frac{v^2}{2} = 39.24\sin 60^\circ = v^2 = 67.97\mathrm{m}^2/\mathrm{s}^2$$

Substituting this result into equation for N_s yields

$$N_s = mg\sin\theta + mv^2/\rho \Rightarrow N_s = 60\times 9.81\frac{\sqrt{3}}{2} + 60\times 67.97/4 = 1529.23\,\mathrm{N}$$

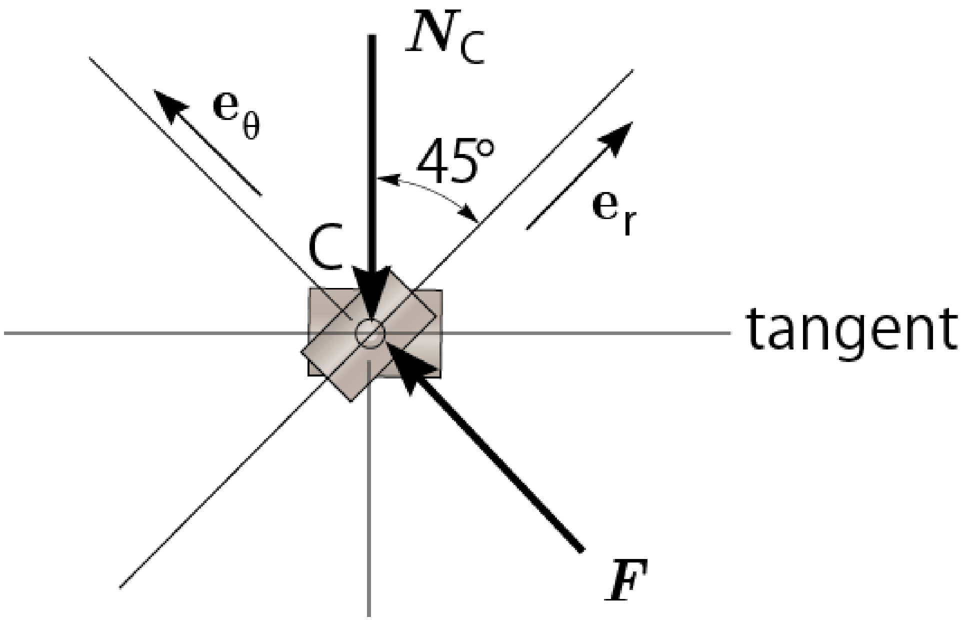
Sample Problem 3

The smooth 0.5 kg double collar can freely slide on arm AB and the circular guide rod. If the arm rotates with a constant angular velocity of 3 rad/s, determine the force the arm exerts on the collar at the instant $\theta = 45^0$. The motion is in the horizontal plane.



Solution

Free-Body Diagram. Choose the proper coordinate system (in this case polar coordinates) and construct the free body diagram



The normal reaction N_C of the circular guide rod and the force F of arm AB act on the collar in the plane of motion. Note that F acts perpendicular to the axis of arm AB, that is, in the direction of the θ axis, while N_C acts perpendicular to the tangent of the circular path at $\theta = 45^o$. The four unknowns are N_C, F, a_r, a_θ .

Solution

Equations of Motion.

$$\sum F_r = ma_r \quad \Rightarrow \quad N_C \cos 45^\circ = ma_r = (0.5\,kg)\,a_r$$

$$\sum F_\theta = ma_\theta \quad \Rightarrow \quad F - N_C \cos 45^\circ = ma_\theta = (0.5\,kg)\,a_\theta$$

Kinematics.

Using the chain rule of calculus, the first and second time derivatives of r when $\theta = 45^\circ$, $\dot{\theta} = 3\,\text{rad}/\text{s}$, $\ddot{\theta} = 0$, are

$$r = 0.8\cos\theta = 0.8\cos 45^\circ = 0.5657\,\text{m}$$

$$\dot{r} = -0.8\,(\sin\theta)\,\dot{\theta} = -0.8\sin 45^\circ \times 3 = -1.6971\,\text{m}/\text{s}$$

$$\ddot{r} = -0.8(\cos\theta)\,\dot{\theta}(\dot{\theta}) - 0.8(\sin\theta)\,\ddot{\theta}$$

$$= -0.8(\cos 45^\circ \times 3 + \cos 45^\circ \times 0) = -5.091\,\text{m}/\text{s}^2$$

Solution

Now we can compute

$$a_r = (\ddot{r} - r\dot{\theta}^2) = -5.091 - 0.5657 \times 3 = -10.18\,\mathrm{m/s^2}$$

$$a_\theta = (r\ddot{\theta} + 2\dot{r}\dot{\theta}) = 0.5657 \times 0 + 2 \times (-1.6971) \times 3 = -10.18\,\mathrm{m/s^2}$$

Substituting these results into motion equations and solving for N_C and F, we get

$$N_C = -ma_r / \cos 45^\circ = 7.20\,\mathrm{N}$$

$$F = ma_\theta + N_C \sin 45^\circ = ma_\theta - ma_r \tan 45^\circ = m(a_\theta - a_r) = 0$$