3/5 Curvilinear Motion

We turn our attention now to the kinetics of particles which move along plane curvilinear paths. In applying Newton's second law, Eq. 3/3, we will make use of the three coordinate descriptions of acceleration in curvilinear motion which we developed in Arts. 2/4, 2/5, and 2/6.

The choice of an appropriate coordinate system depends on the conditions of the problem and is one of the basic decisions to be made in solving curvilinear-motion problems. We now rewrite Eq. 3/3 in three ways, the choice of which depends on which coordinate system is most appropriate.

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Because of the banking in the turn of this track, the normal reaction force provides most of the normal acceleration of the bobsled.

Rectangular coordinates (Art. 2/4, Fig. 2/7)

$$\boxed{\begin{aligned} \Sigma F_x &= ma_x \\ \Sigma F_y &= ma_y \end{aligned}} \qquad \textbf{(3/6)}$$

where $\quad a_x = \ddot{x} \quad$ and $\quad a_y = \ddot{y}$

Normal and tangential coordinates (Art. 2/5, Fig. 2/10)

$$\boxed{\begin{aligned} \Sigma F_n &= ma_n \\ \Sigma F_t &= ma_t \end{aligned}} \qquad \textbf{(3/7)}$$

where $\quad a_n = \rho\dot{\beta}^2 = v^2/\rho = v\dot{\beta}, \quad a_t = \dot{v}, \quad$ and $\quad v = \rho\dot{\beta}$

Polar coordinates (Art. 2/6, Fig. 2/15)

$$\boxed{\begin{aligned} \Sigma F_r &= ma_r \\ \Sigma F_\theta &= ma_\theta \end{aligned}} \qquad \textbf{(3/8)}$$

where $\quad a_r = \ddot{r} - r\dot{\theta}^2 \quad$ and $\quad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

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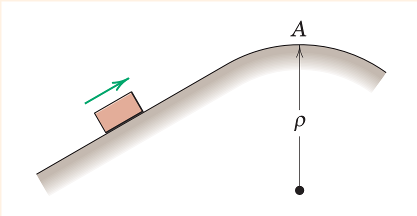
At the highest point of the swing, this child experiences tangential acceleration. An instant later, when she has acquired velocity, she will experience normal acceleration as well.

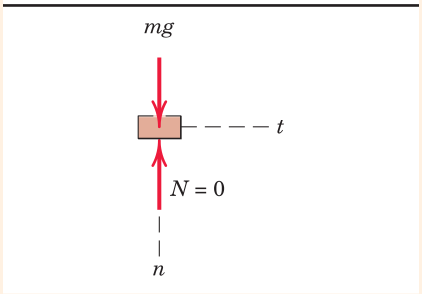
In applying these motion equations to a body treated as a particle, you should follow the general procedure established in the previous article on rectilinear motion. After you identify the motion and choose the coordinate system, draw the free-body diagram of the body. Then obtain the appropriate force summations from this diagram in the usual way. The free-body diagram should be complete to avoid incorrect force summations.

Once you assign reference axes, you must use the expressions for both the forces and the acceleration which are consistent with that assignment. In the first of Eqs. 3/7, for example, the positive sense of the n-axis is *toward* the center of curvature, and so the positive sense of our force summation ΣF_n must also be *toward* the center of curvature to agree with the positive sense of the acceleration $a_n = v^2/\rho$.

Sample Problem 3/6

Determine the maximum speed v which the sliding block may have as it passes point A without losing contact with the surface.





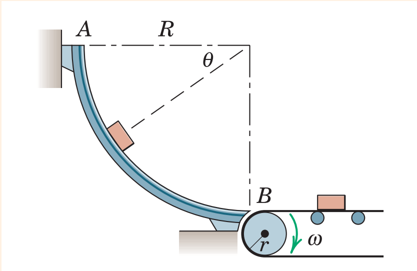
Solution. The condition for loss of contact is that the normal force N which the surface exerts on the block goes to zero. Summing forces in the normal direction gives

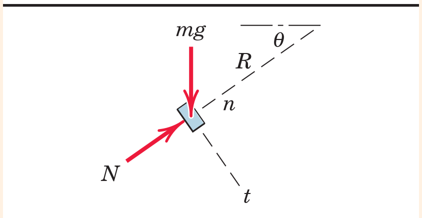
$$[\Sigma F_n = ma_n] \qquad mg = m\frac{v^2}{\rho} \qquad v = \sqrt{g\rho} \qquad \textit{Ans.}$$

If the speed at A were less than $\sqrt{g\rho}$, then an upward normal force exerted by the surface on the block would exist. In order for the block to have a speed at A which is greater than $\sqrt{g\rho}$, some type of constraint, such as a second curved surface above the block, would have to be introduced to provide additional downward force.

Sample Problem 3/7

Small objects are released from rest at A and slide down the smooth circular surface of radius R to a conveyor B. Determine the expression for the normal contact force N between the guide and each object in terms of θ and specify the correct angular velocity ω of the conveyor pulley of radius r to prevent any sliding on the belt as the objects transfer to the conveyor.





Solution. The free-body diagram of the object is shown together with the coordinate directions n and t. The normal force N depends on the n-component of the acceleration which, in turn, depends on the velocity. The velocity will be cumulative according to the tangential acceleration a_t. Hence, we will find a_t first for any general position.

$$[\Sigma F_t = ma_t] \qquad mg\cos\theta = ma_t \qquad a_t = g\cos\theta$$

① Now we can find the velocity by integrating

$$[v\,dv = a_t\,ds] \qquad \int_0^v v\,dv = \int_0^\theta g\cos\theta\,d(R\theta) \qquad v^2 = 2gR\sin\theta$$

We obtain the normal force by summing forces in the positive n-direction, which is the direction of the n-component of acceleration.

$$[\Sigma F_n = ma_n] \qquad N - mg\sin\theta = m\frac{v^2}{R} \qquad N = 3mg\sin\theta \qquad \textit{Ans.}$$

The conveyor pulley must turn at the rate $v = r\omega$ for $\theta = \pi/2$, so that

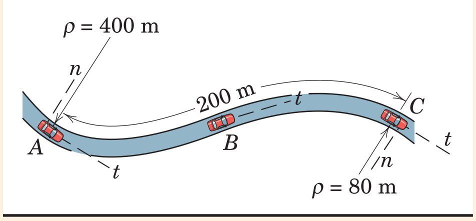
$$\omega = \sqrt{2gR}/r \qquad \textit{Ans.}$$

Helpful Hint

① It is essential here that we recognize the need to express the tangential acceleration as a function of position so that v may be found by integrating the kinematical relation $v\,dv = a_t\,ds$, in which all quantities are measured along the path.

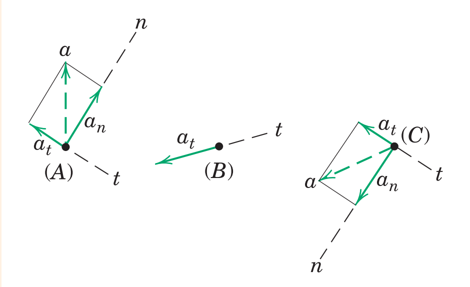
Sample Problem 3/8

A 1500-kg car enters a section of curved road in the horizontal plane and slows down at a uniform rate from a speed of 100 km/h at A to a speed of 50 km/h as it passes C. The radius of curvature ρ of the road at A is 400 m and at C is 80 m. Determine the total horizontal force exerted by the road on the tires at positions A, B, and C. Point B is the inflection point where the curvature changes direction.



Solution. The car will be treated as a particle so that the effect of all forces exerted by the road on the tires will be treated as a single force. Since the motion is described along the direction of the road, normal and tangential coordinates will be used to specify the acceleration of the car. We will then determine the forces from the accelerations.

The constant tangential acceleration is in the negative t-direction, and its magnitude is given by



① $[v_C{}^2 = v_A{}^2 + 2a_t\,\Delta s]$ $$a_t = \left|\frac{(50/3.6)^2 - (100/3.6)^2}{2(200)}\right| = 1.447 \text{ m/s}^2$$

The normal components of acceleration at A, B, and C are

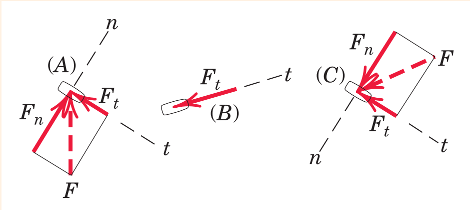
② $[a_n = v^2/\rho]$

At A, $$a_n = \frac{(100/3.6)^2}{400} = 1.929 \text{ m/s}^2$$

At B, $$a_n = 0$$

At C, $$a_n = \frac{(50/3.6)^2}{80} = 2.41 \text{ m/s}^2$$

Application of Newton's second law in both the n- and t-directions to the free-body diagrams of the car gives



$[\Sigma F_t = ma_t]$ $$F_t = 1500(1.447) = 2170 \text{ N}$$

③ $[\Sigma F_n = ma_n]$

At A, $$F_n = 1500(1.929) = 2890 \text{ N}$$

At B, $$F_n = 0$$

At C, $$F_n = 1500(2.41) = 3620 \text{ N}$$

Thus, the total horizontal force acting on the tires becomes

At A, $$F = \sqrt{F_n{}^2 + F_t{}^2} = \sqrt{(2890)^2 + (2170)^2} = 3620 \text{ N}$$ *Ans.*

At B, $$F = F_t = 2170 \text{ N}$$ *Ans.*

④ At C, $$F = \sqrt{F_n{}^2 + F_t{}^2} = \sqrt{(3620)^2 + (2170)^2} = 4220 \text{ N}$$ *Ans.*

Helpful Hints

① Recognize the numerical value of the conversion factor from km/h to m/s as 1000/3600 or 1/3.6.

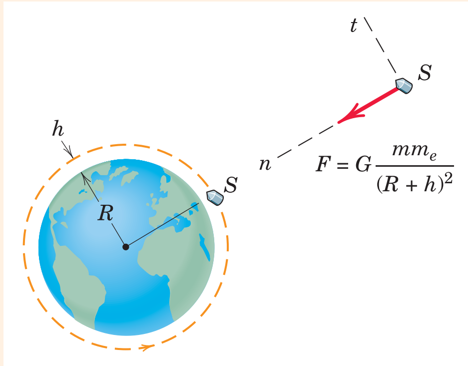
② Note that a_n is always directed toward the center of curvature.

③ Note that the direction of F_n must agree with that of a_n.

④ The angle made by **a** and **F** with the direction of the path can be computed if desired.

Sample Problem 3/9

Compute the magnitude v of the velocity required for the spacecraft S to maintain a circular orbit of altitude 200 mi above the surface of the earth.



Solution. The only external force acting on the spacecraft is the force of grav- ① itational attraction to the earth (i.e., its weight), as shown in the free-body diagram. Summing forces in the normal direction yields

$$[\Sigma F_n = ma_n] \quad G\frac{mm_e}{(R+h)^2} = m\frac{v^2}{(R+h)}, \quad v = \sqrt{\frac{Gm_e}{(R+h)}} = R\sqrt{\frac{g}{(R+h)}}$$

where the substitution $gR^2 = Gm_e$ has been made. Substitution of numbers gives

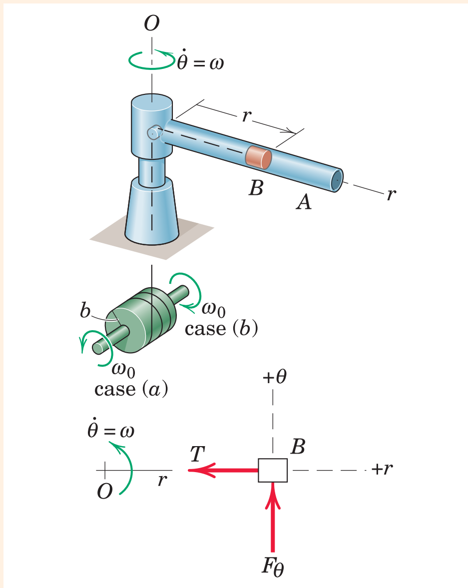
$$v = (3959)(5280)\sqrt{\frac{32.234}{(3959+200)(5280)}} = 25{,}326 \text{ ft/sec} \qquad \textit{Ans.}$$

Helpful Hint

① Note that, for observations made within an inertial frame of reference, there is no such quantity as "centrifugal force" acting in the minus n-direction. Note also that neither the spacecraft nor its occupants are "weightless," because the weight in each case is given by Newton's law of gravitation. For this altitude, the weights are only about 10 percent less than the earth-surface values. Finally, the term "zero-g" is also misleading. It is only when we make our observations with respect to a coordinate system which has an acceleration equal to the gravitational acceleration (such as in an orbiting spacecraft) that we appear to be in a "zero-g" environment. The quantity which does go to zero aboard orbiting spacecraft is the familiar normal force associated with, for example, an object in contact with a horizontal surface within the spacecraft.

Sample Problem 3/10

Tube A rotates about the vertical O-axis with a constant angular rate $\dot{\theta} = \omega$ and contains a small cylindrical plug B of mass m whose radial position is controlled by the cord which passes freely through the tube and shaft and is wound around the drum of radius b. Determine the tension T in the cord and the horizontal component F_θ of force exerted by the tube on the plug if the constant angular rate of rotation of the drum is ω_0 first in the direction for case (a) and second in the direction for case (b). Neglect friction.



Solution. With r a variable, we use the polar-coordinate form of the equations of motion, Eqs. 3/8. The free-body diagram of B is shown in the horizontal plane and discloses only T and F_θ. The equations of motion are

$$[\Sigma F_r = ma_r] \qquad -T = m(\ddot{r} - r\dot{\theta}^2)$$

$$[\Sigma F_\theta = ma_\theta] \qquad F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

Case (a). With $\dot{r} = +b\omega_0$, $\ddot{r} = 0$, and $\ddot{\theta} = 0$, the forces become

$$T = mr\omega^2 \qquad F_\theta = 2mb\omega_0\omega \qquad \textit{Ans.}$$

① ***Case (b).*** With $\dot{r} = -b\omega_0$, $\ddot{r} = 0$, and $\ddot{\theta} = 0$, the forces become

$$T = mr\omega^2 \qquad F_\theta = -2mb\omega_0\omega \qquad \textit{Ans.}$$

Helpful Hint

① The minus sign shows that F_θ is in the direction opposite to that shown on the free-body diagram.