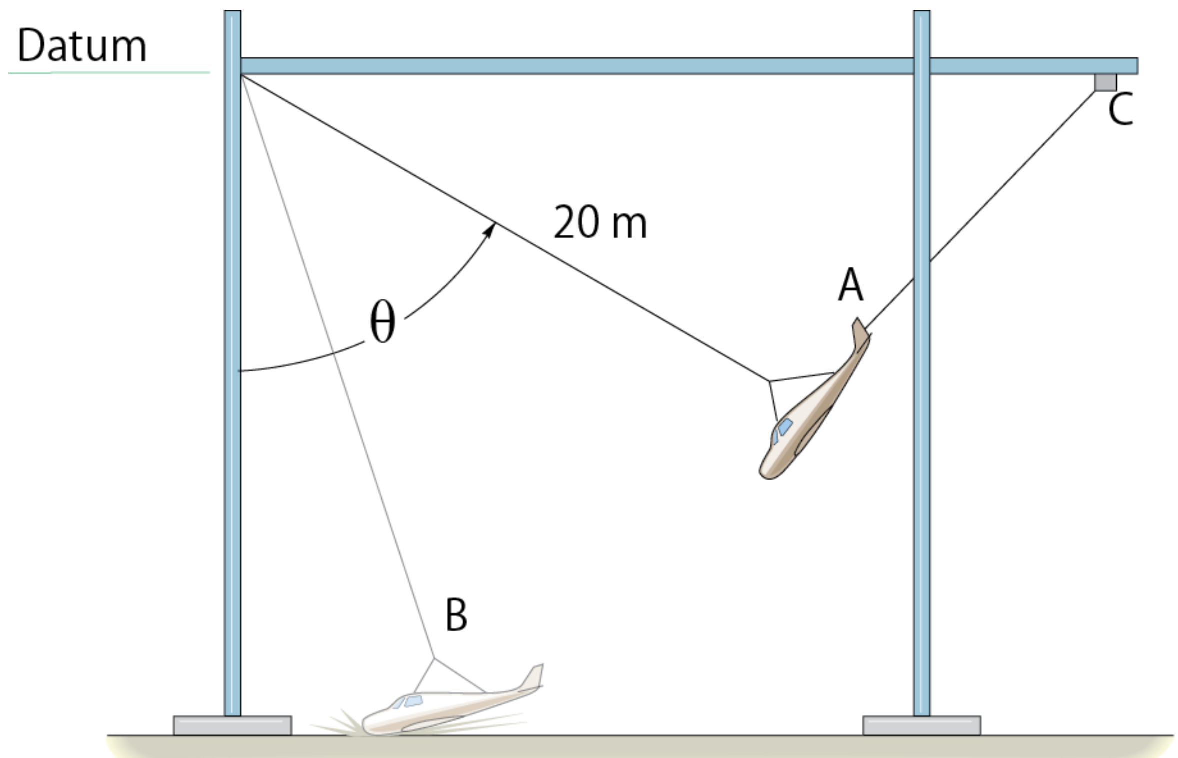
Exercises in Physics

Lecture 11
Potential Energy

Sample Problem 1

The gantry structure in the photo is used to test the response of an airplane during a crash. The plane, having a mass of 8000 kg, is hoisted back until $\theta = 60^{\circ}$, and then the pull-back cable AC is released when the plane is at rest. Determine the speed of the plane just before it crashes into the ground, $\theta = 15^{\circ}$. Also, what is the maximum tension developed in the supporting cable during the motion? Neglect the size of the airplane and the effect of lift caused by the wings during the motion.



Solution

Since the force of the cable does no work on the plane, it must be obtained using the equation of motion. First, however, we must determine the plane's speed at B.

Potential Energy. For convenience, the datum has been established at the top of the gantry (see previous slide)

Conservation of Energy.

$$T_A + V_A = T_B + V_B \quad \Rightarrow$$

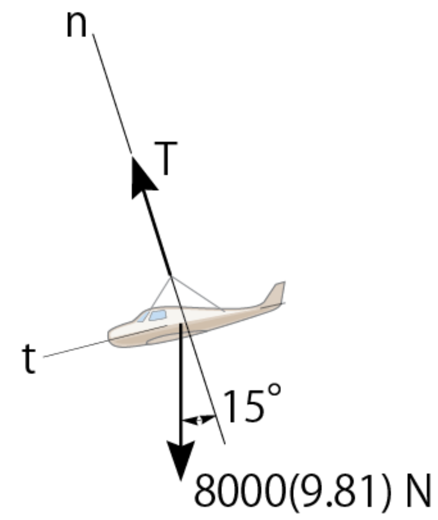
$$0 - 8000\,\text{kg} \times 9.81\,\text{m/s}^2 \times (20\cos 60^\circ\,\text{m}) =$$

$$\frac{1}{2}(8000\,\text{kg})v_B^2 - 8000\,\text{kg} \times 9.81\,\text{m/s}^2 \times (20\cos 15^\circ\,\text{m})$$

$$\Rightarrow \quad v_B = 13.52\,\text{m/s}$$

Solution

Equation of Motion. From the free-body diagram when the plane is at B, we have



$$\sum F_n = ma_n \quad \Rightarrow \quad T - mg\cos 15^{\circ} = ma_n = mv^2/\rho$$

$$\Rightarrow \quad T - 8000\text{kg} \times (9.81\text{m}/\text{s}^2)\cos 15^{\circ} = 8000\text{kg}\frac{(13.52\text{m}/\text{s})^2}{20\text{m}}$$

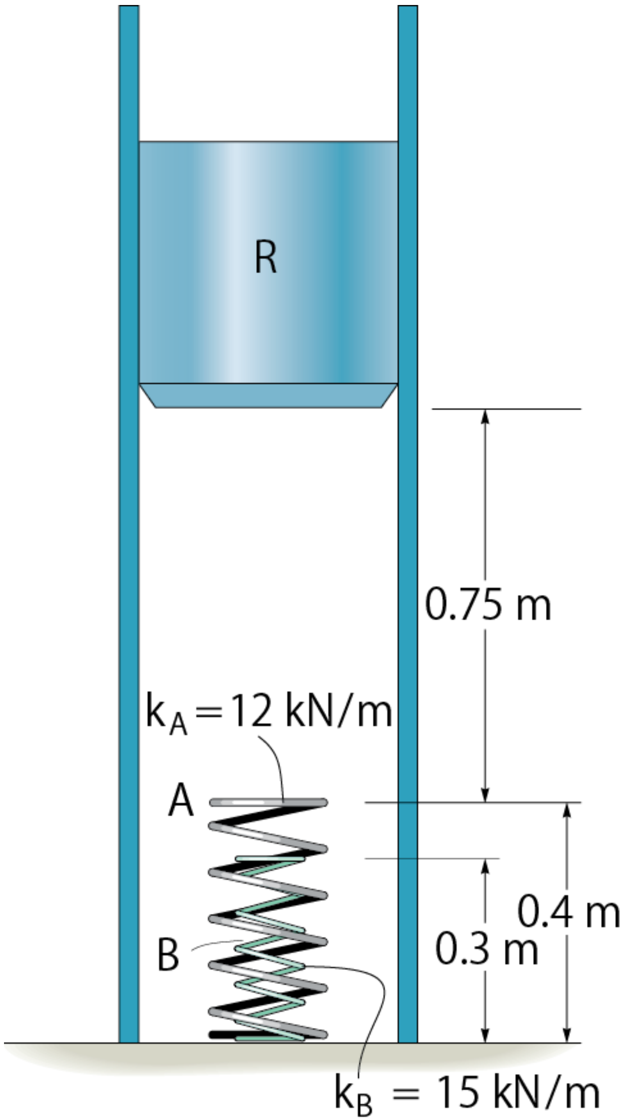
Thus

$$T = 149000\,\text{N}$$

Sample Problem 2

The ram R has a mass of 100 kg and is released from rest 0.75 m from the top of a spring, A, that has a stiffness k_A = 12000 N/m. If a second spring B, having a stiffness k_B =15000 N/m, is “nested” in A, determine the maximum displacement of A needed to stop the downward motion of the ram.

The unstretched length of each spring is indicated in the figure. Neglect the mass of the springs.



Solution

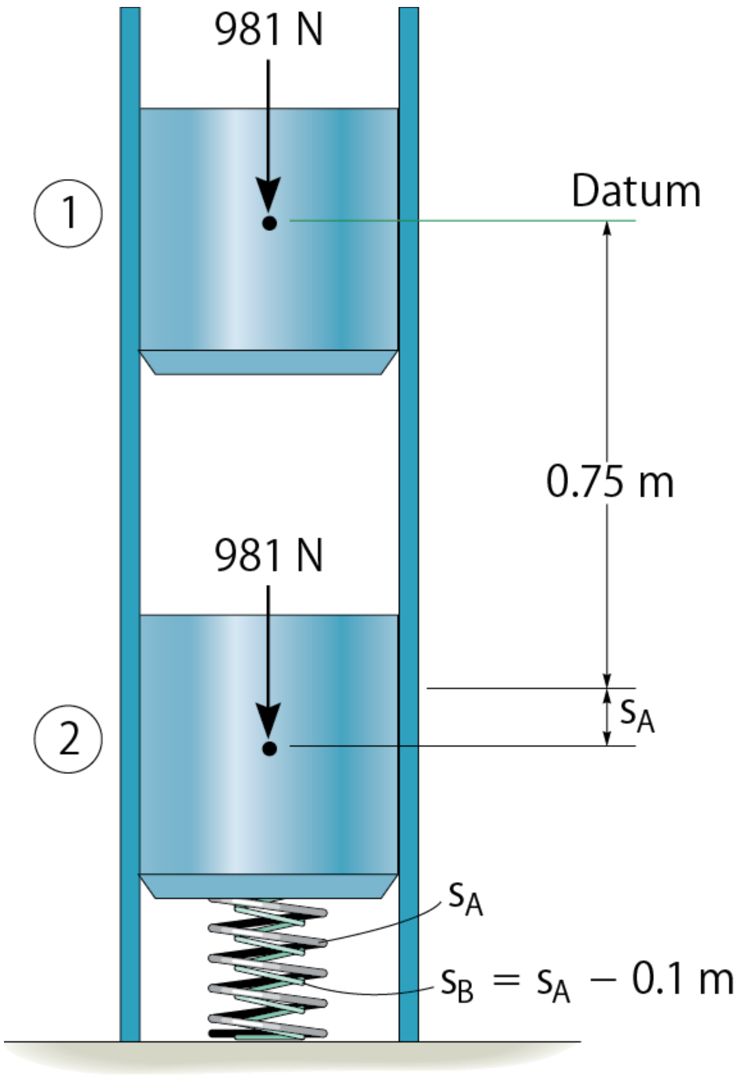
Potential Energy. For convenience, the datum is established through AB.

We will assume that the ram compresses both springs at the instant it comes to rest. The datum is located through the center of gravity of the ram at its initial position. When the kinetic energy is reduced to zero,

$$v_2 = 0,$$

A is compressed a distance s_A and B compresses

$$s_B = s_A - 0.1\text{m}$$



Solution

Conservation of Energy.

$$T_1 + V_1 = T_2 + V_2 \quad \Rightarrow$$

$$0 + 0 = 0 + \left\{ \frac{1}{2} k_A s_A^2 + \frac{1}{2} k_B (s_A - 0.1)^2 - mgh \right\} \quad \Rightarrow$$

$$0 + 0 = 0 + \left\{ \frac{1}{2} (12000\,\mathrm{N/m}) s_A^2 + \frac{1}{2} 15000\,\mathrm{N/m} (s_A - 0.1)^2 \right\}$$

$$- \{ 100 \times 9.81 \times (0.75 + s_A) \} \,\mathrm{Nm}$$

Rearranging the terms

$$13500\, s_A^2 - 2481 s_A - 660.75 = 0$$

Using the quadratic formula and solving for the positive root, we have

$$s_A = 0.331\mathrm{m}$$

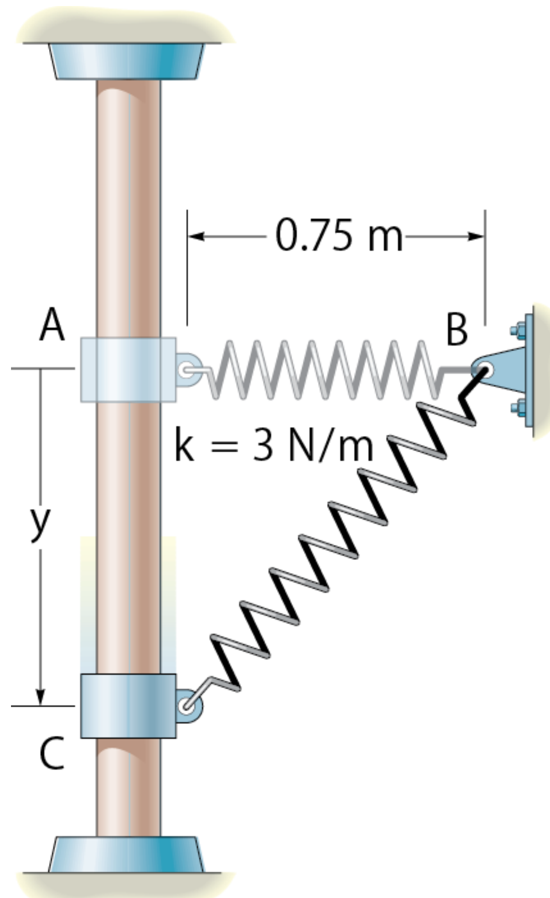
Since $s_B = 0.331\mathrm{m} - 0.1\mathrm{m} = 0.231\mathrm{m}$, which is positive, the assumption that both springs are compressed by the ram is correct.

Solution

NOTE: The second root, $s_A = -0.148\text{m}$, does not represent the physical situation. Since positive s is measured downward, the negative sign indicates that spring A would have to be “extended” by an amount of 0.148 m to stop the ram.

Sample Problem 3

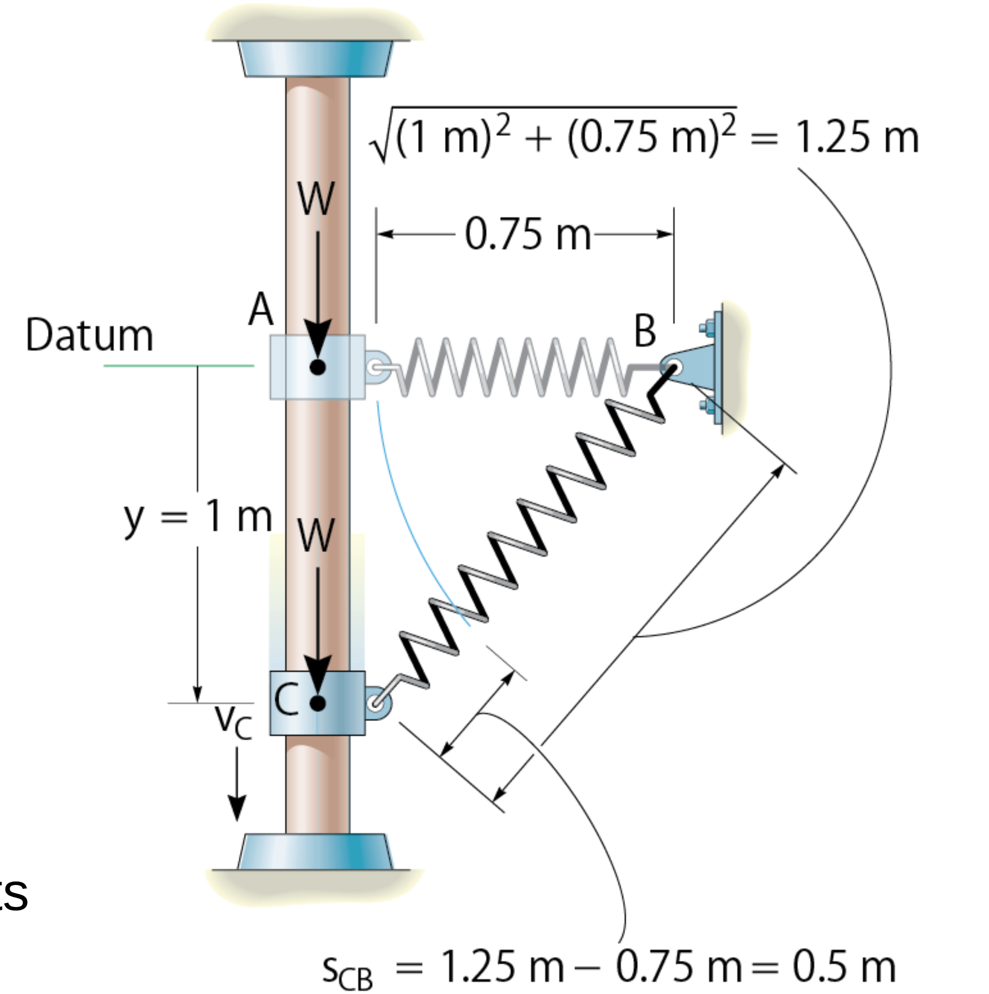
A smooth 2-kg collar fits loosely on the vertical shaft. If the spring is unstretched when the collar is in the position A, determine the speed at which the collar is moving when y = 1 m, if (a) it is released from rest at A, and (b) it is released at A with an upward velocity v_A = 2 m/s.



Solution

Part (a) Potential Energy. For convenience, the datum is established through AB.

When the collar is at C, the gravitational potential energy is $-(mg)y$, since the collar is below the datum, and the elastic potential energy is $\frac{1}{2}ks_{CB}^2$



Here s_{CB} = 0.5 m, which represents the stretch in the spring as shown in the figure.

Solution

Conservation of Energy.

$$T_A + V_A = T_C + V_C \quad \Rightarrow$$

$$0 + 0 = \frac{1}{2} m v_C^2 + \left\{ \frac{1}{2} k s_{CB}^2 - m g y \right\} \quad \Rightarrow$$

$$0 + 0 = \frac{1}{2}(2\text{kg}) v_C^2 + \left\{ \frac{1}{2}(3\text{N}/\text{m})(0.5\text{m})^2 - (2 \times 9.81 \times 1)\text{Nm} \right\} \quad \Rightarrow$$

$$v_C = 4.39 \text{ m}/\text{s}$$

This problem can also be solved by using the equation of motion or the principle of work and energy. Note that for both of these methods the variation of the magnitude and direction of the spring force must be taken into account. Here, however, the above solution is clearly advantageous since the calculations depend only on data calculated at the initial and final points of the path.

Solution

Part (b) Conservation of Energy.

$$T_A + V_A = T_C + V_C \quad \Rightarrow$$

$$\frac{1}{2}mv_A^2 + 0 = \frac{1}{2}mv_C^2 + \left\{\frac{1}{2}ks_{CB}^2 - mgy\right\} \quad \Rightarrow$$

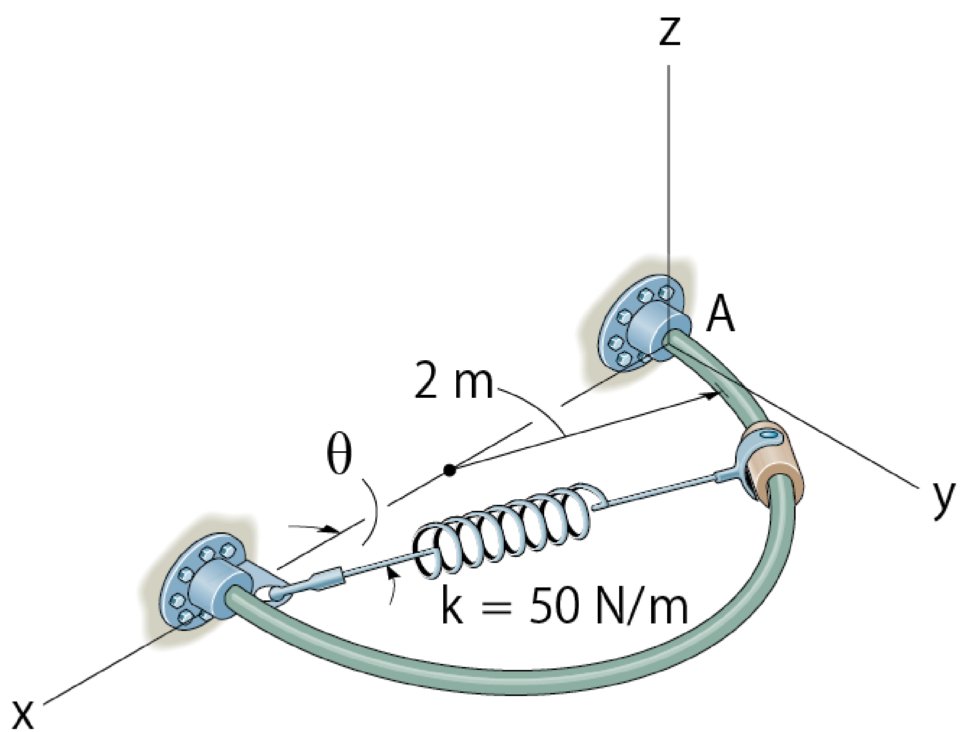
$$\frac{1}{2}(2\mathrm{kg})(2\mathrm{m/s})^2 + 0 = \frac{1}{2}(2\mathrm{kg})v_C^2 + \left\{\frac{1}{2}(3\mathrm{N/m})(0.5\mathrm{m})^2 - (2\times 9.81\times 1)\mathrm{Nm}\right\}$$

$$\Rightarrow \quad v_C = 4.82\ \mathrm{m/s}$$

NOTE: The kinetic energy of the collar depends only on the magnitude of velocity, and therefore it is immaterial if the collar is moving up or down at 2 m/s when released at A.

Sample Problem 4

The spring has a stiffness k = 50 N/m and an unstretched length of 0.3 m. If it is attached to the 2-kg smooth collar and the collar is released from rest at A ($\theta = 0^{o}$), determine the speed of the collar when $\theta = 60^{o}$. The motion occurs in the horizontal plane. Neglect the size of the collar.

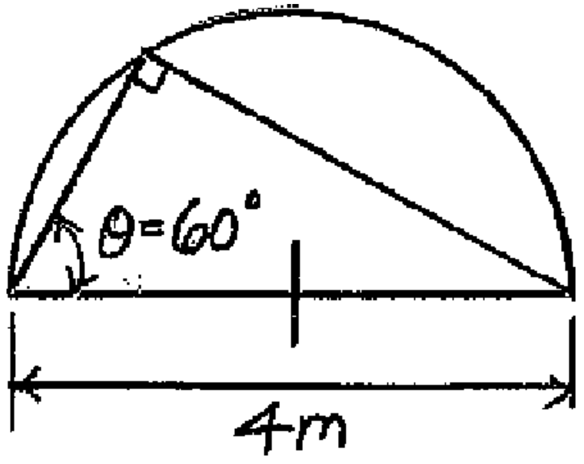


Solution

Potential Energy. Since the motion occurs in the horizontal plane, there will be no change in gravitational potential energy when $\theta = 0^{\circ}$, the spring stretches $x_1 = 4 - 0.3 = 3.7\text{m}$. Referring to the geometry shown in the figure, the spring stretches $x_2 = 4\cos 60^{\circ} - 0.3 = 1.7\text{m}$. Thus, the elastic potential energies in the spring when $\theta = 0^{\circ}$ and $\theta = 60^{\circ}$ are

$$(V_e)_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}50 \times 3.7^2 = 342.25\text{J}$$

$$(V_e)_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}50 \times 1.7^2 = 72.25\text{J}$$



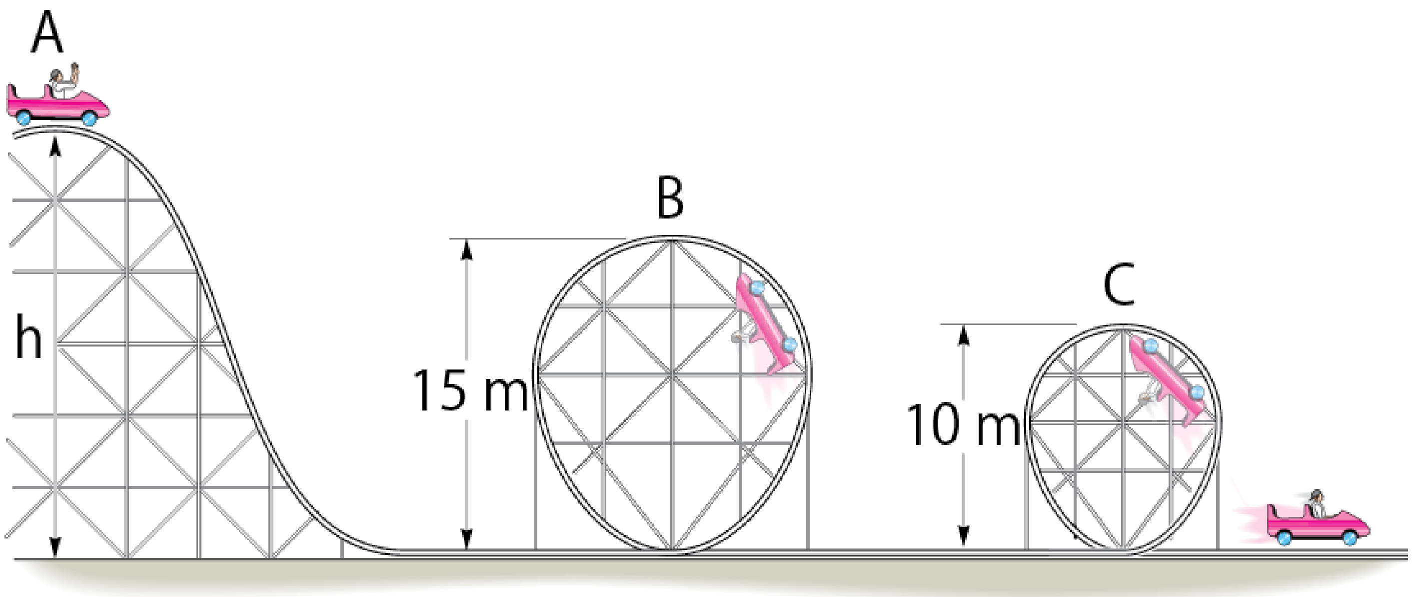
Conservation of Energy. Since the collar is released from rest when $\theta = 0^{\circ}$, $T_1 = 0$.

$$T_1 + V_1 = T_2 + V_2 \quad \Rightarrow$$

$$0 + 342.25 = \frac{1}{2}2v_2^2 + 72.25 \quad \Rightarrow \quad v_2 = 16.43 \text{ m/s}$$

Sample Problem 5

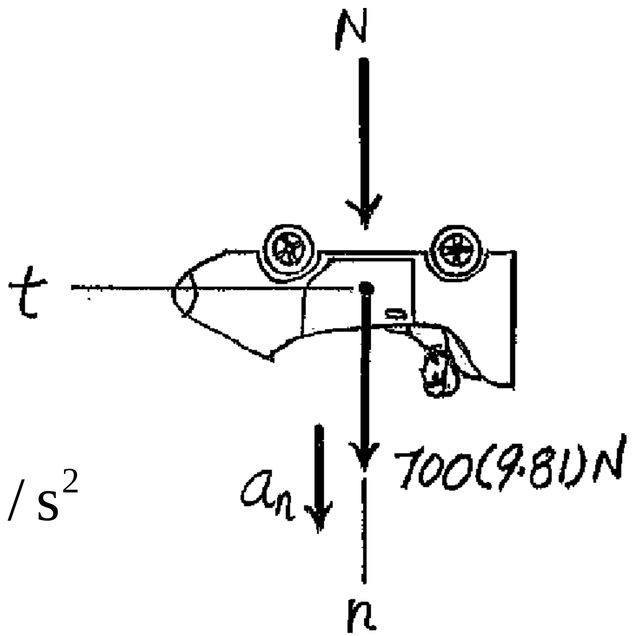
The roller coaster car has a mass of 700 kg, including its passenger. If it is released from rest at the top of the hill A, determine the minimum height h of the hill crest so that the car travels around both inside the loops without leaving the track. Neglect friction, the mass of the wheels, and the size of the car. What is the normal reaction on the car when the car is at B and when it is at C? Take ρ_B = 7.5 m and ρ_C = 5 m.



Solution

Equation of Motion. Referring to the free-body diagram, we have

$$\sum F_n = ma_n \quad \Rightarrow \quad N + mg = ma_n = mv^2 / \rho$$



When the roller-coaster car is about to leave the loop at B and C, N = 0.

$$0 + 700 \times 9.81 = 100 v_B^2 / 7.5 \quad \Rightarrow \quad v_B^2 = 73.575 \text{m}^2 / \text{s}^2$$

and

$$0 + 700 \times 9.81 = 100 v_C^2 / 5 \quad \Rightarrow \quad v_C^2 = 49.05 \text{m}^2 / \text{s}^2$$

Judging from the above result the coaster car will not leave the loop at C provided it passes through B safely. Thus

$$N_B = 0$$

Solution

Conservation of Energy. The datum will be set at the ground level.
From A to B we have

$$T_A + V_A = T_B + V_B \quad \Rightarrow$$

$$0 + 700 \times 9.81 \times h = \frac{1}{2} 700 \times 73.575 + 700 \times 9.81 \times 15 \quad \Rightarrow$$

$$\Rightarrow \quad h = 18.75 \text{ m}$$

From B to C we have

$$T_B + V_B = T_C + V_C \quad \Rightarrow$$

$$\frac{1}{2} 700 \times 73.575 + 700 \times 9.81 \times 15 = \frac{1}{2} 700 v_C^2 + 700 \times 9.81 \times 10 \quad \Rightarrow$$

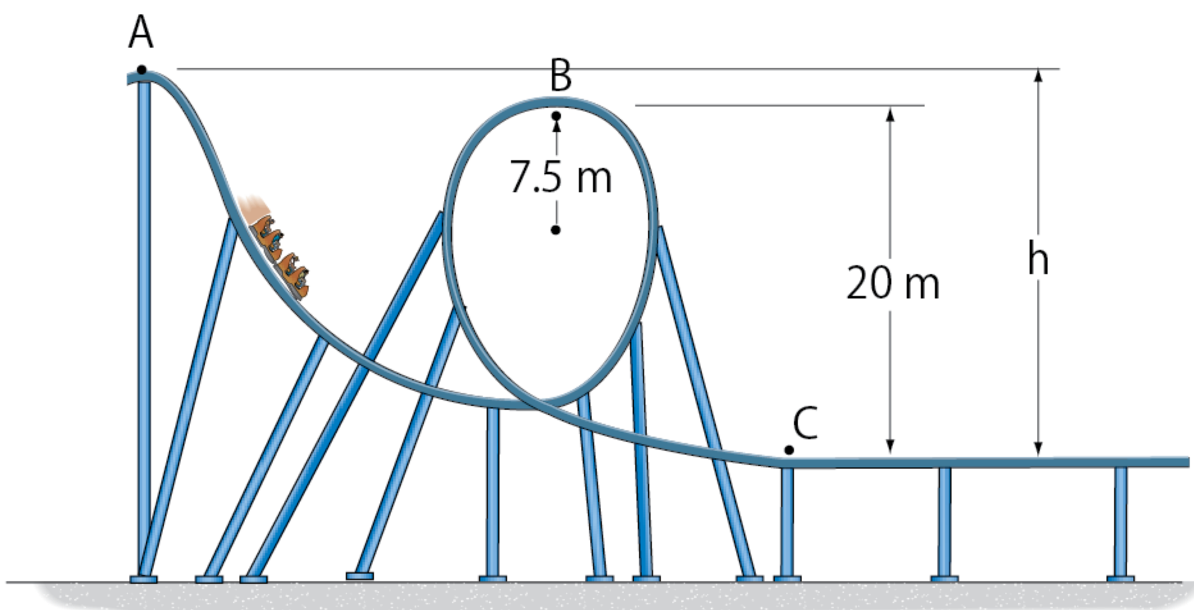
$$\Rightarrow \quad v_C^2 = 171.675 \text{ m}^2/\text{s}^2 > 49.05 \text{ m}^2/\text{s}^2$$

Finally, from motion equation at point C we get

$$N_C + 700 \times 9.81 = 700 \times 171.675/5 \quad \Rightarrow \quad N_C = 17.17 \times 10^3 \text{N}$$

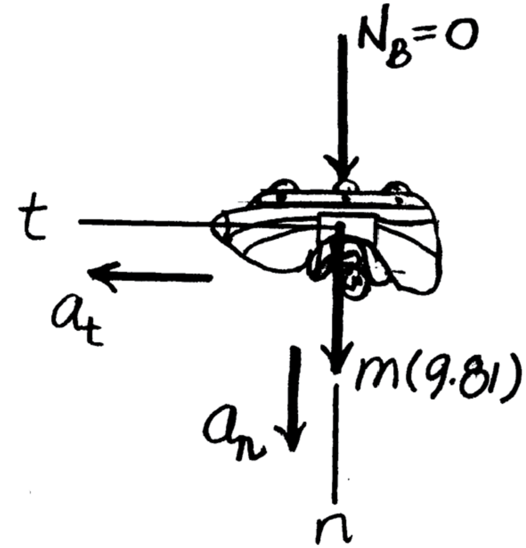
Sample Problem 6

The roller coaster car having a mass m is released from rest at point A. If the track is to be designed so that the car does not leave it at B, determine the required height h. Also, find the speed of the car when it reaches point C. Neglect friction.



Solution

Equation of Motion. Since it is required that the roller coaster car is about to leave the track at B, $N_B = 0$. Here, $a_n = v_B^2 / \rho_B = v_B^2 / 7.5$. By referring to the free-body diagram,



$$\sum F_n = ma_n \quad \Rightarrow \quad N_B + mg = ma_n = mv_B^2 / \rho_B$$

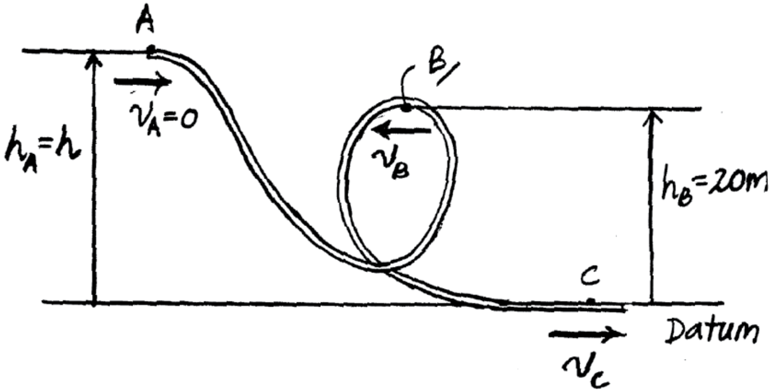
$$0 + mg = mv_B^2 / \rho_B \Rightarrow v_B^2 = \rho g = 73.575\text{m}^2 / \text{s}^2$$

Potential Energy. With reference to the datum set in the figure, the gravitational potential energy of the rollercoaster car at positions *A*, *B*, and *C* are

$$(V_g)_A = mgh_A = 9.81mh_A$$

$$(V_g)_B = mgh_B = 9.81 \times 20m = 196.2m$$

$$(V_g)_C = mgh_C = 0$$



Solution

Conservation of Energy. Using the result of v_B^2 and considering the motion of the car from position A to B,

$$T_A + V_A = T_B + V_B \quad \Rightarrow$$

$$\frac{1}{2}mv_A^2 + (V_g)_A = \frac{1}{2}mv_B^2 + (V_g)_B \quad \Rightarrow$$

$$0 + 9.81mh_A = \frac{1}{2}73.575m + 196.2m \Rightarrow \quad h_A = 23.75 \text{ m}$$

Also, considering the motion of the car from position B to C,

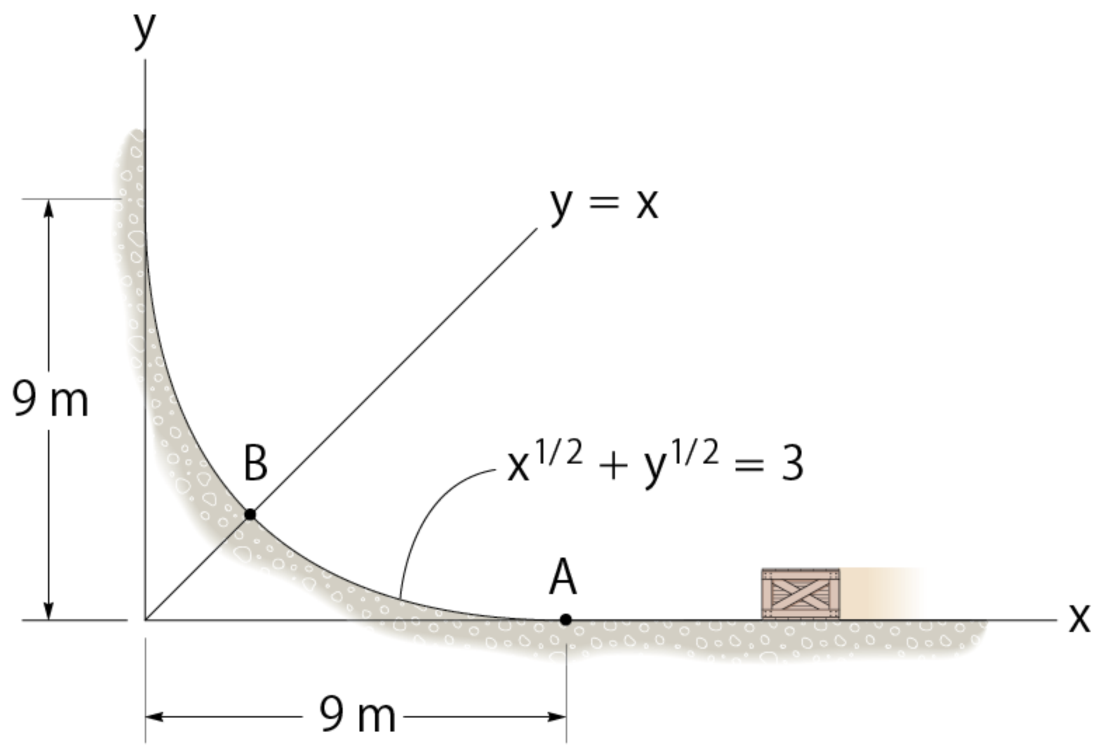
$$T_B + V_B = T_C + V_C \quad \Rightarrow$$

$$\frac{1}{2}mv_B^2 + (V_g)_B = \frac{1}{2}mv_C^2 + (V_g)_C \quad \Rightarrow$$

$$\frac{1}{2}73.575m + 196.2m = \frac{1}{2}mv_C^2 + 0 \Rightarrow \quad v_C = 21.6 \text{ m/s}$$

Sample Problem 7

When the 5-kg box reaches point A it has a speed $v_A = 10$ m/s. Determine the normal force the box exerts on the surface when it reaches point B. Neglect friction and the size of the box.



Solution

Conservation of Energy. At point B, y = x

$$x^{1/2} + y^{1/2} = 9 \Rightarrow x^{1/2} + x^{1/2} = 9 \Rightarrow x = 9/4\text{m}, \;\; y = 9/4\text{m}.$$

With reference to the datum set to coincide with the x axis, the gravitational potential energies of the box at points A and B are

$$(V_g)_A = 0, \;\; (V_g)_B = mgh_B = 5 \times 9.81 \times 9/4 = 110.3625\text{J}$$

Applying the energy equation,

$$T_A + V_A = T_B + V_B \quad \Rightarrow$$

$$\frac{1}{2}mv_A^2 + (V_g)_A = \frac{1}{2}mv_B^2 + (V_g)_B \quad \Rightarrow$$

$$\frac{1}{2}5 \times 10^2 + 0 = \frac{1}{2}5v_B^2 + 110.3625 \Rightarrow \;\; v_B^2 = 55.855 \text{ m}^2/\text{s}^2$$

Solution

Equation of Motion. Here $y = (3 - x^{1/2})^2$. Then

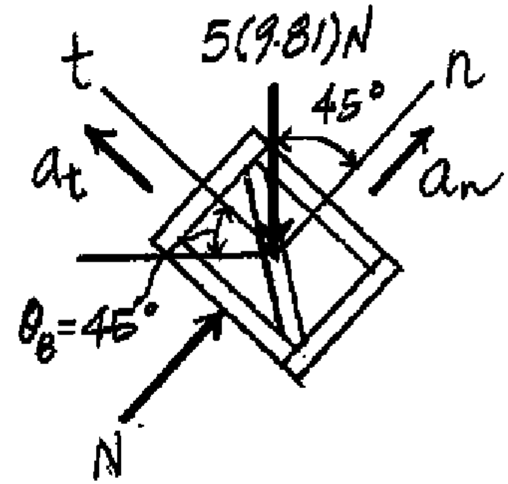
$$dy/dx = 2(3 - x^{1/2})(-\frac{1}{2}x^{-1/2}) = \frac{x^{1/2} - 3}{x^{1/2}} = 1 - \frac{3}{x^{1/2}}$$

$$d^2y/dx^2 = \frac{3}{2}x^{-3/2}$$

The radius of curvature at B is

$$\rho_B = \left.\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}\right|_{x=9/4} = \frac{(1 + (-1)^2)^{3/2}}{|0.4444|} = 6.3640\,\mathrm{m}$$

Referring to the Free Body Diagram



$$\sum F_n = ma_n \Rightarrow N_B - mg\cos 45^{\circ} = ma_n = mv_B^2/\rho_B$$

Thus $N_B = 5 \times 9.81\cos 45^{\circ} + 5\dfrac{55.855}{6.3640} = 78.57\,\mathrm{N}$