Exercises in Physics
Sample Problems

Date Given: June 2, 2022

P1. The 50kg crate is stationary when the force P is applied. Determine the resulting acceleration of the crate if (a) $P = 0$, (b) $P = 150$N, and (c) $P = 300$N. The static and kinetic friction coefficients are respectively $\mu_s = 0.2$ and $\mu_k = 0.15$. (*Hint:* for each loading consider first if the crate can be in equilibrium or not.)

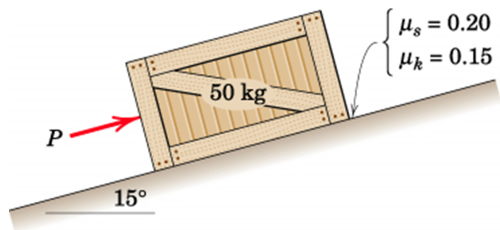

Figure 1: Illustration to Problem 1.

Solution: The forces are sketched in Figure 2. Projection the Newton vector equation ($m\boldsymbol{a} = \boldsymbol{F}$)

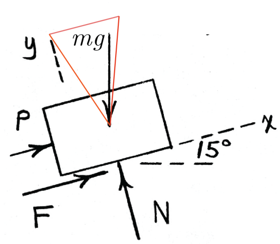

Figure 2: Illustration to Problem 1.

onto x and y axes gives

$$ma_x = P + F - mg\sin\alpha, \quad (1)$$
$$0 = N - mg\cos\alpha, \quad (2)$$

where $m = 50$kg, $g = 9.81\text{m/s}^2$, $\alpha = 15^\circ$, and F stands for the dry friction force. If the block is in equilibrium (does not move and therefore $a_x = 0$) the dry friction is anywhere in $-\mu_s N \le F \le \mu_s N$ and therefore $|F/N| \le \mu_s$. If the block is in motion ($a_x \ne 0$), the dry friction $F = \pm\mu_k N$, where the sign taken opposite to the motion direction[1].

(a) $P = 0$. First, conduct the equilibrium check by setting $a_x = 0$. The equilibrium takes place when the force resulting from F and N is within the friction cone, that is when $|F/N| \le \mu_s$. For $a_x = 0$ we have $F = mg\sin\alpha$, $N = mg\cos\alpha$ and $|F/N| = \tan\alpha \approx 0.268 > \mu_s$. Therefore the equilibrium is not possible and the block is sliding down. Therefore $F = \mu_k N$ and from (1,2) one obtains

$$a_x = \frac{-mg\sin\alpha + \mu_k N}{m} = \frac{-mg\sin\alpha + \mu_k mg\cos\alpha}{m} = g(-\sin\alpha + \mu_k\cos\alpha) \approx -1.118\text{m/s}^2$$

[1] The motion direction is established by the sign of the resulting active force excluding friction. In our problem, it is defined by the sign of $P - mg\sin\alpha$.

(b) $P = 150\,\mathrm{N}$. Conduct the equilibrium check. For $a_x = 0$ we have $F = -P + mg\sin\alpha$, $N = mg\cos\alpha$ and $F/N = \tan\alpha - \frac{P}{mg\cos\alpha} \approx -0.049$. Since $|F/N| < \mu_s$ the block is in equilibrium (does not move) and therefore

$$a_x = 0$$

(c) $P = 300\,\mathrm{N}$. Conduct the equilibrium check. For $a_x = 0$ we have $F = -P + mg\sin\alpha$, $N = mg\cos\alpha$ and $F/N = \tan\alpha - \frac{P}{mg\cos\alpha} \approx -0.365$. Since $|F/N| > \mu_s$ the block cannot be in equilibrium, and the block is sliding up. Therefore $F = -\mu_k N$ and from (1,2) one obtains

$$a_x = \frac{P - mg\sin\alpha - \mu_k N}{m} = \frac{P - mg(\sin\alpha + \mu_k\cos\alpha)}{m} = \frac{P}{m} - g(\sin\alpha + \mu_k\cos\alpha) \approx 2.0396\mathrm{m/s^2}$$

P2. A man pulls himself up the 15° incline by the method shown. If the combined mass of the man and cart is 100kg, determine the acceleration of the cart if the man exerts a pull of 250N on the rope. Neglect all friction and the mass of the rope, pulleys, and wheels.

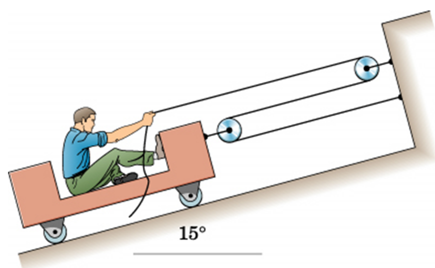

Figure 3: Illustration to Problem 2.

Solution: The forces are sketched in Figure 4. Projecting the Newton vector equation ($m\boldsymbol{a} = \boldsymbol{F}$)

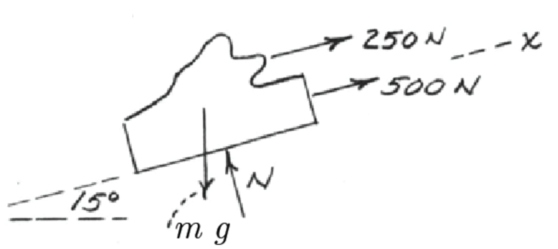

Figure 4: Illustration to Problem 2.

onto x-axis[2] gives

$$ma_x = T + 2T - mg\sin 15^{\mathrm{o}},$$

where the total mass $m = 100$kg, the tension in the rope $T = 250$N, and $g = 9.81\mathrm{m/s^2}$. Thus, $a_x \approx 4.96\mathrm{m/s^2}$.

[2]Projection onto y-axis gives $N - mg\cos 15^{\mathrm{o}} = 0$, from which we can find the normal reaction N. But we are not requested to establish it in this problem.