Exercises in Physics
Assignment # 9

Date Given: June 9, 2022
Date Due: June 16, 2022

P1. (1 point) The car passes over the top of a vertical curve at A with a speed of 60km/h and then passes through the bottom of a dip at B. The radii of curvature of the road at A and B are both 100m. Find the speed of the car at B if the normal force between the road and the tires at B is twice that at A. The height of the mass center of the car is 1m from the road.

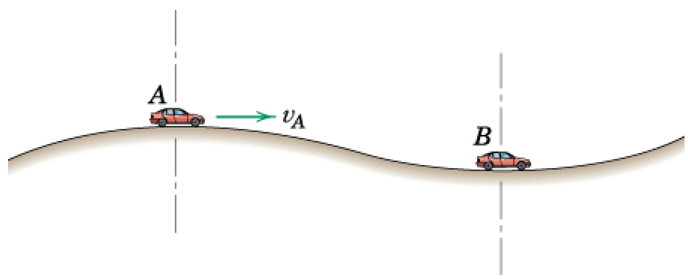

Figure 1: Illustration to Problem 1.

Solution:

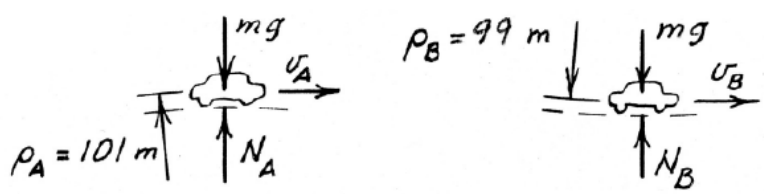

Figure 2: Illustration to Problem 1.

At point A we have, $\sum F_n = ma_n \Longrightarrow mg - N_A = mv_A^2/\rho_A$, and at point B we have, $\sum F_n = ma_n \Longrightarrow N_B - mg = mv_B^2/\rho_B$. Since $N_B = 2N_A$, we obtain

$$m\left(g + \frac{v_B^2}{\rho_B}\right) = 2m\left(\frac{g - v_A^2}{\rho_A}\right) \Longrightarrow v_B^2 = \rho_B g - 2v_A^2\frac{\rho_A}{\rho_B}$$

Therefore, for $v_A^2 \approx 16.666$m/s, $\rho_A = 101$m and $\rho_B = 99$m, we get

$$\boxed{v_B = \sqrt{\rho_B g - 2v_A^2\frac{\rho_A}{\rho_B}} \approx 20.7\text{m/s}}$$

P2. (2 points) The robot arm is elevating and extending simultaneously. At a given instant, $\theta = 30^o$, $\dot\theta = \pi/3\,\text{rad/s}$, and $\ddot\theta = 2\pi/3\,\text{rad/s}^2$ $l = 0.5\,\text{m}$, $\dot l = 0.5\,\text{m/s}$, and $\ddot l = -0.5\,\text{m/s}^2$. Compute the radial and transverse forces F_r and F_θ that the arm must exert on the gripped part P, which has a mass of 1.2 kg. Compare with the case of static equilibrium in the same position.

Solution:

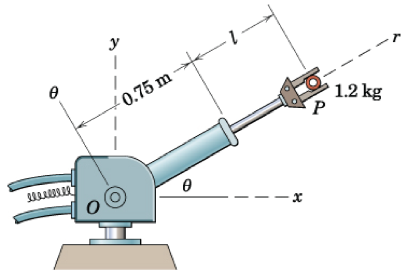

Figure 3: Illustration to Problem 2.

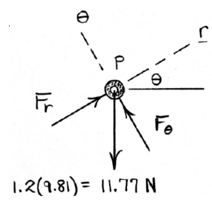

Figure 4: Illustration to Problem 2.

In polar coordinates, we have

$$\begin{array}{lll} r = 1.25\ \text{m}, & \dot{r} = 0.5\ \text{m/s}, & \ddot{r} = -0.5\ \text{m/s}^2, \\ \theta = \pi/6\ \text{rad} & \dot{\theta} = \pi/3\ \text{rad/s}, & \ddot{\theta} = 2\pi/3\ \text{rad/s}^2 \end{array}$$

Therefore

$$a_r = \ddot{r} - r\dot{\theta}^2 \approx -1.87078\,\text{m/s}^2, \quad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \approx 3.66519\,\text{m/s}^2.$$

(a) In the dynamic case, $\sum F_r = ma_r \implies F_r - mg\sin 30^{\text{o}} = ma_r$, and $\sum F_\theta = ma_\theta \implies F_\theta - mg\cos 30^{\text{o}} = ma_\theta$. Therefore

$$\boxed{F_r = mg\sin 30^{\text{o}} + ma_r \approx 3.64107\,\text{N}, \quad F_\theta = mg\cos 30^{\text{o}} + ma_\theta \approx 14.5931\,\text{N}}$$

(b) In the static case, we set $a_r = 0$ and $a_\theta = 0$. Therefore

$$\boxed{F_r = mg\sin 30^{\text{o}} \approx 5.886\,\text{N}, \quad F_\theta = mg\cos 30^{\text{o}} \approx 10.1949\,\text{N}}$$

P3. (3 points) The slotted arm revolves in the horizontal plane[1] about the fixed vertical axis through point O. The 1kg slider C is drawn toward O at the constant rate of 10mm/s by pulling the cord S. At the instant for which $r = 200$mm, the arm has a counterclockwise angular velocity $\omega = 3$rad/s and is speeding up at the rate of 1rad/s^2. For this instant, determine the tension T in the cord and the magnitude N of the force exerted on the slider by the sides of the smooth[2] radial slot. Indicate which side, A or B, of the slot contacts the slider.

Solution: Use polar coordinates. Here we have,

$$\begin{array}{lll} r = 0.2\text{m}, & \dot{r} = -0.01\text{m/s}, & \ddot{r} = 0\ \text{m/s}^2, \\ & \dot{\theta} = \omega = 3\ \text{rad/s}, & \ddot{\theta} = 1\,\text{rad/s}^2 \end{array}$$

For the motion in the radial direction we have

[1]So, gravity can be ignored.
[2]So, friction can be ignored.

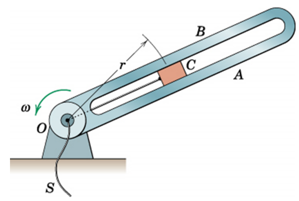

Figure 5: Illustration to Problem 3.

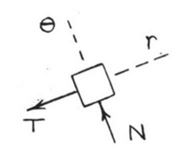

Figure 6: Illustration to Problem 3.

$$ma_r = m(\ddot{r} - r\dot{\theta}^2) = -T \quad \Longrightarrow \quad \boxed{T = 1.8\text{N.}}$$

For the motion in the transversal direction we have

$$ma_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = N \quad \Longrightarrow \quad \boxed{N = 0.14\text{N.}}$$

$\boxed{\text{Since } N \text{ is positive, contact is on side } A.}$

P4. (2 points) Beginning from rest when $\theta = 20^o$, a 35kg child slides with negligible friction down the sliding board which is in the shape of a 2.5m circular arc. Determine the tangential acceleration and speed of the child, and the normal force exerted on her (a) when $\theta = 30^o$ and (b) when $\theta = 90^o$.

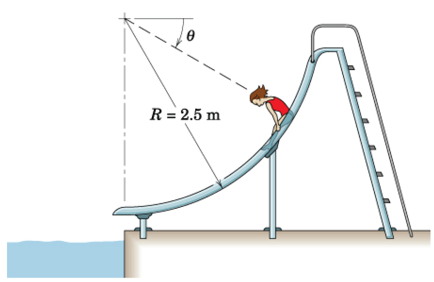

Figure 7: Illustration to Problem 4.

Solution: Treat the child as a particle.

Motion equations in path coordinates are formulated as follows: $\sum F_t = ma_t \Longrightarrow mg\cos\theta = ma_t$. $\sum F_n = ma_n \Longrightarrow N - mg\sin\theta = mv^2/R$ Therefore

$$a_t = g\cos\theta, \qquad N = mg\sin\theta + mv^2/R.$$

The velocity v can be found by integrating $a_t\text{d}s = v\text{d}v$. Note that $\text{d}s = R\text{d}\theta$ (the circular arc length), and therefore

$$\int_{s_0}^{s} a_t\text{d}s = \int_{\theta_0}^{\theta} Ra_t\text{d}\theta = \int_{\theta_0}^{\theta} Rg\cos\theta\text{d}\theta = Rg(\sin\theta - \sin\theta_0) = \int_{v_0}^{v} v\text{d}v = \frac{v^2 - v_0^2}{2},$$

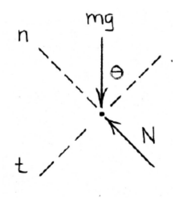

Figure 8: Illustration to Problem 4.

where $v_0 = 0$ since the motion starts from rest. From here we find

$$v = \sqrt{2Rg(\sin\theta - \sin\theta_0)},$$

and then establish

$$N = mg\,(3\sin\theta - 2\sin\theta_0)$$

The computations are conducted for $m = 35\text{kg}, g = 9.81\text{m/s}^2, R = 2.5\text{m}, \theta_0 = 20^o$.

(a) When $\theta = 30^o$ we have,

$$\boxed{v \approx 2.784\text{m/s}, \quad a_t \approx 8.496\text{m/s}^2, \quad N \approx 280.16\text{N}}$$

(b) When $\theta = 90^o$ we have,

$$\boxed{v \approx 5.681\text{m/s}, \quad a_t = 0\text{m/s}^2, \quad N \approx 795.185\text{N}}$$