Exercises in Physics
Assignment # 2

Date Given: April 14, 2022
Date Due: April 21, 2022

P1. (2 points) The acceleration of a particle is given by $a = 2t - 10$, where a is in meters per second squared and t is in seconds. Determine the velocity and displacement as functions of time. The initial displacement at $t = 0$ is $s_0 = -4$m, and the initial velocity is $v_0 = 3$m/s.

Solution: Here we have $a = \mathrm{d}v/\mathrm{d}t = 2t - 10$. Therefore $\mathrm{d}v = (2t - 10)\mathrm{d}t$, and

$$\int_{v_0}^{v} \mathrm{d}v = \int_0^t (2t-10)\mathrm{d}t \Longrightarrow v - v_0 = t^2 - 10t \Longrightarrow v = 3 + t^2 - 10t \text{ (m/s)}$$

Next, from $\mathrm{d}s/\mathrm{d}t = 3 - 10t + t^2$ we have

$$\int_{s_0}^{s} \mathrm{d}s = \int_0^t (3 - 10t + t^2)\mathrm{d}t \Longrightarrow s - s_0 = 3t - 5t^2 + t^3/3 \Longrightarrow s = -4 + 3t - 5t^2 + t^3/3 \text{ (m)}$$

P2. (2 points) The acceleration of a particle is given by $a = -ks^2$, where a is in meters per second squared, k is a constant, and s is in meters. Determine the velocity of the particle as a function of its position s. Evaluate your expression for $s = 5$m if $k = 0.1\mathrm{m}^{-1}\mathrm{s}^{-2}$ and the initial conditions at time $t = 0$ are $s_0 = 3$m and $v_0 = 10$m/s.

Solution: Here we have $a = v\mathrm{d}v/\mathrm{d}s = -ks^2$. Therefore $v\mathrm{d}v = -ks^2\mathrm{d}s$, and

$$\int_{v_0}^{v} v\mathrm{d}v = -\int_{s_0}^{s} ks^2\mathrm{d}s \Longrightarrow \frac{v^2 - v_0^2}{2} = -\frac{k(s^3 - s_0^3)}{3} \Longrightarrow v = \pm\sqrt{v_0^2 - \frac{2}{3}k(s^3 - s_0^3)}$$

Here we must select positive sign because when $s = s_0$ we have $v = v_0 = 10 > 0$. In numbers, for $s = 5$m we have

$$v = \sqrt{v_0^2 - \frac{2}{3}k(s^3 - s_0^3)} = \sqrt{10^2 - \frac{2}{3}0.1(5^3 - 3^3)} \approx 9.67\text{m/s}$$

P3. (2 points) A sprinter reaches his maximum speed $v_{\max}$ in 2 seconds from rest with constant acceleration. He then maintains that speed and finishes the 100 meters in the overall time of 10 seconds. Determine his maximum speed $v_{\max}$.

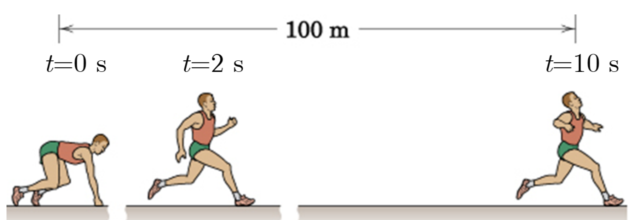

Figure 1: Illustration to Problem 3.

Solution: Divide the overall time interval $t \in [0, 10]$ into two $t \in [0, t_1]$ and $t \in [t_1, 10]$, where $t_1 = 2$s is the time when the sprinter reaches his maximum speed.

1. In the first interval the sprinter starts from the rest ($v_0 = v(0) = 0$ and $s_0 = s(0) = 0$) and runs with constant acceleration a, Therefore $v(t) = at$ and $s(t) = \frac{1}{2}at^2$. At the moment t_1 we have $v_1 = v(t_1) = at_1 = v_{\max}$ and $s_1 = s(t_1) = \frac{1}{2}at_1^2$. Therefore $s_1 = \frac{1}{2}at_1t_1 = \frac{1}{2}v_{\max}t_1 = \frac{1}{2}2v_{\max} = v_{\max}$.
2. In the second interval the sprinter runs with constant velocity (and therefore zero acceleration). The distance corresponding to the second interval is $s_2 = v_{\max}(10-t_1) = (10-2)v_{\max} = 8v_{\max}$.

Since $s_1 + s_2 = 100\text{m}$, we have $(1+8)v_{\max} = 100$ and $v_{\max} = 100/9 \approx 11.11\text{m/s}$.

P4. (2 points) A jet car is originally traveling at a velocity of 10m/s when it is subjected to the acceleration shown. Determine the car's maximum velocity and the time T when it stops.

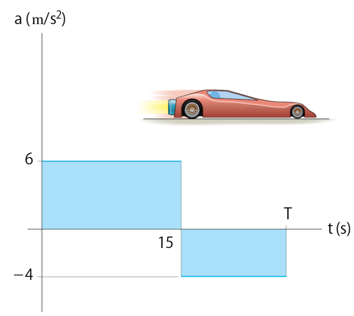

Figure 2: Illustration to Q4.

Solution: The $v - t$ function can be determined by integrating $\mathrm{d}v = a\mathrm{d}t$. For $0 \le t < 15\text{s}$, $a = 6\text{m/s}^2$. Using the initial condition $v = 10\text{m/s}$ at $t = 0$,

$$\int_{10}^{v} \mathrm{d}v = \int_{0}^{15} 6\mathrm{d}t \quad \Longrightarrow \quad v(t) = 10 + 6t$$

The maximum velocity occurs when $t = 15\text{s}$. Then $\boxed{v_{\max} = 100\text{m/s}}$.

For $15 \le t < T\text{s}$, $a = -4\text{m/s}^2$. Using the initial condition $v = 100\text{m/s}$ at $t = 15$,

$$\int_{100}^{v} \mathrm{d}v = -\int_{15}^{t} 4\mathrm{d}t \quad \Longrightarrow \quad v(t) = 160 - 4t$$

The car stops when $v(T) = 0$, therefore $\boxed{T = 40\text{s}}$.