Exercises in Physics
Assignment # 13

Date Given: July 7, 2022
Date Due: July 14, 2022

P1. (2 points) A particle of mass m moves with negligible friction on a horizontal surface and is connected to a light spring fastened at O. At position A the particle has the velocity $v_A = 4\,\mathrm{m/s}$. Determine the velocity v_B of the particle as it passes position B.

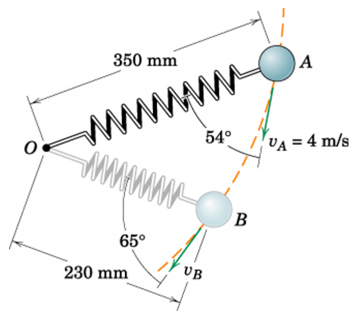

Figure 1: Illustration to Problem 1.

Solution: The particle moves on a horizontal surface so gravity is ignored. The spring force is a central force and it does not create a moment around vertical axis. Therefore $\sum M_{oz} = \dot{H}_{oz} = 0$ and $H_{oz} = \text{const}$.

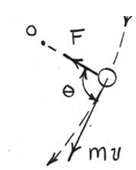

Figure 2: Illustration to Problem 1.

If vectors $\boldsymbol{r}$ and $\boldsymbol{v}$ are in the same plane, $H_{oz} = mrv\sin(\widehat{\boldsymbol{r},\boldsymbol{v}})$. Then, from $(H_{oz})_A = (H_{oz})_B$ we have

$$mv_A L_A \sin 54^\circ = mv_B L_B \sin 65^\circ,$$

where $v_A = 4\,\mathrm{m/s}$, $L_A = 0.350$ m, $L_B = 0.230$ m. Therefore $\boxed{v_B \approx 5.43\mathrm{m/s.}}$

P2. (3 points) The two spheres of equal mass m are able to slide along the horizontal rotating rod. If they are initially latched in position a distance r from the rotating axis with the assembly rotating freely with an angular velocity ω_0, determine the new angular velocity ω after the spheres are released and finally assume positions at the ends of the rod at a radial distance of $2r$. Also find the fraction n of the initial kinetic energy of the system which is lost. Neglect the small mass of the rod and shaft.

Solution: As there is no external moment along the vertical axis (passing through the center of the assembly), the angular momentum about z axis is conserved, $\Delta(H_{oz}) = 0$. At the start point the angular momentum of one particle is[1] $mr^2\omega_0$, and at the end point it is $m(2r)^2\omega$. The conservation of the angular momentum (for the system of two particles) reads

$$2mr^2\omega_0 - 2m(2r)^2\omega = 0.$$

[1]One can use cylindrical coordinates $\boldsymbol{r} \times m\boldsymbol{v} = m(r\boldsymbol{e}_r) \times (\dot{r}\boldsymbol{e}_r + r\omega_0\boldsymbol{e}_\theta) = mr^2\omega_0\boldsymbol{e}_z$.

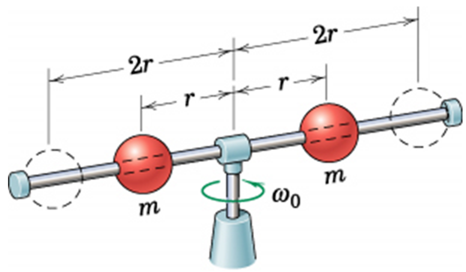

Figure 3: Illustration to Problem 2.

Therefore, we have

$$\boxed{\omega = \omega_0/4\ .}$$

At the start point the kinetic energy of one particle is[2] $\frac{1}{2}m(r\omega_0)^2$, and at the end point it is $\frac{1}{2}m(2r\omega)^2$. The change of kinetic energy (for the system of two particles) is

$$\Delta T = 2\left(\frac{1}{2}m\,[r\omega_0]^2\right) - 2\left(\frac{1}{2}m\,[2r\omega]^2\right) = 2\left(\frac{1}{2}m\,[r\omega_0]^2\right) - 2\left(\frac{1}{2}m\left[2r\frac{\omega_0}{4}\right]^2\right) = \frac{3}{4}mr^2\omega_0^2,$$

and therefore

$$\boxed{n = \Delta T/T = \left(\tfrac{3}{4}mr^2\omega_0^2\right)/\left(mr^2\omega_0^2\right) = \tfrac{3}{4}.}$$

P3. (2 points) The man of mass m_1 and the woman of mass m_2 are standing on opposite ends of the platform of mass m_0 which moves with negligible friction and is initially at rest with $s = 0$. The man and woman begin to approach each other. Derive an expression for the displacement s of the platform when the two meet in terms of the displacement x_1 of the man relative to the platform.

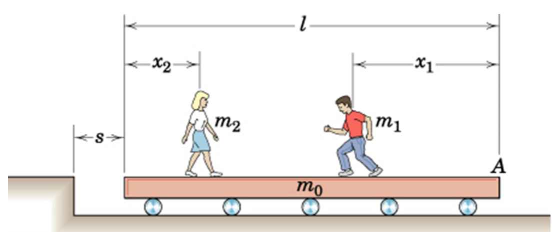

Figure 4: Illustration to Problem 3.

Solution: Since there are no external forces in the motion direction, by conservation of the linear

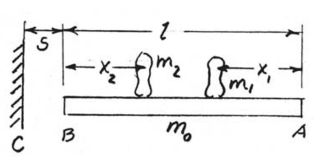

Figure 5: Illustration to Problem 3.

[2]Strictly speaking, $T\frac{1}{2}m(\boldsymbol{v}\cdot\boldsymbol{v}) = \frac{1}{2}m(\dot{r}\boldsymbol{e}_r + r\omega_0\boldsymbol{e}_\theta)\cdot(\dot{r}\boldsymbol{e}_r + r\omega_0\boldsymbol{e}_\theta) = \frac{1}{2}(\dot{r}^2 + (r\omega_0)^2)$. However, when the particle at the start position $\dot{r} = 0$. Also, $\dot{r} = 0$ when the particle assumes the end position.

momentum for the velocity of the center of mass we have

$$v_{\rm cm} = \frac{m_0\dot{s} + m_1(\dot{s} - \dot{x}_1) + m_2(\dot{s} + \dot{x}_2)}{m_0 + m_1 + m_2} = \text{const} = C_1,$$

where $C_1 = 0$ because initially the system is at rest. Since $v_{\rm cm} = 0$, the center of mass,

$$x_{\rm cm} = \frac{m_0(s + \frac{l}{2}) + m_1(s + l - x_1) + m_2(s + x_2)}{m_0 + m_1 + m_2} = \text{const} = C_2. \tag{1}$$

At the initial moment $s = 0, x_1 = 0, x_2 = 0$, and therefore

$$x_{\rm cm} = \frac{m_0\frac{l}{2} + m_1 l}{m_0 + m_1 + m_2} = \text{const} = C_2. \tag{2}$$

From (1) and (2), we obtain

$$s = \frac{m_1x_1 - m_2x_2}{m_0 + m_1 + m_2}.$$

They meet when $x_1 + x_2 = l$, so $x_2 = l - x_1$ and

$$\boxed{s = \frac{(m_1 + m_2)x_1 - m_2 l}{m_0 + m_1 + m_2}.}$$

P4. (3 points) The 10×10^3 kg barge B supports a 2×10^3 kg automobile A. The barge and the automobile are originally at rest. If someone drives the automobile to the other side of the barge, determine how far the barge moves. Neglect the resistance of the water.

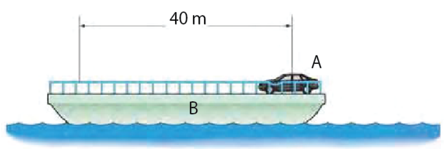

Figure 6: Illustration to Problem 4.

Solution: Let x_A and x_B be the positions of the automobile and the barge, respectively, measured in some inertial reference frame. Since the system is initially at rest, the conservation of linear momentum gives

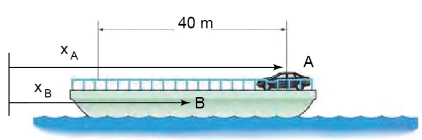

Figure 7: Illustration to Problem 4.

$$v_{\rm cm} = \frac{m_Av_A + m_Bv_B}{m_A + m_B} = 0, \quad \Longrightarrow \quad v_A + 5v_B = 0,$$

where $v_{\rm cm}$ is the velocity of center of mass, v_A, v_B are the velocities of the automobile and the barge, respectively, and m_A, m_B are their masses. Since $v_{\rm cm} = 0$, the center of mass does not change:

$$x_{\rm cm} = \frac{m_Ax_A + m_Bx_B}{m_A + m_B} = \text{const}, \quad \Longrightarrow \quad x_A + 5x_B = \text{const}.$$

Consider this equation at the start (time t=0) and end point (time $t = T$),

$$x_A(0) + 5x_B(0) = x_A(T) + 5x_B(T),$$

and notice that $x_A(0) = x_B(0) + 20$ and $x_A(T) = x_B(T) - 20$. Then the distance the barge moves (to the right) is defined as

$$\boxed{6\left(x_B(T) - x_B(0)\right) = 40, \quad \Longrightarrow \quad x_B(T) - x_B(0) = \tfrac{40}{6} \approx 6.66666\,\mathrm{m/s}.}$$