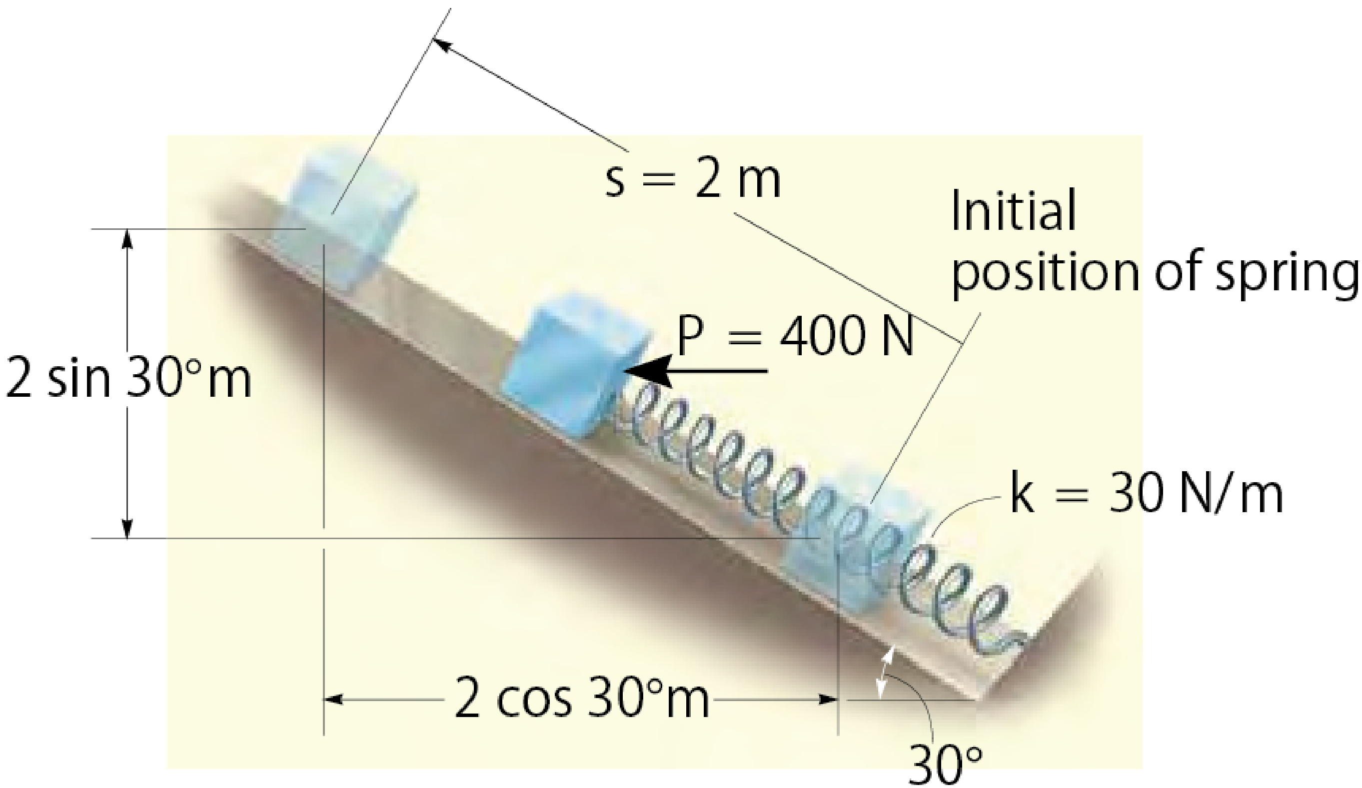
Exercises in Physics

Lecture 10
Introduction to Work

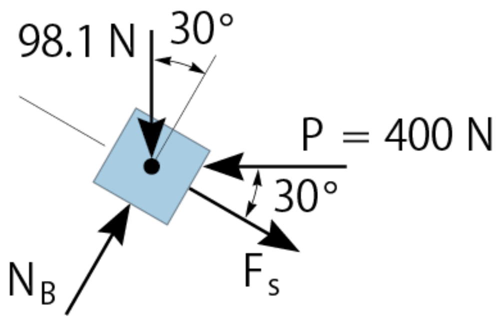
Sample Problem 1

The 10 kg block shown in Figure rests on the smooth incline. If the spring is originally stretched 0.5 m, determine the total work done by all the forces acting on the block when a horizontal force P = 400 N pushes the block up the plane s = 2 m.



Solution

First the free-body diagram of the block is drawn in order to account for all the forces that act on the block



Horisontal Force P. Since this force is constant, the work is determined as the force times the component of displacement in the direction of the force:

$$U_P = 400\,\text{N}\ (2\,\text{m}\cos 30^\circ) = 692.8\ \text{J}$$

or the displacement times the component of force in the direction of displacement:

$$U_P = 400\,\text{N}\ \cos 30^\circ\,(2\,\text{m}) = 692.8\ \text{J}$$

Solution

Spring Force $\boldsymbol{F}_s$. In the initial position the spring is stretched s1 = 0.5 m and in the final position it is stretched s2 = 0.5 m + 2 m = 2.5 m. We require the work to be negative since the force and displacement are opposite to each other. The work of $\boldsymbol{F}_s$ is thus

$$U_s = -\left(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2\right) = -\left(\frac{1}{2}30\frac{\mathrm{N}}{\mathrm{m}}(2.5\mathrm{m})^2 - \frac{1}{2}30\frac{\mathrm{N}}{\mathrm{m}}(0.5\mathrm{m})^2\right) = -90\,\mathrm{J}$$

Weight W. Since the weight ($W = mg$) acts in the opposite sense to its vertical displacement, the work is negative:

$$U_W = -98.1\,\mathrm{N}\ (2\,\mathrm{m}\sin 30^\circ) = -98.1\ \mathrm{J}$$

Note that it is also possible to consider the component of weight in the direction of displacement:

$$U_W = -98.1\sin 30^\circ\ \mathrm{N}\ (2\,\mathrm{m}) = -98.1\ \mathrm{J}$$

Solution

Normal Force N_B . This force does no work since it is always perpendicular to the displacement:

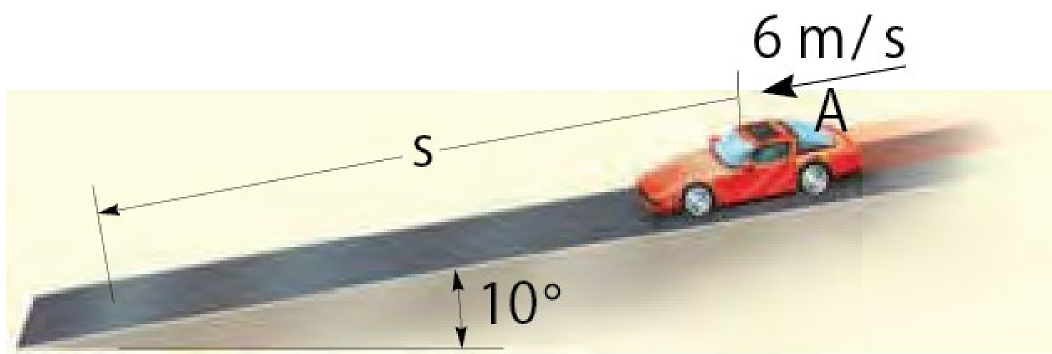
$$U_{N_B} = 0$$

Total work. The work of all the forces when the block is displaced 2 m is therefore:

$$U_T = U_P + U_s + U_W = 692.8\ \text{J} - 90\ \text{J} - 98.1\ \text{J} = 505\ \text{J}$$

Sample Problem 2

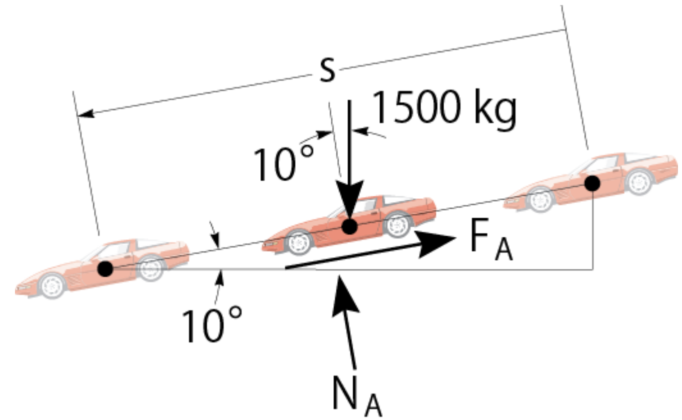
The 1500 kg automobile travels down the 10° inclined road at a speed of 6 m/s. If the driver jams on the brakes, causing his wheels to lock, determine how far s the tires skid on the road. The coefficient of kinetic friction between the wheels and the road is $\mu_k = 0.5$.



SOLUTION. This problem can be solved using the principle of work and energy, since it involves force, velocity, and displacement.

Solution

Work (Free-Body Diagram). As shown in the figure, the normal force N_A does no work since it never undergoes displacement along its line of action. The weight, mg, is displaced $s\sin 10^{\circ}$ and does positive work. Why? The frictional force F_A does both external and internal work when it undergoes a displacement s. This work is negative since it is in the opposite sense of direction to the displacement. Applying the equation of equilibrium normal to the road, we have



$$\sum F_n = ma_n \quad \Rightarrow \quad N_A - mg\cos 10^{\circ} = 0$$

$$N_A = 1500 \times 9.81\cos 10^{\circ} = 14639.2\,\text{N}$$

$$\Rightarrow \quad F_A = \mu_k N_A = 0.5 \times 14639.2 = 7319.58\ \text{N}$$

Solution

Principle of work and energy.

$$T_1 + \sum U_{1-2} = T_2 \qquad \Rightarrow$$

$$\frac{1}{2}mv_1^2 + mgs\sin 10^\circ - \mu_k N_A s = \frac{1}{2}mv_2^2 \quad \Rightarrow$$

$$\frac{1}{2}mv_1^2 + mgs\sin 10^\circ - \mu_k mg\cos 10^\circ s = \frac{1}{2}mv_2^2 \quad \Rightarrow$$

$$\frac{1}{2}v_1^2 + sg(\sin 10^\circ - \mu_k \cos 10^\circ) = 0 \quad \Rightarrow$$

$$\frac{1}{2}6^2 + s \times 9.81(\sin 10^\circ - 0.5\cos 10^\circ) = 0 \quad \Rightarrow$$

$$s = 5.69824 \text{ m}$$

Solution

NOTE: If this problem is solved by using the equation of motion, two steps are involved. First, from the free-body diagram the equation of motion is applied along the incline. This yields

$$\sum F_s = ma_s \quad \Rightarrow \quad mg\sin 10^\circ - N_A = ma_s \quad \Rightarrow$$

$$mg\sin 10^\circ - \mu_k mg\cos 10^\circ = ma_s \quad \Rightarrow$$

$$a_s = g(\sin 10^\circ - \mu_k \cos 10^\circ)$$

Then, since a_s is constant, we have

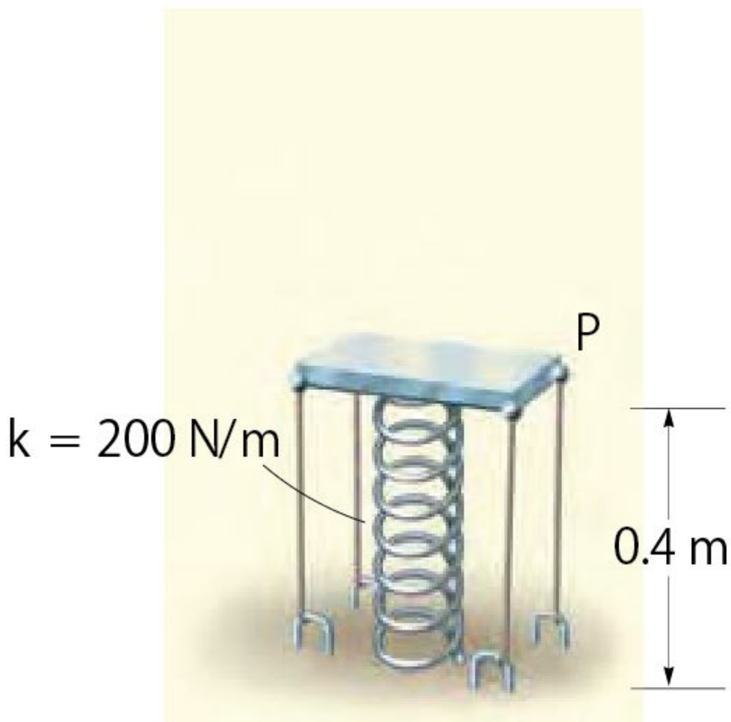
$$v^2 = v_0^2 + 2a_s(s - s_0) \quad \Rightarrow \quad 0 = v_0^2 + 2a_s(s - s_0) \quad \Rightarrow 0 = v_0^2 + 2a_s s$$

$$0 = v_0^2 + 2g(\sin 10^\circ - \mu_k \cos 10^\circ)\ s \quad \Rightarrow$$

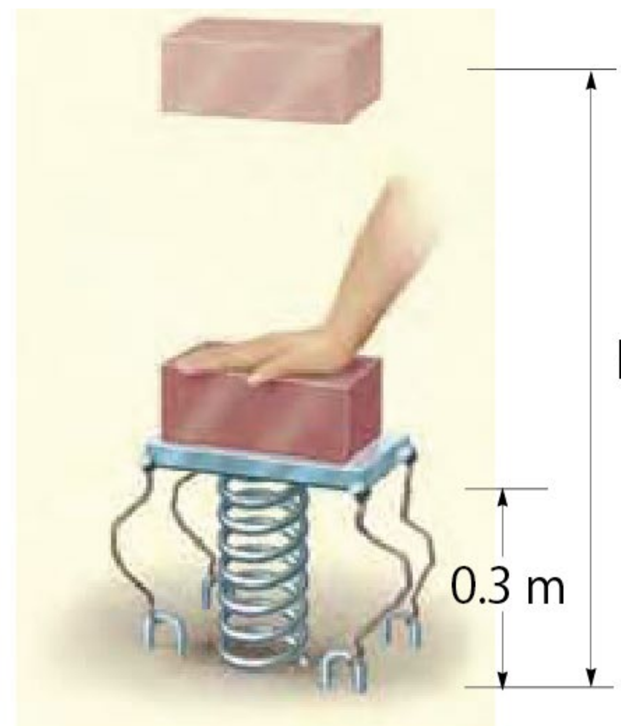
$$s = -\frac{v_0^2}{2g(\sin 10^\circ - \mu_k \cos 10^\circ)} = 5.69824\ \text{m}$$

Sample Problem 3

The platform P has negligible mass and is tied down so that the 0.4-m-long cords keep a 1-m-long spring compressed 0.6 m when nothing is on the platform. If 2-kg block is placed on the platform and released from rest after the platform is pushed down 0.1 m, determine the maximum height h the block rises in the air, measured from the ground.



(a)

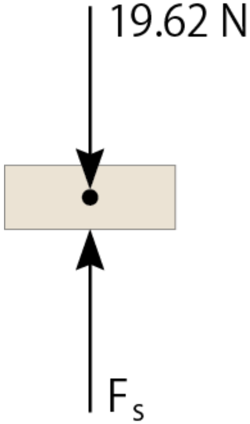


(b)

Solution

Work (Free-Body Diagram). Since the block is released from rest and later reaches its maximum height, the initial and final velocities are zero. The free-body diagram of the block when it is still in contact with the platform is shown in the figure. Note that the weight does negative work and the spring force does positive work. Why?

$$2mg = 19.62\,\mathrm{N}$$



In particular, the initial compression in the spring is s1 = 0.6 m + 0.1 m = 0.7 m. Due to the cords, the spring's final compression is s2 = 0.6 m (after the block leaves the platform). The bottom of the block rises from a height of (0.4 m - 0.1 m) = 0.3 m to a final height h.

Solution

Principle of work-energy.

$$T_1 + \sum U_{1-2} = T_2 \qquad \Rightarrow$$

$$\frac{1}{2}mv_1^2 + \left\{ -\left(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2 \right) - mg\Delta y \right\} = \frac{1}{2}mv_2^2$$

Note that here $s_1 = 0.7\,\mathrm{m}$, $s_2 = 0.6\,\mathrm{m}$, and so the work of the spring will indeed be positive once the calculation is made. Also note that $\Delta y = h - 0.3$ and at the highest point $v_2 = 0$. Thus

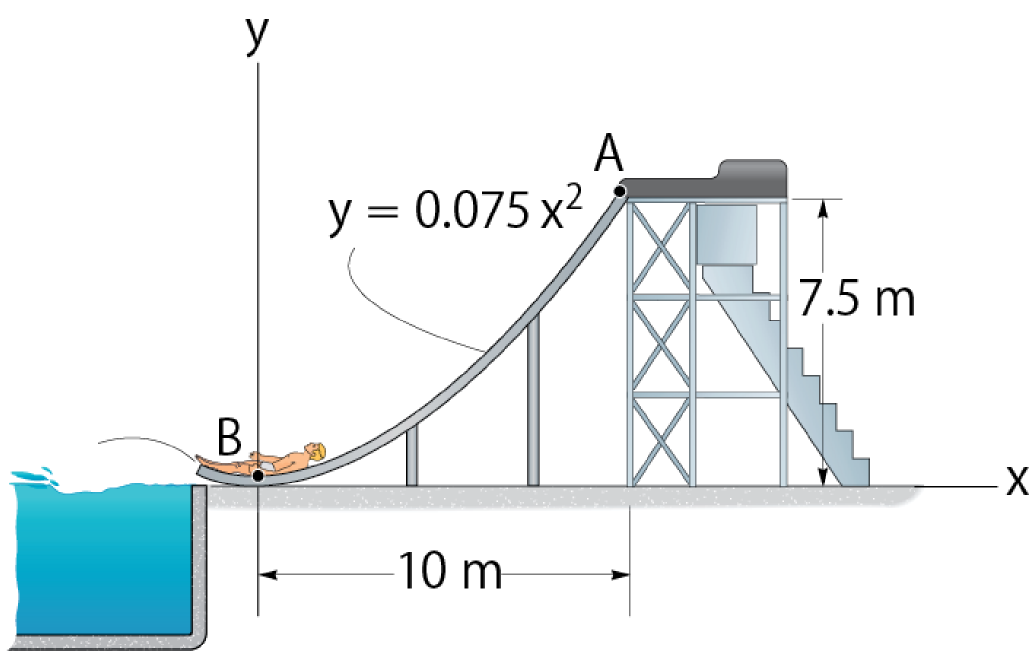
$$0 + \left\{ -\left(\frac{1}{2}200\mathrm{N/m}\,(0.6\,\mathrm{m})^2 - \frac{1}{2}200\mathrm{N/m}\,(0.7\,\mathrm{m})^2 \right) - 19.62\mathrm{N}(h - 0.3)\mathrm{m} \right\} = 0$$

Solving yields

$$h = 0.963\mathrm{m}$$

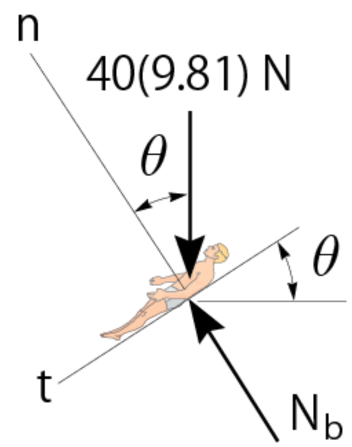
Sample Problem 4

The 40 kg boy slides down the smooth water slide. If he starts from rest at A, determine his speed when he reaches B and the normal reaction the slide exerts on the boy at this position.



Solution

Work (Free-Body Diagram). As shown on the free-body diagram, there are two forces acting on the boy as he goes down the slide. Note that the normal force does no work.



Principle of work-energy.

$$T_A + \sum U_{A-B} = T_B \quad \Rightarrow$$

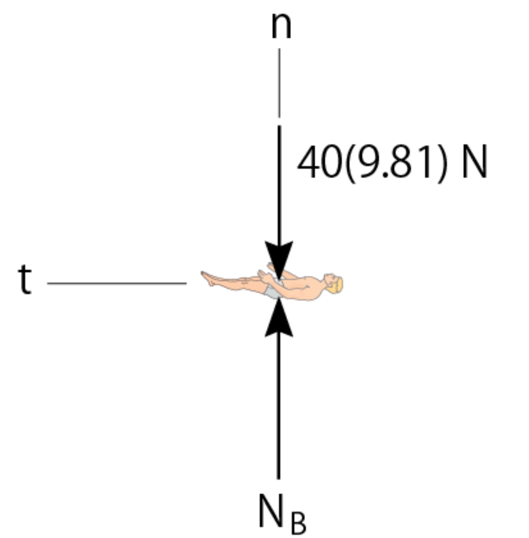
$$\frac{1}{2} m v_A^2 + mgh = \frac{1}{2} m v_B^2 \quad \Rightarrow$$

$$0 + (40 \times 9.81)\text{N} \times 7.5\text{m} = \frac{1}{2}(40\text{kg}) v_B^2 \quad \Rightarrow \quad v_B = 12.13\,\text{m/s}$$

Solution

Equation of Motion. Referring to the free-body diagram of the boy when he is at B, the normal reaction N_B can now be obtained by applying the equation of motion along the n axis. Here the radius of curvature of the path is

$$\rho = \left. \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} \right|_{x=0} = \frac{(1+0.15^2)^{3/2}}{|0.15|} = 6.667\,\mathrm{m}$$



Thus

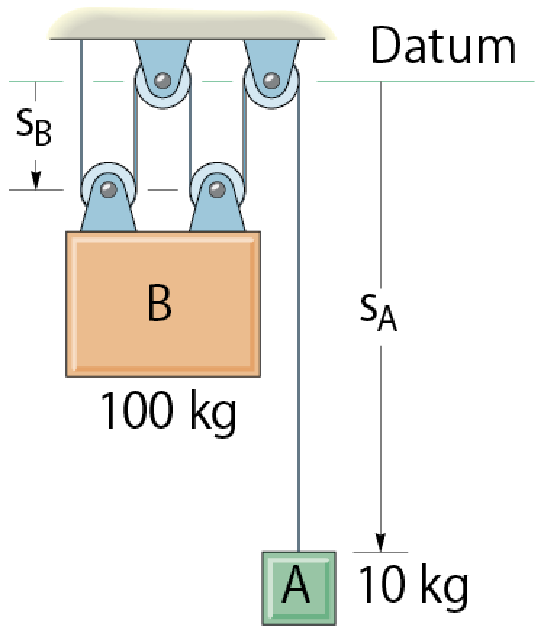
$$\sum F_n = ma_n \quad \Rightarrow \quad N_B - mg = ma_n = mv^2/\rho$$

Thus

$$N_B = (40\times 9.81) + 40(12.31)^2/6.667 = 1275.3\,\mathrm{N}$$

Sample Problem 5

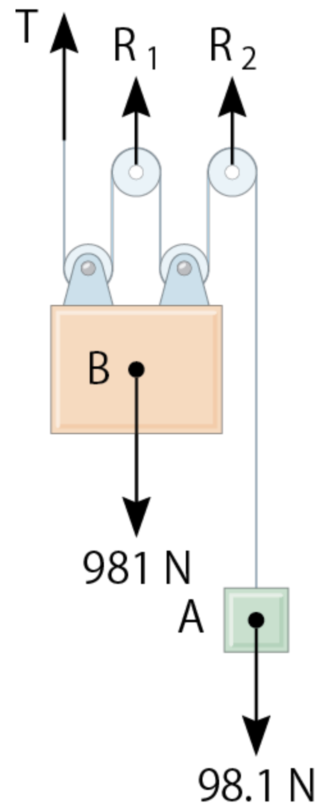
Blocks A and B have a mass of 10 kg and 100 kg, respectively. Determine the distance B travels when it is released from rest to the point where its speed becomes 2 m/s.



SOLUTION. This problem may be solved by considering the blocks separately and applying the principle of work and energy to each block. However, the work of the (unknown) cable tension can be eliminated from the analysis by considering blocks A and B together as a *single system*.

Solution

Work (Free-Body Diagram). As shown on the free-body diagram of the system, the cable force T and reactions R_1 and R_2 do no work, since these forces represent the reactions at the supports and consequently, they do not move while the blocks are displaced. The weights both do positive work if we assume both move downward, in the positive sense of direction of s_A and s_B .



Solution

Principle of work-energy.

$$T_1 + \sum U_{1-2} = T_2 \qquad \Rightarrow$$

$$\frac{1}{2} m_A (v_A^2)_1 + \frac{1}{2} m_B (v_B^2)_1 + \left\{ m_A g \Delta s_A + m_A g \Delta s_B \right\} =$$

$$\frac{1}{2} m_A (v_A^2)_2 + \frac{1}{2} m_B (v_B^2)_2 \qquad \Rightarrow$$

$$0 + 0 + \left\{ 98.1 \Delta s_A + 981 \Delta s_B \right\} = \frac{1}{2} 10 (v_A^2)_2 + \frac{1}{2} 100 (2^2)$$

Solution

Kinematics. It may be seen that the total length l of all the vertical segments of cable may be expressed in terms of the position coordinates s_A and s_B as

$$s_A + 4s_B = l$$

Hence, a change in position yields the displacement equation

$$\Delta s_A + 4\Delta s_B = 0 \quad \Rightarrow \quad \Delta s_A = -4\Delta s_B$$

Here we see that a downward displacement of one block produces an upward displacement of the other block. Note that s_A and s_B must have the same sign convention in all the equations. Taking the time derivative yields

$$v_A = -4v_B = -4 \times 2 = -8\,\mathrm{m/s}$$

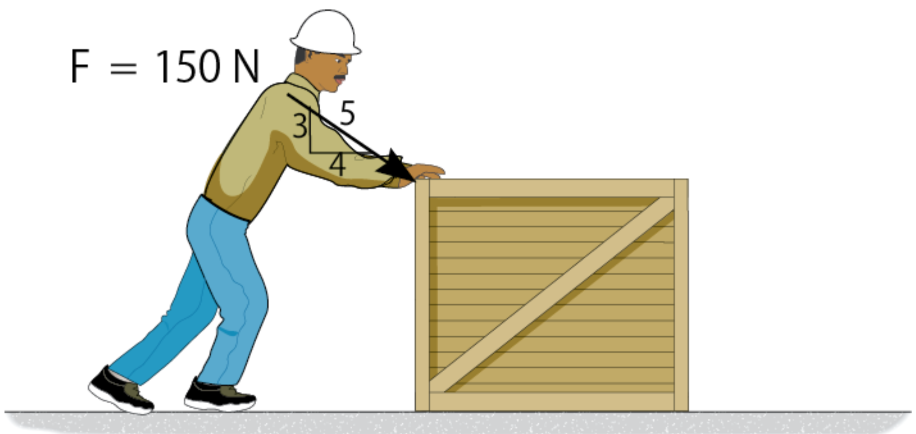
Solving for Δs_B then yields

$$\{-4 \times 98.1\,\Delta s_B + 981\,\Delta s_B\} = \frac{1}{2}10(8^2) + \frac{1}{2}100(2^2)$$

$$\Rightarrow \quad \Delta s_B = 0.883\,\mathrm{m}$$

Sample Problem 6

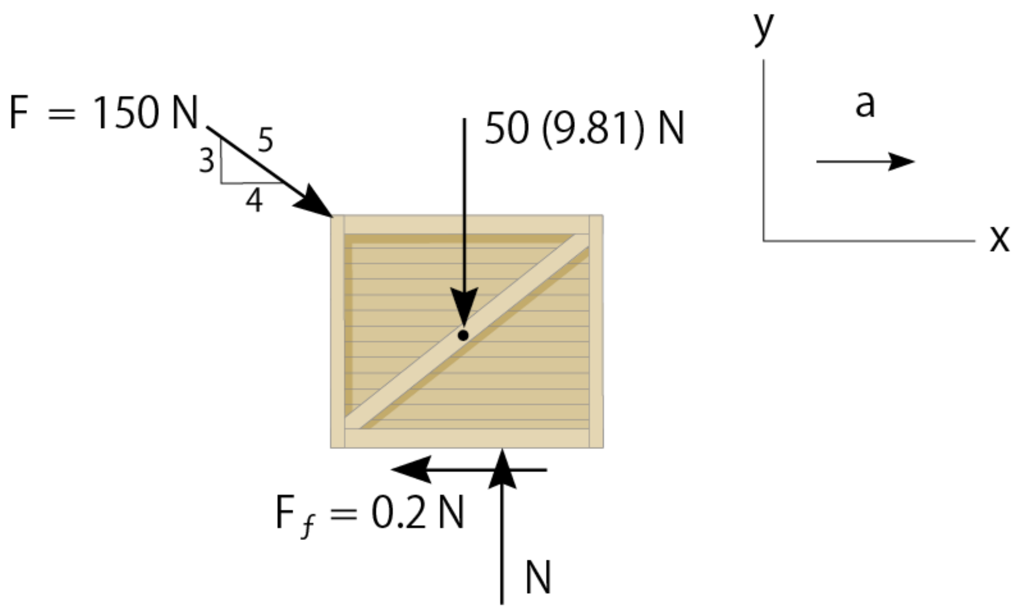
The man in the figure pushes on the 50-kg crate with a force of F = 150 N. Determine the power supplied by the man when t = 4 s. The coefficient of kinetic friction between the floor and the crate is $\mu_k = 0.2$. Initially the create is at rest.



SOLUTION. To determine the power developed by the man, the velocity of the 150-N force must be obtained first.

Solution

Free-Body Diagram. The free-body diagram of the crate is shown below. Applying the equation of motion,



$$\sum F_y = ma_y \;\Rightarrow N - \frac{3}{5}150 - 50\times 9.81 = 0 \;\Rightarrow N = 580.5\,\mathrm{N}$$

$$\sum F_x = ma_x \;\Rightarrow \frac{4}{5}150 - 0.2\times 580.5 = 50a \;\Rightarrow a = 0.078\,\mathrm{m/s^2}$$

Solution

Kinematics. The velocity of the crate when t = 4 s is therefore

$$v = v_0 + a\,t \;\Rightarrow 0 + 0.078 \times 4 = 0.312\,\mathrm{m/s^2}$$

Kinematics. The power supplied to the crate by the man when t = 4 s is therefore

$$P = \boldsymbol{F} \bullet \boldsymbol{v} = F_x\,v \;\Rightarrow \frac{4}{5}150 \times 0.312 = 37.4\,\mathrm{W}$$