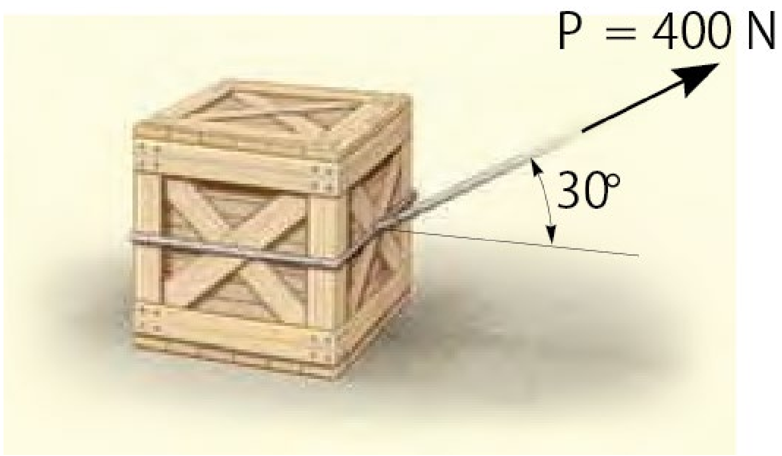
# Lecture 8
# Kinetics: rectangular coordinates

# Sample Problem 1

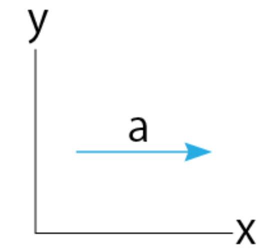
The 50-kg crate rests on a horizontal surface for which the coefficient of kinetic friction is $\mu_k = 0.3$ . If the crate is subjected to a 400-N towing force as shown, determine the velocity of the crate in 3 s starting from rest.




SOLUTION. Using the equations of motion, we can relate the crate's acceleration to the force causing the motion. The crate's velocity can then be determined using kinematics.

# Solution

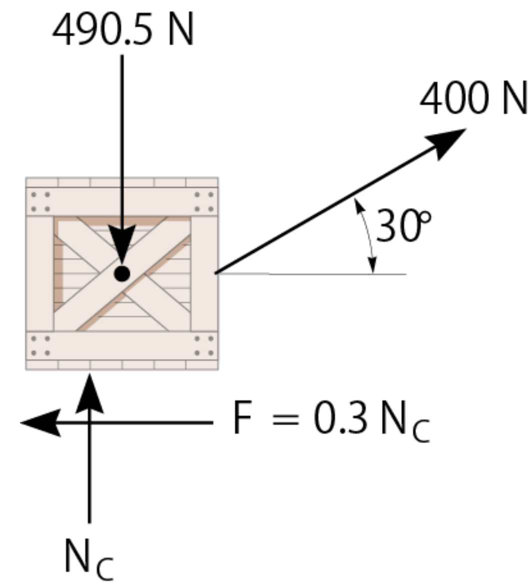
Free-Body Diagram. The weight of the crate is $W = mg = 50\text{kg} \times 9.81\text{m/s}^2 = 490.5\,\text{N}$. As shown in the figure, the frictional force has a magnitude $F = \mu_k N_C$ and acts to the left, since it opposes the motion of the crate. The acceleration $a$ is assumed to act horizontally, in the positive x direction. There are two unknowns, namely $N_C$ and $a$.




Equations of Motion. Using the data shown on the free-body diagram, we have

$$\sum F_x = ma_x \;\Rightarrow\; 400\cos 30^\circ - 0.3N_C = 50ma$$

$$\sum F_y = ma_y \;\Rightarrow\; N_C - 490.5 + 400\sin 30^\circ = 0$$




Solving for $N_C$ and $a$ yields

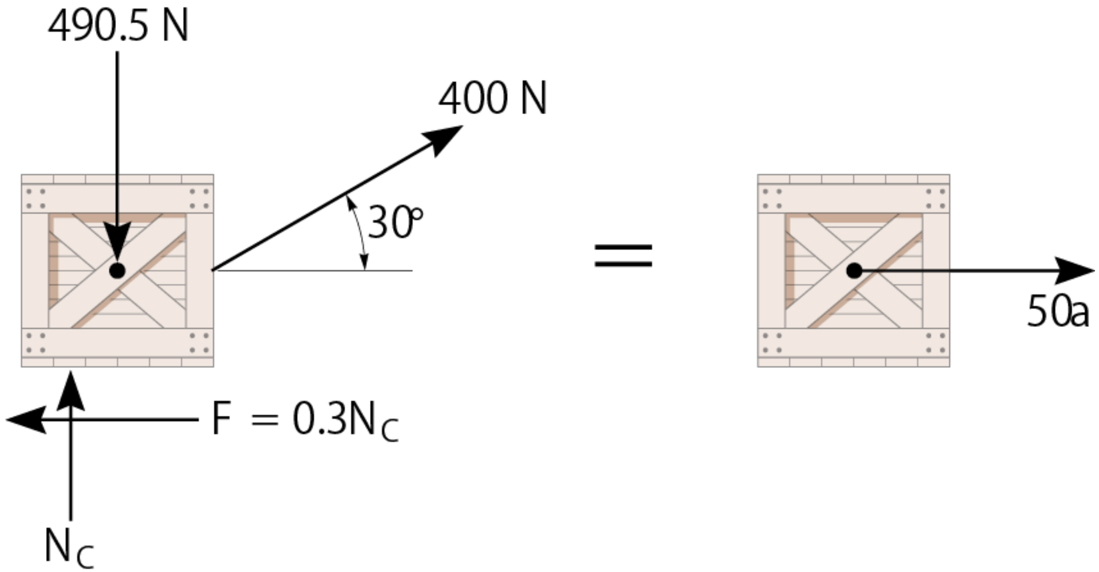
$$N_C = 290.5\,\text{N}, \qquad a = 5.185\text{m/s}^2$$

# Solution

Kinematics. Notice that the acceleration is constant, since the applied force P is constant. Since the initial velocity is zero, the velocity of the crate in 3 s is

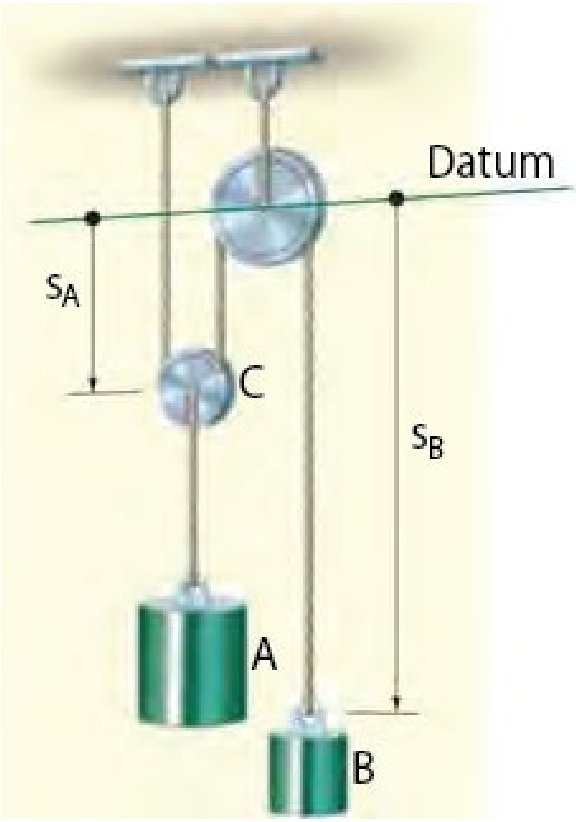
$$v = v_0 + at = 0 + 5.185 \times 3 = 15.6\text{m}/\text{s}$$

Note. We can also use the alternative procedure of drawing the crate's free-body and kinetic diagrams, prior to applying the equations of motion.



# Sample Problem 2

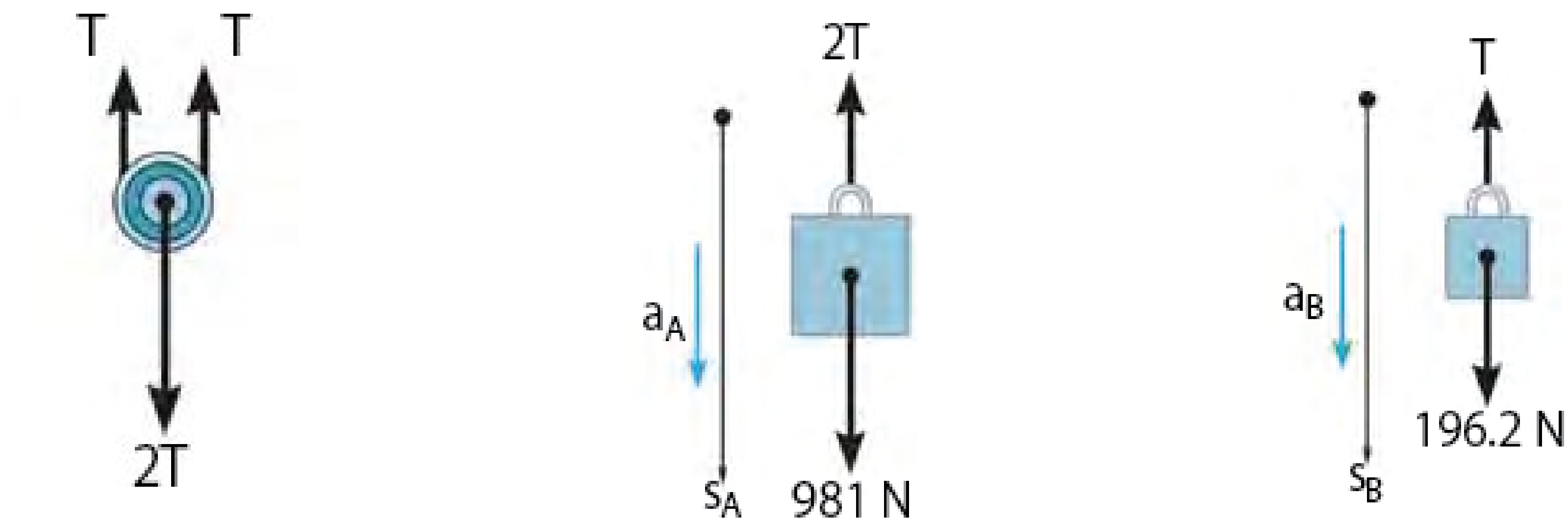
The 100-kg block A is released from rest. If the masses of the pulleys and the cord are neglected, determine the velocity of the 20-kg block B in 2 s.



# Solution

Free-Body Diagram. Since the mass of the pulleys is neglected, then for pulley C, $ma = 0$ and we can apply $\sum F_y = 0$, as shown in the left figure.

The free-body diagrams for blocks A and B are shown in the middle and the right figures, respectively. Notice that for A to remain stationary T = 490.5 N, whereas for B to remain static T = 196.2 N. Hence A will move down while B moves up. Although this is the case, we will assume both blocks accelerate downward, in the direction of $+s_a$ and $+s_b$. The three unknowns are T, $a_A$, and $a_B$.



# Solution

Equations of Motion.

$$\text{Block A}: \quad \sum F_y = ma_y \;\Rightarrow 981 - 2T = 100a_A$$

$$\text{Block B}: \quad \sum F_y = ma_y \;\Rightarrow 196.2 - T = 20a_B$$

Kinematics. The necessary third equation is obtained by relating $a_A$ to $a_B$ using a dependent motion analysis. The coordinates $s_A$ and $s_B$ measure the positions of A and B from the fixed datum. It is seen that

$$2s_A + s_B = l$$

where $l$ is constant and represents the total vertical length of cord. Differentiating this expression twice with respect to time yields

$$2a_A + a_B = 0$$

# Solution

Notice that when writing motion equations, the positive direction was always assumed downward. It is very important to be consistent in this assumption since we are seeking a simultaneous solution of equations. The results are

$$T = 327\mathrm{N}, \quad a_A = 3.27\mathrm{m/s^2}, \quad a_B = -6.54\mathrm{m/s^2}$$

Hence when block A accelerates downward, block B accelerates upward as expected. Since $a_B$ is constant, the velocity of block B in 2 s is thus

$$v = v_0 + a_B t = 0 - 6.54 \times 2 = -13.1\mathrm{m/s}$$

The negative sign indicates that block B is moving upward.