Exercises in Physics
Assignment # 4

Date Given: April 28, 2022
Date Due: May 12, 2022

P1. (2 points) At some instant of time, a particle has velocity $\boldsymbol{v} = \sqrt{3}\boldsymbol{i} + \boldsymbol{j}$ m/s, acceleration magnitude $|\boldsymbol{a}| = 5\,\mathrm{m/s^2}$, and the radius of curvature $\rho = 8/5\,\mathrm{m}$. Find the angle between the velocity and acceleration vectors of the particle at that instant.

Solution: From $\boldsymbol{v} = \sqrt{3}\boldsymbol{i} + \boldsymbol{j} = v\boldsymbol{e}_t$, one finds the speed of the particle $v = \sqrt{3+1} = 2\,\mathrm{m/s}$. The normal component of the acceleration vector (along $\boldsymbol{e}_n$) is $a_n = v^2/\rho = 5/2\,\mathrm{m/s^2}$. Then, since $|\boldsymbol{a}| = \sqrt{a_n^2 + a_t^2} = 5$, one obtains $a_t = 5\sqrt{3}/2$. Next, since

$$\boldsymbol{a} \cdot \boldsymbol{v} = (a_n\boldsymbol{e}_n + a_t\boldsymbol{e}_t) \cdot v\boldsymbol{e}_t = a_t v = 5\sqrt{3},$$

and, on the other hand,

$$\boldsymbol{a} \cdot \boldsymbol{v} = |\boldsymbol{a}||\boldsymbol{v}| \cos(\widehat{\boldsymbol{v},\boldsymbol{a}}) = |\boldsymbol{a}| v \cos(\widehat{\boldsymbol{v},\boldsymbol{a}}) = 10 \cos(\widehat{\boldsymbol{v},\boldsymbol{a}})$$

one defines $\cos(\widehat{\boldsymbol{v},\boldsymbol{a}}) = a_t/|\boldsymbol{a}| = \sqrt{3}/2$, and therefore the angle is 30°.

P2. (2 points) A car travels along the level curved road with a speed which is decreasing at the constant rate of 0.6 m/s each second. The speed of the car as it passes point A is 16 m/s. Calculate the magnitude of the total acceleration of the car as it passes point B which is 120 m along the road from A. The radius of curvature of the road at B is 60 m.

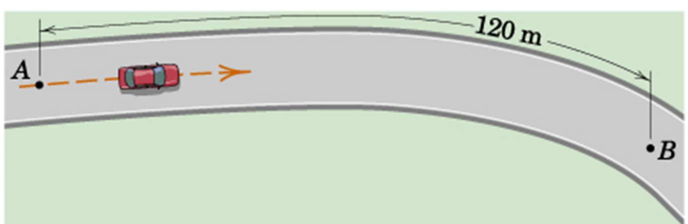

Figure 1: Illustration to Problem 2.

Solution: Since the speed is decreasing, we have $a_t = -0.6\,\mathrm{m/s}$. Since $v\mathrm{d}v = a_t\mathrm{d}s$ (see Sample Problem 2/7 in the textbook), we have

$$\int_{v_A}^{v_B} v\mathrm{d}v = \int_0^s a_t\mathrm{d}s \quad\Longrightarrow\quad \left[\frac{v^2}{2}\right]_{v_A}^{v_B} = a_t\,[s]_0^s \quad\Longrightarrow\quad \frac{v_B^2 - v_A^2}{2} = a_t s \quad\Longrightarrow\quad v_B^2 = v_A^2 + 2a_t s,$$

and, since $v_A = 16\,\mathrm{m/s}$ and $s = 120\,\mathrm{m}$, we find $v_B^2 = v_A^2 + 2a_t s = 16^2 - 2 \times 0.6 \times 120$ and therefore $v_B = 10.58\,\mathrm{m/s}$. Next $a_n = v_B^2/\rho = 1.867\,\mathrm{m/s^2}$ and therefore $a = \sqrt{a_t^2 + a_n^2} = 1.961\,\mathrm{m/s^2}$.

P3. (2 points) The car travels along the circular path such that its speed is increased by $a_t = (4t^2)\mathrm{m/s^2}$, where t is in seconds. Determine the magnitudes of its velocity and acceleration after the car has traveled $s = 27$m starting from rest. Neglect the size of the car.

Solution: Since $\mathrm{d}v = a_t\mathrm{d}t$ and $\mathrm{d}s = v\mathrm{d}t$, we have

$$\int_0^v \mathrm{d}v = \int_0^t a_t\mathrm{d}t = \int_0^t 4t^2\mathrm{d}t \quad\Longrightarrow\quad v = 4t^3/3$$

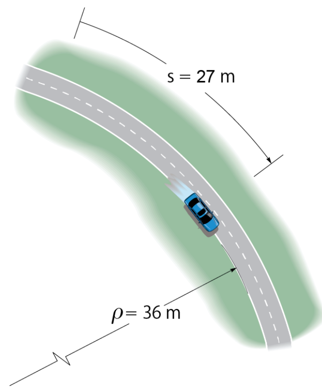

Figure 2: Illustration to Problem 3.

and

$$\int_0^{27} \mathrm{d}s = \int_0^t v\mathrm{d}t = \int_0^t (4t^3/3)\mathrm{d}t \quad \Longrightarrow \quad 27 = t^4/3$$

From these relationship we find

$$\begin{aligned} t &= 3\mathrm{s}, \qquad v = 4t^3/3 = 36\mathrm{m/s}, \\ a_t &= \dot{v} = 4t^2 = 36\mathrm{m/s}^2, \qquad a_n = \frac{v^2}{\rho} = \frac{36^2}{36} = 36\mathrm{m/s}^2, \\ a &= \sqrt{a_t^2 + a_n^2} = \sqrt{36^2 + 36^2} = 36\sqrt{2} \approx 50.9117\mathrm{m/s}^2. \end{aligned}$$

P4. (2 points) If the car passes point A with a speed of 20m/s and begins to increase its speed at a constant rate of $a_t = 0.5\mathrm{m/s}^2$, determine the magnitude of the car's acceleration at point C where $s = 101.68\mathrm{m}$ and $x = 0$.

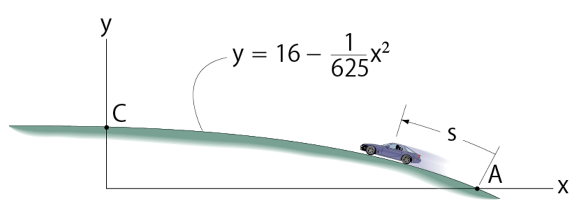

Figure 3: Illustration to Problem 4.

Solution: Since $v\mathrm{d}v = a_t\mathrm{d}s$ (see Sample Problem 2/7 in the textbook), the speed of the car at point C can be defined from

$$\int_{v_A}^{v_C} v\mathrm{d}v = \int_{s_A}^{s_C} a_t\mathrm{d}s \quad \Longrightarrow \quad \left[\frac{v^2}{2}\right]_{v_A}^{v_C} = a_t\,[s]_{s_A}^{s_C} \quad \Longrightarrow \quad \frac{v_C^2 - v_A^2}{2} = a_t(s_C - s_A) \quad \Longrightarrow \quad v_C^2 = v_A^2 + 2a_t(s_C - s_A)$$

Therefore (for $s_A = 0$ and $s_C = 101.68\mathrm{m}$) we get

$$v_C^2 = 20^2 + 2 \times 0.5 \times 101.68 \quad \Longrightarrow \quad v_C = 22.361\mathrm{m/s}$$

Next, the given curve for $y = 16 - x^2/625$ is computed as

$$y = 16 - x^2/625, \qquad \mathrm{d}y/\mathrm{d}x = -3.2 \times 10^{-3}x, \qquad \mathrm{d}^2y/\mathrm{d}x^2 = -3.2 \times 10^{-3}.$$

Next, the radius of the curvature is computed (at $x = 0$) as

$$\rho(x) = \frac{(1 + (\mathrm{d}y/\mathrm{d}x)^2)^{3/2}}{|\mathrm{d}^2y/\mathrm{d}x^2|} = \frac{(1 + (-3.2 \times 10^{-3}x)^2)^{3/2}}{|-3.2 \times 10^{-3}|} \approx 312.5\mathrm{m}$$

Next, we establish the normal and tangential components of the acceleration

$$\begin{aligned} a_n &= \frac{v_C^2}{\rho} = \frac{22.361^2}{312.5} \approx 1.60\mathrm{m/s}^2, \\ a_t &= \dot{v} = 0.5\mathrm{m/s}^2. \end{aligned}$$

Thus, the magnitude of the car's acceleration at point C is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.5^2 + 1.60^2} \approx 1.68\mathrm{m/s}^2.$$