Exercises in Physics
Assignment

Date Given: April 8, 2022
Date Due: April 14, 2022

P1. (1 point) With $\boldsymbol{v} = (1, 1)$ and $\boldsymbol{w} = (1, 5)$ find a number c so that $\boldsymbol{w} - c\boldsymbol{v}$ is perpendicular to $\boldsymbol{v}$.

Solution: $(\boldsymbol{w} - c\boldsymbol{v}) \cdot \boldsymbol{v} = 0$. Therefore $\boldsymbol{v} \cdot \boldsymbol{w} = c\boldsymbol{v} \cdot \boldsymbol{v}$. Therefore $c = (\boldsymbol{v} \cdot \boldsymbol{w})/(\boldsymbol{v} \cdot \boldsymbol{v}) = 6/2 = 3$. Note that $c\boldsymbol{v}$ is the "projection" of $\boldsymbol{w}$ onto the line through vector $\boldsymbol{v}$.)

P2. (1 point) What are the cosines of the angles α, β, γ between the vector $\boldsymbol{v} = (1, 0, -1)$ and the unit vectors $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ along the axes?

Solution: First, find $|\boldsymbol{v}| = \sqrt{\boldsymbol{v} \cdot \boldsymbol{v}} = \sqrt{2}$.

(a) $\boldsymbol{i} \cdot \boldsymbol{v} = (1, 0, 0) \cdot (1, 0, -1) = 1$. On the other hand, $\boldsymbol{i} \cdot \boldsymbol{v} = |\boldsymbol{i}||\boldsymbol{v}| \cos\alpha \Longrightarrow \cos\alpha = 1/\sqrt{2}$.

(b) $\boldsymbol{j} \cdot \boldsymbol{v} = (0, 1, 0) \cdot (1, 0, -1) = 0$. On the other hand, $\boldsymbol{j} \cdot \boldsymbol{v} = |\boldsymbol{j}||\boldsymbol{v}| \cos\beta \Longrightarrow \cos\beta = 0$.

(c) $\boldsymbol{k} \cdot \boldsymbol{v} = (0, 0, 1) \cdot (1, 0, -1) = -1$. On the other hand, $\boldsymbol{k} \cdot \boldsymbol{v} = |\boldsymbol{k}||\boldsymbol{v}| \cos\gamma \Longrightarrow \cos\gamma = -1/\sqrt{2}$.

P3. (3 points) Assume that three unit vectors $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ are such that $\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c} = \boldsymbol{0}$. Compute $\boldsymbol{a}\cdot\boldsymbol{b}+\boldsymbol{b}\cdot\boldsymbol{c}+\boldsymbol{c}\cdot\boldsymbol{a}$.

Solution: It follows from $\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} = \boldsymbol{0}$ that vectors $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ form a triangle. Since the vectors are unit ($|\boldsymbol{a}| = |\boldsymbol{b}| = |\boldsymbol{c}| = 1$), the triangle is equilateral. Therefore, the angles between the sides of the equilateral triangle are $\pi/3$, and the angles between the vectors are $\pi - \pi/3 = 2\pi/3$. Then $\boldsymbol{a} \cdot \boldsymbol{b} + \boldsymbol{b} \cdot \boldsymbol{c} + \boldsymbol{c} \cdot \boldsymbol{a} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos\frac{2\pi}{3} + |\boldsymbol{b}|\,|\boldsymbol{c}| \cos\frac{2\pi}{3} + |\boldsymbol{c}|\,|\boldsymbol{a}| \cos\frac{2\pi}{3} = 3\cos\frac{2\pi}{3} = -3/2$.

P4. (3 points) Compute $\boldsymbol{a} \times \boldsymbol{b}$ for

(a) $\boldsymbol{a} = \boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{b} = -\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$

(b) $\boldsymbol{a} = 6\boldsymbol{i} + \boldsymbol{j}$ and $\boldsymbol{b} = 3\boldsymbol{i} - 2\boldsymbol{j}$

(c) $\boldsymbol{a} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{b} = \boldsymbol{j}$

Solution:

(a) $\boldsymbol{a} \times \boldsymbol{b} = 0\boldsymbol{i} + 0\boldsymbol{j} + 0\boldsymbol{k}$
Note that we can get this result immediately by noticing that $\boldsymbol{a} = -\boldsymbol{b}$ (and therefore $\boldsymbol{a}$ and $\boldsymbol{b}$ are on the same line).

(b) $\boldsymbol{a} \times \boldsymbol{b} = 0\boldsymbol{i} + 0\boldsymbol{j} - 15\boldsymbol{k}$

(c) $\boldsymbol{a} \times \boldsymbol{b} = -1\boldsymbol{i} + 0\boldsymbol{j} + 1\boldsymbol{k}$

P5. (2 points) Assume that three-dimensional vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are not collinear (do not lie on a single straight line). Find a scalar λ such that the vectors $\lambda\boldsymbol{a} + \boldsymbol{b}$ and $3\boldsymbol{a} + \lambda\boldsymbol{b}$ are collinear.

Solution: If two vectors are collinear their cross product is zero vector. Therefore, from $(\lambda\boldsymbol{a}+\boldsymbol{b}) \times (3\boldsymbol{a}+\lambda\boldsymbol{b}) = \lambda\boldsymbol{a}\times(3\boldsymbol{a}+\lambda\boldsymbol{b})+\boldsymbol{b}\times(3\boldsymbol{a}+\lambda\boldsymbol{b}) = 3\lambda\boldsymbol{a}\times\boldsymbol{a}+\lambda^2\boldsymbol{a}\times\boldsymbol{b}+3\boldsymbol{b}\times\boldsymbol{a}+\lambda\boldsymbol{b}\times\boldsymbol{b} = \lambda^2\boldsymbol{a}\times\boldsymbol{b}+3\boldsymbol{b}\times\boldsymbol{a} = \boldsymbol{0}$ one obtains $\lambda^2\boldsymbol{a} \times \boldsymbol{b} = 3\boldsymbol{a} \times \boldsymbol{b}$ and define $\lambda = \pm\sqrt{3}$.