Exercises in Physics
Assignment # 10

Date Given: June 16, 2022
Date Due: June 23, 2022

P1. (2 points) The crate, which has a mass of 100kg, is subjected to the action of the two forces. If it is originally at rest, determine the distance it slides in order to attain a speed of 6m/s. The coefficient of kinetic friction between the crate and the surface is $\mu_k = 0.2$.

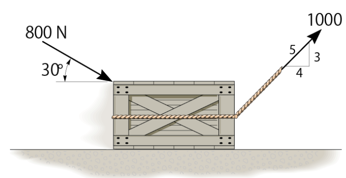

Figure 1: Illustration to Problem 1.

Solution:

(a) Since the crate slides, the friction force developed between the crate and its contact surface is $F_f = \mu_k N$. Since the crate does not move in the normal direction, we have

$$0 = N - mg - 800\sin 30^o + 1000\frac{3}{5} \quad \Longrightarrow \quad N = 781\text{N}$$

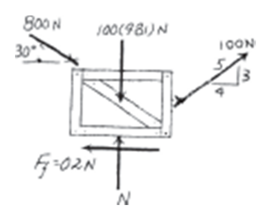

Figure 2: Illustration to Problem 1.

(b) The horizontal components of force 800N and 1000N which act in the direction of displacement do positive work, whereas the friction force $F_f = 0.2(781) = 156.2$N does negative work since it acts in the opposite direction to that of displacement. The normal reaction N, the vertical component of 800N and 1000N force and the weight of the crate do not displace, hence they do no work. Since the crate is originally at rest, $T_1 = 0$. From the work-energy equation, $T_1 + U_{1-2} = T_2$, one gets

$$800\cos 30^o L + 1000\frac{4}{5}L - 156.2L = \frac{1}{2}mv^2 = \frac{1}{2}100\times 6^2 \quad \Longrightarrow \quad \boxed{L \approx 1.347\text{m}}$$

P2. (3 points) The 0.5kg collar C starts from rest at A and slides with negligible friction on the fixed rod in the vertical plane. Determine the velocity v with which the collar strikes end B when acted upon by the force $\boldsymbol{F}$ which is constant in direction and has constant magnitude 5N. Neglect the small size of the collar.

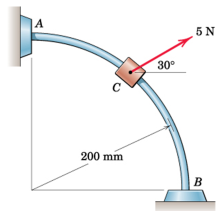

Figure 3: Illustration to Problem 2.

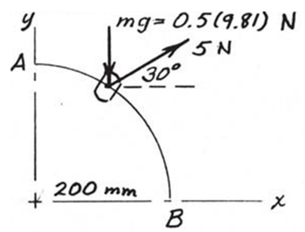

Figure 4: Illustration to Problem 2.

Solution: Let R be the radius of the circular rod. There are two active forces that do work. The gravity force $-mg\boldsymbol{j}$, displacing the collar downward (from A to B) through a distance of R does positive work

$$U_{G_{A-B}} = mgR = 0.981\text{J}.$$

To compute work done by the force $\boldsymbol{F} = 5\cos\frac{\pi}{6}\boldsymbol{i} + 5\sin\frac{\pi}{6}\boldsymbol{j}$, one can use polar coordinates. Since the radius is constant, we have $\boldsymbol{v} = \mathrm{d}\boldsymbol{r}/\mathrm{d}t = R\dot{\theta}\boldsymbol{e}_\theta = R(\mathrm{d}\theta/\mathrm{d}t)\boldsymbol{e}_\theta$ and therefore the small displacement of the collar is $\mathrm{d}\boldsymbol{r} = R\mathrm{d}\theta\boldsymbol{e}_\theta$. Therefore the work done by $\boldsymbol{F}$ is

$$U_{F_{A-B}} = \int_A^B \boldsymbol{F}\cdot\mathrm{d}\boldsymbol{r} = \int_{\pi/2}^0 \boldsymbol{F}\cdot R\mathrm{d}\theta\boldsymbol{e}_\theta = \int_{\pi/2}^0 \boldsymbol{F}\cdot R\mathrm{d}\theta(-\sin\theta\boldsymbol{i} + \cos\theta\boldsymbol{j}) =$$

$$\int_{\pi/2}^0 (5\cos\frac{\pi}{6}\boldsymbol{i} + 5\sin\frac{\pi}{6}\boldsymbol{j})\cdot R\mathrm{d}\theta(-\sin\theta\boldsymbol{i} + \cos\theta\boldsymbol{j}) =$$

$$-5R\cos\frac{\pi}{6}\int_{\pi/2}^0 \sin\theta\mathrm{d}\theta + 5R\sin\frac{\pi}{6}\int_{\pi/2}^0 \cos\theta\mathrm{d}\theta =$$

$$5R\cos\frac{\pi}{6} - 5R\sin\frac{\pi}{6} = 5R\frac{\sqrt{3}-1}{2} = 5\times 0.2\times\frac{\sqrt{3}-1}{2} \approx 0.366025\text{J}$$

Next, from the work-energy equation, $T_A + U_{A-B} = T_B$, where $T_A = 0$ (because the collar is at rest at A) one gets

$$T_B = \frac{1}{2}mv_B^2 = U_{A-B} = U_{F_{A-B}} + U_{G_{A-B}} \quad\Longrightarrow\quad \boxed{v_B = \sqrt{\frac{2(U_{F_{A-B}} + U_{G_{A-B}})}{m}} \approx 2.321\text{m/s}}$$

P3. (3 points) The track is to be designed so that the passengers of the roller coaster experience a certain normal force at points of maxima and minima. Determine the limiting heights h_A and h_C so that the normal force at point C is zero and at point B is four times of the passenger weight. The roller coaster starts from rest at position A. The radii of curvature at the points are indicated. Neglect friction.

Solution: The free-body diagram of the passenger at positions B and C are shown in Figure 6 left and right, respectively.

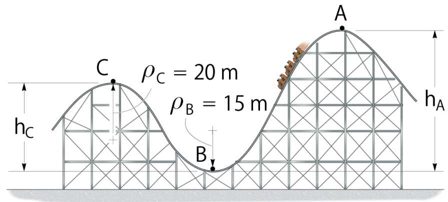

Figure 5: Illustration to Problem 3.

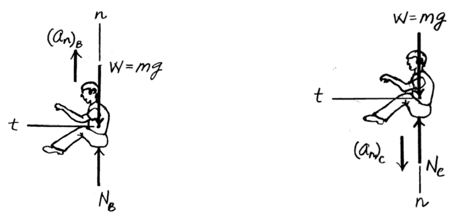

Figure 6: Illustration to Problem 3.

Equations of Motion: in the normal direction we have $ma_n = \sum F_n$, where $a_n = v^2/\rho$ and the total force in the normal direction by the gravity force and reaction force.

(B) The requirement at position B is that $N = 4mg$. By referring to Figure 6 (left), we can write

$$m\frac{v_B^2}{\rho_B} = 4mg - mg \quad \Longrightarrow \quad v_B^2 = 3\rho_B g.$$

(C) The requirement at position B is that $N = 0$. By referring to Figure 6 (right), we can write

$$m\frac{v_C^2}{\rho_C} = mg - 0 \quad \Longrightarrow \quad v_C^2 = \rho_C g.$$

Principle of Work and Energy: The normal reaction N does no work since it always acts perpendicular to the motion.

(B) When the roller coaster moves from position A to B, $W = mg$ displaces vertically downward and does positive work. We have $T_A + U_{A-B} = T_B$ and therefore

$$0 + mgh_A = \frac{1}{2}mv_B^2 = \frac{1}{2}m\,3\rho_B g \quad \Longrightarrow \quad \boxed{h_A = 3\rho_B/2 = 22.5 \text{ m.}}$$

(C) When the roller coaster moves from position B to C, $W = mg$ displaces vertically upward and does negative work. We have $T_B + U_{B-C} = T_C$ and therefore[1]

$$\frac{1}{2}mv_B^2 - mgh_C = \frac{1}{2}mv_C^2 \quad \Longrightarrow \quad \frac{1}{2}m\,3\rho_B - mgh_C = \frac{1}{2}m\rho_C g \quad \Longrightarrow \quad \boxed{h_C = \tfrac{3\rho_B - \rho_C}{2} = 12.5 \text{ m.}}$$

P4. (2 points) The block has a mass of 0.8kg and moves within the smooth vertical slot. If it starts from rest when the attached spring is in the unstretched position at A, determine the constant vertical force F which must be applied to the cord so that the block attains a speed $v_B = 2.5$m/s when it reaches B; $s_B = 0.15$m. Neglect the size and mass of the pulley. (*Hint:* The work of F can be determined as $F\Delta l$, where Δl is the difference in cord lengths AC and BC.)

[1] Alternatively, we could construct the computations in a different way. When the roller coaster moves from position A to C, $W = mg$ displaces vertically downward $h = h_A - h_C$. We have $T_A + U_{A-C} = T_C$ and therefore $0 + mg(h_A - h_C) = \frac{1}{2}mv_C^2 = \frac{1}{2}m\,\rho_C g \implies h_A - h_C = \rho_C/2 \implies h_C = h_A - \rho_C/2 = 12.5$ m.

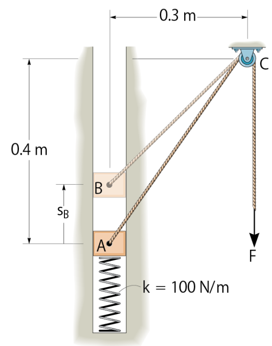

Figure 7: Illustration to Problem 4.

Solution: At state A the block is at rest and its kinetic energy $T_A = 0$. The total work U_{A-B} is done by the gravity force, the spring force, and the pulley force F. The work done by gravity is $-mgs_B$, where $m = 0.8$kg, $s_B = 0.15$m, and $g = 9.81\text{m/s}^2$. Since at state A the spring is unstretched, the work done by the spring force F_s is $-\frac{1}{2}ks_B^2$, where $k = 100$N/m. The work done by the pulling force is $F\Delta l$, where $\Delta l = |AC| - |BC| \approx 0.5 - 0.390512 \approx 0.109488$m.

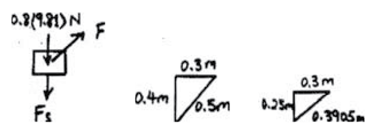

Figure 8: Illustration to Problem 4.

By using the work-energy equation, $T_A + U_{A-B} = T_B$, one obtains

$$F\Delta l - mgs_B - \frac{1}{2}ks_B^2 = \frac{1}{2}mv_B^2 \quad \Longrightarrow \quad \boxed{F = \frac{mgs_B + \frac{1}{2}ks_B^2 + \frac{1}{2}mv_B^2}{\Delta l} \approx 43.9\text{N}}$$