Exercises in Physics
Sample Problems

Date Given: April 14, 2022

P1. A particle moving along a straight line decelerates according to $a = -kv$, where k is a constant and v is velocity. The initial position at time $t = 0$ is $s_0 = 0$. If its initial velocity at time $t = 0$ is $v_0 = 4\text{m/s}$ and its velocity at time $t = 2$ is $v = 1\text{m/s}$, determine the time T and corresponding distance D for the particle speed to be reduced to one-tenth of its initial value.

Solution: Here we have $a = \mathrm{d}v/\mathrm{d}t = -kv$. Therefore $\mathrm{d}v/v = -k\mathrm{d}t$, and

$$-k\int_0^t \mathrm{d}t = \int_{v_0}^{v} \frac{\mathrm{d}v}{v} = \ln v - \ln v_0 = \ln\frac{v}{v_0} \Longrightarrow v = v_0 e^{-kt}$$

For the given conditions $1 = 4e^{-2k}$, and therefore $k \approx 0.693\text{s}^{-1}$. So, $v = v_0 e^{-0.693t}$. When $v = \frac{v_0}{10}$ we have $\frac{v_0}{10} = v_0 e^{-0.693T}$ and therefore $T \approx 3.32\text{s}$.

Next, from $a = -kv = v\mathrm{d}v/\mathrm{d}s$ we have $-k\mathrm{d}s = \mathrm{d}v$, and

$$-k\int_0^s \mathrm{d}s = \int_{v_0}^{v} \mathrm{d}v \Longrightarrow -ks = v - v_0 \Longrightarrow v = v_0 - ks$$

From the given conditions $\frac{v_0}{10} = v_0 - kD$ (taken with $k = k \approx 0.693\text{s}^{-1}$ and $v_0 = 4\text{m/s}$) one establishes $D \approx 5.19\text{m}$.

Note that T is independent of v_0 but D is not.

P2. The aerodynamic resistance to motion of a car is nearly proportional to the square of its velocity. Additional frictional resistance is constant, so that the acceleration of the car when coasting may be written $a = -C_1 - C_2 v^2$, where C_1 and C_2 are constants which depend on the mechanical configuration of the car. If the car has an initial velocity v_0 when the engine is disengaged, derive an expression for the distance D required for the car to coast to a stop.

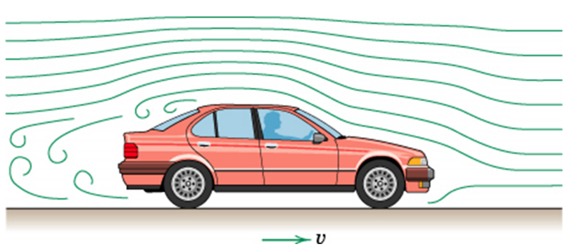

Figure 1: Illustration to Problem 2.

Solution: Here we have $v\mathrm{d}v = a\mathrm{d}s$. Therefore $\mathrm{d}s = v\mathrm{d}v/a = \frac{v\mathrm{d}v}{-C_1 - C_2 v^2}$, and[1]

$$\int_0^s \mathrm{d}s = \int_{v_0}^{v} \frac{v\mathrm{d}v}{-C_1 - C_2 v^2} \Longrightarrow s = -\frac{1}{2C_2}\ln(C_1 + C_2 v^2)\big|_{v_0}^{v} \Longrightarrow s = \frac{1}{2C_2}\ln\frac{C_1 + C_2 v_0^2}{C_1 + C_2 v^2}$$

Next, when $v = 0$ we have

$$s = D = \frac{1}{2C_2}\ln\left(1 + \frac{C_2}{C_1}v_0^2\right)$$

[1] The integral in the velocity part can be taken from the list in Appendix C or computed by hand as follows. Introduce new variable $x = C_1 + C_2 v^2$. Then $\mathrm{d}x = 2C_2 v\mathrm{d}v$ and $v\mathrm{d}v = \frac{\mathrm{d}x}{2C_2}$. Define $x_0 = x = C_1 + C_2 v_0^2$. Then $\int_{v_0}^{v} \frac{v\mathrm{d}v}{-C_1 - C_2 v^2} = -\frac{1}{2C_2}\int_{x_0}^{x}\frac{\mathrm{d}x}{x} = -\frac{1}{2C_2}\ln\frac{x}{x_0} = \frac{1}{2C_2}\ln\frac{C_1 + C_2 v_0^2}{C_1 + C_2 v^2}$.

P3. A motorcycle starts from rest with an initial acceleration of 3m/s^2, and the acceleration then changes with distance as shown. Determine the velocity v of the motorcycle when $s = 200\text{m}$.

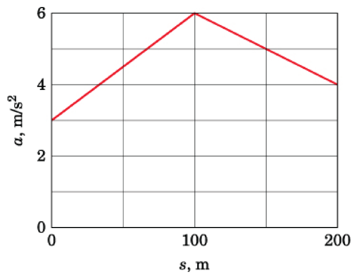

Figure 2: Illustration to Problem 3.

Solution: From $a\mathrm{d}s = v\mathrm{d}v$ we have

$$\int_0^v v\mathrm{d}v = \int_0^{200} a\mathrm{d}s \quad \Longrightarrow \quad \frac{v^2}{2} = \frac{3+6}{2} \times 100 + \frac{6+4}{2} \times 100 = 950,$$

where the second integral is the area under the $a - s$ curve Therefore

$$v = \sqrt{2 \times 950} \approx 43.6\text{m/s}$$