Exercises in Physics
Sample Problems # 5

Date Given: May 12, 2022

P1. The robot arm is elevation and extending simultaneously. At a given instant, $\theta = 30^{\circ}$, $\dot{\theta} = 10^{\circ}/\text{s} = \text{const}$, $l = 0.5$, $\dot{l} = 0.2\text{m/s}$, and $\ddot{l} = -0.3\text{m/s}^2$. Compute the magnitudes of the velocity $\boldsymbol{v}$ and acceleration $\boldsymbol{a}$ of the gripped part P. In addition, express $\boldsymbol{v}$ and $\boldsymbol{a}$ in terms of the unit vectors $\boldsymbol{i}$ and $\boldsymbol{j}$.

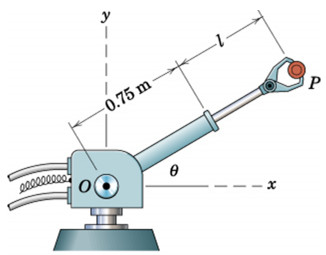

Figure 1: Illustration to Problem 1.

Solution: Here we have

$$
\begin{aligned}
&\theta = 30^{\circ} = \pi/6\,\text{rad} && r = l + 0.75 = 1.25\,\text{m} \\
&\dot{\theta} = 10^{\circ}/\text{s} = 0.1745\,\text{rad/s} && \dot{r} = \dot{l} = 0.2\,\text{m/s} \\
&\ddot{\theta} = 0\,\text{rad/s}^2 && \ddot{r} = \ddot{l} = -0.3\,\text{m/s}^2
\end{aligned}
$$

Therefore

$$\boldsymbol{v} = v_r\boldsymbol{e}_r + v_\theta\boldsymbol{e}_\theta = \dot{r}\boldsymbol{e}_r + r\dot{\theta}\boldsymbol{e}_\theta = 0.2\boldsymbol{e}_r + 0.218\boldsymbol{e}_\theta\,\text{m/s}$$

$$\boxed{v = |\boldsymbol{v}| = \sqrt{v_r^2 + v_\theta^2} \approx 0.296\,\text{m/s}}$$

and

$$\boldsymbol{a} = a_r\boldsymbol{e}_r + a_\theta\boldsymbol{e}_\theta = (\dot{r}r - r\dot{\theta}^2)\boldsymbol{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\boldsymbol{e}_\theta = -0.338\boldsymbol{e}_r + 0.0698\boldsymbol{e}_\theta\,\text{m/s}^2$$

$$\boxed{a = |\boldsymbol{a}| = \sqrt{a_r^2 + a_\theta^2} \approx 0.345\,\text{m/s}^2}$$

To express $\boldsymbol{v}$ and $\boldsymbol{a}$ in terms of the unit vectors $\boldsymbol{i}$ and $\boldsymbol{j}$, recall that

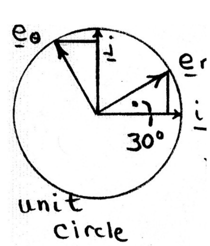

Figure 2: Illustration to Problem 1.

$$\boldsymbol{e}_r = \cos\theta\boldsymbol{i} + \sin\theta\boldsymbol{j}, \qquad \boldsymbol{e}_\theta = -\sin\theta\boldsymbol{i} + \cos\theta\boldsymbol{j}.$$

Therefore

$$\boldsymbol{v} = v_r\boldsymbol{e}_r + v_\theta\boldsymbol{e}_\theta = v_r(\cos\theta\boldsymbol{i} + \sin\theta\boldsymbol{j}) + v_\theta(-\sin\theta\boldsymbol{i} + \cos\theta\boldsymbol{j}) =$$
$$(v_r\cos\theta - v_\theta\sin\theta)\boldsymbol{i} + (v_r\sin\theta + v_\theta\cos\theta)\boldsymbol{j} =$$
$$(0.2\cos\frac{\pi}{6} - 0.218\sin\frac{\pi}{6})\boldsymbol{i} + (0.2\sin\frac{\pi}{6} + 0.218\cos\frac{\pi}{6})\boldsymbol{j} = \boxed{0.064\boldsymbol{i} + 0.289\boldsymbol{j}\ \text{m/s}}$$

Similarly,

$$\boldsymbol{a} = a_r\boldsymbol{e}_r + a_\theta\boldsymbol{e}_\theta = a_r(\cos\theta\boldsymbol{i} + \sin\theta\boldsymbol{j}) + a_\theta(-\sin\theta\boldsymbol{i} + \cos\theta\boldsymbol{j}) =$$
$$(a_r\cos\theta - a_\theta\sin\theta)\boldsymbol{i} + (a_r\sin\theta + a_\theta\cos\theta)\boldsymbol{j} =$$
$$(-0.338\cos\frac{\pi}{6} - 0.0698\sin\frac{\pi}{6})\boldsymbol{i} + (-0.338\sin\frac{\pi}{6} + 0.0698\cos\frac{\pi}{6})\boldsymbol{j} = \boxed{-0.328\boldsymbol{i} - 0.1086\boldsymbol{j}\ \text{m/s}^2}$$

P2. The slotted link is pinned at O, and as a result of the constant angular velocity $\dot{\theta} = 3$rad/s it drives the peg P for a short distance along the spiral guide $r = (0.4\theta)$m, where θ is in radians. Determine the magnitude of the velocity and acceleration of the particle at the instant it leaves the slot in the link, i.e., when $r = 0.5$m.

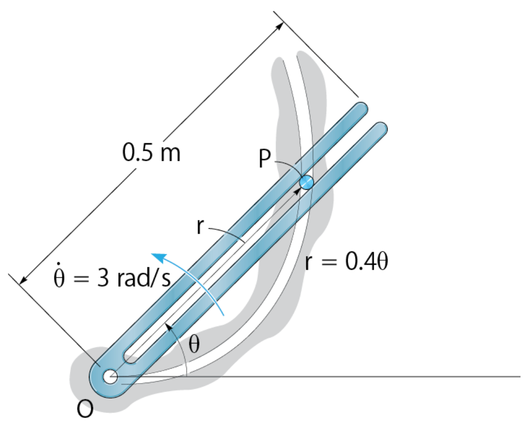

Figure 3: Illustration to Problem 2.

Solution: At $r = 0.5$, $\theta = 0.5/0.4 = 1.25$rad, and therefore

$$\begin{aligned} r &= 0.4\theta = 0.5 & \theta &= 3t, \\ \dot{r} &= 0.4\dot{\theta} = 1.2, & \dot{\theta} &= 3, \\ \ddot{r} &= 0.4\ddot{\theta} = 0, & \ddot{\theta} &= 0. \end{aligned}$$

The components of the velocity and acceleration in polar coordinates are

$$\begin{aligned} v_r &= \dot{r} = 1.2 & a_r &= \ddot{r} - r\dot{\theta}^2 = -4.5 \\ v_\theta &= r\dot{\theta} = 1.5, & a_\theta &= r\ddot{\theta} + 2\dot{r}\dot{\theta} = 7.2 \end{aligned}$$

Therefore

$$v = \sqrt{v_r^2 + v_\theta^2} = 1.92094\ \text{m/s},$$

and

$$a = \sqrt{a_r^2 + a_\theta^2} = 8.49058\ \text{m/s}^2.$$