Systems Biology – Exercises

Week 14: Epidemiological Modeling

Epidemiological Modeling

- You have investigated one of the most widely applied models in deterministic epidemiology, the SEIR model. This model can, as we have seen, be modeled by a system of four differential equations.
- Today's focus will be on a simplified SIR (Susceptible-Infected-Removed) model (Equations 1 to 3), which has one less compartment and does not include birthrates and natural death rates. It is lacking a compartment for modeling the incubation time (often referred to as the Exposed stage). The SIR model is used to model diseases, in which individuals contracting an illness will almost immediately become infectious carrier of said illness, skipping the incubation period.

$$\frac{\mathrm{d}S}{\mathrm{d}t} = -\beta \frac{SI}{N} \tag{1}$$

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \beta \frac{SI}{N} - \gamma I \tag{2}$$

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \gamma I \tag{3}$$

Setting up Initial Values.

- For a simulation, we have to set up a scenario for an epidemic.
- For the Python algorithm below to work, we need to import numpy as np, matplotlib.pyplot as plt and odeint from scipy.integrate.

Code 1. Setting up Number of the Population and Initial Values for Susceptible, Infected, and Removed.

```python
# Total population: N
N = 10000
# Number of infected at time t=0
I0 = 0
# Number of removed (either due to immunity or by death) at time t=0
R0 = 0
# All susceptible individuals at time t=0
S0 = N - I0 - R0
```

Coefficients

- Now let's set the coefficients *β* and *γ*, the contact rate and infectious period respectively.
- The contact rate can be understood as how effective a contagion agent spreads. This one value will have more complex implications in real life, such as the number of people coming in contact with each other on average and the virulence of the illness.
- We use *γ* adjust how long an individual will stay in the infectious compartment—or put differently, how long it takes to recover/die from an illness. The code below has a contact rate of *β* = 0.3 and an infectious period of one week.

Code 2. Equation Coefficients.

```
# Contact rate
beta = 0.3
# Infectious period (inverse of the number of days) or recovery time
gamma = float(1/7)
```

Timeframe

- To prepare for a numerical output of the differential equations, we set a **numpy.linspace** to start at 0 and end at day 100. To get a smooth line, even at steep curves, the increment is set to simulate an hourly rate.

Code 3. Set Timeframe and Increments.

```
# Time from day 0 to day 100 in 1-hour increments
t = np.linspace(0, 100, num=2400)
```

SIR Model Differential Equations.

- In our next step, we define a function that describes the three differential equations. These assignment statements will be interpreted as differential equations by using **odeint** from **scipy.integrate** in the code one after this.

Code 4. SIR Model's Differential Equations and Initial Conditions.

```
# The differential equations of the SIR model
def SIRmodel(compartmentValues, t, N, beta, gamma):
    S, I, R = compartmentValues
    dSdt = -beta * S * I / N
    dIdt = beta * S * I / N - gamma * I
    dRdt = gamma * I
    return dSdt, dIdt, dRdt

# Vector with initial conditions
compartmentValues0 = S0, I0, R0
```

Numerical Solution to the SIR Model.

- The following code section is to integrate the SIR equations. This will pass the initiation values ($S_{t=0}$, $I_{t=0}$, and $R_{t=0}$) and calculates each curve for increasing time *t*. The code will return numerical values for S, I, and R.
- To read up on some basics in solving differential equations with Python, refer to the following URL: https://docs.scipy.org/doc/scipy/reference/generated/scipy.integrate.odeint.html.
- Look also into the variables S, I, and R and figure out their structure and data type.

Code 5. Numerical Solution of the Ordinary Differential Equations.

```
# Integrate the SIR equations over the time grid t
# and return numerical solutions
SIRmodelNumSolve = odeint(
    SIRmodel, compartmentValues0, t, args=(N, beta, gamma))
S, I, R = SIRmodelNumSolve.T
```

Visualization of the SIR Dynamics.

- With the code below, we visualize the three curves corresponding to $S(t)$, $I(t)$, and $R(t)$. We layer all plots and the legend, using one plot in a 1 × 1 grid. Although the dynamic of an epidemic can only be understood with all components in play, particularly interesting is the number of infectious individuals. The *Infected* curve is therefore emphasized with a line-width of 2pt and no opacity to the line.

Visualization code

Code 6. Plotting the Solutions of the Differential Equations.

```
# White background
fig = plt.figure(facecolor="w")
# 1x1 grid with one plot, light gray background
ax = fig.add_subplot(1,1,1, facecolor="#eeeeee", axisbelow=True)
# Plot the data for three separate curves: S(t), I(t) and R(t)
ax.plot(t, S/N, "b", alpha=0.2, lw=1, label="Susceptible")
ax.plot(t, I/N, "r", lw=2, label="Infected")
ax.plot(t, R/N, "g", alpha=0.2, lw=1, label="Removed")
# Axes labeling
ax.set_xlabel("Time in days")
ax.set_ylabel("Individuals (in " + str(N) + "s)")
# Call matplotlib function to print the graph legend
ax.legend()
# The moment of truth
plt.show()
```

Exercises

- **Exercise 1**: Spoiler alert, the code above only produces three stable lines without any apparent dynamics. Think about what needs to be set to start an epidemic?
- **Exercise 2**: After solving the above exercise, set the infectious period to three days. What happens and how can you explain it?
- **Exercise 3**: With the settings of exercise 1 and 2, increase the value of β by 50% - 70%. What value do you have to put in for β to get a peak maximum of 1000 infected individuals?

Homework

• Due next Wednesday (17:00, 13th, Jan. 2021) electronically to manaba+R

• File format: YourStudentID_W14.pdf (ID without hyphen, e.g., 12345678901_W14.pdf).

Q1.

- Write a report describing how you solved exercises 1 to 3.
  - Three figures generated in exercises 1 to 3 must be included into the report.
  - There is no fixed number of words, however, use full sentences and write in an understandable manner.

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Week 15:

Course summary