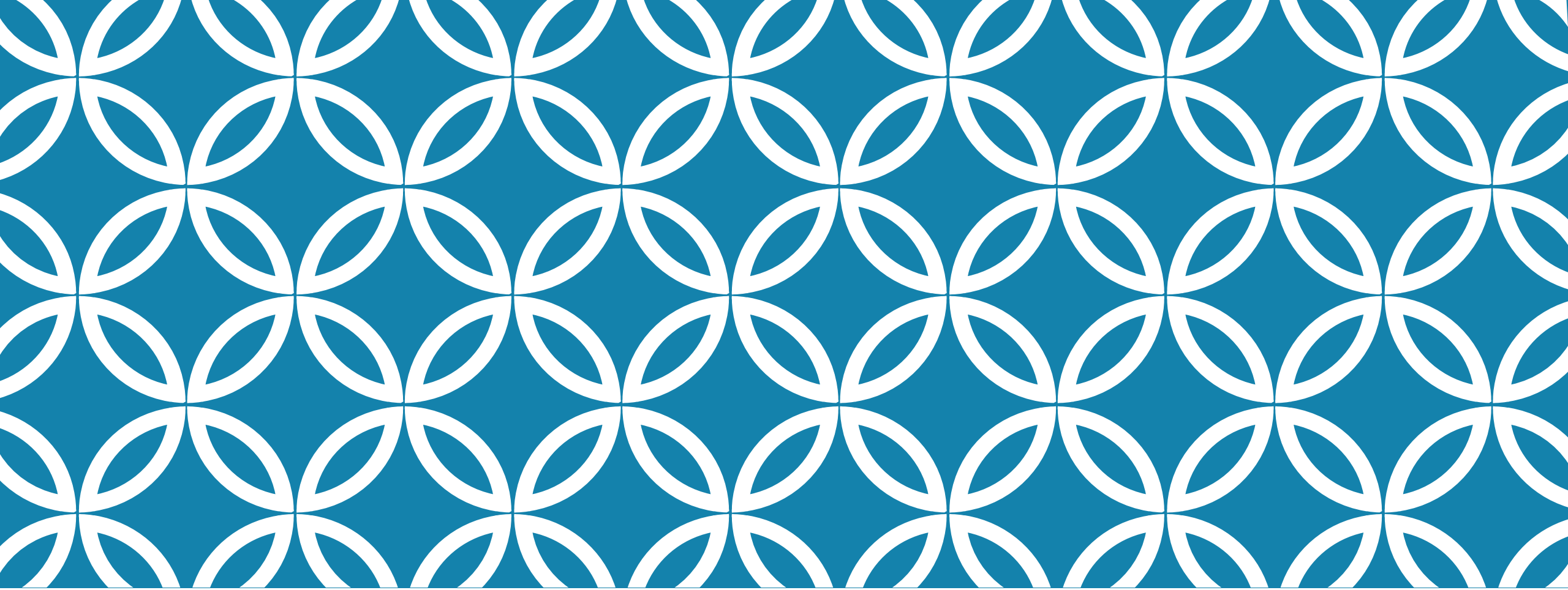

PROBLEM SOLVING FRAMEWORK

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POLYA'S FRAMEWORK*

Understanding the problem

Devising a Plan

Carrying out the plan

Looking back i.e. verifying

*How To Solve It, by George Polya, 2nd ed., Princeton University Press, 1957, ISBN 0-69108097

TWO TYPES OF PROBLEMS

- Find type problem
  - Given some inputs and conditions, find the output
  - Example: “Given your birthday and the current date, calculate your age in days”
- Proof type problem
  - Example: Fermat's Last Theorem states that no three positive integers *a*, *b*, and *c* can satisfy the equation $a^n + b^n = c^n$ for any integer value of *n* greater than two.

UNDERSTANDING FIND TYPE PROBLEM

Extract information from the problem

- What is input?
- What is unknown?
- What is output?
- What are the (hidden) assumptions?
- What are the facts?
- What are the conditions?

How to extract?

EXTRACTING INFORMATION

Read carefully (slowly and repeat if necessary)

Consult definition for unfamiliar (or even familiar) terminologies

Construct one or two simple example to illustrate what the problem says

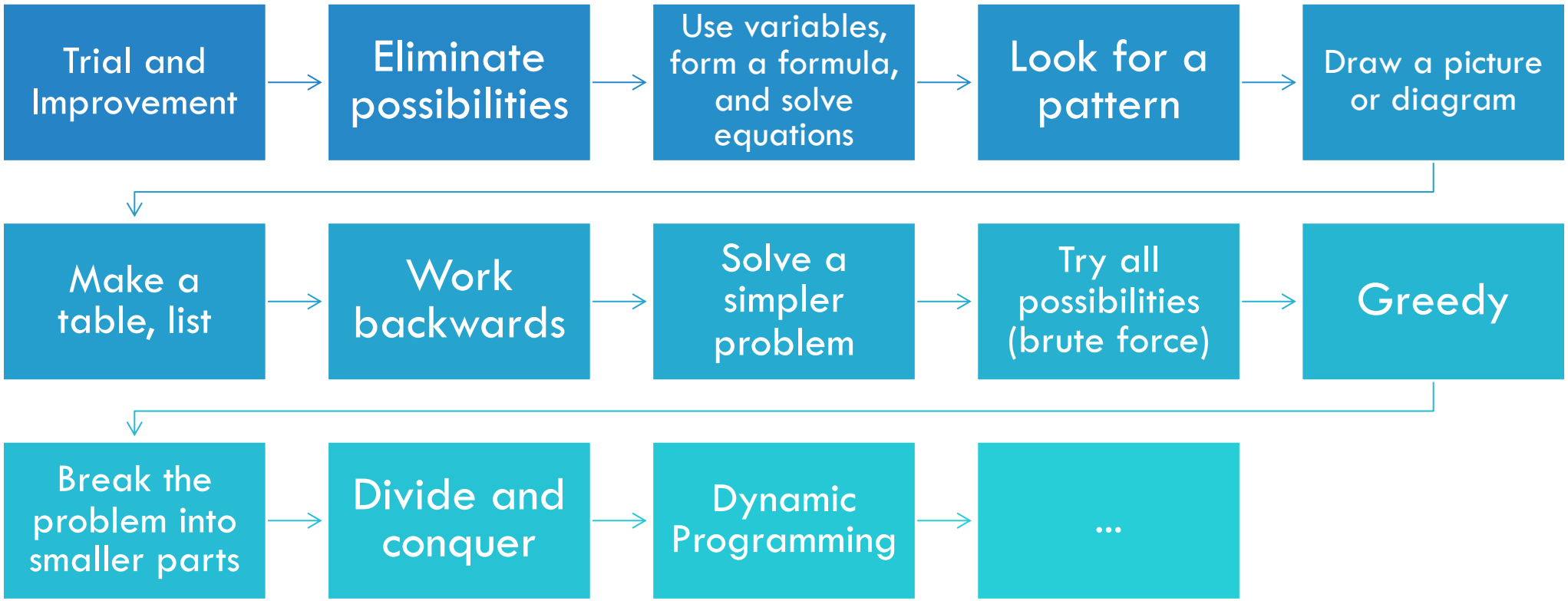
Restate the problem in your own words

Think of the “Inventory of concepts”

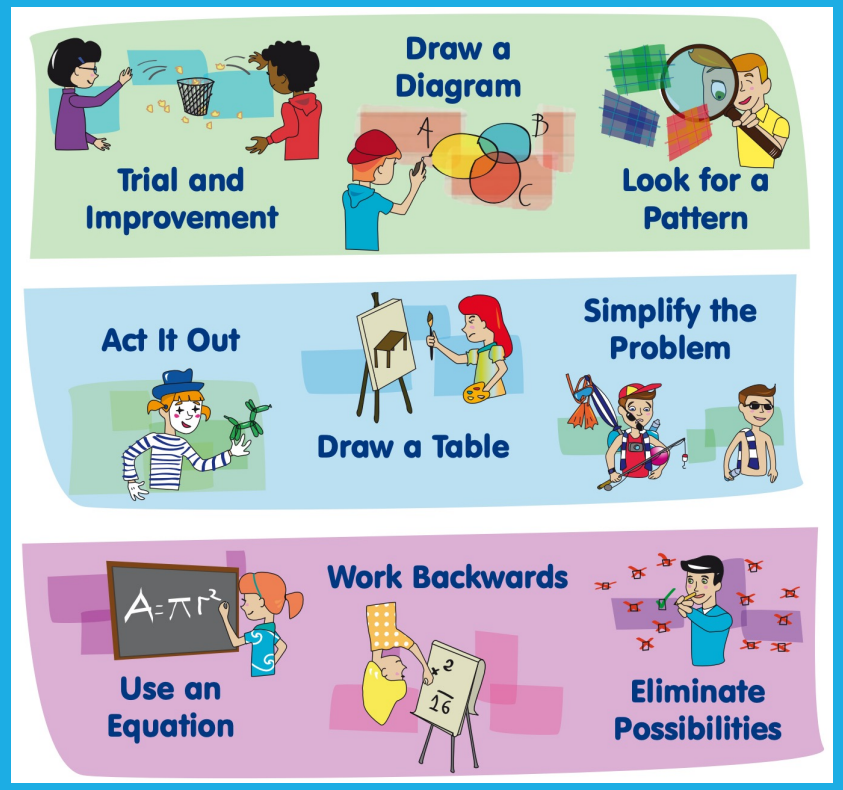
Think of a picture or diagram that might help understand the problem

Write specification of the problem (and/or sub-problems)

DEVISING A PLAN



PROBLEM SOLVING STRATEGIES



Q A pen and a pencil cost €5.10. The pen costs €5 more than the pencil. How much does each cost?

Trial and Improvement

Trial 1: €5.00 + €0.10 = €5.10 but €5.00 - €0.10 = €4.90

Trial 2: €4.95 + €0.15 = €5.10 but €4.95 - €0.15 = €4.80

Trial 3: €5.05 + €0.05 = €5.10 and €5.05 - €0.05 = €5.00

The pen costs €5.05 and the pencil costs €0.05

Use an Equation

$x+(5+x) = €5.10$
$x+x+5 = €5.10$
$2x+5 = €5.10$
$2x = €0.10$
$x = €0.05$

The pencil costs €0.05 and the pen costs €5.05

Q Eddie invested a certain amount of money in the bank and the money earned 4% interest in the first year. He added €1,000 to the investment at the end of the first year and the whole investment earned 3% interest in the second year. This amounted to €6,171.76 at the end of the second year. Work out how much money Eddie invested at the start of the first year.

Work Backwards

Reversing the increase of 3%: (2nd year)
€6,171.76 ÷ 1.03% = €5,992

Reversing the addition of €1,000:
€5,992 – €1,000 = €4,992

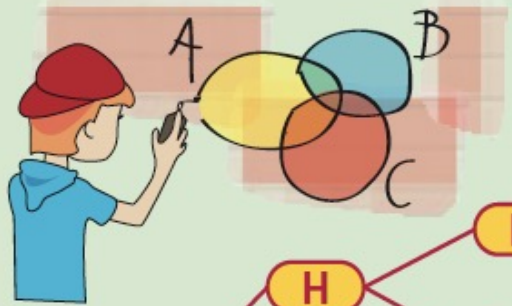
Reversing the increase of 4%: (1st year)
€4,992 ÷ 1.04% = €4,800

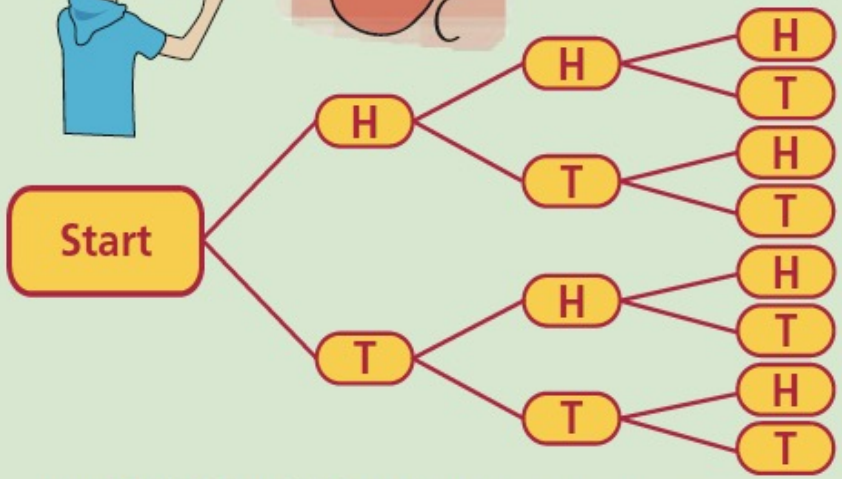
Use an Equation

$$
\begin{aligned}
(x \times 1.04 + €1000) \times 1.03 &= €6171.76 \\
(1.04x + €1000) \times 1.03 &= €6171.76 \\
1.0712x + 1030 &= €6171.76 \\
1.0712x &= €6171.76 - €1030 \\
1.0712x &= €5141.76 \\
x &= \frac{€5141.76}{1.0712} \\
x &= €4800
\end{aligned}
$$

Q If you toss a coin three times, how many outcomes are there with two heads and one tail?

Draw a Diagram





HHH, HHT, HTH, HTT, THH, THT, TTH, TTT are the outcomes.

There are three outcomes with 2 heads and 1 tail.

Q Tony is younger than 60 years of age. When you add the digits of his age the sum is 9. His age lies between two prime numbers. If his age is a multiple of 6, how old is he?

Eliminate Possibilities

His age is divisible by 6 and less than 60.
Only 6, 12, 18, 24, 30, 36, 42, 48, 54 are possible ages.

The sum of the digits is 9, so only 18, 36, and 54 are possible ages.
~~6~~, ~~12~~, 18, ~~24~~, ~~30~~, 36, ~~42~~, ~~48~~, 54.

His age lies between two prime numbers,
so we can eliminate 36 and 54.

Only 18 is possible.

Tony is 18 years of age.

Q Find the sum of the all the whole numbers between 1 and 200 inclusive.

Look for a Pattern

There's a repetitive pattern if we add the 1st number to the last number and add the 2nd number to the 2nd last number etc.

$1+2+3+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+198+199+200$

$1+200 = 201$
$2+199 = 201$
$3+198 = 201$
etc.

We have 200 numbers.
If we pair up the numbers
we get 100 pairs.

Since each pair adds to 201 the total is $201 \times 100 = 20{,}100$

Q There will be 7 players playing in a tournament. Each player must play every other player once. How many games will take place in the tournament?

Simplify the Problem

Case 1: The 2nd person plays the 1st i.e. 1 game
Case 2: The 3rd person plays the 1st and 2nd i.e. 2 games
Case 3: The 4th person plays the 1st, 2nd and 3rd i.e. 3 games
Case 4: The 5th person plays the 1st, 2nd, 3rd and 4th i.e. 4 games

The total number of games for 5 people = 1+2+3+4 = 10 games
Therefore, for 6 people, it would be 10+5 = 15 games
and for 7 people, it would be 15+6 = 21 games

Draw a Table

	1st person	2nd person	3rd person	4th person	5th person	6th person	7th person
1st person		✖	✖	✖	✖	✖	✖
2nd person → 1 game			✖	✖	✖	✖	✖
3rd person → 2 games				✖	✖	✖	✖
4th person → 3 games					✖	✖	✖
5th person → 4 games						✖	✖
6th person → 5 games							✖
7th person → 6 games							

1+2+3+4+5+6=21 games

Q In how many ways can pictures of Angela, Barbara and Colin be hung on a wall?

Act it out

There are three possible ways to hang the first picture, two possible ways to hang the second picture and one possible way to hang the third picture.

3 x 2 x 1 = 6 ways

6 ways

CARRYING OUT THE PLAN

Programming starts here

Be patience

Check each step

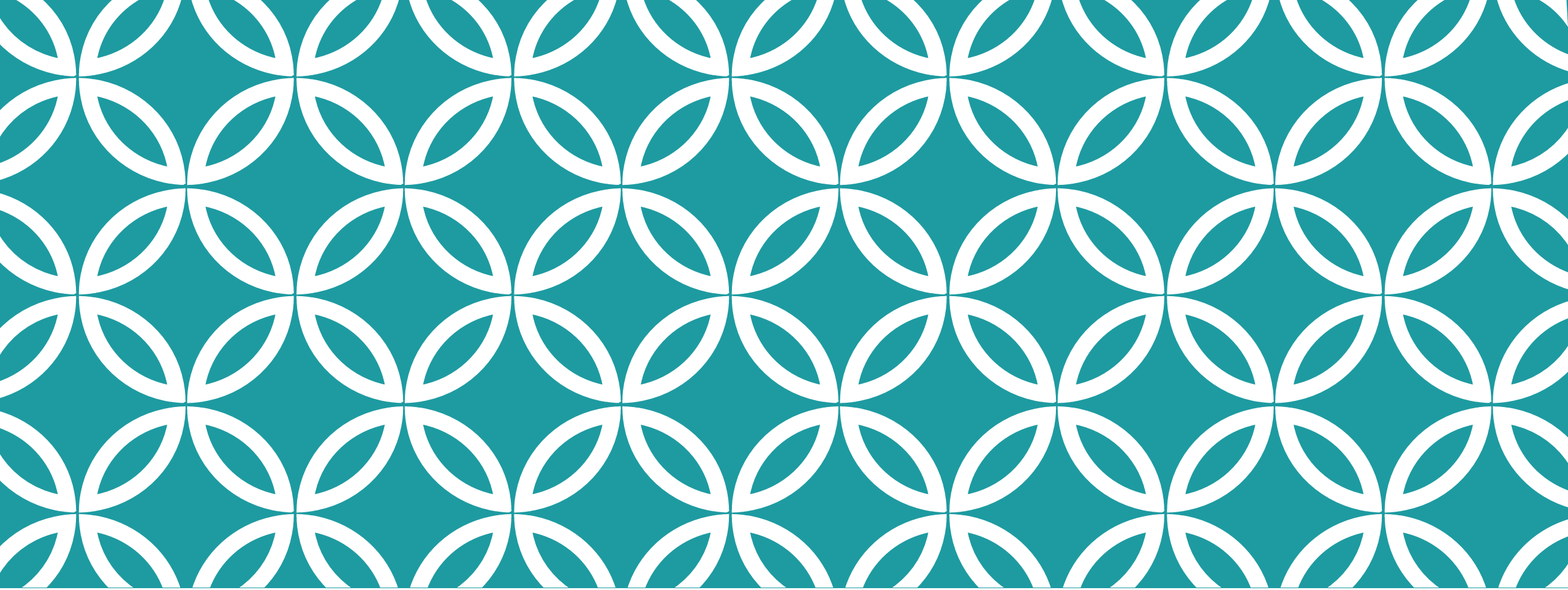
Step through the sample data

LOOKING BACK

Examine the solution

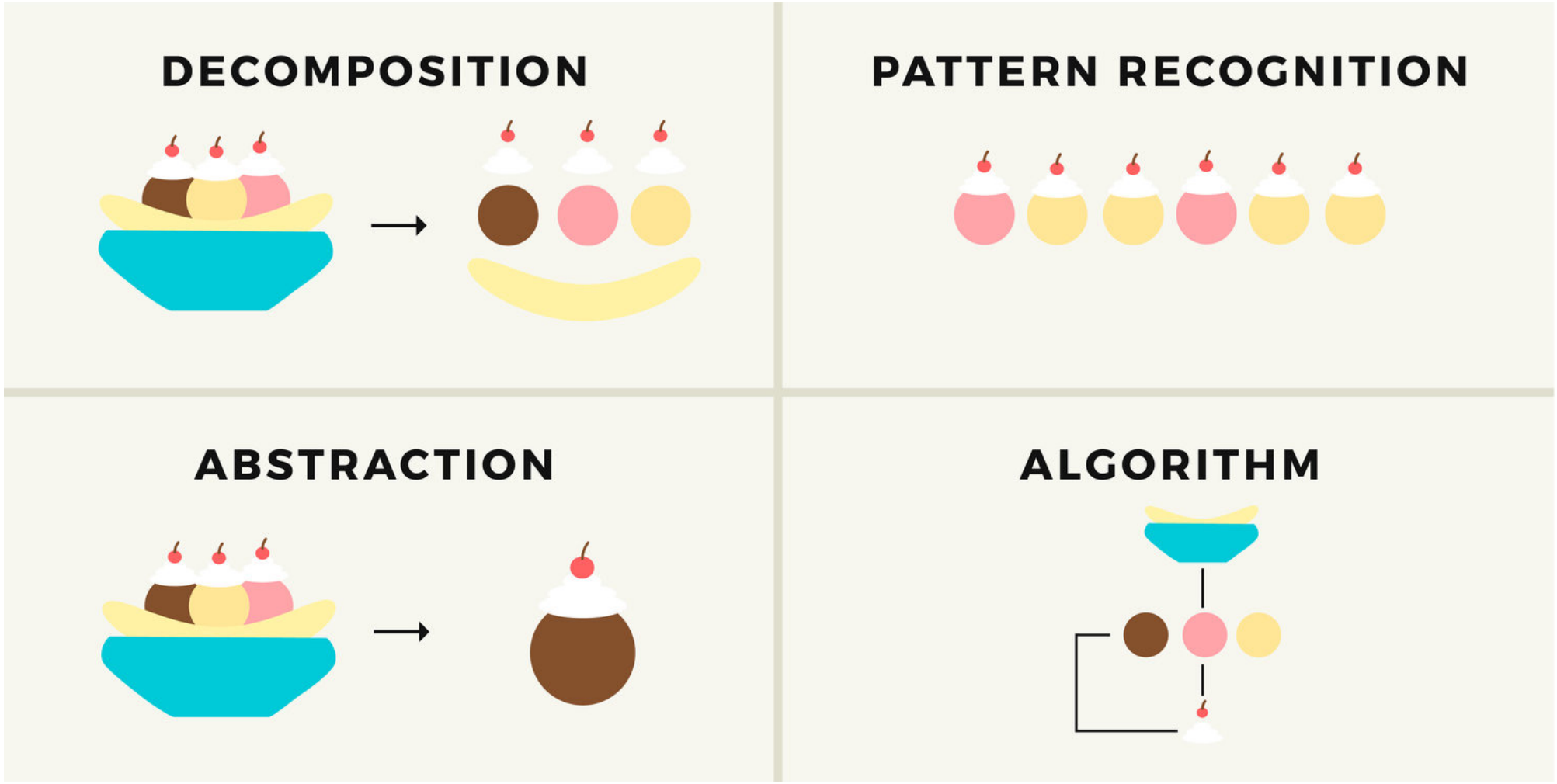
Is it satisfied all the conditions?

Can we derive the solution differently?



COMPUTATIONAL THINKING AND ALGORITHMIC THINKING

COMPUTATIONAL THINKING



COMPUTATIONAL THINKING

4 MAIN ASPECTS OF COMPUTATIONAL THINKING

Decomposition: Breaking down a big problem into small, manageable parts

Pattern Recognition: Observing similarities and patterns in these smaller parts, to help us solve complex problems more efficiently

Abstraction: Identifying and extracting the important parts of the problem

Algorithm: Creating the step-by-step instructions of solving the problem

https://www.tinythinkers.org/benefits

REFERENCES

1. *How To Solve It, by George Polya, 2nd ed., Princeton University Press, 1957, ISBN 0-69108097*

2. *http://math.berkeley.edu/~gmelvin/polya.pdf*

3. *http://www.cs.odu.edu/~toida/nerzic/content/problem_solving/problem_solving.html*

4. *https://jenniereaves.weebly.com/uploads/2/9/2/8/29288005/lets_practice.pdf*

5. https://www.projectmaths.ie/workshops/workshop3/ProblemSolvingPosters.pdf

6..https://math.libretexts.org/Courses/Monroe_Community_College/MTH_155_Mathematics_for_Elementary_Teachers_I_(placeholder)/Chapter_1%3A_Problem_Solving_Strategies/Module_1%3A_Problem_Solving_Strategies

7. *https://www.cs.cmu.edu/~15110-s13/Wing06-ct.pdf*