Find the value of each function

1. $\int_{-1}^{1} \left(14x^6 - \frac{8}{\sqrt[7]{x^3}} + 2\right) dx$

$$\int_{-1}^{1} (14x^6 - 8x^{-3/7} + 2)dx = \frac{14x^7}{7} - \frac{8x^{4/7}}{4/7} + 2x$$

$$= 2x^7 + \frac{7}{4}(-8x^{4/7})^{-2} + 2x$$

$$= \left[2x^7 - 14x^{4/7} + 2x\right]_{-1}^{1}$$

$$= 2 - 14 + 2 - (-2 - 14 - 2)$$

$$= -10 - (-18)$$

$$= -10 + 18 = 8$$

2. $\int_1^5 |x - 4| \, dx$

$f(x) = x - 4$

$x = 4 \longrightarrow f(x) = 0$

$\underset{\text{critical number}}{\downarrow}$

$= -\int_1^4 (x-4) \, dx + \int_4^5 (x-4) \, dx$

$= -\left[\dfrac{x^2}{2} - 4x \right]_1^4 + \left[\dfrac{x^2}{2} - 4x \right]_4^5$

$= -\left[\dfrac{16}{2} - 16 - \left(\dfrac{1}{2} + 4 \right) \right] + \left[\dfrac{25}{2} - 20 - \left(\dfrac{16}{2} + 16 \right) \right]$

$= -\left[\dfrac{15}{2} - 12 \right] + \left[\dfrac{9}{2} - 4 \right]$

$= -\left[\dfrac{15}{2} - \dfrac{24}{2} \right] + \left[\dfrac{9}{2} - \dfrac{8}{2} \right]$

$= \dfrac{9}{2} + \dfrac{1}{2} = \dfrac{10}{2} = 5$

3. $\displaystyle\int \frac{(x^3+1)}{(2x^4+8x)^3}\,dx$

$u = x^3+1$

if $u = g(x)$

then

$\dfrac{du}{dx} = 3x^2$

$\displaystyle\int (2x^4+8x)^{-3}(x^3+1)\,dx$

$\dfrac{1}{8}\displaystyle\int (2x^4 + 8x)^{-3}\, 8(x^3+1)\,dx$

$= \dfrac{1}{8}\dfrac{(2x^4+8x)^{-2}}{-2} + C$

$= \dfrac{-1}{16}(2x^4+8x)^{-2} + C$

$= \dfrac{-1}{16(2x^4+8x)^2} + C$

$= \dfrac{-1}{16}\left[4x^8+32x^5+64x^2\right] + C$ #