Derivative II

Find dy/dx using implicit differentiation

Product Rule

$uv' + u'v$

Quotient Rule

$$\frac{u'v - uv'}{v^2}$$

1. $y^2 = (x - y^2)(x^2 - 1)$

$$2y\frac{dy}{dx} = (x-y^2)\frac{d}{dx}[x^2-1] + \frac{d}{dx}[x-y^2](x^2-1)$$

$$2y\frac{dy}{dx} = (x-y^2)(2x) + (x^2-1)\left(1-2y\frac{dy}{dx}\right)$$

$$2y\frac{dy}{dx} = (x-y^2)(2x) + x^2 - 2x^2y\frac{dy}{dx} - 1 + 2y\frac{dy}{dx}$$

$$\cancel{2y\frac{dy}{dx}} + 2x^2y\frac{dy}{dx} - \cancel{2y\frac{dy}{dx}} = 2x^2 - 2xy^2 + x^2 - 1$$

$$2x^2y\frac{dy}{dx} = 3x^2 - 2xy^2 - 1$$

$$\frac{dy}{dx} = \frac{3x^2 - 2xy^2 - 1}{2x^2y}$$ #

2. $2y\sqrt{x} - 3(\sqrt[3]{y}) = 34$

$$2yx^{\frac{1}{2}} - 3y^{\frac{1}{3}} = 34$$

$$2y\frac{d}{dx}[x^{\frac{1}{2}}] + \frac{d}{dx}[2y]x^{\frac{1}{2}} - \cancel{\frac{1}{3}(3)}y^{-2/3} = 0$$

$$= \cancel{2y}\left(\cancel{\frac{1}{2}}\right)(x^{-\frac{1}{2}}) + 2\frac{dy}{dx}(x^{\frac{1}{2}}) - y^{-2/3} = 0$$

$$2x^{1/2}\cancel{\frac{dy}{dx}} = y^{-2/3} - yx^{-1/2}$$

$$\frac{dy}{dx} = \frac{y^{-\frac{2}{3}} - yx^{-\frac{1}{2}}}{2x^{\frac{1}{2}}}$$ #

3. $\sin(x + 2y) = y$

$$\frac{d}{dx}\left[\sin(x+2y)\right] = \frac{dy}{dx}$$

$$\cos(x+2y)\frac{d}{dx}\left[x+2y\right] = \frac{dy}{dx} \Leftarrow \text{Swapping}$$

$$\frac{dy}{dx} = \cos(x+2y)\left(1 + 2\frac{dy}{dx}\right)$$

$$1\frac{dy}{dx} = \cos(x+2y) + 2\cos(x+2y)\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(1-2\cos(x+2y)\right) = \cos(x+2y)$$

$$\frac{dy}{dx} = \frac{\cos(x+2y)}{1-2\cos(x+2y)} \quad \#$$