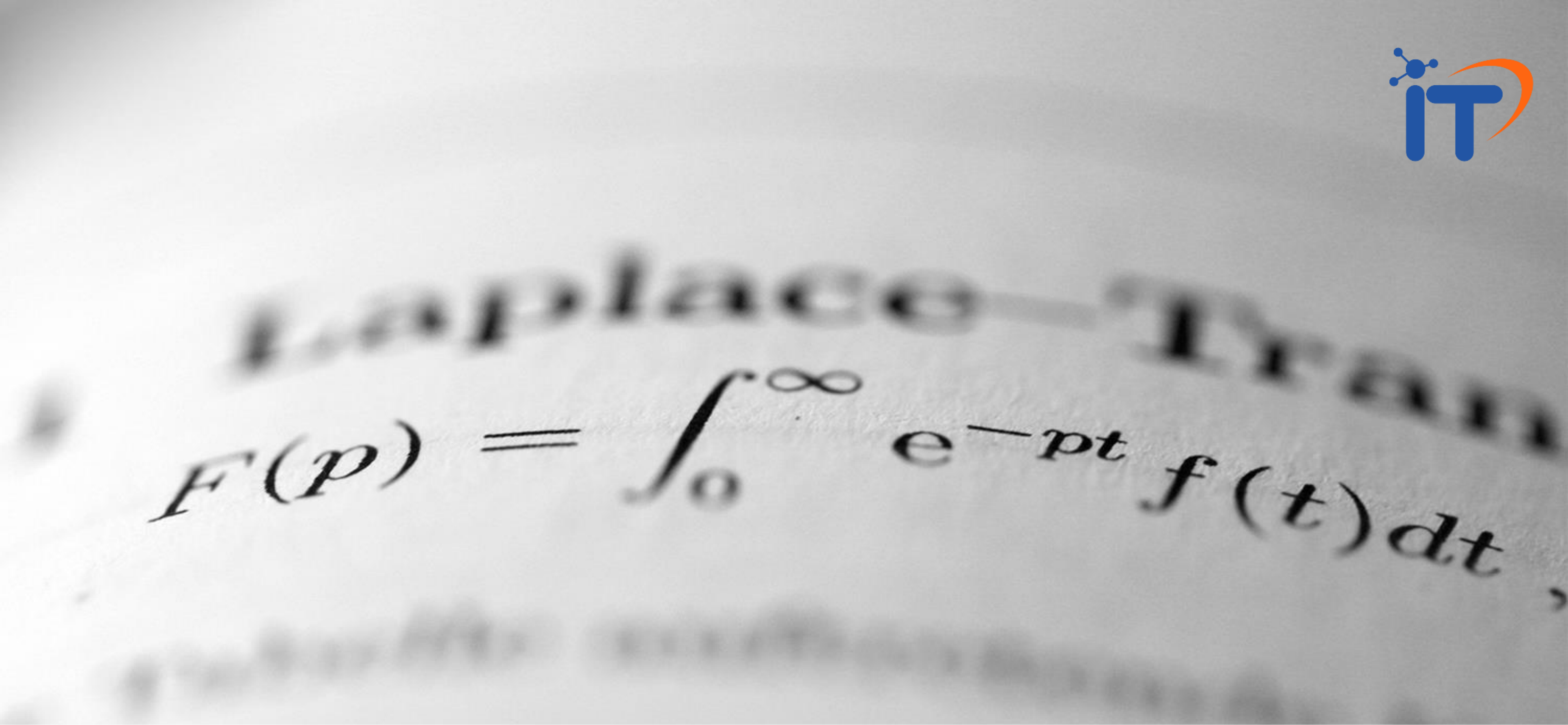


06016401
MATHEMATICS FOR INFORMATION TECHNOLOGY

Instructor: Asst. Prof. Dr. Praphan Pavarangkoon
Office: Room no. 418-5, 4th Floor
Email: praphan@it.kmitl.ac.th
Office hours: Wednesday at 9:00 – 12:00
or as an advance appointment

Outline

- Course Introduction
  - Syllabus
  - Calendar

- Linear Algebra
- Matrices
  - Definition of a Matrix
  - Matrix Addition
  - Scalar Multiplication
  - Matrix Multiplication
  - The Transpose of a Matrix

Course Introduction

- Course: 06016401
- Course Title: Mathematics for Information Technology
- Duration: 17 weeks (including Midterm and Final examination weeks)

lab self-study

- Credit Hours: 3 (3-0-6) credit hours
  - Lecture: 45 hours
  - Self-Study: 90 hours

theory

Team code: 3xkd16e

- Lecturers
  - Asst. Prof. Dr. Somkiat Wangsiripitak
  - Asst. Prof. Dr. Praphan Pavarangkoon
- Microsoft Teams
  - https://teams.microsoft.com/l/channel/19%3Ah99LtcKZVzNSt7R51WJrnL0fKztxC4sBdOyA7pWrEAY1%40thread.tacv2/General?groupId=6b6e18e6-f76c-474f-af83-f0392752b281&tenantId=

Course Introduction (cont.)

- Teaching Assistants
  - 66070100 นายนวชาต องค์เจริญ (นิว)
    - Tuesday: 2 hours at 16:00 - 18:00
  - 66070101 นายนัฐพงษ์ วงค์สมศรี (ไทค์)
    - Thursday: 2 hours at 16:00 - 18:00
  - 66070315 นายสุขุม ฤดีเมธากุล (ขุม)
    - Wednesday: 2 hours at 16:00 - 18:00
- Course Date/Time/Room
  - This class meets every Tuesday and Wednesday (3 hours/week).
    - Tuesday: 3 hours at 9:00 - 12:00 and 13:00 - 16:00
    - Wednesday: 3 hours at 13:00 - 16:00

Course Description

- ลิมิตและความต่อเนื่อง อนุพันธ์ของฟังก์ชัน อนุพันธ์อันดับสูง การประยุกต์ของอนุพันธ์ ปัญหาค่าสูงสุดและต่ำสุดของฟังก์ชัน การอินทิเกรตและเทคนิคของการอินทิเกรต การประยุกต์ของการอินทิเกรต ฟังก์ชันสองตัวแปร อนุพันธ์ย่อย พีชคณิตเชิงเส้น เมทริกซ์และดีเทอร์มิแนนท์ ระบบสมการเชิงเส้น และการหาผลเฉลยของระบบสมการเชิงเส้น เวกเตอร์สเปซ การแปลงเชิงเส้น การแปลงเมทริกซ์ เมทริกซ์เชิงตั้งฉาก

Aims and Objectives

- จุดมุ่งหมาย
  - เพื่อให้ผู้เรียนมีความรู้พื้นฐานทางคณิตศาสตร์ที่จำเป็นสำหรับการเรียนในวิชาต่างๆ เกี่ยวกับเทคโนโลยีสารสนเทศ
- วัตถุประสงค์
  - มีการประยุกต์ใช้ซอฟต์แวร์ที่เหมาะสมนอกเหนือจากการคำนวณแบบธรรมดา เพื่อให้ผู้เรียนสามารถประยุกต์ความรู้ที่ได้เรียนในการแก้ปัญหาทางเทคโนโลยีสารสนเทศได้

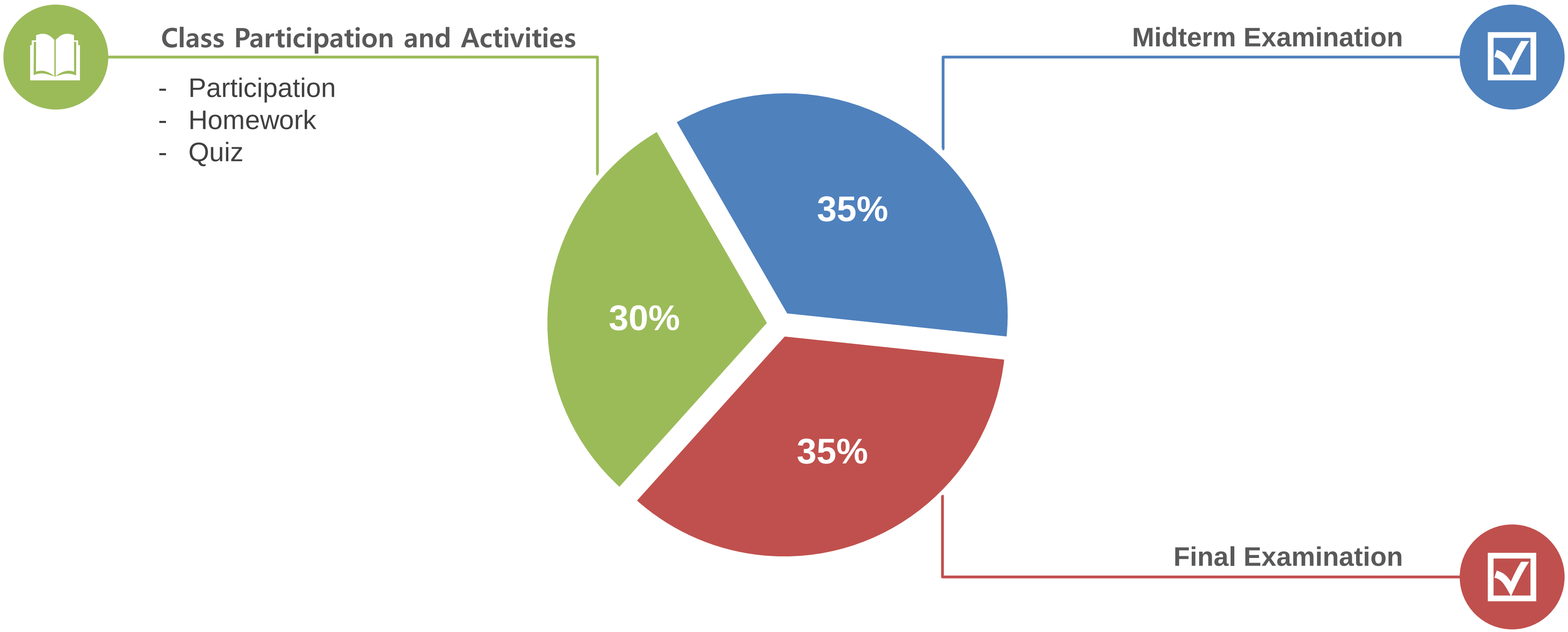
Topics and Details of 17 Weeks

Week	Date	Topics and Details	Lecturer
1	02/07 and 03/07	Linear algebra	Praphan
2	09/07 and 10/07	Matrices and determinants	Praphan
3	16/07 and 17/07	System of linear equations and its solutions	Praphan
4	23/07 and 24/07	Vector space (1)	Praphan
5	30/07 and 31/07	Vector space (2)	Praphan
6	06/08 and 07/08	Linear transformations	Praphan
7	13/08 and 14/08	Orthogonal matrix	Praphan
8	20/08 and 21/08	Function, limit and continuity	Somkiat
9	**Midterm examination (Week 1 – Week 7)** **Friday 30 August at 9:30 – 12:30**		

Topics and Details of 17 Weeks (cont.)

Week	Date	Topics and Details	Lecturer
10	03/09 and 04/09	Derivative of functions	Somkiat
11	10/09 and 11/09	Higher-order derivatives, chain rule, implicit differentiation	Somkiat
12	17/09 and 18/09	Applications of the derivative	Somkiat
13	24/09 and 25/09	Integration (indefinite & definite)	Somkiat
14	01/10 and 02/10	Integration by substitution	Somkiat
15	08/10 and 09/10	Applications of integration	Somkiat
16	15/10 and 16/10	Function of two variables, partial derivatives	Somkiat
17	**Final examination** **Tuesday 29 October at 13:30 – 16:30**		

Course Evaluation and Evaluation Criteria



- Each homework is assigned on Microsoft Teams every Tuesday at 11 AM and is worth 1 point.
- The due date is Sunday at 5 PM:
  - ❖ You may receive 1 point if completed on time, including resubmissions or re-turned in after the due date.
  - ❖ If submitted or turned in after the due date, you will receive 0.5 points.
- The close date is Monday at 5 PM:
  - You cannot submit or turn in after the close date and will receive 0 points.

Course Assessment

Description	Grade		Score (points)
Excellent	A	4.0	80-100
Very Good	B+	3.5	75-79
Good	B	3.0	70-74
Fairly Good	C+	2.5	65-69
Fair	C	2.0	60-64
Poor	D+	1.5	55-59
Very Poor	D	1.0	50-54
Fail	F	0.0	0-49

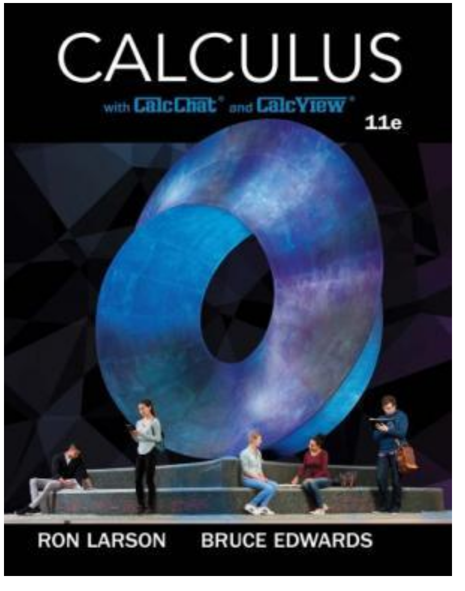
<u>หมายเหตุ</u> ผู้สอนอาจจะพิจารณาแบบอิงกลุ่มร่วมด้วย

Resources

- ตำราและเอกสารหลัก
  - Title: Calculus

    Authors: Larson Edwards and Bruce H. Edwards

    Publisher: Brooks/Cole, Cengage Learning

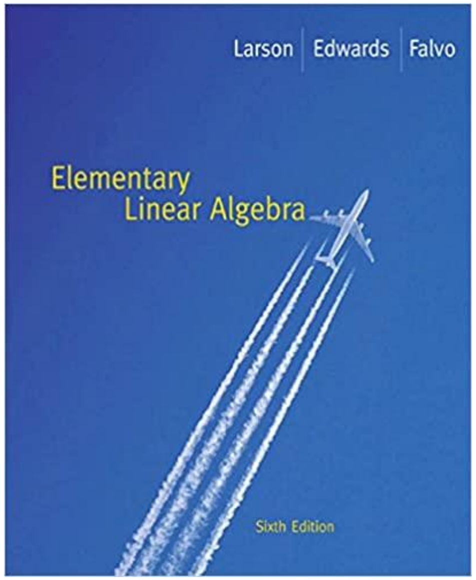


Resources (cont.)

– Title: Elementary Linear Algebra

Authors: Ron Larson and David C. Falvo

Publisher: Houghton Mifflin Harcourt



Note

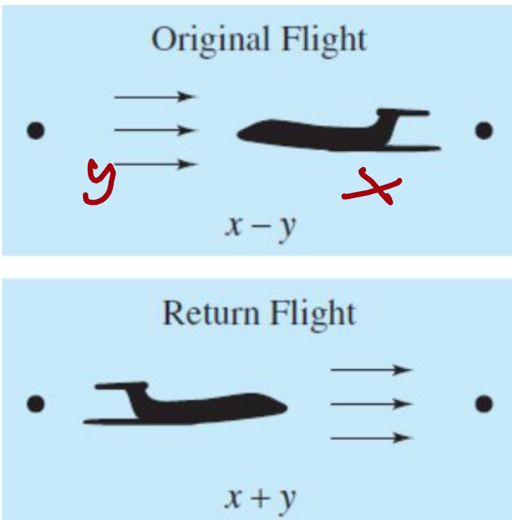
- Student Responsibility, Course Policies (Attendance, Late/Absence/Medical Reasons), Disabilities and Accommodations, Academic Dishonesty (Plagiarism), and KMITL rules and regulations: Please refer to Curriculum Guide

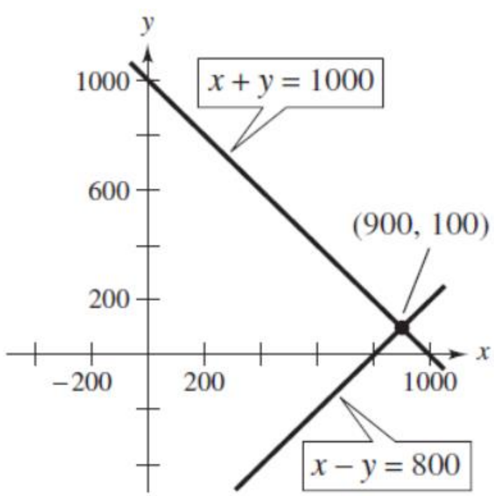
What Is Linear Algebra?

- The most fundamental theme of linear algebra is the theory of systems of linear equations.
- You have probably encountered small systems of linear equations in your previous mathematics courses.
- For example, suppose you travel on an airplane between two cities that are 5000 kilometers apart.
- If the trip one way against a headwind ต้านลม takes 6.25 hours and the return trip the same day in the direction of the wind takes only 5 hours, can you find the ground speed of the plane and the speed of the wind, assuming that both remain constant?

What Is Linear Algebra? (cont.)

$v = \frac{s}{t}$, $vt = s$





The lines intersect at (900, 100).

- If you let x represent the speed of the plane and y the speed of the wind, then the following system models the problem.

$$6.25(x - y) = 5000 \quad (1)$$
$$5(x + y) = 5000 \quad (2)$$

- This system of two equations and two unknowns simplifies to

$$x - y = 800$$
$$x + y = 1000.$$

- and the solution is $x = 900$ kilometers per hour and $y = 100$ kilometers per hour.
- Geometrically, this system represents two lines in the xy-plane.
- You can see in the figure that these lines intersect at the point (900, 100) which verifies the answer that was obtained.

What Is Linear Algebra? (cont.)

- Solving systems of linear equations is one of the most important applications of linear algebra.
- It has been argued that the majority of all mathematical problems encountered in scientific and industrial applications involve solving a linear system at some point.
- Linear applications arise in such diverse areas as engineering, chemistry, economics, business, ecology, biology, and psychology.

Linear Algebra

- Linear algebra is a fairly extensive subject that covers vectors and matrices, determinants, systems of linear equations, vector spaces and linear transformations, eigenvalue problems, and other topics.
- As an area of study it has a broad appeal in that it has many applications in engineering, physics, geometry, computer science, economics, and other areas.
- It also contributes to a deeper understanding of mathematics itself.

Definition of a Matrix

- A matrix is a rectangular array of numbers or functions which we will enclose in brackets.
- The plural of matrix is matrices.

If m and n are positive integers, then an $m \times n$ **matrix** is a rectangular array

$$\left.\underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}}_{n \text{ columns}}\right\} m \text{ rows}$$

สมาชิก / element

in which each **entry,** a_{ij}, of the matrix is a number. An $m \times n$ matrix (read "m by n") has m **rows** (horizontal lines) and n **columns** (vertical lines).

Examples of Matrices

Each matrix has the indicated size.

(a) Size: 1×1

$$[2]$$

(b) Size: 2×2

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(c) Size: $\overset{\text{row}}{1} \times \overset{\text{column}}{4}$

$$\begin{bmatrix} 1 & -3 & 0 & \frac{1}{2} \end{bmatrix}$$

(d) Size: 3×2

$$\begin{bmatrix} e & \pi \\ 2 & \sqrt{2} \\ -7 & 4 \end{bmatrix}$$

Definition of a Matrix (cont.)

เมทริกซ์แต่งเติม = Augmented Matrix

- One very common use of matrices is to represent systems of linear equations.
- The matrix derived from the coefficients and constant terms of a system of linear equations is called the **augmented matrix** of the system.
- The matrix containing only the coefficients of the system is called the **coefficient matrix** of the system. Here is an example.

constant

System

$$\left.\begin{aligned} x - 4y + 3z &= 5 \\ -x + 3y - z &= -3 \\ 2x - 4z &= 6 \end{aligned}\right\}$$

coefficient

constant

Augmented Matrix

$$\begin{bmatrix} 1 & -4 & 3 & 5 \\ -1 & 3 & -1 & -3 \\ 2 & 0 & -4 & 6 \end{bmatrix}$$

Coefficient Matrix

$$\begin{bmatrix} 1 & -4 & 3 \\ -1 & 3 & -1 \\ 2 & 0 & -4 \end{bmatrix}$$

y = 0

Operations with Matrices

- Matrix Addition $+$
- Scalar Multiplication $\cdot$ wi
- Matrix Multiplication

Matrix Addition

- You can **add** two matrices (of the same size) by adding their corresponding entries.

If $A = [a_{ij}]$ and $B = [b_{ij}]$ are matrices of size $m \times n$, then their **sum** is the $m \times n$ matrix given by

$$A + B = [a_{ij} + b_{ij}].$$

The sum of two matrices of different sizes is undefined.

Addition of Matrices

ขนาดเท่าเดีย

(a) $\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -1+1 & 2+3 \\ 0-1 & 1+2 \end{bmatrix} = \begin{bmatrix} 0 & 5 \\ -1 & 3 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 & -2 \\ 1 & 2 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ 1 & 2 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(d) The sum of

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 4 & 0 & -1 \\ 3 & -2 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 1 \\ -1 & 3 \\ 2 & 4 \end{bmatrix}$$

is undefined.

Scalar Multiplication

- When working with matrices, real numbers are referred to as **scalars.**
- You can multiply a matrix *A* by a scalar *c* by multiplying each entry in *A* by *c*.

If $A = [a_{ij}]$ is an $m \times n$ matrix and c is a scalar, then the **scalar multiple** of A by c is the $m \times n$ matrix given by

$$cA = [ca_{ij}].$$

Scalar · element ทุกตัวใน Matrix

You can use $-A$ to represent the scalar product $(-1)A$. If A and B are of the same size, $A - B$ represents the sum of A and $(-1)B$. That is,

$$A - B = A + (-1)B.$$ Subtraction of matrices

$-A = (-1) \cdot A$

Scalar Multiplication and Matrix Subtraction

For the matrices

$$A = \begin{bmatrix} 1 & 2 & 4 \\ -3 & 0 & -1 \\ 2 & 1 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 0 & 0 \\ 1 & -4 & 3 \\ -1 & 3 & 2 \end{bmatrix}$$

find (a) $3A$, (b) $-B$, and (c) $3A - B$.

SOLUTION

(a) $$3A = 3\begin{bmatrix} 1 & 2 & 4 \\ -3 & 0 & -1 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 3(1) & 3(2) & 3(4) \\ 3(-3) & 3(0) & 3(-1) \\ 3(2) & 3(1) & 3(2) \end{bmatrix} = \begin{bmatrix} 3 & 6 & 12 \\ -9 & 0 & -3 \\ 6 & 3 & 6 \end{bmatrix}$$

(b) $$-B = (-1)\begin{bmatrix} 2 & 0 & 0 \\ 1 & -4 & 3 \\ -1 & 3 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 \\ -1 & 4 & -3 \\ 1 & -3 & -2 \end{bmatrix}$$

(c) $$3A - B = \begin{bmatrix} 3 & 6 & 12 \\ -9 & 0 & -3 \\ 6 & 3 & 6 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 1 & -4 & 3 \\ -1 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 6 & 12 \\ -10 & 4 & -6 \\ 7 & 0 & 4 \end{bmatrix}$$

Matrix Multiplication

If $A = [a_{ij}]$ is an $m \times n$ matrix and $B = [b_{ij}]$ is an $n \times p$ matrix, then the **product** AB is an $m \times p$ matrix

$$AB = [c_{ij}]$$

$$A_{m \times n} \times B_{n \times p} = C_{m \times p}$$

where

$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \cdot \cdot \cdot + a_{in}b_{nj}.$$

- This definition means that the entry in the *i*th row and the *j*th column of the product *AB* is obtained by multiplying the entries in the *i*th row of *A* by the corresponding entries in the *j*th column of *B* and then adding the results.

Finding the Product of Two Matrices

Find the product AB, where

$$A = \begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix}_{3\times 2} \quad \text{and} \quad B = \begin{bmatrix} -3 & 2 \\ -4 & 1 \end{bmatrix}_{2\times 2}.$$

SOLUTION First note that the product AB is defined because A has size 3×2 and B has size 2×2. Moreover, the product AB has size 3×2 and will take the form

$$\begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ c_{31} & c_{32} \end{bmatrix}.$$

To find c_{11} (the entry in the first row and first column of the product), multiply corresponding entries in the first row of A and the first column of B. That is,

$$c_{11} = (-1)(-3) + (3)(-4) = -9$$

$$\begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} -9 & c_{12} \\ c_{21} & c_{22} \\ c_{31} & c_{32} \end{bmatrix}.$$

* Summation ของ ผลคูณ • ตัว

Finding the Product of Two Matrices (cont.)

Similarly, to find c_{12}, multiply corresponding entries in the first row of A and the second column of B to obtain

$c_{12} = (-1)(2) + (3)(1) = 1$

$$\begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} -9 & 1 \\ c_{21} & c_{22} \\ c_{31} & c_{32} \end{bmatrix}.$$

Continuing this pattern produces the results shown below.

$$\begin{aligned} c_{21} &= (4)(-3) + (-2)(-4) = -4 \\ c_{22} &= (4)(2) + (-2)(1) = 6 \\ c_{31} &= (5)(-3) + (0)(-4) = -15 \\ c_{32} &= (5)(2) + (0)(1) = 10 \end{aligned}$$

The product is

$$AB = \begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} -9 & 1 \\ -4 & 6 \\ -15 & 10 \end{bmatrix}.$$

Matrix Multiplication (cont.)

- Be sure you understand that for the product of two matrices to be defined, the number of columns of the first matrix must equal the number of rows of the second matrix. That is,

$$\underset{m \times n}{A} \quad \underset{n \times p}{B} \quad = \quad \underset{m \times p}{AB.}$$

equal

size of AB

ใน case นี้คูณกันไม่ได้

- So, the product *BA* is not defined for matrices such as *A* and *B* in the previous example.

Matrix Multiplication (cont.)

- The general pattern for matrix multiplication is as follows. To obtain the element in the *i*th row and the *j*th column of the product *AB*, use the *i*th row of *A* and the *j*th column of *B*.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \cdots & a_{in} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1j} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2j} & \cdots & b_{2p} \\ b_{31} & b_{32} & \cdots & b_{3j} & \cdots & b_{3p} \\ \vdots & \vdots & & \vdots & & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nj} & \cdots & b_{np} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1j} & \cdots & c_{1p} \\ c_{21} & c_{22} & \cdots & c_{2j} & \cdots & c_{2p} \\ \vdots & \vdots & & \vdots & & \vdots \\ c_{i1} & c_{i2} & \cdots & c_{ij} & \cdots & c_{ip} \\ \vdots & \vdots & & \vdots & & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mj} & \cdots & c_{mp} \end{bmatrix}$$

$$a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \cdots + a_{in}b_{nj} = c_{ij}$$

Matrix Multiplication

$$\text{(a)}\ \underbrace{\begin{bmatrix}1 & 0 & 3\\ 2 & -1 & -2\end{bmatrix}}_{2\times 3}\underbrace{\begin{bmatrix}-2 & 4 & 2\\ 1 & 0 & 0\\ -1 & 1 & -1\end{bmatrix}}_{3\times 3}=\underbrace{\begin{bmatrix}-5 & 7 & -1\\ -3 & 6 & 6\end{bmatrix}}_{2\times 3}$$

$$\text{(b)}\ \underbrace{\begin{bmatrix}3 & 4\\ -2 & 5\end{bmatrix}}_{2\times 2}\underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{2\times 2}=\underbrace{\begin{bmatrix}3 & 4\\ -2 & 5\end{bmatrix}}_{2\times 2}$$

$$\text{(c)}\ \underbrace{\begin{bmatrix}1 & 2\\ 1 & 1\end{bmatrix}}_{2\times 2}\underbrace{\begin{bmatrix}-1 & 2\\ 1 & -1\end{bmatrix}}_{2\times 2}=\underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{2\times 2}$$

$$\text{(d)}\ \underbrace{\begin{bmatrix}1 & -2 & -3\end{bmatrix}}_{1\times 3}\underbrace{\begin{bmatrix}2\\ -1\\ 1\end{bmatrix}}_{3\times 1}=\underbrace{\begin{bmatrix}1\end{bmatrix}}_{1\times 1}$$

$$\text{(e)}\ \underbrace{\begin{bmatrix}2\\ -1\\ 1\end{bmatrix}}_{3\times 1}\underbrace{\begin{bmatrix}1 & -2 & -3\end{bmatrix}}_{1\times 3}=\underbrace{\begin{bmatrix}2 & -4 & -6\\ -1 & 2 & 3\\ 1 & -2 & -3\end{bmatrix}}_{3\times 3}$$

c)

$$\begin{bmatrix}1 & 2\\ 1 & 1\end{bmatrix}\begin{bmatrix}-1 & 2\\ 1 & -1\end{bmatrix}$$

$$= \begin{bmatrix}-1+2 & 2-2\\ -1+1 & 2-1\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \#$$

Properties of Matrix Operations

- This section begins to develop the algebra of matrices.
- You will see that this algebra shares many (but not all) of the properties of the algebra of real numbers.

Properties of Matrix Addition and Scalar Multiplication

If A, B, and C are $m \times n$ matrices and c and d are scalars, then the following properties are true.

1. $A + B = B + A$ — Commutative property of addition สลับที่การบวก
2. $A + (B + C) = (A + B) + C$ — Associative property of addition จัดหมู่การบวก
3. $(cd)A = c(dA)$ — Associative property of multiplication
4. $1A = A$ — Multiplicative identity
5. $c(A + B) = cA + cB$ — Distributive property
6. $(c + d)A = cA + dA$ — Distributive property

แจกแจง

Properties of Matrix Multiplication

If A, B, and C are matrices (with sizes such that the given matrix products are defined) and c is a scalar, then the following properties are true.

1. $A(BC) = (AB)C$
2. $A(B + C) = AB + AC$ [illegible]
3. $(A + B)C = AC + BC$
4. $c(AB) = (cA)B = A(cB)$

Properties of the Identity Matrix

เมทริกซ์เอกลักษณ์

- You will now look at a special type of square matrix that has 1's on the main diagonal and 0's elsewhere.

$$I_n = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

For instance, if $n = 1, 2,$ or 3, we have

$$\underbrace{I_1 = [1]}_{1 \times 1}, \qquad \underbrace{I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{2 \times 2}, \qquad \underbrace{I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{3 \times 3}.$$

- When the order of the matrix is understood to be n, you may denote I_n simply as I.

Properties of the Identity Matrix (cont.)

If A is a matrix of size $m \times n$, then the following properties are true.

1. $AI_n = A$
2. $I_mA = A$

As a special case of this theorem, note that if A is a *square* matrix of order n, then

$$AI_n = I_nA = A.$$

$A_{n\times n}$

The Transpose of a Matrix

การสลับเปลี่ยน

The **transpose** of a matrix is formed by writing its columns as rows. For instance, if A is the $m \times n$ matrix shown by

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix},$$

Size: $m \times n$

then the transpose, denoted by A^T, is the $n \times m$ matrix below

$$A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} & \cdots & a_{m1} \\ a_{12} & a_{22} & a_{32} & \cdots & a_{m2} \\ a_{13} & a_{23} & a_{33} & \cdots & a_{m3} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{1n} & a_{2n} & a_{3n} & \cdots & a_{mn} \end{bmatrix}.$$

Size: $n \times m$

The Transpose of a Matrix (cont.)

Find the transpose of each matrix.

(a) $A = \begin{bmatrix} 2 \\ 8 \end{bmatrix}$ (b) $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ (c) $C = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $D = \begin{bmatrix} 0 & 1 \\ 2 & 4 \\ 1 & -1 \end{bmatrix}$

SOLUTION (a) $A^T = \begin{bmatrix} 2 & 8 \end{bmatrix}$ (b) $B^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$ (c) $C^T = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(d) $D^T = \begin{bmatrix} 0 & 2 & 1 \\ 1 & 4 & -1 \end{bmatrix}$

- Note that the square matrix in part (c) of the above example is equal to its transpose.

symmetric matrix = เมทริกซ์สมมาตร

- Such a matrix is called **symmetric.**
- A matrix A is symmetric if $A = A^T$. From this definition it is clear that a symmetric matrix must be square. Also, if $A = [a_{ij}]$ is a symmetric matrix, then $a_{ij} = a_{ji}$ for all $i \neq j$.

Properties of Transposes

If A and B are matrices (with sizes such that the given matrix operations are defined) and c is a scalar, then the following properties are true.

1. $(A^T)^T = A$ — Transpose of a transpose
2. $(A + B)^T = A^T + B^T$ — Transpose of a sum
3. $(cA)^T = c(A^T)$ — Transpose of a scalar multiple
4. $(AB)^T = B^TA^T$ — Transpose of a product

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The Product of a Matrix and Its Transpose

For the matrix

$$A = \begin{bmatrix} 1 & 3 \\ 0 & -2 \\ -2 & -1 \end{bmatrix}$$

find the product AA^T and show that it is symmetric.

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คืออะไร

SOLUTION Because

$$AA^T = \begin{bmatrix} 1 & 3 \\ 0 & -2 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 3 & -2 & -1 \end{bmatrix} = \begin{bmatrix} 10 & -6 & -5 \\ -6 & 4 & 2 \\ -5 & 2 & 5 \end{bmatrix}$$

it follows that $AA^T = (AA^T)^T$, so AA^T is symmetric.

Q & A