3+4

1

$-3C_{31}$

$-3\begin{vmatrix} 3 & 2 \\ -2 & -1 \end{vmatrix}$

2. จงแสดงวิธีทำและหาค่า $|A^{-1}|$

นั่นคือ เริ่มต้นด้วยการหา A^{-1} จากนั้นให้ทำการหาดีเทอร์มิแนนต์ของเมทริกซ์นั้น

$$A = \begin{bmatrix} 1 & 0 & -1 & 3 \\ 1 & 0 & 3 & 2 \\ 2 & 0 & -2 & -1 \\ 1 & -3 & 1 & 2 \end{bmatrix}$$

$M_{11} = \begin{vmatrix} 0 & 3 & 2 \\ 0 & -2 & -1 \\ -3 & 1 & 2 \end{vmatrix} = -3$, $C_{11} = -3$

$M_{12} = \begin{vmatrix} 1 & 3 & 2 \\ 2 & -2 & -1 \\ 1 & 1 & 2 \end{vmatrix} = -10$, $C_{12} = 10$

$M_{13} = \begin{vmatrix} 1 & 0 & 2 \\ 2 & 0 & -1 \\ 1 & -3 & 2 \end{vmatrix} = -15$, $C_{13} = -15$

$M_{14} = \begin{vmatrix} 1 & 0 & 3 \\ 2 & 0 & -2 \\ 1 & -3 & 1 \end{vmatrix} = -24$, $C_{14} = 24$

$M_{21} = \begin{vmatrix} 0 & -1 & 3 \\ 0 & -2 & -1 \\ -3 & 1 & 2 \end{vmatrix} = -21$, $C_{21} = 21$

$M_{22} = \begin{vmatrix} 1 & -1 & 3 \\ 2 & -2 & -1 \\ 1 & 1 & 2 \end{vmatrix} = 14$, $C_{22} = 14$

$M_{23} = \begin{vmatrix} 1 & 0 & 3 \\ 2 & 0 & -1 \\ 1 & -3 & 2 \end{vmatrix} = -21$, $C_{23} = 21$

$M_{24} = \begin{vmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & -3 & 1 \end{vmatrix} = 0$, $C_{24} = 0$

$M_{31} = \begin{vmatrix} 0 & -1 & 3 \\ 0 & 3 & 2 \\ -3 & 1 & 2 \end{vmatrix} = 33$, $C_{31} = 33$

$M_{32} = \begin{vmatrix} 1 & -1 & 3 \\ 1 & 3 & 2 \\ 1 & 1 & 2 \end{vmatrix} = -2$, $C_{32} = 2$

$M_{33} = \begin{vmatrix} 1 & 0 & 3 \\ 1 & 0 & 2 \\ 1 & -3 & 2 \end{vmatrix} = -3$, $C_{33} = -3$

$M_{34} = \begin{vmatrix} 1 & 0 & -1 \\ 1 & 0 & 3 \\ 1 & -3 & 1 \end{vmatrix} = 12$, $C_{34} = -12$

$M_{41} = \begin{vmatrix} 0 & -1 & 3 \\ 0 & 3 & 2 \\ 0 & -2 & -1 \end{vmatrix} = 0$, $C_{41} = 0$

$M_{42} = \begin{vmatrix} 1 & -1 & 3 \\ 1 & 3 & 2 \\ 2 & -2 & -1 \end{vmatrix} = -28$, $C_{42} = -28$

$M_{43} = \begin{vmatrix} 1 & 0 & 3 \\ 1 & 0 & 2 \\ 2 & 0 & -1 \end{vmatrix} = 0$, $C_{43} = 0$

$M_{44} = \begin{vmatrix} 1 & 0 & -1 \\ 1 & 0 & 3 \\ 2 & 0 & -2 \end{vmatrix} = 0$, $C_{44} = 0$

$$\begin{bmatrix} -3 & 10 & -15 & 24 \\ 21 & 14 & 21 & 0 \\ 33 & 2 & 0 & -12 \\ 0 & -28 & 0 & 0 \end{bmatrix}$$

$$adj(A) = \begin{bmatrix} -3 & 21 & 33 & 0 \\ 10 & 14 & 2 & -28 \\ -15 & 21 & -3 & 0 \\ 24 & 0 & -12 & 0 \end{bmatrix}$$

$det(A) = -3 \cdot C_{42} = -3(-28)$

$= 84$

$$A^{-1} = \frac{1}{det(A)} \cdot adj(A)$$

$$= \frac{1}{84}\begin{bmatrix} -3 & 21 & 33 & 0 \\ 10 & 14 & 2 & -28 \\ -15 & 21 & -3 & 0 \\ 24 & 0 & -12 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-1}{28} & \frac{1}{4} & \frac{11}{28} & 0 \\ \frac{5}{42} & \frac{1}{6} & \frac{1}{42} & -\frac{1}{3} \\ \frac{-5}{28} & \frac{1}{4} & \frac{-1}{28} & 0 \\ \frac{2}{7} & 0 & -\frac{1}{7} & 0 \end{bmatrix}$$

$|A^{-1}| = -\frac{1}{3}C_{24} = -\frac{1}{3}\begin{vmatrix} \frac{-1}{28} & \frac{1}{4} & \frac{11}{28} \\ \frac{-5}{28} & \frac{1}{4} & \frac{-1}{28} \\ \frac{2}{7} & 0 & -\frac{1}{7} \end{vmatrix}$

$= -\frac{1}{3}\left(\frac{1}{4}C_{12} + \frac{1}{4}C_{22}\right) = -\frac{1}{3}\left(\frac{-1}{28}\right)$

Answer $= \frac{1}{84}$ #