Finding pure strategies in security games with protection externalities

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Inspired by Jiarui Gan et al. 2015[1]. In that paper, the defender allocates resources to a set of targets, and a resource located at a target has the externality of defending other neighboring targets. The proposed algorithm could take in any adjacency matrix A indicating target adjacencies. A pure strategy for the defender corresponded to an allocation of integer resources/defenders to targets, such that each target was either defended or not defended (1 or 0). The authors acknowledged the limitation of only locating resources at targets and not between targets, but noted that a dummy target with no value to the attacker or defender could be placed at such locations. This prompted us to consider how one might determine which locations ought to have dummy targets – which locations are worth considering placing a resource.

We must first make some additional assumptions in order to say anything about dummy generation. Let the targets correspond to points on a plane, and let each defense resource defend all targets within a unit disc. Each potential resource location on the plane defends some set of targets T. We are only interested in "good" locations for which there is no other location whose T' is a superset of T; that is, locations that are not strictly worse to any other location. Among good locations which defend an identical set of targets, we should choose exactly one location arbitrarily to become a dummy target.

I claim that if all "good" locations have a corresponding dummy target, then we need only consider placing defenders on dummy targets, and no longer need consider placing defenders exactly on actual targets. This is fairly intuitive if one considers that a dummy could be created on exactly the same location as a real target.

Essentially, we hope to find the sets in S which can be used to cover U, and leave the approximate solution to the set cover decision problem to the CLASPE program from [1] or a similar program.

This turns out to correspond to finding all maximal cliques in a unit disk graph, where the discs are centered on actual targets. This is exponential, although polynomial approximations exist [2]. A dummy target can be placed anywhere in the region corresponding to a maximal clique.

So, my original idea was not so great – it reduced to a well-known problem, and turned out to be exponential. If we alter the problem so that a resource guards a square region, then this

should instead be equivalent to finding all maximal cliques in a boxicity 2 graph, which at least have only a polynomial number of maximal cliques unlike unit disk graphs.

My new project idea is to find a small (ideally the same as the number of actual targets) number of maximal-cliques/dummy-targets/allowable-defender-positions which do not allow optimal solutions, but are allow for better solutions than if defenders must locate themselves exactly on a target.

[1] Jiarui Gan, Bo An, and Yevgeniy Vorobeychik. Security games with protection externalities. (AAAI, 2015, to appear).

[2]Gupta, Rajarshi, Jean Walrand, and Olivier Goldschmidt. "Maximal cliques in unit disk graphs: Polynomial approximation." Proceedings INOC. Vol. 2005. 2005.