Investigating the Magnetic Field Produced by Solenoids

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INTRODUCTION

The aim of this experiment is to test a model for the magnetic field produced by a solenoid. We hope to make predictions about the results of tests done on one and two solenoids using equations derived from the Boit-Savart law of magnetic fields created by currents, then test the predictions experimentally.

Solenoids are used as sources of magnetic field in many places. They are often used in switches, actuators, and locks, in automatic gearboxes, and in medical equipment. They are a heavily used part of modern life, so it is critical that their workings are well understood.

Accurately knowing what the magnetic field of any solenoid looks like using a model is vital for these everyday uses and also for further experimentation using solenoids such as particle accelerators. The model we are testing is based on the Boit-Savart law of magnetic fields, discovered in 1820, and will be explained further in the next section.

METHODS

About the model

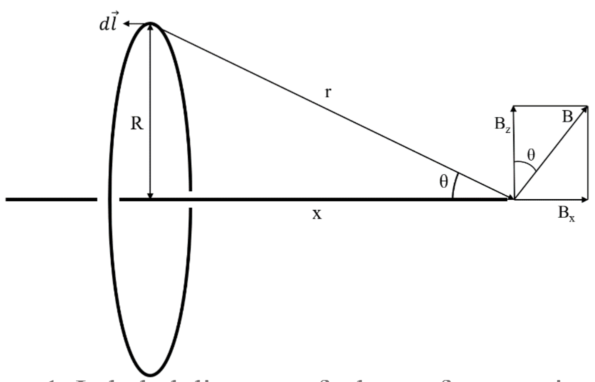

Figure 1. Labeled diagram of a loop of current in a wire

The model for our solenoid comes from the Boit-Savart law for a loop of wire with some current running through it. In this equation, $d\vec{l}$ is a small section of moving charge in the direction of the current, $\vec{r}$ is the vector from $\vec{dl}$ to the point where we are measuring the magnetic field, I is the current, and μ_0 is the vacuum permeability.

$$B(r) = \frac{\mu_0}{4\pi}\int \frac{I\vec{dl} \times \vec{r}}{|\vec{r}|^3}$$

Since our loop of wire has a rotational symmetry, the angle between $\vec{dl}$ and $\vec{r}$ is always 90°, meaning that the top half of our integral is $I|\vec{dl}||\vec{r}|\hat{n}$, where n is the direction vector pointing perpendicular to both $\vec{dl}$ and $\vec{r}$. Due to the symmetry, all vector directions not in the x direction away from the solenoid cancel out, leaving just the B_x which is equal to $B(r)sin\theta$. $sin\theta$ is just $\frac{R}{|\vec{r}|}$ so B_x is

$$B_x = \frac{R}{|\vec{r}|}\frac{\mu_0}{4\pi}\frac{I}{|\vec{r}|^2}\int |\vec{dl}|$$

The absolute value of the r vector is found using Pythagoras to be $\sqrt{x^2 + R^2}$ where x is the straight-line distance to the point where we are evaluating B_x. When we now do the integral, we sum up all the small $|\vec{dl}|$ values which gives the length of the wire loop, $2\pi R$. Putting this all together gives this equation for field strength in the x direction:

$$B_x = \frac{\mu_0}{2}\frac{R^2 I}{(x^2 + R^2)^{\frac{3}{2}}} \tag{1}$$

This equation works well for a single loop, but if we want to measure at any distance x from a solenoid, we will need an equation that takes into account the width of the solenoid. For this, we can use the equation:

$$B_x = \frac{\mu_0 NI}{4l}\left(\frac{x+l}{\sqrt{(x+l)^2 + R^2}} - \frac{x-l}{\sqrt{(x-l)^2 + R^2}}\right) \tag{2}$$

Where l is half the width of the solenoid and N is the number of turns. This is the main equation that I will be using for modeling the results of the second and third parts of the experiment. For the first experiment, x=0, which gives a simplified version of the above formula:

$$B_x = \frac{\mu_0 NI}{2\sqrt{l^2 + R^2}} \tag{3}$$

Finally, the equation below can be used to calculate the uncertainty in any of the B_x equations:

$$\sigma(B_x) = \sqrt{\left(\frac{\partial B_x}{\partial I} \times \sigma(I)\right)^2 + \left(\frac{\partial B_x}{\partial l} \times \sigma(l)\right)^2 + \left(\frac{\partial B_x}{\partial x} \times \sigma(x)\right)^2 + \left(\frac{\partial B_x}{\partial R} \times \sigma(R)\right)^2} \tag{4}$$

Equipment

- Two identical solenoids, mounted on a rail with a ruler to measure distances, with a known number of turns N, radius R, and width $2l$. In this experiment we used solenoids with 397 turns, a 15.5cm radius and a width of 0.47
- Hall effect sensor to measure the magnetic field strength

- A power supply that can operate in constant voltage or constant current mode, and can apply a constant voltage independently to each solenoid
- A current meter

Risks

Be aware that solenoids can get hot, especially when large amounts of current are flowing through them. Be careful when touching the solenoid while it's on or after it's been on. Switch off the current when you are not taking measurements and do not run an excessive amount of current through the solenoid.

You are working with mains power electricity through the power supply, so make sure that all equipment has been safety checked and follows safety regulations. The wires of all your devices should be well isolated.

Testing the model at a single point as current varies

Using equation 3, we know that the magnetic field strength of a wire should be directly proportional to the current through it. Since we know what the model says the proportionality constant between them should be, the aim of the first part of this experiment is to test this relation and see if it is consistent with the model. There may also be some correction factor that can be worked out using these results, which can be applied to later examinations of the model.

The first step is to set up the Hall effect sensor so that it can accurately measure the magnetic field strength produced by the solenoids. This means eliminating any background effects such as the earth's magnetic field. This can be done by adjusting the digital reading of the Hall effect sensor to be zero when there is no current through the solenoids.

Place the probe exactly in the center of the solenoid and take measurements of the magnetic field strength as you increase the current. For this experiment, we used increments of 0.15A in the range from 0A to 1.5A.

Do this for both solenoids to check that they are identical. If they are not identical, then it is harder to model the results, but it is still possible to adjust for it by using an altered version of equation 2.

Testing the model by adjusting the distance to one solenoid

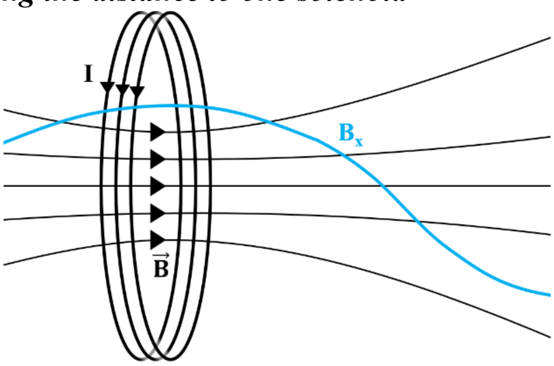

Figure 2. Diagram of the magnetic field created by a single solenoid and the theoretical prediction of the axial magnetic field strength

This part of the experiment requires a constant current. We used a current of 1.5A, however larger currents would give smaller uncertainties in the final results.

Supplying current to only one solenoid, move the Hall effect sensor along the axis of the solenoid and measure the axial magnetic field strength. A distance of 30cm with steps of 3cm was used for this part of the experiment.

Plot the experimental results next to the theoretical predictions obtained with equation 2.

Testing the model by moving around the midpoint of two solenoids

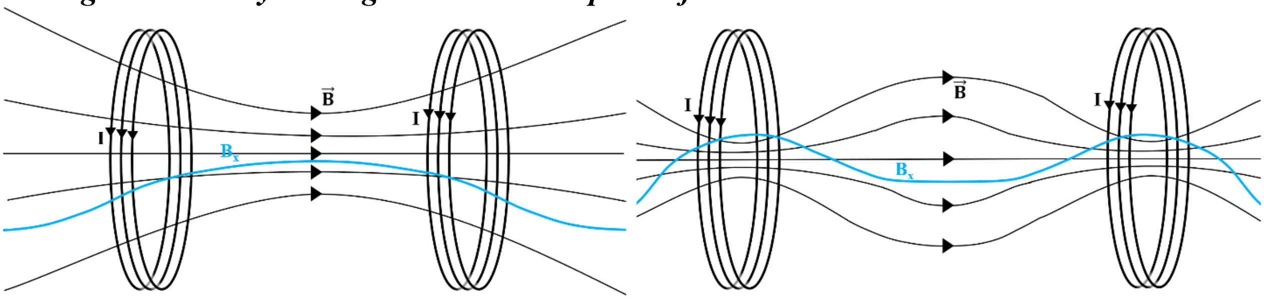

Figure 3. Diagrams of the magnetic field created by two solenoids spaced closely (left) and further apart (right) and the shape of the theoretical predictions of the axial magnetic field strength

The final part of this experiment involves using both solenoids to create a flat section of magnetic field around the midpoint of their separation, then using the Hall effect sensor to measure the magnetic field strength at different points around the midpoint. Do this for 2 different separations of solenoid. For example, we used a 15cm and a 20cm separation. Our data was taken between -5cm and 5cm from the midpoint, with intervals of 1cm, and compared to the results of the theoretical predictions.

A flat section of magnetic field means the rate of change of the magnetic field strength at the midpoint is zero, making it either a local maximum point or a local minimum point. As you can see in Figure 2, the model predicts that the distance between the solenoids will affect whether the midpoint of the magnetic field is a point of local maximum or local minimum. Using equation 2, you can get equation 5 for this experiment, where D is half the separation between the two solenoids and x is the distance from the midpoint:

$$B_x = \frac{\mu_o NI}{4l}\left(\frac{D+x+l}{\sqrt{(D+x+l)^2+R^2}} - \frac{D+x-l}{\sqrt{(D+x-l)^2+R^2}} + \frac{D-x+l}{\sqrt{(D-x+l)^2+R^2}} - \frac{D-x-l}{\sqrt{(D-x-l)^2+R^2}}\right) \qquad (5)$$

If we set the second derivative of equation 5 with respect to x as equal to zero, then let x equal zero and D be a variable, we can find at what distance the field strength at the center should switch from a maxima to a minima, around 16cm for us. This should be where the magnetic field strength around the midpoint is most flat. We will do this part of the experiment twice, once with 15cm separation and once with 20cm separation, so we expect to see a maximum point in the 15cm data and a minimum point in the 20cm data if our model is correct, and in both cases the magnetic field should be relatively constant.

RESULTS

Testing the model at a single point as current varies

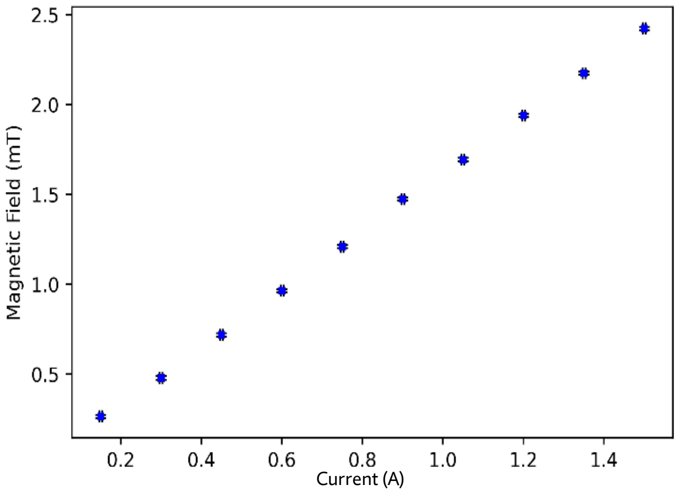

Figure 4. Plot of magnetic field strength (mT) at the center of a solenoid against current (A) in the solenoid.

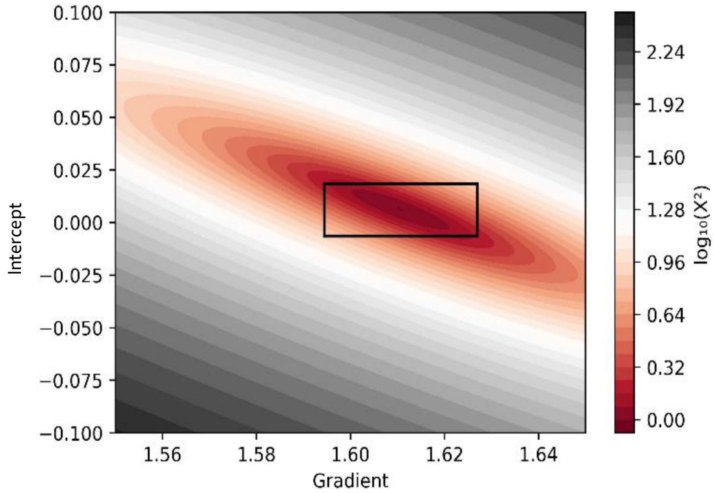

Figure 5. Contour plot of X^2 values for different models to fit the data as well as possible.

As you can see in Figure 4, the relationship between magnetic field strength at the center of a solenoid and the current is linear. Figure 4 shows a strong relationship with tiny errors, the error bars are plotted, but they are small. From equation 5 we expected B_x and I to have a linear relationship.

$$X^2 = \sum_{i=1}^{n} \frac{[y_i - (mx_i + b)]^2}{m^2\sigma_{x,i}^2 + \sigma_{y,i}^2} \tag{6}$$

To find the line of best fit and its uncertainty for my data, I used a reduced X^2 'goodness of fit' test shown in figure 5, where a range of gradients and intercepts that fit the data are run through equation 6 above, and a range is chosen. The standard deviation of a reduced X^2 test is $\sigma = \sqrt{v/2}$, where v is the degrees of freedom of the data. I have also placed a box around reduced X^2 values that are within the standard deviation as a representation of the error. I used a logged representation of reduced X^2 in the graph because the value grows very rapidly and makes it harder to interpret.

Due to the very tiny uncertainties in my data, the initial reduced X^2 values were very large. This is likely due to intrinsic scatter in our data, which could be caused by environmental fluctuations such as nearby electronics, so I introduced an extra uncertainty of 0.0065 mT to my magnetic field strength error, bringing the reduced value of the best fitting line to 0.998, very close to 1. The gradient of this line is 1.61 ± 0.02 mTA^{-1}, or $(1.61 \pm 0.02) \times 10^{-3}$ TA^{-1}.

To find out if this matches our predictions, we can check equation 3. From this equation, we know that the gradient of a graph of B_x against I should give a gradient of:

$$\frac{\mu_0 N}{2\sqrt{l^2 + R^2}}$$

Since we know all of these values with an uncertainty in l, we can find out what our model predicts the gradient should be, and it's 1.59×10^{-3} TA^{-1} with a negligible uncertainty. This is very close to our result, and it's within our error estimates, meaning that the model is, so far, a good prediction of reality. Consequently, we don't need to use a correction factor to adjust the model for future calculations. The below plot comparing the model and the experimental values shows how well the prediction fits reality for this set of data.

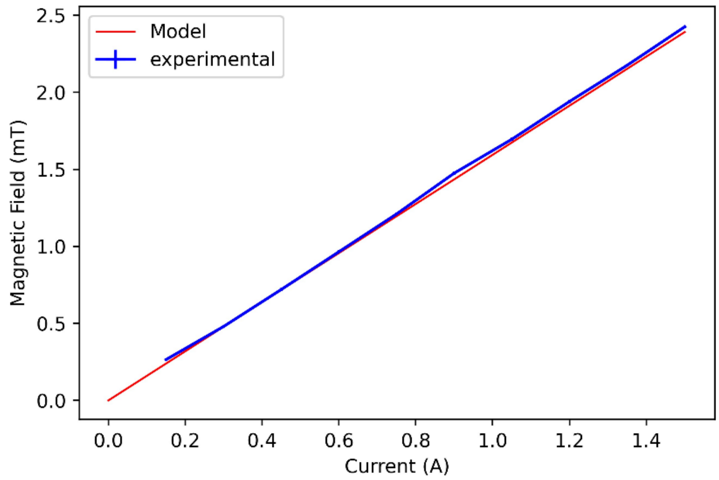

Figure 6. Comparison between the model and the experimental values for the relationship between magnetic field strength in the center of a solenoid and current

Testing the model by adjusting the distance to one solenoid

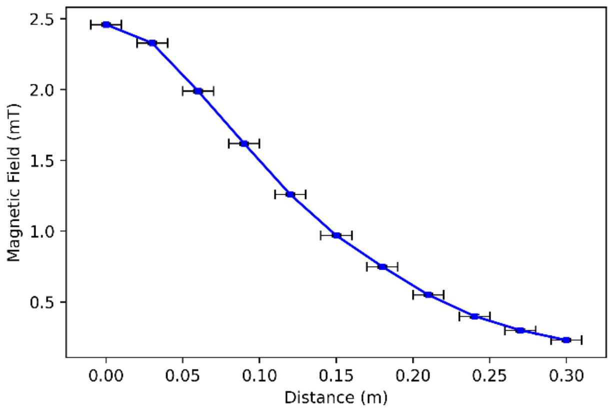

Figure 7. Plot of magnetic field strength (mT) against the distance from the center of the solenoid (m)

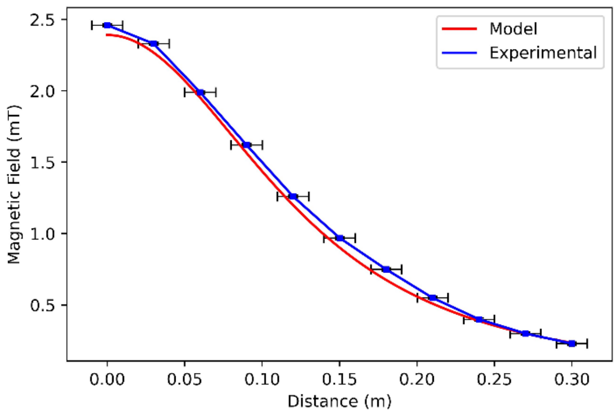

Figure 8. Comparison between the model and experimental values for the distance adjustment experiment

The model in Figure 8 comes from equation 2 evaluated when x is evaluated between 0m and 0.3m from the center of the solenoid, overlayed over Figure 7, the graph of our collected data.

You will notice that the model predicts that the magnetic field strength should be slightly weaker than the obtained experimental results across the whole range of distances.

This discrepancy could be caused by a systematic error with the current measuring equipment used for the experiments. I believe this is the most likely cause due to the upcoming results for the experiment using two solenoids, but it could also be caused by an incorrect assumption for our model or some other unaccounted-for error.

The final two experiments use a constant current of 1.5A. however, the results here more closely fit a current of around 1.55A as shown in Figure 9. The data in the first experiment still closely fits the model if an amount (proportional to how much was added to 1.5A to become 1.55A) is added to each current value, but not quite as well as previously, as you can see in figure 10.

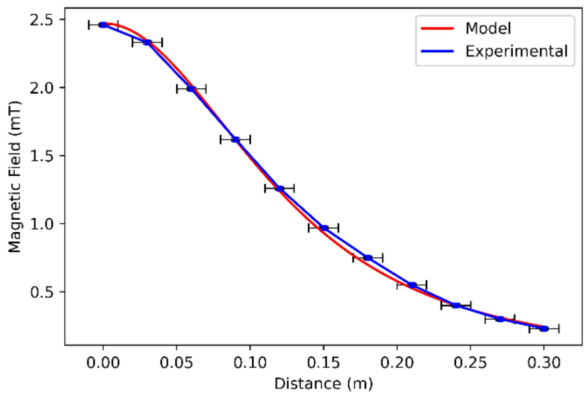

Figure 9. Same as figure 6 but using Current = 1.55A instead of Current = 1.5A

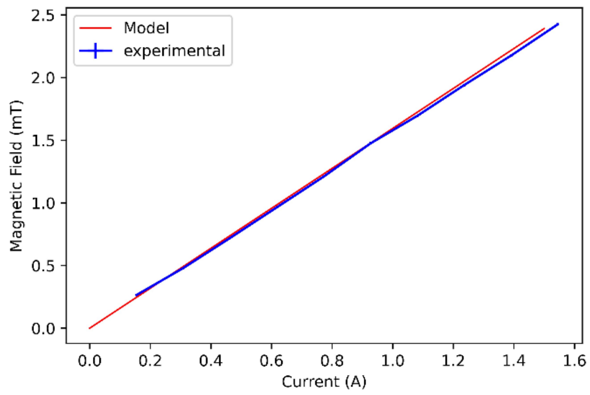

Figure 10. figure 4 but an amount has been added to each current value proportional to its value

Testing the model by moving around the midpoint of two solenoids

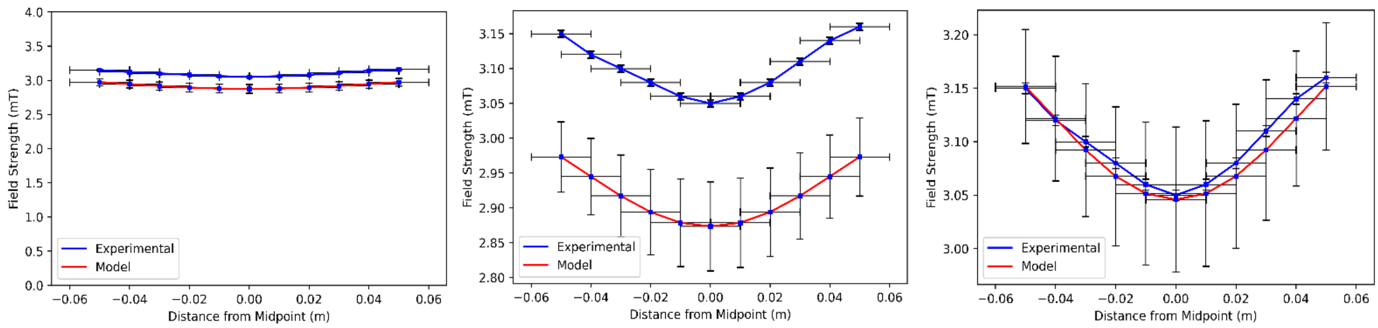

Figure 11. Graphs for the magnetic field strength around the midpoint of two solenoids of radius 15.5cm spaced 20cm apart, the rightmost graph's model uses a current of 1.59A instead of 1.5A

As you can see in Figure 11 and Figure 12, the difference between the model and the experimental results is more apparent in this experiment, and again, it can be explained by a systematic error regarding the measurement of current.

Despite this, looking at the zoomed-out graphs on the left, you can see that the model is relatively close, and it correctly predicted the shape of the graph of magnetic field strength against distance from midpoint for both 20cm and 15cm spaced solenoids. Our prediction from the model, that the magnetic field strength should be relatively constant around the midpoint of the two solenoids, was correct as can be seen in leftmost graphs in Figure 11 and Figure 12.

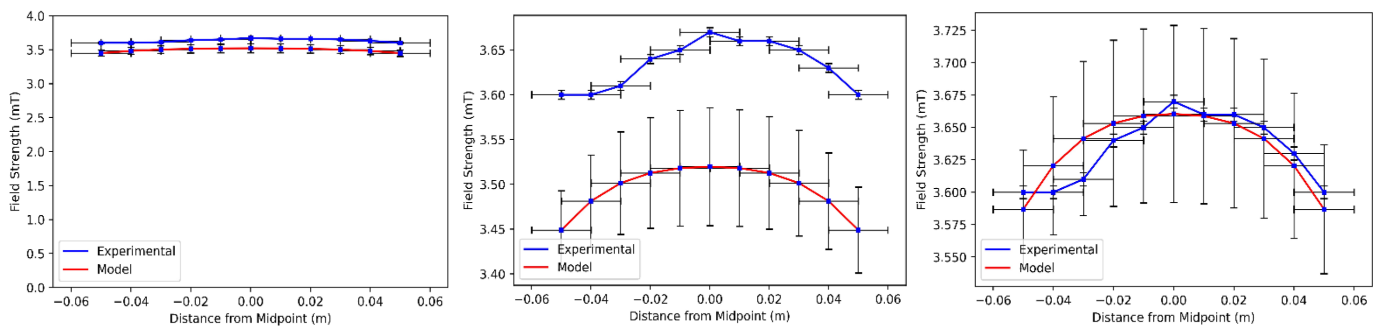

Figure 12. Graphs for the magnetic field strength around the midpoint of two solenoids of radius 15.5cm spaced 15cm apart, the rightmost graph's model uses a current of 1.56A instead of 1.5A

The average percentage change in magnetic field strength 5cm from the midpoint compared to at the midpoint of the solenoids is 3.4% for 20cm separation and 1.9% for 15cm separation. This is in line with our model because not only is the field strength relatively constant for both, but it is also more constant for the 15cm separation, which is closer to the 16cm point where we previously estimated the magnetic field strength would be most constant.

This consistency of the model, despite the problem in our results, is obvious when looking at the graphs, since both the experimental and theoretical results have the same curve shape. The separation that creates the most consistent magnetic field strength is not dependent on current but is dependent on the separation D, the solenoid width $2l$, and the radius R, meaning that these variables are likely not the cause of our issue, making me more confident that a systematic error with current is the most likely explanation for our results.

Conclusion

The first experiment varying current fit the theoretical values very well. The hypothesis that current through a solenoid and magnetic field strength at the center of the solenoid are directly proportional was shown to be correct, as was the hypothesised value of the proportionality constant between them.

After adding a small amount extra random error in the magnetic field value of $\pm$ 0.0065 mT to account for environmental factors, our data fit the theoretical values extremely well, with a reduced X^2 value of 0.998.

The second experiment using a constant current but varying distance to a single solenoid also fit the theoretical values quite well and within the error. Although it fits within the error, the experimental data has a higher-than-expected magnetic field strength, which could be explained by a systematic error in our constant current. A current of 1.55A instead of 1.5A fits the model better.

The final experiment makes the slight difference visible in the last experiment more pronounced. Despite this, our hypothesis about the shape and strength of the magnetic field was still shown to be true, even though the theoretical values were not within the uncertainty of our experimental values. Once again, it is possible that this discrepancy is caused by a systematic error in the constant current, as the theoretical model fits our results far better if the current through our solenoids was around 1.56A or 1.59A for 15cm separation and 20cm separation, respectively. Although it is possible that the issue is caused by an incorrect model or other unaccounted-for uncertainties.

In conclusion, the model has very good prediction power for the relationship between solenoid current and magnetic field strength and for the shape of the magnetic field, but our results don't confirm the model's ability to describe the behavior of the magnetic field strength between two solenoids.

The experiment should be repeated with a more careful focus on a correct constant current through the solenoid, and a higher current to remove some uncertainty. If I were to do this again, along with what I just mentioned, I would take more measurements of each magnetic field strength value, and I would do the third experiment with many additional solenoid separation distances.

References:

Muskens, O. and Barkovic, S. (2023) ‘PHYS1017: PHYSICS SKILLS 1’.

Erlichson, H. (1998). ‘The experiments of Biot and Savart concerning the force exerted by a current on a magnetic needle’. *American Journal of Physics* doi:https://doi.org/10.1119/1.18878.