Effects of Power Input and Temperature Difference on the Efficiency of Stirling Heat Engines

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Abstract

This work investigates how the efficiency of a Stirling heat engine is affected by changes in temperature difference and power input. Heat was provided to the Stirling engine through a heating element and efficiency was measured experimentally using power output found through pressure against volume graphs. In the second part of the experiment, we use ice in the cooling system to investigate how the engine reacts to higher temperature differences than what is possible through just the input power.

We found that for a constant temperature difference, the efficiency of the engine increased with additional input power at a rate of 0.008% ± 0.002% per watt between 150W and 300W. I theorized that this is likely due to the constant temperature resulting in a constant rate of heat loss by radiation when varying power input, resulting in a lower proportion of heat loss to total power input.

We also found that raising the temperature difference beyond what can be sustained by the input power alone does not result in the efficiency continuing to rise in the same way as it does for lower temperature differences, and that the efficiency starts to level off and stop increasing.

Future investigation into this topic should involve measuring efficiency while increasing power input without restricting temperature difference, as well as further data taken around increasing temperature difference higher than what is possible through just the input power.

Introduction

This experiment will investigate Stirling heat engines, it will look specifically at how efficiency changes with power input, and the relation between efficiency and engine temperature difference. We will make predictions about how the engines should operate based on theory, and then test an engine to see how closely it acts to how we predicted.

Heat engines are at the center of our modern world, and our lives are based in large part around the operation of them. For this reason, investigation into their workings and characteristics is extremely important. They are used in electricity generation, vehicles and machinery, refrigeration, and heat pumps.

Stirling engines specifically are used in applications requiring quiet operation, or low temperature differences. They are well suited for underwater propulsion due to being able to run without atmospheric oxygen, and have been used on Swedish and Japanese submarines, allowing multiple week underwater operation previously only possible on nuclear submarines.

Knowing the effects of power input and temperature difference on the efficiency of a heat engine is therefore vital information for many people around the world, and can be used to maximize efficiencies of power generation or to give a further insight into the workings of heat engines, allowing for innovations and improvements.

Theory

A Stirling heat engine, like the one used in this experiment, works using the forces provided through the heating and cooling of gasses to output mechanical power. The heating in our engine is provided by an electrical heating element supplied with a voltage V and a current I, the power of our input, is therefore given by equation 1.

$$P_{input} = IV \tag{1}$$

Starting from a cool, room temperature gas in the engine, the gas is heated by the heating element, and exerts more pressure on the engine's piston, which can be approximated as isochore heating (constant volume but increasing pressure). The increased pressure then pushes the piston outwards, and the gas expands to fill the new volume, this can be approximated as isothermal expansion (constant temperature, decreasing pressure and increasing volume). The gas now cools down and the pressure it applies to the piston decreases, the opposite of step 1, isochore cooling (constant volume decreasing pressure). The final step happens now where the lower pressure gas is compressed by the piston until it is back at the starting point, approximated by isothermal compression (constant temperature, increasing pressure and decreasing volume). This all can be put together to create a graph of pressure against volume where the engine cycle forms a loop, shown in figure 1.

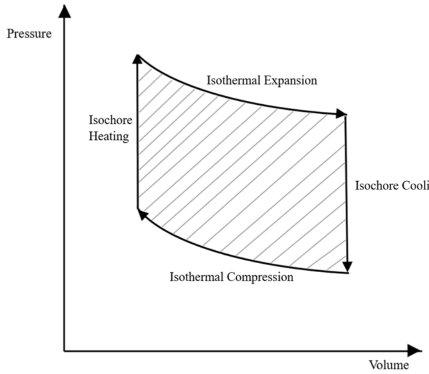

Figure 1. PV diagram of a Stirling cycle heat engine

The area inside this loop is the mechanical work done by the engine in the single cycle. This means that if you can obtain values for the pressure and volume inside a heat engine for one cycle you can find the mechanical power output of that cycle.

Power is work done per second, so finding the frequency of the cycle of the engine gives you enough information to find the power output of the engine using equation 2 where W is work done per cycle and f is the frequency of cycling of the engine.

$$P_{output} = Wf \tag{2}$$

Efficiency is final energy over initial energy which we can now work out using equation 3.

$$\eta_{engine} = \frac{P_{output}}{P_{input}} \tag{3}$$

It can also be shown that the maximum theoretical efficiency depends only on the hot and cold temperatures of the engine and the equation is given below with T_c being the cold temperature and T_h being the hot temperature both in kelvin. You can see that the only way to get 100% efficiency is for the cold temperature of the engine to be 0k, which is not possible.

$$\eta_{max\ theoretical} = 1 - \frac{T_c}{T_h}$$

In reality, the PV diagram does not look as clean, because the approximations do not perfectly describe reality and because the parts aren't all separate, they blend together. Figure 2 on the next page shows a real example of a PV diagram from this experiment. You can see that the shape is similar to the theoretical image, however the sections are not so cleanly defined, and the volume is smaller because of it; the isochore heating is happening at the same time as isothermal compression and the isochore cooling is happening at the same time as the isothermal expansion.

The area of the inside of the graph will be in the units cm^3hPa. Using some manipulation of units, 10,000 cm^3hPa = 1J, which can be used to convert to more useful units.

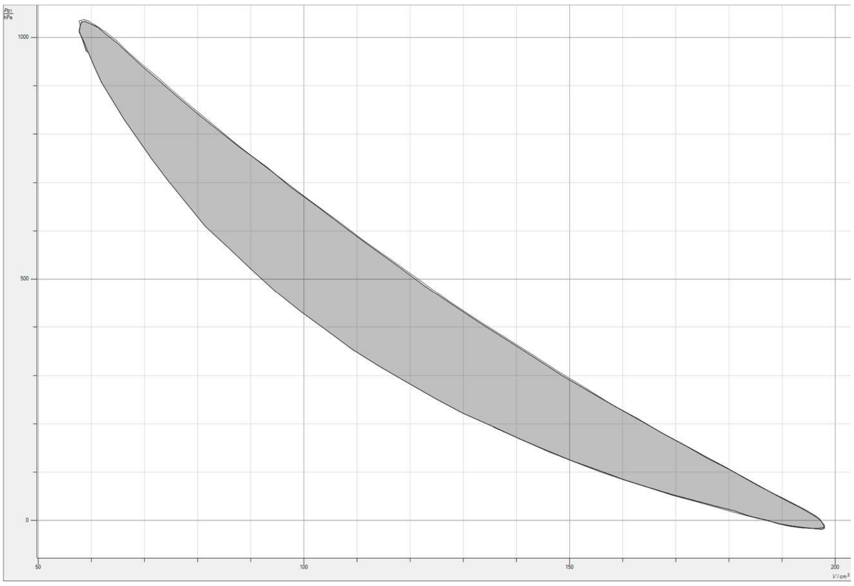

Figure 2. Measured PV diagram for a heat engine, x-axis is volume in cm^3, y-axis is pressure in hPa. The shaded volume is the work done in the cycle shown.

The amount of heat lost to the environment per second is given by power input minus power output, which will be useful later. This encapsulates all lost energy during the process, the lost energy to escaped heat, sound, and friction. In theory, the energy lost to heat radiating into the environment should stay constant if there is a constant hot and cold temperature in the engine (and room temperature) no matter how much energy is provided.

If heat loss through radiation is a major source of the inefficiency of a Stirling engine, increasing power will not increase the heat loss of the engine much, which should result in a higher efficiency with an increased power input.

Methods

For this experiment we used a Stirling heat engine set up as shown in figure 3, as well as a datalogger. For the second part we also needed a large amount of ice.

Prior to this extension project, I had measured the change in efficiency depending on the temperature difference within the engine for a single constant input voltage of 10v (and constant power).

For the first experiment then, a constant temperature difference (and close to constant cold and hot temps) was kept, while input voltage was varied. Varying the input voltage also varies the input current, so this was noted down for each measurement. Since the voltage displayed on the power supply was not completely accurate, we kept note of the actual voltage provided using the datalogger. We used the CassY lab software to do this, providing it with readings from the electrical input, the pressure sensor, and the position of the piston, which was used to calculate the current volume of the inside of the engine.

We used the software to plot pressure against volume. CassY has a function which finds the area of the PV graph for you (work done per cycle), studying the other graph provided by CassY with time on the x-axis lets you find the frequency of the cycling of the engine.

We used a water pump cooling system to keep the engine cool and we closely monitored multiple sensors in the engine to make sure the experiment was carried out safely. We also carried out multiple test runs before beginning our experiment to make sure our equipment was set up correctly and working as we expected it to.

Now we did the first experiment, measuring work done per cycle, frequency, voltage, current, cold temperature, and hot temperature multiple times for each value of voltage on our power supply: 8v, 12v, 14v, and 16v. it was not safe to go over 16v using our Stirling engine. We used a single engine in all our tests so that results were consistent; we had access to multiple engines and may have been able to gather more data had we used more than one, but decided against it incase different engines gave slightly different results.

In the second part of the experiment, we used ice to cool the cold part of the engine to lower than room temperature. We wanted to see what would happen to the efficiency at temperature differences greater than the input power could get the engine to.

As mentioned earlier, prior to this extension project, I had measured the change in efficiency depending on the temperature difference within the engine for a single constant input voltage of 10v. This gave a graph which seemed to show that efficiency rose in an exponential fashion as the temperature difference grew, shown later in figure 6. As this reached the maximum temperature difference that the heating element could manage, the increase in efficiency started to plateau. At the time I stated that I believed with higher temperature differences over what the heating element could produce, you'd see the efficiency flatten out.

One way to reach this temperature would be to make the hot part of the engine hotter, maybe done through insulation or some other method. We decided this would be more dangerous and harder than the simpler method, to cool down the cold part of the engine.

We used a constant voltage of 8.9v and therefore a constant current and power input. We let the engine run for a while until it reached the maximum temperature that the heating element was able to reach, at a temperature difference of about 70K.

We took the measurements of Voltage, current, hot temperature, cold temperature, work done per cycle, and frequency, for this value of temperature difference.

A few handfuls of ice were tipped into the water reservoir used to cool the engine. More ice was added each time the ice in the reservoir had fully melted, ensuring that the temperature kept decreasing. We took measurements of the engine at milestones of the cold temperature, for example 20ºC, 15ºC, 10ºC, and 5ºC. A slower rate of cooling is optimal over a faster rate of cooling since it gives time for the whole cold section of the engine to reach that temperature, rather than the temperature of the engine lagging behind your temperature reading of the coolant water in the engine.

If the increase in efficiency does in fact level out after the maximum temperature different that the heating element can manage, then a plot of efficiency against temperature difference of these results should show the plateau part of the efficiency graph.

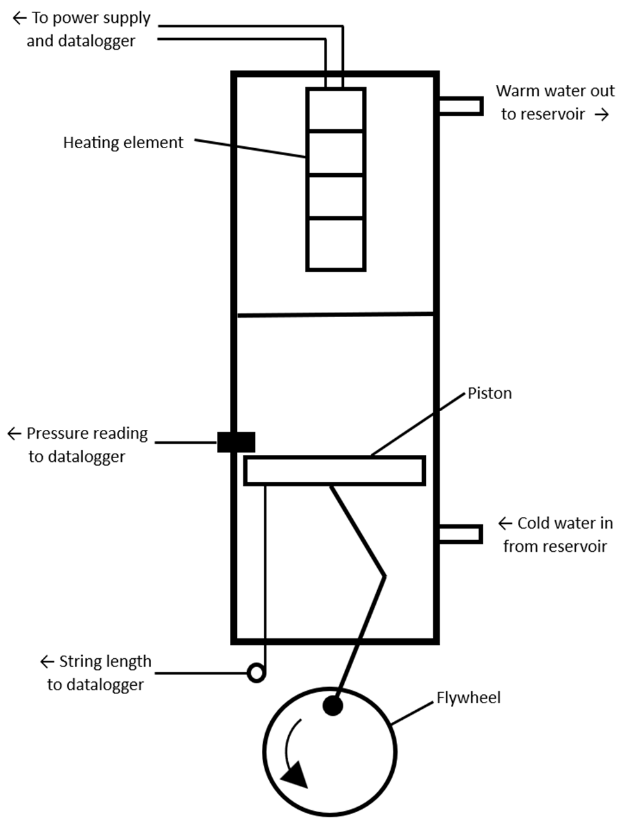

Figure 3. Labeled Diagram of the Stirling engine setup used in the experiment

Results

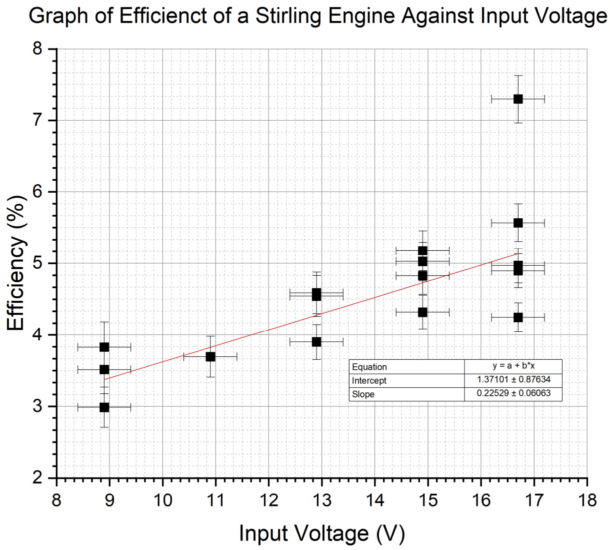

Figure 4. Graph of efficiency against input voltage of the Stirling engine using a constant temperature difference of around 308K (35ºC)

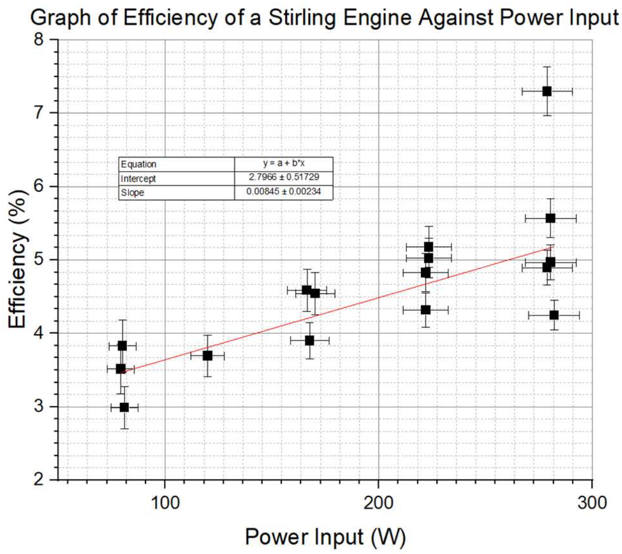

Figure 5. Graph of efficiency against power input of the Stirling engine using a constant temperature difference of around 308K (35ºC)

Figures 4 and 5 show the results of our measurements of efficiency of the Stirling engine while adjusting input voltage, which also causes changes in current, and therefore different power input. As power input increases it is clear than the engine's efficiency also increases. The graphs show a measured rate of increase in efficiency of 0.22 ± 0.06 percent per volt or 0.008 ± 0.002 percent per watt.

Although this implies linearity, and I plotted a line on the graphs, the true change in efficiency with power input is not linear and so efficiency outside of the range that I measured in my experiment cannot be reliably extrapolated to. It is likely that the line would eventually plateau, maybe not far after the 300 watts we went up to in the experiment.

This result is similar to the expected result. Since the temperature difference is kept constant across the different power input tests, the temperature gradient between the engine and the outside of the engine is mostly constant, which means heat loss should be approximately the same across each measurement. This results in a larger proportion of the provided power being used to operate the engine, instead of being lost to the environment. If the experiment did not use constant temperature difference, and instead used the highest temperature that could be reached for each different power input, the results may look very different, with efficiency being constant with increasing power input, or even decreasing with increasing power input.

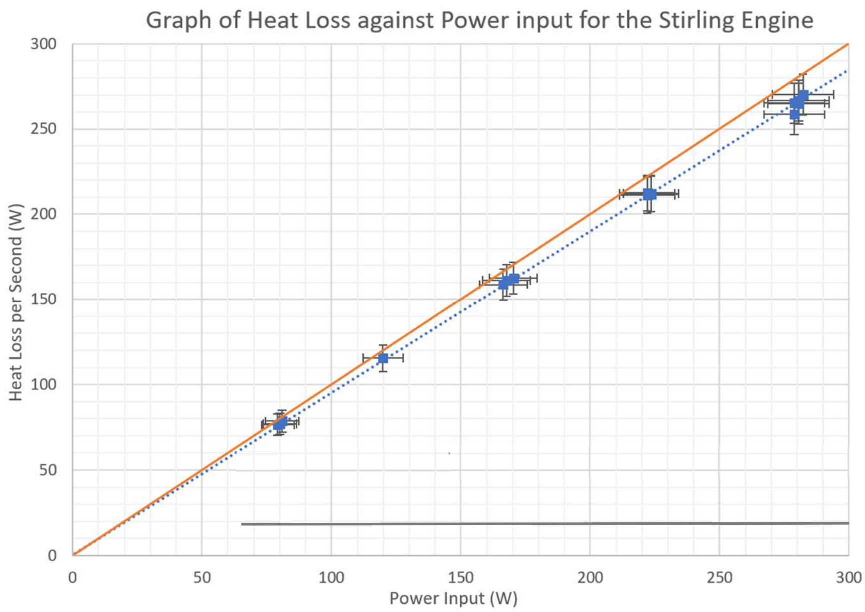

Figure 6. Graph of heat loss for the Stirling engine against power input. Orange line is where heat loss is equal to power input, gray line is estimated heat loss due to radiation.

Looking at the graph of heat loss in the engine (figure 6), you can see that, of course, the amount of energy lost to the environment increases as you supply more energy to the engine. However, the amount of heat lost rises slower than the power input rises (shown by the orange line), which is obvious since the efficiency of the engine has been shown to be increasing. You can see this as the orange and blue lines are diverging. This is likely happening due to the reasons I stated previously about heat radiation.

Making some large approximations, we can estimate the amount of heat lost by radiation. considering the engine cylinder as a black body, at 60ºC, with a 3cm radius a 10cm height, it likely radiates somewhere around 20W of power, marked on figure 6 with a gray line. This being constant is probably a large contributor to the increase in efficiency.

The extra heat loss likely comes from the increase in friction and noise as the engine rotates faster and with more force with higher power input.

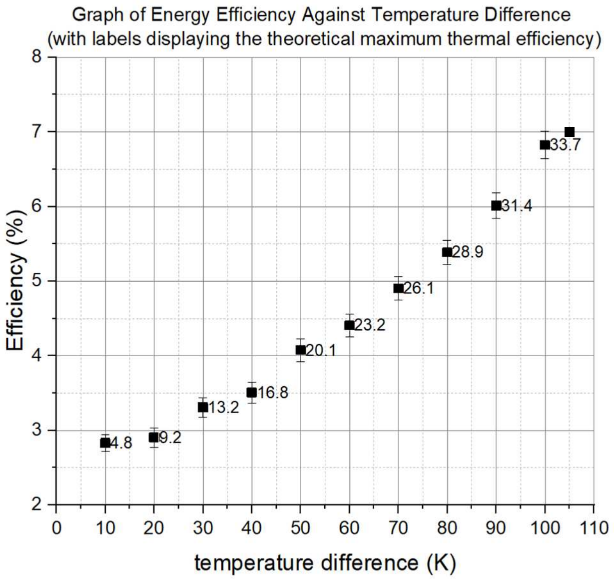

Figure 7. Graph of efficiency against temperature difference for 10v supply up to maximum temperature difference

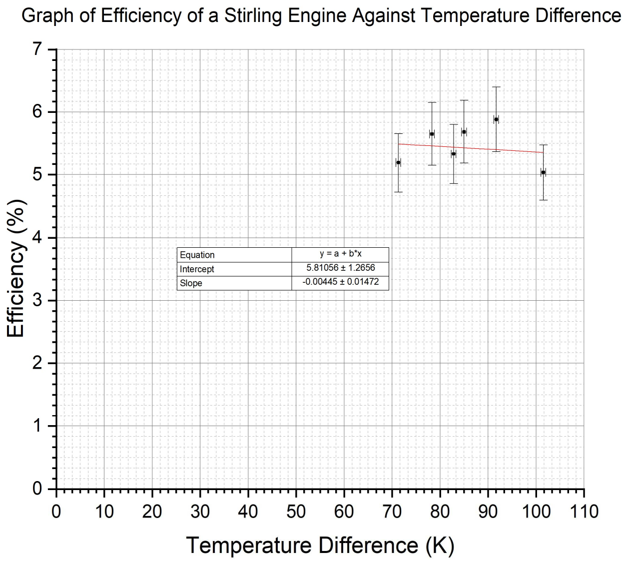

Figure 8. Graph of mechanical power output against temperature difference for 8.9v supply above maximum temperature difference

In the second part of the experiment, we looked at how the efficiency changes after the maximum temperature difference that the heating element can provide, around 70K. We expected that the efficiency would level out after the maximum temperature difference, and figure 8 shows that it does.

The graph shows a slight decrease in efficiency over all the measurements taken, a change in efficiency of -0.004 ± 0.01 percent per Kelvin, however the error bars on this value also go into the positive, so the decrease is likely a statistical anomaly. Looking at the graph, the last point at ~100K is anomalously low, the rest of the values seem to be more consistent with our theory, there is a slight rise and then the plot levels out. If you remove the anomalous result as in figure 9, the trend is upwards at 0.03 ± 0.01 percent per kelvin.

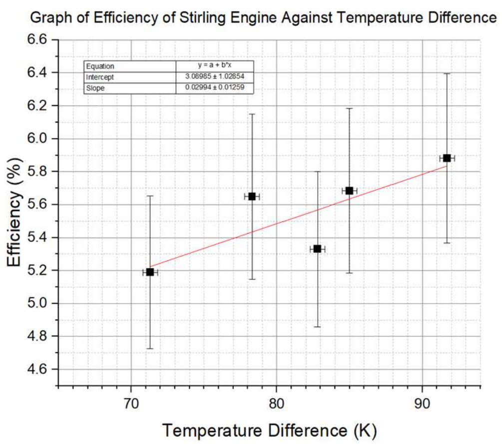

Figure 9. Graph of efficiency against temperature difference of 8.9v source with anomaly removed

Although the last point is slightly anomalous, it is a good idea to take it into account since it is not far off the main trend. It provides some proof that the efficiency does not continue to increase as it did before the temperature difference was larger than the heating element could produce as in figure 7. Therefore, the pattern shown in figure 7 does not continue forever as you increase temperature difference further.

Discussion

There are multiple experiments which should be done to further investigate heat engines in the ways that this experiment missed. Firstly, more data is needed for the second experiment, 5 datapoints is not enough to definitively state the pattern in the data, this is where it may have been useful to use multiple engines rather than just one to take the data. The error in efficiency in this section was also not optimal, this is due to rather large uncertainties in the values for provided voltage and current, due to these values fluctuating during experimentation. With a more precise power supply or better way of reading current than a clamp meter, smaller uncertainties are possible. It would also be preferable to start from a small temperature difference in this experiment instead of starting at the maximum, to obtain a full graph of efficiency against temperature difference. We did not do this because we didn't have time and already had separate data about what happens before the maximum temperature difference, but with a full graph the whole curve could be characterized, and it would more clearly show the level off.

Secondly, I mentioned previously that in experiment one, instead of using a constant temperature difference for different power inputs, you could use the maximum temperature difference reached by each different power input. Higher power inputs would reach higher temperature differences, meaning larger power outputs, but the temperature would be hotter, meaning heat loss due to radiation would increase, perhaps resulting in seeing a constant efficiency across power inputs, or even a decrease in efficiency with power input. This kind of experiment would give a more practical understanding of heat engines usage in the real world since most of the time a heat engine will be operating at the maximum temperature difference it can manage given the provided heat input.

Finally, experiment one can be further investigated. In this experiment, we showed the relationship between efficiency, and power input, but only in the range between ~150W and ~300W. My data does not say much about what happens outside of this range (although I expect the line will plateau after some amount of power input). A larger range of values for power input should be investigated, going as low as possible while the engine still runs, and as high as is safe using an engine which can safely run at high power inputs, we went as high as was advised to run on the engine we were using.

Conclusion

We showed that, as predicted, the efficiency of our Stirling engine rose as we increased the power input, and we found the rate of increase in efficiency while changing power input to be 0.008 ± 0.002 percent per watt between 150W and 300W.

As mentioned in the method section, I had previously predicted that we should see the graph of efficiency against temperature difference level off as temperature difference goes above what can be maintained by the power input. In experiment 2 we found evidence to suggest that this is the case, although further data is needed to characterize the shape of the full graph and give extra evidence to the conclusion.

The second experiment should be repeated starting as low as the engine can run and using ice to reach the largest temperature difference possible, so that a full picture of the efficiency against temperature difference graph can be seen instead of different sections of the graph separately.

An experiment investigating the relation between efficiency and power input without limiting the temperature difference should also be done, since it would be more useful for describing the operation of heat engines used outside of a lab setting.

References

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