

Assignment 0 - LS

MnP Club

July 5, 2023

Here is the assignment for the first week of the course! For the theory questions, you can either write them down, or \LaTeX them. For the coding questions, you can submit either a Python file or a Jupyter Notebook (ipynb file)

The instructions to submit the assignment will be given shortly :)

1. We're going to give you some matrices, and your job is to find their eigenvalues (no calculator, and write down how you find them, because that is what we want to know)

$$(a) \begin{bmatrix} 0 & 5 & 0 & 4 \\ 5 & 0 & 4 & 0 \\ 0 & 3 & 0 & 2 \\ 3 & 0 & 2 & 0 \end{bmatrix}$$

$$(b) \begin{bmatrix} 0 & 0 & 5 & 4 \\ 0 & 0 & 3 & 2 \\ 5 & 4 & 0 & 0 \\ 3 & 2 & 0 & 0 \end{bmatrix}$$

$$(c) \begin{bmatrix} 25 & 20 & 20 & 16 \\ 15 & 10 & 12 & 8 \\ 15 & 12 & 10 & 8 \\ 9 & 6 & 6 & 4 \end{bmatrix}$$

2. Suppose that O is a (orthogonally) diagonalizable operator with eigenvalues ± 1 Show that

$$P_{\pm 1} = \frac{I \pm O}{2}$$

where P_{λ} is the projector on the eigenspace of O associated with eigenvalue λ . (Even though this is an exercise in the reference, it's pretty nice and relevant, so here you go)

3. Let V be a *finite* dimensional inner product space, and let A be an operator on V . The following are equivalent:

Prove that an operator that is norm preserving (for every vector $x \in V$, $|Ax| = |x|$ is equivalent to a unitary operator.

Hint : Try using the fact that the below condition is also equivalent to the above two :

$$\langle Ax | Ay \rangle = \langle x | y \rangle$$

for all $x, y \in V$

4. Write code (using qiskit) to plot the following states on the Bloch Sphere

$$|0\rangle, |1\rangle, \frac{|0\rangle + |1\rangle}{\sqrt{2}}, \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

Justify their directions, and do you notice some similarities among the first two and the last two?

5. Make a Quantum Circuit (code it up in qiskit) that prepares the state $\frac{|000\rangle + |111\rangle}{\sqrt{2}}$ from $|000\rangle$ and only with basic gates such as H , X and CNOT gates.

Hint : First prepare the state $\frac{|00\rangle + |11\rangle}{\sqrt{2}}$ and try to generalise it

Can you generalise this further to prepare the state $\frac{|0\dots 0\rangle + |1\dots 1\rangle}{\sqrt{2}}$?