Assignment 1 - LS

MnP Club

July 20, 2023

Here is the second assignment for the course! For the theory questions, you can either write them down, or LaTeX them. For the coding questions, you can submit either a Python file or a Jupyter Notebook (ipynb file)

The instructions to submit the assignment will be given shortly :)

1. Some basic operations!

   (a) Code up the circuit to swap the states of two qubits. You should have seen the circuit already in QCQI/Qiskit Textbook

   (b) Given a three digit binary number *abc*, code up a circuit to increment the number by 1 (mod 8) The result should be stored in-place, i.e., in the same qubits that are used for the inputs. The inputs will be of the form $|a\rangle \otimes |b\rangle \otimes |c\rangle$ where a, b, c are each either 0 or 1.

   (c) (Optional) If you want to take this further, can you code up a circuit that takes two 3 digit binary numbers and adds them up (mod 8), storing the result in the qubits for the second number?

   Try doing this first using some *ancilla* qubits, i.e., extra qubits pre-initialised to some fixed state, which can serve as some place to store information. A challenge would be to do it without using any extra qubits.

   Also, it's possible to do the previous two parts just with the Pauli-X, CNOT and Toffoli gates!

   (d) The **Hamming Weight** of a binary number is the number of 1s in its binary representation.

   For a binary number with 3 bits, construct a circuit that takes $|x\rangle |0\rangle$ to $|x\rangle |w(x)\rangle$ where $w(x)$ is the Hamming weight of (x).

   Optional : Can you extend this to a general n bit number?

2. Implement either the Deutsch-Josza or Bernstein Vazirani algorithms (try not to copy Qiskit Textbook, please :))

3. Consider the following game, called the GHZ game. There are three players, Alice, Bob, and Charlie, and a Referee who mediates the game. The Referee will give one bit each to the players such that even number of 1 are given. Let us call the bits given to Alice, Bob, and Charlie as r, s, t respectively. Then each of the 3 players must give one bit back to the Referee, call the bits given by them as a, b, c respectively. Then Alice, Bob and Charlie win if $a \oplus b \oplus c = r \vee s \vee t$, else the Referee wins.

   (a) Classical Case: Alice, Bob and Charlie get to discuss their strategy beforehand. They are not allowed to communicate during the game. Show that the best strategy for them is able to win $\frac{3}{4}$ th of the time, and they can not do better.

   (b) Quantum Case: Alice, Bob and Charlie are allowed to discuss and share qubits beforehand. During the game they can not communicate with each other, but they can perform operations on their own qubits. Find a strategy that allows them to always win the game. Here are some hints for the same:

- They have to share entangled qubits, non-entangled qubits would be useless. In particular the GHZ state.
- They must perform some operation in case they get a 1 from the referee. They must not do this operation if they get a 0.
- Once everyone is done performing/not performing their operation, they must all measure and tell the referee three bits a, b, c based on the measurement

4. Implement the following in qiskit as well (Implementation always helps you understand it better)
   - Quantum Fourier Transform
   - Quantum Phase Estimation
   - Shor's Algorithm
   - Grover's Search Algorithm