MIS 381N Homework 2

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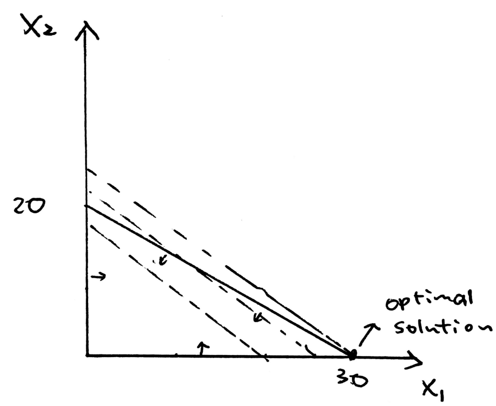
Question 1

Let number of tortes and apple pies max takes be x_1 and x_2. Then the objective is to maximize

$$4x_1 + 5x_2$$

subject to the constraints

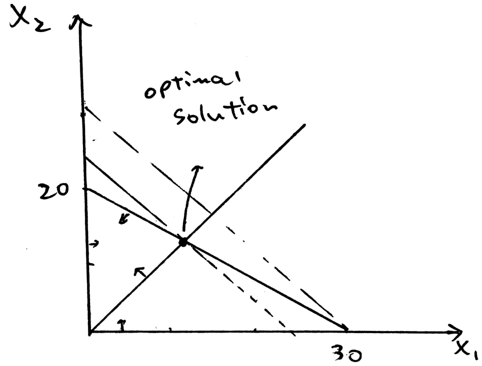
$$\begin{cases} 2x_1 + 3x_2 & \leq 60 \\ x_1, x_2 & \geq 0 \end{cases}$$



Based on the graph, we could see that the optimal solution given the constraint and isoprofit lines is $x_1 = 30$ and $x_2 = 0$.

Now, given that the number of pies should be greater than or equal to number of tortes, or $x_1 \leq x_2$, the constraints changes to

$$\begin{cases} 2x_1 + 3x_2 & \leq 60 \\ x_1 - x_2 & \leq 0 \\ x_1, x_2 & \geq 0 \end{cases}$$



Based on the graph, we could see that the optimal solution given the constraint and isoprofit lines is $x_1 = 12$ and $x_2 = 12$.

Question 2

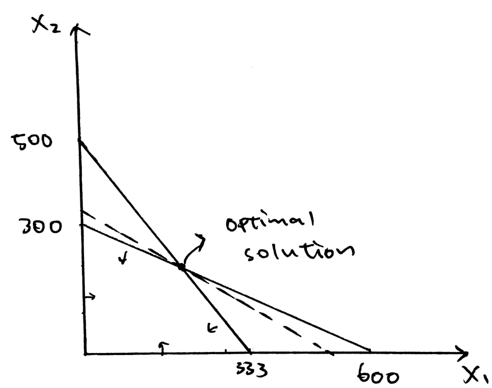
a)

Let acreage of wheat and corn planted be x_1 and x_2. Then the objective is to maximize

$$2000x_1 + 3000x_2$$

subject to the constraints

$$\begin{cases} 3x_1 + 2x_2 & \leq 1000 \\ 2x_1 + 4x_2 & \leq 1200 \\ x_1, x_2 & \geq 0 \end{cases}$$



Based on the graph, we could see that the optimal solution given the constraint and isoprofit lines is $x_1 = 200$ and $x_2 = 200$.

b)

```r
c <- c(2000,3000)
A <- matrix(c(3,2,2,4),2,2)
dir <- rep('<=',2)
b <- c(1000,1200)
s <- lp('max',c,A,dir,b)
print(s$solution)
```

```
## [1] 200 200
```

```r
print(s$objval)
```

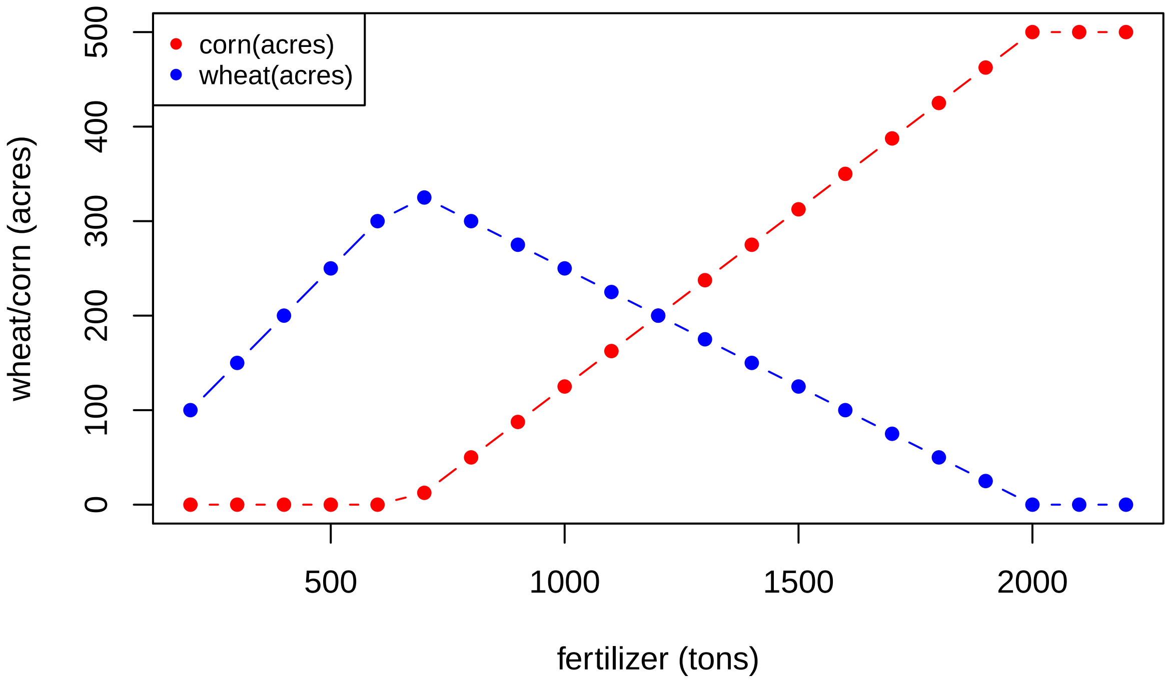
```
## [1] 1e+06
```

Solutions are the same as solving by graph.

c)

```r
corn = rep(0,21)
wheat = rep(0,21)
fert = seq(200,2200,by=100)
for (i in 1:21){
  b <- c(1000,fert[i])
  s <- lp('max',c,A,dir,b)
  wheat[i] = s$solution[1]
  corn[i] = s$solution[2]
}

plot(fert,corn,xlab = 'fertilizer (tons)', ylab = 'wheat/corn (acres)',col='red',type="b",pch = 16)
points(fert, wheat, col='blue', type="b",pch = 16)
legend("topleft", pch =c(16,16), col = c('red','blue'),legend = c('corn(acres)','wheat(acres)'), cex = (
```



As shown, the farmer discontinue producing wheat when fertilizer availability reaches 2,000 tons. The farmer stop producing corns when the fertilizer availability is below 600 tons.

Question 3

Let fraction of investment for each investment opportunity be x_1,x_2,x_3,x_4,x_5. Then the objective is to maximize

$$13x_1 + 16x_2 + 16x_3 + 14x_4 + 39x_5$$

subject to the constraints

$$\begin{cases} 11x_1 + 53x_2 + 5x_3 + 5x_4 + 29x_5 & \leq 40 \\ 3x_1 + 6x_2 + 5x_3 + 5x_4 + 34x_5 & \leq 20 \\ x_1, x_2, x_3, x_4, x_5 & \leq 1 \\ x_1, x_2, x_3, x_4, x_5 & \geq 0 \end{cases}$$

We could rewrite the objective function and contraints in matrix forms $\mathbf{c}^T\mathbf{x}$ and $A\mathbf{x} \leq \mathbf{b}$ respectively, where

$$\mathbf{c} = \begin{bmatrix} 13 \\ 16 \\ 16 \\ 14 \\ 39 \end{bmatrix}, \quad A = \begin{bmatrix} 11 & 53 & 5 & 5 & 29 \\ 3 & 6 & 5 & 1 & 34 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 40 \\ 20 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

we could use R to solve for $\mathbf{x}$,

```
c <- c(13,16,16,14,39)
A <- rbind(matrix(c(11,3,53,6,5,5,5,1,29,34),2,5),diag(5))
dir <- rep('<=',7)
b <- c(40,20,1,1,1,1,1)
s <- lp('max',c,A,dir,b)
print(s$solution)
```

```
## [1] 1.0000000 0.2008600 1.0000000 1.0000000 0.2880835
```

```
print(s$objval)
```

```
## [1] 57.44902
```

Hence,

$$\mathbf{x} = \begin{bmatrix} 1 \\ 0.20 \\ 1 \\ 1 \\ 0.29 \end{bmatrix}$$

Question 4

Let number of servings for each food be x_1,x_2,x_3. Then the objective is to maximize

$$0.18x_1 + 0.23x_2 + 0.05x_3$$

subject to the constraints

$$\begin{cases} 107x_1 + 500x_2 & \geq 5000 \\ 107x_1 + 500x_2 & \leq 50000 \\ 72x_1 + 121x_2 + 65x_3 & \geq 2000 \\ 72x_1 + 121x_2 + 65x_3 & \leq 2250 \\ x_1, x_2, x_3 & \leq 10 \\ x_1, x_2, x_3 & \geq 0 \end{cases}$$

We could rewrite the objective function and contraints in matrix forms $\mathbf{c}^T\mathbf{x}$ and $A\mathbf{x} \leq \mathbf{b}$ respectively, where

$$\mathbf{c} = \begin{bmatrix} 0.18 \\ 0.23 \\ 0.05 \end{bmatrix}, \quad A = \begin{bmatrix} -107 & -500 & 0 \\ 107 & 500 & 0 \\ -72 & -121 & -65 \\ 72 & 121 & 65 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -5000 \\ 50000 \\ -2000 \\ 2250 \\ 10 \\ 10 \\ 10 \end{bmatrix}$$

we could use R to solve for $\mathbf{x}$,

```r
c <- c(0.18,0.23,0.05)
A <- rbind(matrix(c(-107,107,-72,72,-500,500,-121,121,0,0,-65,65),4,3),diag(3))
dir <- rep('<=',7)
b <- c(-5000,50000,-2000,2250,10,10,10)
s <- lp('min',c,A,dir,b)
print(s$solution)
```

```
## [1]  1.944444 10.000000 10.000000
```

```r
print(s$objval)
```

```
## [1] 3.15
```

Hence,

$$\mathbf{x} = \begin{bmatrix} 1.94 \\ 10 \\ 10 \end{bmatrix}$$

Question 5

Let number of acreage cutting per year for forest unit 1 be x_1,x_2,x_3 and for forest unit 2 be y_1,y_2,y_3. Then the objective is to maximize

$$x_1 + 1.3x_2 + 1.4x_3 + y_1 + 1.2y_2 + 1.6y_3$$

subject to the constraints

$$\begin{cases} x_1 + x_2 + x_3 & \leq 2 \\ y_1 + y_2 + y_3 & \leq 3 \\ 1.2 \leq x_1 + y_1 & \leq 2 \\ 1.5 \leq 1.3x_2 + 1.2y_2 & \leq 2 \\ 2 \leq 1.4x_3 + 1.6y_3 & \leq 3 \\ x_1, x_2, x_3, y_1, y_2, y_3 & \geq 0 \end{cases}$$

We could rewrite the objective function and contraints in matrix forms $\mathbf{c}^T\mathbf{x}$ and $A\mathbf{x} \leq \mathbf{b}$ respectively, where

$$\mathbf{c} = \begin{bmatrix} 1 \\ 1.3 \\ 1.4 \\ 1 \\ 1.2 \\ 1.6 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ -1 & 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ -1.3 & 0 & 0 & -1.2 & 0 & 0 \\ 1.3 & 0 & 0 & 1.2 & 0 & 0 \\ -1.4 & 0 & 0 & -1.6 & 0 & 0 \\ 1.4 & 0 & 0 & 1.6 & 0 & 0 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ 3 \\ -1.2 \\ 2 \\ -1.5 \\ 2 \\ -2 \\ 3 \end{bmatrix}$$

we could use R to solve for $\mathbf{x}$,

```r
c <- c(1,1.3,1.4,1,1.2,1.6)
A <- rbind(c(1,1,1,0,0,0),c(0,0,0,1,1,1),
           c(-1,0,0,-1,0,0),c(1,0,0,1,0,0),
           c(0,-1.3,0,0,-1.2,0),c(0,1.3,0,0,1.2,0),
           c(0,0,-1.4,0,0,-1.6),c(0,0,1.4,0,0,1.6))
dir <- rep('<=',8)
b <- c(2,3,-1.2,2,-1.5,2,-2,3)
s <- lp('max',c,A,dir,b)
print(s$solution)
```

```
## [1] 0.4615385 1.5384615 0.0000000 1.1250000 0.0000000 1.8750000
```

```r
print(s$objval)
```

```
## [1] 6.586538
```

Hence,

$$\mathbf{x} = \begin{bmatrix} 0.46 \\ 1.54 \\ 0 \\ 1.125 \\ 0 \\ 1.875 \end{bmatrix}$$