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 */

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1. Answer: Running time is $O(n)$

Explanation: There is one instruction in the first line that executes once. There is one instruction in the loop header that executes $(100n + 1) + 1$ times (including the last loop check). There is 1 instruction in the body of the loop that executes $100n + 1$ times. The total of the run times is:

$$\begin{aligned}
 &(1) + ((100n + 1) + 1) + (100n + 1) \\
 &= 1 + 2(100n + 1) + 1 \\
 &= 200n + 4 \\
 &= O(n)
 \end{aligned}$$

2. Answer: Running time is $O(n^3)$

Explanation: There is one instruction in the first line that executes once. There is one instruction in the loop header that executes $((n^3)/4 + 1)$ times (including the last loop check). There is 1 instruction in the body of the loop that executes $((n^3)/4)$ times. The total of the runtimes is:

$$\begin{aligned}
 &(1) + ((n^3)/4 + 1) + ((n^3)/4) \\
 &= 1 + 2((n^3)/4) + 1 \\
 &= (n^3)/2 + 4 \\
 &= O(n^3)
 \end{aligned}$$

3. Answer: Running time is $O(\log n)$

Explanation: There is one instruction in the first line that executes once. There is one instruction in the loop header that executes $(\log n + 1) + 1$ (base 2) times (including the last loop check). There is 1 instruction in the body of the loop that executes $\log n + 1$ times. The total of the runtimes is:

$$\begin{aligned}
 &(1) + (\log n + 1) + 1 + (\log n + 1) \\
 &= 1 + 2(\log n + 1) + 1 \\
 &= 2\log n + 4 \\
 &= O(\log n)
 \end{aligned}$$

4. Answer: Running time is $O(1)$

Explanation: There is one instruction in the first line that executes once. There is one instruction in the loop header that executes $100 + 1$ times (including the last loop check). There is 1 loop in the body of the first loop that executes $\log 1000 + 1$ times (including last check) instruction in the body of the loop that executes $\log 1000$ times. The total of the runtimes is:

$$\begin{aligned}
 &1 + (100 + 1) + (\log 1000 + 1) + \log 1000 \\
 &= 1 + 101 + 2\log 1000 + 1 \\
 &= 103 + 2\log 1000 \\
 &= O(1)
 \end{aligned}$$

5. Answer: Running time is $O(n^2)$

Explanation: There is one instruction in the first line that executes once. There is one instruction in the first loop header that executes $n + 1$ times (including the last loop check). There is 1 instruction in the second loop in the body of the first loop that executes $1 + 2 + 3 + 4 + \dots + n$ times with for each different loop the first loop loops. If $n = 10$, the second loop would have looped a total of $1 + 2 + \dots + 10 = 55$ times. The average times that the second loop loops each time would be $55/10$, which is $5.5 = (11/2) = (10+1)/2$. We can say that the runtime of the second loop is $O((n+1)/2)$, for each time the first loop loops. The body of the second for loop therefore also has run time $O((n+1)/2)$. Combining the runtimes of the two loops, we have:

$$\begin{aligned} & n((n + 1)/2) \\ &= (n^2 + n)/2 \\ &= O(n^2 + n) \\ &= O(n^2) \end{aligned}$$

6. Answer: Running time is $O(n \log n)$

Explanation: There is one instruction in the first line that executes once. There is one instruction in the loop header that executes $\log n + 1$ (base 2) times (including the last loop check). There is 1 instruction in the second loop in the body of the first loop that executes $(n/4 + 1) + 1$ times including the last loop check each time the outer loop loops. The body of the inner loop executes $(n/4 + 1)$ times each time the outer loop loops. The total of the runtimes is:

$$\begin{aligned} & O(1 + (\log n + 1)((n/4 + 1) + 1) + (\log n + 1)(n/4 + 1)) \\ &= O((\log n)(n/4) + (\log n)(n/4)) \\ &= O(2(\log n)(n)) \\ &= O(n \log n) \end{aligned}$$

7. Answer: Running time is $O(n^3)$

Explanation: There is one instruction in the first loop header that executes $2n + 1$ times (including the last loop check). There is 1 instruction in the second loop that executes $(n^2 + 1) + 1$ times (including last loop check). The loop in the second loop executes $(n + 1) + 1$ times (including last loop check). The runtime of the second loop and nested loop body is $O(((n^2 + 1) + 1)((n + 1) + 1))$. We can simplify this to $O((n^2)(n))$ or $O(n^3)$. Combining the runtimes with the first loop, we have $O((2n + 1) + n^3)$, which simplifies to $O(n^3)$.

8. Answer: Running time is $O(n^3)$

Explanation: There is one instruction in the first line that executes one time. There is an instruction in the first loop header that executes $n + 1$ times (including the last loop check). The loop in the first loop executes an average of $(n^2)/2$ times each time the outer loop loops. The runtime of the first loop and nested loop body is $O((n + 1)((n^2)/2))$. We can simplify this to $O((n)(n^2))$ or $O(n^3)$.

9. Answer: Running time is $O(n^2)$

Explanation: The first loop executes n times including the last check. The nested loop executes an average of $(n/2)$ times for each time the outer loop loops. The runtime for the first loop and its nested loop combined is then $O((n)(n/2))$, simplified to $O(n^2)$.

10. Answer: Running time is $O((\log n)^2)$

Explanation: The first loop executes $\log n$ times. The nested for loop executes $\log n$ times in the worst case where $i = n$. We can consider that the simplified runtime for the nested loop is $O(\log n)$. The total runtime would hence be $O((\log n)(\log n))$ or simplified to $O((\log n)^2)$.