Lab 3

Exponential Random Graph Models

What are ERGMs?

ERGMs are used to **model the probability of observing a specific network or graph structure**, taking into account various network characteristics.

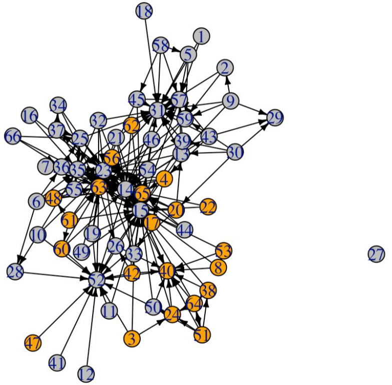
Agenda

- Overview of Lab 3
- Hypothesis Testing
- Model Convergence
- Goodness-of-Fit
- Q & A

Advice-Seeking Network

Dependent variable:

"List up to 5 employees who you rely on the most for help or advice at work."



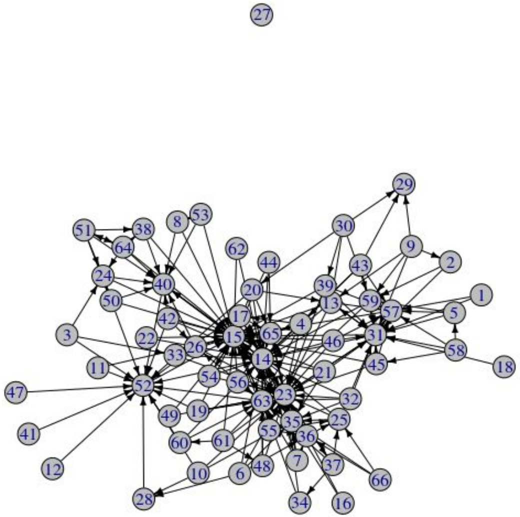
Predictor variables:

- Endogenous patterns (in advice seeking)
- Exogenous node attributes (e.g., office location)
- Exogenous ties – Messaging frequency over six weeks

Part I: Building and Visualizing the Networks

Part I Tips

- Plot the base network and include it in your report. Explain whether this plot supports or rejects Hypothesis 1 (Indegree popularity effects)? In this example, we show a “buy-in” network, rather than the advice network



A. This question is asking what you think about H1 based on the plot. That said, there is no right and wrong to support or reject H1. <u>Your reasoning is more important.</u>

B. Focus on <u>whether there are few employees receiving more ties than others</u> (that’s what indegree popularity effects are).

Part II: Model Estimation

Hypothesis Testing

- A **hypothesis**: a conjecture statement about how two or more variables are related based on "educated guess"

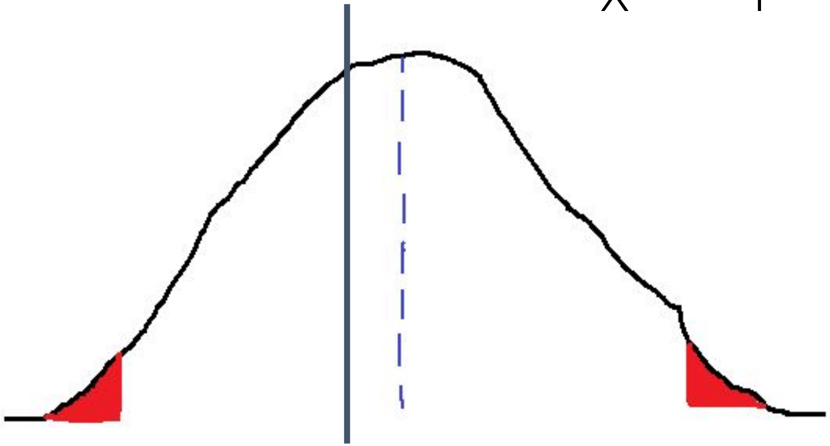
- H_A: Individuals are *less* likely to report buy-in ties from others than not to report them.
- → The probability of having a buy-in tie in the network is *lower* than 50% (random chance).
- → There is a *negative* relationship between the number of buy-in ties and the network.

$$Network \sim -\theta * Edges$$

Hypothesis Testing

H_0(null): X and Y are the same ($\mu_X = \mu_Y$) if $p \geq .05$

H_A(alternative): X and Y are different ($\mu_X \neq \mu_Y$) if $p < .05$



Fail to reject Null

$\mu_X = \mu_y$

μ: the mean of X or Y

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Hypothesis Testing

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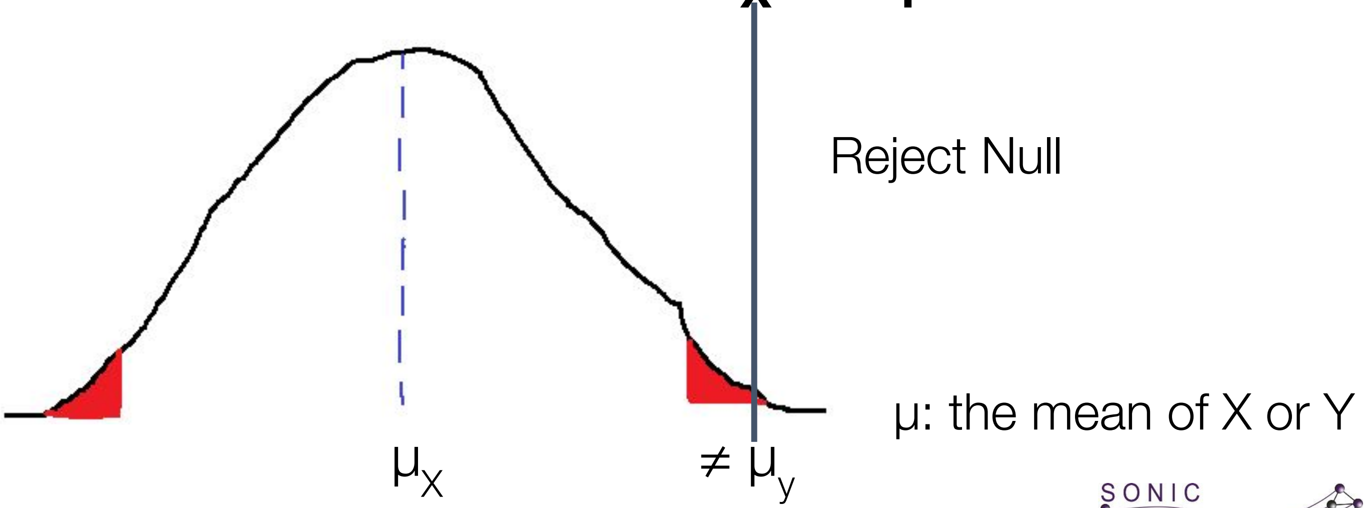

Hypothesis Testing

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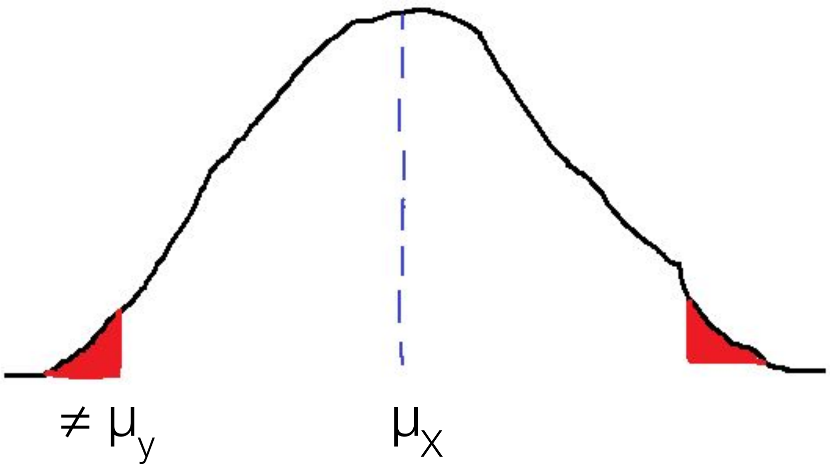


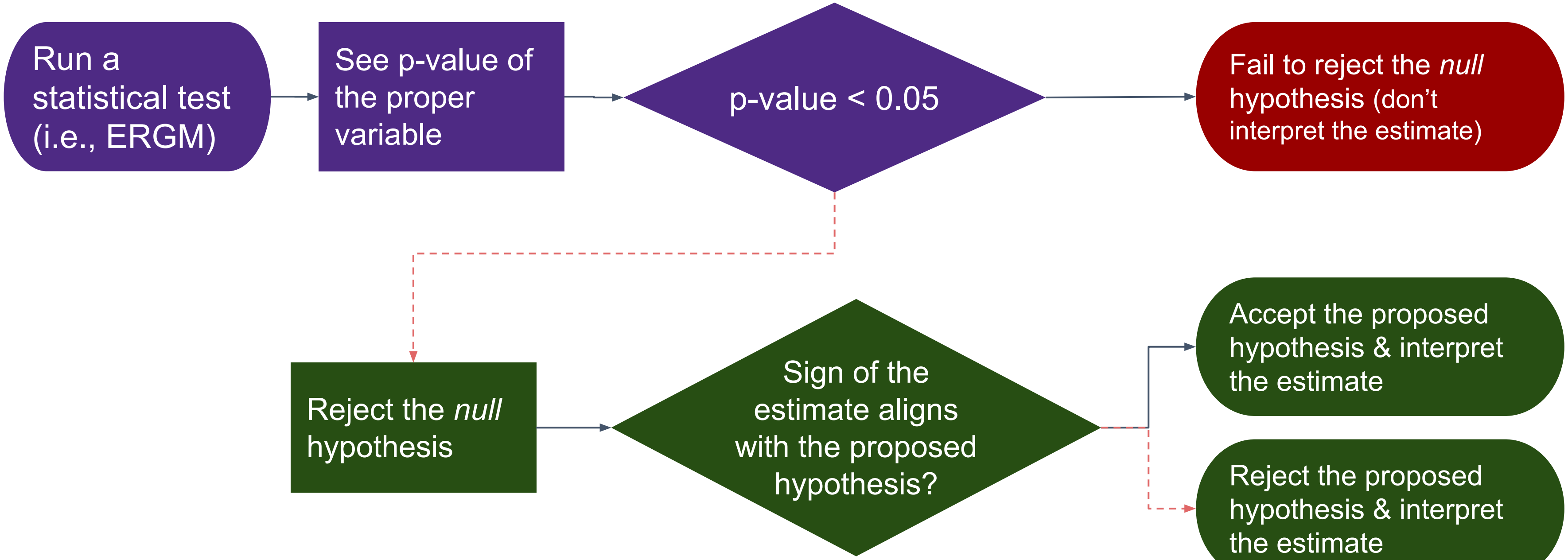
μ: the mean of X or Y

Testing Hypotheses

Yes

No

- H_A: Individuals are *less* likely to report buy-in ties from others than not to report them.



Testing Hypotheses

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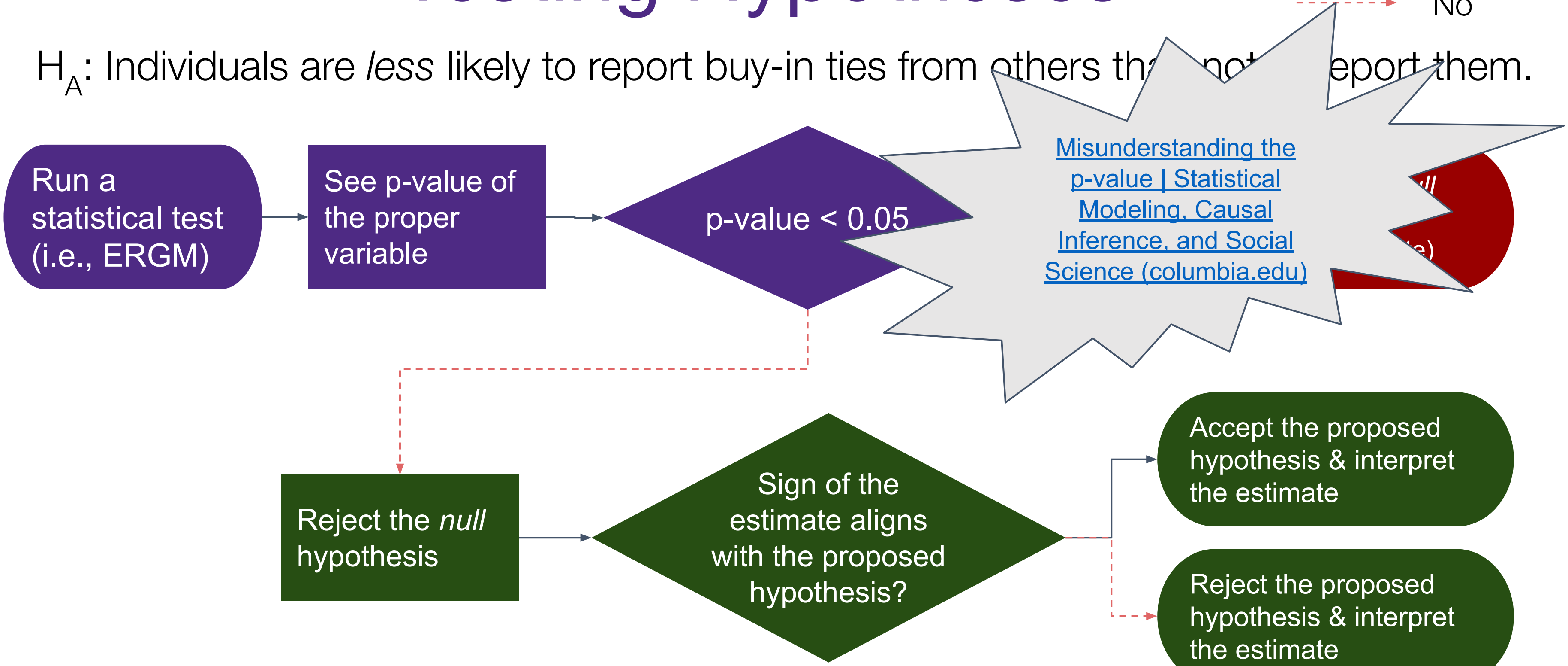


Example Model Results

```
> summary(model1)

==========================
Summary of model fit
==========================

Formula:   buyIn ~ edges + mutual + edgecov(hundreds_messages)

Iterations:  5 out of 20

Monte Carlo MLE Results:
                           Estimate Std. Error MCMC % z value Pr(>|z|)
edges                      -2.73196    0.10011      0 -27.288  < 1e-04 ***
mutual                      0.76882    0.31794      0   2.418 0.015601 *
edgecov.hundreds_messages   0.31061    0.08494      0   3.657 0.000255 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

     Null Deviance:    0.0  on 4290  degrees of freedom
 Residual Deviance: -389.5  on 4287  degrees of freedom

Note that the null model likelihood and deviance are defined to be 0. This means that
all likelihood-based inference (LRT, Analysis of Deviance, AIC, BIC, etc.) is only valid
between models with the same reference distribution and constraints.

AIC: -383.5    BIC: -364.4    (Smaller is better.)
```

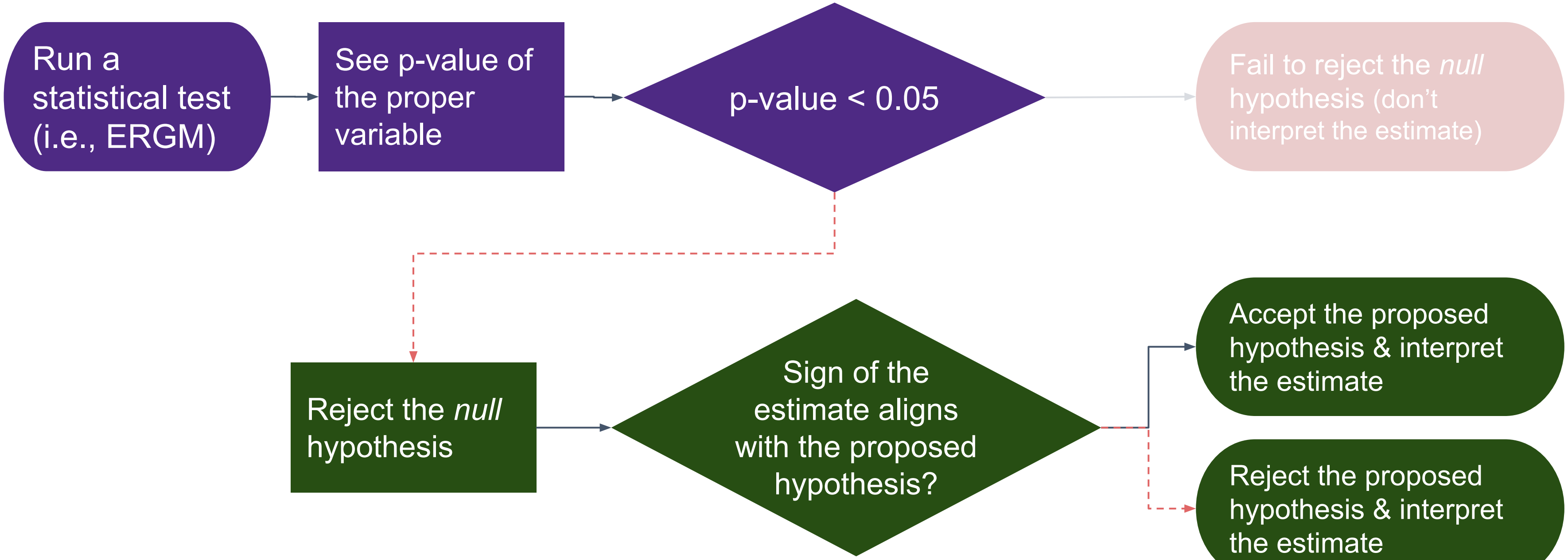
***p* < .05 (meaning the estimate is not 0 (θ ≠ 0))**

Testing Hypotheses

Yes

No

- H_A: Individuals are *less* likely to report buy-in ties from others than not to report them.



Testing Hypothesis A

```
> summary(model1)

==========================
Summary of model fit
==========================

Formula:   buyIn ~ edges + mutual + edgecov(hundreds_messages)

Iterations:  5 out of 20

Monte Carlo MLE Results:
                           Estimate Std. Error MCMC % z value Pr(>|z|)    
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AIC: -383.5    BIC: -364.4    (Smaller is better.)
```

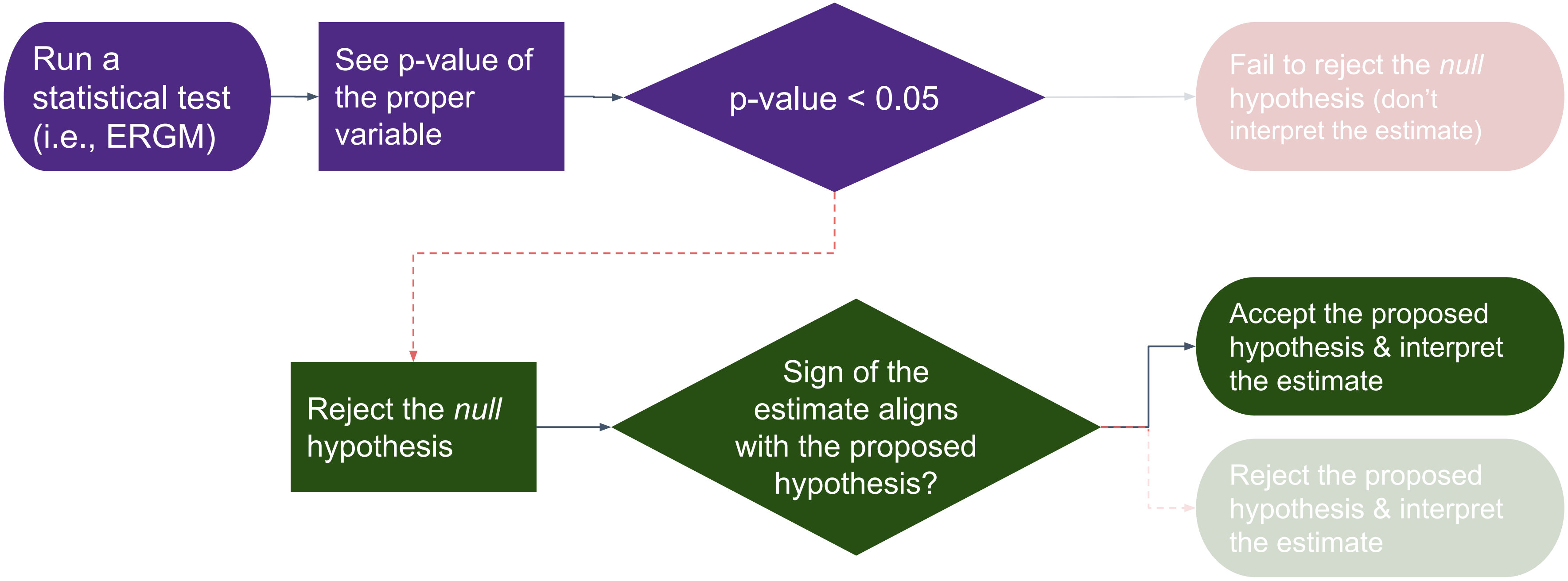
Negative → support the hypothesis

Testing Hypotheses

Yes

No

- H_A: Individuals are *less* likely to report buy-in ties from others than not to report them.



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Interpret the Estimate of Edges

Run `exp(-2.73196)`

→ 0.06509159

- The estimate (**θ**): Conditional log-odds ratio

$$Network \sim -2.73196 * Edges$$

- Exponential: **odds ratio (if an odds ratio is 1, odds are even)**
  - The odds of individuals reporting buy-in ties are 0.07 times *lower* than the odds of individuals *not* reporting them

Interpret the Estimate of Edges

Run `exp(-2.73196)/(1+exp(-2.73196))`

→ 0.0611136

- The estimate (**θ**): Conditional log-odds ratio

$$Network \sim -2.73196 * Edges$$

- Inverse logit (`plogis`): **probability (random chance is 50%)**
  - The probability of having a buy-in tie in the network is <u>6%</u>.

One More Example

```
Formula:   buyIn ~ edges + mutual + gwidegree(1.06, fixed = T) + gwodegree(log(2),
    fixed = T) + dgwesp(log(2), type = "OTP", fixed = T) + dgwdsp(log(2),
    type = "RTP", fixed = T) + nodematch("female") + nodemix("leader",
    base = 3) + nodematch("department") + nodeicov("office") +
    nodeocov("office") + diff("tenure") + edgecov(hundreds_messages)

Iterations:  5 out of 20

Monte Carlo MLE Results:
                                  Estimate Std. Error MCMC % z value Pr(>|z|)
edges                             -3.24101    0.40986      0  -7.908  < 1e-04 ***
mutual                             0.79745    0.67111      0   1.188  0.23474
gwideg.fixed.1.06                 -2.25771    0.35076      0  -6.437  < 1e-04 ***
gwodeg.fixed.0.693147180559945     0.25273    0.64754      0   0.390  0.69632
gwesp.OTP.fixed.0.693147180559945  0.92603    0.14131      0   6.553  < 1e-04 ***
gwdsp.RTP.fixed.0.693147180559945 -1.37944    0.60425      0  -2.283  0.02244 *
nodematch.female                   0.19908    0.15834      0   1.257  0.20865
mix.leader.0.0                    -0.66742    0.21215      0  -3.146  0.00166 **
mix.leader.1.0                    -1.42147    0.61474      0  -2.312  0.02076 *
mix.leader.1.1                    -0.50916    0.63961      0  -0.796  0.42601
nodematch.department               2.02933    0.17996      0  11.276  < 1e-04 ***
nodeicov.office                   -0.30094    0.14439      0  -2.084  0.03714 *
nodeocov.office                    0.20150    0.21665      0   0.930  0.35233
diff.t-h.tenure                   -0.13790    0.02374      0  -5.809  < 1e-04 ***
edgecov.hundreds_messages          0.39560    0.09560      0   4.138  < 1e-04 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

One More Example

- H1: There will be *indegree popularity effects* (tendency of a small number of nodes to receive many ties) in who people report is capable of getting buy in from them.

Geometrically weighted indegree measures a tendency *<u>against</u>* indegree preferential attachment

```
Monte Carlo MLE Results:
                                    Estimate Std. Error MCMC % z value Pr(>|z|)
edges                               -3.24101    0.40986      0  -7.908  < 1e-04 ***
mutual                               0.79745    0.67111      0   1.188  0.23474
gwideg.fixed.1.06                   -2.25771    0.35076      0  -6.437  < 1e-04 ***
gwodeg.fixed.0.693147180559945       0.25273    0.64754      0   0.390  0.69632
gwesp.OTP.fixed.0.693147180559945    0.92603    0.14131      0   6.553  < 1e-04 ***
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---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Interpret the Estimate of H1

Run `exp(-2.25771)/(1+exp(-2.25771))`

→ 0.0946865

Or run `exp(2.25771)/(1+exp(2.25771))`

→ 0.9053135

- Inverse logit (`plogis`): **probability**
  - The probability of those who are *not* popular receiving a tie in the network is 9%.
  - The probability of those who are popular receiving a tie in the network is 91%.

Part III: Model Diagnostics

Part III: Model Diagnostics

- **Model convergence:** examining whether the estimated process has converged or not

- **Goodness-of-fit test:** examining whether your model estimates represent your data

What are ERGMs?

ERGMs are used to **model the probability of observing a specific network or graph structure**, taking into account various network characteristics.

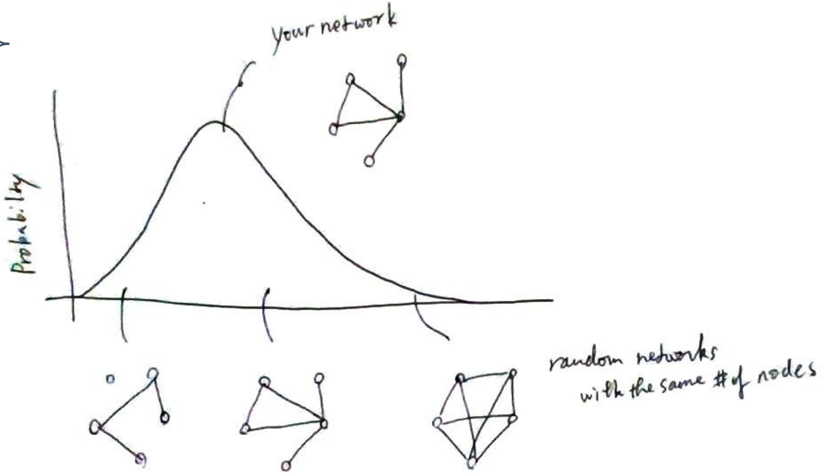
Model Convergence

- When you run ergm(buyIn ~ edges +...) in R

```
The log-likelihood improved by 0.006663.
Starting Monte Carlo maximum likelihood estimation (MCMLE):
Iteration 1 of at most 20:
Optimizing with step length 0.201157773508558.
The log-likelihood improved by 3.067.
Iteration 2 of at most 20:
Optimizing with step length 0.234619875876688.
The log-likelihood improved by 3.125.
Iteration 3 of at most 20:
Optimizing with step length 0.451674345548064.
The log-likelihood improved by 3.155.
Iteration 4 of at most 20:
Optimizing with step length 1.
The log-likelihood improved by 3.055.
Step length converged once. Increasing MCMC sample size.
Iteration 5 of at most 20:
NOTE: Messages 'Error in mcexit(0L)...' may appear; please disregard them.
Optimizing with step length 1.
The log-likelihood improved by 1.101.
Step length converged twice. Stopping.
Finished MCMLE.
Note: The constraint on the sample space is not dyad-independent. Null model likelihood
is only implemented for dyad-independent constraints at this time. Number of
observations is similarly poorly defined.  This means that all likelihood-based
inference (LRT, Analysis of Deviance, AIC, BIC, etc.) is only valid between models with
the same reference distribution and constraints.
Evaluating log-likelihood at the estimate. Using 20 bridges: 1 2 3 4 5 6 7 8 9 10 11 12 13 14
18 19 20 .
This model was fit using MCMC.  To examine model diagnostics and check for degeneracy,
use the mcmc.diagnostics() function.
```

MCMC-MLE

- Markov Chain: because it simulates network Yt+1 randomly based on Yt
- Monte Carlo: because of the computational implementation of the "randomly" generated part
- Maximum Likelihood Estimation



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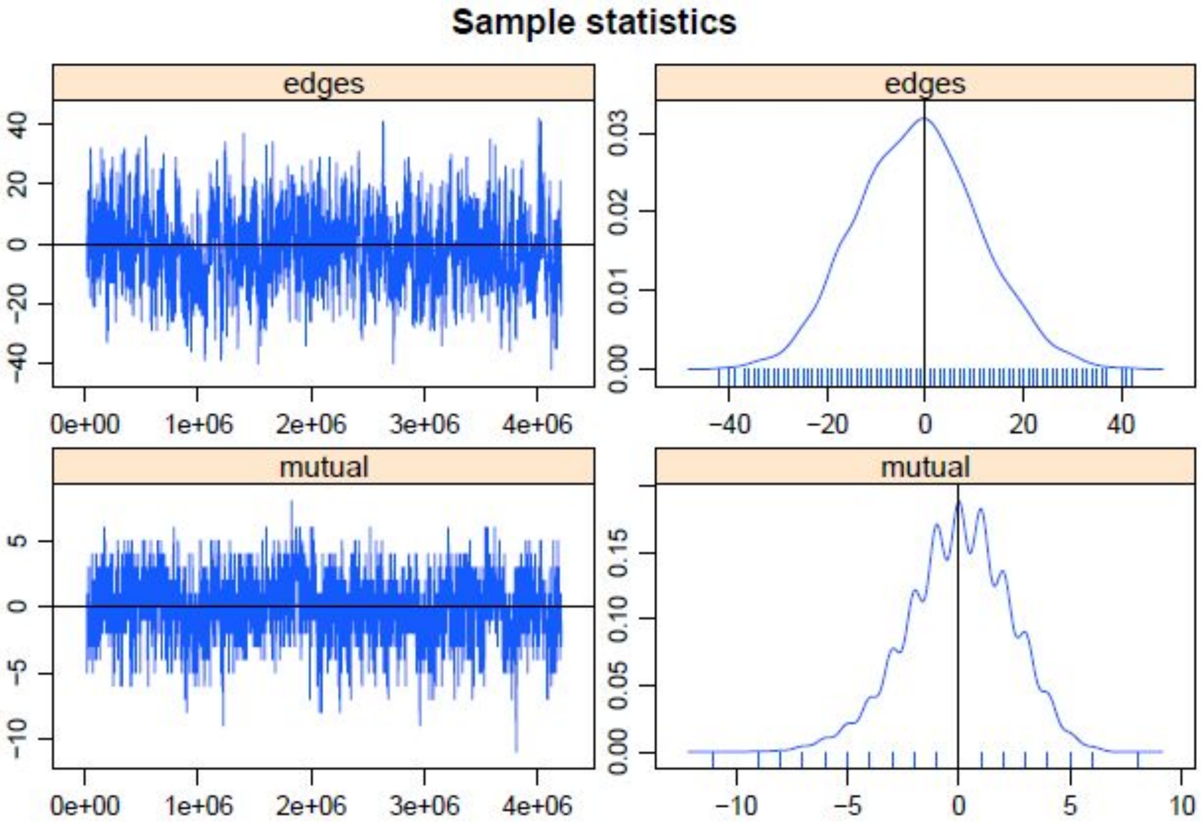
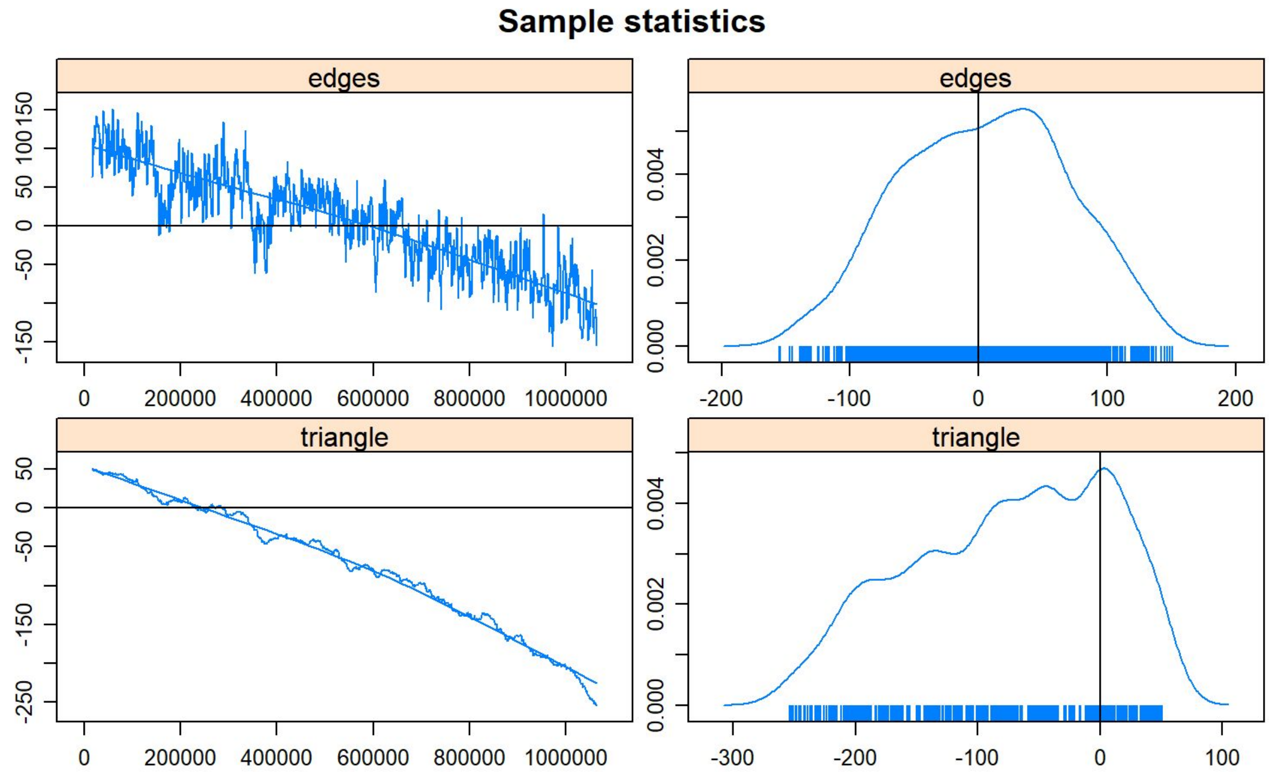
Producing a PDF file

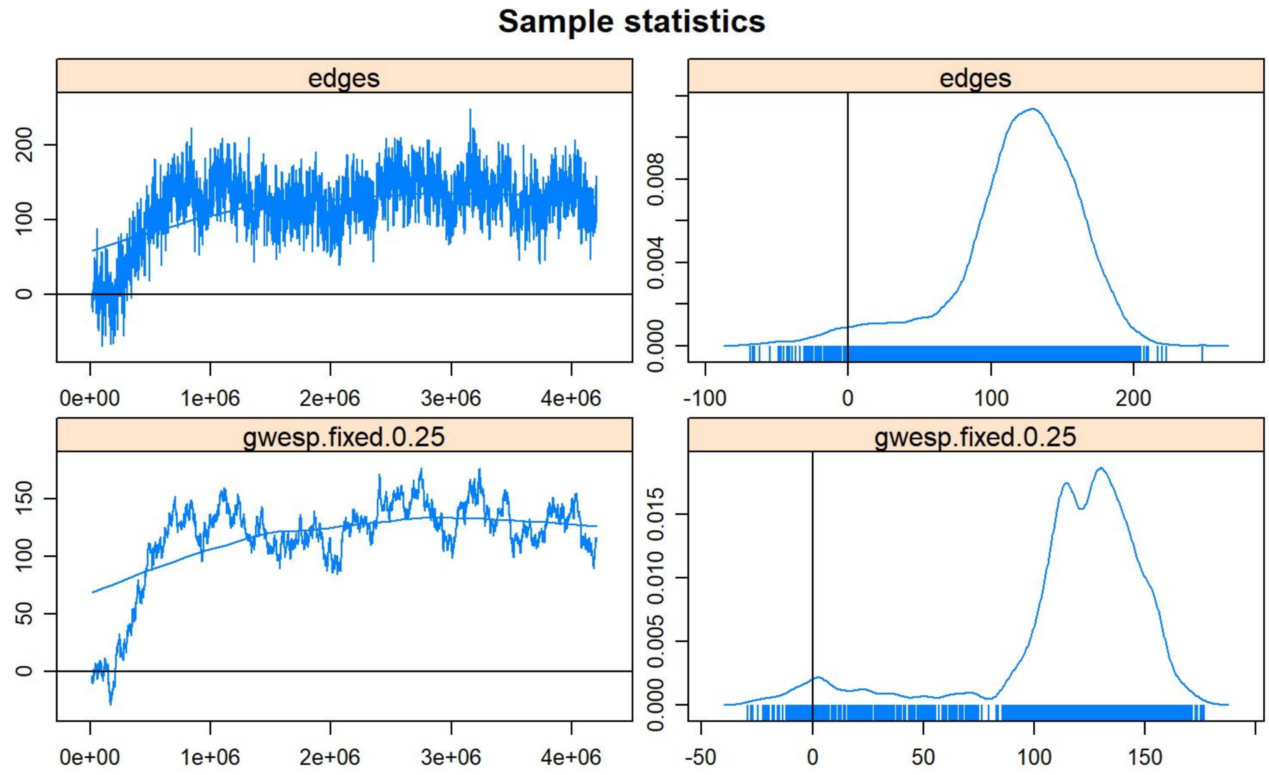


Bad Cases



http://statnet.org/Workshops/ergm_tutorial.html#7_diagnostics:_troubleshooting_and_checking_for_model_degeneracy

Bad Cases



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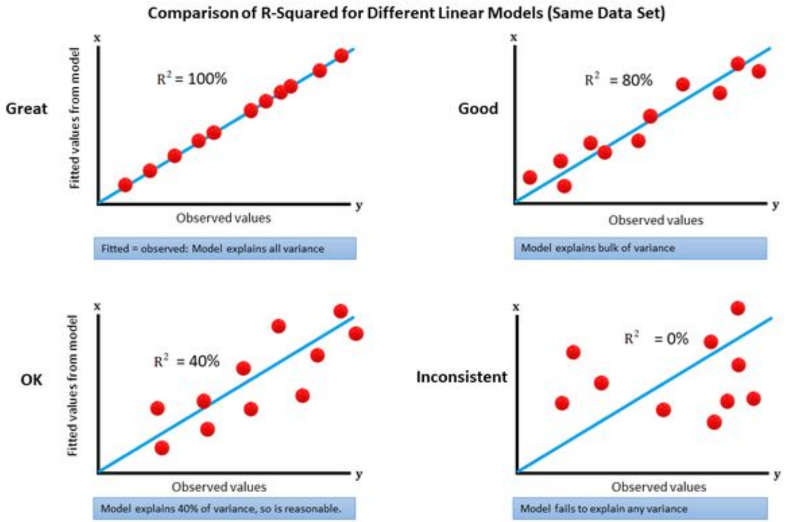
Goodness-of-fit Test

- **Does my model (i.e., the results of ERGM) fit well with my network data?**
- Do not think of a goodness-of-fit test as R-squared in regression



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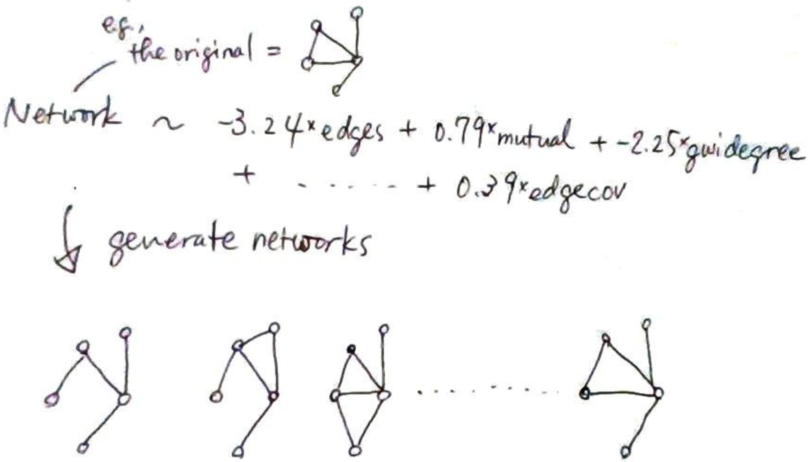
Goodness-of-fit Test

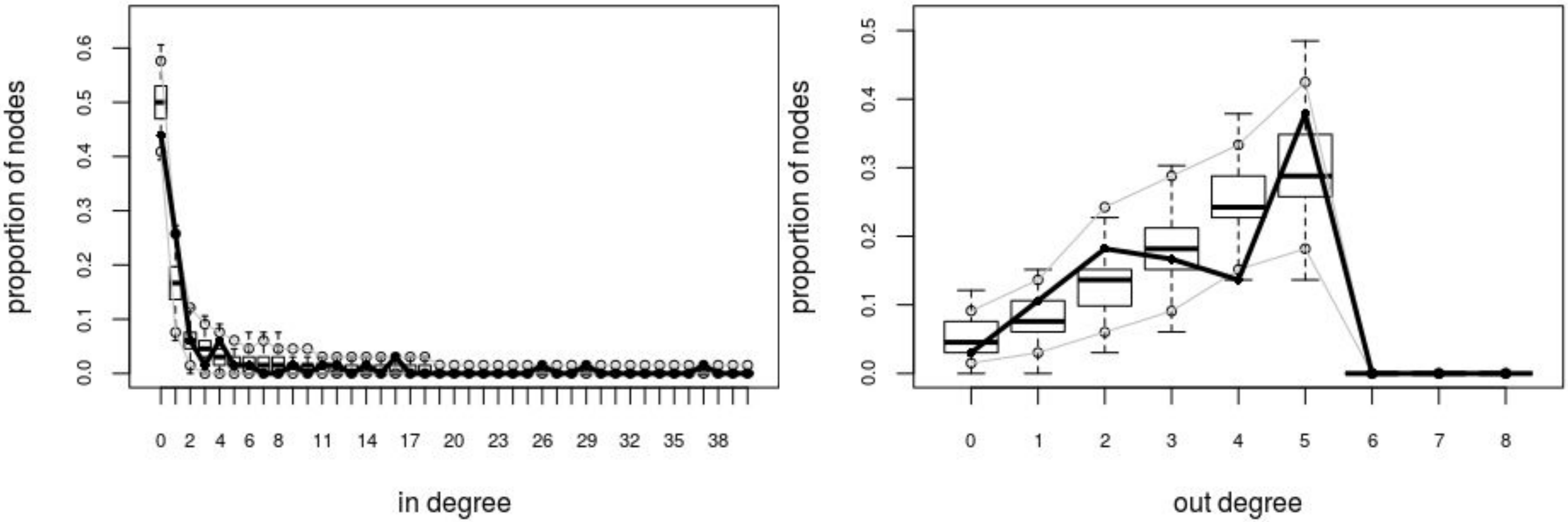
If you run gof(model1 ~ …)

```
> gof <- gof(model ~ idegree + odegree + espartners + distance, verbose=T, burnin=1e+5, interval=1e+5,
control = control.gof.ergm(nsim = 200))
Starting GOF for the given ERGM formula.
Starting GOF for the given ERGM formula.
Calculating observed network statistics.
Starting simulations.
Sim 1 of 200: Starting MCMC iterations to generate  1  network
Finished simulation 1 of 1.
Sim 2 of 200: Starting MCMC iterations to generate  1  network
Finished simulation 1 of 1.
Sim 3 of 200: Starting MCMC iterations to generate  1  network
Finished simulation 1 of 1.
Sim 4 of 200: Starting MCMC iterations to generate  1  network
Finished simulation 1 of 1.
Sim 5 of 200: Starting MCMC iterations to generate  1  network
Finished simulation 1 of 1.
Sim 6 of 200: Starting MCMC iterations to generate  1  network
Finished simulation 1 of 1.
Sim 7 of 200: Starting MCMC iterations to generate  1  network
Finished simulation 1 of 1.
Sim 8 of 200: Starting MCMC iterations to generate  1  network
Finished simulation 1 of 1.
Sim 9 of 200: Starting MCMC iterations to generate  1  network
Finished simulation 1 of 1.
Sim 10 of 200: Starting MCMC iterations to generate  1  network
Finished simulation 1 of 1.
Sim 11 of 200: Starting MCMC iterations to generate  1  network
Finished simulation 1 of 1.
Sim 12 of 200: Starting MCMC iterations to generate  1  network
```



Produce Plots in Your Plots Panel

If you run plot(gof)



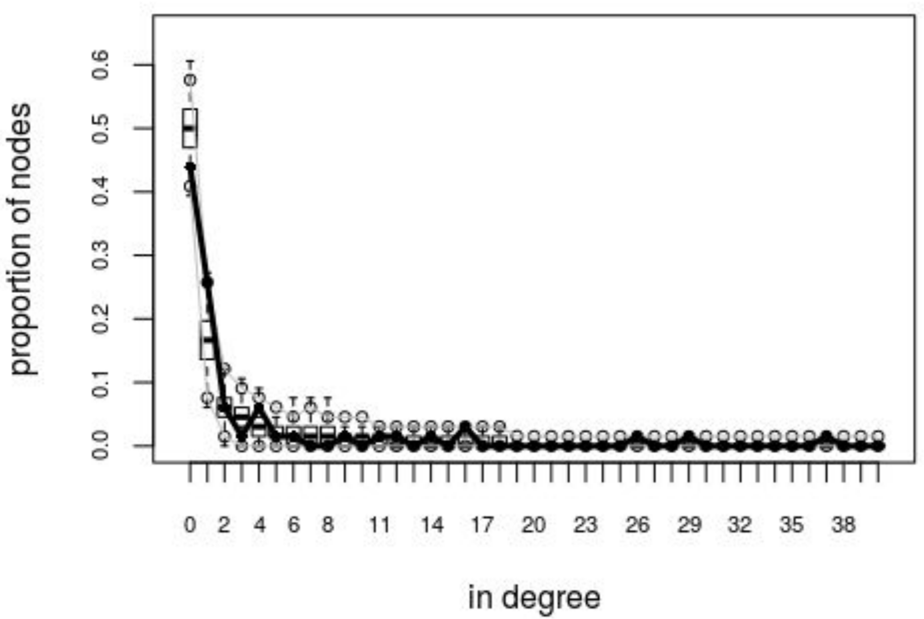
Interpret the Output of GOF

If you run gof

Goodness-of-fit for in-degree

	obs	min	mean	max	MC p-value
0	29	23	32.905	43	0.27
1	17	4	10.685	20	0.08
2	4	0	4.100	10	1.00
3	1	0	2.765	8	0.45
4	4	0	1.875	6	0.21
5	1	0	1.640	6	1.00
6	1	0	1.255	5	1.00
7	0	0	1.335	5	0.59
8	0	0	1.050	5	0.64
9	1	0	0.930	5	1.00
10	0	0	0.745	4	0.92
11	1	0	0.735	3	1.00
12	1	0	0.570	4	0.90
13	0	0	0.435	3	1.00
14	1	0	0.395	3	0.69
15	0	0	0.360	3	1.00
16	2	0	0.375	3	0.09
17	0	0	0.380	2	1.00
18	0	0	0.295	3	1.00
19	0	0	0.260	2	1.00
20	0	0	0.185	2	1.00
21	0	0	0.200	2	1.00
22	0	0	0.165	2	1.00
23	0	0	0.180	2	1.00
24	0	0	0.140	1	1.00

=



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Interpret the Output of GOF

If you run gof

```
Goodness-of-fit for in-degree

   obs min   mean max MC p-value
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1   17   4 10.685  20       0.08
2    4   0  4.100  10       1.00
3    1   0  2.765   8       0.45
4    4   0  1.875   6       0.21
5    1   0  1.640   6       1.00
6    1   0  1.255   5       1.00
7    0   0  1.335   5       0.59
8    0   0  1.050   5       0.64
9    1   0  0.930   5       1.00
10   0   0  0.745   4       0.92
11   1   0  0.735   3       1.00
12   1   0  0.570   4       0.90
13   0   0  0.435   3       1.00
14   1   0  0.395   3       0.69
15   0   0  0.360   3       1.00
16   2   0  0.375   3       0.09
17   0   0  0.380   2       1.00
18   0   0  0.295   3       1.00
19   0   0  0.260   2       1.00
20   0   0  0.185   2       1.00
21   0   0  0.200   2       1.00
22   0   0  0.165   2       1.00
23   0   0  0.180   2       1.00
24   0   0  0.140   1       1.00
```

Actual count from the original network

Counts from the simulated network based on the model (i.e., the results of ERGM)

p-value

Interpret the Output of GOF

If you run gof

Goodness-of-fit for in-degree

	obs	min	mean	max	MC p-value
0	29	23	32.905	43	0.27
1	17	4	10.685	20	0.08
2	4	0	4.100	10	1.00
3	1	0	2.765	8	0.45
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6	1	0	1.255	5	1.00
7	0	0	1.335	5	0.59
8	0	0	1.050	5	0.64
9	1	0	0.930	5	1.00
10	0	0	0.745	4	0.92
11	1	0	0.735	3	1.00
12	1	0	0.570	4	0.90
13	0	0	0.435	3	1.00
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16	2	0	0.375	3	0.09
17	0	0	0.380	2	1.00
18	0	0	0.295	3	1.00
19	0	0	0.260	2	1.00
20	0	0	0.185	2	1.00
21	0	0	0.200	2	1.00
22	0	0	0.165	2	1.00
23	0	0	0.180	2	1.00
24	0	0	0.140	1	1.00

If p > .05, it indicates that the model fits well with the original network in terms of the in-degree distribution

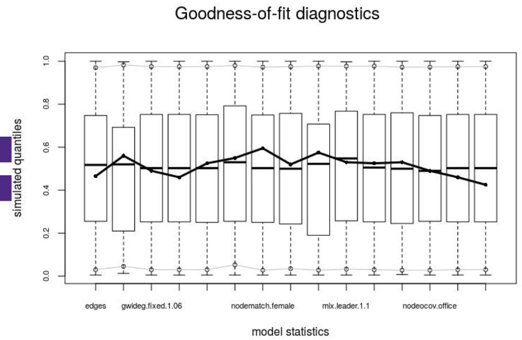
Interpret the Output of GOF

If you run gof

Goodness-of-fit for model statistics

	obs	min	mean	max	MC p-value
edges	225.00000	192.00000	223.74500	255.00000	0.93
mutual	12.00000	7.00000	11.89000	18.00000	1.00
gwideg.fixed.1.06	66.19405	51.63342	65.82389	81.20335	0.98
gwodeg.fixed.0.693147180559945	109.56250	99.50000	108.90937	116.68750	0.92
gwesp.OTP.fixed.0.693147180559945	214.12500	161.37500	214.08812	266.75000	0.95
gwdsp.RTP.fixed.0.693147180559945	5.00000	0.00000	5.12625	14.00000	1.00
nodematch.female	162.00000	132.00000	163.50500	197.00000	0.96
mix.leader.0.0	94.00000	62.00000	93.81500	132.00000	1.00
mix.leader.1.0	8.00000	3.00000	8.00000	17.00000	1.00
mix.leader.1.1	12.00000	5.00000	11.85500	18.00000	1.00
nodematch.department	112.00000	87.00000	111.38500	142.00000	1.00
nodeicov.office	182.00000	155.00000	182.89000	222.00000	1.00
nodeocov.office	173.00000	152.00000	172.70000	193.00000	0.98
diff.t-h.tenure	-737.25753	-938.29315	-750.30788	-594.08219	0.92
edgecov.hundreds_messages	56.69000	25.12000	53.87505	86.44000	0.85

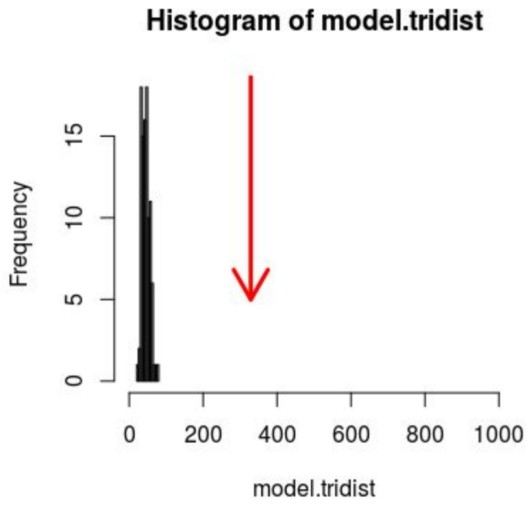
=



If p > .05, it indicates that the model fits well with the original network in terms of the variables

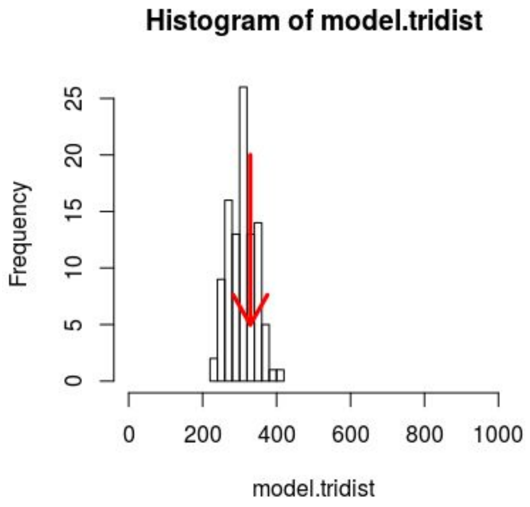
GOF for Triangles

Model 1



t-statistic = 26.3656
→ p < .001

Model 2



t-statistic = 0.6205
→ p > .50

If p > .05, it indicates that the model fits well with the original network in terms of the triangles

```
2*pt(t, df=100, lower.tail = FALSE)
```

Interested in learning more about ERGMs?

Watch this supplemental lecture providing an overview of the ERGM estimation procedures and algorithms!

The slide deck is also posted on Canvas for your reference.

https://bit.ly/ERGM_Explanations

Q & A

ERGM Terms

- This is not for Lab 2, but if you use ERGMs for your final project, please take a look at this page: https://cran.r-project.org/web/packages/ergm/vignettes/ergm-term-crossRef.html

Basic / Frequently-used term category matrix

For convenience, this table lists a subset of the most commonly-used ergm terms and categories.

Term name	binary	valued	directed	undirected	bipartite	dyad-independent
absdiff	✓		✓	✓		✓
b1cov	✓			✓	✓	✓
b1cov		✓		✓	✓	✓
b1degree	✓			✓	✓	
b1factor	✓			✓	✓	✓
b1factor		✓		✓	✓	✓
b1nodematch	✓			✓	✓	✓
b2concurrent	✓			✓	✓	