Search Frictions and International Trade Intermediation*

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Abstract

This paper studies the interaction of matching frictions and intermediation in international trade in a general equilibrium model. Heterogeneous firms both decide whether to export and, if so, whether to use an intermediary or export directly. Matching frictions force direct exporting firms to invest in market penetration to reach foreign consumers, but intermediated firms face lower consumer demand. This trade-off generates productivity sorting in the export mode: the most productive firms export directly, the least productive do not export and the moderately productive firms use an intermediary. Second, I show how the interaction of search frictions and intermediation can explain several well-established empirical regularities of trade networks. The main contribution consists of novel predictions on the response of intermediaries to trade liberalization. I show that intermediaries increase the number of products they export while simultaneously reducing the product diversity.

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I. Introduction

Informal trade barriers, or barriers beyond policy and transport costs, have become an increasingly important explanation for international trade costs (Rauch, 1999). One salient example is matching frictions, which arise when economic agents do not observe all potential buyers or if they have incomplete information on these partners (Chade et al., 2017). Understanding matching frictions in trade is important since their existence is detrimental to welfare (Arkolakis, 2010; Eaton et al., 2016). Also, they affect firms heterogeneously: larger and more productive firms are better able to independently mitigate such frictions (Allen, 2014). Of interest is then how smaller firms can operate in this frictional trade environment. One potential mechanism, as formalized in the rich

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microeconomic literature on matching frictions, is intermediation. An intermediary is broadly defined as a third party able to facilitate trade beyond the capabilities of buyers and sellers (e.g. Rubinstein and Wolinsky (1987)). The prevalence of intermediaries operating internationally – intermediaries represent 10 to 35 percent of the total export value of a country[1] – is certainly suggestive of this being relevant in international trade. However, the interaction between matching frictions and intermediaries in international trade has been studied in only a few papers.

The contribution of my paper is to study this interaction in a tractable general equilibrium trade model. I assume, following Arkolakis (2010), the trading environment is characterized by search frictions, which means that firms cannot readily observe potential consumers and vice versa. To acquire consumers, firms need to carry out informative advertising as in Grossman and Shapiro (1984). Firms both decide whether and how much to advertise in a destination market: the higher the investment in advertising, the larger the share of consumers reached. Profit maximization culminates in every firm deciding on both the optimal price and the optimal level of advertising, or market penetration, for each destination. It turns out that for the least productive firms, the additional costs of advertising always exceed the benefits, such that these firms will not export. This result is analogous to the export threshold found in Arkolakis (2010).

This threshold equilibrium is altered when intermediaries are introduced. In my model, intermediaries face the same frictions when they set up a distribution network but they profit from economies of scope as they, unlike firms, can process multiple goods through the same distribution channel.[2] In order to cover the exporting cost, intermediaries impose a mark-up on the procurement price of the varieties they handle. This causes a trade-off for the firm engaging in indirect exporting: while the firm does not have to invest in a distribution network to sell its goods abroad, it forgoes demand due to the higher consumer price. Because the lost profits increase faster in productivity than the savings in distribution cost, the most productive firms still export directly. However, the least productive direct exporting firms switch to using an intermediary, as does a subset of domestic firms. Thus, my model generates productivity sorting across export *strategies*. Note that this model does not capture all export strategies, most notably FDI. However, since this is an export strategy that is opted for only by the most productive firms, I feel comfortable in abstracting from it in this paper.[3]

This paper contributes to the literature in two ways. First, my framework, despite its simplicity, captures all of the important stylized facts on the role of intemediaries in international trade. First, export sales per variety are lower for intermediaries than for firms because they charge an additional mark up (Feenstra and Hanson, 2004; Ahn et al., 2011). Second, for markets where the cost of exporting is relatively large, the share of intermediaries in trade will be larger (Bernard et al., 2010). In such markets, the comparative advantage of intermediaries in exporting is more pronounced, thereby increasing the demand for intermediation. Third, the level of trade intermediation is higher if the goods are more homogeneous (Bernard et al., 2015). In that case, the intermediary charges lower additional mark-ups such that exporting indirectly is more attractive.

[1] The exact fraction varies across countries; see for example Bernard et al. (2010), Blum et al. (2010) or Ahn et al. (2011).

[2] The assumption that intermediaries profit from economies of scope above and beyond manufacturing firms is consistent with empirical evidence. Intermediaries export more products than manufacturing firms even when controlling for firm size (Akerman, 2018). More suggestive is evidence on the behaviour of intermediaries. For example, while (multi-product) firms generally specialize in certain products, intermediaries specialize in countries, handling products that span unrelated sectors (Ahn et al., 2011). Finally, the degree of intermediation is higher for countries that have high trade costs, suggesting that intermediaries can facilitate trade to more difficult to reach markets by pooling resources (Bernard et al., 2010).

[3] See for example the seminal work by Grossman and Helpman (2002) or Krautheim (2013).

Fourth, firms can employ different strategies for different destination markets: exporting directly to one destination and indirectly to another are not mutually exclusive since trade barriers – formal and informal – vary across destination markets (Abel-Koch, 2013). Finally, my model can reconcile empirical regularities of trade networks with the presence of intermediation, since the resulting trade network exhibits negative assortative matching both in terms of number of trade partners and firm size (see e.g. Bernard and Moxnes (2018) or Blum et al. (2010)).

My second contribution is a novel prediction on the effect of trade liberalization in an environment with intermediation. In my model, in contrast to canonical models such as Melitz (2003) and Arkolakis (2010), the export strategy chosen by firms is influenced by the trading environment. This directly implies that changes in the trading environment also affect a firm's export strategy. I show this when I compute the partial elasticity of a firm's sales with respect to trade costs for both direct and indirect exporters. As already established by Arkolakis (2010), for a direct exporter, this elasticity can be decomposed into two effects: the intensive margin, showing how individual consumer demand reacts to this trade cost change, and the new consumers margin, which captures the change in the optimal market penetration. Intuitively, a decrease in trade costs will not only increase the amount sold per foreign consumer by reducing the consumer price, but also increase the amount of foreign consumers reached as the marginal revenue of advertising increases.

The corresponding elasticity for an indirect exporter is smaller in absolute terms, which means that the sales of an intermediated firm are less responsive to changes in trade costs. This is because the changed environment affects the export strategy of firms, and thereby the set of firms that are served by an intermediary. The intermediary adjusts its marketing strategy to maximize profits, taking the new composition of the intermediary's portfolio into account. I refer to this as the *composition effect*. The notion that the change in sales in response to a trade cost shock is less pronounced for an intermediated firm than for an exporting firm, which is broadly in line with Bernard et al. (2015) who show this in the context of a currency shock. To better understand the impact of this portfolio adjustment, I decompose the composition effect into measurable statistics that describe why the average productivity of the intermediary's portfolio changes. I find that trade liberalization reduces the diversity of intermediated products but increases the number of intermediated firms. Thus, the intermediary's portfolio contains more firms but fewer different types of firms.

My paper is not the first to study the role of intermediaries in international trade. The existing theoretical literature generally assigns a transactional benefit to intermediaries: either they are the only avenue through which firms can access the goods market as in Antràs and Costinot (2011) or Fernandez-Blanco (2012), or they profit from economies of scope as in Ahn et al. (2011) and Akerman (2018). The transaction cost mechanism, provided that the (opportunity) costs of intermediation increase in productivity, is sufficient to generate the export sorting patterns we see in the data. The implication that firms exporting via an intermediary are indeed less productive than those exporting directly has also been confirmed empirically (Bernard et al., 2010; Abel-Koch, 2013). This paper goes beyond this by reconciling stylized facts on trade networks with the presence of intermediation, showing how intermediaries are a key element in their formation. For example, in Akerman (2018) and related papers, there is a bimodal distribution of consumers over firm size. Specifically, an exporting firm either has one customer in a market (the intermediary) or serves the full market. However, several empirical studies of trade networks (see e.g. Chaney (2014); Eaton et al. (2016)) have shown that trade connections are characterized by imperfect matching. Other papers document a positive correlation between firm size and the number of foreign connections, with only a few very large firms serving many consumers (Blum et al., 2010).

This implies that larger firms are more capable in mitigating the effects of these frictions. Through the introduction of endogenous market-specific costs of export, my model produces this positive correspondence between firm productivity and the number of foreign consumers; more productive firms can afford to advertise more. Thus, the interaction between intermediation and matching frictions generates additional empirically important predictions beyond the models based on a transaction cost motive.

Other papers have explored the consequences of international trade intermediation in settings with matching frictions, many of which in partial equilibrium.[4] My model is most closely related to Blum et al. (2009), who analyse a heterogeneous-firm model with costly consumer matching where firms choose a distribution technology endogenously. A crucial difference with my approach is the modelling of matching frictions. Blum et al. (2009) assume that the cost of matching with a customer follows a step function, assigning a low matching cost to large firms and a large cost to small firms. In my model, the matching technology is identical across all firms. Nevertheless, my model produces the positive correlation between productivity and number of consumers as larger, more profitable, firms endogenously advertise more. Moreover, the assumption on matching costs generates a counterintuitive prediction in Blum et al. (2009): intermediaries are less important when goods are more homogeneous.[5] Instead, in my model the marginal revenue for an intermediated firm is higher for more homogeneous goods. Therefore, more homogeneous sectors will be associated with higher levels of intermediation.

In the next section, both the baseline model without intermediaries and the setting including intermediaries will be presented. I define the equilibrium and solve for the thresholds explicitly for a simple case. Section IV addresses the qualitative and quantitative testable implications following from the generalized model. Section V contains the empirical analysis. Finally, section VI will provide a conclusion and suggestions for future research.

II. Model

The model builds on the paper of Arkolakis (2010), which integrates heterogeneous firm productivity and endogenous costs of exporting in a framework of monopolistic competition. Before introducing the full-fledged model with intermediaries, I will briefly layout the baseline model.

i. *Search frictions and heterogeneous firms*

Consumer problem. There are N possibly asymmetric countries where each country has a measure of identical inhabitants equal to L_i. In what follows, I will refer to the source country as i and the destination country as j. Each country has two sectors: an agricultural sector and a manufacturing sector. The agricultural sector produces a freely traded homogeneous good H with labour as the only production factor. Production of the homogeneous good in country i yields w_i units for each unit of labour. Setting the homogeneous good as the numeraire, the wage in country i is equal to w_i. The manufacturing sector is characterized by a continuum of potential varieties indexed by ω, produced using a constant returns to scale technology.

[4] Examples include Dasgupta and Mondria (2014), who show how intermediaries help alleviate information asymmetry on quality, or Petropoulou (2011) who considers search frictions.

[5] They assume an efficient equilibrium in which foreign consumers are willing to pay a share of the matching cost to increase the mass of varieties they consume. The share of the matching cost to be paid by the firm is increasing in the elasticity of substitution. Given the nature of the matching technology, increased homogeneity allows the large firms to expand at the expense of small firms. This reduces the share of intermediated trade.

Consumers face a two-tier Cobb-Douglas utility function over good H and a manufacturing good C, which is a CES composite of all varieties the consumer imports. Each consumer in country j faces a potentially different set of varieties Ω_j. Then, utility of a representative consumer in country j is given by:

$$U_j = \left[\int_{\omega \in \Omega_j} c_j(\omega)^{\rho} \; \mathrm{d}\omega \right]^{\mu/\rho} H_j^{1-\mu}, \qquad \mu, \rho \in (0,1).$$

Note that the elasticity of substitution of manufacturing goods is given by $\sigma = \frac{1}{1-\rho} > 1$. Consumers in country j maximize their utility subject to a budget constraint, and generate income from two sources. First, they obtain income from labour, which equals the exogenously pinned down wage w_j. Moreover, as in Chaney (2008), I assume that the net profits of all firms are accumulated in a global fund and redistributed to consumers as dividends based on their wage income. Denoting the dividend per share in the fund by d, the income of a representative consumer in country j thus equals $y_j = (1+d)w_j$.

Following Dixit and Stiglitz (1977), the representative consumer in country j has the following optimal demand for each variety ω, provided $\omega \in \Omega_j$:

$$c_j(\omega) = \frac{\mu y_j}{P_j} \left[\frac{p(\omega)}{P_j} \right]^{-\sigma} \equiv A_j p(\omega)^{-\sigma}, \tag{1}$$

where $p(\omega)$ is the price of the variety and P_j the aggregate price level in country j, which will be defined explicitly later.

Firm problem Every country $i \in N$ has a continuum of potential firm entrants for the manufacturing sector with measure J_i, which is proportional to the size of its economy $w_i L_i$. Firms are heterogeneous: every potential entrant is characterized by its productivity level $\phi \in [1, \infty)$, which is drawn ex ante from a distribution function $g(\phi)$ with cumulative distribution function $G(\phi)$. Firms face a production technology with constant returns to scale in ϕ, where labour is the sole factor of production.[6] This implies that the marginal cost of production is decreasing in productivity. Transportation is costly: I impose the standard iceberg cost assumption, which states that delivering one unit of a variety from country i to j requires shipping $\tau_{ij} \geq 1$ units. Without loss of generality, I set $\tau_{ii} = 1$ for all $i \in N$.[7]

To parsimoniously introduce search frictions, I build on the market penetration model by Arkolakis (2010). In this setting, firms do not readily observe potential consumers. This means that besides the costs of production and variable trade costs, firms also face a market-specific cost of exporting, encompassing those costs related to setting up and operating a distribution network, and product promotion. Based on this market penetration decision, the firm then reaches a corresponding fraction of consumers. Specifically, the costs of advertising in order to reach a consumer in j with probability n is a function of population L_j and wages in both the origin and

[6] I assume each firm only produces one good, as this facilitates tractability. Of course, in reality manufacturing firms can produce multiple goods and thus also enjoy economies of scope in the cost of exporting. Still, intermediaries generally export more products than manufacturers (Bernard et al., 2012; Akerman, 2018). A related literature explicitly discusses multi-product firms, see for example Bernard et al. (2012) or Mayer et al. (2014).

[7] Moreover, I assume $\tau_{iv}\tau_{vj} \geq \tau_{ij} \;\; \forall i, j, v \in N$ as to prevent transport arbitrage opportunities.

destination country:

$$\Lambda(n, L) \equiv w_j^{\gamma} w_i^{1-\gamma} \frac{L^{\alpha}}{\psi} \frac{1-(1-n)^{1-\beta}}{1-\beta}, \tag{2}$$

where $\alpha \in [0,1]$, $\beta \in [0,1) \cup (1,\infty)$ and $1/\psi > 0$. Here, α captures the returns to scale in advertising, asserting that the cost of market penetration is lower for more populated markets. Note that for $\beta > 0$, $\Lambda(n, L)$ is increasing and convex in n and $\lim_{n\to 1} \Lambda'(n) = \infty$. This implies that the marginal cost of reaching a consumer is always positive and that firms with different revenues make different market penetration decisions. Also, no firm can saturate the market. Put differently, β represents the diminishing returns to advertising: reaching additional potential consumers becomes increasingly difficult as the consumer base increases. Exporting is thus an increasing-cost activity for firms.

From now on, I restrict my attention to symmetric equilibria, where every firm from i with productivity ϕ chooses the same price $p_{ij}(\phi)$ and the same probability of reaching consumers $n_{ij}(\phi) \in [0,1]$. Then, because of the large mass of firms, every consumer from country j faces the same distribution of prices for goods of different types. Moreover, given our continuum of consumers, the fraction of consumers reached is given by $n_{ij}(\phi)L_j$.[8] Based on these insights and optimal consumer demand as given in (1), I can write the total effective demand in country j for a ϕ-type firm from country i as

$$q_{ij}(\phi) = n_{ij} L_j A_j p_{ij}(\phi)^{-\sigma}. \tag{3}$$

Each firm maximizes its profits conditional on entering market j, taking its productivity ϕ and consumer demand as given. The optimal pricing decision is the constant mark-up rule standard in the monopolistic competition framework:

$$p_{ij}(\phi) = \frac{\sigma}{\sigma-1} \frac{\tau_{ij} w_i}{\phi}. \tag{4}$$

Given the optimal price, the firm determines the optimal market penetration strategy, which is given by:

$$n_{ij}(\phi) = \max\left\{1 - \left(\frac{\phi_{ij}^*}{\phi}\right)^{\frac{\sigma-1}{\beta}}, 0\right\} \tag{5}$$

$$\left(\phi_{ij}^*\right)^{\sigma-1} = \frac{w_j^{\gamma} w_i^{1-\gamma} L_j^{\alpha-1} \sigma}{A_j \psi} \left(\frac{\sigma}{\sigma-1} \tau_{ij} w_i\right)^{\sigma-1},$$

where ϕ_{ij}^* is the productivity of the marginal exporting firm.

Optimal investment in market penetration is thus characterized by a threshold equilibrium, where $n_{ij}(\phi) = 0$ for all $\phi < \phi_{ij}^*$ and $0 < n_{ij}(\phi) < 1$ otherwise. Intuitively, firms with higher productivity will set a higher market penetration probability: because they face lower marginal costs of production, they can extract relatively higher marginal revenue from each consumer.

[8] This is a direct application of the Glivenko-Cantelli theorem, which states that for a set of identically and independently distributed random variables, the empirical distribution converges almost surely to the underlying cumulative distribution (Tucker, 1959). To apply the theorem, I assume firms reach consumers independently from one another. I refer the interested reader to Arkolakis (2010) for a more detailed discussion on the Glivenko-Cantelli theorem.

In contrast, for those firms that are least productive, the marginal cost of advertising for the first consumer already exceeds the marginal revenue of selling to that customer. Accordingly, these firms will not enter market j. Note that if $\beta = 0$, the marginal cost to reach additional consumers is constant such that firms with $\phi \geq \phi^*_{ij}$ choose to enter the market and set $n_{ij} = 1$, which is observationally equivalent to assuming a fixed entry cost as in Melitz (2003).[9] Another important observation to make is that ϕ^*_{ij} also contains ϕ^*_{ii}. This implies that, for certain values of L_j and A_j, it is theoretically possible that a company exports to other markets but does not sell domestically. For ease of exposition, I will refer to all sales, including domestic sales, as exporting in the remainder of this paper.

ii. Intermediaries

Having established the distribution network mechanism in the baseline model, I will now show how the presence of intermediaries affects economic decision-making and the equilibrium outcomes.

I define an intermediary as a firm that does not produce, but possesses an intermediation technology that allows it to source a set of varieties with mass m^I_{ij} from home country i and ship them to j. Intermediaries are fully homogeneous and the sector is characterized by free entry. Also, intermediaries face the same trade barriers as firms, meaning that they both face an iceberg transportation cost and need to invest in market penetration in each destination market.

To ensure tractability, I have to impose three further assumptions on the intermediary sector. First of all, I assume that firms are matched randomly with an intermediary. This ensures that in expectation, each intermediary sources the same mass of varieties and is characterized by the same productivity distribution. Second, conditional on being approached by an intermediary, consumers do not observe the exact set of sourced varieties ex ante. Consequently, in expectation, firms still approach consumers independently if they opt for an intermediary. This means I can apply the Glivenko-Cantelli theorem, which ensures the model remains tractable. Finally, I assume intermediaries are still sufficiently small as to not affect the price level.

Intermediary's problem The intermediary's marginal cost of shipping a good from country i to j consists of two distinct components: (i) variable trade costs τ_{ij} and (ii) the procurement price $p_{I,ii}(\phi)$. I assume that the intermediary has full bargaining power. Then, given the outside option of the firm, the intermediary will offer the domestic selling price to the firm which the firm will accept. This implies that the procurement price is $p_{I,ii}(\phi) = p_{ii}(\phi) = \frac{\sigma}{\sigma-1}\frac{w_i}{\phi}$.[10]

I will show that – as in the baseline model – the equilibrium exhibits productivity sorting. Specifically, the range of firms represented by an intermediary is $\phi \in \left[\phi^W_{ij}, \phi^X_{ij}\right]$, where ϕ^W_{ij} is the indirect exporting productivity threshold and ϕ^X_{ij} the direct exporting counterpart. Evidently, the profits of the intermediary depend on this range. What complicates matters is that the thresholds follow from firms' profit maximization decisions. Thus, the intermediary's profits and the firm's profits are determined simultaneously. To impose some structure on this simultaneity, I assume

[9] For a complete proof of this result, see Arkolakis (2010).

[10] In Akerman (2018), this procurement price is derived differently. He asserts the intermediary takes final demand as given and then uses Shephard's Lemma on the intermediary's cost function to derive its demand for a variety. However, in this setting the intermediary internalizes how the consumer demand responds to changes in the procurement price. Combined with the fact that the intermediary is not necessarily a price taker implies Shephard's Lemma cannot be used in this setting.

the firms solve their maximization problem before the intermediaries do so. This means that in the intermediary's optimization problem, the firms' best replies are taken into account. I can write the expected operating profit of an intermediary per each variety it exports as:

$$\tilde{\pi}^I_{ij}(\phi^W_{ij}, \phi^X_{ij}) = \frac{1}{G(\phi^X_{ij}) - G(\phi^W_{ij})} \int_{\phi^W_{ij}}^{\phi^X_{ij}} n^I L_j A_j p^{-\sigma} \left(p - \frac{\sigma}{\sigma - 1} \frac{w_i}{\phi} \tau_{ij} \right) \; \mathrm{d}G(\phi).$$

I assume, in line with empirical evidence, that finding an intermediary and setting up a contract is costly. Specifically, I impose that firms have to incur a fixed cost f_{ij}, upon which they are matched with one intermediary. This intermediary receives f_{ij} and the exclusive right to sell the manufacturing variety abroad. Then, an intermediary's profits from exporting a mass m^I_{ij} of goods from i to j are given by:

$$\pi^I_{ij}(p^I, n^I; m^I_{ij}, \phi^W_{ij}, \phi^X_{ij}) = m^I_{ij}\tilde{\pi}^I_{ij} - w_j^\gamma w_i^{1-\gamma} \frac{L_j^\alpha}{\psi} \frac{1 - (1 - n^I)^{1-\beta}}{1 - \beta} + m^I_{ij} f_{ij}.$$

Like the firm, the intermediary maximizes its profits by setting the optimal price and the optimal market penetration probability. The first order condition on the price yields:

$$p^I_{ij}(\phi^W_{ij}, \phi^X_{ij}) = \left(\frac{\sigma}{\sigma - 1} \right)^2 \int_{\phi^W_{ij}}^{\phi^X_{ij}} \frac{\tau_{ij} w_i}{\phi} \; \mathrm{d}G(\phi), \tag{6}$$

which is effectively the average price of the goods the intermediary sells. Note that the intermediary, because it faces CES demand, also follows the constant mark-up over marginal costs rule. Using the expression above, I can derive the price for each individual variety the intermediary carries:

$$p^I_{ij}(\phi) = \left(\frac{\sigma}{\sigma - 1} \right)^2 \frac{\tau_{ij} w_i}{\phi}, \tag{7}$$

such that $\int_{\phi^W_{ij}}^{\phi^X_{ij}} p^I_{ij}(\phi) \; \mathrm{d}G(\phi) = p^I_{ij}$. The intermediary charges a mark-up over the marginal cost of each variety it exports. Thus, in equilibrium intermediation implies double marginalization. Conditional on the optimal price strategy, the intermediary decides on the optimal market penetration. This is determined by the first-order condition on n^I_{ij} if the intermediary enters the market:

$$m^I_{ij} \frac{1}{G(\phi^X_{ij}) - G(\phi^W_{ij})} \int_{\phi^W_{ij}}^{\phi^X_{ij}} L_j A_j \frac{1}{\sigma} \left[\left(\frac{\sigma}{\sigma - 1} \right)^2 \frac{\tau_{ij} w_i}{\phi} \right]^{1-\sigma} \; \mathrm{d}G(\phi) = w_j^\gamma w_i^{1-\gamma} \frac{L_j^\alpha}{\psi} \frac{1}{(1 - n^I_{ij})^\beta}. \tag{8}$$

In the setting without intermediaries, the choice rule for the firm was based on its productivity; the more productive a firm is, the more it is willing to invest in advertising. Intermediaries adopt a similar rule, but they consider the average productivity of the firms in their portfolio instead. In the remainder of the paper, I will refer to this measure as the the average productivity of the intermediary.[11]

Definition 1 (Average productivity intermediary)**.** If the productivity thresholds are given by ϕ^W_{ij} and ϕ^X_{ij}, this implies that the average productivity of the intermediary is given by:

$$\tilde{\phi}^I(\phi^W_{ij}, \phi^X_{ij}) = \frac{1}{G(\phi^X_{ij}) - G(\phi^W_{ij})} \int_{\phi^W_{ij}}^{\phi^X_{ij}} \phi^{\sigma-1} \; \mathrm{d}G(\phi)$$

[11] This is slight abuse of terminology, since it is not the intermediary which has a certain productivity. Nevertheless, for ease of exposition this shorthand notation will be used in the rest of the paper.

In the remainder of the paper, I will refrain from listing the arguments of $\tilde{\phi}^I$ for notational convenience; nevertheless, it is important to understand that the average productivity is a function of both thresholds and potentially differs across destination markets. As expected, the higher the average productivity, the more the intermediary would like to invest in market penetration as the marginal revenue of doing so is higher. Then, setting $n^I_{ij} = 0$ and solving for $\tilde{\phi}^I$ gives the market penetration threshold for the intermediary:

$$\tilde{\phi}^{I^*}_{ij} \equiv \frac{1}{G(\phi^X_{ij}) - G(\phi^W_{ij})} \int_{\phi^W_{ij}}^{\phi^X_{ij}} \phi^{\sigma-1} \, \mathrm{d}G(\phi) = \frac{w_j^\gamma w_i^{1-\gamma} L_j^{\alpha-1}\sigma}{m^I_{ij} A_j \psi} \left[\left(\frac{\sigma}{\sigma-1}\right)^2 \tau_{ij} w_i \right]^{\sigma-1}. \tag{9}$$

To better understand the intermediary's market penetration strategy, I compare this to the corresponding threshold for the firm, ϕ^*_{ij}:

$$\frac{\tilde{\phi}^{I^*}_{ij}}{(\phi^*_{ij})^{\sigma-1}} = \frac{1}{m^I_{ij}} \left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}.$$

The ratio of the market penetration thresholds of the intermediary and the firm is determined by two terms. First, the ratio is falling in the mass of goods an intermediary exports. This term reflects the relative advantage for the intermediary in terms of economies of scope in market penetration: the more varieties an intermediary exports, the higher its incentive to increase its market penetration. Second, the ratio is increasing in the elasticity of substitution: the threshold of the intermediary is higher if goods are more homogeneous. Intuitively, the higher elasticity of substitution implies that the mark-up of the intermediary on average is lower, such that the marginal revenue of increasing the consumer base is lower as well. This disincentivizes the intermediary to invest substantially in market penetration.

To obtain an explicit expression for the intermediary's market penetration, I substitute 9 into (8) and solve for n^I_{ij}, which leads to the following proposition:

Proposition 1 (Market Penetration Intermediary)**.** *Given the average productivity of the firms in its portfolio, $\tilde{\phi}^I$, the intermediary sets the market penetration probability according to the following decision rule:*

$$n^I_{ij}(\tilde{\phi}^I) = \begin{cases} 1 - \left(\dfrac{\tilde{\phi}^{I^*}}{\tilde{\phi}^I}\right)^{1/\beta} & \text{if } \tilde{\phi}^I \geq \tilde{\phi}^{I^*} \\ 0 & \text{otherwise.} \end{cases} \tag{10}$$

Proof. See appendix. □

Again, the optimal investment in market penetration is characterized by a origin-destination-specific threshold equilibrium. The intuition for this result is also similar to that of the decision rule for direct exporting firms. Given the threshold-level, the intermediary is incentivized to invest more in market penetration when the average productivity of the firms in its portfolio, i.e. the potential marginal revenue for selling a bundle to another customer, is higher.

Lastly, as the intermediary sector is characterized by free entry, the intermediary's profits are zero. We can rewrite the free entry condition as follows:

$$m^I_{ij}(\tilde{\pi}_{ij}(\phi^W_{ij}, \phi^X_{ij}) + f_{ij}) = w_j^\gamma w_i^{1-\gamma} \frac{L_j^\alpha}{\psi} \frac{1 - (1 - n^I_{ij}(\tilde{\phi}^I))^{1-\beta}}{1-\beta}. \tag{11}$$

This condition highlights the balance between its average productivity and the mass of goods in its portfolio. If the expected profits per variety are low, fewer intermediaries will be incentivized to enter the market. Accordingly, the mass of goods sourced by each intermediary increases, thus increasing the average total profit such that it is large enough to incentivize market penetration. In effect, the lower expected per-variety profits are compensated by a larger scope.

Having addressed the problem of the intermediary, and having devised its optimal strategy in terms of both price and market penetration, I can now revisit the firm problem.

Firm problem with intermediaries. The introduction of intermediaries extends the strategy set of firms: a firm can either not sell its good in market j, it can directly sell to market j or it can employ an intermediary to sell in market j. For ease of exposition, I will refer to these choices as not exporting, exporting directly and exporting indirectly respectively, but note that this analysis subsumes selling domestically. If, as posited in the previous section, the choice of export mode is determined by a threshold equilibrium, the productivity thresholds follow from the profit maximization problem of the firm.

If the firm exports directly, it sets its market penetration level according to (5). Then, the profit of direct exporting to country j for a ϕ-type firm in country i equals:

$$\pi_{ij}^{X}(\phi) = n_{ij}(\phi) L_j A_j \frac{1}{\sigma} \left(\frac{\sigma}{\sigma-1} \frac{\tau_{ij} w_i}{\phi} \right)^{1-\sigma} - w_j^{\gamma} w_i^{1-\gamma} \frac{L_j^{\alpha}}{\psi} \frac{1-(1-n_{ij}(\phi))^{1-\beta}}{1-\beta}. \tag{12}$$

Note that the profit of direct exporting is increasing in ϕ; the more productive firms both extract higher marginal revenue per consumer and invest more in market penetration, such that they reach more consumers. A firm can also export to country j indirectly. While this reduces the market-specific cost of exporting to f_{ij}, it reduces consumer demand because the intermediary charges an additional mark-up. Therefore, this option is not attractive for the most productive firms.

Any firm that exports indirectly takes the advertising effort of the intermediary into account, such that its profits would be:

$$\pi_{ij}^{W}(\phi) = n_{ij}^{I}(\tilde{\phi}^{I}) L_j A_j \frac{1}{\sigma} \left(\frac{\sigma}{\sigma-1} \frac{\tau_{ij} w_i}{\phi} \right)^{1-\sigma} \left(\frac{\sigma}{\sigma-1} \right)^{-\sigma} - f_{ij}. \tag{13}$$

The productivity threshold for exporting directly, ϕ_{ij}^{X}, is implicitly determined by the equality of these two equations. Note that there is one intersection only: π_{ij}^{X} monotonically increases faster in ϕ than $\pi_{ij}^{W}(\phi)$, and the intercept of π_{ij}^{X} is below that of π_{ij}^{W}. Thus, the direct exporting productivity threshold is uniquely determined.

The productivity threshold for exporting indirectly, ϕ_{ij}^{W}, is the level of productivity for which a firm is indifferent between exporting indirectly and not exporting at all:

$$n_{ij}^{I}(\tilde{\phi}^{I}) L_j A_j \frac{1}{\sigma} \left(\frac{\sigma}{\sigma-1} \frac{\tau_{ij} w_i}{\phi_{ij}^{W}} \right)^{1-\sigma} \left(\frac{\sigma}{\sigma-1} \right)^{-\sigma} - f_{ij} = 0, \tag{14}$$

This threshold is also unique, because π_{ij}^{W} is increasing in ϕ and the right-hand side is a constant.

Having established that the thresholds are unique, what remains to analyze is the equilibrium export strategy choices that arise for bilateral trade between i and j. There are two distinct possibilities depending on the origin-destination-specific parameters. First, it could be that there are no intermediaries. This occurs when $\phi_{ij}^W > \phi_{ij}^*$, which implies that exporting directly is always dominated by exporting indirectly. This is possible if and only if the fixed costs of intermediation f_{ij} are prohibitively high. In that case, intermediaries are not active in trade between this origin-destination pair and the model collapses into Arkolakis (2010). In the second, arguably more interesting, case, the environment is such that $\phi_{ij}^W < \phi_{ij}^* < \phi_{ij}^X$. Then, firms with productivities between ϕ_{ij}^W and ϕ_{ij}^X will employ an intermediary. This intuition is formalized in the following proposition:

Proposition 2 (Firm export strategy decision)**.** *Define the prohibitive level of fixed costs of intermediation as $\bar{f}_{ij}$. The firm's export decision depends solely on its productivity ϕ and is characterized by a threshold equilibrium:*

1. ***Intermediation***
   If $f_{ij} < \bar{f}_{ij}$, there exist a unique ϕ_{ij}^W, ϕ_{ij}^X such that firms in i with
   $\phi \in [1, \phi_{ij}^W)$ *do not export to j;*
   $\phi \in [\phi_{ij}^W, \phi_{ij}^X)$ *export indirectly to j;*
   $\phi \in [\phi_{ij}^X, \infty)$ *export directly to j.*

1. ***No intermediation***
   If $f_{ij} \geq \bar{f}_{ij}$, there exists a unique ϕ_{ij}^* such that firms with
   $\phi \in [1, \phi_{ij}^*)$ *do not export to j;*
   $\phi \in [\phi_{ij}^*, \infty)$ *export directly to j.*

Proof. See appendix. □

iii. Equilibrium

To close the model, I need to define the measures of active firms and the aggregate price levels. The measure of firms in i that export directly to j is given by the measure of existing firms conditional on their productivity exceeding ϕ_{ij}^X:

$$M_{ij}^X = J_i[1 - G(\phi_{ij}^X)]. \tag{15}$$

Similarly, the measure of firms that use an intermediary is given by:

$$M_{ij}^W = J_i[G(\phi_{ij}^X) - G(\phi_{ij}^W)]. \tag{16}$$

Using the free entry condition, the mass of intermediaries is defined as the mass of firms exporting indirectly divided by the scope of the intermediary:

$$I_{ij} = \frac{M_{ij}^W}{m_{ij}^I}. \tag{17}$$

Using the fact that intermediaries are homogeneous, I can define the aggregate price level in country j by aggregating over all destinations:

$$P_j^{1-\sigma} = \sum_{i=1}^{N} J_i \left[\int_{\phi_{ij}^X}^{\infty} p_{ij}(\phi)^{1-\sigma} n_{ij}(\phi) g(\phi) \ \mathrm{d}\phi + \int_{\phi_{ij}^W}^{\phi_{ij}^X} p_{ij}^I(\phi)^{1-\sigma} n_{ij}^I(\tilde{\phi}^I) g(\phi) \ \mathrm{d}\phi \right]. \tag{18}$$

Finally, I can express the export volume in country j of a ϕ-firm from i as:

$$r_{ij}(\phi) = \begin{cases} n_{ij}(\phi) L_j A_j \left[\frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{\phi} \right]^{1-\sigma} & \text{if } \phi \geq \phi_{ij}^X \\ n_{ij}^I(\tilde{\phi}^I) L_j A_j \left[\left(\frac{\sigma}{\sigma - 1} \right)^2 \frac{\tau_{ij} w_i}{\phi} \right]^{1-\sigma} & \text{if } \phi_{ij}^W \leq \phi < \phi_{ij}^X \\ 0 & \text{otherwise.} \end{cases} \tag{19}$$

The last step of defining the equilibrium of this model is to derive the labour market-clearing condition. First, I calculate the value of a share in the global profits fund d:

$$d = \frac{\sum_{i=1}^N w_i L_i \sum_{v=1}^N \left[\int_0^\infty \pi_{iv}^X(\phi) \ \mathrm{d}G(\phi) + \int_{\phi_{iv}^W}^{\phi_{iv}^X} \pi_{iv}^W(\phi) \ \mathrm{d}G(\phi) \right]}{\sum_{i=1}^N w_i L_i}. \tag{20}$$

Note that trade balance implies that the total income in country i equals total spending in this country. Thus, we have:

$$(1 + d) w_i L_i = \sum_{v=1}^N J_v \left[\int_{\phi_{vi}^X}^\infty r_{vi}^X(\phi) \ \mathrm{d}G(\phi) + \int_{\phi_{vi}^W}^{\phi_{vi}^X} r_{vi}^W(\phi) \ \mathrm{d}G(\phi) \right]. \tag{21}$$

If this equation holds, the labour market in i is in equilibrium. The global equilibrium can then be formulated as follows:

Equilibrium. Given τ_{ij}, w_i, L_i and definitions (15), (16) and (17), for all $i, j = 1, ..., N$, an equilibrium is a set of allocations for the representative consumer, $\hat{c}_{ij}(\phi)$, allocations for the representative firm, $\hat{p}_{ij}(\phi)$[12] and $\hat{n}_{ij}(\phi)$, and allocations for the representative intermediary, $\hat{p}_{ij}^I(\phi)$ and $\hat{n}_{ij}^I$, such that (1), (4), (5), (7), (10), (14), (18), (11), (21) hold and (12) and (13) are equal to each other.

This general model, nesting Arkolakis (2010) and Ahn et al. (2011) , also nests the main predictions from these models. The next section will discuss the implications of intermediation in international trade according to my model, both in terms of export patterns as well as trade networks. Section IV outlines novel predictions on how the consequences of trade liberalization differ between direct exporters and intermediated firms. In Section V, these hypotheses are brought to the data.

III. IMPLICATIONS OF INTERMEDIATION IN INTERNATIONAL TRADE

This simple framework is able to capture the important stylized facts on the role of trade intermediation in international trade and their consequences for the formation of trade networks; I list these seven stylized facts below. The main novelty here is that my model simultaneously explains these empirical regularities, in contrast to earlier models that focused on subsets of these.

Prediction 1: Firms may use different export strategies for different destination markets. The thresholds for exporting directly and indirectly vary across regions. Thus, a firm may use different export strategies for different markets depending on its productivity. Indeed, Abel-Koch (2013) shows how 15 percent of Turkish exporting firms engages in both direct and indirect exporting.

[12] Note that the exogenous wages guarantee positive prices.

Prediction 2: Intermediaries charge higher prices for any variety they export compared to direct exporters. This result is generated through two channels: (i) firms exporting indirectly are less productive than those exporting directly and therefore the marginal costs are higher and (ii) the intermediaries charge an additional mark-up. This stylized fact has been established in different contexts such as China (Feenstra and Hanson, 2004; Ahn et al., 2011) and Sweden (Akerman, 2018).

Prediction 3: Intermediaries sell less of each variety than firms that export directly. This follows from the fact that intermediaries charge higher prices. Indeed, Akerman (2018) finds that the export volume per good of Swedish intermediaries on average is between one-third and one-half lower than that of directly exporting firms.

Prediction 4: Small sellers never sell to small buyers. In trade models without intermediation, such as Melitz (2003) and Arkolakis (2010), even the smallest exporters match directly with consumers. In my model, small exporters endogenously choose to transact with an intermediary over selling to multiple consumers. This mechanism can explain the absence of trade relationships between small exporters and small importers as documented in Blum et al. (2010). Instead, they find that more than 90 percent of trade linkages between Colombian and Cuban firms comprise of a large and a small partner.

Prediction 5: Only a small fraction of exporting firms serves many buyers. Conditional on entering a market, the number of downstream trade partners per country varies with firm productivity: larger firms, i.e. those with greater productivity, invest more in market penetration and thereby reach more downstream consumers. In combination with the option of intermediation this results in a setting where most exporting firms have few consumers – they either use an intermediary or they invest modestly in market penetration – and only a few firms export to many clients. This mirrors the findings in Blum et al. (2009).

The remaining two predictions concern comparative statics regarding the export strategy thresholds. Since the general model does not admit closed-form solutions for these, I consider a special case of the model with $\beta = 0$. Note that in this version of the model, economies of scope are the only reason why intermediaries may prove beneficial to firms. Nevertheless, given the fact that intermediaries and firms face the same search frictions, the qualitative implications generalize to the original model. If there are no diminishing returns in advertising, the marginal cost is constant in the market penetration probability. This implies that for all firms with $\phi < \phi^*_{ij}$, $n_{ij} = 0$, and otherwise $n_{ij} = 1$. Equivalently, intermediaries do not export if the average productivity of those choosing an intermediary is below $\tilde{\phi}^{I^*}_{ij}$ as given by (9); otherwise it will export to all consumers in a country. The free entry condition ensures that the intermediary's average productivity is always at least equal to this threshold.

For the special case I can define the productivity thresholds explicitly. First, the equality of (12) and (13) for $n_{ij}(\phi), n^I_{ij}(\tilde{\phi}^I) = 1$, gives the new export cut-off ϕ^X_{ij}:

$$(\phi^X_{ij})^{\sigma-1} = \frac{\sigma\left(w_j^{\gamma} w_i^{1-\gamma}\frac{L_j^{\alpha}}{\psi} - f_{ij}\right)}{L_j A_j\left[1-\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\right]}\left(\frac{\sigma}{\sigma-1}\tau_{ij}w_i\right)^{\sigma-1}. \tag{22}$$

Note that if fixed costs are not prohibitively high:

$$f_{ij} < w_j^{\gamma} w_i^{1-\gamma} \frac{L_j^{\alpha}}{\psi} \left(\frac{\sigma-1}{\sigma}\right)^{\sigma} \equiv \bar{f}_{ij}, \tag{A1}$$

we have $\phi_{ij}^{X} > \phi_{ij}^{*}$: the introduction of intermediaries incentivizes some firms that previously exported directly to opt for an intermediary. In the remainder of the paper, I will assume f_{ij} is sufficiently low such that this is satisfied.

Next, I can define the productivity threshold for using an intermediary ϕ_{ij}^{W}. To that end, I adapt (14) and solve for ϕ:

$$(\phi_{ij}^{W})^{\sigma-1} = \frac{\sigma f_{ij}}{L_j A_j} \left(\frac{\sigma}{\sigma-1}\tau_{ij} w_i\right)^{\sigma-1} \left(\frac{\sigma}{\sigma-1}\right)^{\sigma}. \tag{23}$$

If assumption A1 holds, the productivity threshold of exporting indirectly ϕ_{ij}^{W} is lower than the threshold of exporting directly ϕ_{ij}^{X} and we have an equilibrium where intermediaries are active. Now, I can do some simple comparative statics, as summarized in the following corollary:

Corollary 2.1 (Comparative statics thresholds)**.** *Assume $\beta = 0$. Then:*

1. *An increase in the fixed costs of using an intermediary f_{ij}, or a decrease in the (constant) marginal cost of advertising $w_j^{\gamma} w_i^{1-\gamma} \frac{L_j^{\alpha}}{\psi}$, increases ϕ_{ij}^{W} and decreases ϕ_{ij}^{X}.*

2. *An increase in the elasticity of substitution σ decreases ϕ_{ij}^{W} and increases ϕ_{ij}^{X}.*

Proof. These statements readily follow from equations (22) and (23). □

This analysis thus generates the following predictions:

Prediction 6: The share of intermediation increases in market-specific costs. Ceteris paribus, an increase in the cost of market penetration for a certain destination will increase the corresponding direct exporting threshold. Thus, an increase in market-specific entry costs or a decrease in the cost of using an intermediary will induce firms that previously exported by themselves to find an intermediary instead. The result that intermediaries are more prevalent in markets with high costs of exporting has been confirmed by a multitude of studies such as Bernard et al. (2010); Ahn et al. (2011); Akerman (2018).

Prediction 7: More competition, i.e. more homogeneous products, are correlated with more intermediation. If goods are more homogeneous, which in this model is proxied by higher elasticity of substitution between varieties, the mark-ups on prices are lower. This implies that double marginalization is less impactful, and therefore that the fall in consumer demand is less pronounced. Thus, the incentive for firms to export indirectly is stronger if there is more competition. Again, this is consistent with empirical evidence as in Ahn et al. (2011) or Bernard et al. (2015), and contrary to the predictions from Blum et al. (2009).

IV. INTERMEDIATION AND THE CONSEQUENCES OF TRADE LIBERALIZATION

This subsection addresses novel testable implications regarding the effect of trade liberalization in a tradiong environment with intermediation. Interestingly, most previous papers on trade intermediation do not consider explicitly this question. Common sense dictates that a decrease in trade costs incentivizes firms to export: I show that the most productive intermediated firms start exporting directly and that the most productive non-exporters now export through an intermediary. I refer to this change in the intermediary's portfolio as the composition effect. Interestingly, this negatively impacts the intermediary's market penetration strategy, and thereby mutes the sales growth for an individual indirect exporter compared to a direct exporter. Consequently, intermediated firms are less responsive to trade shocks than direct exporting firms.

i. *Composition effect*

In this model, where the fixed cost of export is endogenous, a fall in the variable trade cost has two effects on the sales of a direct exporter. This can be shown by computing the partial elasticity of these revenues with respect to variable trade costs:

$$\begin{aligned}\varepsilon_{ij}^{X}(\phi) &= -\frac{\partial \ln r_{ij}^{X}(\phi)}{\partial \ln \tau_{ij}} \\ &= \underbrace{\sigma - 1}_{\text{Intensive margin}} + \underbrace{\frac{\sigma-1}{\beta}\left[\left(\frac{\phi}{\phi_{ij}^{*}}\right)^{\frac{\sigma-1}{\beta}} - 1\right]^{-1} \frac{\partial \ln \phi_{ij}^{*}}{\partial \ln \tau_{ij}}}_{\text{New consumers margin}}. \end{aligned} \tag{24}$$

First, the sales to existing consumers grow proportional to the elasticity of demand $\sigma - 1$. This effect on the *intensive margin* is standard in models with heterogeneity in firm productivity. Second, the decrease in trade costs incentivizes firms to increase their investment in advertising and thus their customer base, which is referred to as the *new consumers margin*. This will disproportionally benefit the firms with relatively low initial trade, as the marginal cost of advertising is slowly increasing. A fall in trade costs will thus lead to larger benefits for firms that have lower levels of market penetration.

I derive the corresponding elasticity for an intermediated firm in a similar fashion. For ease of exposition, I first formalize this proposition below:

Proposition 3 (Intermediary's sales reacting to trade costs)**.** *The impact of the new consumers margin is decreasing in $\tilde{\phi}^{I}$, as is the impact of the composition effect.*

Proof. I calculate the partial elasticity of an intermediated firm's sales in country j with respect to iceberg trading costs τ_{ij}:

$$\begin{aligned}\varepsilon_{ij}^{W}(\phi) &= -\frac{\partial \ln r_{ij}^{W}(\phi)}{\partial \ln \tau_{ij}} \\ &= \underbrace{\sigma - 1}_{\text{Intensive margin}} + \underbrace{\frac{1}{\beta}\left[\left(\frac{\tilde{\phi}_{ij}^{I}}{\tilde{\phi}_{ij}^{I^*}}\right)^{\frac{1}{\beta}} - 1\right]^{-1} \frac{\partial \ln \tilde{\phi}_{ij}^{I^*}}{\partial \ln \tau_{ij}}}_{\text{New consumers margin}} - \underbrace{\frac{1}{\beta}\left[\left(\frac{\tilde{\phi}_{ij}^{I}}{\tilde{\phi}_{ij}^{I^*}}\right)^{\frac{1}{\beta}} - 1\right]^{-1} \frac{\partial \ln \tilde{\phi}_{ij}^{I}}{\partial \ln \tau_{ij}}}_{\text{Composition effect}}. \end{aligned} \tag{25}$$

Because $\frac{\tilde{\phi}^I_{ij}}{\tilde{\phi}^{I*}_{ij}}$ is increasing in $\tilde{\phi}^I$, the effect of both the new consumers margin and the composition effect will be less pronounced if the average productivity of the intermediary is higher. □

Comparing this elasticity to 24 reveals a novel term that only affects the sale of intermediated firms: the *composition effect*. The composition effect captures the change in the composition of goods in the intermediary's portfolio, as firms reassess their export strategy choice. Consider for example a decrease in trade costs. This reduces the market-specific costs of exporting, such that some previously intermediated firms are incentivized to start exporting directly. This is characterized by a lower ϕ^X_{ij}. On the other side of the spectrum, there are new firms that previously did not export but now find it profitable to export through an intermediary. This is captured by the lower level of the indirect productivity threshold ϕ^W_{ij}. The fact that the intermediary loses its most productive clients but gains less productive firms implies that the composition effect impacts the level of market penetration – and thus the indirectly exporting firm's revenues – negatively: the marginal revenue per additional consumer will be lower.

The loss in firm sales due to the composition effect are less pronounced if the average productivity of the intermediary is lower, since the optimal level of market penetration is concave in average productivity.

ii. Implications of the composition effect

The composition effect shows how the decrease in average intermediary productivity negatively affects its market penetration decision. However, the analysis in the general model cannot shed light on how this decrease comes about: trade liberalization affects not only the types of firms that are intermediated, but also the number of firms that export indirectly. To that end, I again restrict my attention to the special case where $\beta = 0$. Note that in this setting the composition effect strictly speaking does not exist, since the market penetration decision now only concerns the extensive margin. Instead, this analysis serves to determine how the average intermediary productivity changes upon trade liberalization.

The composition elasticity, or the partial elasticity of $\tilde{\phi}$ with respect to the variable trade costs, is given by:

$$\frac{\partial \ln(\tilde{\phi}^I)}{\partial \ln(\tau_{ij})} = \sigma - 1 > 0.$$

This derivation shows that a decrease in the variable trade costs lowers the average productivity of the goods sourced by the intermediary. This decrease can be decomposed in two margins: the change in the range of varieties an intermediary offers and the mass of firms per variety type in its portfolio.

The first margin reflects how trade liberalization affects the set of firms sourced by the intermediary. A decrease in variable trade costs results in a fall in both the direct and indirect exporting threshold, but so far these changes have not been quantified. This would be a proxy for the *diversity* of products, or the range of varieties, in the intermediary's portfolio. To that end, I compute the derivative of $\phi^X_{ij} - \phi^W_{ij}$ with respect to variable trade costs:

$$\frac{\partial\ (\phi^X_{ij} - \phi^W_{ij})}{\partial \tau_{ij}} = \frac{\phi^X_{ij} - \phi^W_{ij}}{\tau_{ij}} > 0.$$

Thus, intermediaries will reduce their product diversification in case of trade liberalization. This implies the fall in ϕ_{ij}^X dominates the fall in ϕ_{ij}^W, which is in line with the idea that the sales and thereby profit of a directly exporting firm is more responsive to changes in trade costs than that of an intermediated firm. This result is independent from the firm productivity distribution.

To address the size of the intermediary's portfolio, I need to take a stance on the productivity distribution. I assume, as is standard in the literature, productivity follows a Pareto distribution with shape parameter $k > \sigma - 1$.[13] First, the measure of intermediated firms increases following a decrease in trade costs:

$$M_{ij}^W = J_i \left[(\phi_{ij}^W)^{-k} - (\phi_{ij}^X)^{-k} \right]$$
$$\frac{\partial \ \ln(M_{ij}^W)}{\partial \ \ln(\tau_{ij})} = -k.$$

A decrease in trade costs results in a more-than-proportional increase in the mass of intermediated firms. Also, since the probability distribution function of productivity is right-skewed (by assumption), the intermediary serves not just relatively less productive firms but also relatively more firms that are less productive. In essence, the lower average productivity of the intermediary is compensated for by a larger mass of firms served by the intermediary. Thus, the intermediary exports more products but reduces its product scope after a reduction in the variable trade cost.

There are two related papers that complement these two findings. First, Bernard et al. (2015) analyze the differences in responses of wholesalers and firms after a common currency shock. They find that the effect on sales to the currency shock is less pronounced for wholesalers, because wholesalers are more flexible with adding and dropping products. My paper provides a theoretical explanation for this behaviour, and shows that sales of intermediated firms are indeed less responsive than those of direct exporters. Second, Bernard et al. (2011) analyze the consequences of trade liberalization for multi-product firms with endogenous product scope. They uncover a similar response to trade liberalization; that is, multi-product firms reduce their product range in case of trade liberalization, focusing on their core competencies. I show how intermediaries also adjust their product range, but instead offer higher-priced goods. This model therefore suggests that while intermediaries and multi-product firms are both more flexible than smaller exporters in response to an exogenous shock, they differ in how they adapt to changes in the environment.

V. Conclusion

In this paper, a microeconomic foundation for intermediaries in international trade is presented. I extend a general equilibrium model with search frictions and firm heterogeneity by introducing intermediaries. To overcome the search frictions, firms have to invest in informative advertising to export to foreign consumers. Alternatively, firms can save on advertising by selling their good to an intermediary. However, while the intermediary brings the advantage of a lower market-specific cost, it charges an additional mark-up over the price, reducing consumer demand. This trade-off generates productivity sorting in the export mode. As standard in the literature, the least productive firms will not export at all. However, a subset of firms that are not productive enough to export directly, can now make positive profits if they opt for intermediation. Second, some firms that previously exported directly can increase their profits if they start exporting through an

[13] This assumption on the shape parameter is necessary to ensure the distribution has a finite mean.

intermediary because the market penetration probability is higher than they can afford. Finally, the most productive firms will still export directly, as the effect of foregone demand is more pronounced than the gain in market-specific costs. Thus, intermediaries improve transactional efficiency and facilitate trade by allowing previously non-exporting firms to access foreign markets.

The predictions from the model are consistent with existing empirical evidence. It predicts that intermediaries are more prevalent in countries where the market-specific costs of exporting are higher and that they are more important for more homogeneous goods. Also, it matches empirical evidence on pricing and export volume per variety.

Finally, I derive novel predictions on the consequences of trade liberalization. I show that sales of an intermediated firm are less responsive to trade cost shocks than those of direct exporters because the intermediary reoptimizes its portfolio. Specifically, trade liberalization results in the intermediary increasing the number of goods it exports while reducing the product diversity.

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APPENDIX

i. Proof of Proposition 1

Proof. I will now prove that n^I_{ij} as given in Proposition 1 is the unique and optimal solution to the intermediary's problem. In order to do so, I set up the Lagrangian for the intermediary. The intermediary maximizes its profits subject to the condition that the market penetration probability is between zero and one. However, note that the advertising costs set 1 as a natural limit for n^I_{ij}, since $\lim_{n^I_{ij}\to 1} w_j^\gamma w_i^{1-\gamma}\frac{L^\alpha}{\psi}\frac{1-(1-n^I_{ij})^{1-\beta}}{1-\beta} = \infty$. Thus, the only condition I need is $n^I_{ij} > 0$. The Lagrangian becomes:

$$\mathcal{L}^I_{ij} = m^I_{ij}\frac{1}{G(\phi^X_{ij}) - G(\phi^W_{ij})}\int_{\phi^W_{ij}}^{\phi^X_{ij}} n^I L_j A_j p^{-\sigma}\left(p - \frac{\sigma}{\sigma-1}\frac{w_i}{\phi}\tau_{ij}\right)dG(\phi)$$
$$- w_j^\gamma w_i^{1-\gamma}\frac{L_j^\alpha}{\psi}\frac{1-(1-n^I)^{1-\beta}}{1-\beta} + m^I_{ij} f + \Lambda n^I_{ij},$$

where Λ is the Lagrange multiplier on the constraint $n^I_{ij} > 0$. The first order condition on the price is the same as (6), but the first order condition for market penetration probability, after substituting in the first order condition on the intermediary's price, becomes:

$$\frac{\partial \mathcal{L}^I_{ij}}{\partial n^I_{ij}} = \frac{m^I_{ij}}{G(\phi^X_{ij}) - G(\phi^W_{ij})}\int_{\phi^W_{ij}}^{\phi^X_{ij}}\frac{L_j A_j}{\sigma}\left[\left(\frac{\sigma}{\sigma-1}\right)^2\frac{\tau_{ij}w_i}{\phi}\right]^{1-\sigma} dG(\phi) - w_j^\gamma w_i^{1-\gamma}\frac{L_j^\alpha}{\psi}\frac{1}{(1-n^I_{ij})^\beta} + \Lambda = 0,$$

with complementary slackness condition $\Lambda n^I_{ij} = 0,\ \Lambda \geq 0$. Note that for all $\tilde{\phi}^I \leq \tilde{\phi}^{I^*}$, this equation only holds for $\Lambda > 0$, such that $n^I_{ij} = 0$. On the other hand, if $\tilde{\phi}^I > \tilde{\phi}^{I^*}$, we get $n^I_{ij} \in (0,1)$, which implies that $\Lambda = 0$. Thus, n^I_{ij} as formulated in Proposition 1 is indeed the solution to the intermediary's maximization problem.

To see whether the solution is unique, I assess the sign of the second order conditions. The Hessian is given by:

$$H = \begin{pmatrix} \frac{\partial^2 \pi^I_{ij}}{\partial p^{I2}_{ij}} & \frac{\partial^2 \pi^I_{ij}}{\partial p^I_{ij}\partial n^I_{ij}} \\ \frac{\partial^2 \pi^I_{ij}}{\partial n^I_{ij}\partial p^I_{ij}} & \frac{\partial^2 \pi^I_{ij}}{\partial n^{I2}_{ij}} \end{pmatrix}.$$

where

$$\frac{\partial^2 \pi^I_{ij}}{\partial^2 p^{I2}_{ij}} = \frac{m^I_{ij}}{G(\phi^X_{ij}) - G(\phi^W_{ij})}\int_{\phi^W_{ij}}^{\phi^X_{ij}} n^I_{ij} L_j A_j\left[\sigma(\sigma-1)p^{-\sigma-1} - \sigma(\sigma+1)p^{-\sigma-2}\frac{\sigma}{\sigma-1}\frac{\tau_{ij}w_i}{\phi}\right]dG(\phi) < 0$$

$$\frac{\partial^2 \pi^I_{ij}}{\partial p^I_{ij}\partial n^I_{ij}} = \frac{\partial^2 \pi^I_{ij}}{\partial n^I_{ij}\partial p^I_{ij}} = \frac{m^I_{ij}}{G(\phi^X_{ij}) - G(\phi^W_{ij})}\int_{\phi^W_{ij}}^{\phi^X_{ij}} L_j A_j\left[(1-\sigma)p^{-\sigma} + \sigma p^{-\sigma-1}\frac{\sigma}{\sigma-1}\frac{\tau_{ij}w_i}{\phi}\right]dG(\phi) = 0$$

$$\frac{\partial^2 \pi^I_{ij}}{\partial n^{I2}_{ij}} = -\beta w_j^\gamma w_i^{1-\gamma}\frac{L_j^\alpha}{\psi}\frac{1}{(1-n^I_{ij})^{\beta-1}} < 0 \quad \text{if } \beta > 0.$$

Because the second order conditions are satisfied, the pair (p^I_{ij}, n^I_{ij}) that solves the first order conditions of the intermediary for $\tilde{\phi}^I > \tilde{\phi}^{I^*}$, is the unique maximum of the profit maximization problem. Moreover, for $\tilde{\phi}^I \leq \tilde{\phi}^{I^*}$, $n^I_{ij} = 0$. $\square$

ii. Proof of Proposition 2

Proof. This subsection proves that the export strategies equilibrium is as described in Proposition 2, with thresholds ϕ_{ij}^W, ϕ_{ij}^X implicitly defined by:

$$\pi^W(\phi_{ij}^W) = 0 \tag{ii.1}$$
$$\pi^X(\phi_{ij}^X) = \pi^W(\phi_{ij}^X). \tag{ii.2}$$

Let the prohibitive fixed cost of intermediation be denoted by $\bar{f}_{ij}$, and be defined as:

$$\bar{f}_{ij} \equiv \{f_{ij} \in \mathbb{R} \mid \pi^X(\phi_{ij}^W) = 0\}. \tag{ii.3}$$

I will first prove the first statement in the proposition, which claims that for $f_{ij} < \bar{f}_{ij}$, the export strategy equilibrium can be characterized by unique thresholds ϕ_{ij}^W, ϕ_{ij}^X. First, note that given the definition of $\bar{f}_{ij}$ and the fact that $\pi^W(\phi)$ decreases in f_{ij} implies that $\pi^W(\phi_{ij}^W) = 0 > \pi^X(\phi_{ij}^W)$. This establishes that there is a lower bound on productivity level for which the firm would export indirectly. Moreover, since $\pi^X(\phi)$ increases faster in ϕ then $\pi^W(\phi)$, there exists an ϵ such that $\pi^W(\phi_{ij}^W + \epsilon) = \pi^X(\phi_{ij}^W + \epsilon)$ and that for all $\hat{\phi} > \phi_{ij}^W + \epsilon$, $\pi^W(\hat{\phi}) < \pi^X(\hat{\phi})$, such that (II.2) indeed defines the upper bound on productivity for which a firm would opt for intermediation.

The second assertion states that for $f_{ij} \geq \bar{f}_{ij}$, the prevailing equilibrium has no intermediation. Note that the level of fixed costs now implies that $\phi_{ij}^W \geq \phi_{ij}^*$, since ϕ_{ij}^* is the productivity level such that $\pi^X(\phi_{ij}^*) = 0$. Therefore, since $\pi^X(\phi)$ is increasing in ϕ, we have $\pi^X(\phi_{ij}^W) \geq 0$. Thus, a firm with productivity level ϕ_{ij}^W would be better off if it would export directly instead of indirectly. More generally, exporting directly always dominates exporting indirectly in terms of profits, such that there will be no intermediation in equilibrium. □