“By Example ”introduction into the Mathematica PairwiseComparisons` Package

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To install the package on the computer start Mathematica and choose File/Install..., then select PairwiseComparisons.m as the package source and press OK.

To install the package at the Raspberry PI just copy them into /opt/Wolfram/WolframEngine/10.0/AddOns/ExtraPackages and restart the application

Introduction

Loading the package

In[121]:=
```
<< PairwiseComparisons`
```

Printing function usage

In[122]:=
```
? KoczkodajIdx
```

KoczkodajIdx[M] returns the value of the Koczkodaj inconsistency computed for the matrix M

Printing full (with implementation) function usage

In[123]:=
```
?? KoczkodajIdx
```

KoczkodajIdx[M] returns the value of the Koczkodaj inconsistency computed for the matrix M

```
KoczkodajIdx[PairwiseComparisons`Private`matrix_] :=
 Last[KoczkodajTheWorstTriad[PairwiseComparisons`Private`matrix]]
```

Print all the public functions in the package

In[124]:=
```
?PairwiseComparisons`*
```

▼ PairwiseComparisons`

COP1Check	HREConstantTermVector	KoczkodajImproveMatrixStep
COP1ViolationList	HREFullRank	KoczkodajTheWorstTriad
COP2Check	HREGeomConstantTermVector	KoczkodajTheWorstTriads
COP2ViolationList	HREGeomFullRank	KoczkodajTriadIdx
DeleteColumns	HREGeomIntermediateRank	LocalDiscrepancyMatrix
DeleteRows	HREGeomMatrix	PrincipalEigenValue
DeleteRowsAndColumns	HREGeomPartialRank	PrincipalEigenVector
EigenvalueRank	HREGeomRescaledRank	RankOrder
ErrorMatrix	HREMatrix	RankToVector
GeometricRank	HREPartialRank	RecreatePCMatrix
GeometricRescaledRank	HRERescaledRank	SaatyIdx
GetMatrixEntry	KoczkodajConsistentTriad	SetDiagonal
GlobalDiscrepancy	KoczkodajIdx	

Assign the matrix to the variable M

In[125]:= $M = \begin{pmatrix} 1 & 2 & 3 \\ \frac{1}{2} & 1 & 4 \\ \frac{1}{3} & \frac{1}{4} & 1 \end{pmatrix}$

Out[125]= $\left\{\{1, 2, 3\}, \left\{\frac{1}{2}, 1, 4\right\}, \left\{\frac{1}{3}, \frac{1}{4}, 1\right\}\right\}$

Eigenvalue based method

Calculate the principal eigenvalue of the matrix M

In[126]:=
```
PrincipalEigenValue[M]
```

Out[126]= 3.108

Calculate the principal eigenvector of the matrix M

In[127]:=
```
PrincipalEigenVector[M]
```

Out[127]= {4.16, 2.884, 1.}

Calculate the value of the Saaty (eigenvalue based) inconsistency index of M

In[128]:=
```
SaatyIdx[M]
```

Out[128]= 0.05392

Calculate the eigenvalue based ranking based on M

In[129]:=
```
EigenvalueRank[M]
```

Out[129]= {0.5171, 0.3586, 0.1243}

Calculate the geometric mean based ranking based on M

Geometric Mean method (Logarithmic Least Square Method)

In[130]:=
```
N[GeometricRank[M]]
```
Out[130]=
```
{1.817, 1.26, 0.4368}
```

Calculate the geometric mean based ranking based on M rescaled so that the sum of entries is 1

In[131]:=
```
N[GeometricRescaledRank[M]]
```
Out[131]=
```
{0.5171, 0.3586, 0.1243}
```

Heuristic Rating Estimation Method (additive)

Calculate the HRE Matrix for the given matrix M where the unknown concepts are $\{c_1, c_2, c_3\}$ and the known concepts are $\{c_4 = 5, c_5 = 9\}$. It is assumed that the unknown concepts has value 0 whilst the known concepts have the values greater than 0.

Further references could be found in papers :
* Konrad Kułakowski,
Heuristic Rating Estimation Approach to The Pairwise Comparisons Method
http : // arxiv.org / abs / 1309.0386
* Konrad Kułakowski, A heuristic rating estimation algorithm for the pairwise comparisons method
http : // dx.doi.org / 10.1007 / s10100 – 013 – 0311 – x

In[132]:=
$$\text{HREMatrix}\left[\begin{pmatrix} m_{1,1} & m_{1,2} & m_{1,3} & m_{1,4} & m_{1,5} \\ m_{2,1} & m_{2,2} & m_{2,3} & m_{2,4} & m_{2,5} \\ m_{3,1} & m_{3,2} & m_{3,3} & m_{3,4} & m_{3,5} \\ m_{4,1} & m_{4,2} & m_{4,3} & m_{4,4} & m_{4,5} \\ m_{5,1} & m_{5,2} & m_{5,3} & m_{5,4} & m_{5,5} \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 5 \\ 9 \end{pmatrix}\right]$$

Out[132]=
$$\begin{pmatrix} 1 & -\frac{m_{1,2}}{4} & -\frac{m_{1,3}}{4} \\ -\frac{m_{2,1}}{4} & 1 & -\frac{m_{2,3}}{4} \\ -\frac{m_{3,1}}{4} & -\frac{m_{3,2}}{4} & 1 \end{pmatrix}$$

Calculate the HRE constant term vector for M
where C_U equals $\{c_1, c_2, c_3\}$ and C_K equals $\{c_4 = 5, c_5 = 9\}$

In[133]:=
$$\text{HREConstantTermVector}\left[\begin{pmatrix} m_{1,1} & m_{1,2} & m_{1,3} & m_{1,4} & m_{1,5} \\ m_{2,1} & m_{2,2} & m_{2,3} & m_{2,4} & m_{2,5} \\ m_{3,1} & m_{3,2} & m_{3,3} & m_{3,4} & m_{3,5} \\ m_{4,1} & m_{4,2} & m_{4,3} & m_{4,4} & m_{4,5} \\ m_{5,1} & m_{5,2} & m_{5,3} & m_{5,4} & m_{5,5} \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 5 \\ 9 \end{pmatrix}\right]$$

Out[133]=
$$\begin{pmatrix} \frac{5 m_{1,4}}{4} + \frac{9 m_{1,5}}{4} \\ \frac{5 m_{2,4}}{4} + \frac{9 m_{2,5}}{4} \\ \frac{5 m_{3,4}}{4} + \frac{9 m_{3,5}}{4} \end{pmatrix}$$

Auxiliary function that transform an upper triangle matrix into a full and reciprocal matrix

In[134]:= M = RecreatePCMatrix$\left[\begin{pmatrix} 1 & \frac{3}{5} & \frac{4}{7} & \frac{5}{8} & \frac{5}{10} \\ 0 & 1 & \frac{5}{7} & \frac{5}{2} & \frac{10}{3} \\ 0 & 0 & 1 & \frac{7}{2} & 4 \\ 0 & 0 & 0 & 1 & \frac{4}{3} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}\right]$

Out[134]= $\left\{\left\{1, \frac{3}{5}, \frac{4}{7}, \frac{5}{8}, \frac{1}{2}\right\}, \left\{\frac{5}{3}, 1, \frac{5}{7}, \frac{5}{2}, \frac{10}{3}\right\}, \left\{\frac{7}{4}, \frac{7}{5}, 1, \frac{7}{2}, 4\right\}, \left\{\frac{8}{5}, \frac{2}{5}, \frac{2}{7}, 1, \frac{4}{3}\right\}, \left\{2, \frac{3}{10}, \frac{1}{4}, \frac{3}{4}, 1\right\}\right\}$

In[135]:=

Defining known and unknown alternatives. It is assumed that c_2 and c_3 are known and equal 5 and 7 correspondingly

In[136]:= mk = $\begin{pmatrix} 0 \\ 5 \\ 7 \\ 0 \\ 0 \end{pmatrix}$

Out[136]= {{0}, {5}, {7}, {0}, {0}}

Calculate the HRE ranking vector for unknown alternatives i.e. for c_1, c_4 and c_5 only

In[137]:= mu = N[HREPartialRank[M, mk]]

Out[137]= $\begin{pmatrix} 2.528 \\ 2.883 \\ 2.617 \end{pmatrix}$

Calculate the full HRE ranking for the given input matrix M and the vector mk

In[138]:= mu = N[HREFullRank[M, mk]]

Out[138]= {2.528, 5., 7., 2.883, 2.617}

Calculate the full HRE ranking rescaled so that all its entries sum up to 1

In[139]:= **mu = N[HRERescaledRank[M, mk]]**

Out[139]= {0.1262, 0.2497, 0.3495, 0.144, 0.1307}

Heuristic Rating Estimation Method (multiplicative/geometric)

Further references could be found in papers:
 * Konrad Kułakowski, Grobler-Dębska Katarzyna, Wąs Jarosław, Heuristic rating estimation - geometric approach, http://arxiv.org/abs/1404.6981

In[140]:= **HREGeomMatrix** $\left[\begin{pmatrix} m_{1,1} & m_{1,2} & m_{1,3} & m_{1,4} & m_{1,5} \\ m_{2,1} & m_{2,2} & m_{2,3} & m_{2,4} & m_{2,5} \\ m_{3,1} & m_{3,2} & m_{3,3} & m_{3,4} & m_{3,5} \\ m_{4,1} & m_{4,2} & m_{4,3} & m_{4,4} & m_{4,5} \\ m_{5,1} & m_{5,2} & m_{5,3} & m_{5,4} & m_{5,5} \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ q_4 \\ q_5 \end{pmatrix}\right]$

Out[140]= $\begin{pmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{pmatrix}$

Calculate the HRE constant term vector

In[141]:= **HREGeomConstantTermVector** $\left[\begin{pmatrix} m_{1,1} & m_{1,2} & m_{1,3} & m_{1,4} & m_{1,5} \\ m_{2,1} & m_{2,2} & m_{2,3} & m_{2,4} & m_{2,5} \\ m_{3,1} & m_{3,2} & m_{3,3} & m_{3,4} & m_{3,5} \\ m_{4,1} & m_{4,2} & m_{4,3} & m_{4,4} & m_{4,5} \\ m_{5,1} & m_{5,2} & m_{5,3} & m_{5,4} & m_{5,5} \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ q_4 \\ q_5 \end{pmatrix}\right]$

Out[141]= $\begin{pmatrix} \frac{\log(q_4 q_5 m_{1,2} m_{1,3} m_{1,4} m_{1,5})}{\log(10)} \\ \frac{\log(q_4 q_5 m_{2,1} m_{2,3} m_{2,4} m_{2,5})}{\log(10)} \\ \frac{\log(q_4 q_5 m_{3,1} m_{3,2} m_{3,4} m_{3,5})}{\log(10)} \end{pmatrix}$

Calculate the HRE geometric ranking vector for unknown alternatives i.e. for c_1, c_4 and c_5 only

In[142]:= `mu = N[HREGeomPartialRank[M, mk]]`

Out[142]= $\begin{pmatrix} 2.113 \\ 2.49 \\ 2.133 \end{pmatrix}$

Calculate the full HRE geometric ranking for the given input matrix M and the vector mk

In[143]:= `mu = N[HREGeomFullRank[M, mk]]`

Out[143]= `{2.113, 5., 7., 2.49, 2.133}`

Calculate the full HRE geometric ranking, rescaled so that all its entries sum up to 1

In[144]:= `mu = N[HREGeomRescaledRank[M, mk]]`

Out[144]= `{0.1128, 0.2669, 0.3736, 0.1329, 0.1139}`

Show intermediate (before raisng up to the power) HRE geometric partial rank vector

In[145]:= **`N[HREGeomIntermediateRank[M, mk]]`**

Out[145]= $\begin{pmatrix} 0.3248 \\ 0.3963 \\ 0.3291 \end{pmatrix}$

Bana e Costa and Vansnick ̊s Condition of Order Preservation test

Let calculate the eigenvalue ranking

In[146]:= `rank = EigenvalueRank[M]`

Out[146]= `{0.1191, 0.2748, 0.3565, 0.131, 0.1187}`

.... and check whether the first Bana e Costa and Vansnick condition “condition of order preservation - COP" is satisfied

In[147]:=
```
COP1Check[M, mu]
```

Out[147]=
```
True
```

then check whether the second Bana e Costa and Vansnick condition (preserving intensity of preferences postulate) is satisfied

In[148]:=
```
COP2Check[M, mu]
```

Out[148]=
```
False
```

Prints the list of pairs for which the 1st COP is not satisfied

In[149]:=
```
COP1ViolationList[M, mu]
```

Out[149]=
```
{}
```

Prints the list of pairs of pairs for which the 2nd COP is not satisfied

In[150]:=
```
COP2ViolationList[M, mu]
```

Out[150]=
```
{{False, {{1, 2}, {1, 5}}}, {False, {{1, 2}, {4, 2}}}, {False, {{1, 2}, {5, 2}}},
 {False, {{1, 3}, {1, 5}}}, {False, {{1, 3}, {4, 2}}}, {False, {{1, 3}, {4, 3}}},
 {False, {{1, 3}, {5, 2}}}, {False, {{1, 3}, {5, 3}}}, {False, {{1, 4}, {1, 5}}},
 {False, {{1, 4}, {2, 3}}}, {False, {{1, 5}, {1, 2}}}, {False, {{1, 5}, {1, 3}}},
 {False, {{1, 5}, {1, 4}}}, {False, {{1, 5}, {2, 3}}}, {False, {{1, 5}, {5, 4}}},
 {False, {{2, 1}, {2, 4}}}, {False, {{2, 1}, {2, 5}}}, {False, {{2, 1}, {5, 1}}},
 {False, {{2, 3}, {1, 4}}}, {False, {{2, 3}, {1, 5}}}, {False, {{2, 4}, {2, 1}}},
 {False, {{2, 4}, {3, 1}}}, {False, {{2, 5}, {2, 1}}}, {False, {{2, 5}, {3, 1}}},
 {False, {{3, 1}, {2, 4}}}, {False, {{3, 1}, {2, 5}}}, {False, {{3, 1}, {3, 4}}},
 {False, {{3, 1}, {3, 5}}}, {False, {{3, 1}, {5, 1}}}, {False, {{3, 2}, {4, 1}}},
 {False, {{3, 2}, {5, 1}}}, {False, {{3, 4}, {3, 1}}}, {False, {{3, 5}, {3, 1}}},
 {False, {{4, 1}, {3, 2}}}, {False, {{4, 1}, {5, 1}}}, {False, {{4, 2}, {1, 2}}},
 {False, {{4, 2}, {1, 3}}}, {False, {{4, 3}, {1, 3}}}, {False, {{4, 5}, {5, 1}}},
 {False, {{5, 1}, {2, 1}}}, {False, {{5, 1}, {3, 1}}}, {False, {{5, 1}, {3, 2}}},
 {False, {{5, 1}, {4, 1}}}, {False, {{5, 1}, {4, 5}}}, {False, {{5, 2}, {1, 2}}},
 {False, {{5, 2}, {1, 3}}}, {False, {{5, 3}, {1, 3}}}, {False, {{5, 4}, {1, 5}}}}
```

Koczkodaj's Iterative Inconsistency Reduction algorithm

Calculate the value of the Koczkodaj inconsistency index

In[151]:=
```
N[KoczkodajIdx[M]]
```

Out[151]=
```
0.7813
```

Prints the worst Koczkodaj triad in M. As we can see it is $m_{5,3} == \frac{1}{4}$, $m_{3,1} == \frac{7}{4}$, $m_{5,1} == 2$. The value of inconsistency introduced by this triad is $\frac{25}{32}$

In[152]:=
```
KoczkodajTheWorstTriad[M]
```

Out[152]= $\left\{\{5, 3, 1\}, \left\{\frac{1}{4}, \frac{7}{4}, 2\right\}, \frac{25}{32}\right\}$

Perform one step of the Koczkodaj inconsistency reduction algorithm. On the output there is a new slightly modified matrix M2 that is expected to be more consistent than M

In[153]:=
```
M2 = N[KoczkodajImproveMatrixStep[M]]
```

Out[153]=

$$\begin{pmatrix} 1. & 0.6 & 0.3443 & 0.625 & 0.8298 \\ 1.667 & 1. & 0.7143 & 2.5 & 3.333 \\ 2.904 & 1.4 & 1. & 3.5 & 2.41 \\ 1.6 & 0.4 & 0.2857 & 1. & 1.333 \\ 1.205 & 0.3 & 0.4149 & 0.75 & 1. \end{pmatrix}$$

In[154]:=
```
KoczkodajIdx[M2]
```

Out[154]=
```
0.5851
```