Homework 2 solutions

Problem 1 (10.1)

a) Starting from the LHS of the equation, note that

$$\frac{1}{|C_k|}\sum_{i,i'\in C_k}\sum_{j=1}^{p}(x_{ij}-x_{i'j})^2 = \frac{1}{|C_k|}\sum_{i,i'\in C_k}\sum_{j=1}^{p}(x_{ij}-\bar{x}_{kj}+\bar{x}_{kj}-x_{i'j})^2$$

$$= \frac{1}{|C_k|}\sum_{i,i'\in C_k}\sum_{j=1}^{p}(x_{ij}-\bar{x}_{kj})^2 + \frac{1}{|C_k|}\sum_{i,i'\in C_k}\sum_{j=1}^{p}2(x_{ij}-\bar{x}_{kj})(\bar{x}_{kj}-x_{i'j}) + \frac{1}{|C_k|}\sum_{i,i'\in C_k}\sum_{j=1}^{p}(\bar{x}_{kj}-x_{i'j})^2$$

The first and third terms in the expression above are equal, differing only by a choice in indexing notation between i and i'. Hence the expression above is equal to:

$$\frac{2}{|C_k|}\sum_{i,i'\in C_k}\sum_{j=1}^{p}(x_{ij}-\bar{x}_{kj})^2 + \frac{2}{|C_k|}\sum_{i,i'\in C_k}\sum_{j=1}^{p}(x_{ij}-\bar{x}_{kj})(\bar{x}_{kj}-x_{i'j})$$

$$= \frac{2}{|C_k|}\sum_{i\in C_k}\sum_{i'\in C_k}\sum_{j=1}^{p}(x_{ij}-\bar{x}_{kj})^2 + \frac{2}{|C_k|}\sum_{i\in C_k}\sum_{i'\in C_k}\sum_{j=1}^{p}(x_{ij}-\bar{x}_{kj})(\bar{x}_{kj}-x_{i'j})$$

$$= \frac{2}{|C_k|}\sum_{i\in C_k}\sum_{j=1}^{p}\sum_{i'\in C_k}(x_{ij}-\bar{x}_{kj})^2 + \frac{2}{|C_k|}\sum_{i\in C_k}\sum_{j=1}^{p}\sum_{i'\in C_k}(x_{ij}-\bar{x}_{kj})(\bar{x}_{kj}-x_{i'j})$$

$$= \frac{2}{|C_k|}\sum_{i\in C_k}\sum_{j=1}^{p}(x_{ij}-\bar{x}_{kj})^2\left(\sum_{i'\in C_k}1\right) + \frac{2}{|C_k|}\sum_{i\in C_k}\sum_{j=1}^{p}(x_{ij}-\bar{x}_{kj})\left(\sum_{i'\in C_k}(\bar{x}_{kj}-x_{i'j})\right)$$

$$= \frac{2}{|C_k|}\sum_{i\in C_k}\sum_{j=1}^{p}(x_{ij}-\bar{x}_{kj})^2\,(|C_k|) + \frac{2}{|C_k|}\sum_{i\in C_k}\sum_{j=1}^{p}(x_{ij}-\bar{x}_{kj})\,(0)$$

$$= 2\sum_{i\in C_k}\sum_{j=1}^{p}(x_{ij}-\bar{x}_{kj})^2 \quad \blacksquare$$

b) The result of part (a) is that objective (10.11) is equivalent to the objective:

$$\min_{C_1,\ldots,C_K}\left\{\sum_{k=1}^{K}\sum_{i\in C_k}\sum_{j=1}^{p}(x_{ij}-\bar{x}_{kj})^2\right\}$$

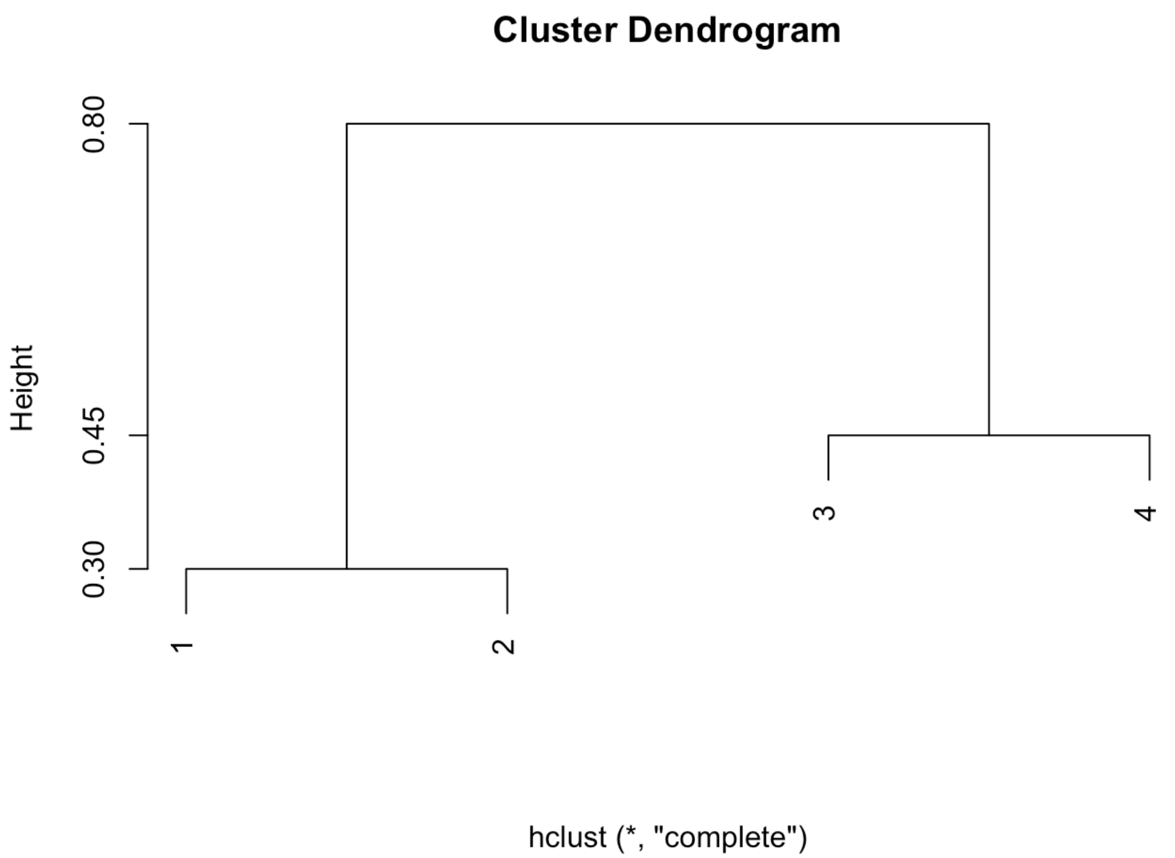
That is, minimize the sum of distances from observations to their centroids. Note that in Algorithm 10.1, with each iteration there are two steps: (a) compute each cluster centroid and (b) assign each observation to the nearest centroid. Step (a) necessarily decreases this objective because the cluster centroid is the point which minimizes the sum of squared distances to its members, and step (b) necessarily decreases this objective because assigning each observation to the nearest centroid minimizes the distance from it to the centroid.

Problem 2 (10.2)

```
dissimilarity = as.dist(matrix(c(0, 0.3, 0.4, 0.7, 0.3, 0, 0.5, 0.8, 0.4, 0.5,
    0, 0.45, 0.7, 0.8, 0.45, 0), 4, 4))
```

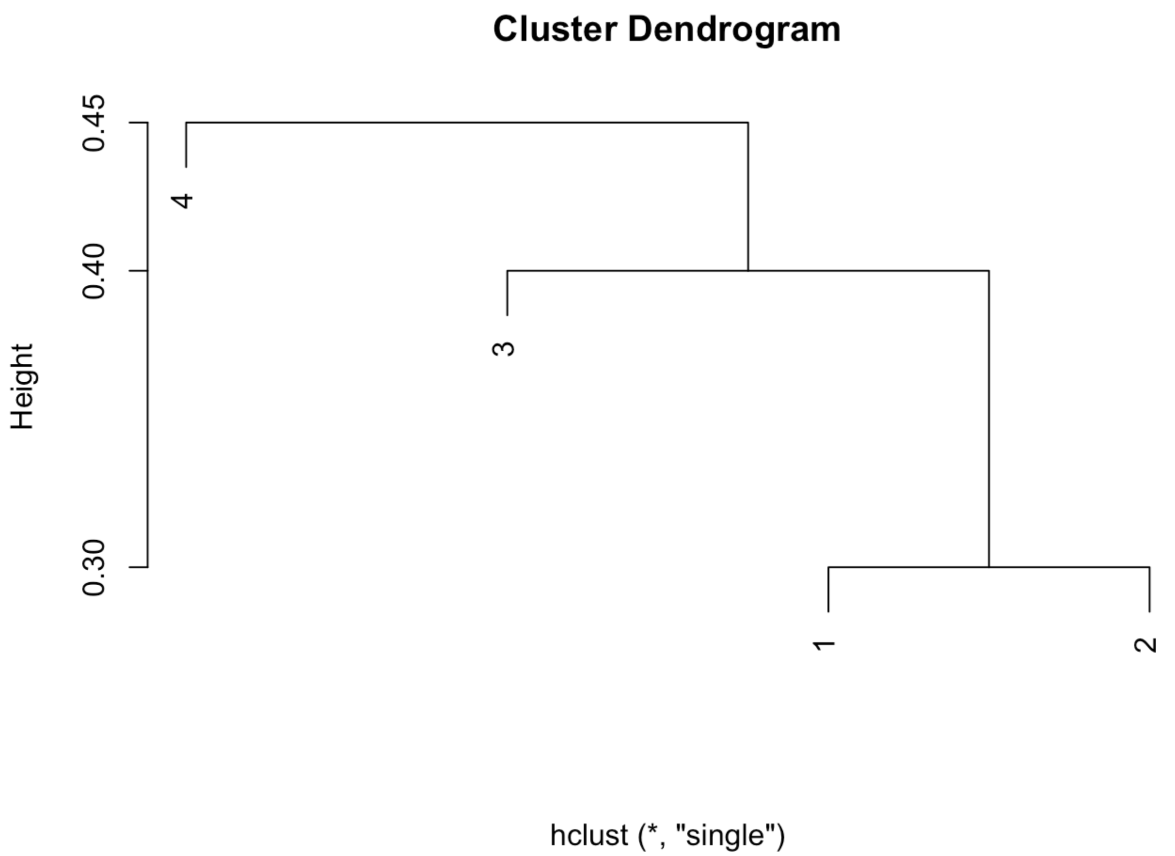
a. Complete linkage:

```
plot(hclust(dissimilarity), axes = F, xlab = NA)
axis(2, at = c(0.3, 0.45, 0.8))
```



b. Single linkage:

```
plot(hclust(dissimilarity, method = "single"), axes = F, xlab = NA)
axis(2, at = c(0.3, 0.4, 0.45))
```

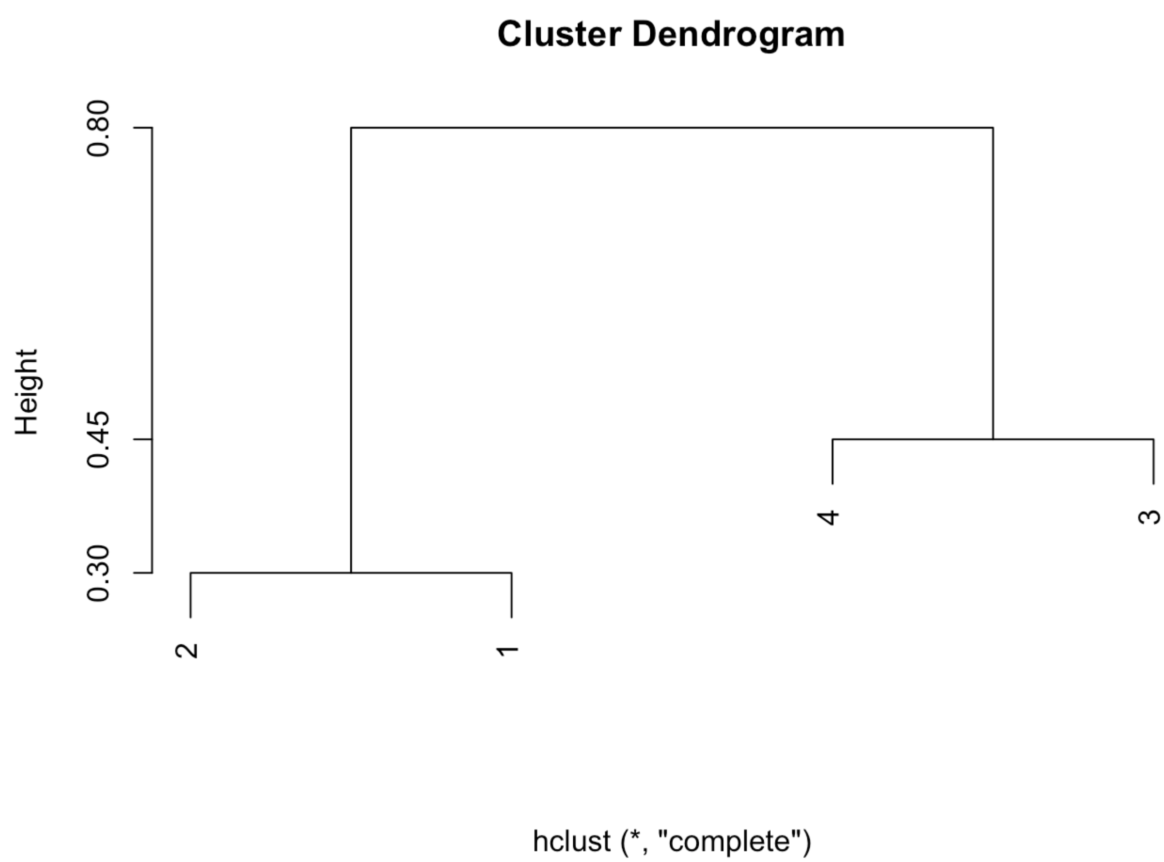


c. The two clusters would be {1,2} and {3,4}.

d. The two clusters would be {1,2,3} and {4}.

e. Below is one possible solution to this problem, but answers may vary greatly.

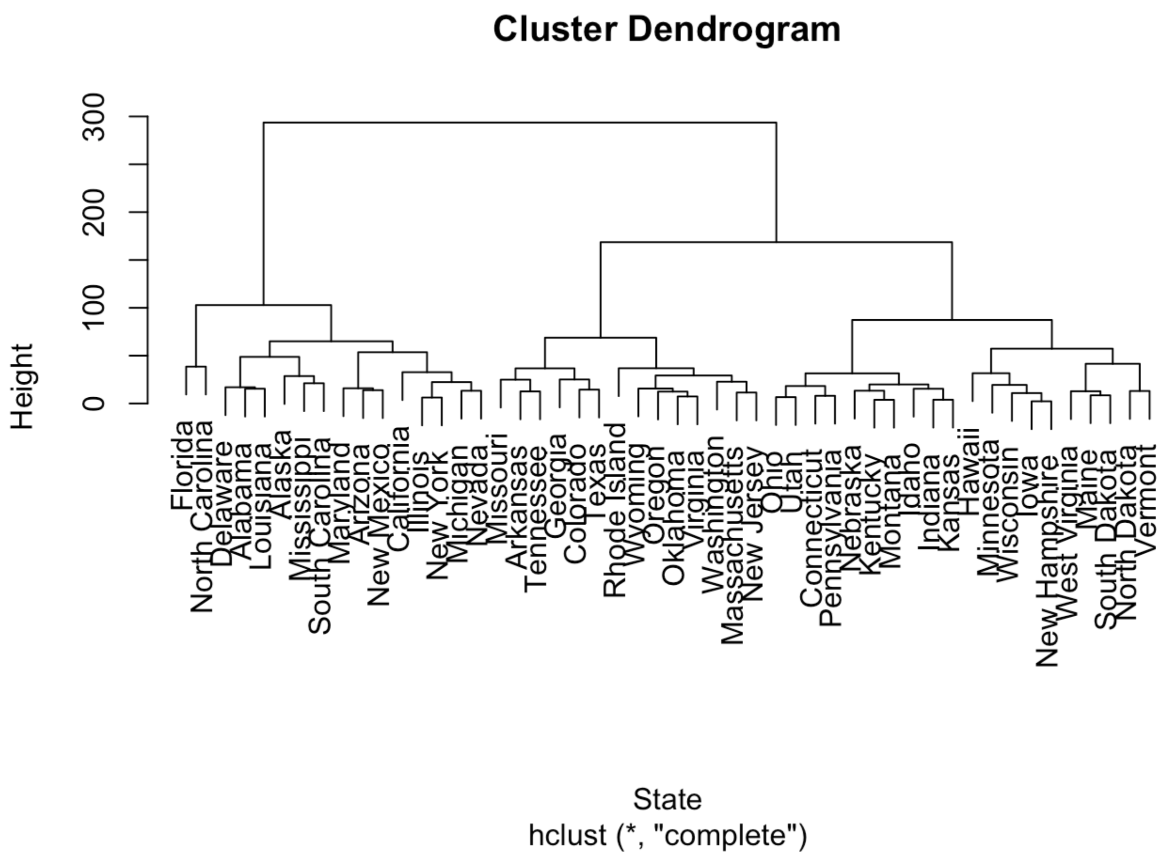
```
plot(hclust(dissimilarity), labels = c("2", "1", "4", "3"), axes = F, xlab = N
A)
axis(2, at = c(0.3, 0.45, 0.8))
```



Problem 3 (10.9)

a. Clustering before scaling variables:

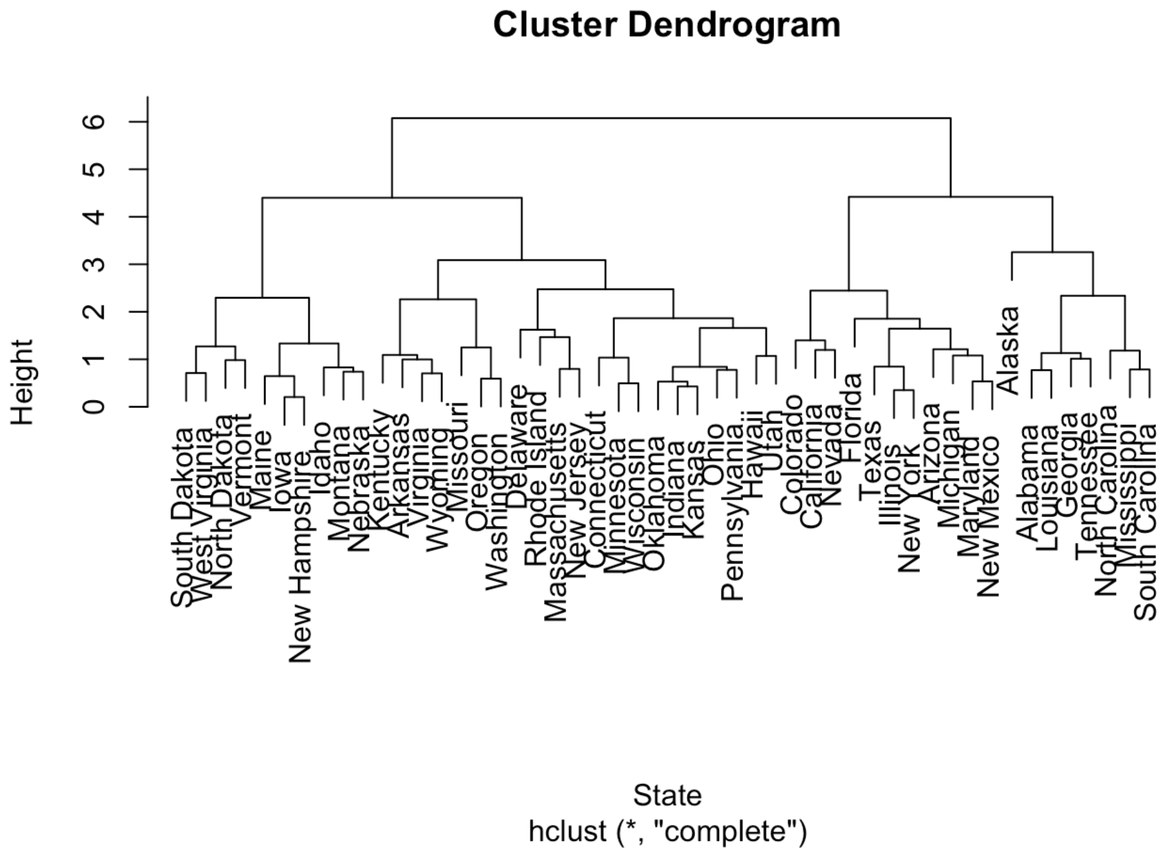
```
plot(hclust(dist(USArrests)), xlab = "State")
```



b. The three clusters are: {FL,NC,DE,AL,LA,AK,MS,SC,MD,AZ,NM,CA,IL,NY,MI,NV}
{MO,AR,TN,GA,CO,TX,RI,WY,OR,OK,VA,WA,MA,NJ}
{OH,UT,CT,PA,NE,KY,MT,ID,IN,KS,HI,MN,WI,IA,NH,WV,ME,SD,ND,VT}

c. Clustering after scaling variables:

```
plot(hclust(dist(scale(USArrests, center = T))), xlab = "State")
```



d. Scaling the variables leads to a different clustering of the states. Before scaling, it was the assaults variable that was primarily responsible for the clustering because it had by far the largest standard deviation. Variables should be scaled before inter-observation dissimilarities are computed because that way each of the variables has comparable weight in the clustering.

Problem 5 (3.4)

a) We would expect the training RSS for the **cubic regression** to be lower than for the linear regression because fitting the cubic regression model minimizes training RSS over all cubic functions, which include all linear function as a subset.

b) We would expect the test RSS for the **linear regression** to be lower because the cubic regression model might over fit the training data, leading to high variance in the estimate of the function. However, if n is large, the two estimates could be close, making their test RSS similar.

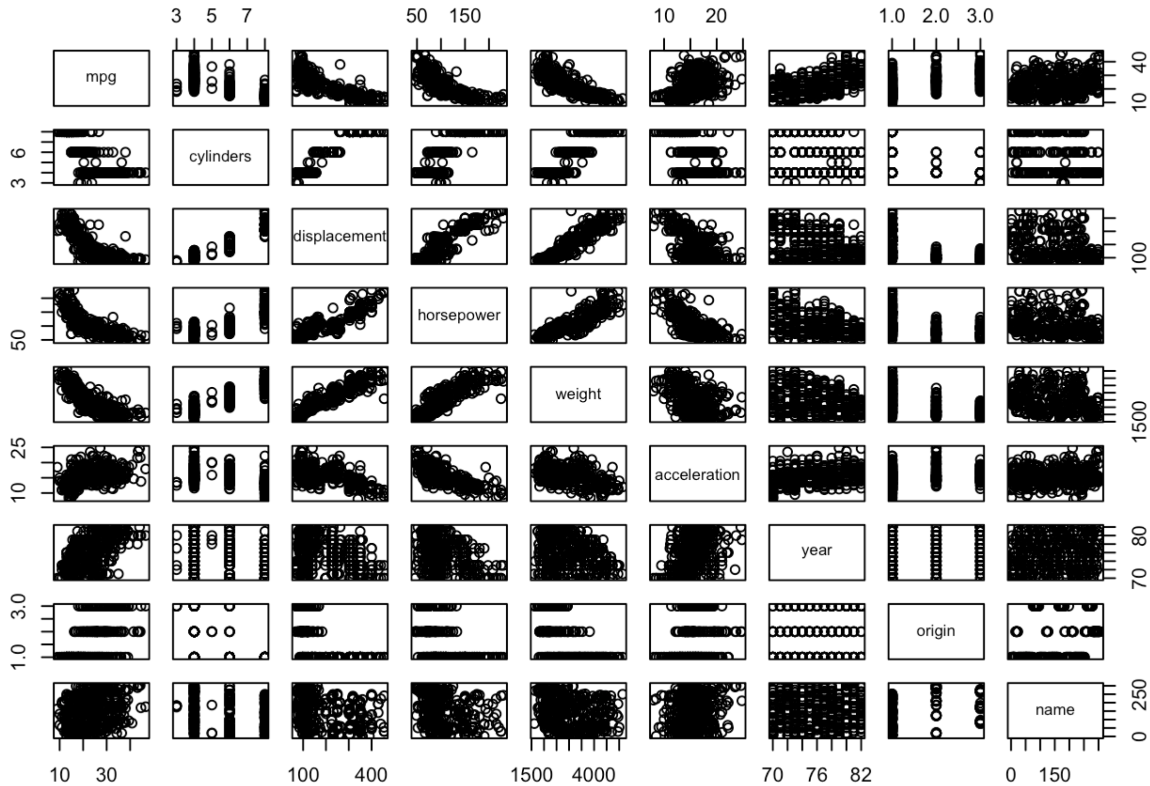
c) Again we would expect the training RSS for the **cubic regression** to be lower than for the linear regression for the same reasoning as in part (a).

d) Here, there is **not enough information to tell**. If the true relationship between X and Y is cubic, then we would expect the test RSS to be lower for the cubic regression model. However, if the true relationship is quadratic, for example, the test RSS could be lower for the linear model.

Problem 6

a. Pairwise scatterplot matrix:

```r
library(ISLR)
Auto = na.omit(Auto)
pairs(Auto)
```



b. Correlation matrix:

```r
round(cor(Auto[, -9]), digits = 3)
```

```
##                 mpg cylinders displacement horsepower weight acceleration
## mpg           1.000    -0.778       -0.805     -0.778 -0.832        0.423
## cylinders    -0.778     1.000        0.951      0.843  0.898       -0.505
## displacement -0.805     0.951        1.000      0.897  0.933       -0.544
## horsepower   -0.778     0.843        0.897      1.000  0.865       -0.689
## weight       -0.832     0.898        0.933      0.865  1.000       -0.417
## acceleration  0.423    -0.505       -0.544     -0.689 -0.417        1.000
## year          0.581    -0.346       -0.370     -0.416 -0.309        0.290
## origin        0.565    -0.569       -0.615     -0.455 -0.585        0.213
##                year origin
## mpg           0.581  0.565
## cylinders    -0.346 -0.569
## displacement -0.370 -0.615
## horsepower   -0.416 -0.455
## weight       -0.309 -0.585
## acceleration  0.290  0.213
## year          1.000  0.182
## origin        0.182  1.000
```

c. Note that the variable origin is categorical, but has been coded as a quantitative variable. We should correct this before moving on with the regression analysis:

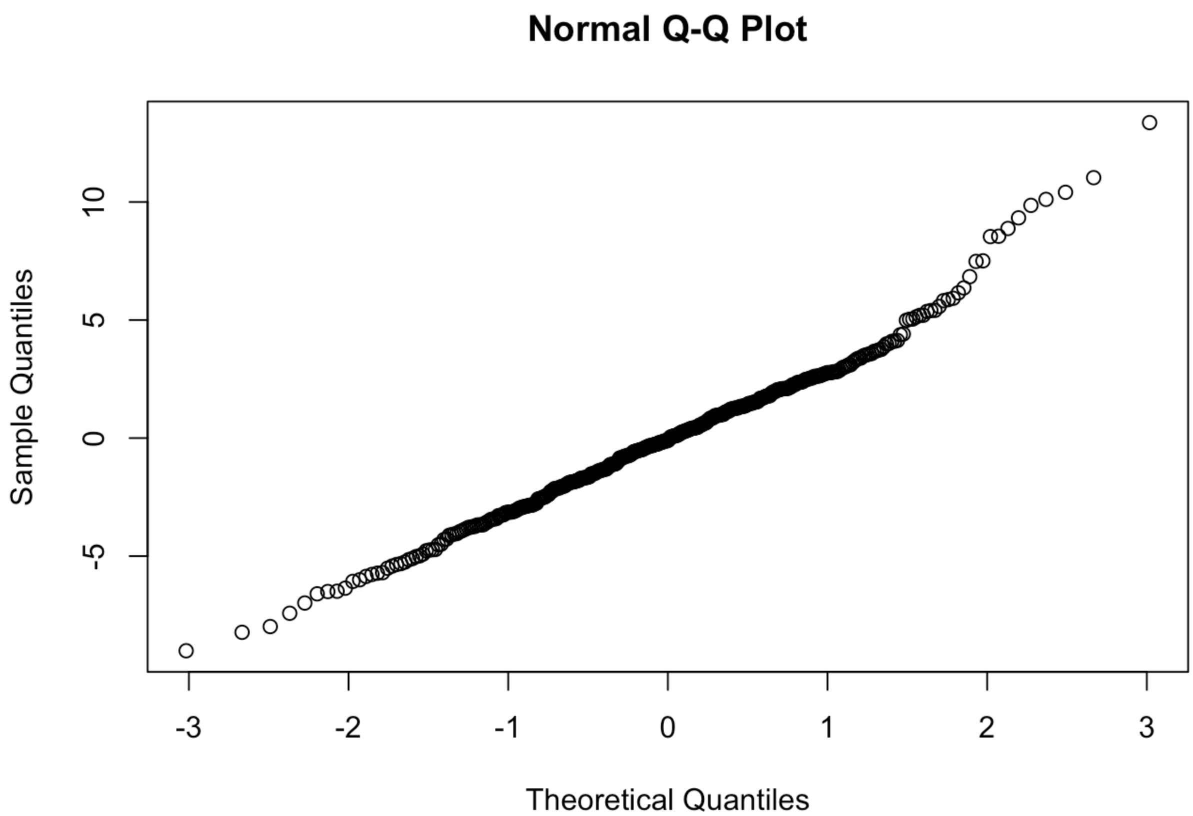
```
Auto$origin = factor(Auto$origin)
autoLinearModel = lm(mpg ~ . - name, data = Auto)
summary(autoLinearModel)
```

```
##
## Call:
## lm(formula = mpg ~ . - name, data = Auto)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.0095 -2.0785 -0.0982  1.9856 13.3608
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -1.795e+01  4.677e+00  -3.839 0.000145 ***
## cylinders    -4.897e-01  3.212e-01  -1.524 0.128215
## displacement  2.398e-02  7.653e-03   3.133 0.001863 **
## horsepower   -1.818e-02  1.371e-02  -1.326 0.185488
## weight       -6.710e-03  6.551e-04 -10.243  < 2e-16 ***
## acceleration  7.910e-02  9.822e-02   0.805 0.421101
## year          7.770e-01  5.178e-02  15.005  < 2e-16 ***
## origin2       2.630e+00  5.664e-01   4.643 4.72e-06 ***
## origin3       2.853e+00  5.527e-01   5.162 3.93e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.307 on 383 degrees of freedom
## Multiple R-squared:  0.8242, Adjusted R-squared:  0.8205
## F-statistic: 224.5 on 8 and 383 DF,  p-value: < 2.2e-16
```

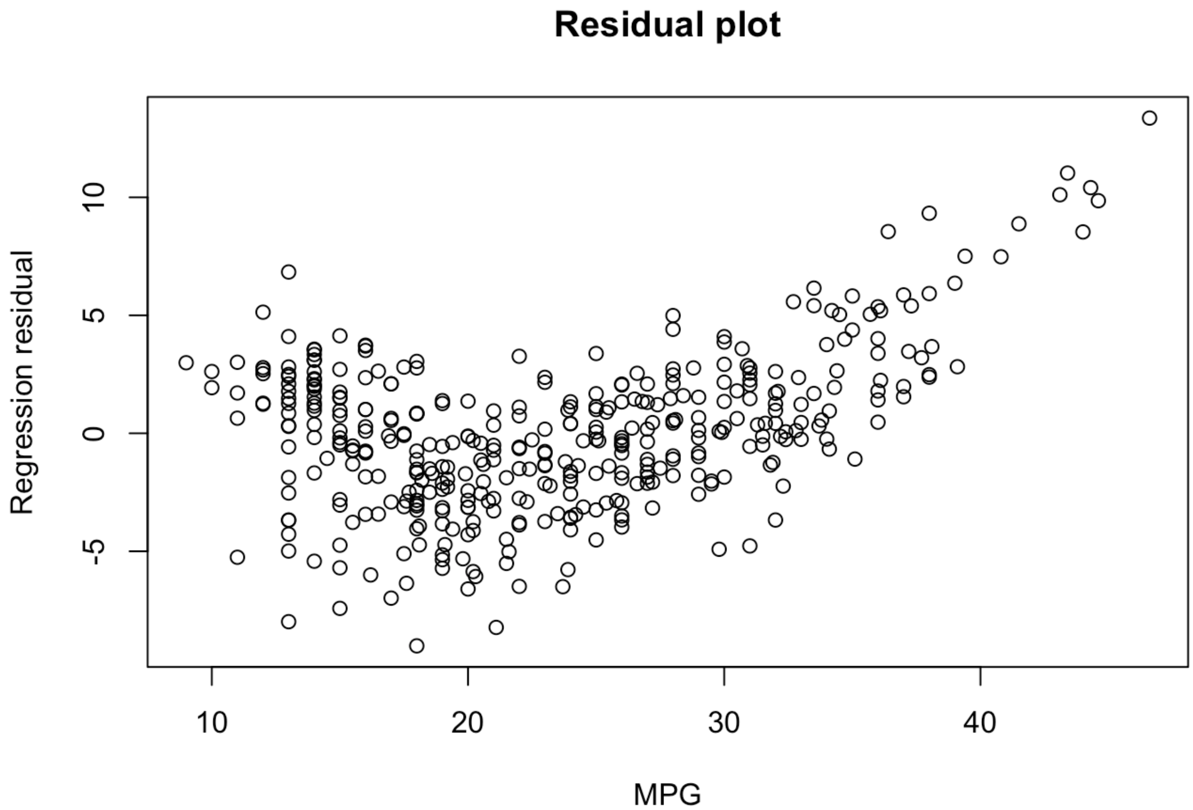
We note that:

There is a strong linear relationship between the predictors and the response. The predictors explain 82% of the variability in the response. The variables which appear to have the most significant relationship with the response are displacement, weight, year and origin. The positive estimated coefficient for the variable year suggests that, all else equal, gas efficiency improves over time. d)

```
qqnorm(autoLinearModel$residuals)
```




```
plot(Auto[, 1], autoLinearModel$residuals, xlab = "MPG", ylab = "Regression res
idual",
    main = "Residual plot")
```



The first plot, a normal quantile-quantile plot of the regression residuals, shows that the residuals are approximately normally distributed and that there are no extremely surprising outliers. However, the other plot, the residual plot, shows a problem with the fit of the model. There is a dependence between the response (MPG) and the residual. For small and large MPGs, the model seems to underestimate, and for MPGs in the middle, the model seems to overestimate.

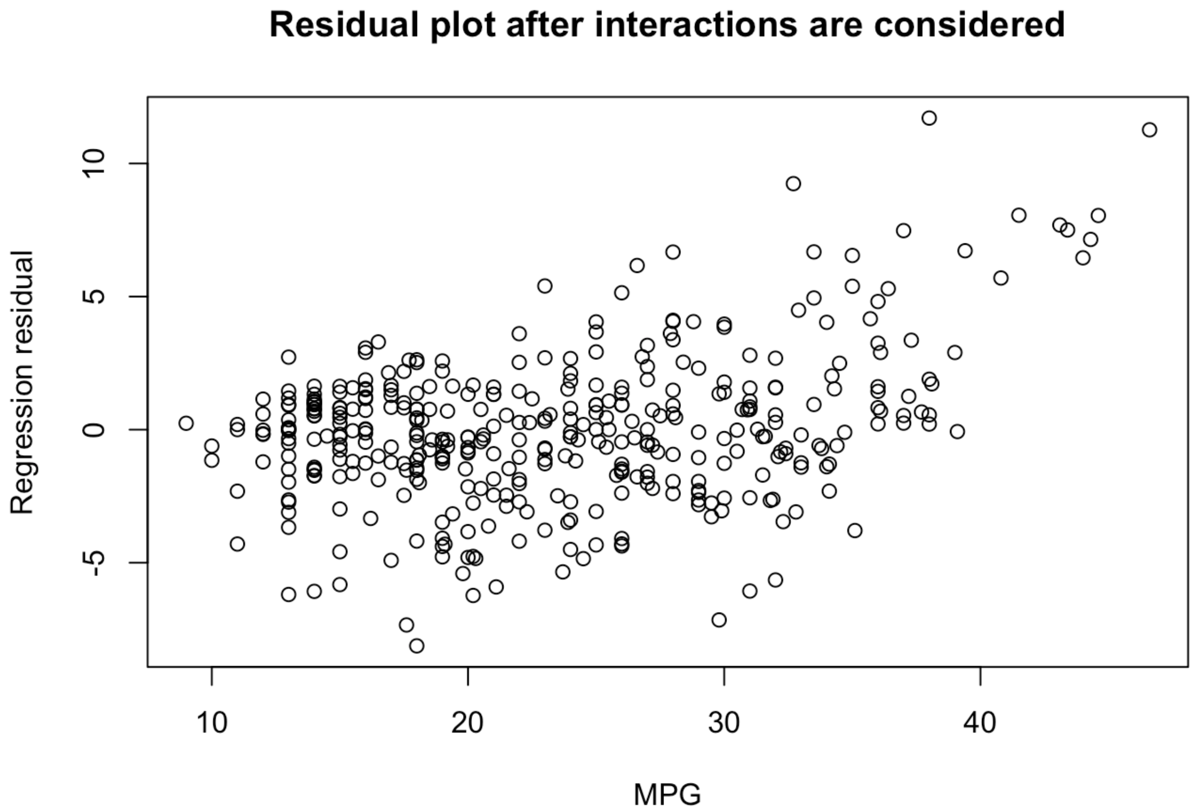
e. In this setting, there are (72)=32 pairwise interactions to consider. Hence there are 235=34359738368 choices of “models with interaction effects” (more if you consider three-way interactions, four-way interactions, etc.). It is not feasible to consider such a large set of models, so we can approach this problem in a greedy fashion: Consider all 35 pairwise interactions and include the one with the largest (standardized) effect. Then consider the remaining 34 pairwise interactions and again include the one with the largest effect. You could do this repeatedly until the effects being added to the linear model are no longer significant.

In this problem, the most significant interaction is between displacement and horsepower. Once that interaction is included in the model, the next most significant interaction is between horsepower and year. One could continue from there, but the next interaction is of dubious significance. To answer the question posed by the textbook, yes, there are definitely interactions that appear to be significant. However, including these interaction terms does not entirely fix (see plot below) the diagnostic problem from part (d). In order to fix this problem, one should think critically about a physical model for gas consumption and intelligently choose transformations of the variables to reflect this, as the next part of this exercise is getting at.

```
autoInteractionModel = lm(mpg ~ . - name + displacement:horsepower + horsepowe
r:year,
    data = Auto)
summary(autoInteractionModel)
```

```
##
## Call:
## lm(formula = mpg ~ . - name + displacement:horsepower + horsepower:year,
##     data = Auto)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.1231 -1.4969 -0.0565  1.3339 11.7067
##
## Coefficients:
##                           Estimate Std. Error t value Pr(>|t|)
## (Intercept)             -5.235e+01  1.141e+01  -4.588 6.09e-06 ***
## cylinders                6.949e-01  2.946e-01   2.359  0.01885 *
## displacement            -5.784e-02  1.153e-02  -5.016 8.12e-07 ***
## horsepower               3.255e-01  1.111e-01   2.931  0.00358 **
## weight                  -3.396e-03  6.477e-04  -5.243 2.63e-07 ***
## acceleration            -2.034e-01  8.836e-02  -2.302  0.02189 *
## year                     1.378e+00  1.402e-01   9.834  < 2e-16 ***
## origin2                  1.239e+00  5.124e-01   2.417  0.01611 *
## origin3                  1.461e+00  4.929e-01   2.964  0.00322 **
## displacement:horsepower  3.919e-04  5.459e-05   7.178 3.72e-12 ***
## horsepower:year         -6.612e-03  1.385e-03  -4.773 2.59e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.835 on 381 degrees of freedom
## Multiple R-squared:  0.8714, Adjusted R-squared:  0.8681
## F-statistic: 258.2 on 10 and 381 DF,  p-value: < 2.2e-16
```

```
plot(Auto[, 1], autoInteractionModel$residuals, xlab = "MPG", ylab = "Regressio
n residual",
    main = "Residual plot after interactions are considered")
```



f. This question is quite open-ended. However, from the scatter plot matrix in part (a), it seems there is a quadratic relationship between MPG and displacement, and MPG and weight.

```
autoLinearModel = lm(mpg ~ . - name + I(weight^2) + I(displacement^2), data = A
uto)
summary(autoLinearModel)
```

```
##
## Call:
## lm(formula = mpg ~ . - name + I(weight^2) + I(displacement^2),
##     data = Auto)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.5345 -1.7279  0.0206  1.6162 12.3679
##
## Coefficients:
##                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)       -2.232e+00  4.648e+00  -0.480  0.63138
## cylinders          2.048e-01  3.353e-01   0.611  0.54171
## displacement      -3.950e-02  2.139e-02  -1.847  0.06557 .
## horsepower        -4.184e-02  1.356e-02  -3.085  0.00218 **
## weight            -1.476e-02  2.600e-03  -5.678 2.70e-08 ***
## acceleration       2.136e-02  8.890e-02   0.240  0.81029
## year               8.034e-01  4.697e-02  17.106  < 2e-16 ***
## origin2            1.195e+00  5.619e-01   2.127  0.03409 *
## origin3            1.003e+00  5.413e-01   1.853  0.06465 .
## I(weight^2)        1.468e-06  3.540e-07   4.147 4.16e-05 ***
## I(displacement^2)  9.662e-05  3.435e-05   2.813  0.00516 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.948 on 381 degrees of freedom
## Multiple R-squared:  0.861,  Adjusted R-squared:  0.8574
## F-statistic:   236 on 10 and 381 DF,  p-value: < 2.2e-16
```

Both of the quadratic terms included appear to be significant and improve the R2 and adjusted R2 statistics.

Problem 7 (3.14)

a) The form of the linear model is $Y_i = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \epsilon_i$ for $i = 1, \ldots, 100$, where $\epsilon_i \sim$ i.i.d. $\mathcal{N}(0, 1)$. In this case, the regression coefficients are $\beta_0 = 2, \beta_1 = 2$ and $\beta_2 = 0.3$.

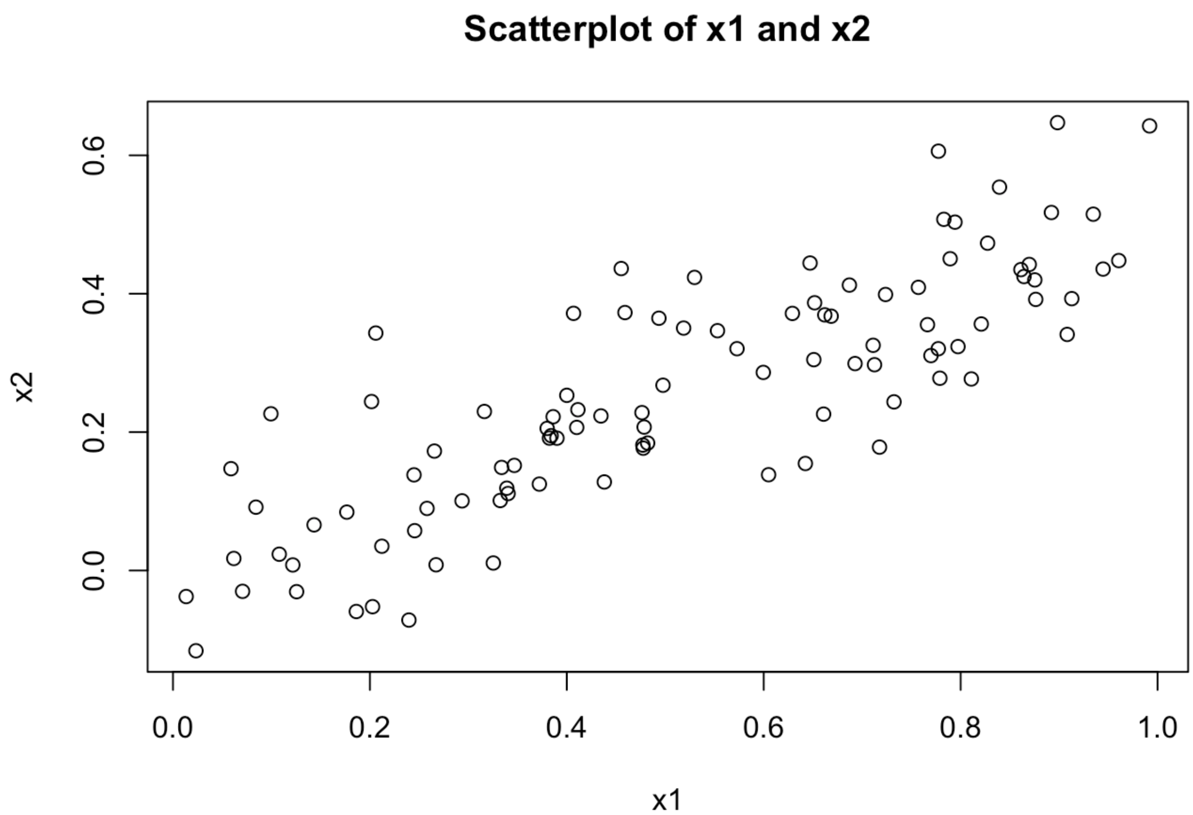
```
set.seed(1)  # We set the seed to obtain the same result
# every time the script is run.
x1 = runif(100)
x2 = 0.5 * x1 + rnorm(100)/10
y = 2 + 2 * x1 + 0.3 * x2 + rnorm(100)
```

b.

```
cor(x1, x2)
```

```
## [1] 0.8351212
```

```
plot(x1, x2, main = "Scatterplot of x1 and x2")
```



c)

```
summary(lm(y ~ x1 + x2))
```

```
##
## Call:
## lm(formula = y ~ x1 + x2)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.8311 -0.7273 -0.0537  0.6338  2.3359
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.1305     0.2319   9.188 7.61e-15 ***
## x1            1.4396     0.7212   1.996   0.0487 *
## x2            1.0097     1.1337   0.891   0.3754
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.056 on 97 degrees of freedom
## Multiple R-squared:  0.2088, Adjusted R-squared:  0.1925
## F-statistic:  12.8 on 2 and 97 DF,  p-value: 1.164e-05
```

The result of fitting the model is $\hat{\beta}_0 = 2.13, \hat{\beta}_1 = 1.44$ and $\hat{\beta}_2 = 1.01$. The estimate of β_0 (2) is close, but the estimate of β_1 (2) is too low, and the estimate of β_2 (0.3) is too high. At α-level .05, you can reject the null hypothesis $H_0 : \beta_1 = 0$ but not the null hypothesis $H_1 : \beta_2 = 0$. It seems that because of the high correlation between x_1 and x_2, their effects are confounded, but hypothesis testing still reflects that the effect of x_1 is more significant.

d.

```
summary(lm(y ~ x1))
```

```
##
## Call:
## lm(formula = y ~ x1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.89495 -0.66874 -0.07785  0.59221  2.45560
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.1124     0.2307   9.155 8.27e-15 ***
## x1            1.9759     0.3963   4.986 2.66e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.055 on 98 degrees of freedom
## Multiple R-squared:  0.2024, Adjusted R-squared:  0.1942
## F-statistic: 24.86 on 1 and 98 DF,  p-value: 2.661e-06
```

The result of fitting the model with only x_1 as a predictor is $\hat{\beta}_1 = 1.98$, with standard error 0.40. Yes, you can reject the null hypothesis $H_0 : \beta_1 = 0$ at any reasonable significance level. This is consistent with the result of part ©.

e.

```
summary(lm(y ~ x2))
```

```
##
## Call:
## lm(formula = y ~ x2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.62687 -0.75156 -0.03598  0.72383  2.44890
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.3899     0.1949   12.26  < 2e-16 ***
## x2            2.8996     0.6330    4.58 1.37e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.072 on 98 degrees of freedom
## Multiple R-squared:  0.1763, Adjusted R-squared:  0.1679
## F-statistic: 20.98 on 1 and 98 DF,  p-value: 1.366e-05
```

The result of fitting the model with only x_2 as a predictor is $\hat{\beta}_1 = 2.90$, with standard error 0.63. Yes, you can reject the null hypothesis $H_0 : \beta_1 = 0$ at any reasonable significance level. This is *not* consistent with the result of part ©.

f. The result of part (e) is contradictory to the result of part (c) and shows how the significance test for one variable can be affected by the inclusion of another variable in the model.

g.

```
x1 = c(x1, 0.1)
x2 = c(x2, 0.8)
y = c(y, 6)
summary(lm(y ~ x1 + x2))
```

```
##
## Call:
## lm(formula = y ~ x1 + x2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.73348 -0.69318 -0.05263  0.66385  2.30619
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.2267     0.2314   9.624 7.91e-16 ***
## x1            0.5394     0.5922   0.911  0.36458
## x2            2.5146     0.8977   2.801  0.00614 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.075 on 98 degrees of freedom
## Multiple R-squared:  0.2188, Adjusted R-squared:  0.2029
## F-statistic: 13.72 on 2 and 98 DF,  p-value: 5.564e-06
```

```r
summary(lm(y ~ x1))
```

```
##
## Call:
## lm(formula = y ~ x1)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.8897 -0.6556 -0.0909  0.5682  3.5665
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.2569     0.2390   9.445 1.78e-15 ***
## x1            1.7657     0.4124   4.282 4.29e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.111 on 99 degrees of freedom
## Multiple R-squared:  0.1562, Adjusted R-squared:  0.1477
## F-statistic: 18.33 on 1 and 99 DF,  p-value: 4.295e-05
```

```r
summary(lm(y ~ x2))
```

```
##
## Call:
## lm(formula = y ~ x2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.64729 -0.71021 -0.06899  0.72699  2.38074
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.3451     0.1912  12.264  < 2e-16 ***
## x2            3.1190     0.6040   5.164 1.25e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.074 on 99 degrees of freedom
## Multiple R-squared:  0.2122, Adjusted R-squared:  0.2042
## F-statistic: 26.66 on 1 and 99 DF,  p-value: 1.253e-06
```

```r
influence(lm(y ~ x1 + x2))$hat[101]
```

```
##       101
## 0.4147284
```

```r
influence(lm(y ~ x1))$hat[101]
```

```
##        101
## 0.03347156
```

```
influence(lm(y ~ x2))$hat[101]
```

```
##       101
## 0.1013106
```

Introducing the mis-measured observation does not change the hypothesis testing result of either of the single-predictor regressions, but in the full regression, the significance is switched: You can reject the null hypothesis $H_0 : \beta_2 = 0$ but not the null hypothesis $H_1 : \beta_1 = 0$. Inspection of the histograms of the three variables reveals that the new observation is not an outlier in any of the three models. However, this observation is a point of high leverage in the full regression model (leverage statistic $h_{101} = 0.41$) and the regression model with just x_2 as a predictor ($h_{101} = 0.10$), not in the regression model with just x_1 as a predictor ($h_{101} = 0.03$).

Problem 8 (3.15)

```
library(MASS)
```

a. We ran simple linear regressions with the following commands (output suppressed). The response has a significant relationship with all predictors except chad.

```
summary(lm(crim ~ zn, data = Boston))
summary(lm(crim ~ indus, data = Boston))
summary(lm(crim ~ chas, data = Boston))
summary(lm(crim ~ nox, data = Boston))
summary(lm(crim ~ rm, data = Boston))
summary(lm(crim ~ age, data = Boston))
summary(lm(crim ~ dis, data = Boston))
summary(lm(crim ~ rad, data = Boston))
summary(lm(crim ~ tax, data = Boston))
summary(lm(crim ~ ptratio, data = Boston))
summary(lm(crim ~ black, data = Boston))
summary(lm(crim ~ lstat, data = Boston))
summary(lm(crim ~ medv, data = Boston))
```

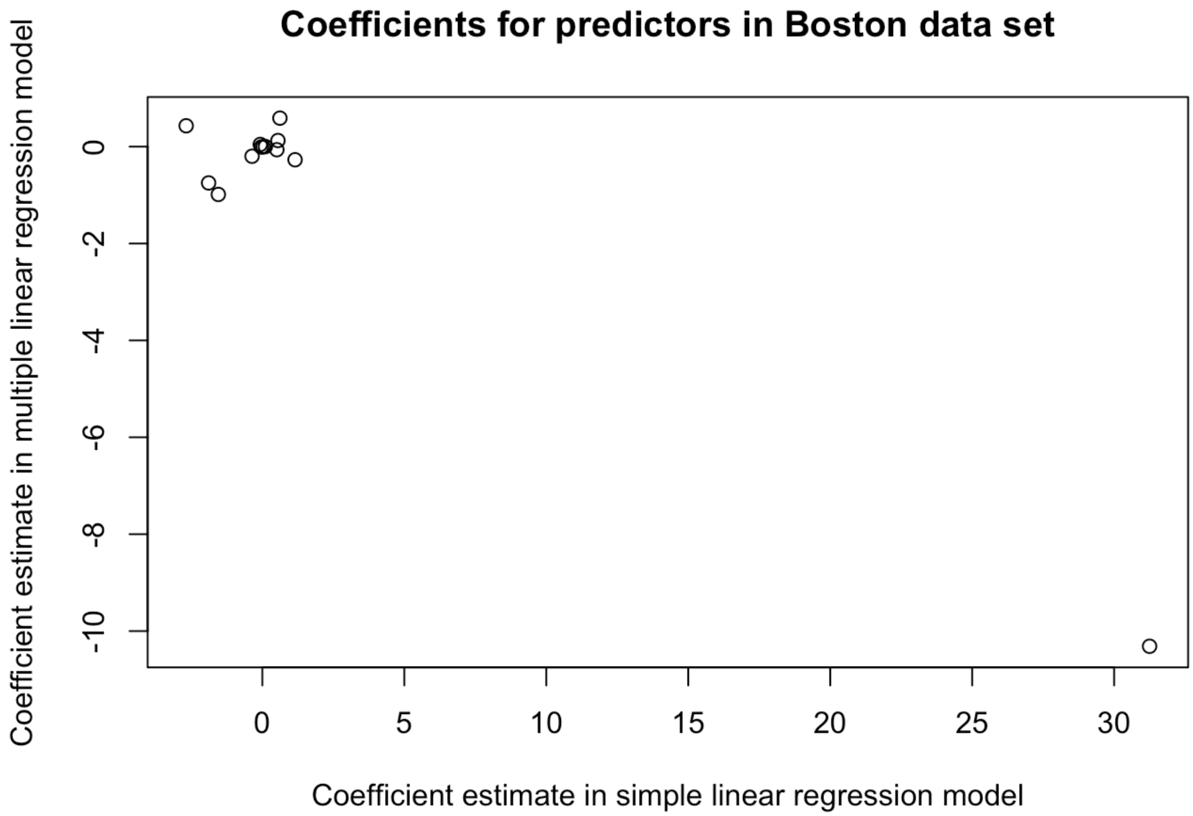
b) The predictors for which we can reject the null hypothesis $H_0 : \beta_j = 0$ at significance level .05 are zn, dis, rad, black and medv.

```
summary(lm(crim ~ ., data = Boston))
```

```
##
## Call:
## lm(formula = crim ~ ., data = Boston)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -9.924 -2.120 -0.353  1.019 75.051
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  17.033228   7.234903   2.354 0.018949 *
## zn            0.044855   0.018734   2.394 0.017025 *
## indus        -0.063855   0.083407  -0.766 0.444294
## chas         -0.749134   1.180147  -0.635 0.525867
## nox         -10.313535   5.275536  -1.955 0.051152 .
## rm            0.430131   0.612830   0.702 0.483089
## age           0.001452   0.017925   0.081 0.935488
## dis          -0.987176   0.281817  -3.503 0.000502 ***
## rad           0.588209   0.088049   6.680 6.46e-11 ***
## tax          -0.003780   0.005156  -0.733 0.463793
## ptratio      -0.271081   0.186450  -1.454 0.146611
## black        -0.007538   0.003673  -2.052 0.040702 *
## lstat         0.126211   0.075725   1.667 0.096208 .
## medv         -0.198887   0.060516  -3.287 0.001087 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.439 on 492 degrees of freedom
## Multiple R-squared:  0.454,  Adjusted R-squared:  0.4396
## F-statistic: 31.47 on 13 and 492 DF,  p-value: < 2.2e-16
```

c)The results are quite different, which makes sense because including a large number of additional predictors can change the estimated effect of each predictor.

```
y = lm(crim ~ ., data = Boston)$coef[2:14]
x = c(-0.073, 0.51, -1.893, 31.249, -2.684, 0.108, -1.551, 0.618, 0.03, 1.152,
    -0.036, 0.549, -0.363)
plot(x, y, xlab = "Coefficient estimate in simple linear regression model",
    ylab = "Coefficient estimate in multiple linear regression model", main =
"Coefficients for predictors in Boston data set")
```



d. Yes, there is evidence of non-linear association between the response and all of the predictors except black (details not given) and chas (since chas is binary). That is, for all but the model for the variable black, there is a significant higher-order term (squared or cubed).

```
summary(lm(crim ~ poly(zn, 3), data = Boston))
summary(lm(crim ~ poly(indus, 3), data = Boston))
summary(lm(crim ~ poly(nox, 3), data = Boston))
summary(lm(crim ~ poly(rm, 3), data = Boston))
summary(lm(crim ~ poly(age, 3), data = Boston))
summary(lm(crim ~ poly(dis, 3), data = Boston))
summary(lm(crim ~ poly(rad, 3), data = Boston))
summary(lm(crim ~ poly(tax, 3), data = Boston))
summary(lm(crim ~ poly(ptratio, 3), data = Boston))
summary(lm(crim ~ poly(black, 3), data = Boston))
summary(lm(crim ~ poly(lstat, 3), data = Boston))
summary(lm(crim ~ poly(medv, 3), data = Boston))
```

Question 4

a.

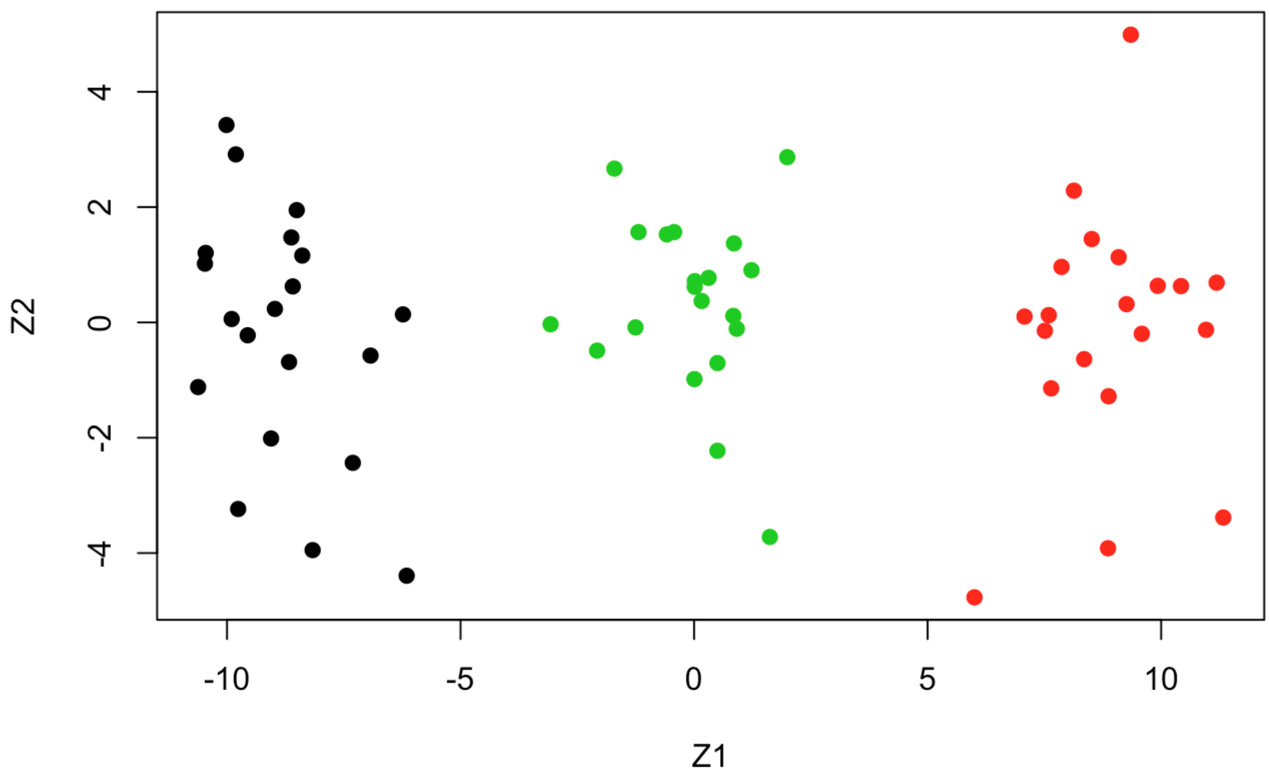
```
X <- matrix(rnorm(3000), 60, 50)
X[1:20, 1] <- X[1:20, 1] + 4;
X[21:40, 1] <- X[21:40, 1] - 4;

X[1:20, 2] <- X[1:20, 1] + 4;
X[21:40, 2] <- X[21:40, 1] - 4;
```

b.

```
pca = prcomp(X)
summary(pca)
```

```
## Importance of components:
##                           PC1    PC2     PC3     PC4     PC5     PC6
## Standard deviation     7.4017 1.9492 1.79083 1.71476 1.67233 1.63812
## Proportion of Variance 0.5205 0.0361 0.03047 0.02793 0.02657 0.02549
## Cumulative Proportion  0.5205 0.5566 0.58704 0.61498 0.64155 0.66704
##                            PC7     PC8     PC9    PC10   PC11   PC12
## Standard deviation     1.54143 1.51214 1.47083 1.40467 1.3991 1.3727
## Proportion of Variance 0.02257 0.02172 0.02055 0.01875 0.0186 0.0179
## Cumulative Proportion  0.68961 0.71134 0.73189 0.75064 0.7692 0.7871
##                           PC13    PC14    PC15   PC16   PC17    PC18
## Standard deviation     1.31985 1.24097 1.21163 1.1833 1.1606 1.10798
## Proportion of Variance 0.01655 0.01463 0.01395 0.0133 0.0128 0.01166
## Cumulative Proportion  0.80368 0.81831 0.83226 0.8456 0.8584 0.87002
##                           PC19    PC20    PC21    PC22    PC23   PC24
## Standard deviation     1.08057 1.06285 1.03186 0.96037 0.92612 0.9000
## Proportion of Variance 0.01109 0.01073 0.01012 0.00876 0.00815 0.0077
## Cumulative Proportion  0.88112 0.89185 0.90196 0.91073 0.91888 0.9266
##                          PC25    PC26    PC27    PC28    PC29    PC30
## Standard deviation     0.8584 0.82377 0.81513 0.79853 0.76326 0.73840
## Proportion of Variance 0.0070 0.00645 0.00631 0.00606 0.00553 0.00518
## Cumulative Proportion  0.9336 0.94002 0.94633 0.95239 0.95792 0.96310
##                           PC31    PC32    PC33    PC34    PC35    PC36
## Standard deviation     0.71328 0.63926 0.58763 0.57476 0.55882 0.53012
## Proportion of Variance 0.00483 0.00388 0.00328 0.00314 0.00297 0.00267
## Cumulative Proportion  0.96794 0.97182 0.97510 0.97824 0.98120 0.98387
##                           PC37   PC38    PC39    PC40    PC41    PC42
## Standard deviation     0.52050 0.4923 0.45681 0.44048 0.40283 0.35663
## Proportion of Variance 0.00257 0.0023 0.00198 0.00184 0.00154 0.00121
## Cumulative Proportion  0.98645 0.9888 0.99073 0.99258 0.99412 0.99533
##                           PC43    PC44    PC45    PC46    PC47    PC48
## Standard deviation     0.34336 0.31146 0.28258 0.27698 0.22234 0.19458
## Proportion of Variance 0.00112 0.00092 0.00076 0.00073 0.00047 0.00036
## Cumulative Proportion  0.99645 0.99737 0.99813 0.99886 0.99932 0.99968
##                           PC49    PC50
## Standard deviation     0.17127 0.06217
## Proportion of Variance 0.00028 0.00004
## Cumulative Proportion  0.99996 1.00000
```

```
Y = pca$x[,1:2]
plot(Y, col=c(rep(1,20), rep(2,20), rep(3,20)), xlab="Z1", ylab="Z2", pch=19)
```



(c)(d)(e) For $K = 3$, K-means does a good job and all points are clustered in the correct group. For $K = 4$, K-means seperates one of the cluster into two. For $K = 2$, K-means combines two clusters into one.

```
km.out <-  kmeans(X, 3, nstart=20)
table(km.out$cluster, c(rep(1,20), rep(2,20), rep(3,20)))
```

```
##
##      1  2  3
##   1  0 20  0
##   2  0  0 20
##   3 20  0  0
```

```
km.out <-  kmeans(X, 2, nstart=20)
table(km.out$cluster, c(rep(1,20), rep(2,20), rep(3,20)))
```

```
##
##      1  2  3
##   1 20  0 20
##   2  0 20  0
```

```
km.out <-  kmeans(X, 4, nstart=20)
table(km.out$cluster, c(rep(1,20), rep(2,20), rep(3,20)))
```

```
##
##       1  2  3
##    1  0  0 20
##    2  8  0  0
##    3 12  0  0
##    4  0 20  0
```

f. With only two principle components, the clustering result are still correct. This shows the first two components captures most of the information in the original dataset.

```
km.out <-  kmeans(Y, 3, nstart=20)
table(km.out$cluster, c(rep(1,20), rep(2,20), rep(3,20)))
```

```
##
##       1  2  3
##    1  0 20  0
##    2  0  0 20
##    3 20  0  0
```

g. The correctness is not as good as the result without scaling. In genearl, scaling may or may not enhance the performance.

```
km.out <-  kmeans(scale(X), 3, nstart=20)
table(km.out$cluster, c(rep(1,20), rep(2,20), rep(3,20)))
```

```
##
##       1  2  3
##    1  9  0  8
##    2  4 17  9
##    3  7  3  3
```