STAT110 Probability – Assignment 2

Strategic Practice

1.1 For each season to have at least one people out of 7 people, we first choose 7 out of 4 which gives us $\binom{7}{4}$ ways, then we organize 4 people to 4 season which gives us 4! Ways. In total, to have at least 1 people in each season, we have $\binom{7}{4}$ * 4! Ways.

$$\frac{\binom{7}{4} * 4!}{4^7} = 0.513$$

1.2 let A_i be the event when there is not class in weekday i (Monday = 1, Tuesday = 2, …).
At least one day there is not class could be represented as
$A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5$, where the complement will be the event we are interested in.
P($A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5$) =

$$\sum_{i=1}^{5} P(A_i) - \sum_{i<j} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) - \sum_{i<j<k<l} P(A_i \cap A_j \cap A_k \cap A_k) + \sum_{i<j<k<l<m} P(A_i \cap A_j \cap A_k \cap A_k \cap A_m)$$

$$= 5 * \frac{\binom{24}{7}}{\binom{30}{7}} + \binom{5}{2} * \frac{\binom{18}{7}}{\binom{30}{7}} + \binom{5}{3} * \frac{\binom{12}{7}}{\binom{30}{7}} = 0.697612732$$

Ans: 1 – 0.697612732 = 0.302
p.s.

$$P(A_i) = \frac{\binom{24}{7}}{\binom{30}{7}}, \; P(A_i \cap A_j) = \frac{\binom{18}{7}}{\binom{30}{7}}, \; P(A_i \cap A_j \cap A_k) = \frac{\binom{12}{7}}{\binom{30}{7}},$$

$$P(A_i \cap A_j \cap A_k \cap A_l) = 0, P(A_i \cap A_j \cap A_k \cap A_l \cap A_m) = 0$$

2.1 yes. Let event a be the get a head when flipping a coin. Getting the first head is independent to getting the second head.

2.2

2.3

2.4

3.1 if the marble is blue, the probability to see the green marble taken out is 50% * 50% = ¼
If the marble is green, the probability to see the green marble taken out is 50% * 100% = ½

If the remaining marble is still green, then the original marble must be green. So the probability will be (1/2) / (1/2 + ¼) = 2/3

3.2 let event S be spam email, let event M be the email contains "free money".

$$P(S) = 0.8, P(S^c) = 0.2, P(M|S) = 0.1, P(M|S^c) = 0.01$$
$$P(S|M) = \frac{P(M|S)P(S)}{P(M|S)P(S) + P(M|S^c)P(S^c)} = \frac{0.1 * 0.8}{0.1 * 0.8 + 0.2 * 0.01} = 0.9756$$

3.3

4 Let event G be A or B is guilty, let event B be target blood type.
We know that the guilty party has the target blood type,
$$P(G_A) = P(G_B) = 0.5, P(B) = 0.1, P(B|G) = 1, P(G|B^c) = 0$$

a.
$$P(G_A|B) = \frac{P(B|G_A)P(G_A)}{P(B|G_A)P(G_A) + P(B|G_B)P(G_B)} = \frac{1 * 0.5}{1 * 0.5 + 0.1 * 0.5} = 0.909$$

b. Let event B be B's blood type matches.

5. let event W be win the game, let event O_1, O_2, O_3 be event the opponent be beginner, intermediate, or master, $P(O_i) = 1/3$, $P(W|O_1) = 0.9, P(W|O_2) = 0.5, P(W|O_3) = 0.3$,

a.
$P(W) = P(W, O_1) + P(W, O_2) + P(W, 0_3) = 0.9 * 1/3 + 0.5 * 1/3 + 0.3 * 1/3 = 0.567$

a.
$$P(O_1|W_1) = \frac{P(W_1|O_1)P(O_1)}{P(W_1|O_1)P(O_1) + P(W_1|O_1^c)P(O_1^c)} = \frac{0.9 * 1/3}{0.567} = 0.529$$

$$P(O_2|W_1) = \frac{P(W_1|O_2)P(O_2)}{P(W_1|O_2)P(O_2) + P(W_1|O_2^c)P(O_2^c)} = \frac{0.5 * 1/3}{0.567} = 0.294$$
$$P(O_3|W_1) = \frac{P(W_1|O_3)P(O_3)}{P(W_1|O_3)P(O_3) + P(W_1|O_3^c)P(O_3^c)} = \frac{0.3 * 1/3}{0.567} = 0.176$$

$$P(W_2|W_1) = P(W_2 \cap O_1|W_1) + P(W_2 \cap O_2|W_1) + P(W_2 \cap O_3|W_1)$$
$$= P(W_2|O_1, W_1)P(O_1|W_1) + P(W_2|O_2, W_1)P(O_2|W_1) + P(W_2|O_3, W_1)P(O_3|W_1)$$
$$= P(W_2|O_1)P(O_1|W_1) + P(W_2|O_2)P(O_2|W_1) + P(W_2|O_3)P(O_3|W_1)$$
$$= 0.9 * 0.529 + 0.5 * 0.294 + 0.3 * 0.176 = 0.6759$$

b. Independent assume the 1st game does not provide any information to the 2nd game. While it may be true if you play with the same person where each game is considered independent (conditional independent on same opponent). However, winning the 1st game helps you guessing the level of your opponent. So after the 1st winning, your confidence towards you opponent is a beginner increase. As a result, it affects the result of 2nd time winning probability.

Homework

1. Lets assume there are 3 events, A,B,C which are equally likely to happen during a day. By the end of the day, one of the event will occur. Now Arby is buying 3 certificates, each for 1000.

   If he purchase 1000, 2/3 of times he will loss 2/3 * 1000
   If he purchase 2000, 2/3 will breakeven, 1/3 will loss, 1/3 * 2000
   If he purchase 3000, he will loss 2000
2. Hold
3. ~~a. one example: A is 18 years old, B is 9 years old, C is 10 years old~~
   ~~second example: A is 18 years old, B is 9 years old, C is 20 years old.~~
   ~~So A>B gives no information to weather A is older than C.~~

c. the possible combination is as follows:
   B A C,
   A B C,
   A C B
   Therefore, it is 2/3
4. let K be # of times get heads, n is # of toss. F be coins is fair, B be coin is biased.
   a.

$$P(K = 2|F) = \binom{2}{2}(1/2)^2(1/2)^0 = 1/4$$

$$P(K=2|B) = \binom{2}{2}(1/4)^2(3/4)^0 = 1/16$$

$$Ans = \frac{1/4}{1/16} = 4/5$$

b. No. when the first toss is heads, it updates ones believe in coin C to be the fair or biased, which impact the second toss of coin c is heads.

c.

$$Ans = 0.5 * \binom{10}{3}(1/2)^3(1/2)^7 + 0.5 * \binom{10}{3}(1/4)^3(3/4)^7 = 0.169$$