Statistics 203
Introduction to Regression and Analysis of Variance
Assignment #1
Due Thursday, January 20

Prof. J. Taylor

USE R FOR ALL CALCULATIONS. PROVIDE COPIES OF YOUR CODE IN THE ASSIGNMENT.

Q. 1) **(MP 2.7)** The purity of oxygen produced by fractionation is thought to be related to the percentage of hydrocarbons in the main condensor of the processing unit. The data can be found at

http://www-stat/~jtaylo/courses/stats203/data/oxygen.table

(a) Fit a simple linear regression model to the data.

(b) Test the hypothesis $H_0 : \beta_1 = 0$.

(c) Calculate R^2.

(d) Find a 95% confidence interval on the slope.

(e) Find a 95% confidence interval on the mean purity when the hydrocarbon percentage is 1.0.

Q. 2) **(MP 2.19)** Consider the simple linear regression model

$$y = \beta_0 + \beta_1 x + \varepsilon$$

where the intercept β_0 is known.

(a) Find the least squares estimator of β_1 for this model. Does this answer seem reasonable?

(b) What is $\text{Var}(\widehat{\beta}_1)$ for the least squares estimator found in (a)?

(c) Find a $100(1-\alpha)\%$ confidence interval for β_1. Is this interval narrower than the estimator for the case where both slope and intercept are unkown?

Q. 3) **(MP 3.10)** Consider the soft drink delivery time data found in the example

http://www-stat/~jtaylo/courses/stats203/R/multiple/multiple.R.html

(a) Compute the residuals and the standardized residuals for this model. (HINT: TRY `help(rstandard)` IN `R`)

(b) Observation 9 has an unusually large residual. Assess the impact of this observation on the model by plotting the fitted model in `R`. On inspection of the covariates in this observation, is there anything unusual about this observation?

Q. 4) **(MP 4.24)** The matrix

$$H = X(X^tX)^{-1}X^t$$

is usually called the *hat matrix* because it maps Y to $\widehat{Y}$ the vector of fitted values. Show that in the multiple linear regression model

$$\mathrm{Var}(\widehat{Y}) = \sigma^2 H.$$

Q. 5) **(MP 4.25)** Prove that the matrices H and $I - H$ are idempotent, that is

$$H^2 = H$$
$$(I - H)^2 = (I - H)$$