MATH 232 Lecture 1 pre-lecture comments

Key points for today:

1. What are scalars? Vectors?
2. How do we represent vectors algebraically? geometrically?
3. How do we manipulate vectors algebraically? geometrically?
4. What are matrices?

Terminology:

1. scalar
2. vector
3. linear combination
4. matrix

Definition

e.g. 2, 3, -1, $\sqrt{2}$

A **scalar** is an object that can be described entirely by a number.
A **vector** is an object that can be described by a number value for its length (**magnitude**) and the direction it points. eg $\nearrow$ 2 units north-east

Example

scalars:

temperature, price, speed, weight

vectors:

Notation

$a, b, c, k, l, m, u, v, w$ scalar

normal letter (not bold / no arrow)

$\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{k}, \mathbf{l}, \mathbf{m}, \mathbf{u}, \mathbf{v}, \mathbf{w}$ vector

bold in print

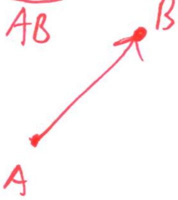
or $\vec{a}$ $\leftarrow$ arrow on top

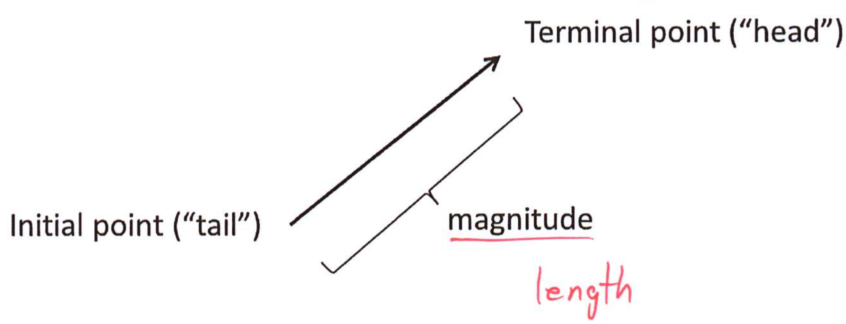
Geometrically, a vector points from one point place to another: $\mathbf{v} = \vec{AB}$.

start at A

end at B

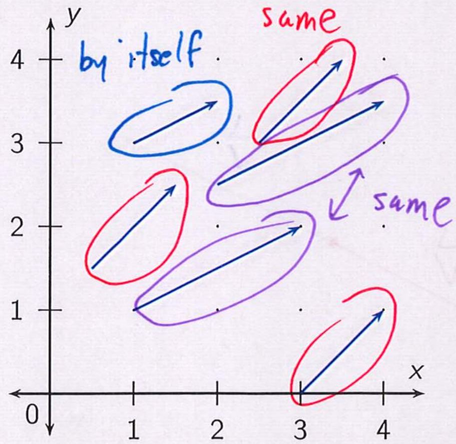
$\vec{AB}$





Two vectors are **equal** if they have the same magnitude and direction.

Example: How many vectors?



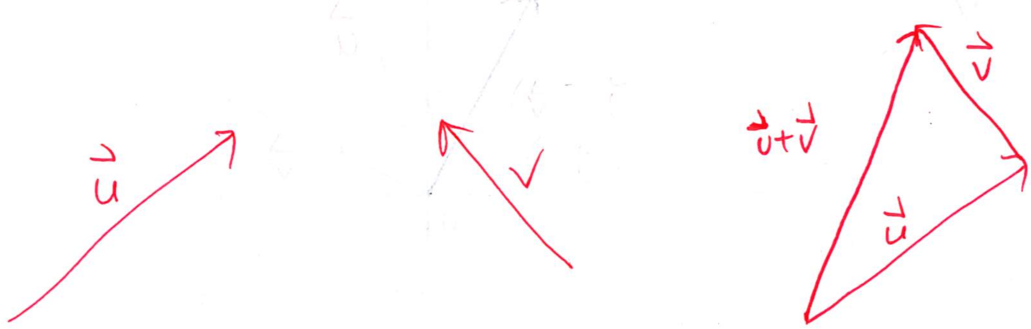
Three different vectors.

Zero vector $\vec{0}$

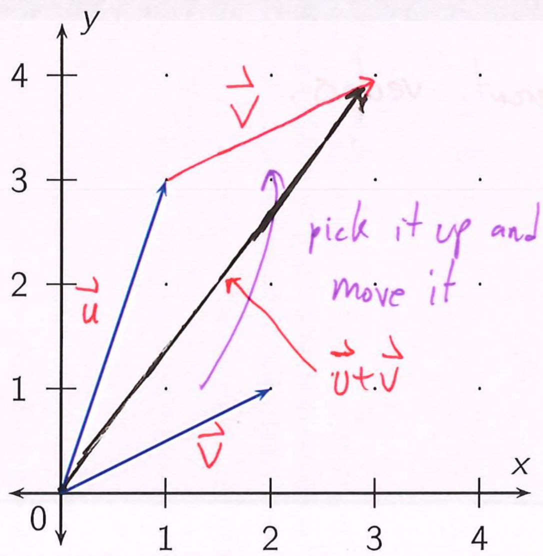
collapse into single point → • start point = end point

$\vec{0}$ has length 0 and direction doesn't matter

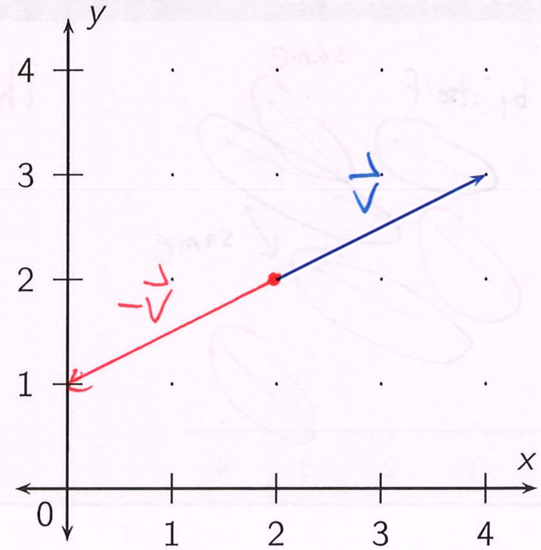
Geometrically we add vectors "head" to "tail".



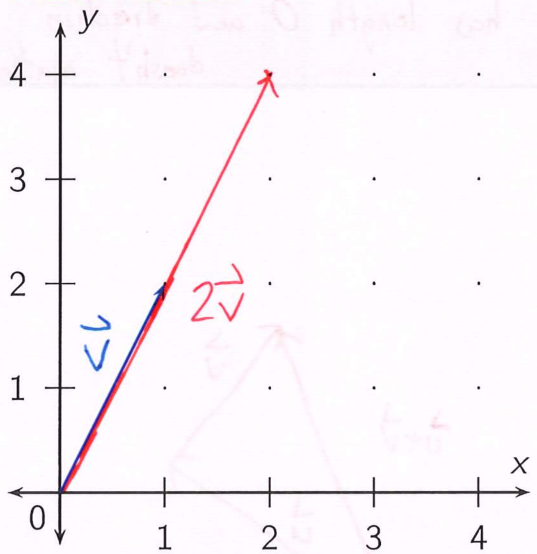
Geometric manipulation of vectors



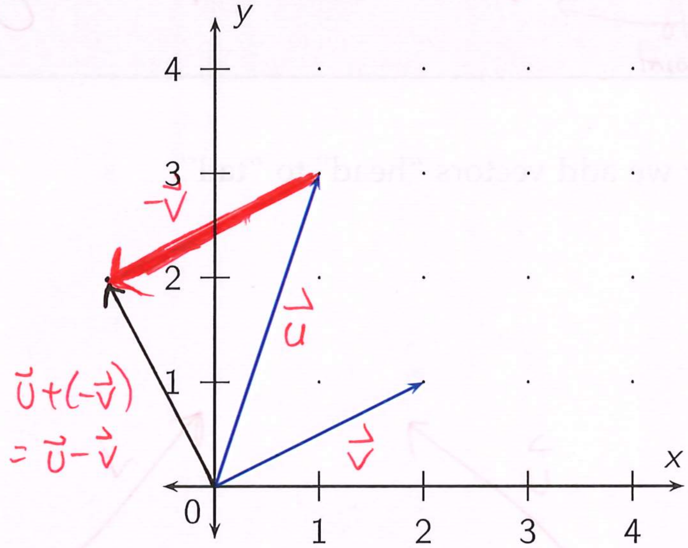
Vector addition



Negative of a vector

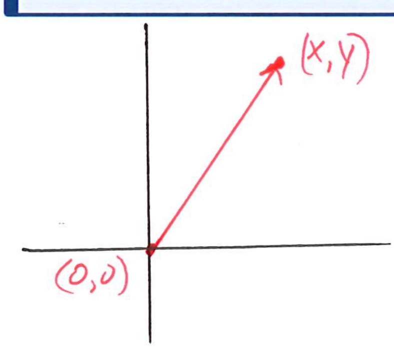


Scalar multiplication



Vector subtraction

Coordinates In 2-space (2D) R^2, given a vector $\mathbf{v}$, place the initial point of the vector at $(0,0)$. The coordinates (x,y) of the terminal points are called *components* or *coordinates* of $\mathbf{v}$.



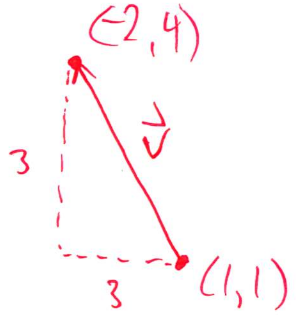
Notation for vectors in coordinate form:

$\vec{v} = (x,y)$ $\quad \vec{v} = \langle x,y \rangle$

$\vec{v} = [x \ y]$ $\quad \vec{v} = \begin{bmatrix} x \\ y \end{bmatrix}$

matrix notation

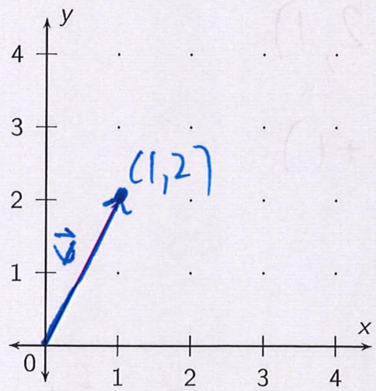
Ex:



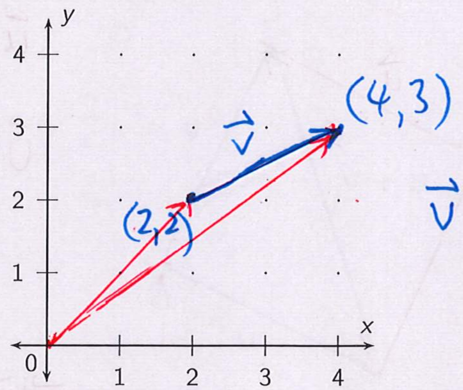
$\vec{v} = (-3, 3)$

(3 to the left, 3 up)

Example. What are the coordinates?



$\vec{v} = (1,2)$



$\vec{v} = (2,1)$

n-dimensional space

Vectors in $\mathbf{R}^n$ $\quad \vec{\mathbf{u}} = (u_1, u_2, u_3, \ldots, u_n)$

$\underline{n \text{ components}}$

$\mathbb{R}^2: \vec{u} = (x, y) \qquad \mathbb{R}^3: \vec{u} = (x, y, z)$

The zero vector $\vec{\mathbf{0}} = (0, 0, \ldots, 0)$

Equality of vectors

When are the vectors $\vec{\mathbf{u}} = (u_1, u_2, \ldots, u_n)$ and $\vec{\mathbf{v}} = (v_1, v_2, \ldots, v_n)$ equal?

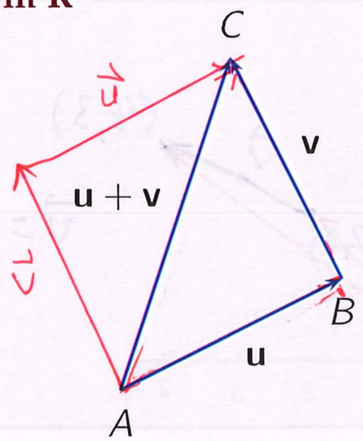
$\vec{u}$ and $\vec{v}$ are equal if: $u_1 = v_1$

$u_2 = v_2$

$u_3 = v_3$ $\ldots$ $u_n = v_n$

(each component is the same)

Addition in $\mathbf{R}^2$



$\vec{u} = (1,3) \quad \vec{v} = (-2,1)$

$\vec{u}+\vec{v} = (1+(-2), 3+1)$
$= (-1,4)$

$\vec{u}+\vec{v} = \vec{v}+\vec{u}$ (order doesn't matter for addition)

Algebraic operation examples

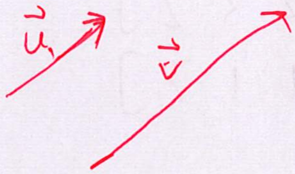
Algebraic operations in $\mathbf{R}^n$ Given two vectors $\mathbf{u} = (u_1, \dots, u_n)$ and $\mathbf{v} = (v_1, \dots, v_n)$ and a real number (scalar) $c \in R$:

- $\mathbf{u}+\mathbf{v} =$ $(u_1+v_1, u_2+v_2, \dots, u_n+v_n)$
- $c\mathbf{u} =$ $(cu_1, cu_2, \dots, cu_n)$
- $\mathbf{u}-\mathbf{v} =$ $\vec{u}+(-\vec{v}) = (u_1-v_1, u_2-v_2, \dots, u_n-v_n)$

Properties (Theorems 1.1.5 and 1.1.6)

- $\mathbf{u}+\mathbf{v} = \mathbf{v}+\mathbf{u}$
- $(\mathbf{u}+\mathbf{v})+\mathbf{w} = \mathbf{u}+(\mathbf{v}+\mathbf{w})$
- $\mathbf{u}+\mathbf{0} = \mathbf{u}$
- $(k+l)\mathbf{u} = k\mathbf{u}+l\mathbf{u}$
- $k(\mathbf{u}+\mathbf{v}) = k\mathbf{u}+k\mathbf{v}$
- $(kl)\mathbf{u} = k(l\mathbf{u})$
- $1\mathbf{u} = \mathbf{u}$
- $(-1)\mathbf{u} = -\mathbf{u}$
- $0\mathbf{u} = \mathbf{0}$
- $k\mathbf{0} = \mathbf{0}$

Parallel vectors

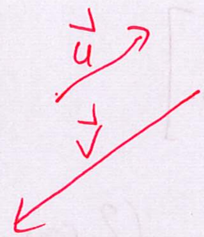


$\vec{u}, \vec{v}$ same direction

$\vec{u} = (1,2)$

$\vec{v} = (2,4)$

$\vec{v} = 2\vec{u}$



$\vec{u}, \vec{v}$ opposite direction

$\vec{u} = (1,2)$

$\vec{v} = (-3,-6)$

$\vec{v} = -3\vec{u}$

Linear dependence

$\vec{w}$ is a linear combination of $\vec{u}$ and $\vec{v}$ if

$\vec{w} = c\vec{u} + d\vec{v}$ for some $c, d \in \mathbb{R}$ scalars

Example $\vec{u} = (1,4)$
$\vec{v} = (1,5)$
$\vec{w} = (0,2)$

is $\vec{w}$ a linear combination of $\vec{u}, \vec{v}$?

Yes. $\vec{w} = 2\vec{v} - 2\vec{u}$

$= 2(1,5) - 2(1,4)$

$= (2,10) - (2,8)$

$= (0,2)$ ✓

Matrix A **matrix** is a rectangular array of numbers. Matrix with m rows and n columns has **size** $m \times n$. An example of a **matrix** with **size** 3×4:

$$M = \begin{bmatrix} 1 & 2 & -1 & 5 \\ -2 & 0 & 1 & 0 \\ -3 & 3 & 0 & 2 \end{bmatrix}$$

Matrix examples

Ex 1: $\begin{bmatrix} 2 & 3 & 2 \\ 1 & 1 & 7 \end{bmatrix}$ 2 rows of size 3: $\begin{bmatrix} 2 & 3 & 2 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 & 7 \end{bmatrix}$

size 2×3 (2 rows, 3 columns)

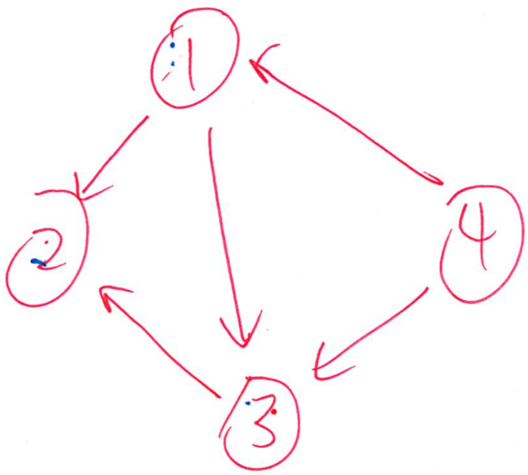
3 columns of size 2: $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 2 \\ 7 \end{bmatrix}$

Ex 2:

column vector: $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ size 3×1

Ex 3: row vector: $\begin{bmatrix} 0 & -1 & -3 \end{bmatrix}$ size 1×3

Digraphs and the adjacency matrix



To

$$
\text{from}\quad
\begin{array}{c}
 \\ 1 \\ 2 \\ 3 \\ 4
\end{array}
\begin{array}{c}
\begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
\begin{bmatrix}
0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
1 & 0 & 1 & 0
\end{bmatrix}
\end{array}
$$

node 1 goes to 2, 3

node 2 goes nowhere

node 3 goes to 2

node 4 goes to 1, 3