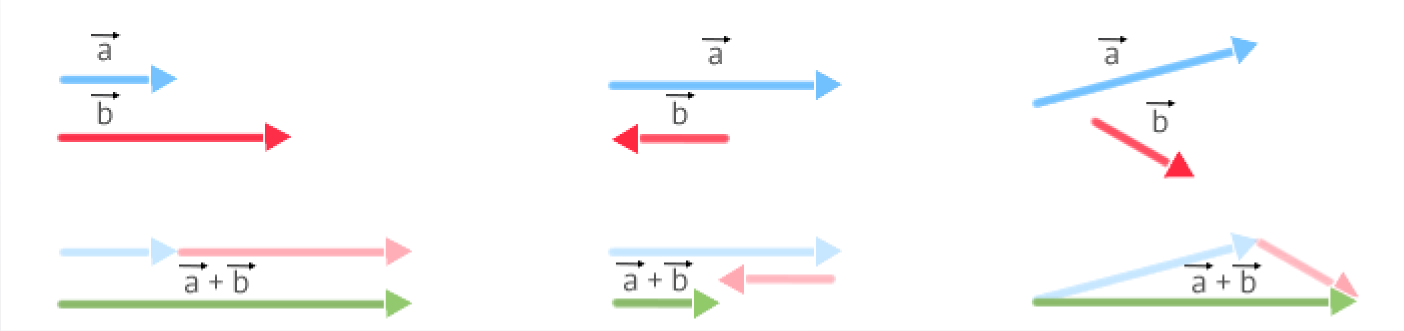
0. **VECTOR CALCULUS** REVIEW

GENERAL PROPERTIES OF VECTORS

- **Vector addition**
  - 1D: algebraic addition
  - 2D or 3D: geometric addition

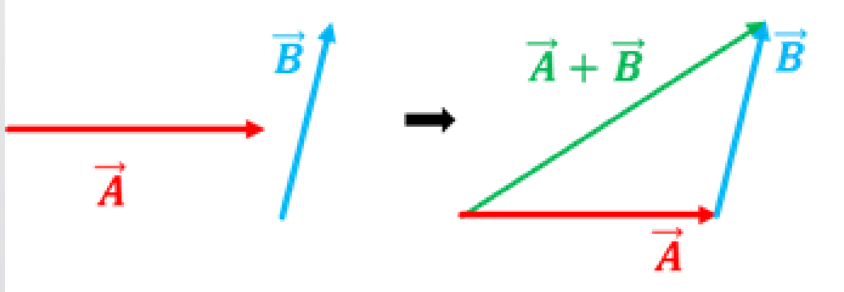


Same angle and direction

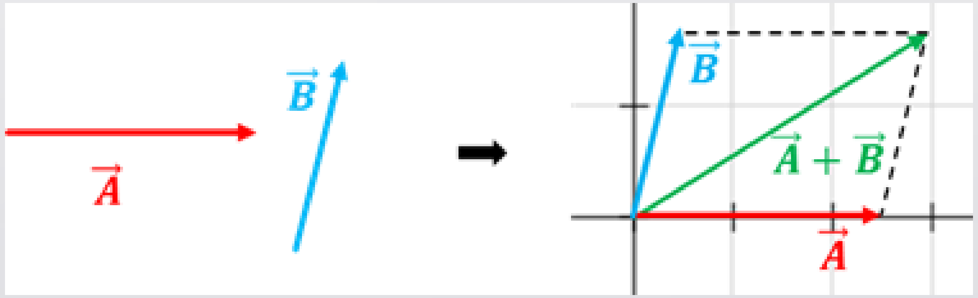
Same angle but opposite direction

Different angle

One vector after another:

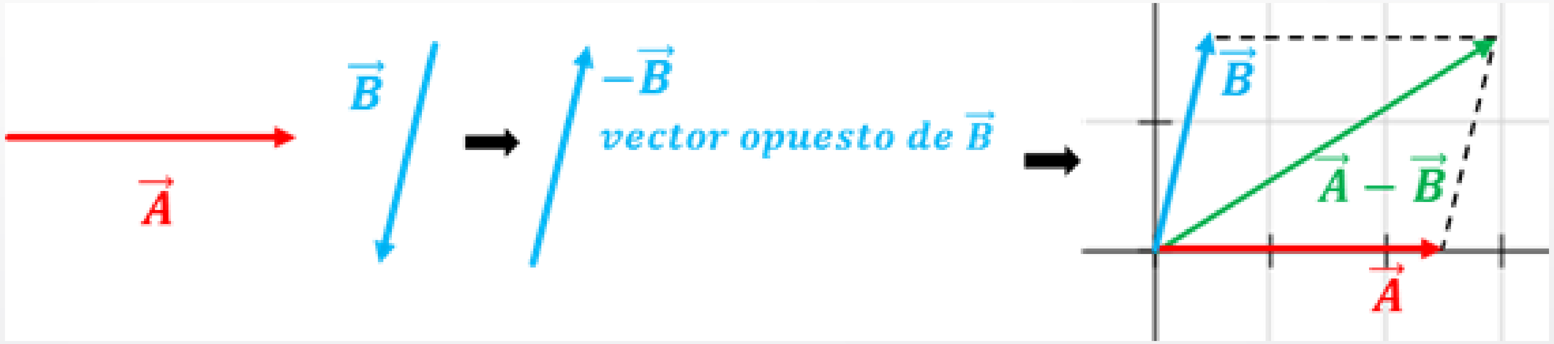


Or by the
Parallelogram law:

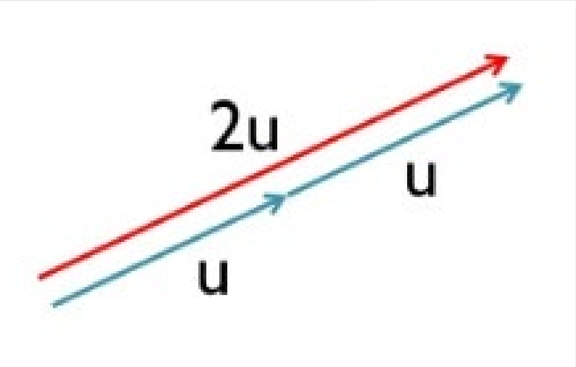


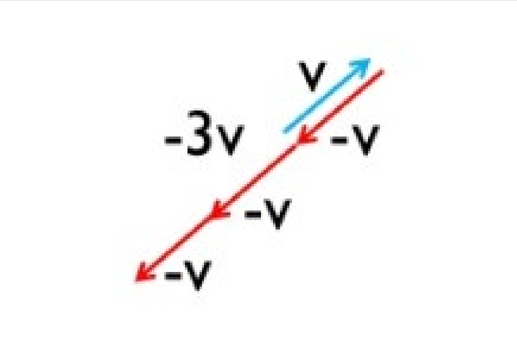
GENERAL PROPERTIES OF VECTORS

- **Vector subtraction**



- **Scalar multiplication: multiplication of a scalar and a vector**



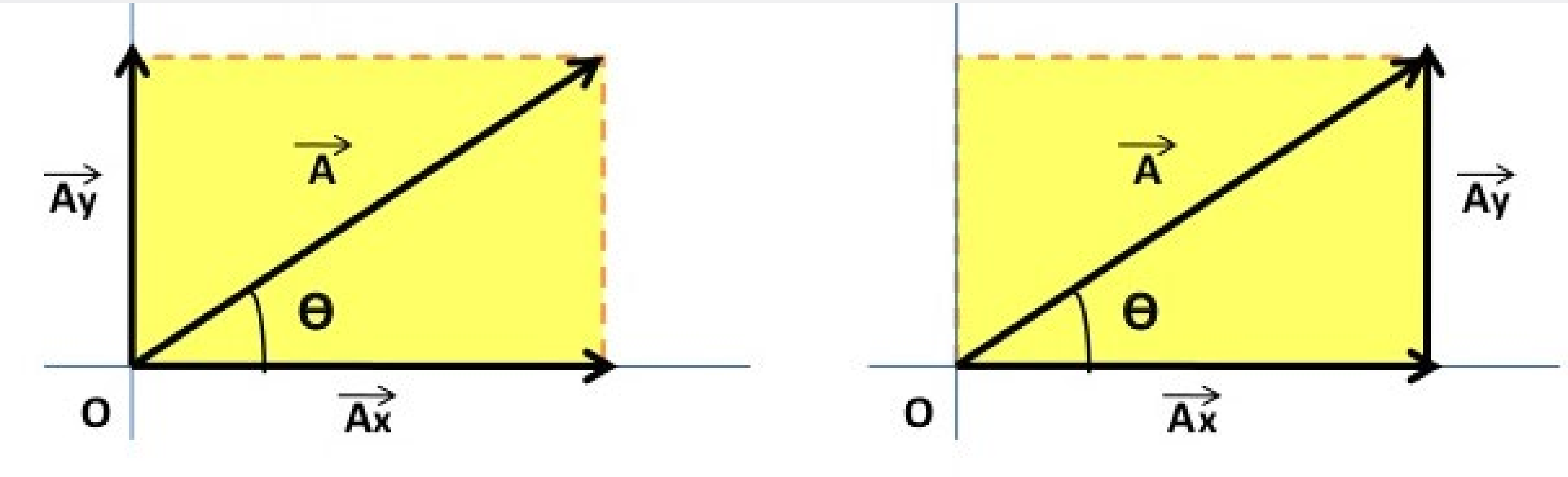


GENERAL PROPERTIES OF VECTORS

- **Components of a vector**

A component of a vector is the projection of this vector in any of the axes.

If we are considering the cartesian axes, these components will be called rectangular or cartesian components.



$$\vec{\boldsymbol{A}} = \overrightarrow{\boldsymbol{A_x}} + \overrightarrow{\boldsymbol{A_y}}$$
$$A_x = A \cdot cos\theta$$
$$A_y = A \cdot sin\theta$$
$$tg\theta = \frac{A_y}{A_x}$$
$$A = \sqrt{A_x^2 + A_y^2}$$

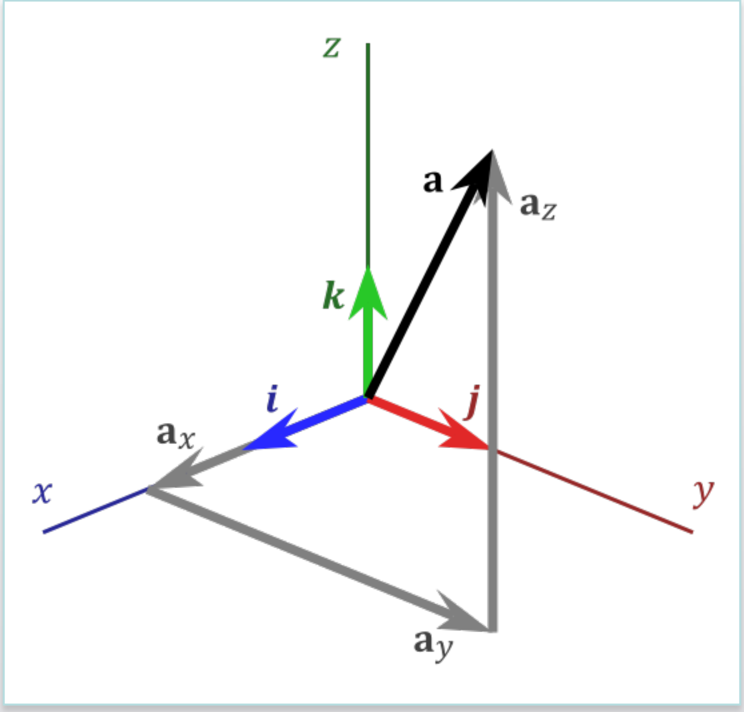
GENERAL PROPERTIES OF VECTORS

- **Components of a vector**

Orthonormal basis in 3D space:

An orthonormal basis (in 3D) is that in which we condider 3 unit vectors (modulus = 1), perpendicular to each other.

The basis formed by the vectors $\{\vec{\imath},\vec{\jmath},\vec{k}\}$, in the X, Y and Z axis (respectively), is called **canonic basis**.

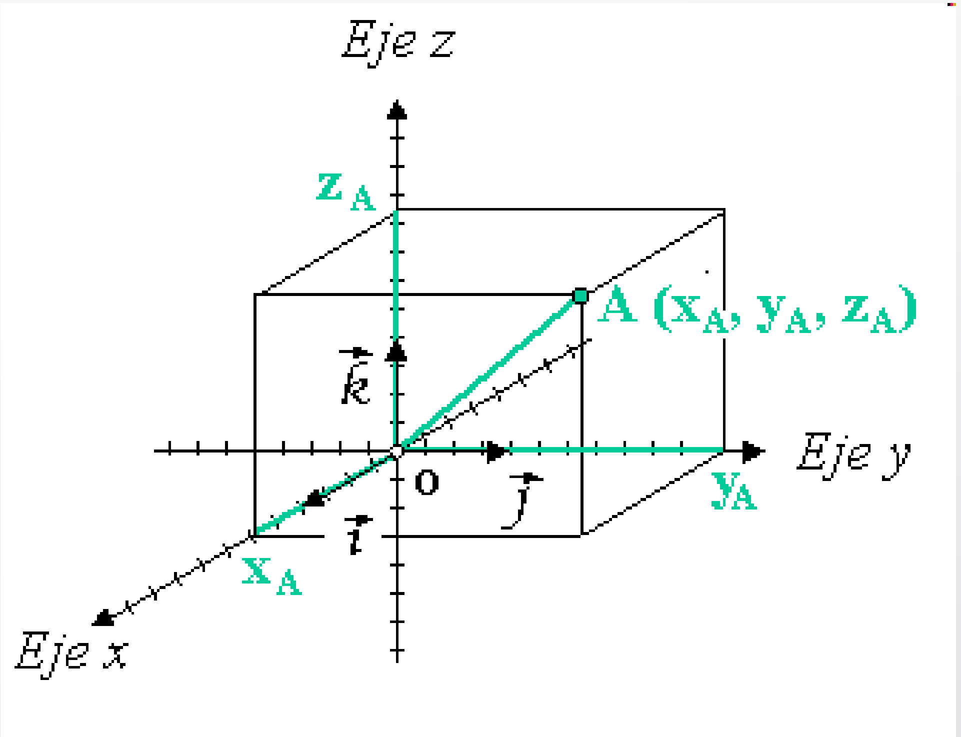


GENERAL PROPERTIES OF VECTORS

- **Components of a vector**

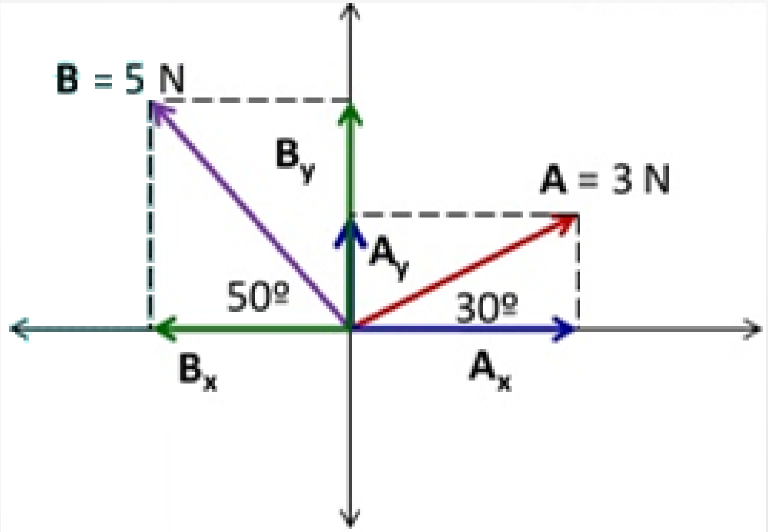
$$\vec{A} = x_A\vec{i} + y_A\vec{j} + z_A\vec{k}$$

(x_A, y_A, z_A): cartesian components of a vector



GENERAL PROPERTIES OF VECTORS

- **Vectors addition, by components:**



GENERAL PROPERTIES OF VECTORS

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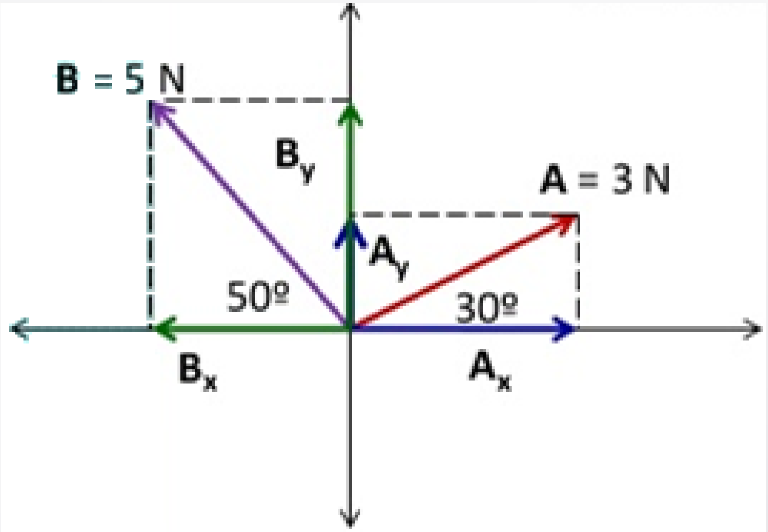


$$A_x = Acos30º = 3cos30º = 2.58N$$

$$A_y = Asin30º = 1.5N$$

$$B_x = -Bcos50º = -5Ncos50º = -3.21N$$

$$B_y = Bsin50º = 3.83N$$

$R_x = A_x + B_x$

$R_x = (2.58\ N) + (-3.21\ N)$

$R_x = -0.63N$

$R_y = A_y + B_y$

$R_y = (1.5\ N) + (3.83\ N)$

$R_y = 5.33\ N$

$R^2 = (R_x)^2 + (R_y)^2$

$R^2 = (-0.63N)^2 + (5.33N)^2$

$R^2 = 0.39 + 28.4$

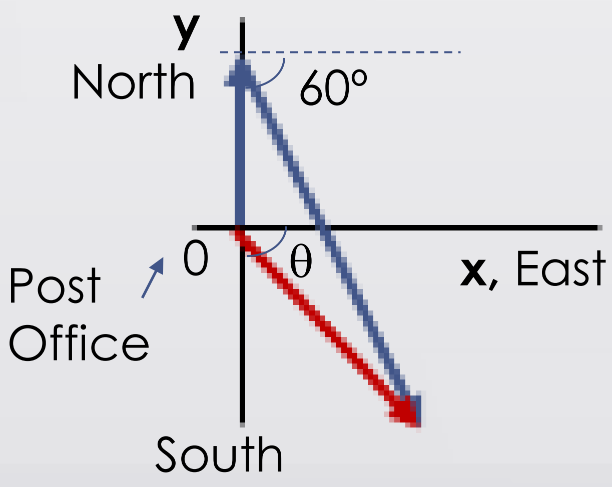
$R^2 = 28.79$

$R = 5.36N$

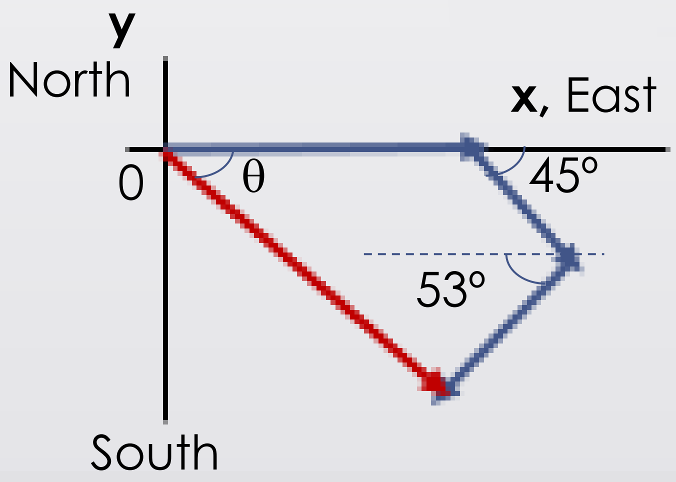
GENERAL PROPERTIES OF VECTORS

Examples:

A postman travels 22.0 km North and then 47.0 km Southeast (forming an angle of 60º with East direction. Which is the displacement of the postman since he left the Post Office?



A plane makes 3 consecutive trips: in the 1st one, it travels 620 km East, in the 2nd one, it travels 440 km Southeast (45º) and in the 3rd, it travels 550 km Southwest (53º with West direction) (see drawing). Which is the displacement since the beginning?



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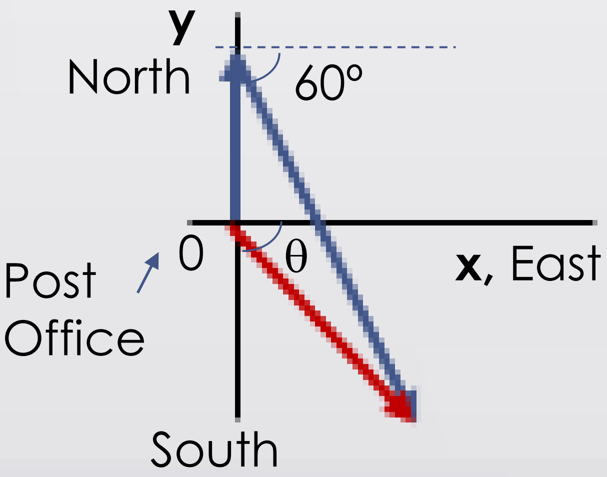
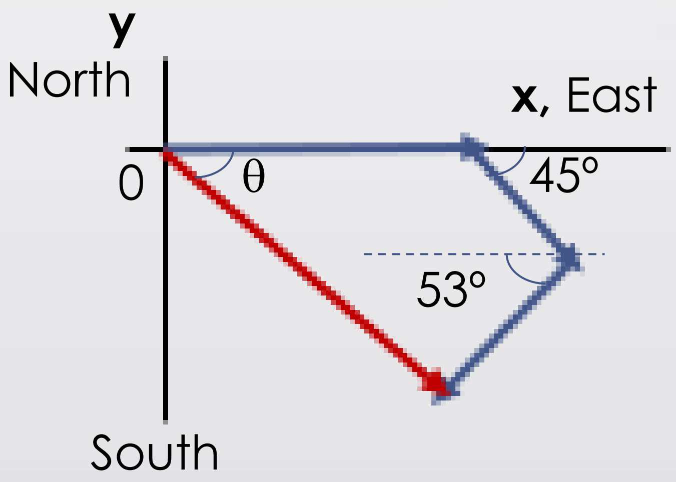


$$D_x = D_{1x} + D_{2x}$$
$$= 0 + 47cos60º$$
$$= 23.5\ km$$
$$D_y = D_{1y} + D_{2y}$$
$$= 22 - 47sin60º$$
$$= 22 - 40.7 = -18.7\ km$$
$$D = \sqrt{D_x^2 + D_y^2}$$
$$= \sqrt{23.5^2 + (-18.7)^2}$$
$$= 30km$$



GENERAL PROPERTIES OF VECTORS

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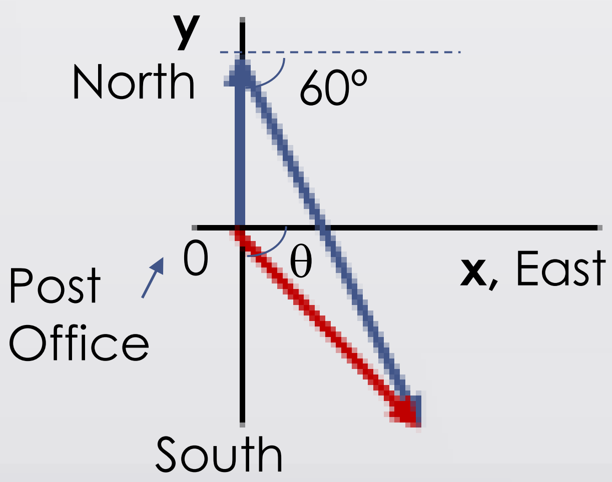

$$
\begin{aligned}
D_x &= D_{1x} + D_{2x} \\
&= 0 + 47cos60º \\
&= 23.5\ km \\
D_y &= D_{1y} + D_{2y} \\
&= 22 - 47sin60º \\
&= 22 - 40.7 = -18.7\ km \\
D &= \sqrt{D_x^2 + D_y^2} \\
&= \sqrt{23.5^2 + (-18.7)^2} \\
&= 30km
\end{aligned}
$$

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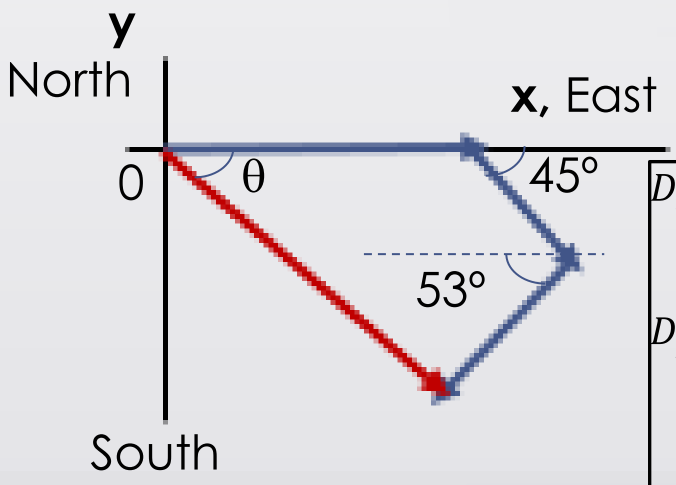


$$
\begin{aligned}
D_x &= D_{1x} + D_{2x} + D_{3x} \\
&= 620 + 440cos45º - 550cos53º \\
&= 620 + 311 - 331 = 600\ km \\
D_y &= D_{1y} + D_{2y} + D_{3y} \\
&= 0 - 40sin45º - 550sin53º \\
&= -311 - 439 = -750\ km \\
D &= \sqrt{D_x^2 + D_y^2} = \sqrt{600^2 + (-750)^2} \\
&= 960km
\end{aligned}
$$

GENERAL PROPERTIES OF VECTORS

- Dot product of two vectors: it yields a scalar.

$$\vec{a} = x_A\vec{i} + y_A\vec{j} + z_A\vec{k}$$

$$\vec{b} = x_B\vec{i} + y_B\vec{j} + z_B\vec{k}$$

$$\vec{a} \cdot \vec{b} = x_A x_B + y_A y_B + z_A z_B$$

Commutative property:

$$\vec{\boldsymbol{a}} \cdot \vec{\boldsymbol{b}} = \vec{\boldsymbol{b}} \cdot \vec{\boldsymbol{a}}$$

Associative property:

$$\alpha(\vec{\boldsymbol{a}} \cdot \vec{\boldsymbol{b}}) = (\alpha\vec{\boldsymbol{a}}) \cdot \vec{\boldsymbol{b}} = \vec{\boldsymbol{a}} \cdot (\alpha\vec{\boldsymbol{b}})$$

Distritutive property:

$$\vec{\boldsymbol{a}} \cdot (\vec{\boldsymbol{b}} + \vec{\boldsymbol{c}}) = \vec{\boldsymbol{a}} \cdot \vec{\boldsymbol{b}} + \vec{\boldsymbol{a}} \cdot \vec{\boldsymbol{c}}$$

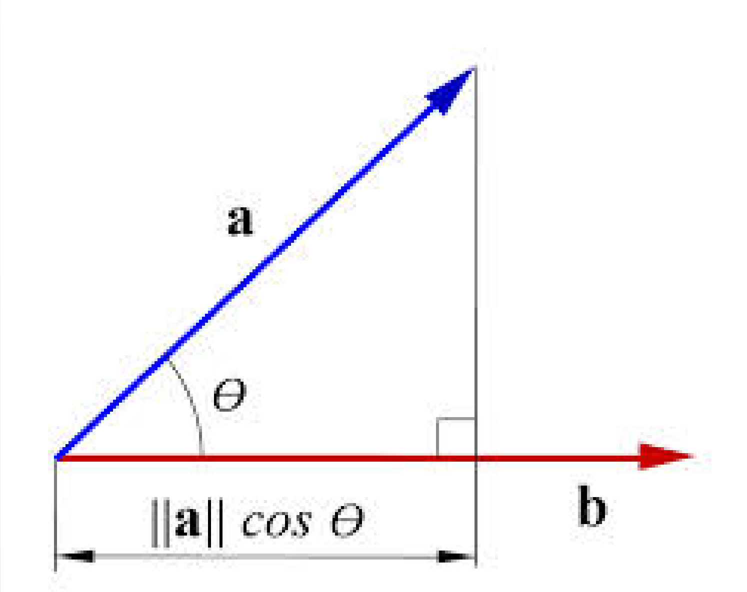
Dot product of the vectors of the canonical basis:

$$\vec{\boldsymbol{i}} \cdot \vec{\boldsymbol{i}} = \vec{\boldsymbol{j}} \cdot \vec{\boldsymbol{j}} = \vec{\boldsymbol{k}} \cdot \vec{\boldsymbol{k}} = 1$$

$$\vec{\boldsymbol{i}} \cdot \vec{\boldsymbol{j}} = \vec{\boldsymbol{j}} \cdot \vec{\boldsymbol{k}} = \vec{\boldsymbol{k}} \cdot \vec{\boldsymbol{i}} = 0$$

GENERAL PROPERTIES OF VECTORS

$\vec{a} \cdot \vec{b}$ **is the product of the modulus of** $\vec{a}$ **by the modulus of** $\vec{b}$**, multiplied by the cosine of the angle between** $\vec{a}$ **and** $\vec{b}$**:**



$$\vec{a} \cdot \vec{b} = abcos\theta$$

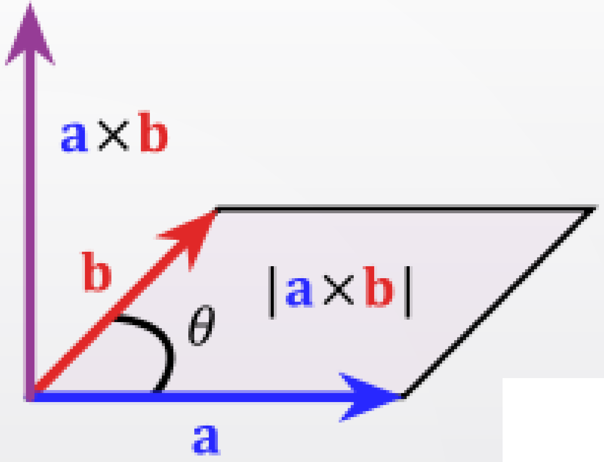
If the dot product of two vectors is 0, and the modulus of any of the two vectors is different from 0, then the two vectors are perpendicular:

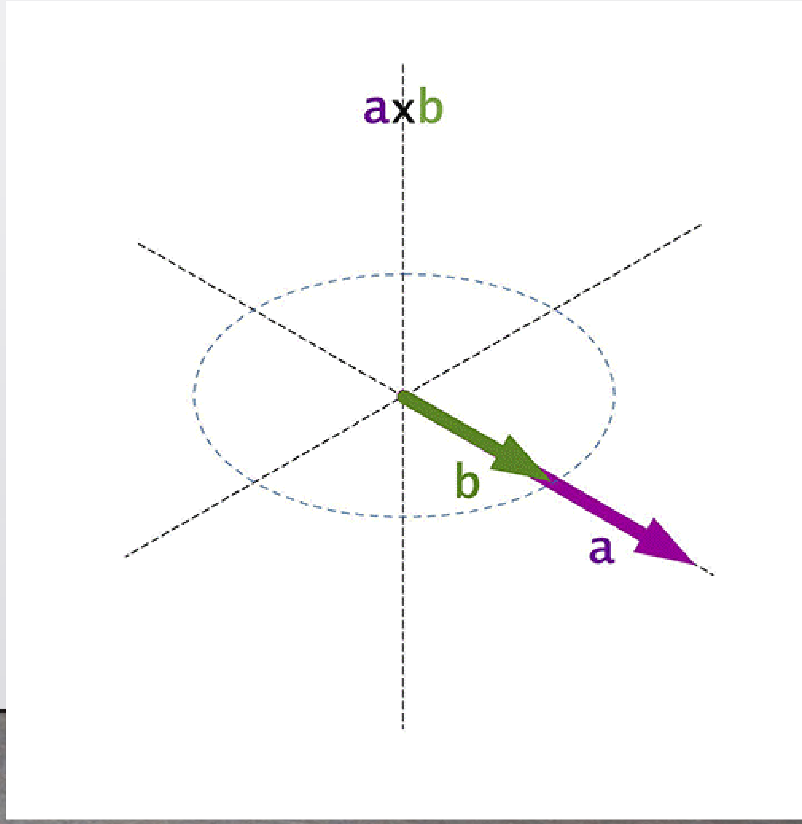
$$\text{If } \vec{a} \cdot \vec{b} = 0 \text{ and } |\vec{a}| \neq 0 \text{ and } \left|\vec{b}\right| \neq 0 \Rightarrow \vec{a} \perp \vec{b}$$

GENERAL PROPERTIES OF VECTORS

- **Cross product: it yields a vector.**

$$\vec{a} = x_A\vec{i} + y_A\vec{j} + z_A\vec{k}$$
$$\vec{b} = x_B\vec{i} + y_B\vec{j} + z_B\vec{k}$$





Cross product of the vectors of the canonical basis:

$$\vec{i}\times\vec{i} = \vec{j}\times\vec{j} = \vec{k}\times\vec{k} = 0; \quad \vec{i}\times\vec{j} = \vec{k}; \quad \vec{j}\times\vec{k} = \vec{i}; \quad \vec{k}\times\vec{i} = \vec{j}$$

$$\vec{a}\times\vec{b} = \left(x_A\vec{i} + y_A\vec{j} + z_A\vec{k}\right)\times\left(x_B\vec{i} + y_B\vec{j} + z_B\vec{k}\right) =$$
$$= \vec{i}(y_Az_B - z_Ay_B) + \vec{j}(z_Ax_B - x_Az_B) + \vec{k}(x_Ay_B - y_Ax_B)$$

GENERAL PROPERTIES OF VECTORS

- Cross product: it yields a vector.

$$\vec{a} = x_A\vec{i} + y_A\vec{j} + z_A\vec{k}$$
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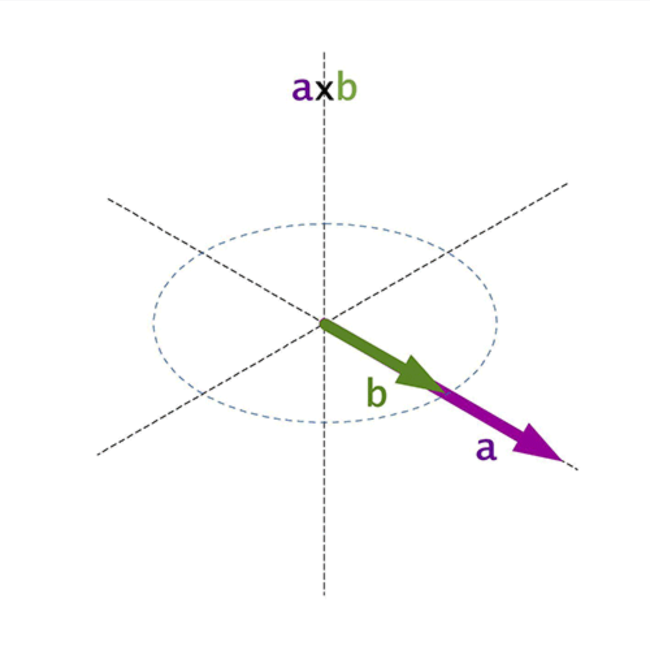

$$\vec{\boldsymbol{a}} \times \vec{\boldsymbol{b}} = \left(x_A\vec{i} + y_A\vec{j} + z_A\vec{k}\right) \times \left(x_B\vec{i} + y_B\vec{j} + z_B\vec{k}\right) =$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_A & y_A & z_A \\ x_B & y_B & z_B \end{vmatrix} =$$

$$= \vec{i}(y_A z_B - z_A y_B) + \vec{j}(z_A x_B - x_A z_B) + \vec{k}(x_A y_B - y_A x_B)$$

GENERAL PROPERTIES OF VECTORS

Exercise:

Given these vectors: $\vec{u} = (1,2,3), \vec{v} = (2,0,1)\ and\ \vec{w} = (-1,3,0)$, determine:

a) $\vec{u} \cdot \vec{v}$, $\vec{v} \cdot \vec{w}$, $\vec{w} \cdot \vec{u}$ and $\vec{v} \cdot \vec{u}$

b) $\vec{u} \times \vec{v}$, $\vec{v} \times \vec{w}$, $\vec{u} \times \vec{w}$ and $\vec{v} \times \vec{u}$

c) $(\vec{u} \times \vec{v}) \cdot \vec{w}\ and\ (\vec{v} \times \vec{w}) \cdot \vec{u}$

d) $|\vec{u}|, |\vec{v}|\ and\ |\vec{w}|$