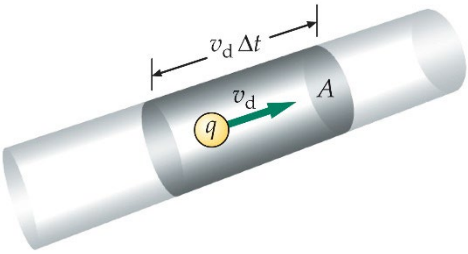
CHAPTER 2. ELECTRIC CURRENT

CURRENT (INTENSITY) AND DENSITY CURRENT

Electric current is defined as the flow of electric charges that, per unit of time, pass through a cross-sectional area.



$$I = \frac{dQ}{dt}$$

$$I = \frac{C}{s} = Amperes = A$$

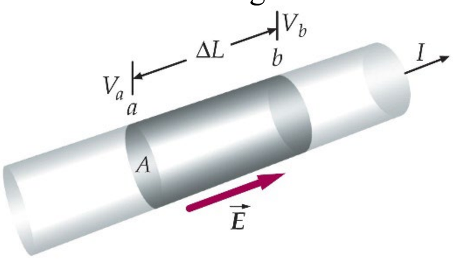
The direction of electric current was considered to be the flow of positive charges. This convention was established before it was known that free electrons, which are negatively charged, are the particles that actually move and produce the current in a conducting wire. The movement of negatively charged electrons in one direction is equivalent to the flow of positive charges in the opposite direction. Therefore, electrons move in the opposite direction to the current.

OHM'S LAW. RESISTANCE

Consider a segment of cable with a length ΔL and cross-sectional area A through which a current I flows. Since the electric field is always directed from regions of higher potential to regions of lower potential, the potential at point *a* is higher than at point *b*. If we consider the current as the flow of positive charges, these positive charges move in the direction in which the potential decreases. Assuming that the electric field $\vec{E}$ is constant across the segment, the **potential difference V** between points *a* and *b* is

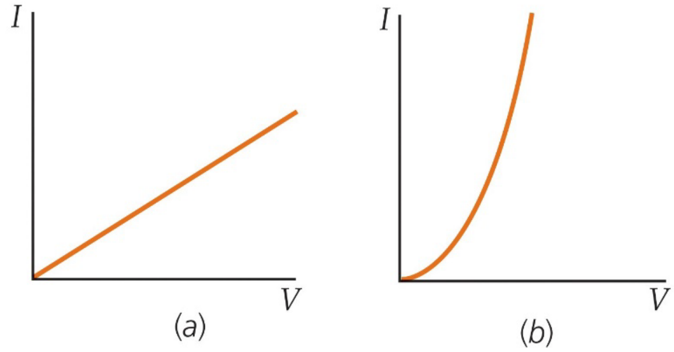
$$V = V_a - V_b = E \cdot \Delta L$$

The ratio of the potential drop in the direction of the current to the current intensity is called the **resistance** of the segment:



$\boxed{R = \frac{\Delta V}{I}}$ The unit for the resistance in the SI is **ohm (Ω)**

For many materials, resistance does not depend on the voltage drop or current intensity. These materials, including most metals, are referred to as **ohmic materials**. In many ohmic materials, resistance remains constant over a wide range of conditions. In ohmic materials, the potential drop across a portion of the conductor is proportional to the current. In ohmic materials, the relationship is linear (with a slope equal to the resistance, R), whereas in non-ohmic materials, it is nonlinear (with a slope equal to V/I = R).



Graphs of I versus V for (a) ohmic materials and (b) non-ohmic materials.

The resistance of a conducting wire is proportional to its length and inversely proportional to its cross-sectional area.

$\boxed{R = \rho \frac{L}{A}}$ being ρ the resistivity of the conductor material.

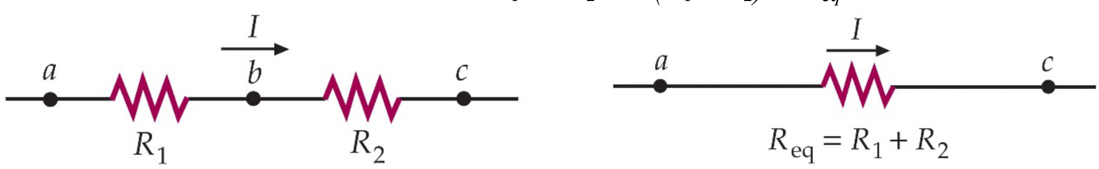
The resistivity of any metal depends on the temperature: $\rho(T) = \rho_0\left[1 + \alpha\left(T - T_0\right)\right]$

being ρ_0 the resistivity at temperature T_0 *y* ρ the resistivity at temperature T

Resistors combination

- **In series combination:** When two or more resistances are connected in a way that the same current I flows through them.

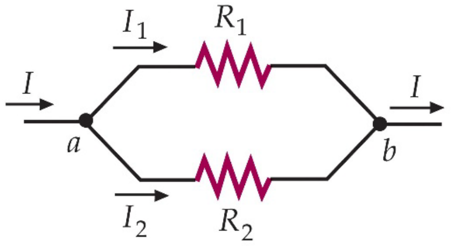
$$V = IR_1 + IR_2 = I\,(R_1 + R_2) = IR_{eq}$$

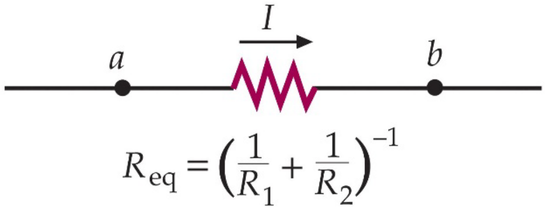


- **In parallel combination:** When two or more resistances are connected in a way that the same potential difference exists across them.

$I = I_1 + I_2$ $V = I_1 R_1$ $V = I_2 R_2$

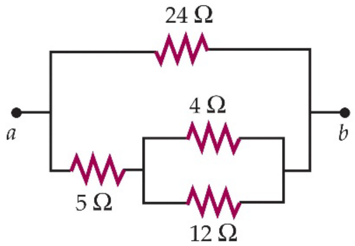
$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$





Example:

Determine the equivalent resistance between points *a* and *b* for the combination of resistors in the figure.

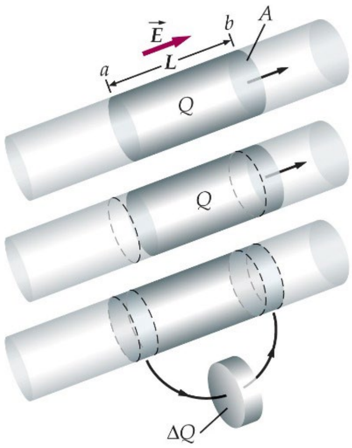


Solution: $R_{eq} = 6\ \Omega$

ENERGY IN ELECTRICAL CIRCUITS. JOULE EFFECT

When there is an electric current in a conductor, electrical energy continuously converts into thermal energy. The electric field in the conductor accelerates free electrons for a short period, giving them an increase in kinetic energy that quickly turns into thermal energy in collisions between electrons and the crystalline lattice ions of the material. The mechanism by which the increase in internal energy of the conductor results in a temperature rise is called the Jou**le effect**.

Let's consider a wire segment of length L and cross-sectional area A. The free charge in the segment is initially Q, and during a time interval Δt, this charge undergoes a small displacement to the right. This displacement is equivalent to a charge amount ΔQ moving from the left end, where the potential energy is ΔQV_a, to the right end of the cable segment, where the potential energy is ΔQV_b. The net change in potential energy for Q is



$$\Delta U = \Delta Q(V_a - V_b) = \Delta QV$$

$$\frac{\Delta U}{\Delta t} = \frac{\Delta Q}{\Delta t}\mathrm{V} = \mathrm{IV}$$

$$P_{lost} = IV = I^2R$$

(Power dissipated by the conductor)

GENERADORES: FUERZA ELECTROMOTRIZ

In order to maintain a steady current in a conductor, we need a supply of electrical energy. An apparatus or device that supplies electrical energy is called an electromotive force (*emf*) source. This device converts chemical or mechanical energy, among other forms of energy, into electrical energy. It is often a battery or cell, converting chemical energy into electrical energy, or a generator that converts mechanical energy into electrical energy.

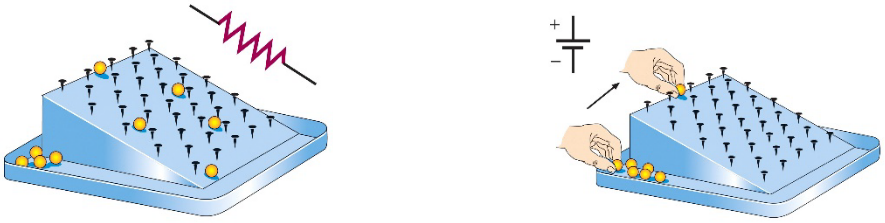
An *emf* source does work on the charge passing through it, raising the potential energy of the charge. This increase in potential energy per unit charge is called the electromotive force, *emf* (ε), of the source.

When the charge ΔQ flows through an emf source, its potential energy increases by an amount ΔQε.

$$[\epsilon] = V$$

Within an *emf* source, the charge flows from the lower potential to the higher potential. When the charge ΔQ flows through the *emf* source ε, its potential energy is increased by the amount ΔQε.

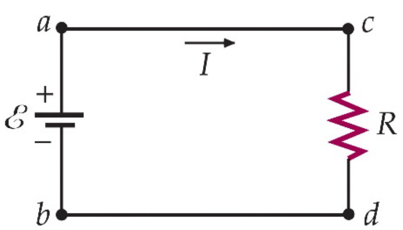
Supplied Power: $P = \frac{\Delta Q \epsilon}{\Delta t} = \epsilon I$



Mechanical analogy of a simple circuit formed by a resistor and an *emf* source.

Ideal battery

An **ideal battery** is a source of electromotive force (*emf*) that maintains a constant potential difference between its two terminals regardless of the flow of charge between them. (It does not depend on I). The potential difference between the terminals of an ideal battery is equal, in absolute value, to the *emf* of the battery.

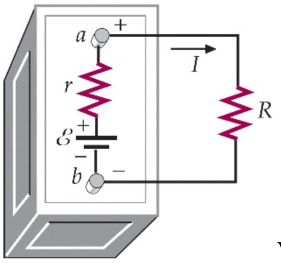


$V_a - V_b = \varepsilon$

$V_{ac} = 0$
$V_{bd} = 0$
$V_{ab} = \varepsilon = V_{cd} = IR \quad \Longrightarrow \quad I = \varepsilon / R$

Real battery

The potential difference between the terminals of the battery, referred to as terminal voltage, is not simply equal to the value of the *emf* of the battery (it depends on I). A real battery can be considered as an ideal battery with *emf* ε plus a small resistance r, called the internal resistance of the battery.



$V_a = V_b + \varepsilon - Ir$ $\qquad$ $V_a - V_b = \varepsilon - Ir$

$$\boxed{I = \frac{\varepsilon}{R + r}}$$

DIRECT CURRENT CIRCUITS

Kirchhoff's rules

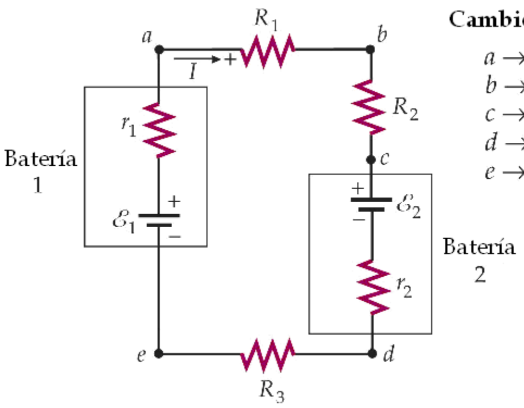
1. The algebraic sum of the potential differences along any loop or mesh in the circuit must be equal to zero. (Mesh Rule)

$$\sum I = 0$$

2. At a branching point or node in a circuit where the current can divide, the sum of the currents entering the node must be equal to the sum of the currents leaving it. (Node Rule)

$$\sum I_{entering} = \sum I_{leaving}$$

Single mesh circuits



Cambios en el Potencial

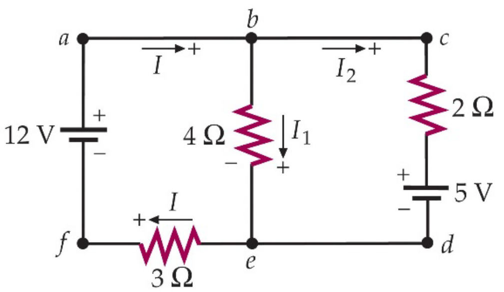
$a \to b$	Pierde IR_1
$b \to c$	Pierde IR_2
$c \to d$	Pierde $\varepsilon_2 + Ir_2$
$d \to e$	Pierde Ir_3
$e \to a$	Aumenta $\varepsilon_1 - I r_1$

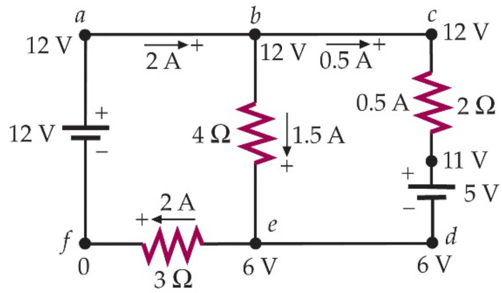
$$-IR_1 - IR_2 - \varepsilon_2 - Ir_2 - IR_3 + \varepsilon_1 - Ir_1 = 0$$

$$\boxed{I = \frac{\varepsilon_1 - \varepsilon_2}{R_1 + R_2 + R_3 + r_1 + r_2}}$$

Several mesh circuits

Determine the current intensity in each branch of the circuit.



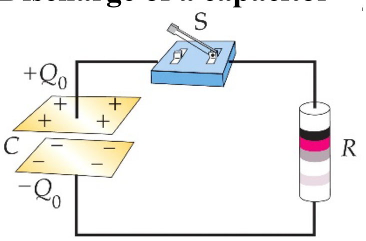


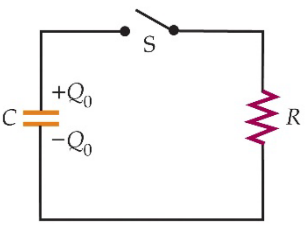
1. Replace any combination of resistors or capacitors with their equivalent resistance.
2. Choose a direction for the current and indicate the positive direction with an arrow. Specify the currents in each branch. Add plus and minus signs to indicate the ends of the higher and lower potential terminals of each emf source.
3. Apply the node rule to each junction where the current divides.
4. Apply the mesh rule to each closed loop until obtaining as many equations as unknowns.
5. Solve the equations to deduce the values of the unknowns.
6. Interpret the results.

RC CIRCUITS

A circuit that involves both a resistor and a capacitor is referred to as an **RC circuit**. In an RC circuit, the current flows in one direction, similar to DC circuits, but the current intensity varies over time.

Discharge of a capacitor





The figure shows a capacitor with an initial charge $+Q_0$ on the upper plate and a charge $-Q_0$ on the lower plate. It is connected to a resistor R and a switch S that is initially open. The potential difference across the capacitor is initially $V_0 = Q_0/C$, where C is the capacitance. Let's close the switch at time $t = 0$. Since there is now a potential difference across the ends of the resistor, current must flow through it. The initial current is:

$$I_0 = \frac{V_0}{R} = \frac{Q_0}{RC}$$

The current is due to the flow of charge from the positive plate to the negative plate passing through the resistor, and thus, after a certain time, the charge on the capacitor is reduced. If we take the current in the clockwise direction as positive, the current at any moment is equal to the decrease in this charge per unit of time. If Q is the charge on the capacitor at a time t, the current at that moment is

$$I = -\frac{dQ}{dt}$$

The first Kirchhoff's rule is:

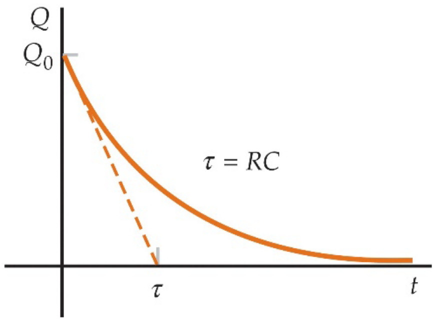
$$\frac{Q}{C} - IR = 0 \rightarrow \frac{Q}{C} + R\frac{dQ}{dt} = 0$$

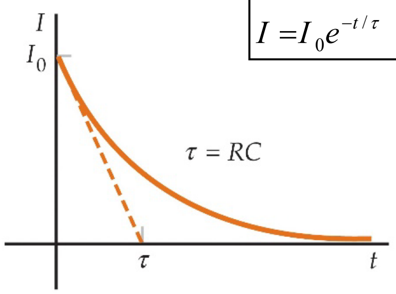
$$\frac{dQ}{dt} = -\frac{1}{RC}dt \Rightarrow \int_{Q_0}^{Q}\frac{dQ}{Q} = -\frac{1}{RC}\int_0^t dt \Rightarrow \ln\frac{Q}{Q_0} = -\frac{t}{RC} \Rightarrow Q(t) = Q_0 e^{-t/RC}$$

$$\boxed{Q(t) = Q_0 e^{-t/RC} = Q_0 e^{-t/\tau}}$$

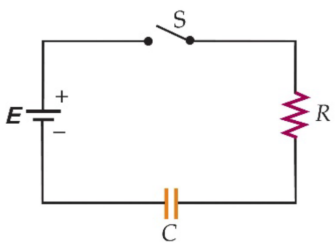
τ =RC   time constant

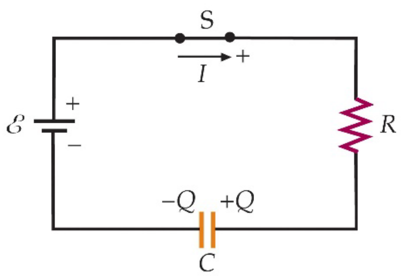
$$\boxed{\begin{array}{l} I = -\dfrac{dQ}{dt} = \dfrac{Q_0}{RC}e^{-t/RC} \\ I = I_0 e^{-t/\tau} \end{array}}$$





Charge of a capacitor





The switch S, initially open, is closed at time t = 0. The charge starts to flow immediately through the resistor, depositing on the positive plate of the capacitor.

Applying Kirchhoff's rule:

$$\varepsilon - IR - \frac{Q}{C} = 0$$

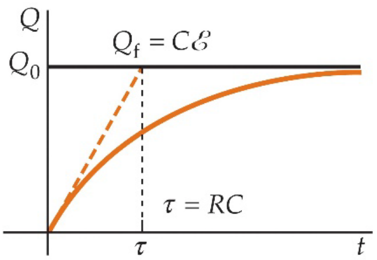
$$I = +\frac{dQ}{dt} \qquad \varepsilon - R\frac{dQ}{dt} - \frac{Q}{C} = 0$$

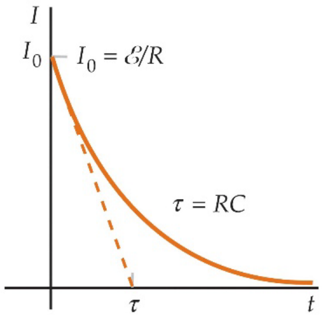
$$\boxed{Q = C\varepsilon(1 - e^{-t/(RC)}) = Q_f(1 - e^{-t/\tau})}$$

being $Q_f = C\varepsilon$ the final charge.

$$I = \frac{dQ}{dt} = C\varepsilon\left[-\frac{-1}{RC}e^{-t/(RC)}\right] = \frac{\varepsilon}{R}e^{-t/(RC)}$$

$$I = \frac{\varepsilon}{R}e^{-t/(RC)} = I_0 e^{-t/\tau}$$

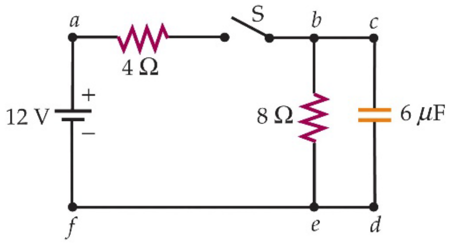




$$\boxed{I = \frac{\varepsilon}{R} e^{-t/(RC)} = I_0 e^{-t/\tau}}$$

Example

The 6 μF capacitor in the circuit is initially discharged. Calculate the current through the 4 Ω and 8 Ω resistors (a) immediately after the switch has been closed and (b) a long time after the switch has been closed. (c) Determine the charge on the capacitor a long time after the switch has been closed.



Solution: a) $I_{4\Omega} = 3A$, $I_{8\Omega} = 0$, b) $I_f = 1$ A, c) $Q_f = 48$ μC

Example

A 4.6 μF capacitor initially uncharged is connected in series with a 7.5 kΩ resistor and a source ε = 125 V. Right after closing the switch: (a) voltage drop across the capacitor; (b) voltage drop across the resistor; (c) charge on the capacitor; (d) current through the capacitor; (e) the same questions after a long time has elapsed.

Solution: $V_C= 0$; $V_R = 125$ V; Q = 0; $I_R= 0.017$ A. $V_C = 125$ V; $V_R= 0$; Q = 575 μC; $I_R= 0$.