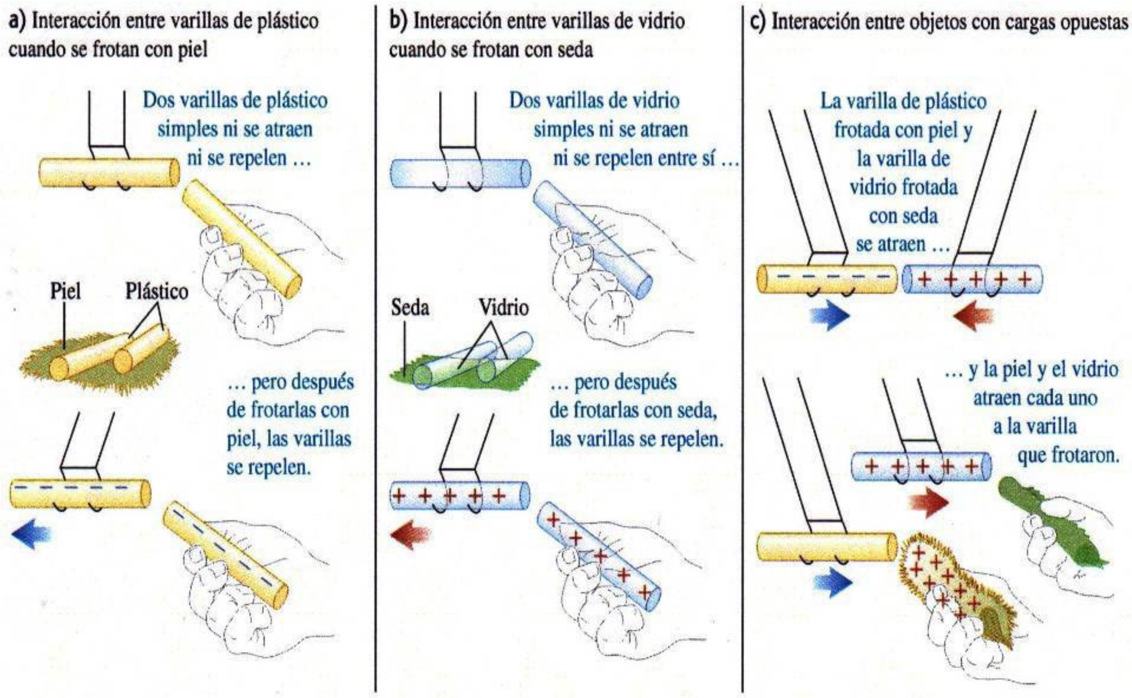
CHAPTER 1. ELECTROSTATICS

ELECTRICAL CHARGE

- **Two positive charges repel each other, as do two negative charges. A positive and a negative charge attract each other.**

Let's consider an experiment involving electrostatic attraction.



Franklin: "*Every object has a normal amount of electricity and when two objects rub together, part of the electricity is transferred from one body to the other; Thus, one has an excess and the other a deficiency of equal value.*"

- **The algebraic sum of all electric charges in any closed system is constant (law of conservation of charge)**

Matter is made up of electrically neutral atoms. Each atom has a nucleus that contains protons and neutrons, with positive and zero charge, respectively. Surrounding the nucleus is an equal number of negatively charged electrons (than protons).

$m_p = 2000 m_e$

$Q_p = +e$; $Q_e = -e$

The magnitude of the charge on the electron or proton is the natural unit of charge: $e = 1.6\times10^{-19}$ C.

- **Any charge is presented in whole units of the fundamental unit of charge e. The charge is quantized: $Q = \pm ne$**

Conductors

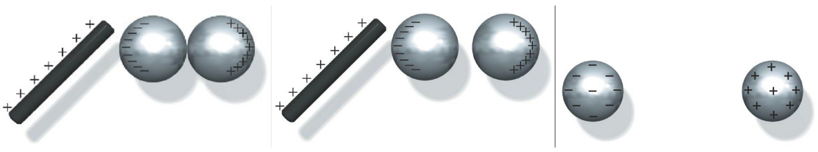
Electrons can move freely within the material. Examples: copper, metals, etc.

Insulators

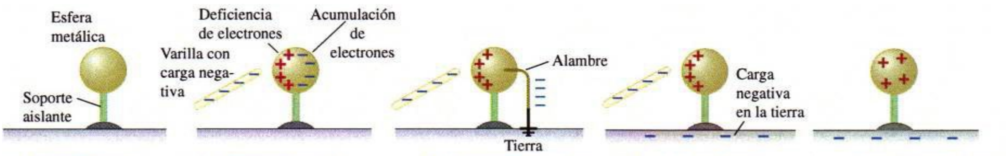
Electrons are bound to nearby atoms and none can move freely. Examples: glass, wood, etc.

Induction charging

Let's suppose two uncharged metal spheres that are in contact. When a charged rod is brought close to one of the spheres, free electrons flow from one sphere to the other. If the rod is positively charged, it attracts negatively charged electrons and the sphere closest to the rod acquires electrons from the other. If the spheres are separated before removing the bar, they will be left with equal and opposite charges. The spheres have been charged without being touched by the bar and the charge is not modified. This process is called **Electrostatic induction or induction charging**.



The Earth itself constitutes a conductor. When a conductor comes into contact with the ground it is said to be grounded.



a) Esfera metálica sin carga.

b) La carga negativa en la varilla repele a los electrones, lo que crea zonas de **carga inducida** negativa y positiva.

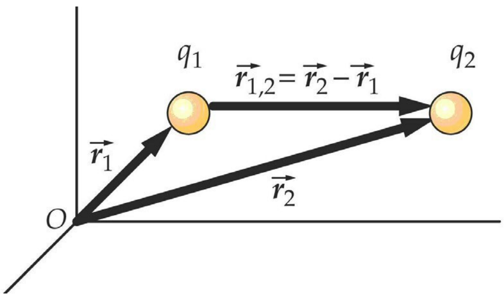
c) El alambre permite que los electrones acumulados (carga negativa inducida) fluyan hacia la tierra.

d) Se quita el conductor; ahora, la esfera tiene sólo una región con deficiencia de electrones, con carga positiva.

e) Se quita la varilla; los electrones se reacomodan por sí solos, y toda la esfera tiene una deficiencia de electrones (carga neta positiva).

COULOMB'S LAW

The force exerted by one point charge on another is directed along the line joining them. The force varies inversely with the square of the distance separating the charges and is proportional to the product of the charges. It is repulsive if the charges have the same sign and attractive if the charges have opposite signs.



$$\boxed{\vec{F}_{1,2} = k\,\frac{q_1 q_2}{r_{1,2}^2}\,\vec{u}_{1,2}}$$

k = Coulomb constant = 8.99×10^9 N·m²/C²

Comparison between the electric force and the gravitational force.

- The electric force can be attractive or repulsive and is proportional to the product of the charges. The gravitational force is proportional to the product of the masses of the particles and is always attractive.

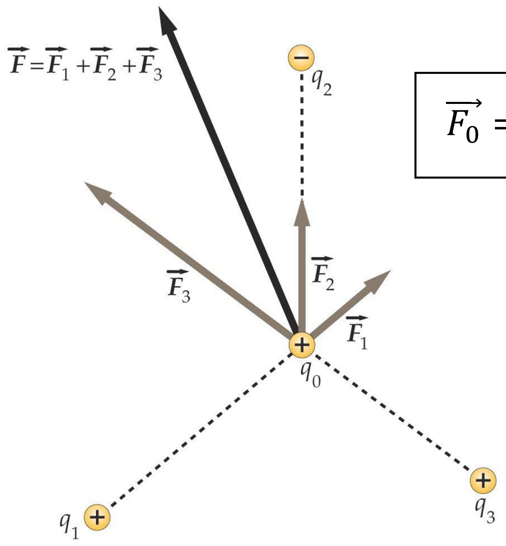
Then, the interaction force between two protons:

$$\left.\begin{array}{l} F_E = \dfrac{ke^2}{r^2} \\ F_G = \dfrac{Gm^2}{r^2} \end{array}\right\} \qquad \frac{F_E}{F_G} = 1.24 \times 10^{36} \qquad \begin{array}{c} m = 1.67 \times 10^{-27} kg \\ G = 6.67 \times 10^{-11}\ Nm^2/kg^2 \\ k = 8.99 \times 10^{9} Nm^2/C^2 \end{array}$$

➔ The gravitational force can be neglected when describing their interactions.

Superposition Principle

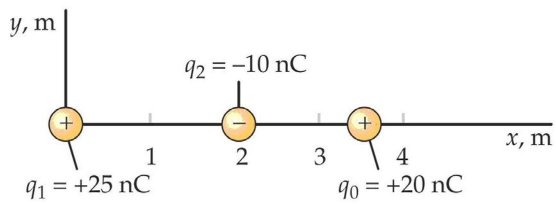
In a system of charges, each of them exerts a force on the rest. So t*he net force on a charge is the vector sum of the individual forces exerted on said charge by the remaining charges of the system.*



$$\boxed{\vec{F_0} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \cdots = \sum_i \vec{F_i}}$$

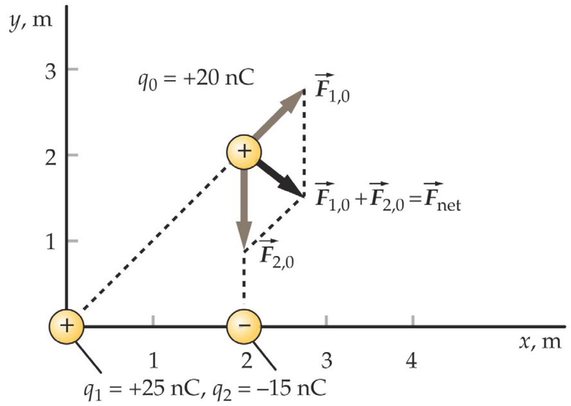
<u>Examples</u>:

1.1Calculate the net force on q_0:



Solution: $\vec{F}_{net} = -0.432\ \mu N\ \vec{\imath}$

1.2Calculate the vector of the resultant force on q_0:



Solution: $\vec{F}_{net} = (0.397\,\vec{\imath} - 0.277\,\vec{\jmath})\ \mu N$

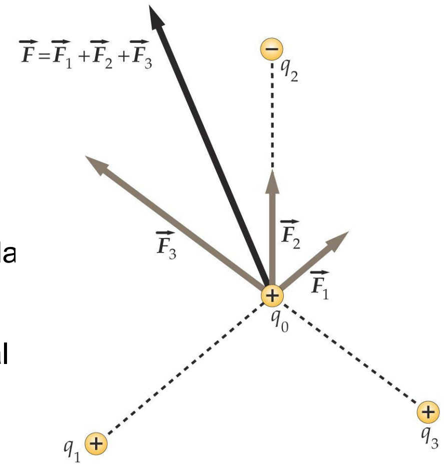
ELECTRIC FIELD

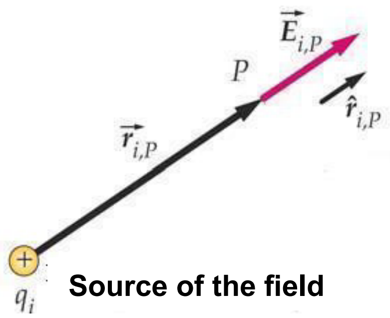
The electric force exerted by one charge on another is an example of action at a distance, similar to the gravitational force exerted by one body on another.

A charge creates an electric field $\vec{E}$ throughout space and this field exerts a force on another charge.

The force is thus exerted by the field at the position of the second charge, rather than by the first charge itself which is at some distance.

La fuerza neta ejercida sobre q_0 es la suma vectorial de las fuerzas individuales ejercidas sobre q_0 por cada una de las otras cargas del sistema. Según la ley de Coulomb, cada una de estas fuerzas es proporcional a q_0 y por tanto, la fuerza resultante será proporcional a q_0.





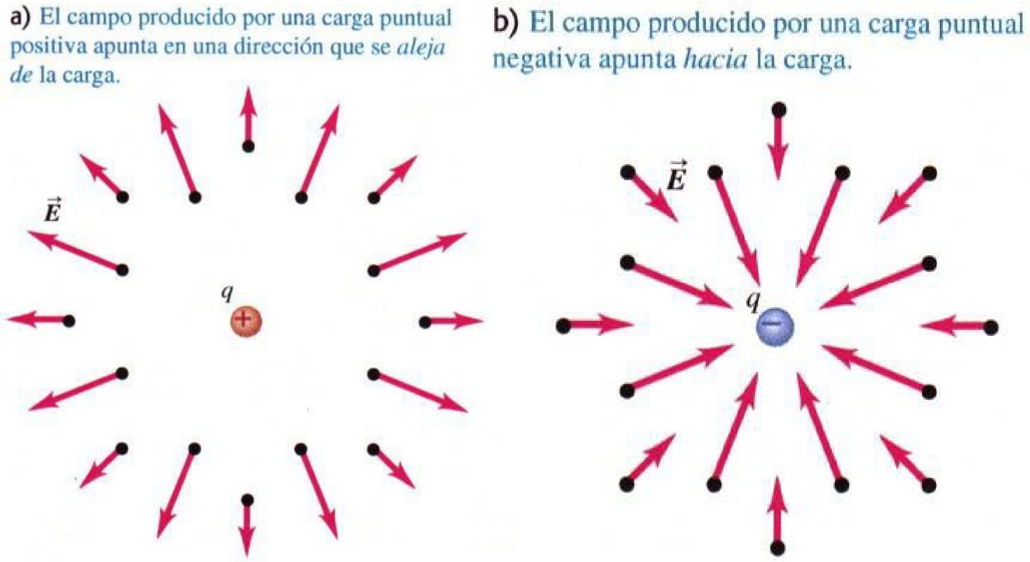
Source of the field

$$\vec{E} = \frac{\vec{F}}{q_0} \qquad (q_0 \text{ small})$$

Electric field $\vec{E}$ at a point P due to the charge q_0 placed at a point i

Electric field $\vec{E}$ produced by a point charge:

$$\boxed{\vec{E} = k\frac{q}{r_i^2}\ \vec{u}_i}$$

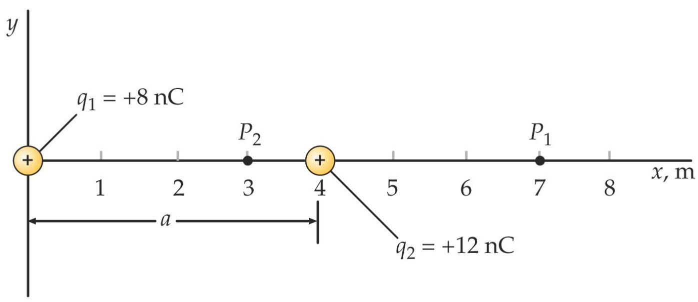


Electric field $\vec{E}$ at a point P due to a system of charges

$$\boxed{\overrightarrow{E_P} = \sum_{i=1}^{N} \overrightarrow{E_{i,P}} = \sum_{i=1}^{N} k\frac{q_i}{r_{i,P}^2}\ \vec{u}_{i,P}}$$

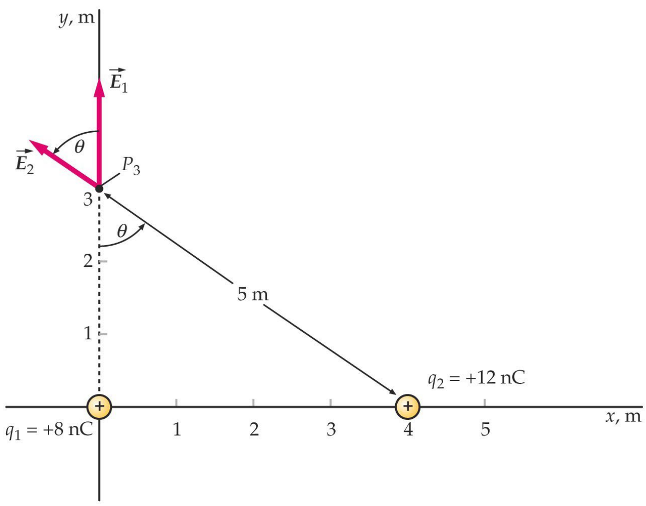
Examples:

1.3 Determine the electric field at points P_1 and P_2.



Solution: $\vec{E}_{P_1} = 13.5\ N/C\ \vec{\imath}$; $\vec{E}_{P_2} = -100\ N/C\ \vec{\imath}$

1.4 Determine the electric field on the y axis at y =3 for the system of charges

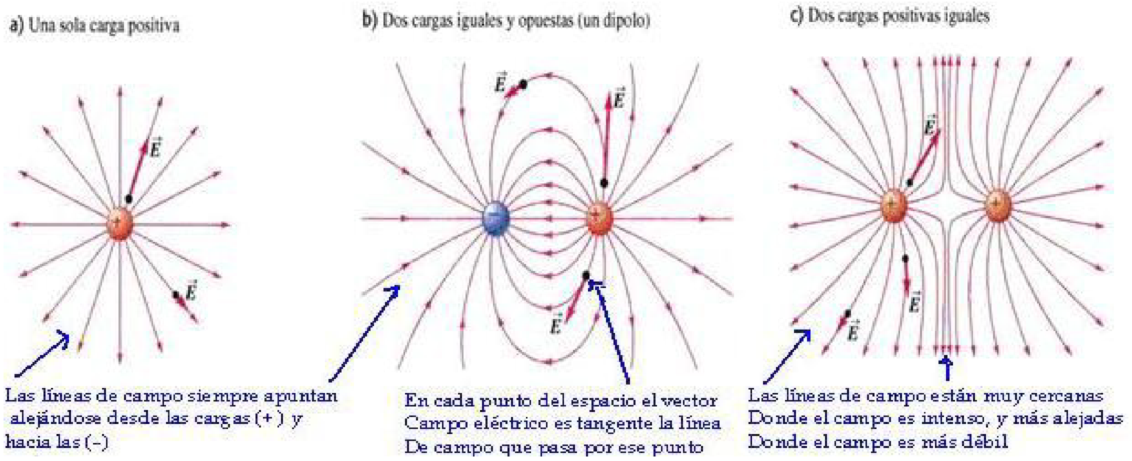


Solution: $\vec{E} = (-3.46\,\vec{\imath} + 10.6\,\vec{\jmath})N/C$

Electric field lines

The electric field can be represented by drawing lines that indicate its direction.**A field line is a path in which the electric field is tangent to it at every point.** At any point near a positive charge, the electric field points radially away from the charge; Likewise, the electric field lines converge towards a point occupied by a negative charge.

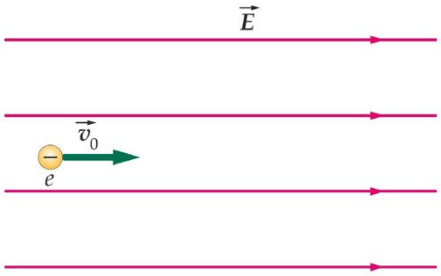
- The lines are drawn evenly spaced and exiting or entering the charge.
- The number of lines that leave a positive charge or enter a negative charge is proportional to the magnitude of the charge.
- The density of lines (number of them per unit area perpendicular to them) at a point is proportional to the value of the field module at that point.
- At large distances from a system of charges, the field lines are equally spaced and radial, as if coming from a single point charge equal to the net charge of the system.
- Two field lines can never intersect. (If two field lines crossed, this would indicate two directions for $\vec{E}$ at the point of intersection, which is impossible)



Movement of point charges in an electric field

Electron moving parallel to a uniform electric field:

-When a particle with charge q is placed in an electric field $\vec{E}$, it experiences the action of a force qE. If the electric force is only significant force acting on the particle, it acquires an acceleration.



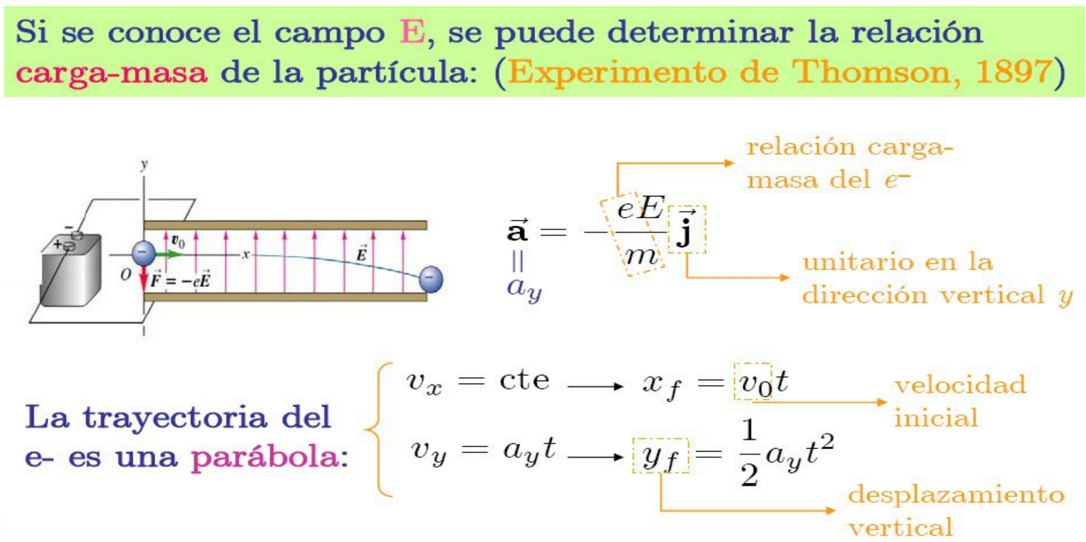
$$\boxed{\vec{a} = \frac{\sum \vec{F}}{m} = \frac{q}{m}\,\vec{E}}$$

Example:

1.5 An electron is projected into a uniform electric field $\vec{E} = 1000\,\vec{\imath}\,N/C$, with an initial speed $v_0 = (2 \times 10^6)\mathrm{m}/s$ in the direction of the field. How far will the electron travel before it momentarily remains at rest?

Solution: 1.14 cm.

Electron moving perpendicular to a uniform electric field:



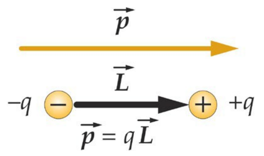
Example:

1.6 An electron is projected into a uniform electric field $\vec{E} = -2000\,\vec{\imath}\,N/C$, with a speed $\vec{v_0} = (2 \times 10^6\,\vec{\imath})\mathrm{m}/s$ perpendicular to the field. How much will the electron have deviated if it has traveled 1 cm in the x direction?

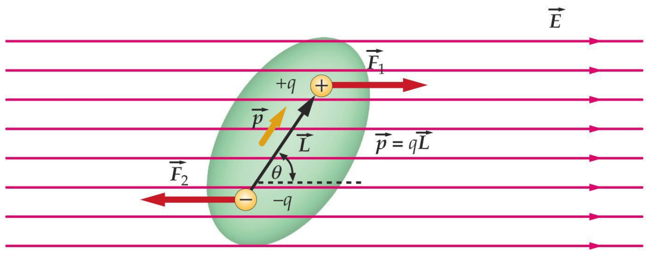
Solution:1.76 cm

ELECTRICAL DIPOLE

A system of two equal and opposite charges q separated by a small distance L is called **electric dipole**. Its intensity and orientation are described by the electric dipolar momentum vector $\vec{p}$, which points from the negative charge to the positive charge and whose module is the product qL:



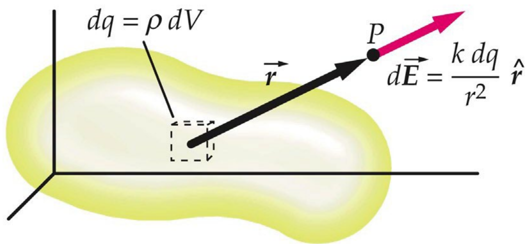
Electric dipoles in electric fields



$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

A dipole in a uniform electric field experiences equal and opposite forces that tend to rotate the dipole, so that its dipole moment tends to align with the electric field.

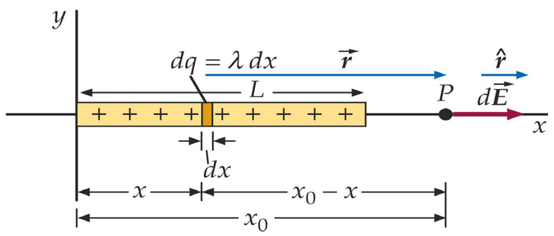
Electric field $\vec{E}$ due to a continuous distribution of charge



$$\boxed{\vec{E} = \int k \frac{dq}{r^2}\, \vec{u}_r}$$

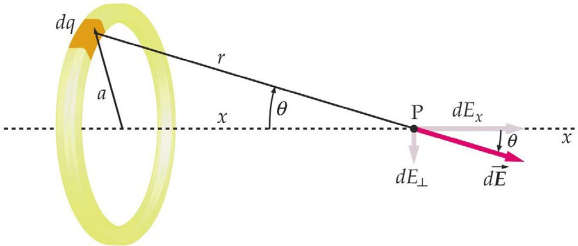
Examples:

1.7 Electric field $\vec{E}$ calculation on the axis produced by a finite linear charge distribution.



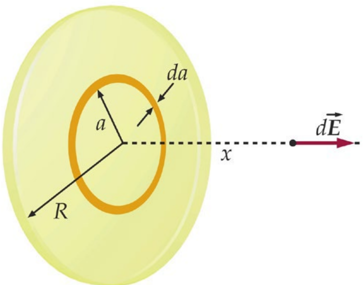
Solution: $E_x = \frac{kQ}{x_0(x_0 - L)}$

1.8 Electric field $\vec{E}$ calculation on the axis produced by a ring.



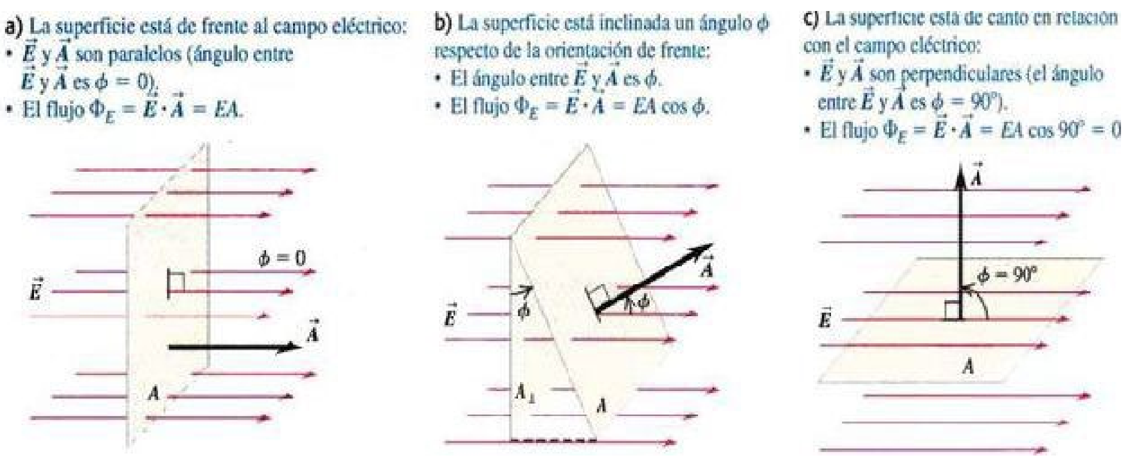
Solution: $E_x = \frac{kQx}{(x^2+a^2)^{\frac{3}{2}}}$

1.9 Electric field $\vec{E}$ calculation on the axis produced by a uniformly charged disk.

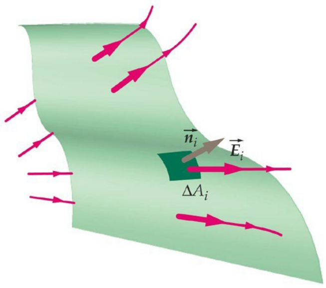


Solution: $E_x = -2\pi k\sigma\left(1 - \frac{x}{\sqrt{1+\frac{R^2}{x^2}}}\right)$

GAUSS'S LAW



The mathematical magnitude that is related to the number of field lines that cross a surface is called **electric flux, Φ**. The units for the electric flux in the S.I. are Nm^2/C. Since the electric field is proportional to the number of lines per unit area, the electric flux is proportional to the number of field lines passing through the area.



Let’s consider an arbitrary surface on which the electric field $\vec{E}$ may vary. If the area ΔA_i of the area element we choose is small enough, we can consider it as a plane and the variation of the electric field through the surface element can be neglected. Then, the electric flow for the entire surface is:

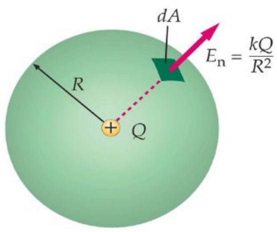
$$\boxed{\phi = \lim_{\Delta A_i \to 0} \sum_i \vec{E}_i \cdot \vec{n}_i \, \Delta A_i = \int_S \vec{E} \cdot \vec{n} \, dA}$$

In the case of electric field flow through closed surfaces, there is the convention of taking always the unit vector $\vec{n}$ directed towards the outside of the surface at each point on it.

$$\boxed{\phi = \oint_S \vec{E} \cdot \vec{n} \, dA = \oint_S E_n \, dA}$$

The net flux through any closed surface is positive or negative depending on whether it is directed predominantly outward or inward of the surface.

The electric field at any point on the surface is perpendicular to the surface and has the modulus:

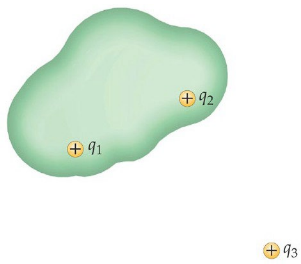


$$E_n = k \frac{Q}{R^2}$$

The net electric flux of $\vec{E}$ through this surface is:

$$\phi_{net} = \oint_S E_n dA = E_n \oint_S dA = \frac{kQ}{R^2} 4\pi R^2 = 4\pi kQ = Q/\epsilon_0$$

Generalizing to a system of point charges q_1,q_2,q_3, the flux through the surface is $4\pi k(q_1+q_2)$ which can be positive, negative or zero depending on the signs and values of the two charges.

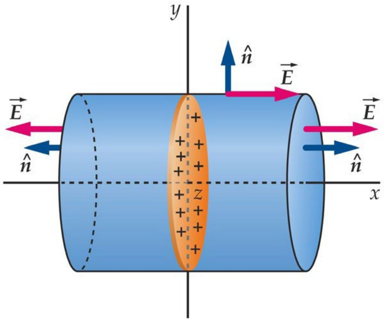


The electric flow through a closed surface is proportional to the charge enclosed in said surface.

$$\boxed{\phi_{net} = \oint_S \overrightarrow{E_n} \cdot \vec{n} dA = \oint_S E_n dA = \frac{Q_{enclosed}}{\epsilon_0}}$$

Applications to Gauss's law

- **Plane symmetry** (if the charge density depends only on the distance from a plane)

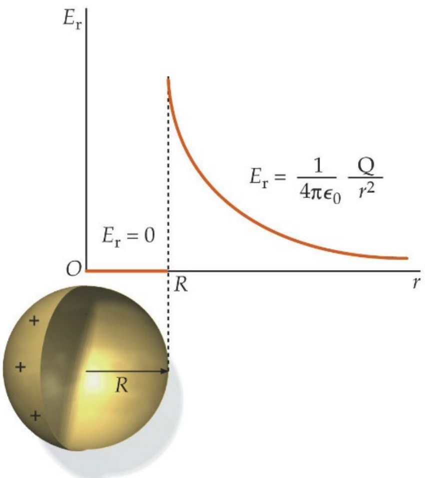


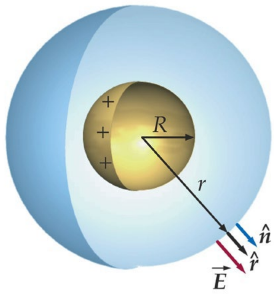
$$\boxed{E_n = \frac{\sigma}{2\epsilon_0}}$$

The magnitude is constant all over space.

- **Spherical symmetry** (if the charge density depends only on the distance to a point)

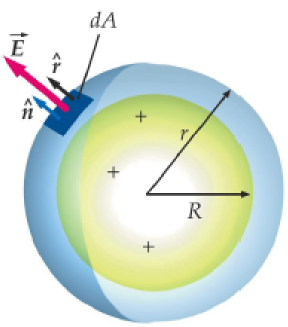
  - **Spherical shell (crust, cortex)**



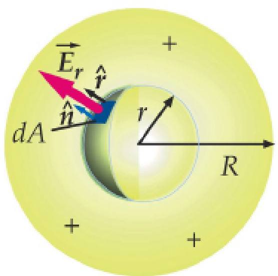


$$\begin{cases} E_r = 0 & r<R \\ E_r = \dfrac{1}{4\pi\varepsilon_o}\dfrac{Q}{r^2} & r>R \end{cases}$$

  - **Uniformly charged solid sphere**

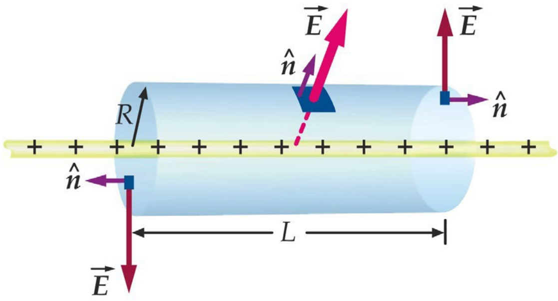


$$\boxed{E_r = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}} \quad r>R$$



$$\boxed{E_r = \frac{1}{4\pi\varepsilon_o}\frac{Q}{R^3}r} \quad r<R$$

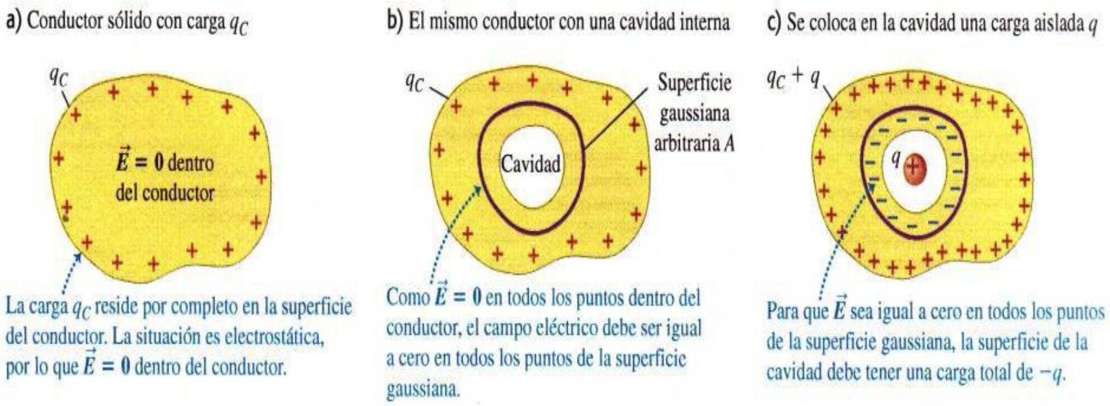
- **Cylindrical symmetry** (if the charge density depends only on the distance from a plane)

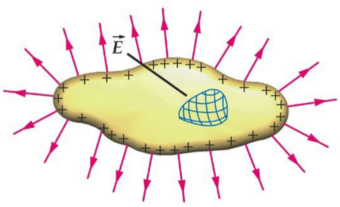


$$\boxed{E_r = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}}$$

CONDUCTORS

Charge and field on the surface of conductors

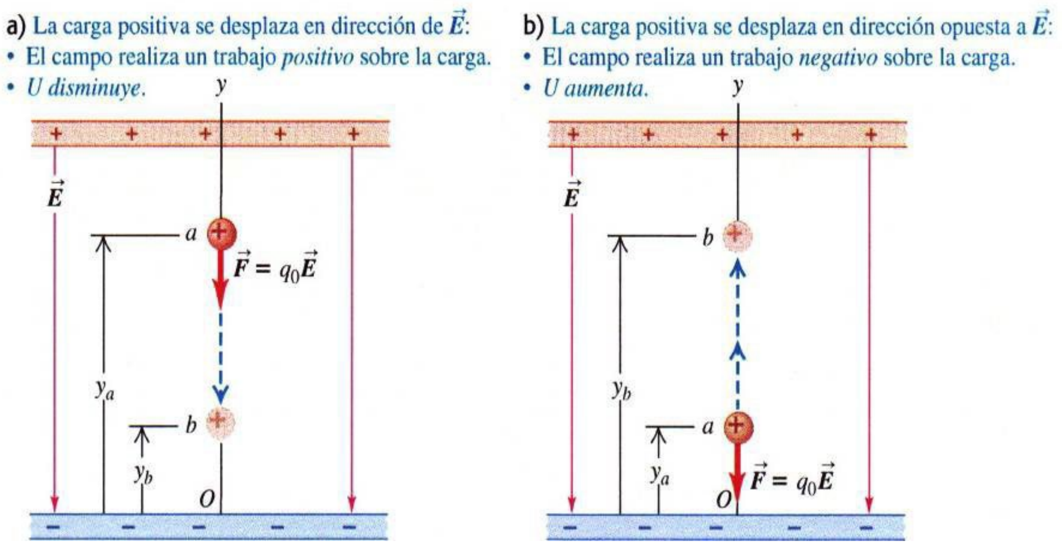




- The electric field inside a conductor in electrostatic equilibrium is zero.
- The electric field is normal to the surface of the conductor and its magnitude is: $E_n = \sigma/\epsilon_0$
- The charge on a conductor in electrostatic equilibrium resides on its surface.

ELECTRICAL POTENTIAL ENERGY

Uniform electric field



$$W_{a\to b} = U_a - U_b = -(U_b - U_a) = -\Delta U$$

- <u>Work done by a conservative force</u>

The potential energy for the electric force $F_y = -q_0 E$, is:

$$U = q_0 E y$$

When the test charge moves from the height y_a and to the height y_b, the work done on the charge by the electric field is given by:

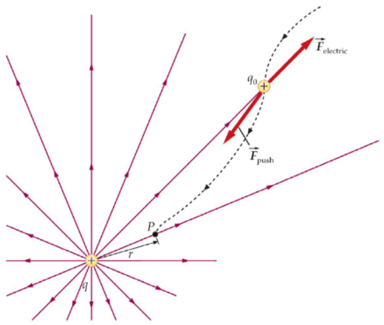
$$W_{a\to b} = U_a - U_b = -(U_b - U_a) = -\Delta U = -(q_0 E y_b - q_0 E y_a) = q_0 E(y_a - y_b)$$

Not uniform electric field

The work done on a test charge *q₀* that moves in the electric field caused by a single stationary point charge *q:*

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}\,\vec{u}_r$$

If q and q_0 are of the same sign, the force is repulsive and F is positive; if charges have opposite signs, the force is attractive and F is negative. The magnitude of the force is not constant so that in order to obtain the work $W_{a\to b}$ that the force exerts on q_0 when q_0 moves from *a* to *b*, it is necessary to integrate:



$$W_{a\to b} = \int_{r_a}^{r_b} \vec{F}\cdot d\vec{r} = \int_{r_a}^{r_b} \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}\,\vec{u}_r\cdot d\vec{r}$$
$$= \frac{qq_0}{4\pi\epsilon_0}\left(\frac{1}{r_a} + \frac{1}{r_b}\right) = U_a - U_b$$

The potential energy between two point charges q and q_0:

Generalizing to n point charges $q_1, q_2, \ldots, q_n$ at distances $r_1, r_2, \ldots, r_n$ from the charge q_0:

$$\boxed{U = \frac{q_0}{4\pi\varepsilon_o}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots\right) = \frac{q_0}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}}$$

ELECTRIC POTENTIAL

It is the potential energy per unit charge. The potential V at any point is defined as the potential energy U per unit charge associated with a test charge q_0

$$V = \frac{U}{q_0}$$

Potential due to a point charge:

$$W = \int_r^{\infty} \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\,dr = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r} = U$$

$$\boxed{V = \frac{U}{q_o} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}}$$

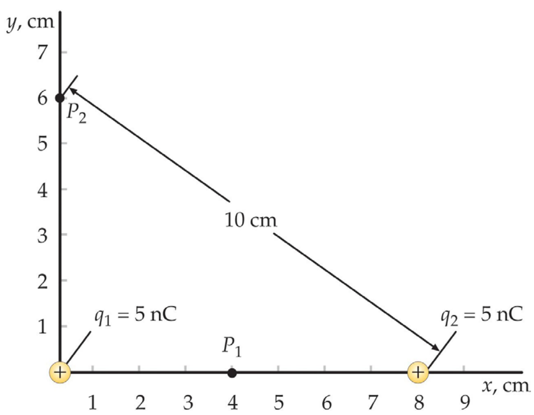
The potential is positive or negative, depending on the charge.

Potential due to various point charges:

$$\boxed{V = \frac{U}{q_o} = \frac{1}{4\pi\varepsilon_o}\sum_i \frac{q_i}{r_i}}$$

Example:

1.10 Determine the potential at points P_1 and P_2



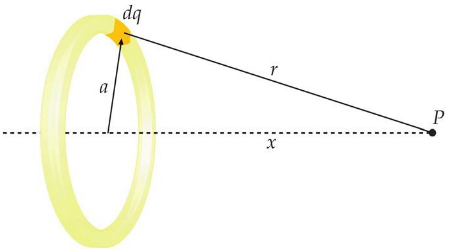
Solution: V_{P1}= 2.25kV, V_{P2}= 1.20kV

Potential due to continuous charge distributions

$$\boxed{V = \frac{1}{4\pi\varepsilon_o}\int\frac{dq}{r}}$$

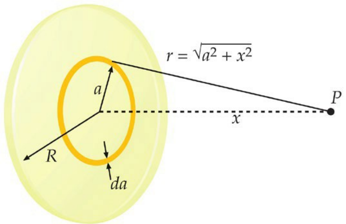
Examples:

1.11 Potential along the axis of a uniformly charged ring



$$V = \frac{1}{4\pi\varepsilon_o}\int\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{\sqrt{x^2+a^2}} = \frac{1}{4\pi\varepsilon_0\sqrt{a^2+x^2}}\int dq = \frac{Q}{4\pi\varepsilon_0\sqrt{a^2+x^2}}$$

1.12 Potential on the axis of the uniformly loaded disk.



$$V = \int\frac{kdq}{r} = \int\frac{k\sigma 2\pi a da}{\sqrt{x^2+a^2}} = 2\pi\sigma k\int_0^R (x^2+a^2)^{-1/2} a da = 2\pi\sigma k\left[(x^2+a^2)^{1/2}\right]_0^R =$$
$$= 2\pi\sigma k\left[(x^2+R^2)^{1/2} - x\right]$$

Electric field and potential.

Obtention of $\vec{E}$, known V:

$$dU = -\vec{F}\cdot d\vec{l} = -q\vec{E}\cdot d\vec{l}$$
$$dV = \frac{dU}{q} = -\vec{E}\cdot d\vec{l}$$

$$\boxed{\Delta V = V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{l}}$$

Applications:

- Potential and electric field on the axis of a dipole at points very far from it. Consider a charge be q (0,0,a) and a charge $-q$ (0,0,-a). The potential at point P (0,0,z) is:

$$V_P = \frac{kq}{z-a} - \frac{kq}{z+a}$$

If z >>a $\qquad V_P = \frac{kq}{z^2}$

- V potential inside and outside a charged spherical crust

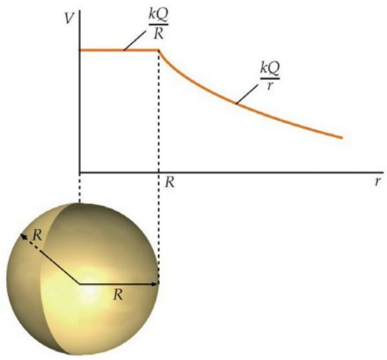
Outside the spherical shell, the electric field is radial and is the same as if the entire charge Q were punctual and located at the origin:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\vec{u}_r$$

$$dV = -\vec{E}\cdot d\vec{l} = -\frac{kQ}{r^2}\vec{u}_r\cdot d\vec{l} = -\frac{kQ}{r^2}dr \implies V = \int_\infty^r -\vec{E}\cdot d\vec{l} = -\int_\infty^r -\frac{kQ}{r^2}dr = \frac{kQ}{r}$$

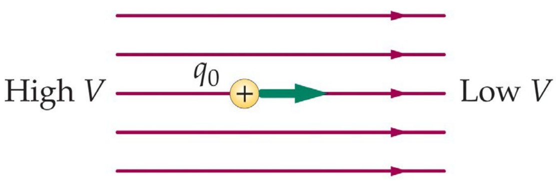
For r ≥ R: $\qquad V = \frac{kQ}{r}$

For r ≤ R $\qquad V = \frac{kQ}{R}$



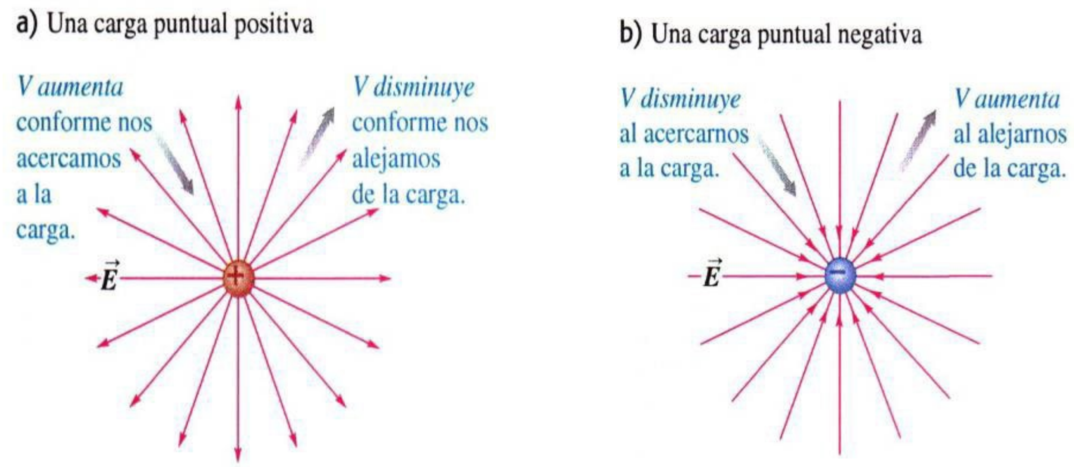
Potential and Electric Field Lines

If we place a positive test charge in an electric field $\vec{E}$ and we set it free, it will accelerate in the direction of the electric field, along the field line.



The kinetic energy of the charge will increase and its potential energy will decrease. Thus, the charge moves toward a region of less potential energy. For a point control charge, a region of less potential energy is a region of less potential

Therefore, the electric field lines will point in the direction in which the electric potential decreases.



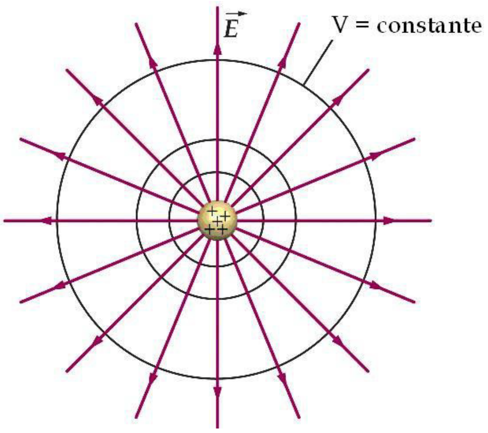
Equipotential surfaces

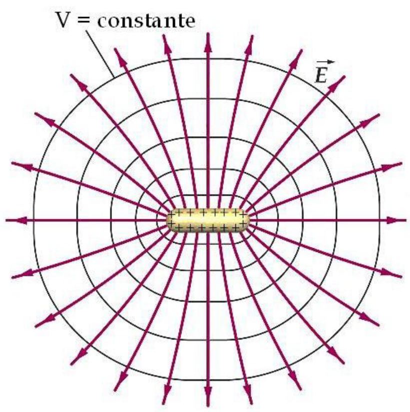
Since there is no electric field inside a conductor that is in electrostatic equilibrium, the change in potential from one point to another inside the conductor is zero. The electric potential is, therefore, the same throughout the conductor, that is, the conductor occupies a equipotential volume and its surface is a equipotential surface.

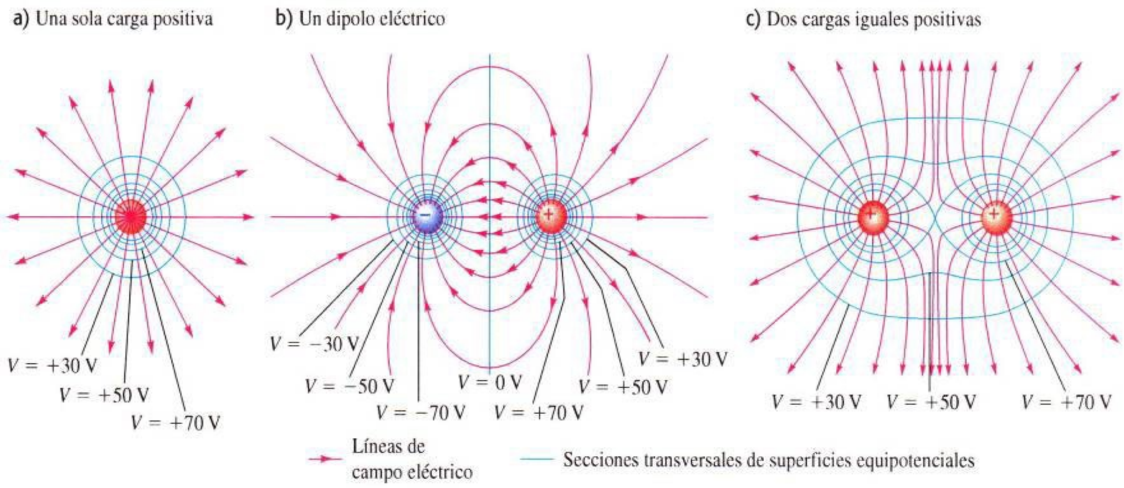
Since the potential is constant over such a surface, the change in V when a test charge experiences a displacement $d\vec{l}$ that is parallel to the surface is:

$$dV = -\vec{E} \cdot d\vec{l}$$

As $\vec{E} \cdot d\vec{l} = 0$ for any $d\vec{l}$ that is parallel to the surface, $\vec{E}$ must be perpendicular to every $d\vec{l}$ parallel to this surface. The only way for the $\vec{E}$ to be perpendicular to every $d\vec{l}$ parallel to the surface is that the electric field $\vec{E}$ is actually perpendicular to that surface at every point. Consequently, it can be concluded that the electric field lines are normal to any equipotential surface.

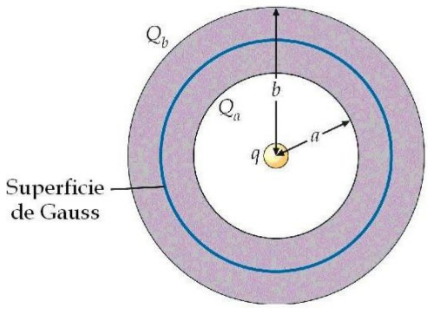




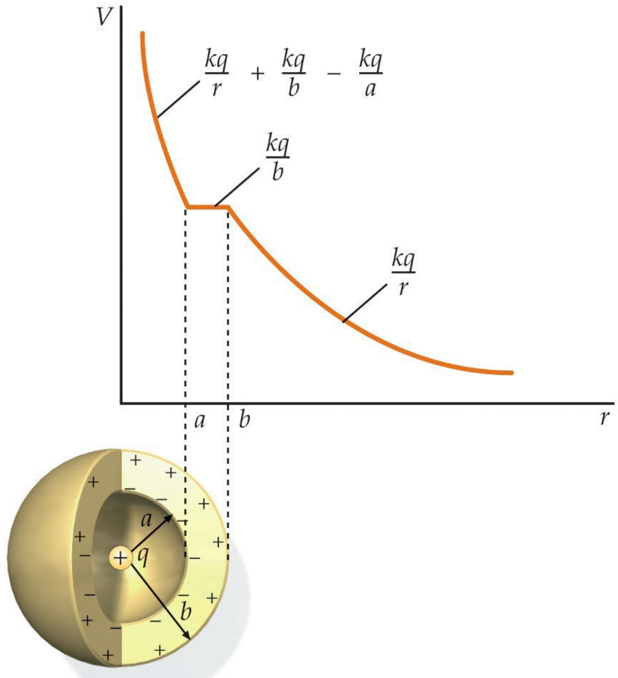


Example:

An unchanrged hollow spherical conductor has an internal radius *a* and an external radius *b*. In the center of the spherical cavity there is a point charge +*q* (as shown in the figure). (a) Determine the charge existing on each surface of the conductor. b) Determine the potential V(r) at any point assuming that V= 0 for r=∞.



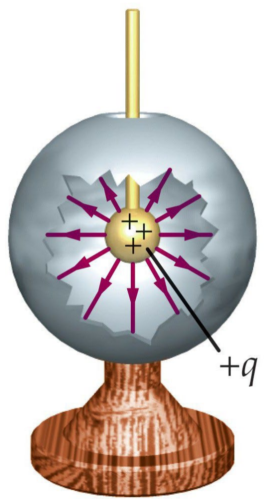
Solution: Q_a= -q; Q_b= +q;



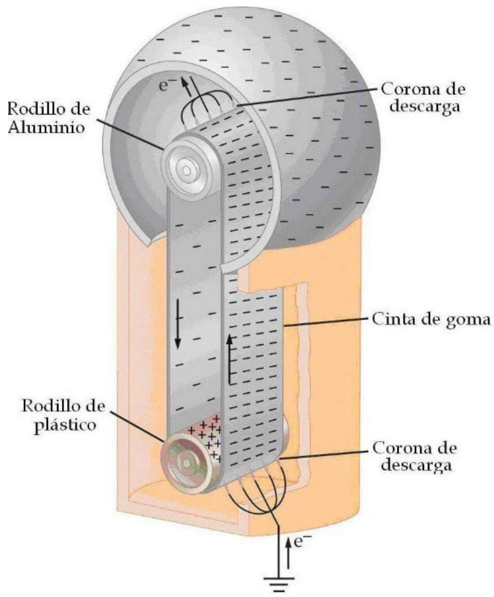
Van de Graaff generator

Consider a small conductor that has a positive charge *q*. This conductor is located inside the cavity of another second larger conductor. At equilibrium, the electric field is zero inside the conductive material of both conductors. The lines of force coming from the positive charge *q* must end at the inner surface of the large conductor (see left figure below). This should occur no matter what charge is located on the external surface of the larger conductor. Regardless of the charge on the large conductor, the small conductor in the cavity is at a higher potential because the electric field lines run from this conductor to the larger conductor. If the conductors are then connected, for example with a thin conducting wire, all the charge originally placed on the smaller conductor will flow to the larger one. When the connection is broken, there will be no charge on the small conductor inside the cavity and there will be no field lines between the conductors. The positive charge transferred from the smaller conductor to the larger conductor resides entirely on the outer surface of the latter. If we put more positive charge on the smaller conductor of the cavity and again connect the conductors with a thin wire, we again transfer all the charge to the outer conductor. This procedure can be repeated indefinitely. This method is used to produce large potentials in the Van de Graff generator, in which charge is brought to the inner surface of a very large spherical conductor by a continuous conveyor belt (see right figure below).

A Van de Graff accelerator is a device that uses the intense electric field produced by a Van de Graff generator to accelerate positive particles, such as protons.



Small conductor that has a (+) charge inside a larger conductor



Schematic diagram of a generator in the by Van de Graaff.

CAPACITORS

Capacitance

The potential of a single isolated conductor, containing a charge Q, is proportional to this charge and depends on the size and shape of the conductor. In general, the larger the surface area of the conductor, the greater the amount of charge it can store for a given potential.

The potential of a spherical potential of radius *R* and charge *Q*: $V = \frac{kQ}{R}$

The relation *Q/V* between charge and potential of an isolated conductor is called **capacitance C**.

A capacitor it is a device made up of two conductors, one of them with a Q charge and the other with a –*Q* charge. The relationship between the charge *Q* and the potential difference between the two conductors is defined as the capacitance of the capacitor.

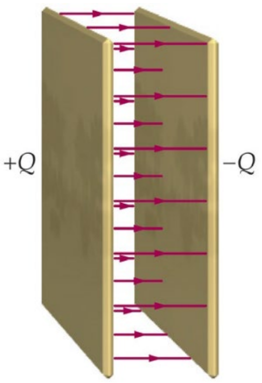
$$\boxed{C = \frac{Q}{V}}$$

This magnitude measures the "capacity" to store charge for a given potential difference.

$$C = \frac{Q}{V} = \frac{Q}{kQ/R} = \frac{R}{k} = 4\pi\epsilon_0 R$$

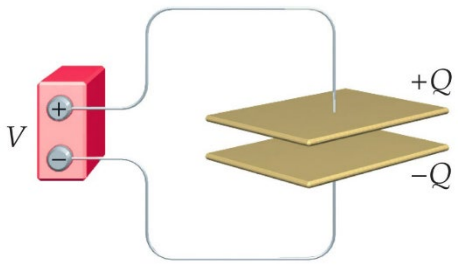
Types of capacitors

- Plane-parallel plate capacitor: it is formed by two large parallel conducting plates, where the area of each plate is *A*, the separation distance *d*, and +*Q* the charge on one plate and –*Q* the charge on the other. Since the plates are very close together, the field at any point between them is approximately equal to the field due to two infinite planes with equal and opposite charges. Each plate contributes a uniform field of module $E = \sigma/(2\varepsilon_0)$.



$$V = Ed = \frac{\sigma}{\epsilon_0} d = \frac{Qd}{A\epsilon_0}$$

$$\boxed{C = \frac{Q}{V} = \frac{\epsilon_0 A}{d}}$$



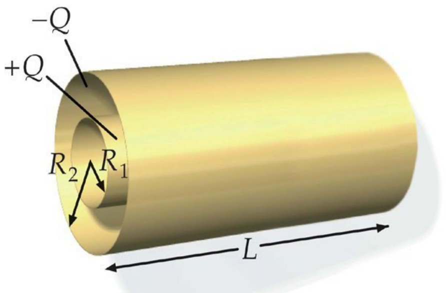
In general, the capacitance depends on the size, shape, geometry and relative position of the conductors and also on the properties of the insulating medium that separates them.

Example:

A plane-parallel plate capacitor is formed by two square conductors with sides 10 cm separated by 1 mm of distance. (a) Calculate the capacity. (b) If this capacitor is charged with 12 V, how much charge is transferred from one plate to the other?

Solution: 88.5 pF, 1.06 nC.

- **Cylindrical capacitor:** It consists of a small cylinder or conducting wire of radius R_1 (or a small cortex of radius R_1) and a larger cylindrical cortex of radius R_2 concentric with the previous one.



$$V = \frac{Q}{2\pi L \varepsilon_0} \ln \frac{R_2}{R_1}$$

$$\boxed{C = \frac{2\pi\varepsilon_o L}{\ln\left(R_2/R_1\right)}}$$

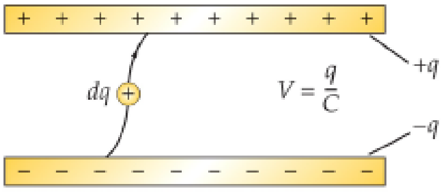
ELECTRICAL ENERGY STORAGE. ELECTRIC FIELD ENERGY

When a capacitor charges, electrons are transferred from the positively charged conductor to the negatively charged conductor. Thus, the positively charged conductor has a deficit of electrons whose value is identical to the charge excess (*surplus*) of the conductor that is negatively charged.

We initially consider two discharged conductors that are not in contact with each other. The charge transferred after a certain time during the process of charging the capacitor will be *q*. The potential difference is then *V=q/C*. If a small additional amount of charge *dq* is now transferred from the negative conductor at zero potential to the positive conductor at a potential V, the potential energy U of the capacitor increases by

$$dU = Vdq = \frac{q}{C}dq$$

$$U = \int dU = \int_0^Q \frac{q}{C}dq = \frac{1}{2}\frac{Q^2}{C}$$



$$\boxed{U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2}$$

-Electrostatic field energy

In the process of charging a capacitor, an electric field is created between the plates. The work necessary to charge the capacitor can be considered as that required to create the electric field. That is, the energy stored in the capacitor resides in the electric field and, therefore, is called **energy of the electrostatic field**. Let's consider a parallel plate capacitor.

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)(Ed)^2 = \frac{1}{2}\epsilon_0 E^2 (Ad)$$

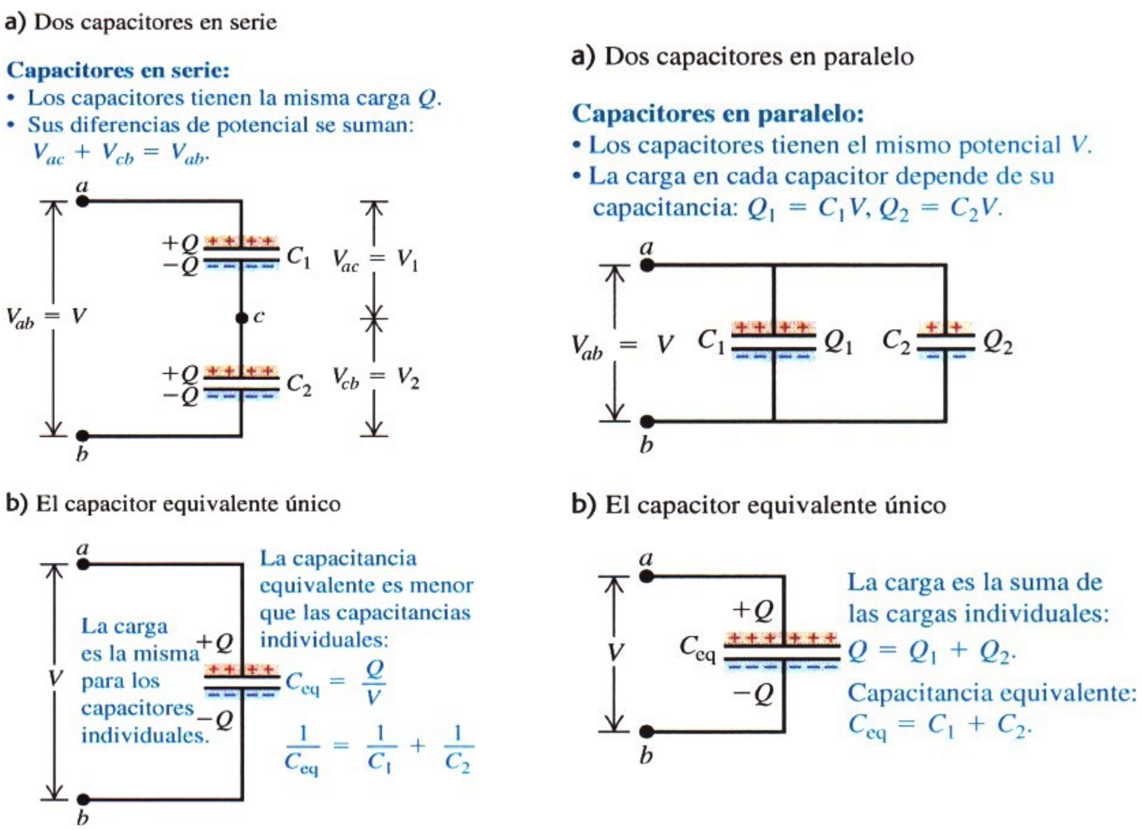
The energy density of an electrostatic field: $\rho_e = \frac{energy}{volume} = \frac{1}{2}\epsilon_0 E^2$

Example:

A capacitor with squared, plane-parallel plates, measuring 14 cm and spaced 2 mm apart, is connected to a battery and charged to 12 V. (a) What is the charge of the capacitor? (b) How much energy is originally stored in the capacitor? (c) If the battery of the capacitor is diconnected and the plate separation is increased to 3.5 mm, by how much does the energy increase by changing the separation of the plates?

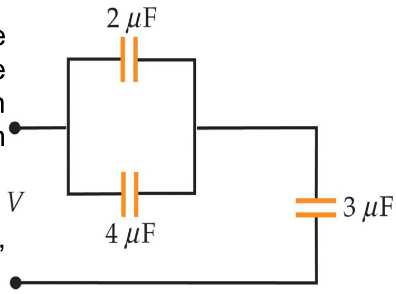
Solution:1.04 nC; 6.24 nJ; 4.68 nJ.

Capacitors Association



Examples:

1. a) Determine the equivalent capacitance of the circuit formed by the three capacitors. b) Initially the capacitors are discharged. Determine the charge on each capacitor and the voltage drop across it when the system is connected to a 6V battery.



Solution: C_{eq}= 2 μF, $V_{2\mu F,\ 4\mu F}$= 2 V, $V_{3\mu F}$= 4 V, $Q_{2\mu F}$= 4 μC, $Q_{4\mu F}$= 8 μC

2. Two parallel plate capacitors, each with a capacity of C_1=C_2= 2μF are connected in parallel through a 12 V battery. Determine: a) the charge on each capacitor and b) the total energy stored in the capacitors.

The capacitors are then disconnected from the battery and between the C_2 capacitor plates a dielectric of constant ε_r= 2.5, determine c) the potential difference between the capacitor plates, d) the charge deposited on each of them and e) the total energy stored by both.

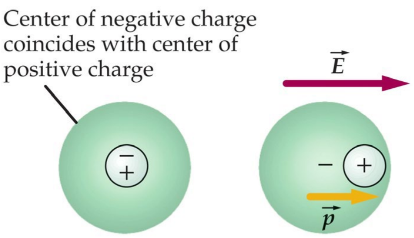
Solution: a) $Q_{2\mu F}$= 24 μC, b) U= 288 μJ, c) V = 6.86 V, d) Q_1 = 13.7 μC, Q_2 = 34.3 μC, e) U = 165 μJ.

DIELECTRICS, PHENOMENOLOGICAL DESCRIPTION

A non-conductive (insulating) material such as glass, paper or wood is called a “dielectric”. Faraday discovered that when the space between the two conductors of a capacitor is occupied by a dielectric, the capacitance increases by a factor ε_r which is characteristic of the dielectric. The reason for this increase is that the electric field between the plates of a capacitor is weakened by the dielectric. Thus, for a given charge on the plates, the potential difference is reduced and the Q/V ratio is increased.

A dielectric weakens the electric field between the capacitor plates because, in the presence of an external electric field, the dielectric molecules produce an additional electric field in the opposite direction to that of the external field.

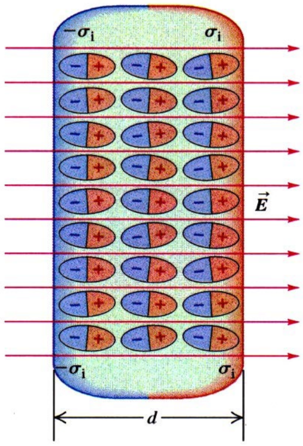
In some atoms and molecules, the electron cloud is spherically symmetrical, so that its “center of charges” is in the center of the atom or molecule, coinciding with the positive charge.



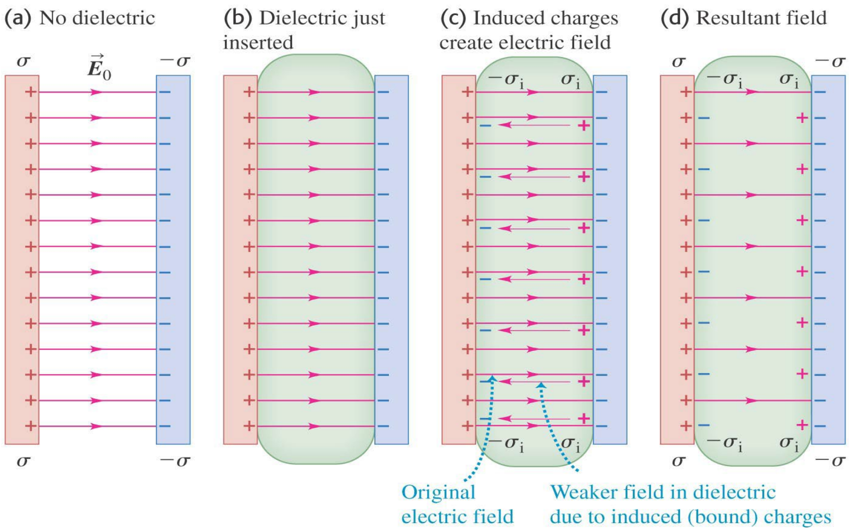
An atom or molecule of this type has a zero dipole moment and is called nonpolar. However, in the presence of an external electric field, the positive charge and the negative charge experience forces in opposite directions. The molecule is then said to be polarized and behaves like an electric dipole.

In some molecules (for example HCl and H_2O), the centers of positive and negative charge do not coincide, even in the absence of an external electric field. These polar molecules have a permanent electric dipole moment.

When a dielectric is placed in the field of a capacitor, its molecules become polarized in such a way that a net dipole moment parallel to the field is produced. If the molecules are polar, their dipole moments, originally oriented randomly, tend to align due to the moment of force exerted by the field. If the molecules are nonpolar, the field induces dipole moments that are parallel to the field. In either case, the dielectric molecules become polarized in the direction of the external field.



Polarization of a dielectric in an electric field



The net effect of polarizing a homogeneous dielectric in a capacitor is the creation of a surface charge on the faces of the dielectric next to the plates. This surface charge, linked to the dielectric, is called **induced surface charge density $\boldsymbol{\sigma_i}$**. It is bound to the molecules of the dielectric and cannot move like the free charge that exists on the conductive plates of the capacitor. The induced charge produces an electric field opposite to the direction of the field produced by the free charge of the conductors.
Thus, the net electric field between the plates weakens.

$$E = \frac{E_0}{\epsilon_r}$$

$$V = Ed = \frac{E_0 d}{\epsilon_r} = \frac{V_0}{\epsilon_r}$$

$$C = \frac{Q}{V} = \frac{Q}{\frac{V_0}{\epsilon_r}} = \epsilon_r \frac{Q}{V_0}$$

Thus,

$$\boxed{C = \epsilon_r C_0}$$

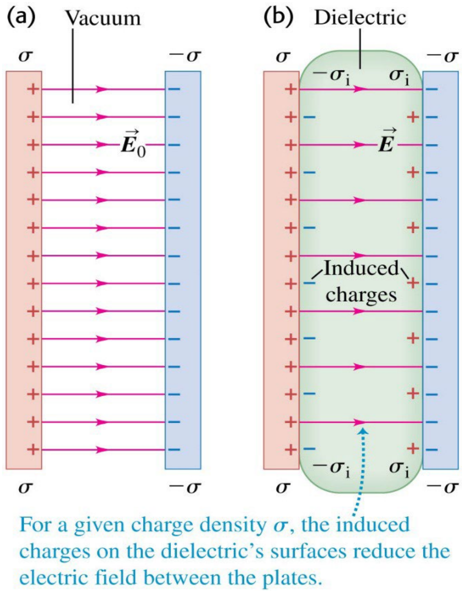
<u>Example</u>:

A flat capacitor has square plates with a side of 10 cm and a separation d = 4 mm. A dielectric block of constant ε_r= 2 has dimensions 10cm×10cm×4mm is inserted between the plates. (a) What is the capacity without dielectric? (b) What is the capacity if the dielectric block fills the space between the plates?

Solution: 22 pF, 44 pF

Bound and free charge

The surface density of the induced charge σ_i the dielectric surfaces is related to the dielectric constant ε_r and the free charge density σ the plates.



$$E_0 = \frac{\sigma}{\epsilon_0}$$

$$E_i = \frac{\sigma_i}{\epsilon_0}$$

$$\boxed{\sigma_i = \sigma\left(1 - \frac{1}{\epsilon_r}\right)}$$

$$E = E_0 - E_i = \frac{E_0}{\epsilon_r}$$

$$\boxed{E_i = \left(1 - \frac{1}{\epsilon_r}\right) E_0}$$

Therefore, the surface induced charge density σ_i is always less than the free charge density σ located in the sheets of the capacitor and is zero if ε_r=1, which is the case of lack of dielectric (air, vacuum). For a conducting block, $\varepsilon_r= \infty$ and $\sigma_i= \sigma$.