Matrices

- eigen value & eigen vectors
- diagonalisable
- property based sums
- cayley-Hamilton theorem

eigen value & eigen vector

characteristic eqn

$|A - \lambda I| = 0$

distinct

$\lambda = 1, 2, 3$

(eigen values)

↓

apply Crammer's Rule

↓

Repeated

$\lambda = 1, 2, 2$

(eigen values)

↓

Reduction method

↓

diagonalisable matrix

characteristic eqn

$|A - \lambda I| = 0$

distinct

$\lambda = 1, 2, 3$

(eigen values

↓

eigen vector by crammer's Rule

↓

λ is distinct so matrix is diagonalisable

Repeated

$\lambda = 1, 2, 2$

(eigen vectors)

↓

Reduction method (col-Row)

↓

λ is repeated we will check for $AM = GM$

↓

AM = no. of repeated values

$GM = n - r$ (Rank)

↓

no. of unknowns

---

for checking the repeat value using you solve $x_2 = s$, $x_3 = t$

$k_1(x_1) + k_2(x_2)$

eg: $k_1(-2, 1, 0) + k_2(-1, 0, 1)$

$-2k_1 - k_2 = 0$, $k_1 = 0$, $k_2 = 0$

put $k_1 = k_2 = 0$

$$\left.\begin{aligned} -2k - k_2 &= 0 \\ 2(0) - (0) &= 0 \\ 0 &= 0 \end{aligned}\right\}$$ it is linearly independent means answer is right

How to solve eigen values & eigen vectors

Steps : 3×3 matrix

characteristic eqn

$|A - \lambda I| = 0$

$\lambda^3 - S1\lambda^2 + S2\lambda - |A| = 0$

2×2 matrix

For 2×2 $\begin{vmatrix} 0 & \\ & \end{vmatrix} \begin{vmatrix} & 0 \\ & \end{vmatrix}$

if there is double likes then λ will find directly

S1 = diagonal elements add karne kal!

S2 = $\begin{vmatrix} \text{first row} \\ \text{first col} \end{vmatrix} + \begin{vmatrix} \text{second row} \\ \text{2nd col} \end{vmatrix} + \begin{vmatrix} \text{3rd row} \\ \text{3rd col} \end{vmatrix}$

|A| = through calc → mode → 6 → 1 → 1 → add value then Ac
shift + 4 then 7
shift + 4 then 3
boom!

λ = — — — — —
eigen values

distinct

λ = — — — $\lambda = [A - \lambda I]X = 0$

by Crammers Rule

$\frac{x1}{|\quad|} = \frac{-x2}{|\quad|} = \frac{x3}{|\quad|} = t$

$\frac{x1}{\sim} = \frac{x2}{\sim} = \frac{x3}{\sim}$

$\begin{bmatrix} - \\ - \\ - \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix}$

repeated

$\lambda = [A - \lambda I]X = 0$

if it's repeated then make row transformation and make last two row zero using R1

and assume $x2 = s$
$x3 = t$

and find out $x1$ =

$\begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} = \begin{bmatrix} t\text{ side} + s\text{ side} \\ + \\ + \end{bmatrix}$

"How to solve diagonalisable?"

Same steps of eigen values & eigen vector

Note: "if eigen value is distinct then it's diagonalisable"

and if it's repeated then we have to check for AM = GM

AM → repeated values

GM → n − r → rank
↳ no. of unknowns

for λ = AM = & GM =

for λ = AM = & GM =

A is diagonalisable.

for M → transforming / model / matric

$$M = \begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ \left[\begin{matrix} - & - & - \\ - & - & - \\ - & - & - \end{matrix}\right. & & \left.\right] \\ x_1 & x_2 & x_3 \\ \uparrow & \uparrow & \uparrow \\ \text{eigen} & \text{eigen} & \text{eigen} \end{array}$$

$$\underbrace{\text{eigen} \quad \text{eigen} \quad \text{eigen}}_{\text{vectors}}$$

D → Diagonalisable matrix

$D = M^{-1}AM$

$$\begin{bmatrix} - & 0 & 0 \\ 0 & - & 0 \\ 0 & 0 & - \end{bmatrix}$$

put in diagonal → λ = eigen values and rest of them are zero

"Property Based sum"

$$\left.\begin{bmatrix} & & \\ 0 & & \\ 0 & 0 & \end{bmatrix} \begin{bmatrix} & 0 & 0 \\ & & 0 \\ & & \end{bmatrix}\right\} \text{triangular matrix}$$

(1) sum of eigen value = sum of diagonal elements

(2) product of eigen value = $|A|$

(3) $A^{-1} = \lambda^{-1} = \frac{1}{\lambda}$ or $A^2 = \lambda^2$ and $A^3 = \lambda^3$

(4) Adjoint of $A = \frac{|A|}{\lambda}$

(5) If matrix is triangular then eigen value are it's diagonal elements and if it's not then solve manually using characteristic eqn. $|A - \lambda I| = 0$

"Cayley-Hamilton Theorem"

for verify and find for A^{-1}, A^{-2}, A^4

for matrix and $A^6 - 5A^5 + A^4 + \ldots$ something like that

C-H theorem bolta hai, eigen values nahi nikalneka.

characteristic eqn tak hi nikalna. $|A - \lambda I| = 0$

then $\lambda = A$

Steps: characteristic eqn $|A - \lambda I| = 0$

- $\lambda^3 - S_1\lambda \mp S_2\lambda - |A| = 0$

$S_1 =$
$S_2 =$
$|A| =$

you know how to solve previously we saw!

eg: $\lambda^3 - 5\lambda^2 + 9\lambda - 1 = 0$

then $A^3 - 5A^2 + 9A - I = 0$ — ①

and prove them

- for find A^{-1}

multiply by eqn ① by A^{-1}

$$\frac{A^3}{A} - \frac{5A^2}{A} + \frac{9A}{A} - \frac{I}{A} = 0$$

$$A^2 - 5A + 9I - A^{-1} = 0 \text{ — eqn ②}$$

for find A^{-2}

multiply by eqn ② by A^{-1}

$$\frac{A^2}{A} - \frac{5A}{A} + \frac{9I}{A} - \frac{A^{-1}}{A} = 0$$

$$A - 5I + 9A^{-1} - A^{-2} = 0$$

for find A^4

multiply eqn ① by A

$$A^4 - 5A^3 + 9A^2 - A = 0$$

Same here we have to prove then solve for

eg: ~~6A^3 + 9A~~ $A^6 - 6A^5 + 9A^4 + 4A^3 - 12A^2 + 2A - I$

λ wala.

Q

D

$$\frac{A^6}{A^3} = A^3$$

$$\frac{A^5}{A^3} = A^2$$

R ← $2A - I$ → stop here

then

$D = D \times Q + R$

$= (\quad) \times (\quad) + R$

$2A - I$

$$2\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

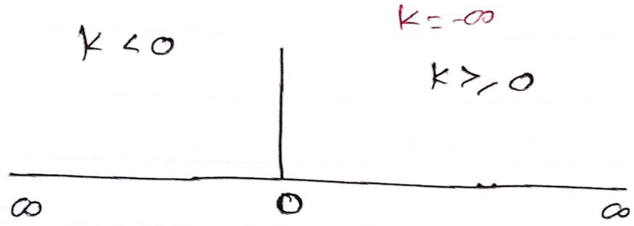
Solve it!

Z- transform

- → Basic $Z[a^k]$, $Z[a^{|k|}]$
- → property based / trigo
- → convolution
- → Inverse of Z transform.

(1) Basic :

$$Z[f(k)] = \sum_{k=-\infty}^{\infty} f(k)\, z^{-k}$$



$a^{|k|} \xrightarrow{\text{means}} a^{\pm k}$

$k<0 \Rightarrow -\infty$ to -1 and $k \geq 0 \rightarrow 0$ to ∞

Steps : formula put karo and solve karo

$1 + a + a^2 + a^3 + \cdots \Rightarrow \dfrac{1}{1-a}$

$a + ar + ar^2 + ar^3 + \cdots \Rightarrow \dfrac{a}{1-r}$ → common ratio

$1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \dfrac{x^4}{4!} + \cdots \Rightarrow e^x$

$(1+z)^{-1} = 1 - z + z^2 - z^3 + z^4 \cdots$

$(1-z)^{-1} = 1 + z + z^2 + z^3 + z^4 + \cdots$

$(1+z)^{-2} = \cancel{1-2x+3x} \cdot 1 - 2x + 3x - 4x + 5x \cdots$

$(1-z)^{-2} = 1 + 2x + 3x + 4x + 5x \cdots$

For range

Jo pehele hai woh bada hai!

eg: $\dfrac{1}{(b-z)(z-a)}$ $\quad b>z, \quad z>a$

16

always positive solving

★ Convolutions: (*) $Z[F(k) * g(k)]$

Note: $U(k) = 1$ $k \geq 0$ → positive side (most asked)

$U(k) = 0$ $k < 0$ → negative side

Steps:

$$Z[F(k)] = \sum_{k=-\infty}^{\infty} F(k) z^{-k} \quad \text{—①}$$

$$Z(g(k)) = \sum_{k=-\infty}^{\infty} g(k) z^{-k} \quad \text{—②}$$

solve like Basic ques we solve.

then after solving,

by convolution property

$[Z[F(k)] * Z[g(k)]]$

put eqn ① & ② value and ~~right~~ write range.

Z-transform property based.

(1) change of scale. : $Z[a^k f(k)] = F\left(\frac{z}{a}\right)$

(2) shifting : $Z(f(k+n)) = z^n F(z)$

$Z[f(k-n)) = z^{-n} F(z)$

(3) multiplication by k:

$Z[k \cdot f(k)] = -z \frac{d}{dz} F(z)$

$Z(k^n \cdot f(k)) = \left(-z \frac{d}{dz}\right)^n F(z)$

(4) division by k :

$Z\left(\frac{f(k)}{k}\right) = \int_z^\infty \frac{1}{z} \cdot F(z)\, dz$

① $Z[U(k)] = \frac{z}{z-1}$ ② $Z(a^k) = \frac{z}{z-a}$

③ $Z[\sin \alpha k] = \frac{z \sin \alpha}{z^2 - 2z\cos\alpha + 1}$

④ $Z[\cos \alpha k] = \frac{z^2 - z\cos\alpha}{z^2 - 2z\cos\alpha + 1}$

⑤ $Z[\sinh \alpha k] = \frac{z \sinh\alpha}{z^2 - 2z\cosh\alpha + 1}$

⑥ $Z(\cosh \alpha k) = \frac{z^2 - z\cosh\alpha}{z^2 - 2z\cosh\alpha + 1}$

Inverse Z-transform:

Partial fraction:

① $$\frac{1}{(s-a)(s-b)(s-c)} = \frac{A}{(s-a)} + \frac{B}{(s-b)} + \frac{C}{(s-c)}$$

② $$\frac{1}{(s-a)(s-b)^2} = \frac{A}{s-a} + \frac{B}{(s-b)} + \frac{C}{(s-b)^2}$$

③ $$\frac{1}{(z+a)(z^2+b)} = \frac{A}{(z+a)} + \frac{Bs+C}{(z^2+b)}$$

$$A(z^2+b) + (Bs+C)(z+a)$$

$$\frac{1}{(s-a)(s-6)^2} = \frac{A}{(z-3)} + \frac{B}{(z-3)} + \frac{C}{(z-3)^2}$$

$$A(z-3)^2 + B(z-3)(z-3) + C(z-3)$$

ROC condn

① $|z| <$

② $< |z| <$

③ $|z| >$

How to solve z^{-1} transform

\# ②

always remember constant always outside and only multiply by z

Step 1: partial fraction

and find out A, B, C

then ROC $|z| <$

$< |z| <$

$|z| >$

so bada → hoga voh bahar nikalega.

then:

Step 1: taken common based on this Rule.

Step 2: convert into $(1+z)^{-1}$ or $(1-z)^{-1}$

Step 3: take it up and multiple by − if it's not in above type.

Step 4: expand.

constant always outside if $\frac{1}{z}$ then only multiply.

then. eg: $1 + z^{-1}z + z^{-2}z^{2} + \cdots z^{-k}z^{k}$

z^{-k} → hona chahiye

if z^{k} hoga toh usko z^{-k} karke z^{k} positive likh do.

then last

$z^{-1}[F(z)] =$ coefficient of z^{-k}

eg: $-2^{k-1} + 3^{k-1}$

if $2z^3 + 3z^3 + 4z^2$ — then $(k-1)$

and if $-\frac{1}{3} \times 3^{k}$ hoga toh usko -3^{k-1} likhna ka.

Complex Integration

- Laurent & Taylor Series (Imp)
- Line Integral : ~~[illegible]~~ circle
  parabola or line
- Cauchy Integral formula
- Cauchy Residue formula.

① Taylor & Laurent Series:

Type I

$|z| = 0$

Type II

$z = \pm a$

$z = +a$

$|z-a| = 0$

$z = -a$

$|z+a| = 0$

same for both type: Steps: ① Partial fractional
Find out A, B --- etc

② Cond^n : denometor decide → 3 cond^n

Type I : $z=0$ or kuch nahi diya hoga

eg: $f(z) = \dfrac{z}{(z-1)(z+3)}$ → Her jo bada hoga means pehale voh bahar jayega (based on cond^n)

min (under $(z-1)$), max (under $(z+3)$)

based on cond^n

① $|z|<1$, $|z|<3$

② $1 < |z| < 3$

③ $|z|>1$, $|z|>3$

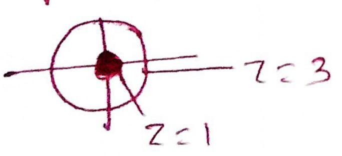
vahi fir z^{-1} transform jaisa solve karne ka.

→ taken common

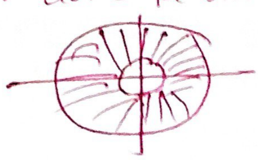
→ convert into $(1+z)^{-1}$ or $(1-z)^{-1}$
Alternate sign positive

→ expand and stop it

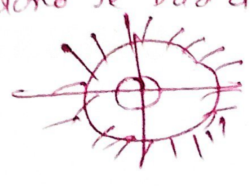
For $|z|<1$ & $|z|<3$ dono se chota



for dono ke bich me



dono se bada



Type 2 :

$z = 1, 2, 3 \ldots$ whatever

For $z = +a$

$|z-a| = 0$

eg: $f(z) = \dfrac{z}{(z-1)(z+3)}$

$\dfrac{1}{(z+1)} + \dfrac{z}{(z+3)}$

$\dfrac{1}{(z-1)+1-1} + \dfrac{z}{(z-1)+1+3}$

min max

For $z = -a$

$|z+a| = 0$

For eg: $\dfrac{z}{(z-1)(z-3)}$

$\dfrac{1}{(z-1)} + \dfrac{z}{(z+3)}$

Here $|z+a|$ so bala

$\dfrac{1}{(z+1)-1-1} + \dfrac{z}{(z+3)-1+3}$

min max

So then Same three 3 condⁿ

(1) $|z| < 1$ & $|z| < 3$

(2) $1 < |z| < 3$

(3) $|z| > 1$, $|z| > 3$

vahi same z^{-1} ki steps

(bada wala bahar jayega)

→ take common

→ convert it to $(1+z)^{-1}$ or $(1-z)^{-1}$

→ expand and stop then draw circle

Line Integral

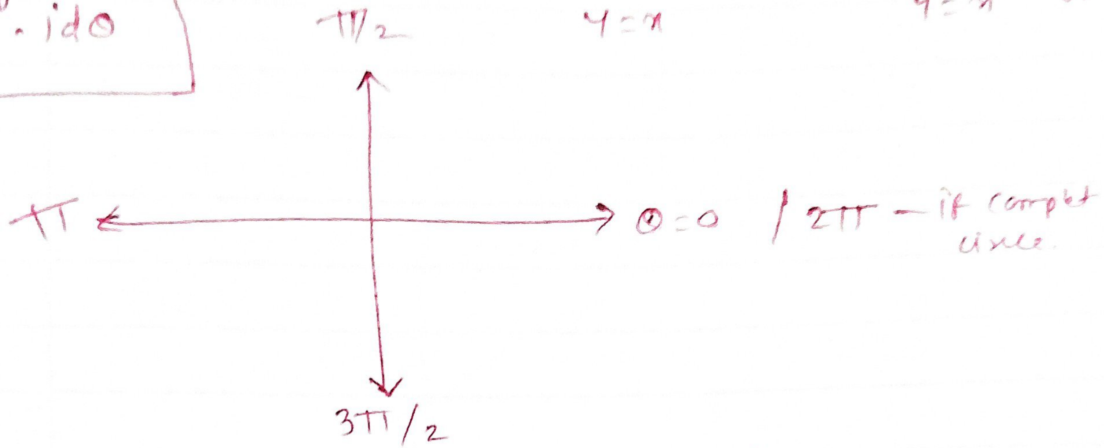
$z = x + iy$

$z = re^{i\theta}$ ⟵ circle $z = re^{i\theta}$

$dz = re^{i\theta} \cdot i\,d\theta$

line: $y = x$

parabola: $y = x^2$ or $x = y^2$



- **Line**

$y = x$

$dy = dx$

$z = x + iy$

$dz = dx + i\,dy$

$dz = dx + i\,dx$

$dz = (1+i)\,dx$

Parabola

$y = x^2$

$dy = 2x\,dx$

$z = x + iy$

$dz = dx + i\,dy$

$dz = dx + i\,2x\,dx$

$dz = (1+2i)\,dx$

$x = y^2$

$dx = 2y\,dy$

$z = x + iy$

$dz = dx + i\,dy$

$dz = 2y\,dy + i\,dy$

$dz = (2y+i)\,dy$

- **circle**

$z = x + iy \Rightarrow z = re^{i\theta}$

$\bar{z} = x - iy \Rightarrow \bar{z} = re^{-i\theta}$

$\overline{dz} = dx - i\,dy$

$e^{i\theta} = \cos\theta + i\sin\theta$

$e^{-i\theta} = \cos\theta - i\sin\theta$

$\cos(\text{even } \pi) = +1$

$\cos(\text{odd } \pi) = -1$

$r = |z| = 1$

$|z| = \sqrt{x^2+y^2}$

$r = \sqrt{x^2+y^2}$

$\cos 0 = 1$

$\sin 0 = 0$

$\cos \pi/2 = 0$

$\sin \pi/2 = 1$

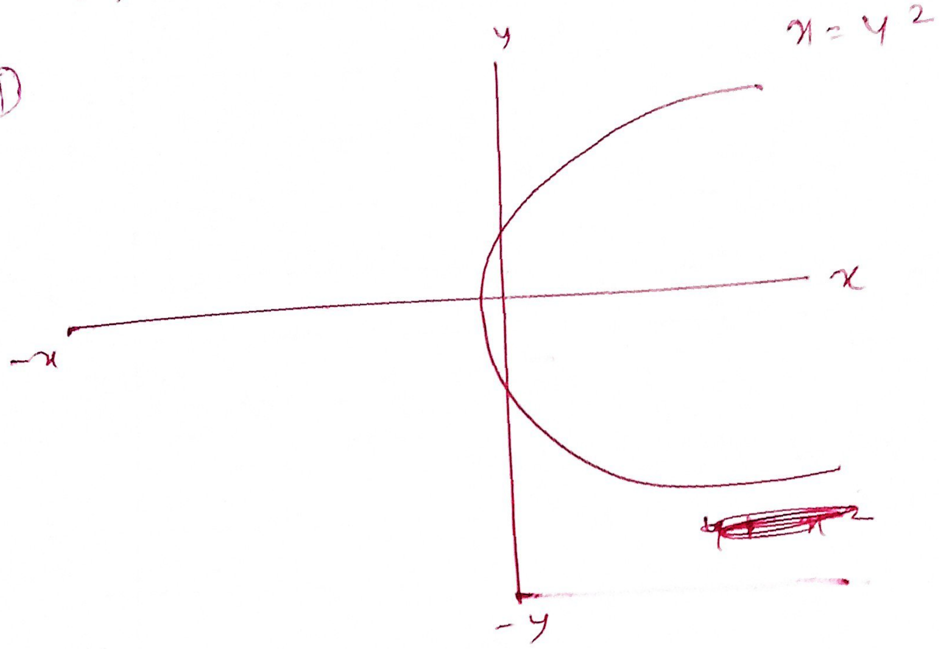
$\int uv = u\int v - u_{du} \int v + u_{d \cdot d} \int\int\int \cdots$

Zero aaya toh stop

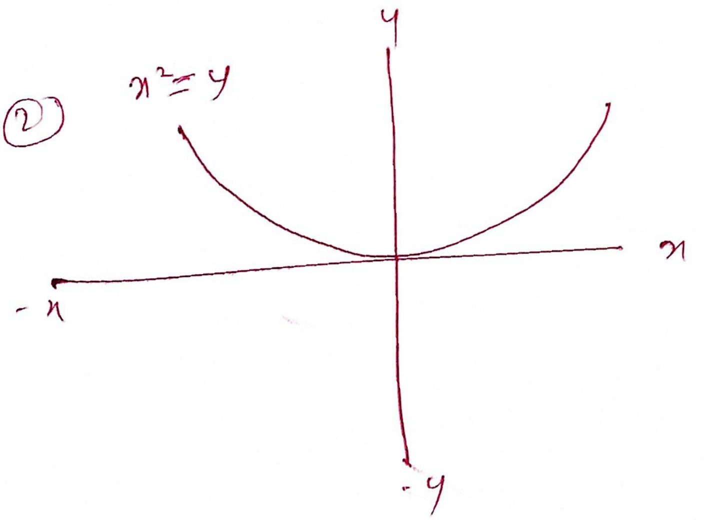
② $\log m^n = n \log m$

$\log e = 1$

①



②



Probability distribution

Poisson's

$p(X=x) = \frac{e^{-m} \cdot m^x}{x!}$

$m \to$ parameter

$x \to 0, 1, 2, 3 \ldots$

→ defective } to solve
→ Accident }

mean = m
variance = m

$\boxed{\text{mean} = \text{variance} = m}$

→ Avg also means mean

if m is not give

$m = np$

Recurrence formula:

$p(x+1) = \frac{m}{x+1} p(x)$

atleast → $p(X \geq \ )$

atmost → $p(X \leq \ )$

exact → $p(X = \ )$

more than → $p(x > \ )$

less than → $p(x < \ )$

Normal.

Standard normal variate

$z = \frac{X - m}{\sigma}$

$m \to$ mean

$\sigma \to$ standard deviation

forward (Normal)

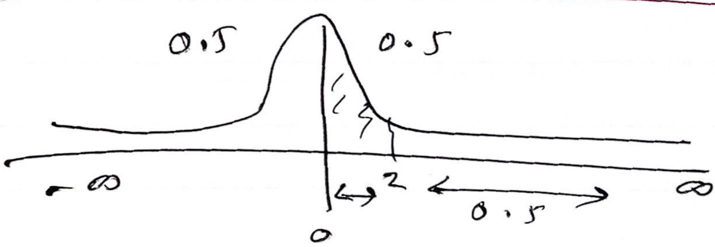
x, m, σ diya hoga

↓

type (5) find z ka vale in table.

Reverse

z ya x nahi diya hoga nikalneka hota!



for 0 → z we can find in z table

for → z → ∞ use 0.5 − (0 → z)

0 → z → nikalne hai toh table me z ka column me 0.0 to 2.0 tak jam ka or side ki value

Agar 1.36 hoga toh → 1.3 z me dikhne ka or 0.6 Row me upper ki

★ How to solve Poisson's

$$p(x = x) = \frac{e^{-m} \cdot m^{x}}{x!}$$

value put karo ho gaya!

and condⁿ based hoga toh solve karo.

★ Fit poisson

x		f
↓	× × ×	↓
Σx	Σf	Σxf

- $\bar{x} = m = \dfrac{\Sigma xf}{\Sigma f} =$

~~mean~~

then

$$p(x = x) = \frac{e^{-m} \cdot m^{x}}{x!}$$

expected freq = $N \times p(x)$

(Σf) 200 × —— something solve kar

Now for $x = 0, 1, 2, 3$ question me diya hoga.

- $p(x = 0) = \dfrac{e^{(\ )} \times (\ )^{0}}{0!} = \longrightarrow$ Round off karo.

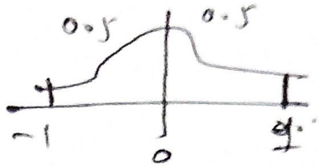
continue

Normal distribution

① Forward sum: (m & σ given, x → z find karte)

$z = \frac{x-m}{\sigma}$

Find $z \le x \le 4.5$



and if $z > 1$

so then

0.5 −' (z = 0 to z = 1)

and if $z < 1$

so then 0.5 − (z = 0 to z = 1)

② Reversed sum: Probability diya hoga → z nikalneka (m & σ is given z find karte)

if p = 5% = 0.05

so = 0.5 − 0.05

= 0.45

&

then yeh value z ke table me dikhenge ka.

z = value aayegi.

0.5 − probability %

Z ki value = $\frac{X - 300}{20}$ something

m & n find karte.

③ Type ⑤ Find z value in table.

① 7% below 35

50 − 7 = 43% ⇒ 0.43   35 < m

② 89% below 63

89 − 50 = 39% ⇒ 0.39

so look 0.43 & 0.39 value in z table

and write down "Remember first below value is always negative)

if ~~....~~ 0

mean = mean than one hoga, 1,4,3

usko add karo

σ = more than two 1,4,3

$(1)^2 + (2)^2 + (3)^2 =$

$\sqrt{\ } =$

then

$z = \frac{x-m}{\sigma}$, $z = \frac{x-m}{\sigma}$

eqn ① & ② substract

then we will get σ value & then

put in eqn ① σ value we will get

$\boxed{m}$

small sample test (t- distribution)

$n < 30$

single population | double population | chi-test χ^2

steps for t-dist and χ^2 dist

step ①: Null hypo H_0 : $\mu = 0$ or $\mu_1 = \mu_2$

Altend Hypo H_a : $\mu \neq 0$ or $\mu_1 \neq \mu_2$

step ②: calculation of statistics;

step ③: level of significal - by default 5% = 0.05

degree of freedom → $n-1$

step ④: critical value:

t_α

① calculated value > table value

Null hypo Rejected

② calculated value < table value

Null hypo accepted.

step 5: decision

$$S = \sqrt{\frac{\Sigma (x-\bar{x})^2}{n}}$$

$$t = \frac{\bar{x} - \mu}{\frac{S}{\sqrt{n-1}}} \rightarrow |t| =$$

★ How to solve single population

n, $\bar{x}$. diya hoga.

$u =$

$u \neq$

first find $S =$

$S = \sqrt{\dfrac{(x_1 - \bar{x})^2}{n}}$ $S =$

then

follow the steps:

in step ② statistic calc means

$$t = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n-1}}}$$

if single population table wala aaya toh: μ is not given

$\mu = 0$

Steps: $n =$

$x_1 =$ eg: 1, 2, ...

$\bar{x}_1 = \dfrac{\Sigma x_1}{n} = 0.2$

$\Sigma (x_1 - \bar{x})^2 = \Sigma (1 - 0.2)^2$ ————

then $S = \sqrt{\dfrac{\Sigma (x_1 - \bar{x})}{n}}$

then $t = \dfrac{\bar{x} - u}{\frac{S}{\sqrt{n-1}}}$

then steps follow karo.

"How to solve double population"

$x_1 =$ $\quad x_2 =$ $\quad n_1 =$

$\Sigma(x_1-\bar{x}_1)^2 =$ $\quad \Sigma(x_2-\bar{x}_2)^2$ $\quad n_2 =$

$\mu_1 = \mu_2$

$\mu_1 \neq \mu_2$

★ for sum of square :

$$S_P = \sqrt{\frac{\Sigma(x_1-\bar{x}_1)^2 + \Sigma(x_2-\bar{x}_2)^2}{n_1+n_2-2}}$$

$S_P =$

Standard error $(S.E) = S_P \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

$S.E =$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S.E} \Rightarrow |t| =$$

★ for Unbaised sum : (likh ke aayega)

ismaebhi sab kuch hoga

$$S_P = \cancel{\sqrt{(n_1-s_1)^2+(n_2-s_2)^2}} \sqrt{\frac{(n_1-1)\overbrace{s_1^2 + (n_2-1)s_2}^{S.D}{}^2}{n_1+n_2-2}}$$

★ for sab kuch diya hoga toh : $\bar{x}_1 =$

$\bar{x}_2 =$

$$S_P = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1+n_2-2}}$$

$\left.\begin{array}{l} s_1 = \\ s_2 = \end{array}\right\} S.D$ $\quad n_1 =$ $\quad n_2 =$

then follow the same steps:

double population if asked in Table (like suilons)

$\overline{X1} = \frac{\Sigma X1}{n1}$ $n1 =$

$\overline{X2} = \frac{\Sigma x2}{n2}$ $n2 =$

$\Sigma(X1 - \overline{X1})^2 =$

$\Sigma(X2 - \overline{X2})^2 =$

$$SP = \sqrt{\frac{\Sigma(X1-\overline{X1})^2 + \Sigma(X2-\overline{X2})^2}{n1 + n2 - 2}}$$

Standard error $S.E = SP \times \sqrt{\frac{1}{n1} + \frac{1}{n2}}$

$$t = \frac{\overline{X1} - \overline{X2}}{S.E} \qquad |t|$$

then follow the same steps

χ^2 – Test

① Test of goodness fit (table wala) — type ①

$$E = \frac{\text{observe freq (EF)}}{N}$$

$$\chi^2 = \frac{\Sigma(O-E)^2}{E}$$

table	O	E	$(O-E)^2$

total =

$$\frac{\Sigma(O-E)^2}{E} = \chi^2_{\text{calculated}}$$

Here NH : always assume true yes

AH : NO

DOF = n-1

LOC = 5% = 0.05

$\chi^2_{\text{calculate}} < \chi^2_{\text{table}} \longrightarrow$ Accepted

$\chi^2_{\text{calculate}} > \chi^2_{\text{table}} \longrightarrow$ Rejected

Remaining steps are same.

(2) Test of goodness fit (Ratio wal'a) 9 : 3 : 3 : 1 distributed.

~~A~~ First add ratio = E =

NH : Yes
AH : NO

A → $\frac{9}{E} \times$ total given =
B → =
C → =
D → =

O	E	$(O-E)^2$	$\left(\frac{O-E}{E}\right)^2$

or else find manually

$$\chi^2 = \frac{\Sigma (O-E)^2}{E}$$

Type (2) : Test of Independent Attributes :

Here NH : NO
AH : Yes (true)

$$DOF = \underbrace{(C-1)}_{col}\ \underbrace{(r-1)}_{row}$$

and don't count total col and row

$$E = \frac{RT \times CT}{T}$$

$\frac{\text{Row total} * \text{col total}}{\text{total}}$

O	E	$(O-E)^2$	$(O-E)^2/E$
question	new value		

Remaing same steps.

LPP

- Basic sol^n
- simplex method
- dual simplex
- find dual of ~~[illegible]~~

① Basic sol^n (find all Basic sol^n)

- feasible sol^n : both ans has positive elements
- Degenerate sol^n : dono me se ek ka ans zero hona chahiye or positive element hona chahiye
- optimal :· if max (most asked) : biggest value of z table
  if min (never asked) : smallest value of z table.

if $z = x_1 - 2x_2 + 4x_3$ (3 variable honge) toh no. of basic sol^n = 3

Non basic variable is 3 → $x_1, x_2, x_3 = 0$

if $z = x_1 + x_2 + x_3 + x_4$ (4 variable honge) toh no. of basic sol^n = 6

Non basic variable : x_1 x_2 x_3 x_4

$x_1, x_2 = 0$
$x_1, x_3 = 0$
$x_1, x_4 = 0$
$x_2, x_3 = 0$
$x_2, x_4 = 0$
$x_3, x_4 = 0$

table structure :

No. of basic sol^n	Non-basic variable	basic variable	eq^n	is it feasible?	is it degenerate	value of z	optimal ?
		Remaining variable	Based on				

Note: problem is always in maximization type ($\leq$) constraint if it's not then convert into max and $\leq$ by multiple by (—)

② Simplex Method (Imp):

firstly Add slack variables in 'Z' $-0S1-0S2-0S3$: eg

slack variable, how many ? :→ total no. of eqn in subject to

key column ⟶ sabse jyada negative value in Z

~~key~~ ~~row~~ → Ratio $\frac{RHS}{\text{key col}^n}$

table structure:

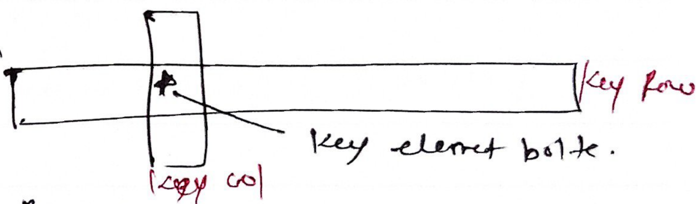
Iteration no	Basic variables	coefficient of here based on Z variable	R.H.S soln	Ratio
0	Z			
Slack variables	S1 S2 S3			

Iteration zero: filled out basic details:

fill karne ke baad key column find karne ka,

then Ratio me sabse smallest positive number find karne ka

$\left(\frac{RHS}{\text{key col}^n}\right)$ then



Iteration 1: you need to divide Entire key Row by key element.

then key col^n ko zero banane ka.

already zero hoga (iteration zero me) toh as it is copy paste karo.

key Row-col (Reduction method)

R1 Relation with help of key Row in new Iteration means 1 wala row

Z → R1
S1 → R2
S2 → R3
S3 → R4

R1 → R1 (+) R2 ——— ~~R~~ 1 wala usko ★ karne ka usko sath operation perform karne ka!

if Z has all positive value then stop it! | and if Ratio me sab negative value hoga toh stop it → No soln.

Note: problem is always be in minimization type (≤) if it's not then make it — if more than then multi by (−) and make it min

③ Dual- simplex method:

Firstly add slack variables in 'z' $- 0s_1 - 0s_2 - 0s_3$; eg

slack variables how many :→ total no. of eqⁿ in subject to

- here we have to find key row → In RHS select most negative value.

key column → $\frac{z}{\text{key row}}$ and find smallest positive.

table structure:

No. of Iteration	Basic variables	Coefficient of based on z variable	R.H.S
	z		
slack variable	s_1		
	s_2		
ratio			

Iteration zero: first filled the values

key row → in R.H.S most negative value

key col → $\frac{z}{\text{key row}}$ (and find smallest positive) (In Ratio)

Iteration one: Jaha key elements hai, udhar pure key row se divide karna hai and key element make it = 1

key colⁿ ko zero bananeka and simplex method jaise solve karneka.

In RHS all value is positive numbers then stop!!

Find dual of LPP

Maximization
(constraint must be $\leq$)

minimization
(constraint must be $\geq$)

if $=$ hoga toh
usko $\geq$ and $\leq$ karne ka
and then if mini hoga toh $\geq$ and max hoga toh $\leq$ karne ka.

and if $\geq$ hoga toh
usko $-$ se multiple karne ka.

steps: first express maximization $\leq$ constraint type.

max

subject to

Then uska matrix (dual hone ke bad)

$$\begin{array}{ccc} x_1 & x_2 & x_3 \ldots \\ \end{array} \begin{bmatrix} & & \\ & & \end{bmatrix}$$

Then required dual is

min $w =$ (subject to eqn ke $\leq$ value)
$w = y_1 \quad y_2 \quad y_3$

if $=$ hoga toh usko $y_3' - y_3''$

subject to : y_1 ke form me matrix likhe ka

eg: $-y_1 - 3y_2 + 2y_3' - 4y_3'' \geq$ here x ke value in R.H.S

(vertically lene ka hai)

Now put if $y_3' - y_3''$ is y_3

min $w =$

subject to $=$

y_1, y_2 and y_3 is unrestricted.

and if $\leq$ hoga toh usko $-$ se multiply karne ka.

steps: first express minimization constraint ($\geq$) type

min :
subject to : Then uska matrix (dual hone ke baad) $\longrightarrow$

$$\begin{array}{ccc} x_1 & x_2 & x_3 \\ \end{array} \begin{bmatrix} & & \\ & & \end{bmatrix}$$

if $=$ hoga toh $y_3' - y_3''$

The required dual is

max : $w =$ (subject to eqn $\geq$ RHS)
subject to : matrix ke eqn (vertically)

Now put if $y_3' - y_3'' = y_3$

max : $w =$

subject to :

y_1, y_2, y_3 is unrestricted.

NLPP

(1) Lagranage's multiplier (equality constraints)

- → 2 variable & 1 constraints
- → 2 variable & 2 constraints
- → 3 varaible & 1 constraints

} most asked

(2) Kuhn-Tucker's (Inequality constraints)

- → 2 variable & 1 constraints → (most asked)
- → 2 variables & 2 constraints

(1) Lagranage's method: 2 variables and 1 constraints

$$L(x_1, x_2 ; \lambda) = \underbrace{Z}_{\text{optimize}} - \lambda [\underbrace{h}_{\text{subject to}}]$$

$$\frac{\partial L}{\partial x_1} = 0 \quad , \quad \frac{\partial L}{\partial x_2} = 0 \quad \text{and} \quad \frac{\partial L}{\partial \lambda} = 0$$

Find x_1, x_2, (λ) → must be same as in subject to eqn

put x_1, x_2 in eqn (3) means λ

so will get $\lambda =$, $\underbrace{x_1 = \quad , \quad x_2 =}_{\text{Put eqn (1), (2) and } \lambda \text{ value.}}$ } you can solve using calc

$X_0 (x_1, x_2)$ → milega

$$\Delta_3 = \begin{array}{c|ccc|} & 0 & x_1 & x_2 \\ 0 & 0 & \frac{\partial h}{\partial x_1} & \frac{\partial h}{\partial x_2} \\ x_1 & \frac{\partial h_1}{\partial x_2} & \frac{\partial^2 L}{\partial x_1^2} & \frac{\partial^2 L}{\partial x_1 \partial x_2} \\ x_2 & \frac{\partial h_1}{\partial x_2} & \frac{\partial^2 L}{\partial x_2 \partial x_1} & \frac{\partial^2 L}{\partial x_2^2} \end{array}$$

$\boxed{\text{most of answer is } 0}$

$\Delta = -ve$ → minima

$\Delta = +ve$ → maxima

Put x_1, x_2 value in Z

you will get Z

$\frac{\partial h}{\partial x_1}, \frac{\partial h}{\partial x_2}$ = follow h → eqn

$\frac{\partial^2 L}{\partial x_1^2}, \frac{\partial^2 L}{\partial x_2^2}$ = follow L →

(2) Lagrange's method: 3 variables and 1 constraints:

Same steps: $L(x_1, x_2, x_3, \lambda) = Z - \lambda[h]$

$$\frac{\partial L}{\partial x_1} = 0, \quad \frac{\partial L}{\partial x_2} = 0, \quad \frac{\partial L}{\partial x_3} = 0, \quad \frac{\partial L}{\partial \lambda} = 0$$

Add eqn (1), (2), (3) (x_1, x_2, x_3)

you will get λ ka value

put λ ka value in eqn (1), (2), (3) (x_1, x_2, x_3)

you will get $x_1 =$, $x_2 =$, $x_3 =$

$x_0(x_1, x_2, x_3)$

- Now

$$\Delta_4 = \begin{array}{c|cccc|} & 0 & x_1 & x_2 & x_3 \\ \hline 0 & 0 & \frac{\partial h}{\partial x_1} & \frac{\partial h}{\partial x_2} & \frac{\partial h}{\partial x_3} \\ x_1 & \frac{\partial h}{\partial x_1} & \frac{\partial^2 L}{\partial x_1^2} & \frac{\partial^2 L}{\partial x_1 \partial x_2} & \frac{\partial^2 L}{\partial x_1 \partial x_3} \\ x_2 & \frac{\partial h}{\partial x_2} & \frac{\partial^2 L}{\partial x_2 \partial x_1} & \frac{\partial^2 L}{\partial x_2^2} & \frac{\partial^2 L}{\partial x_1 \partial x_3} \\ x_3 & \frac{\partial h}{\partial x_2} & \frac{\partial^2 L}{\partial x_3 \partial x_1} & \frac{\partial^2 L}{\partial x_3 \partial x_2} & \frac{\partial^2 L}{\partial x_3^2} \end{array}$$

If Δ_3 & Δ_4 both are negative $\Rightarrow$ minima

Δ_3 (+ve) & Δ_4 (−ve) means Alternate $\Rightarrow$ maxima

- First solve Δ_3 — using calc.

For Δ_4

$$\left| \quad \right|_{\text{first col, first row}} - \left| \quad \right|_{\text{2nd col, first row}} + \left| \quad \right|_{\text{3rd col, first row}} - \left| \quad \right|_{\text{4th col, first row}}$$

put x_1, x_2, x_3 value in Z and find out.

$Z =$

(3) Lagrange's method : 2 variables and 2 constraints

$$L(x_1, x_2, \lambda) = Z - \lambda_1 [h] - \lambda_2 [h]$$

$\frac{\partial L}{\partial x_1} = 0$ $\frac{\partial L}{\partial x_2} = 0$, $\frac{\partial L}{\partial \lambda_1} = 0$ $\frac{\partial L}{\partial \lambda_2} = 0$

↳ (1) ↳ (2) ↳ (3) ↳ (4)

solve eqn (1) × ~ } here balance through λ_1
eqn (2) × ~

~~Solve eqn~~. then add eqn (1) & (2)

You will get something ~ $\lambda_1 + \lambda_2$ = ~ eqn (5)

solve eqn (1) × ~ } here balance through λ_2
eqn (2) × ~
then add eqn (1) & (2)
You will get something ~ $\lambda_1 + \lambda_2$ = ~ eqn (6)

solving eqn (5) & eqn (6)
you will get x_1 & x_2
$X_0(x_1, x_2)$

if m = 2 variables & n = constraint
m = n Hessian matrix can't use
" if not then solve by Hessian matrix"

principal of minor elements

$$\begin{vmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{vmatrix}$$

eg:
$A_1 = -2$ $|-2|$
$A_2 = +4$

A_1 & A_2 is positive (Alternate elements)
↳ maxima

if A_1 & A_2 both negative then minima

then find Z =

Kuhn - Tucker's (K-T) condn: (Inequality)

- 2 variable & 1 constraint (most asked)
- 3 variable & 1 constraint.

1 Kuhn-Tucker : 2 variable & 1 constraint

$f(x_1, x_2) \rightarrow Z$

$h(x_1, x_2) \rightarrow h$

$L(x_1, x_2, \lambda) = Z - \lambda(h)$

According to K-T condn:

$\frac{\partial L}{\partial x_1} = 0 \longrightarrow (1)$

$\frac{\partial L}{\partial x_2} = 0 \longrightarrow (2)$

$h(x_1, x_2) \leq \rightarrow (3)$

$\lambda h(x_1, x_2) = 0 \longrightarrow (4)$

$x_1, x_2 \geq 0 \longrightarrow (5)$

$\lambda \geq 0$

(if Z is minima then $\lambda \leq 0$)

Case I : $\lambda = 0$

eqn (1) & eqn (2)

you will get $x_1 =$, $x_2 =$

put x_1 & x_2 in eqn (3)

check if it's satisfy

if it's not then "stop" "rejected"!

and if it's satisfy :

then put in Z - (satisfy) check for

you will get Z =

" if it's not satisfy then case II will satisfy "

Case II: $\lambda \neq 0$

eqn (4)
eqn (1)
eqn (2)

you will get x_1, x_2, λ

then find Z =

but here also check for satisfy

Put x_1, x_2 in eqn (3)

if satisfy then put in Z here it's also satisfy

2 Kuhn-Tucker: 2 variable & 2 constraints:

$F(x_1, x_2, \lambda_1, \lambda_2) = z$

$h(x_1, x_2) = h$

$\frac{\partial L}{\partial x_1} = 0$ —— ①, $\frac{\partial L}{\partial x_2} = 0 \rightarrow$ ②

$h_1(x_1, x_2) \le 0$ —— ③

$h_2(x_1, x_2) \le 0$ —— ④

$\lambda h_1(x_1, x_2) = 0$ —— ⑤

$\lambda h_2(x_1, x_2) = 0$ —— ⑥

$x_1, x_2, \geq 0 \quad \lambda_1 \geq 0 \quad \lambda_2 \geq 0$ —— ⑦

There are 4 cases:

Case I: $\lambda_1 = 0$, $\lambda_2 = 0$

eqn ①, ② $\rightarrow x_1 = \quad x_2 =$

put in eqn ③ & ④ check for satisfy → if No reject

if yes

put in z

Case III: $\lambda_1 \neq$, $\lambda_2 = 0$

eqn ①, ② & ⑤

find out x_1, x_2, λ

put in eqn ③ & ④

check for satisfy

Yes: put in z

→ No → Rejected

Case II: $\lambda_1 = 0$, $\lambda_2 \neq 0$

eqn ① & ② & ⑥

find out x_1, x_2, λ

put in eqn ③ & ④

check for satisfy

if yes

put in z

No → Reject

Case IV: $\lambda_1 \neq 0$, $\lambda_2 \neq 0$

eqn ⑤ & ⑥

find x_1 & x_2

put in eqn ③ & ④

check for satisfy

if yes —— if No → Rejected

put x_1 & x_2 in eqn ① & ②

find $\lambda_1 =$

$\lambda_2 =$