

~~Q1) what is greedy algorithm with example.~~

Ans:

- The greedy method is one of the strategies like divide and conquer algorithms used to solve problems
- this method used solving optimization problems that demands either maximum or minimum results.

~~A greedy algorithm is an approach for solving a problem by selecting the best option available at the moment~~

- It doesn't worry whether the current best result will bring the overall optimal result.
- it's works in a top-down approach
- This algorithm may not produce the best result for all the problems. It's because it always goes for the local best choices to produce the global best result.

~~\* characteristic of greedy algorithms~~

- greedy choice property: A globally optimal solution can be arrived at by choosing a locally optimal choice
- optimal substructure: A problem has an optimal substructure if an optimal soln to the problem contains optimal soln to its subproblems.

Example: Coin change problem (Greedy Approach)

problem: suppose you need to make ₹ 93 using the ~~less~~ number of Indian currency coins

₹ 1, 2, 5, 10, 20, 50 //

(Q3)

greedy approach:

- pick the largest denomination possible without exceeding ₹ 93
- subtract this value from ₹ 93
- Repeat until the amount becomes 0 //

Steps:

- ① take ₹ 50 → Remaining 43
- ② take ₹ 20 → Remaining 23
- ③ take ₹ 20 → Remaining 3
- ④ take ₹ 2 → Remaining 1
- ⑤ take ₹ 1 → Remaining 0 //

coins used: 50, 20, 20, 2, 1 (total 5 coins)

Q) What is Dijkstra's Algorithm with example

Ans:

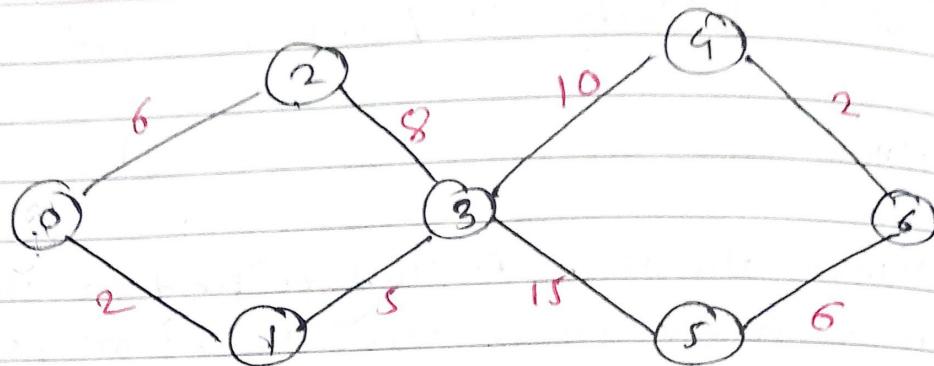
Dijkstra

- Dijkstra algorithm is used to find the shortest path from a single source vertex to all other vertices in a weighted graph
- It is greedy algorithm because it always picks the shortest known path at each step.

Steps of Dijkstra algorithm:

- (1) initialize: Set the distance of the source node 0 and all other nodes to  $\infty$  infinity
- (2) choose the minimum distances node:  
pick the node with smallest known distance that hasn't been processed yet.
- (3) update distance's
- (4) Repeat until all nodes have been processed.

example :



$$0 \rightarrow 1 = .$$

$$0 \rightarrow 2$$

$$0 \rightarrow 3$$

$$0 \rightarrow 4$$

$$0 \rightarrow 5$$

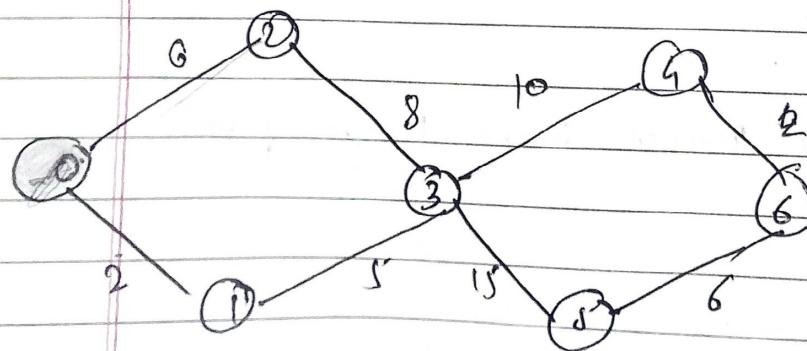
$$0 \rightarrow 6$$

The distance from source node to itself is zero

source to other node infinity ( $\infty$ )

Step ① Start from Node 0 and mark Unvisited Node  
Node as visited.

~~{1, 2, 3, 4, 5}~~



distance

0 : 0 ✓

1 : ~~∞~~ 2

2 : ~~∞~~ 6

3 : ~~∞~~ 7

4 : ~~∞~~ 17

5 : ~~∞~~ 22

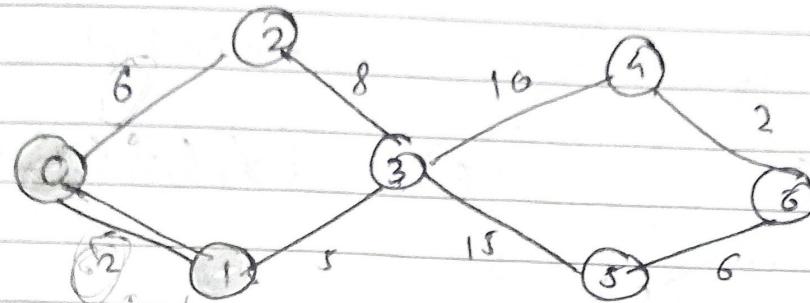
6 : ~~∞~~ 17

check for the adjacent Node, we have two choice either Node ① or ② choose minimum distance.

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for Node ①

Step ②



unvisited Nodes

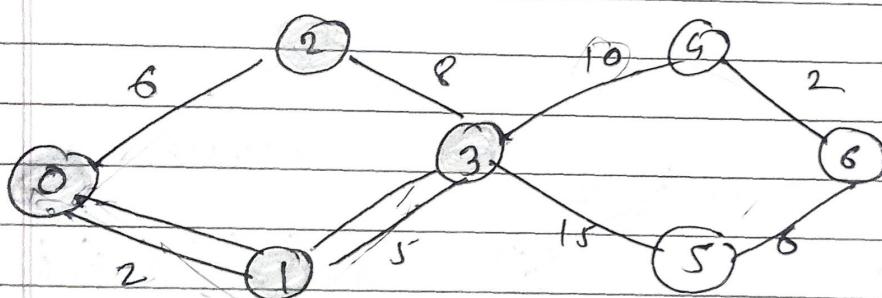
{6, 1, 2, 3, 4, 5, c3}

distance :  $0 \rightarrow 1$

$$= 2$$

Step ③ Then move forward and check adjacent dist Node

which is ③



unvisited Nodes

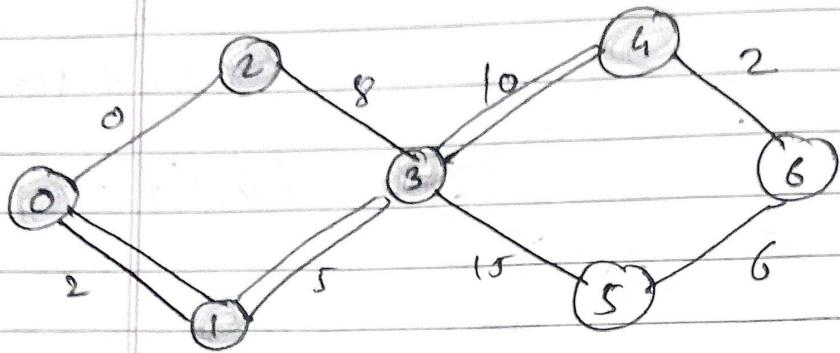
{6, 1, 2, 3, 4, 5, c3}

distance :  $0 \rightarrow 1 \rightarrow 3 = 2 + 5 = 7$

\* Since we can't go  $0 \rightarrow 2 \rightarrow 3 = 6 + 8 = 14$

Step ④ Again we have two choice for adjacent nodes so choose the minimum Node with minimum distance

which is Node ④

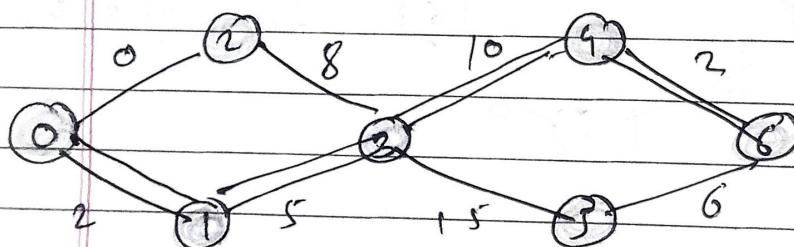


unvisited

{0, 1, 2, 3, 4, 5, 6}

$$\text{distance: } 0 \rightarrow 1 \rightarrow 3 \rightarrow 4 = 2 + 5 + 10 = 17$$

Step 5 Again mark unvisited and check for adjacent node which is 6



Visited node

{0, 1, 2, 3, 4, 5, 6}

$$\text{distance: } 0 \rightarrow 1 \rightarrow 3 \rightarrow 4 \rightarrow 6 = 2 + 5 + 10 + 2 = 19$$

so the shortest distance from 0 to 6 is 19

which is optimal

(q) Diff b/w Kruskal & Prim's algorithm

NO:

Prim's algorithm

Kruskal's Algorithm

(1)	At first check for the feasibility and then for optimality	At first checks for the optimality, and then for the feasibility
(2)	No sorting of edges of a graph is required	It arrange all edges of a graph in ascending order.
(3)	At every edge stage, one and only one connected component is formed	At every stage, one or more than one connected components are formed.
(4)	generally used data structure is a weighted adjacency matrix	Generally used data structure is priority queue.
(5)	Starts from an arbitrary vertex	Start with all vertices as separate trees (forest)
(6)	dense graph	sparse graphs
(7)	difficult to parallelize	easier to parallelize
(8)	TC: $O(E \log V)$	$T( : O(E \log E)$

\* A min spanning tree (MST) is a spanning tree of weighted graph that has the minimum possible sum of edge weight among all vertices without forming any cycle.

(g) Define minimum spanning tree

Ans:

- A spanning tree of connected graph is it's connected a cyclic subgraph (tree) that contains all the vertices of a graph.
- MST of weighted connected graph is a spanning tree is defined as the sum of the weights on all edges
- The MST problem is a problem of finding MST for given weighted connected graph.

\* Algorithms to find MST

- Kruskal's
- Prim's

Both are greedy approach.

(g) Why AOA is imp

Ans:

Analysis of Algorithms (AOA) is essential in computer science because it helps in understanding the efficiency of different algorithms in terms of time and space.

It allows developer to choose the best algorithm for a given problem, ensuring optimal performance.

- Key Reason why AOA is important
  - Performance optimization
  - Scalability
  - Comparison of Algorithms
  - Cost ~~Reduction~~ Reduction
  - Theoretical Understanding
  - Real world application
  - Competitive programming & interviews

Understanding AOA ensures that software runs faster, consumes fewer resources and scale well for large inputs.

It is essential skill for every programmer and software engineer.

~~IMP~~ ① array with arr

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## \* Sorting algorithms:

(1) Merge sort: A divide and conquer sorting algorithm that splits an array into two halves recursively sort each half, and then merges them in sorted order.

Algorithm:

1. If the array has one or zero elements, return
2. Divide the array into two halves
3. Recursively sort both halves using Merge sort
4. Merge the sorted halves back into a single sorted array

TC  $\rightarrow \Theta(n \log n)$  - Best

SC  $\rightarrow \Theta(n)$  - Auxiliary Space

give example

of each algo!

(2) quick sort : A divide and conquer algorithm that selects a pivot, partitions the array around the pivot and recursively sorts both partitions.

Algorithm :

1. Select the pivot element from the array (highest/lowest)
2. Partition the array such that elements smaller than pivot are on the left and larger than ones on the right.
3. Recursively apply quick sort to the left and right sub arrays.

$T \rightarrow O(n \log n) \rightarrow$  Best

$S \rightarrow O(n \log n) \rightarrow$  due to recursion

Comparison-based



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(3) Selection Sort : A simple comparison-based sorting algorithm that repeatedly selects the smallest elements from the unsorted part of the array and swaps it with first unsorted element.

Algorithm :

1. find the smallest elements in the array
2. swap it with the first element
3. Repeat the process for the remaining unsorted part of the array.

TC  $\rightarrow$   $O(n^2)$

SC  $\rightarrow$   ~~$O(n^2)$~~   $O(1)$

### (a) insertion sort:

A simple and efficient sorting method for small datasets where elements are picked one by one and placed in the correct position within the sorted part of the array.

#### Algorithm:

1. Start with from the second elements, assume the first one is sorted
2. compare the element with previous ones and shift elements if needed
3. insert the elements at the correct position
4. Repeat until the entire array is sorted

$$T \rightarrow O(n)$$

$$S \rightarrow \cancel{O(n^2)} O(1)$$

~~Q2~~ What is Recurrence? Various Method to solve Recurrence

Ans:

A recurrence relation is a mathematical expression that defines a sequence in terms of its previous terms.

In context of algorithmic analysis, it is often used to model the time complexity of recursive algorithm.

~~#~~ The general form:

$$a_n = P(a_{n-1}, a_{n-2}, \dots, a_{n-k})$$

where  $P$  is function, that defines the relationship between the current term and previous terms.

The recurrence relation play significant role in analysing and optimising the complexity of algorithms.

- Three ways to solving recurrence:

① Substitution method

② Recurrence Tree method

③ Master Method

### (1) Substitution Method: (guess and verify)

- guess the sol<sup>n</sup> form and use mathematical induction to prove it.

$$T(n) = 2T\left(\frac{n}{2}\right) + O(n) \quad \approx$$

assume  $T(n) = O(n \log n)$

### (2) Recursion Tree method:

- expand the recurrence into a tree to identify patterns and derive the complexity

$$\text{eg: } T(n) = 2T\left(\frac{n}{2}\right) + O(n)$$

$$T(n) = 2T\left(\frac{n}{2}\right) + n$$

$$= 4T\left(\frac{n}{4}\right) + 2n$$

$$= 8T\left(\frac{n}{8}\right) + 3n$$

which leads to  $O(n \log n)$

### (3) Master theorem:

- used for divide and conquer recurrence of the form

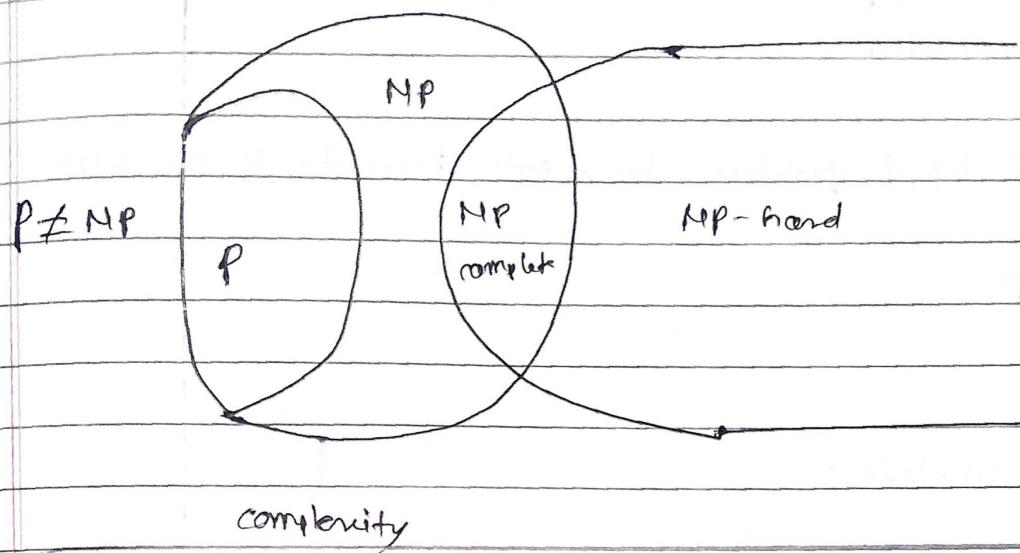
$$T(n) = aT(n/b) + f(n)$$

$f(n) = kn -$   
recursive work

$a \Rightarrow$  no. of sub problems

$b \Rightarrow$  factor by which size is divided

Q) describe the relationship along P, NP, NP-hard,  
NP-complete



(1) P (polynomial time) :

The class of decision problems that can be solved by deterministic Turing machine in polynomial time.

eg: BS, sorting

(2) NP (Non-deterministic Polynomial Time) :

This is a class of decision problem where a given solution can be verified in polynomial time, even if finding the soln might take longer.

eg: sum of subset problem

### (3) NP-Hard :

- A class of problem at least as hard as the hardest problem in NP
- An NP-hard problem doesn't have to be in NP

eg: TSP

### (4) NP-complete :

A problem is NP-complete if it satisfies two conditions:

- it belongs to NP
- it is hard as every problem in NP

eg: SAT

Q) What is Asymptotic analysis and define big o, big omega and big theta notation.

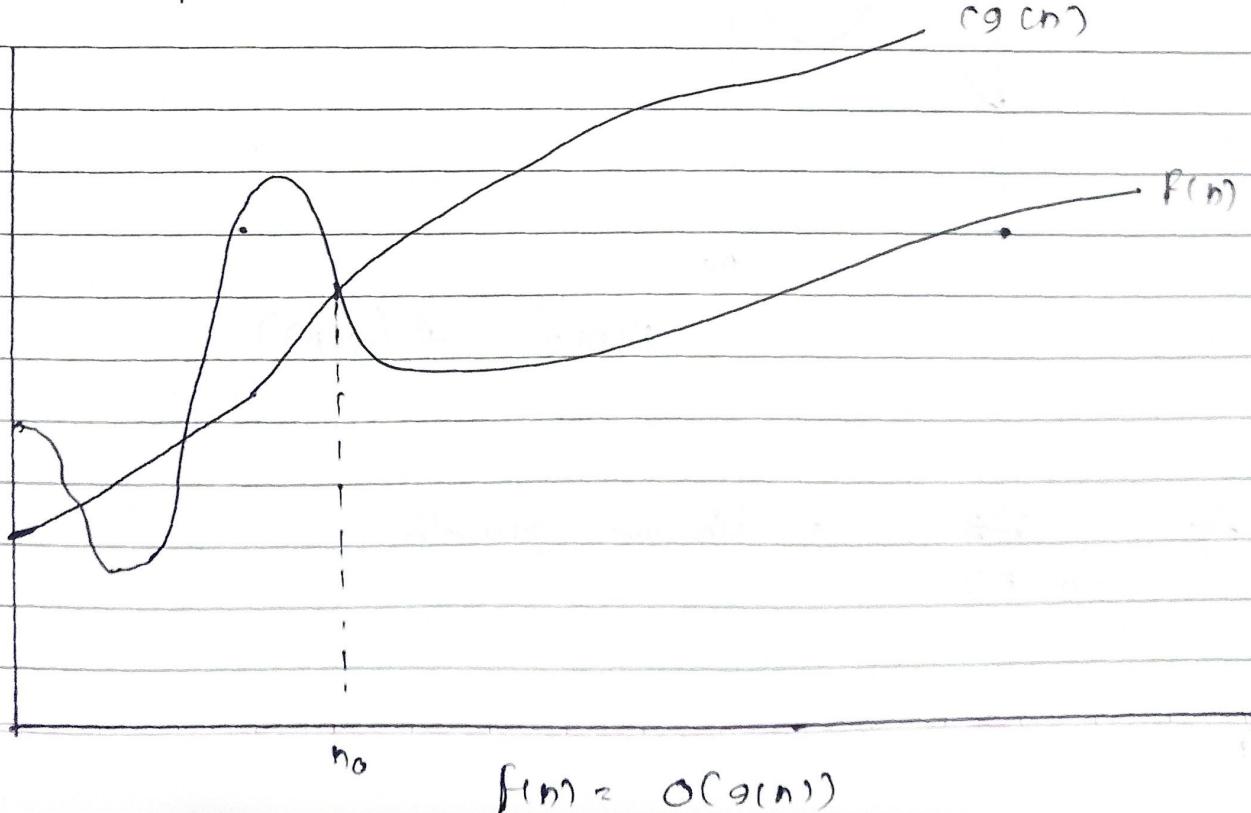
Ans:

Asymptotic Analysis is a method of describing the efficiency of an algorithm as the input size ( $n$ ) becomes very large.

It helps us understand the time or space complexity of an algorithm without worrying about machine specific constant or small inputs.

(1) Big - O Notation ( $O$ ): Worst case:

- It is used to describe an upper bound of the running time.
- It tells us the maximum time an algorithm can take for any input.
- It represents worst case.



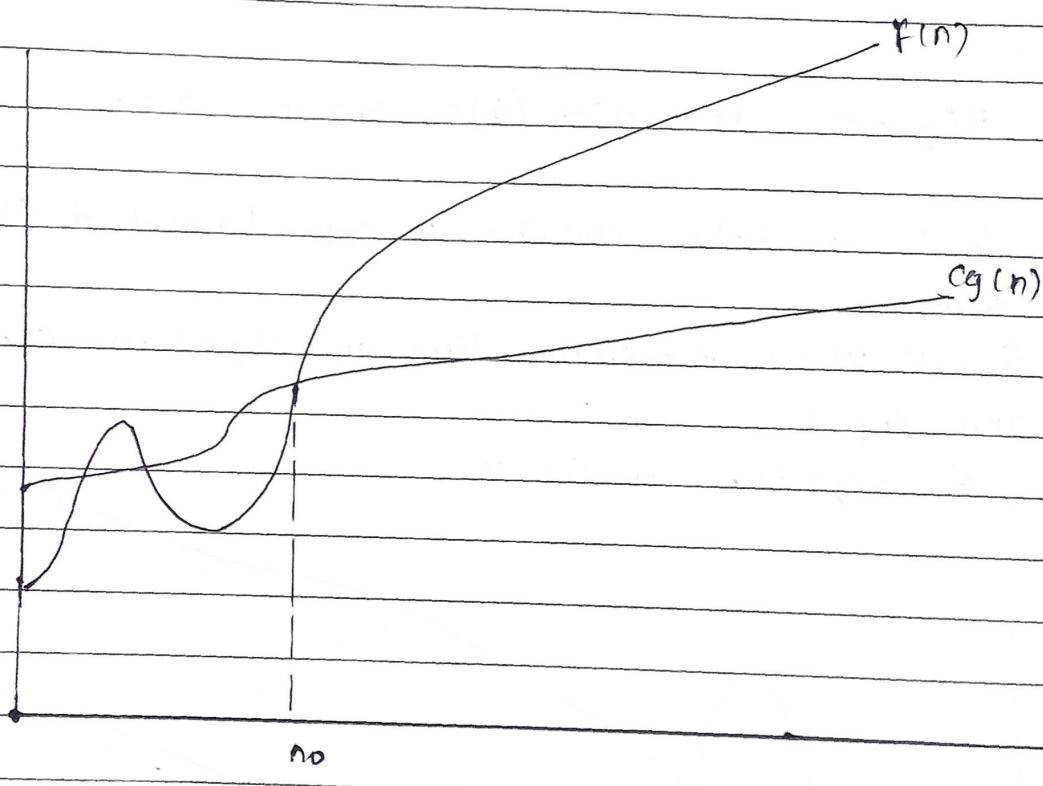
eg:

$O(n) \rightarrow$	Linear
$O(1) \rightarrow$	constant
$O(n^2) \rightarrow$	quadratic

$\} TC$

(2) Big - Omega Notation ( $\Omega$ ): Best case

- it is used to describe the lower bound of the running time.
- it tells minimum time an algorithm will take.
- it represents the best case scenario.

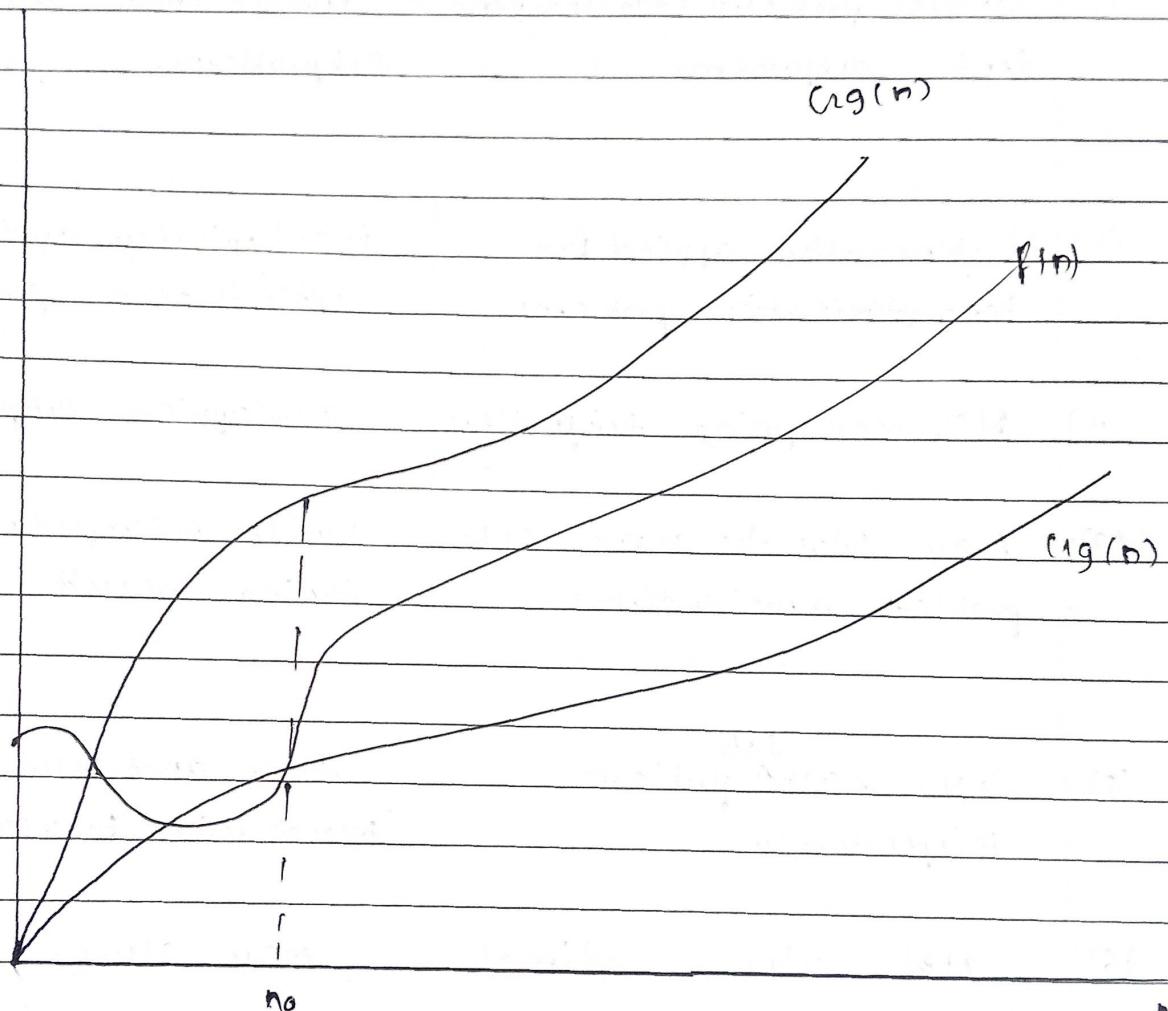


$$f(n) = \Omega(g(n))$$

eg:  ~~$\Theta(n)$~~   $\rightarrow$  linear growth

### (B) Theta Notation ( $\Theta$ ): Average case

- it is used to describe exact/tight bound of the running time
- it gives both upper and lower bound
- it represents average case scenario.



$$f(n) = \Theta(g(n))$$

Eg:  $\Theta(n \log n)$

### Q) diff b/w divide & conquer & Dynamic programming

SR. NO	divide and conquer	dynamic programming
(1)	divides problem into independent subproblems	divides into overlapping subproblems.
(2)	it is generally applied for non-optimization problems	it is typically applied for optimization problem.
(3)	No, overlapping subproblem	overlapping subproblems
(4)	May solve the same sub - problem multiple times	Avoids re-computation by storing result.
(5)	Solves each <sup>sub</sup> problem independently	Stores and re-uses solution (memoization / tabulation)
(6)	Not always optimal	may always be optimal
(7)	eg: Merge sort, quicksort, Binary Search	eg: Fibonacci series, 0/1 knapsack

### Q) Greedy knapsack and 0/1 knapsack

→ Sr. No

**Greedy knapsack  
(Fractional)**

**0/1 knapsack, (DP)**

(1) Item can be broken into fractions item must be taken completely or not at all

(2) Greedy approach use

dynamic programming use

(3) <sup>not</sup> always give optimal solution

gives optimal solution

(4) Require less memory

Require more memory due to DP table.

(5) Strategy : pick item with highest value / weight ratio first

Strategy : explore all possible combination

(6) TC :  $O(n \log n)$  due to sorting

$O(n + W)$  (when  $W$  is capacity)

(7) SC :  $O(1)$  or  $O(n)$

$O(n + W)$

(8) WE can : when fractions are allowed

when only whole item can be chosen.

e.g. gold

e.g. laptop

Q) Explain Multi-Stage graph with example.

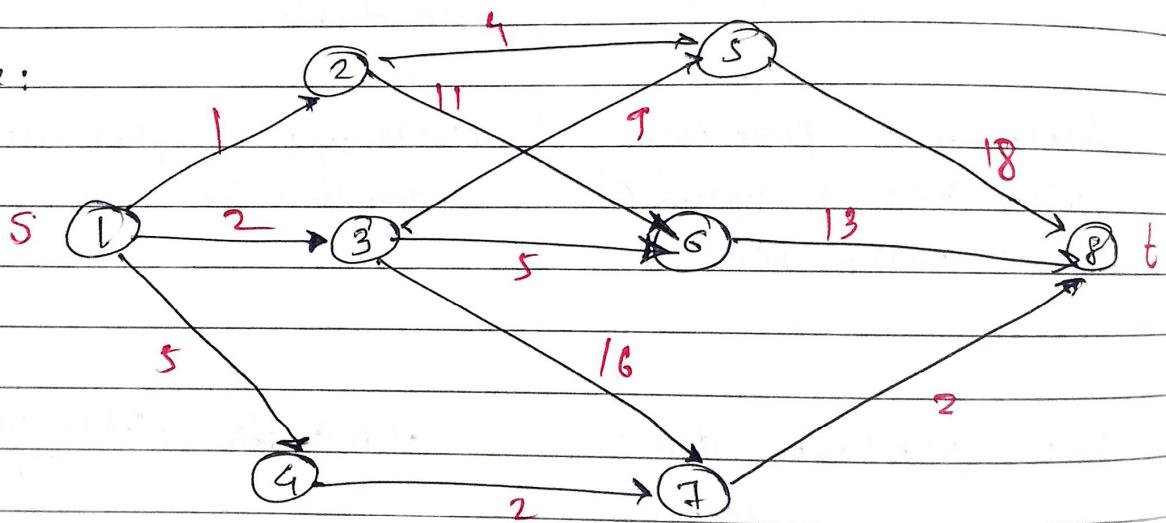


A multistage graph is a directed, weighted graph that is divided into multiple stages where,

- Nodes are arranged in stages from start to end
- Edges go only from one stage to next stage (never backward)
- The goal is usually to find the shortest path from the source node (in stage 1) to destination node (in the last stage)

(or longest)  
A

Example:



$$f\text{cost}(i, p) \geq \min \{ c(p, q) + f\text{cost}(i+1, q) \}$$

$f\text{cost}(i, p)$  gives the cost of the shortest path from node  $p$  in stage  $i$  to the sink node.

Q) Write an abstract algorithm for a greedy design method.

Algorithm Greedy-Subset ( $P, n$ )

input:  $P[1:n]$  is a set of  $n$  inputs of a given problem instances.

output:

{

$S := \emptyset$

for ( $i := 1$ ;  $i \leq n$ ;  $i + 1$ ) {

$q := \text{Select}(P);$

if ( $\text{Feasible}(S, q)$ );

$S := \text{Union}(S, q);$

}

return  $S;$

}

Q) Write an algorithm to find min & max value using divide & conquer and also derive its time complexity. & S.C.

To find out minimum and maximum number of an array by performing lesser operation the divide and conquer strategy is used.

Approach: The divide and conquer approach works by splitting the array into two halves, recursively finding the minimum and maximum in each half and then merging the results.

Algorithm:

MinMax (A, low, high) :

if (low == high):

Only one element

return (A[low], A[low])

else if high == low + 1 : // two elements

if A[low] < A[high]:

return (A[low], A[high])

else:

return (A[high], A[low])

else :

mid = (low + high)/2 :

(min1, max1) = MinMax (A, low, mid)

(min2, max2) = MinMax (A, mid+1, high)

min\_final = min(min1, min2);

max\_final = max(max1, max2);

return (min\_final, max\_final)

derive TC :

let  $T(n)$  be the no. of comparison of an array size  $n$

$$T(n) = T(n/2) + T(n/2) + 2$$

$$= 2T(n/2) + 2$$

$$\therefore T(n) = 3n/2 - 2$$

Worst/Avg case =  $\Theta(n)$

Best Case =  $\Theta(1)$

Q) Write an algorithm and derive its TC & SC of binary search.

**Binary Search:** Binary Search is an efficient searching algorithm used to find the position of a target element in a sorted array by repeatedly dividing the search interval in half.

**Algorithm:** BinarySearch ( A, key ) :

$$\text{low} = 0$$

$$\text{high} = \text{length}(A) - 1$$

$$\text{while } \text{low} \leq \text{high}$$

$$\text{mid} = (\text{low} + \text{high}) / 2$$

$$\text{if } (A[\text{mid}]) == \text{key}$$

return mid

$$\text{else if } (\text{key} < A[\text{mid}])$$

$$\text{high} = \text{mid} - 1$$

else

$$\text{low} = \text{mid} + 1$$

return -1      // element not found.

\* TC: Let  $n$  be the no. of elements

At each step, the array is divided into half  
 So no. of comparisons  $\approx \log_2(n)$

Best case:  $O(1)$

Worst/Avg:  $O(\log n)$

\* SC:  $O(1) \rightarrow$  iterative

$O(\log n) \rightarrow$  recursive.

Q) Write Kruskal's algorithm for finding a minimum spanning tree. Explain its working with an example

(a) Compute TC

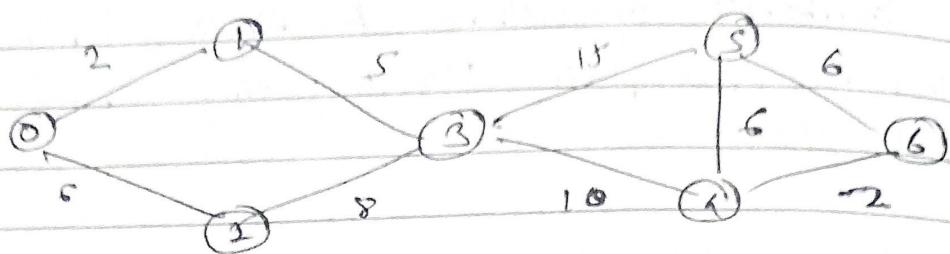
Kruskal algorithm is a classic algorithm used in graph theory to find MST of a connected, directed graph.

The MST is subset of the edges that connect all the vertices without any cycles and with the minimum possible total edge weight.

\* Steps of Kruskal's algorithm:

- ① Sort all the edges
- ② Initialise subset
- ③ Iterate
- ④ Sort edges
- ⑤ Repeat until MST is complete.

e.g:



$$0 \rightarrow 1 = 2$$

$$0 \rightarrow 2 = 6$$

$$1 \rightarrow 3 = 5$$

$$2 \rightarrow 3 = 8$$

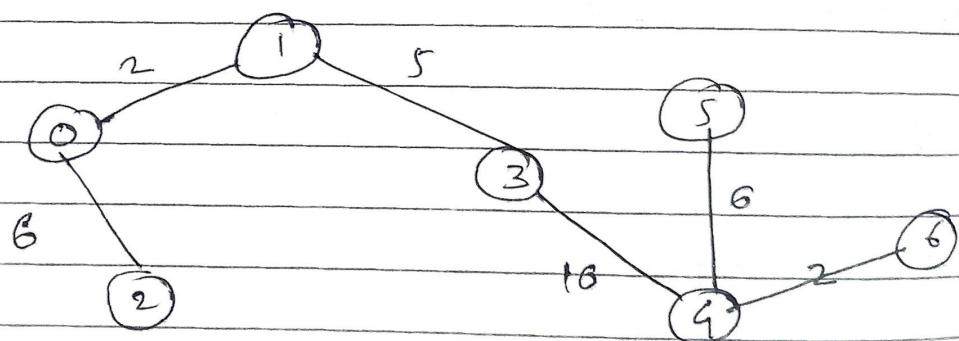
$$3 \rightarrow 5 = 15$$

$$3 \rightarrow 4 = 10$$

$$5 \rightarrow 4 = 6$$

$$5 \rightarrow 6 = 6$$

$$4 \rightarrow 6 = 4$$



$$2 + 6 + 5 + 10 + 6 + 2$$

$$= 30$$

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\* TC:

$V = \text{No. of vertices}$

$E = \text{No. of edges}$

$$O(E \log E) \approx O(E \log V)$$

\* SC:  $O(V)$

(8) Write a algorithm to solve N Queen problem using backtracking.

Ans:

place N queens into  $N \times N$  chessboard such that no two queens attack each other (i.e no same row, column or diagonal)

Algorithm: Solve NQueen (board, now, N):

```
if now == N:
    print board / store solution
    return
```

for col = 0 to N-1:

if isSafe (board, now, col, N):

board [now] [col] = 1 // place queen

Solve NQueen (board, now+1, N)

board [now] [col] = 0 // backtrack

isSafe (board, now, col, N):

// checks vertical column

for i = 0 to now-1:

if board [i] [col] == 1

return false

// check left diagonal

for i = now-1, j = col-1 to i >= 0 & j >= 0

if board [i] [j] == 1

return false

	Q		
			Q
Q			
	Q		

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// check right diagonal

for  $i = 0 \text{ to } n-1$ ,  $j = \text{col} + 1 \text{ to } i+1$ ,  $0 \leq j \leq N$

if board[i][j] == 1

return false

return true.

### \* Working

- Place a queen ~~row~~<sup>row</sup> by row
- For each row, try all columns and only place the queen if it's safe
- If all queen are placed ( $\text{row} == N$ ) You've found a valid solution
- Backtrack if no valid column is found for a row

$$TC = O(N)$$

$$SC = O(M^N)$$

Q) Explain travelling salesman problem using Branch & Bound

Ans:

The travelling salesman problem (TSP) is a classic optimization problem in computer science and operation research task,

A sales man must visit a set of cities exactly once, and return to the starting city. The goal is to find the shortest possible route that visit all city exactly ones and return to the origin.

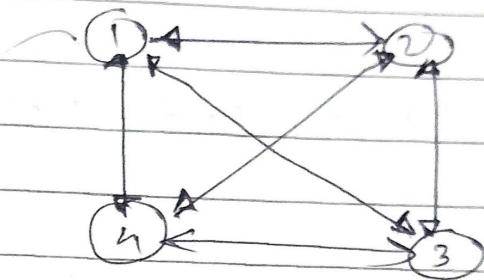
### \* Using branch and bound Method

- ① Start with one city and explore all possible routes to other cities.
- ② But instead of checking all path we,
  - calculate a minimum possible cost (called bound) for each path
  - skip path that already cost more than the best solution we have so far.
- ③ always explore the cheapest-looking path using a priority queue.

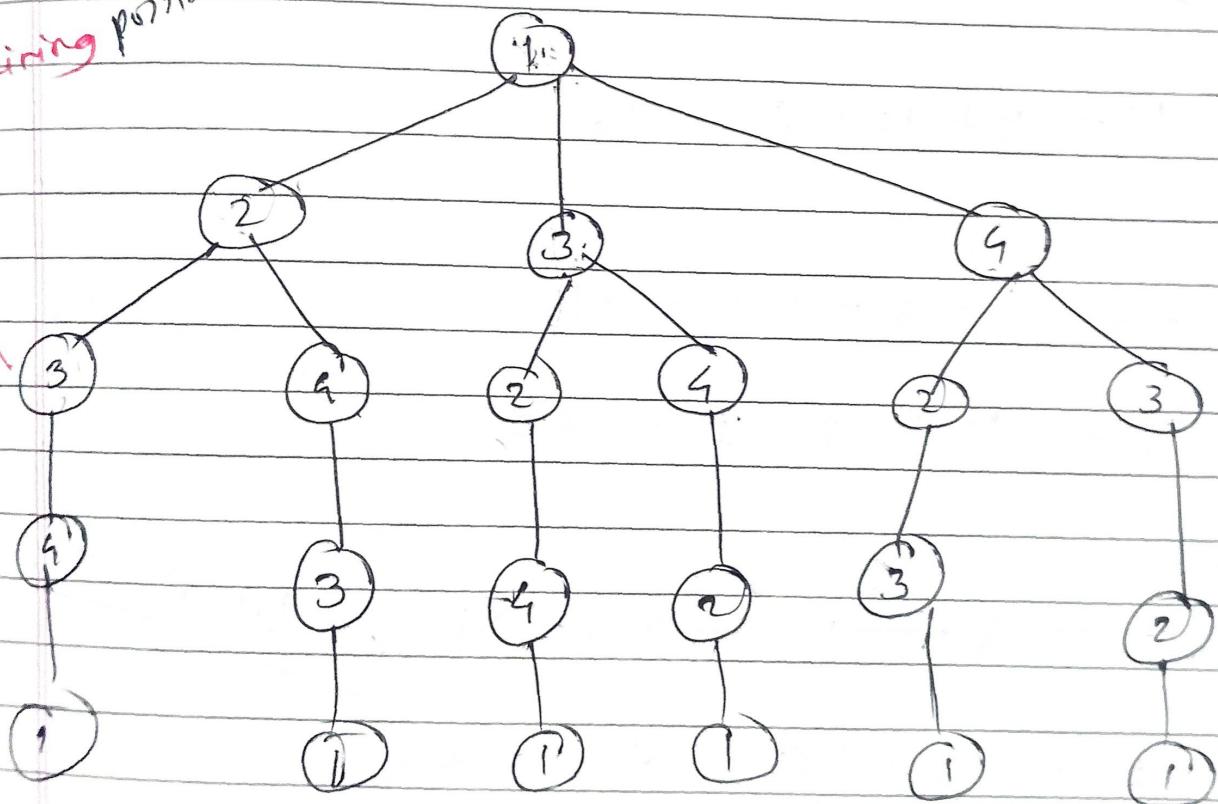
the origin

eq:

	1	2	3	4
1	0	1.0	15	20
2	5	0	9	10
3	6	13	0	12
4	8	8	9	0



remaining possibilities



All possible ways

$$\textcircled{1} \quad 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1$$

$$\text{(cost)} \quad 10 + 9 + 12 + 8 = 39$$

$$\textcircled{2} \quad 1 \rightarrow 2 \rightarrow 4 \rightarrow 3 \rightarrow 1$$

$$\text{(cost)} \quad 10 + 10 + 9 + 6 = 35$$

$$\textcircled{3} \quad 1 \rightarrow 3 \rightarrow 2 \rightarrow 4 \rightarrow 1$$

$$\text{(cost)} \quad 15 + 13 + 10 + 8 = 46$$

$$\textcircled{4} \quad 1 \rightarrow 3 \rightarrow 4 \rightarrow 2 \rightarrow 1$$

$$\text{(cost)} \quad 15 + 12 + 8 + 5 = 40$$

$$\textcircled{5} \quad 1 \rightarrow 4 \rightarrow 2 \rightarrow 3 \rightarrow 1$$

$$\text{(cost)} \quad 20 + 8 + 0 + 6 = 34$$

$$\textcircled{6} \quad 1 \rightarrow 4 \rightarrow 3 \rightarrow 2 \rightarrow 1$$

$$\text{(cost)} \quad 20 + 9 + 13 + 5 = 47$$

## AOA UT-2

### ~~Q2) Naive string Matching algorithm~~

An:

The naive string matching algorithm is the simplest method for searching a pattern string within a text string.

~~It checks for the pattern at every possible position in the text, without using any preprocessing or optimisation.~~

~~This algorithm follows the brute force approach~~

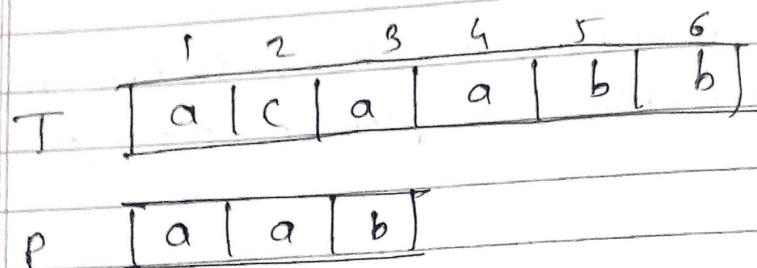
Step1: let

$T = \text{text of length } n$

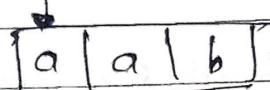
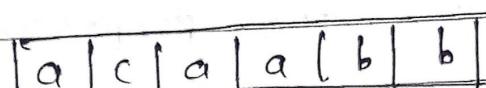
$P = \text{Pattern of length } m$

- ① Start at index  $i=0$  to the next
- ② for each position  $i$  from 0 to  $n-m$ ;
- ③ - Compare  $p[0--m-1]$  with  $T[i--itm-1]$ 
  - if all character match, 'H' is match
  - if not move to the next position  $i+1$

example:  $T = a.caaabb$   
 $P = aab$

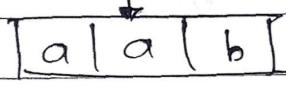
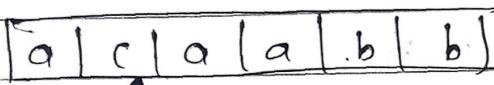


Step 1:



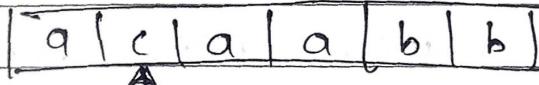
$T[1] = P[1]$  then move next pointer  $p++$

Step 2:



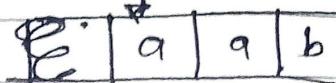
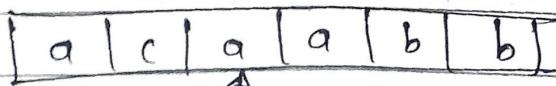
$T[2] \neq P[2]$  then move text pointer  $t++$  and decrease  $p--$

Step 3:



$T[3] \neq P[1]$  so  $t++$

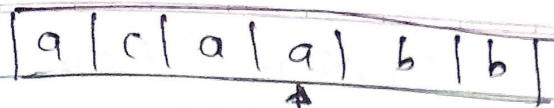
Step 4:



$T[4] = P[1]$  so move  $p++$

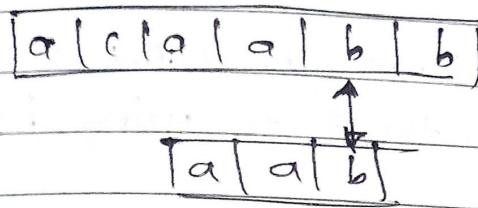
$p++$

Step 5:



$T(q) = P(2)$  so  
p++;

Step 6:



$T(s) = P(3)$

match found at index 3 and 5

check Remaining possibility in answer sheet

Time complexity:

① Best case:  $O(n) \rightarrow$  when pattern doesn't match often

② Worst case:  $O(n+m) \rightarrow$  when pattern and text share many similar characters

\* Q) What is Robin-Karp algorithm? How is it better than Naive string matching algorithm. Write its TC & SC

- Robin-Karp is an algorithm used for searching a matching pattern in the text using hash function.

Q) How is it better than Naive?

Instead of comparing characters one by one at each step/position (as in the naive algorithm)

Rabin Karp compares hash values of strings. This makes it much faster in practice, especially when searching for multiple patterns.

e.g.:       $\begin{matrix} 1 & 3 & 1 & 1 & 2 & 1 & 1 \end{matrix}$       pattern = ab

$a = 1$

$b = 2$

$c = 3$

$$aca = 5$$

$$caa = 5$$

$$aab = 4 \checkmark$$

$$aba = 4$$

$$baa = 4$$

In above string matching there are 3 string matching the hash value,  
 aab, aba & baa

but only aab match the pattern, so aab  
 is the matching string.

### \* Time complexity (TC):

- Avg case:  $O(n+m)$
- ~~Worst case~~:  $O(n*m) \rightarrow$  due to hash collision  
 requiring character checks

### \* Space complexity (SC):

- $O(1)$   $\rightarrow$  single pattern
- $O(k)$   $\rightarrow$  if multiple pattern

~~Optimized~~

\* Q) Explain Knuth-Morris-Pratt algorithm with an example.

Ans:

- The KMP (Knuth-Morris-Pratt) is an efficient string matching algorithm that avoids unnecessary comparison by preprocessing the pattern.
- It uses prefix table (also called LPS array - longest prefix which is also suffix) to skip characters while matching improving performance over the naive algorithm.

\* Algorithm:

- ① Define the prefix function.
- ② Slide the pattern over the text for comparison.
- ③ If all the characters are matched, we found the matching string.
- ④ If not,  
we set a prefix function to skip unnecessary comparison.

If the LPS values of previous character from mismatched character is '0' then start comparison from index 0 of pattern with next character in the text.

However if the LPS values is more than 0.  
 Start the comparison from index value, equal to  
 LPS value of the previously mismatched character.

example:

a b c d a b e a b F

ind:	1	2	3	4	5	6	7	8	9	0
	a	b	c	d	a	b	e	a	b	F
	0	0	0	0	1	2	0	5	6	0

pattern abea

occur again

so put the index

[4 | 2 | 0 | 5] → string matches at ind → 5 to 8

\* TC  $\rightarrow$   $O(m) \rightarrow$  LPS building

$O(p) \rightarrow$  pattern matching

Overall TC  $\rightarrow$   $O(n+m)$

Q8) Explain Floyd-Warshall Algorithm.  
(All pair shortest path)

- The Floyd-Warshall algorithm is a dynamic programming algorithm used to find the shortest distances between all pairs of vertices in a weighted directed graph (can also work for undirected graph)
- It works for graphs with positive or negative edge weights, but no negative weighted cycle.

example:

$$\text{Formula: } D^k[i, j] = \min \{ D^k[i, j], D^{k-1}[i, k] + D^{k-1}[k, j] \}$$

Where  $D^k$  = distance matrix after  $k$ th iteration

Algo:

for  $k = 0$  to  $v-1$

  for  $i = 0$  to  $v-1$

    for  $j = 0$  to  $v-1$

$$D[i, j]^k = \min (D[i, j]^k, D[i, k]^{k-1} + D[k, j]^{k-1})$$

  end

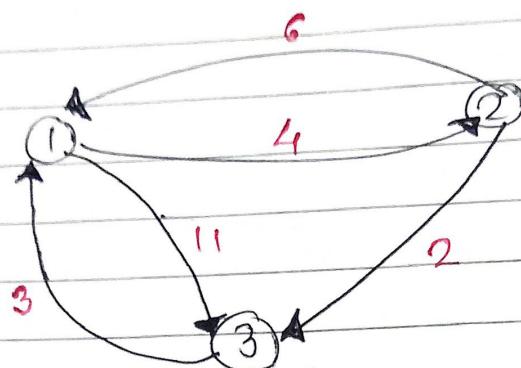
end

end

return 0;

*v=1 first column  
and first row on ij<sup>th</sup> baki ka change  
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\* if asked example then :



	1	2	3
1	0	4	11
2	6	0	2
3	3	$\infty$	0

We used sequence formula.

$$D^k(i, j) = \min \left\{ D^k(i, j), D^{k-1}(i, k) + D^{k-1}(k, j) \right\}$$

Iteration = 1      ( $k=1$ )

	1	2	3
1	0	4	11
2	6	0	2
3	3	7	0

①  $2 \rightarrow 3$

②  $3 \rightarrow 2$

$$P^*(2,3) = \min \{ P^*(2,3); P^*(2,1) + P^*(1,3) \}$$

$$= \min \{ 2, 6+1 \}$$

$$= \min \{ 2, 17 \}$$

$$P^*(2,3) = 2$$

$$P^*(3,2) = \min \{ P^*(3,2), P^*(3,1) + P^*(1,2) \}$$

$$\min \{ \infty, 3+4 \}$$

$$\min \{ \infty, 7 \}$$

$$\min = 7 //$$

Iteration 2 ( $k=2$ )

	1	2	3
1	0	6	6
2	6	0	2
3	3	7	0

( $\rightarrow 3, 3 \rightarrow 1$ )

$$D^2[1,3] = \min \{ D^2(1,3), P^*(1,2) + P^*(2,3) \}$$

$$= \min \{ 11, 9+2 \}$$

$$\min \{ 11, 9 \}$$

$$= 6 //$$

Now  $D^2(3,1) = \min \{ D^2(3,1), D^2(3,2) + D^2(2,1) \}$   
 $= \min \{ 3, 7+6 \}$

$\min \{ 3, 13 \}$

$\min = 3$

Iteration 3:

	1	2	3
1	0	5	6
2	5	0	2
3	3	7	0

$1 \rightarrow 2$   
 $2 \rightarrow 1$

$D^3(1,2) = \min \{ D^3(1,1), D^3(1,3) + D^2(3,2) \}$

$= \min \{ 9, 6+7 \}$

$\min = \{ 9, 13 \}$

$= 4 //$

Now  $D^3(2,1) = \min \{ D^3(2,1), D^2(2,3) + D^2(3,1) \}$

$\min \{ 6, 7+3 \}$

$\min \{ 6, 10 \}$

$\min = 6 //$

g) explain 15 puzzle problem (may be optional)

Ans:

- the 15 puzzle problem is a classic sliding puzzle consisting of:

① A  $4 \times 4$  grid (total 16 positions)

② 15 numbered tiles (from 1 to 15)

③ 1 empty space (used to slide tiles)

The goal is to rearrange the tiles in order from 1 to 15, with the empty space in the bottom-right corner.

① How it's works?

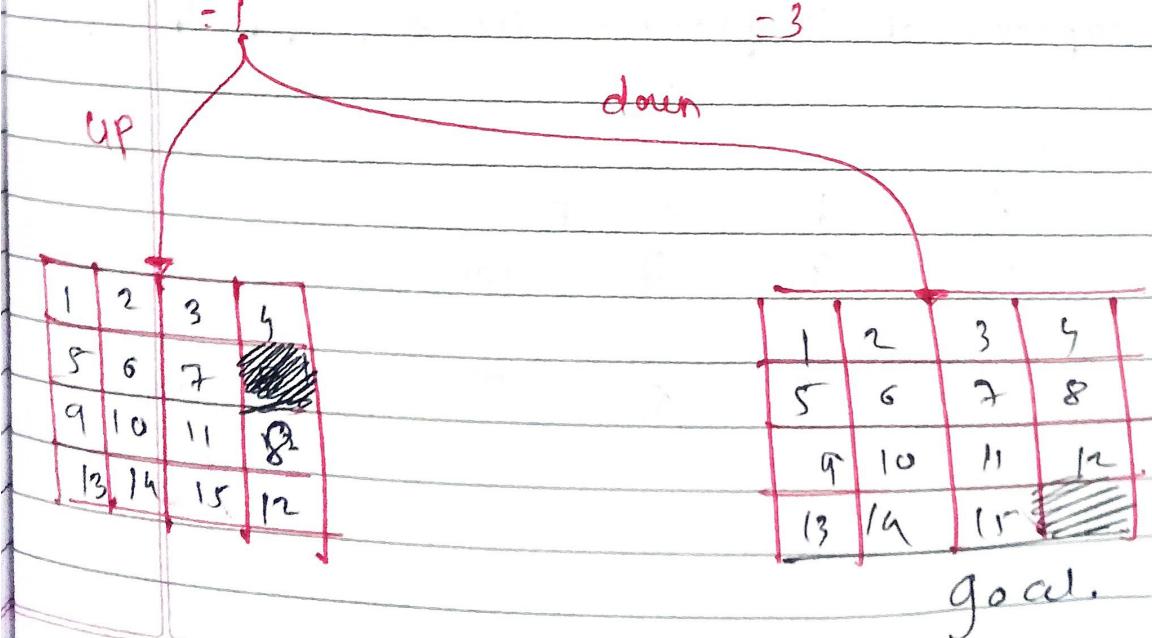
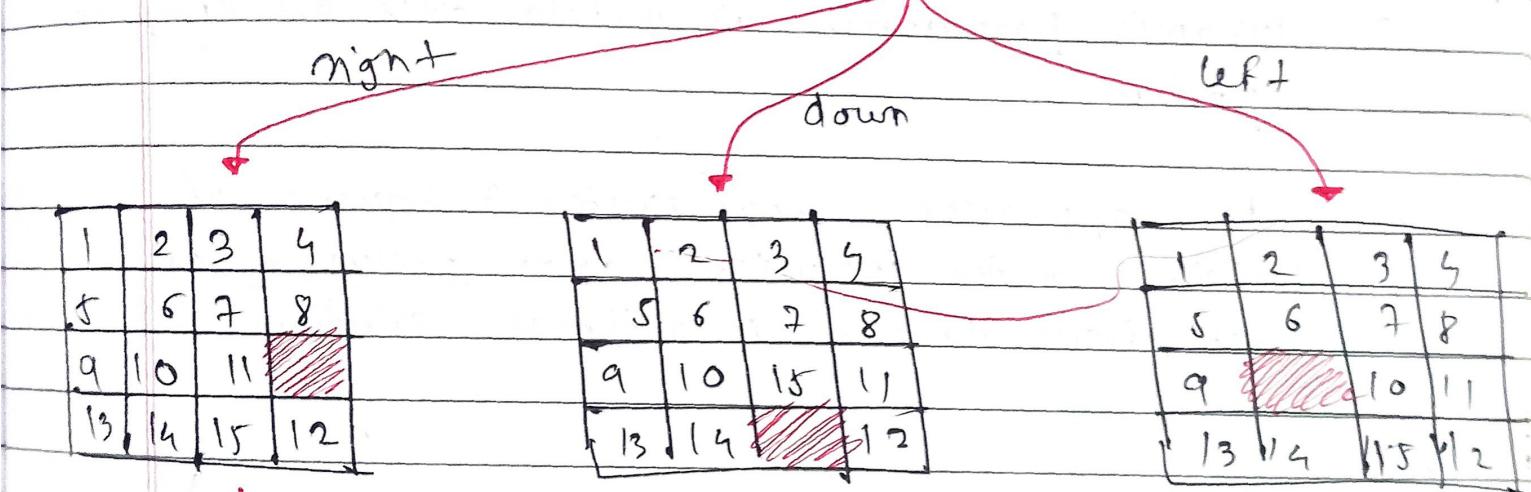
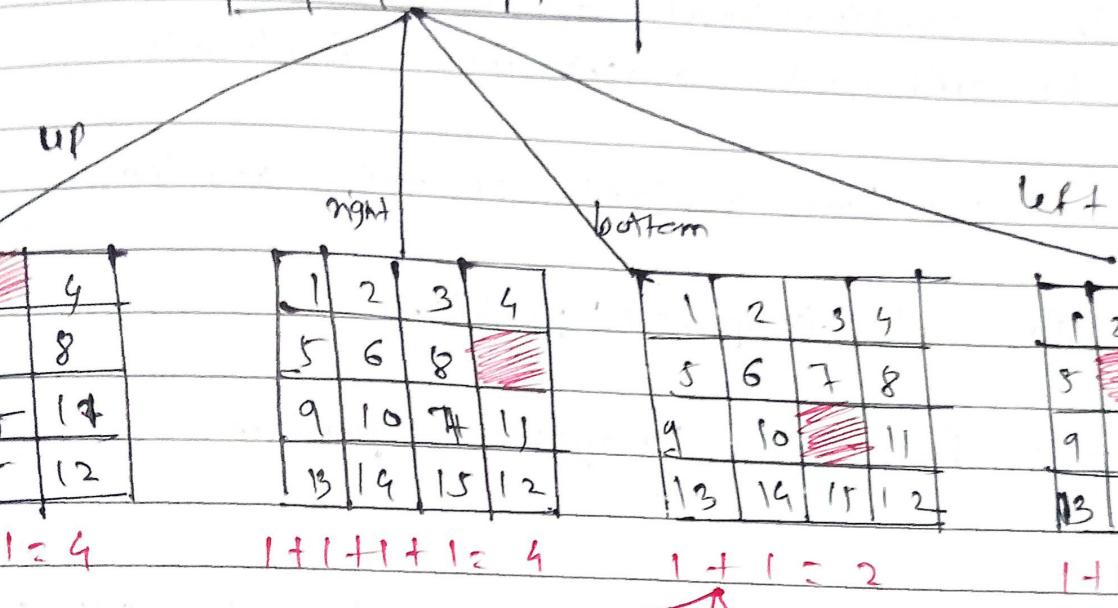
① You can slide adjacent tiles (up, down, left, right) into the empty space.

② only one tile can be moved at a time.

③ The challenge is to reach the goal configuration using the ~~the~~ least number of moves.

example:

1	2	3	4
5	6		8
9	10	7	11
13	14	15	12



(Q) What is Backtracking and Branch & Bound?  
give 2 example example of each.

Ans:

\* Backtracking: Backtracking is a method of solving problems by trying out all possible options and discarding those that do not satisfy the problem's conditions.

It builds solutions step-by-step and goes back (backtrack) when it hits a dead end.

Branch & Bound: Branch and bound is an algorithm used for solving optimization problems

It divides the problems into subproblems (branching) and uses limits (bounds) to avoid exploring parts of the solution space that cannot give better results.

Branching: divide problem into subproblem

Bounding: calculate upper / lower bounds to  
eliminate bad branches.

example of ~~backtracking~~: branch & bound.

- ① 0/1 knapsack problem (optimal) ✓
- ② Travelling Salesman problem (TSP)

example of ~~branch & Bound~~: Backtracking

- ① N-queens problem
- ② Sudoku solver
- ③ sum of subset
- ④ graph coloring
- ⑤ 15-puzzle

~~What is DP? two elements of DP? application of DP?~~

Ans:

- dynamic programming is a method for solving complex problem by breaking them down into smaller overlapping subproblem and solving each subproblem only once, storing the results for future use.

- It also known as tabulation or memoization

• Two elements of DP

① optimal structure: the soln to a problem can be built from the solution of its subproblem

② Overlapping subproblems: the problem can be broken into subproblem which are solved multiple times.

## ⑥ Application of DP

- ① 0/1 knapsack problem
- ② longest common subsequence (LCS)
- ③ coin change problem
- ④ shortest path algorithm ( Floyd-Warshall, Bellman-Ford )

## ⑦ Elements of DP

These are 3 elements ~~of~~ that characterize a dp algorithm

### ① Substructure :

decompose the given problem into smaller problems.

### ② Table structure:

after solving the sub problems , store the result to the subproblems in a table.

### ③ Bottom - up computation:

Using table , combine the soln for smaller sub problems to solve large problem & eventually arrives at a solution to complete problem

## Q) Advantage of DP

- solves problems efficiently that have overlapping subproblems
- reduce TC by avoiding redundant calculation
- Useful in optimal problems

## Q) disadvantage of DP

- High space complexity  
(use large memory tables)
- May be hard to design the correct recurrence relation
- Not suitable if subproblems do not overlap.

## Q) Explain sum of subset using Backtracking.

Ans:

definition: The sum of subset problem is a decision problem where, given a set of positive integers and target sum  $S$ , we are asked to find all subsets of the set whose elements add up exactly to  $S$ .

You must use backtracking to explore all possible subsets, but stop exploring if the current sum exceeding the target.

### \* Backtracking Steps:

① Start from index 0

② At each index, we choose or don't choose the current element.

(b) if  $\text{sum} > \text{target} \rightarrow \text{backtrack}$

(c) if  $\text{sum} = \text{target} \rightarrow \text{print the current subset}$

Example:

problem:      set = {2, 3, 5}  
 target sum = 5 //

Step ① Start with element 2

include 2

current sum = 2 , subset = {2}

Step ② Go to element 3

include : 3

(2+3)

current sum = 5 , subset = {2, 3}

This is valid soln, print it and remove it through back track.

Back track  $\rightarrow$  remove 3

subset = {2}      current sum = 2

remove  $\rightarrow 2$

cumsum = 0

subset {}

Step (3) Go to next element 5

include 5

cumsum = 5

subset = {5}

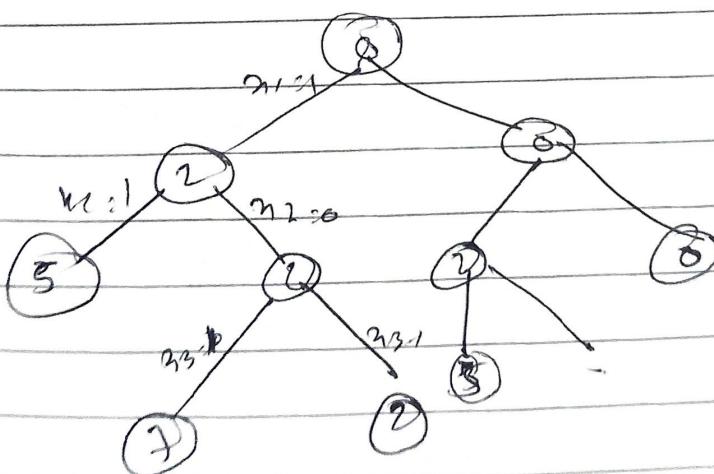
valid solution!

~~final answer:~~

~~{2, 3, 5}~~

( $2, 3, 5$ )

~~{5}~~



### (g) Sum of subset problem:

A = {10, 20, 30, 40}

sum = 50

