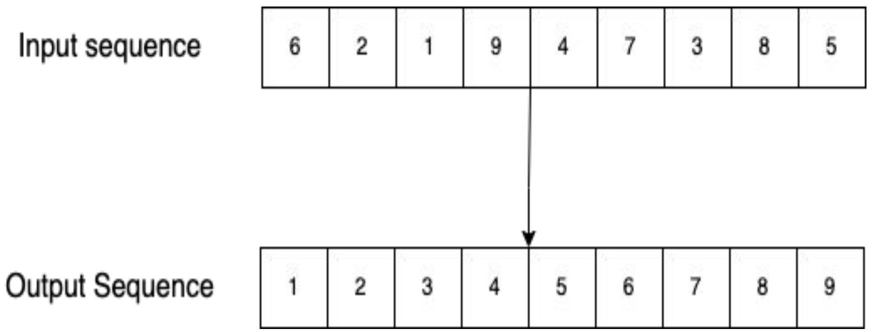
Histogram Sort with Sampling

Course: COMP 5704
Parallel Algorithms and Application in Data Science
Megha Agarwal

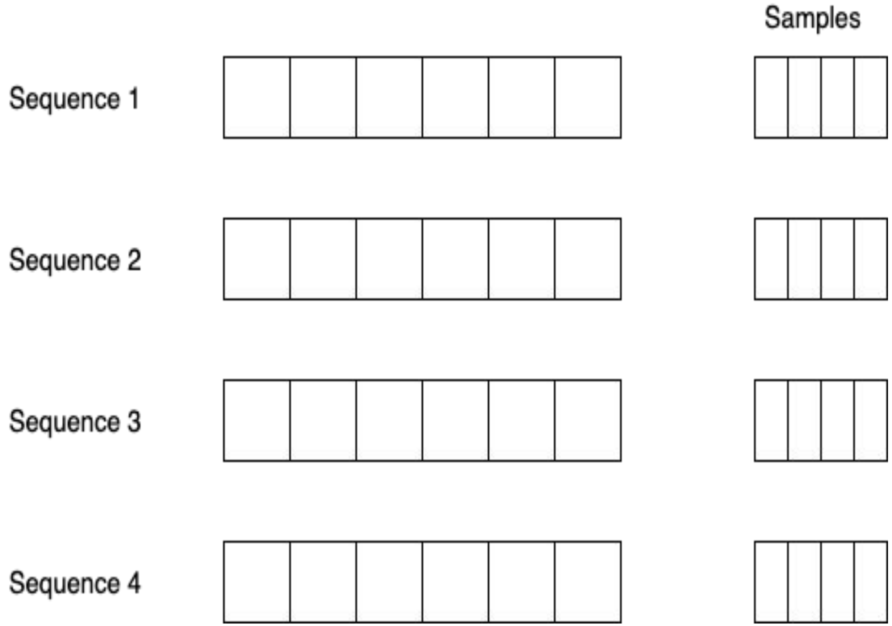
Introduction

- What is Sorting?
- Why do we need it?
- Types of Sorting:
  - Sequential
  - Parallel
- Chief goal of parallel sorting



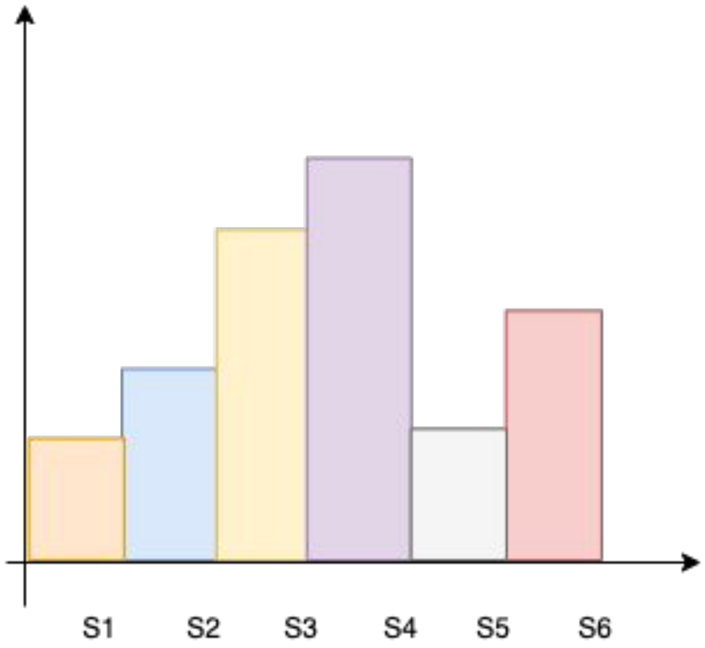
Sample Sort

1. Sample: *ps* keys
2. Split: *p-1* keys are selected
3. Exchange data: p^{th} bucket to p^{th} processor



Histogram Sort

1. Broadcast a probe
2. Create local histogram
3. Sum to form global histogram
4. Finalize splitters



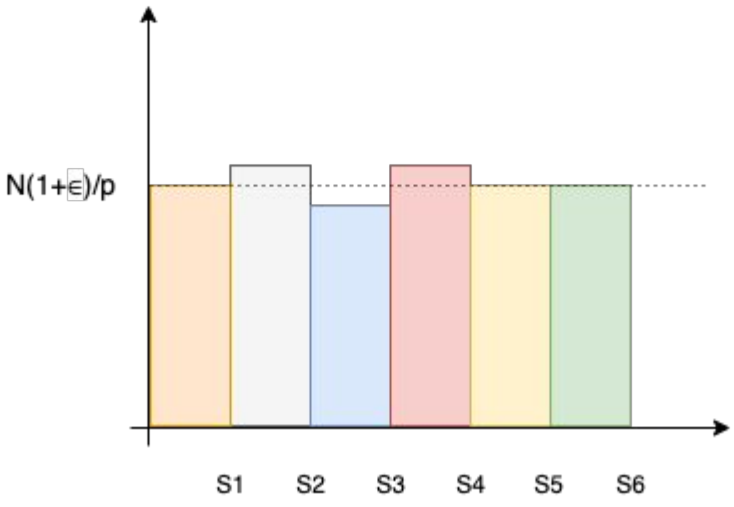
Histogram Sort with Sampling

- Goal: Approximate Splitting => $N(1+\varepsilon)/p$
- Processor *i* owns all keys greater than equal to *S(i)* and less than *S(i+1)* := Global balancing
- Two Major steps:
    - Histogramming
    - Sampling
- Input keys: A(0),...,A(N-1)
- Redistributed to: I(0),...,I(N-1), where if A(j)=I(k), it has rank *k*.

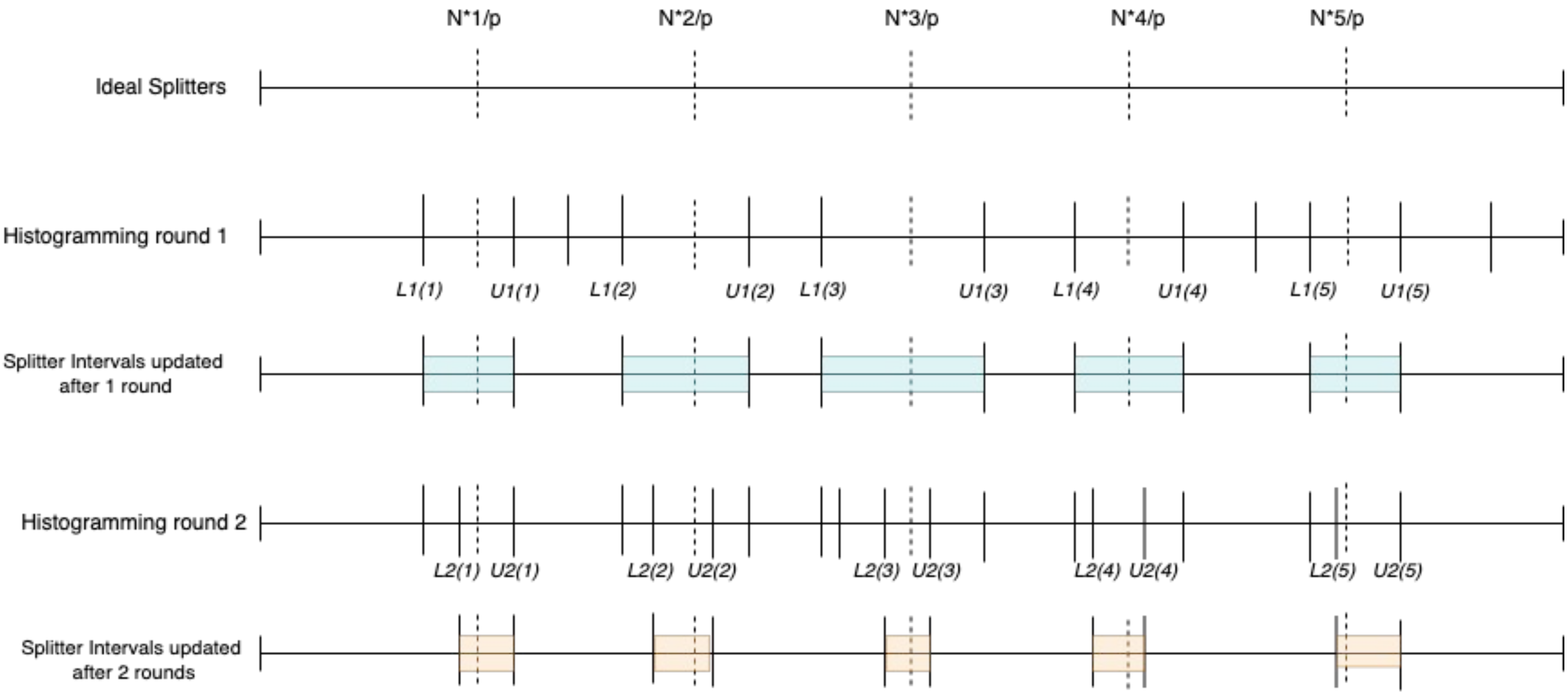
- Satisfactory splitter: S(i) = I(χ(i)), where χ(i) $\in \mathcal{T}_i = \left[\frac{Ni}{p} - \frac{N\epsilon}{2p}, \frac{Ni}{p} + \frac{N\epsilon}{2p}\right]$

Histogram Sort with Sampling

1. Each processor picks samples with probability ps_1/N and broadcast.
2. Create a local histogram at each processor and summed at central processor
3. Maintain a lower and upper bound $L_j(i)$ and $U_j(i)$ and update splitter intervals
4. Sample using new intervals for j+1th round.
5. If $j=k$,
    a. Histogramming phase is complete and move to next step.
    b. Else, if $j<k$, samples are collected at central processor, and move towards next round of histogramming.
6. Finally, the key closest to Ni/p is chosen as the ith splitter.



Ideal Situation



N*1/p
N*2/p
N*3/p
N*4/p
N*5/p
Ideal Splitters
Histogramming round 1
L1(1)
U1(1)
L1(2)
U1(2)
L1(3)
U1(3)
L1(4)
U1(4)
L1(5)
U1(5)
Splitter Intervals updated after 1 round
Histogramming round 2
L2(1)
U2(1)
L2(2)
U2(2)
L2(3)
U2(3)
L2(4)
U2(4)
L2(5)
U2(5)
Splitter Intervals updated after 2 rounds

Experimental setup

Charm++

- C++ based
- Supports MPI communication protocol
- Divides into processor elements called *chares*.
- Steps:
    - Local Sorting
    - Splitter Determination
    - Data exchange

Expected results of HSS

- Sampling ratio s= $O\left(p\sqrt[k]{\frac{\log p}{\epsilon}}\right)$ and *k= log(log p/ϵ)*
- *O(p)* samples can achieve global sorting in *O(log N/p + log log p)* rounds
- Costs:
  - Computation in local Sorting: *O((N/p) log N/p)*
  - Sampling: *O(S)* at local processor and *O(S log p)* for sorting at central processor
  - Computing local histogram: *O(S log N/p)*
  - Computation of sampling and histogramming per stage: *O(r log((log r)/ϵ)) log N)*
- Communication overhead due to multiple stage sorting.

HSS compared to other algorithms

<u>AMS-sort</u>

- Better for one round of histogramming by $\Theta(min(log\ p,\ 1/\varepsilon))$
- HSS achieves a globally-balanced splitting, making it easily generalizable
- Takes approximately *3x* time for splitting phase than HSS

<u>HykSort</u>

- Requires at least $\mathbf{\Omega}(log(p)/log^2 log(p))$ times more samples.
- Faster convergence of splitters

Thank you!