

# Energy Aggregation using Product of Experts

Dr. Haimonti Dutta  
Center for Computational Learning System,  
Columbia University  
New York, NY 10115  
haimonti@ccls.columbia.edu

Megha Gupta  
Dept. of Computer Science,  
IIIT-D  
Delhi, India.  
meghag@iiitd.ac.in

**Abstract**—The abstract goes here. DO NOT USE SPECIAL CHARACTERS, SYMBOLS, OR MATH IN YOUR TITLE OR ABSTRACT.

**Keywords**—product of experts; energy aggregation; contrastive divergence;

## I. INTRODUCTION

## II. RELATED WORK

### A. Automata and their products

Distributed networks can be modelled using interacting automata. Benveniste defines automaton as a quadruple,  $\hat{A} = (X, X_0, A, T)$  where  $X$  is a finite state of sets,  $X_0$  is the subset of initial states,  $A$  is a finite set of messages,  $T$  is a set of transitions of the form  $t = \{x_-, a, x\}$  where  $x_-$  is the previous state,  $a$  is the message label on which the state transitions to the next state  $x$ . The figure 1 below explains the automata with an example.

For automaton R,  $X_R = \{2; R1, R2\}$ ,  $X_{0R} = \{R1\}$ ,  $A_R = \{a, b\}$ ,  $T_R = \{R1, a, R1; R1, b, R2; R2, a, R2; R2, b, R1\}$   
For automaton S,  $X_S = \{3; S1, S2, S3\}$ ,  $X_{0S} = \{S1\}$ ,  $A_S = \{a, b\}$ ,  $T_S = \{S1, a, S1; S1, b, S2; S2, a, S2; S2, b, S3; S3, a, S3; S3, b, S1\}$

The product of two automata  $\hat{A} = R \times S$  is defined as follows:

$$X = X_R \times X_S$$

$$X_0 = X_{0R} \times X_{0S}$$

$$A = A_R \cup A_S$$

Benveniste uses a notion of stuttering transition which helps to distinguish between local and global time by inserting dummy transitions between two transitions of a local automaton attached to a node. This stuttering transition does nothing but lets the rest of the world progress.

A	R1	R2
R1	0.6	0.4
R2	0.3	0.7

Table I  
TRANSITION PROBABILITY, A

Talking in terms of HMM, requires us to equip products of automata with probabilities. Benveniste defines HMM as

B	a	b
R1	0.2	0.8
R2	0.5	0.5

Table II  
OBSERVED PROBABILITY, B

	R1	R2
$\pi$	0.4	0.6

Table III  
INITIAL STATE PROBABILITY,  $\pi$

a triple  $(\hat{A}, \mu, \pi)$  where  $\hat{A} = (X, X_0, A, T)$  is an automaton,  $\mu$  is the initial state probability,  $\pi$  is factored as state transition probability  $\pi_x$  and message transition probability  $\pi_A$ . He uses a random arbiter  $\alpha$ , with values first, second, third to choose automaton to initiate transition. If  $\alpha = \text{first}$  then first automaton chooses any transition having a private message whereas second automaton performs a stuttering transition, and vice versa for  $\alpha = \text{second}$ . If  $\alpha = \text{both}$ , then both automata agree on some shared message and move accordingly.

Using the traditional HMM notation of the parameters  $\lambda = \{A, B, \pi\}$  where  $A$  is the transition probability,  $B$  is the observed probability,  $\pi$  is the initial state probability. For automata R, we have the values of  $A, B, \pi$  as shown in table I, II, III respectively.

### B. Product of HMM

Product of HMM is a way of combining HMM's to form distributed state time series model. The figure 2 is a product of two HMMs shown in 1. For  $P = R \times S$ , the quadruple becomes

$$X = \{6; R1S1, R1S2, R1S3, R2S1, R2S2, R2S3\}$$

$$X_0 = \{R1S1\}$$

$$A = \{a, b\}$$

The rules for synchronised product construction are :

1.  $\langle p, q \rangle \rightarrow -a \rightarrow \langle p', q \rangle$  if  $a \in A_R \cap A_S$  and  $p \rightarrow -a \rightarrow p'$  and  $q \rightarrow -a \rightarrow q'$
2.  $\langle p, q \rangle \rightarrow -a \rightarrow \langle p', q \rangle$  if  $a \in A_R$ ,  $a \notin A_S$  and  $p \rightarrow -a \rightarrow p'$

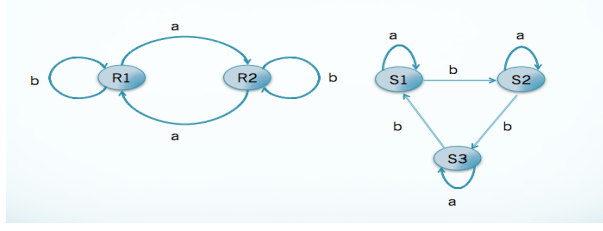


Figure 1. Automata R and S

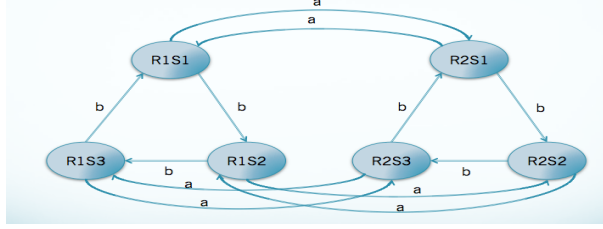


Figure 2. Product of HMMs,  $P = R \times S$

3.  $\langle p, q \rangle \xrightarrow{-a} \langle p, q' \rangle$  if  $a \notin A_R$ ,  $a \in A_S$  and  $q \xrightarrow{-a} q'$

### III. METHODOLOGY

#### A. Training product of experts by minimising contrastive divergence

High dimensional distributions are approximated as a product of one dimensional distributions. The product of individual distributions which is uniguassian or multivariate guassian will also be multivariate guassian. If the individual models are more complicated and contain one or more hidden variables, multiplying their distributions together and renormalizing them can be very powerful. These individual models are called "experts". The product of experts produce sharper distribution than the individual distributions [2].

#### IV. PROOF OF CONCEPT ON REDD HOUSE 2

##### A. Aim

To represent streams of energy consumption data from  $n^1$  appliances by product(s) of  $k$  HMMs.

##### B. Method

- **Data** The Reference Energy Disaggregation Data Set (REDD) is used in empirical analysis. The data contains power consumption from real homes, for the whole house as well as for each individual circuit in the house (labeled by the main type of appliance on that circuit). It is intended for use in developing disaggregation methods, which can predict, from only the whole-home signal, which devices are being used. The REDD data set contains two main types of home electricity data: high-frequency current/voltage waveform data of

the two power mains (as well as the voltage signal for a single phase), and lower-frequency power data including the mains and individual, labeled circuits in the house. The main directory consists of several house\_i directories, each of which contain all the power readings for a single house. Each house subdirectory consists of a labels.dat and several channels\_i.dat files. The labels file contains channel numbers and a text label indicating the general category of device on this channel. Each channel\_i.dat file has two columns containing UTC timestamps (as integers) and power readings (recording the apparent power of the circuit) for the channel. Experiments reported here use the House 2 data from REDD. It has 11 channels where each channel corresponds to the following appliance:

- 1) mains\_1
- 2) mains\_2
- 3) kitchen\_1
- 4) lighting
- 5) stove
- 6) microwave
- 7) washer\_dryer
- 8) kitchen\_2
- 9) refrigerator
- 10) dishwasher
- 11) disposal

The dataset has 318759 records and 2 columns. We randomly sample 300 records for our initial experiment. Time series data from two appliances are represented as product of  $k$  HMMs.

- **Time Series** : The time series data of the microwave, dryer, kitchen\_2 and refrigerator are plotted below in Figures 3, 4, 5, 6.

<sup>1</sup> $n=2$

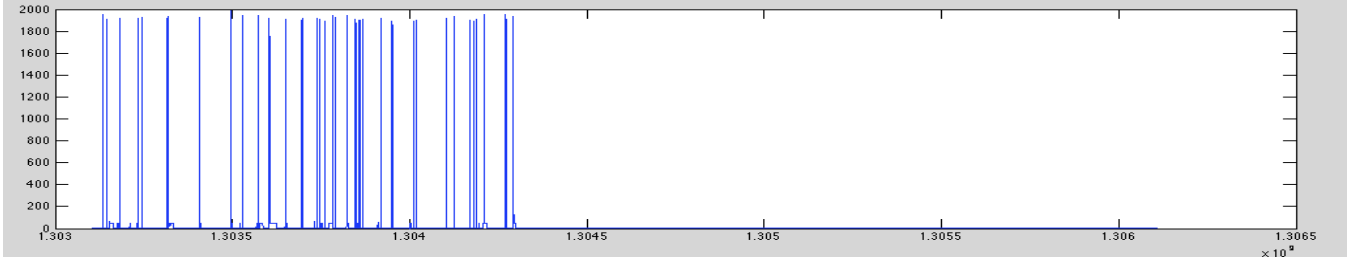


Figure 3. Microwave

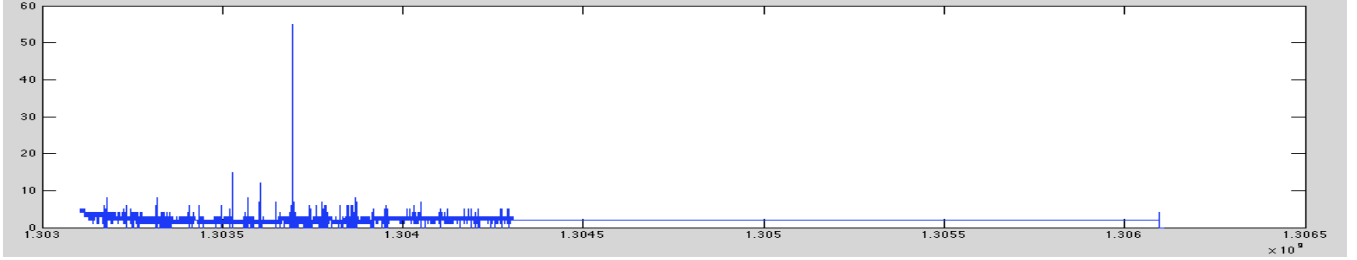


Figure 4. washer\_dryer

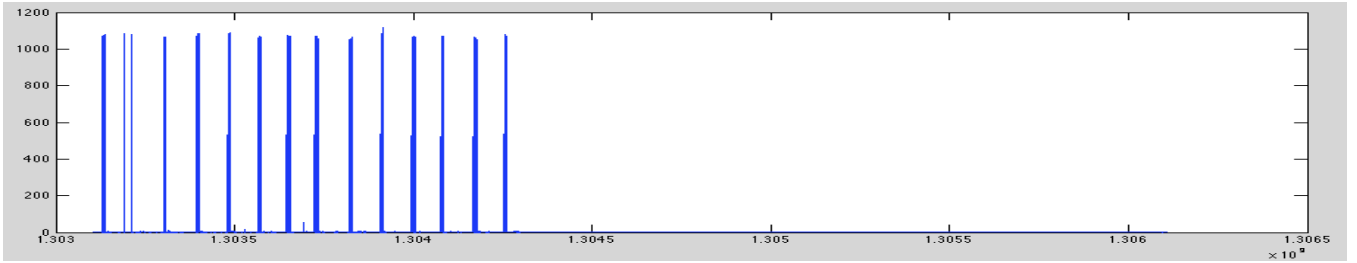


Figure 5. Kitchen\_2

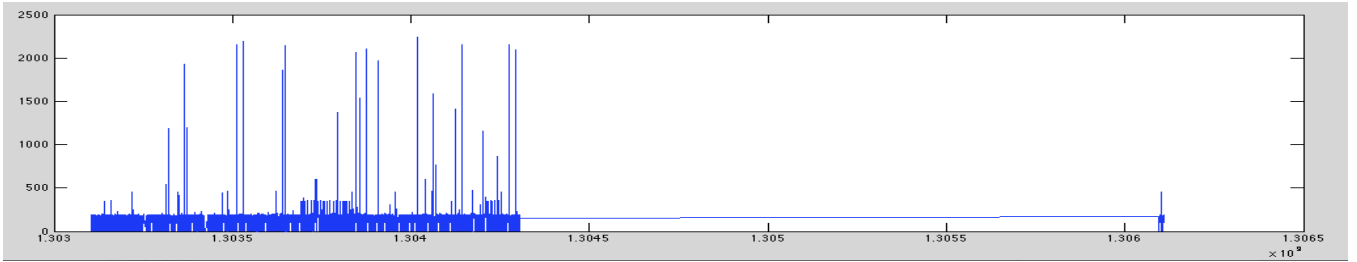


Figure 6. refrigerator

- **Code** The implementation of the product of experts model is obtained from Iain Murray's website [3]. It implements the technique described in Geoff Hinton's paper [2].
- **Additional details** Some additional details regarding experiments:
  - 1) The product of HMMs model (PoHMM) minimizes "contrastive divergence" as described in the paper [2].
  - 2) The number of experts,  $k$  used here is 15. This is

set somewhat arbitrarily and needs to be experimented on.

- 3) Learning rate is  $\epsilon = \frac{1}{300}$ .

### C. Experimental Setup for REDD house 2

Experiments are performed on the REDD which contains 9 appliances each containing 318759 rows of energy consumption data. Experiments are subdivided into 4 parts, in the first part the number of data samples are varied corresponding to which the values of KL Divergence and

convergence time are noted down. In the second part, the number of experts are varied keeping the best value of the sample from the first part constant. In the third part, number of iterations are varied keeping the best values from part 1 and 2 constant. In the fourth part, the no. of appliances to be aggregated are varied keeping the best values from above parts constant.

Samples	$KLDiv$	$T(sec)$	$Iterations$
300	2.4864	186.212 $\pm$ 9.087	18600
500	0.6761	106.564 $\pm$ 10.046	10200
1000	1.1088	158.521 $\pm$ 1.97	11200
1500	3.8829	92.896 $\pm$ 8.075	5300
2000	1.8686	130.98 $\pm$ 1.932	6900
2500	0.4733	215.563 $\pm$ 2.471	9900
3000	2.8204	258.213 $\pm$ 1.918	11000
3500	1.2332	204.661 $\pm$ 1.713	7900
4000	0.8959	292.666 $\pm$ 0.619	10400
4500	1.1118	222.558 $\pm$ 1.967	7200
8000	6.392	381.635 $\pm$ 2.952	8100
10000	8.276	887.932 $\pm$ 13.824	10500
15000	0.7201	1368.514 $\pm$ 13.605	9400

Table IV  
EFFECT OF VARYING SAMPLES ON KL DIV AND TIME

Experts	$KLDiv$	$T(sec)$	$Iterations$
5	0.774	72.968 $\pm$ 1.177	5200
10	1.424	117.482 $\pm$ 1.966	6700
15	0.473	210.249 $\pm$ 1.258	9900
20	1.56	217.739 $\pm$ 10.452	9000
25	7.469	347.019 $\pm$ 8.23	12100
30	2.4968	413.802 $\pm$ 7.304	12900
35	1.5012	348.906 $\pm$ 14.651	11300

Table V  
EFFECT OF VARYING EXPERTS ON KL DIV AND TIME

#### D. Results

The evaluation of how well the learning has taken place is done by using a Kullback-Leibler divergence metric which gives the difference between two probability distributions. The two probability distributions in the REDD example refer to the expert probabilities in real and fantasy data. The learned parameters from the training are fitted to the fantasy data to measure the information lost when fantasy data is used to approximate real data. This is denoted as  $D_{KL}(\text{real data} || \text{fantasy})$ .

Threshold	$KLDiv$	$T(sec)$	$Iterations$
.1	0.473	210.6 $\pm$ 1.493	9900
.05	0.443	240.607 $\pm$ 2.436	10900
.01	0.454	431.536 $\pm$ 14.509	18000
.005	0.509	1167.243 $\pm$ 43.412	49800

Table VI  
EFFECT OF VARYING MIN THRESHOLD ON KL DIV AND TIME

Appliances	$KLDiv$	$T(sec)$	$Iterations$
3	5.559	233.664 $\pm$ 0.579	10700
4	0.188	465.634 $\pm$ 5.275	19900
5	.432	338.416 $\pm$ 3.988	13400
6	8.736	606.062 $\pm$ 7.534	28100
7	5.054	411.457 $\pm$ 10.051	17300
8	0.436	260.544 $\pm$ 27.862	10700
9	0.15	474.579 $\pm$ 14.619	20600

Table VII  
EFFECT OF VARYING APPLIANCES ON KL DIV AND TIME

## V. PROOF OF CONCEPT ON FACULTY HOUSING DATA

### A. Aim

To represent streams of energy consumption data from all the floors of faculty housing as a product(s) of  $k$  HMMs.

### B. Method

- **Data** This data represents the energy consumed by the IIIT Delhi faculty housing building. As a part of research, a team from IIIT Delhi has installed various temperature, light and motion sensors to perform real world studies and to analyse user preferences for energy conservation. For our analysis, we selected one month's historical data ranging from 01-01-2014, 00:01 hours to 31-01-2014, 23:59 hours. The two smart meters installed captures the data from all the floors. The first meter gives out readings from floors 0 to 5 and the second meter gives out readings from floors 6 to 11. The dataset includes timestamp and power consumed in watts and 84133 records. Time series data from two streams are modelled as a product of  $k$  HMMs. We also have the total power consumed by the faculty housing building which would serve as the ground truth to compare product of  $k$  HMMs with. The data is obtained from the website whose screenshot is shown in 7
- **Code** The implementation of the product of experts model is obtained from Iain Murray's website [3]. It implements the technique described in Geoff Hinton's paper [2].
- **Time Series** : The time series data of the energy consumption of floor 0 to 5, floor 6 to 11 and total power are plotted below in Figures ??, 9, 10.
- **Additional Details**

### C. Experimental Setup

Each of the data stream is modelled as a HMM individually. There are three streams of data, the first stream D1 corresponds to the data from floor 0-5, D2 corresponds to floor 6-11 and D3 represents the total power from the faculty housing and a fixed test set, T representing the data D3. Firstly, the stream D1 is used to train the model by minimising contrastive divergence. The parameters learnt during learning are provided to the test set T to obtain the conditional probability of the gaussians given the data

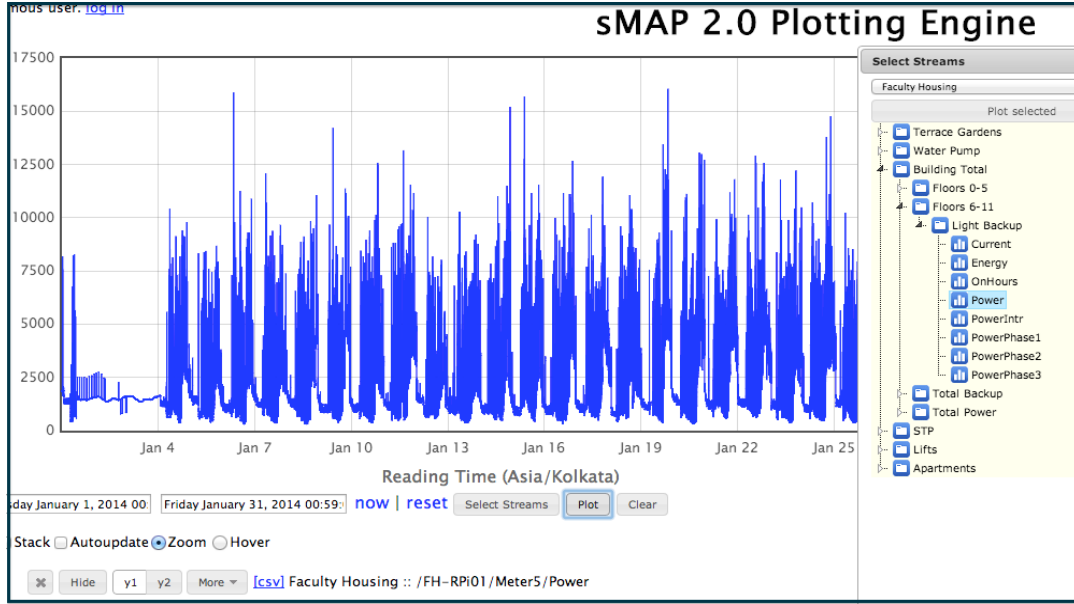


Figure 7. Screen shot of the webpage

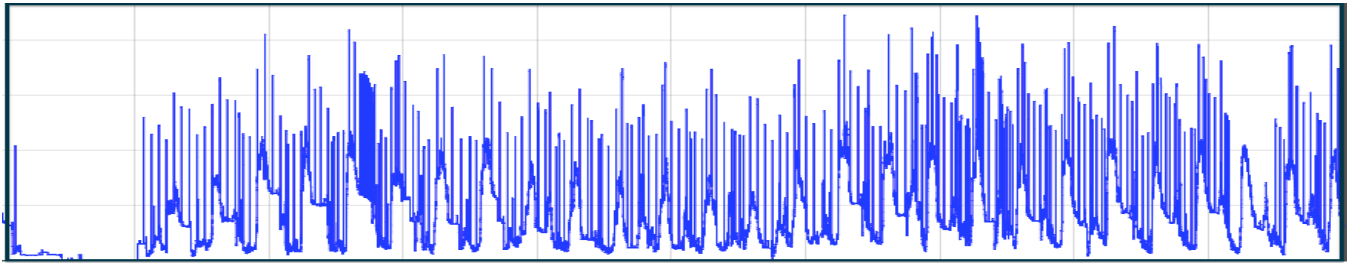


Figure 8. Stream 1: Power consumption of floors 0-5

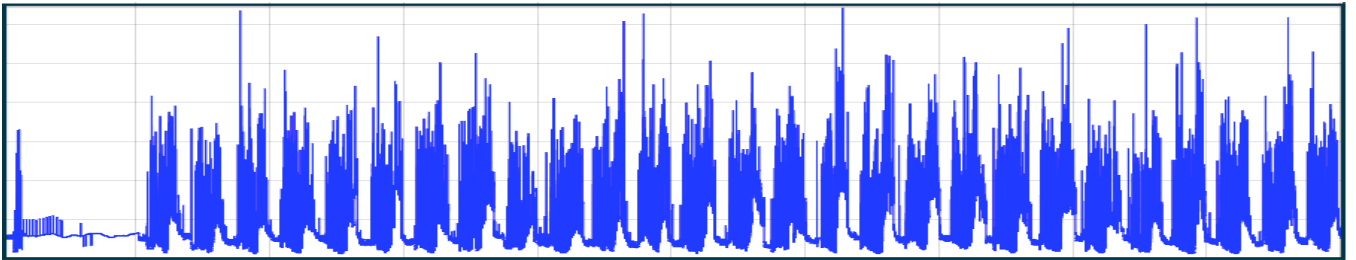


Figure 9. Stream 2: Power consumption of floors 6-11

D1 as pgauss1. Similarly, the second stream of data, D2 collected from floor 6-11, is used to learn the parameters of the model during the training phase which are then again provided to the test set T to obtain conditional probability of the gaussians given the data D2 as pgauss2. Finally the data D3 is used to learn the model and parameters which are then applied to the test set T to obtain the gaussian probability as pgauss3. Now, as we know that the total power consumption of the building should be approximately equal to the product of HMMs, hence the value of the product of pgauss1 and

pgauss2 should be as close as possible to pgauss3.

#### D. Results

#### VI. CONCLUSION & FUTURE WORK

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#### ACKNOWLEDGMENT

The authors would like to thank... more thanks here

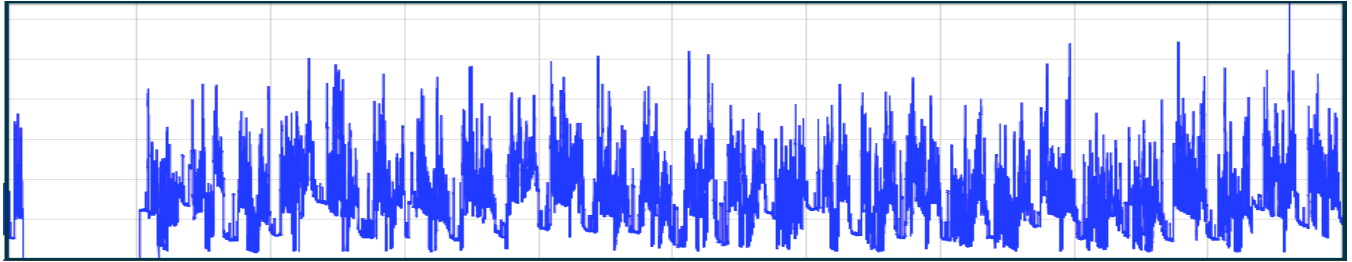


Figure 10. Total Power of the building

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