In[∘]:= `Quit[]`

Metric : modified Kerr-Newman :

For Figure 1 :

Defining the metrics involved:

Metric : modified Kerr-Newman :

```
In[∘]:= coord = {t, r, θ, ϕ}
```

Out[∘]= {t, r, θ, ϕ}

```
In[∘]:= n = Length[coords];

In[∘]:= ρ[r_] = r^2 + a^2 (Cos[θ])^2;

In[∘]:= Δ[r_] = r^2 + a^2 - 2 r e^(-k/r);

In[∘]:= tt[r_] = -{1 - 2 r e^(-k/r) /ρ[r]};

In[∘]:= rr[r_] = ρ[r] / Δ[r];

In[∘]:= θθ[r_] = ρ[r];

In[∘]:= ϕϕ[r_] = (r^2 + a^2 + (2 a^2 r e^(-k/r) (Sin[θ])^2/ρ[r])) (Sin[θ])^2;

In[∘]:= tϕ[r_] = -2 a r e^(-k/r) (Sin[θ])^2/ρ[r];

In[∘]:= metric = {{2 r e^(-k/r) /ρ[r] - 1, 0, 0, -2 a r e^(-k/r) (Sin[θ])^2/ρ[r]},
    {0, ρ[r] / Δ[r], 0, 0}, {0, 0, ρ[r], 0}, {-2 a r e^(-k/r) (Sin[θ])^2/ρ[r],
     0, 0, (r^2 + a^2 + (2 a^2 r e^(-k/r) (Sin[θ])^2/ρ[r])) (Sin[θ])^2}};

In[∘]:= metric // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -1+\frac{2 e^{-\frac{k}{r}} r}{r^2+a^2 \mathrm{Cos}[\theta]^2} & 0 & 0 & -\frac{2 a e^{-\frac{k}{r}} r \mathrm{Sin}[\theta]^2}{r^2+a^2 \mathrm{Cos}[\theta]^2} \\ 0 & \frac{r^2+a^2 \mathrm{Cos}[\theta]^2}{a^2-2 e^{-\frac{k}{r}} r+r^2} & 0 & 0 \\ 0 & 0 & r^2+a^2 \mathrm{Cos}[\theta]^2 & 0 \\ -\frac{2 a e^{-\frac{k}{r}} r \mathrm{Sin}[\theta]^2}{r^2+a^2 \mathrm{Cos}[\theta]^2} & 0 & 0 & \mathrm{Sin}[\theta]^2 \left(a^2+r^2+\frac{2 a^2 e^{-\frac{k}{r}} r \mathrm{Sin}[\theta]^2}{r^2+a^2 \mathrm{Cos}[\theta]^2}\right) \end{pmatrix}$$

large output | show less | show more | show all | set size limit...

```
In[∘]:= inversemetric = Simplify[Inverse[metric]];
```

In[∘]:= inversemetric // MatrixForm

Out[∘]//MatrixForm=

$$\begin{pmatrix} -\frac{2\left(a^2 e^{k/r}\left(a^2+r^2\right)\text{Cos}[\theta]^2+r\left(e^{k/r} r\left(a^2+r^2\right)+2a^2\text{Sin}[\theta]^2\right)\right)}{\left(a^2 e^{k/r}+r\left(-2+e^{k/r} r\right)\right)\left(a^2+2r^2+a^2\text{Cos}[2\theta]\right)} & 0 & 0 & -\frac{4ar}{\left(a^2 e^{k/r}+r\left(-2+e^{k/r} r\right)\right)\left(a^2+2r^2+a^2\text{Co}\right.} \\ 0 & \frac{a^2-2e^{-\frac{k}{r}}r+r^2}{r^2+a^2\text{Cos}[\theta]^2} & 0 & 0 \\ 0 & 0 & \frac{1}{r^2+a^2\text{Cos}[\theta]^2} & 0 \\ -\frac{4ar}{\left(a^2 e^{k/r}+r\left(-2+e^{k/r} r\right)\right)\left(a^2+2r^2+a^2\text{Cos}[2\theta]\right)} & 0 & 0 & \frac{2\left(r\left(-2+e^{k/r} r\right)+a^2 e^{k/r}\text{Cos}[\theta]^2\right)\text{Csc}}{\left(a^2 e^{k/r}+r\left(-2+e^{k/r} r\right)\right)\left(a^2+2r^2+a^2\text{Cos}\right.} \end{pmatrix}$$

large output | show less | show more | show all | set size limit...

For a=0.7

Geodesic equations for the equatorial plane :

1. dt/dτ →dtdτ[r, Em, L]

In[∘]:= a = 0.7;

In[∘]:= dtdτ [r_, Em_, L_] :=
(1 / Δ) [(r^2 + a^2 + (2 a^2 r e^(-k / r) / ρ[r])) Em - (-2 a r e^(-k / r) / ρ[r]) L]

In[∘]:= dtdτ [r, Em, L]

Out[∘]=

$$\frac{1}{\Delta}\left[\frac{1.4\, e^{-\frac{k}{r}}\, L\, r}{r^2+0.49\,\text{Cos}[\theta]^2}+\text{Em}\left(0.49+r^2+\frac{0.98\, e^{-\frac{k}{r}}\, r}{r^2+0.49\,\text{Cos}[\theta]^2}\right)\right]$$

large output | show less | show more | show all | set size limit...

2. dϕ/dτ →dϕdτ [r, L, Em]

In[∘]:= a = 0.7;

In[∘]:= dϕdτ [r_, Em_, L_] := (1 / Δ) [(1 - (2 r e^(-k / r) / ρ[r])) L + (-2 a r e^(-k / r) / ρ[r]) Em]

In[∘]:= dϕdτ [r, Em, L]

Out[∘]=

$$\frac{1}{\Delta}\left[-\frac{1.4\, e^{-\frac{k}{r}}\,\text{Em}\, r}{r^2+0.49\,\text{Cos}[\theta]^2}+L\left(1-\frac{2\, e^{-\frac{k}{r}}\, r}{r^2+0.49\,\text{Cos}[\theta]^2}\right)\right]$$

large output | show less | show more | show all | set size limit...

3. dr/dτ →drdτ [r, L, Em]

In[∘]:=
```
a = 0.7;
```

In[∘]:=
```
drdτ[r_, Em_, L_] :=
  √rr[r]^-1 [-1 - {Em^2 (tt[r] × (dtdτ [r, Em, L]) ^2)} + {2 L × Em × tϕ[r] × (dtdτ [r, Em, L]) ^2 ×
      (dϕdτ [r, Em, L]) ^2} - {L^2 × ϕϕ[r] × (dϕdτ [r, Em, L]) ^2}]
```

In[∘]:=
```
drdτ [r, Em, L]
```

Out[∘]=

$$\sqrt{\frac{0.49-2\,e^{-\frac{k}{r}}\,r+r^2}{r^2+0.49\,\mathrm{Cos}[\theta]^2}\Bigg[\Bigg\{\Bigg\{-1-L^2\,\mathrm{Sin}[\theta]^2\left(0.49+r^2+\frac{0.98\,e^{-\frac{k}{r}}\,r\,\mathrm{Sin}[\theta]^2}{r^2+0.49\,\mathrm{Cos}[\theta]^2}\right)\frac{1}{\Delta}\left[-\frac{1.4\,e^{-\frac{k}{r}}\,\mathrm{Em}\,r}{r^2+0.49\,\mathrm{Cos}[\theta]^2}+L\left(1-\frac{2\,e^{-\frac{k}{r}}\,r}{r^2+0.49\,\cdots 1\cdots^2}\right)\right]^2 - \mathrm{Em}^2\left(-1+\frac{2\,e^{-\frac{k}{r}}\,r}{r^2+0.49\,\cdots 1\cdots^2}\right)\frac{1}{\Delta}\left[\frac{1.4\,e^{\cdots 1\cdots}\,L\,r}{r^2+\cdots 20\cdots\,\cdots 1\cdots}+\mathrm{Em}\,(\cdots 1\cdots)\right]^2 - \frac{2.8\,e^{-\frac{k}{r}}\,\mathrm{Em}\,L\,r\,\mathrm{Sin}[\theta]^2\,\frac{1}{\Delta}\left[-\frac{1.4\,e^{\cdots 1\cdots}\,\mathrm{Em}\,r}{r^2+\cdots 20\cdots\,\cdots 1\cdots}+L\,(1-\cdots 1\cdots)\right]^2\frac{1}{\Delta}\left[\frac{1.4\,e^{-\frac{k}{r}}\,L\,r}{r^2+0.49\,\cdots 1\cdots^2}+\mathrm{Em}\left(0.49+r^2+\frac{\cdots 1\cdots}{r^2+\cdots 1\cdots}\right)\right]^2}{r^2+0.49\,\mathrm{Cos}[\theta]^2}\Bigg\}\Bigg\}\Bigg]}$$

large output | show less | show more | show all | set size limit...

In[∘]:=
```
drdτ [r, Em, L] ^2 // Expand
```

Out[∘]=

$$\frac{0.49-2\,e^{-\frac{k}{r}}\,r+r^2}{r^2+0.49\,\mathrm{Cos}[\theta]^2}\Bigg[\Bigg\{\Bigg\{-1-L^2\,\mathrm{Sin}[\theta]^2\left(0.49+r^2+\frac{0.98\,e^{-\frac{k}{r}}\,r\,\mathrm{Sin}[\theta]^2}{r^2+0.49\,\mathrm{Cos}[\theta]^2}\right)\frac{1}{\Delta}\left[-\frac{1.4\,e^{-\frac{k}{r}}\,\mathrm{Em}\,r}{r^2+0.49\,\mathrm{Cos}[\theta]^2}+L\left(1-\frac{2\,e^{-\frac{k}{r}}\,r}{r^2+0.49\,\cdots 1\cdots^2}\right)\right]^2 - \mathrm{Em}^2\left(-1+\frac{2\,e^{-\frac{k}{r}}\,r}{r^2+0.49\,\cdots 1\cdots^2}\right)\frac{1}{\Delta}\left[\frac{1.4\,e^{\cdots 1\cdots}\,L\,r}{r^2+\cdots 20\cdots\,\cdots 1\cdots}+\mathrm{Em}\,(\cdots 1\cdots)\right]^2 - \frac{2.8\,e^{-\frac{k}{r}}\,\mathrm{Em}\,L\,r\,\mathrm{Sin}[\theta]^2\,\frac{1}{\Delta}\left[-\frac{1.4\,e^{\cdots 1\cdots}\,\mathrm{Em}\,r}{r^2+\cdots 20\cdots\,\cdots 1\cdots}+L\,(1-\cdots 1\cdots)\right]^2\frac{1}{\Delta}\left[\frac{1.4\,e^{-\frac{k}{r}}\,L\,r}{r^2+0.49\,\cdots 1\cdots^2}+\mathrm{Em}\left(0.49+r^2+\frac{\cdots 1\cdots}{r^2+\cdots 1\cdots}\right)\right]^2}{r^2+0.49\,\mathrm{Cos}[\theta]^2}\Bigg\}\Bigg\}\Bigg]$$

large output | show less | show more | show all | set size limit...

For the epicyclic frequencies (ω):

In[∘]:=
```
a = 0.7;
```

In[∘]:=
```
θ = π / 2;
```

In[∘]:=
```
tt[r_] = -{1 - 2 r e^(-k / r) / ρ[r]};
```

In[ ]:=
```
D[tt[r], r] // FullSimplify
```

Out[ ]=

$$\left\{-\frac{2\, e^{-\frac{k}{r}}\, (r - k\, \text{Log}[e])}{r^3}\right\}$$

large output | show less | show more | show all | set size limit...

In[ ]:=
```
tϕ[r_] = -2 a r e^(-k / r) (Sin[θ])^2 / ρ[r];
```

In[ ]:=
```
D[tϕ[r], r] // FullSimplify
```

Out[ ]=

$$\frac{e^{-\frac{k}{r}}\, (1.4\, r - 1.4\, k\, \text{Log}[e])}{r^3}$$

large output | show less | show more | show all | set size limit...

In[ ]:=
```
ϕϕ[r_] = (r^2 + a^2 + (2 a^2 r e^(-k / r) (Sin[θ])^2 / ρ[r])) (Sin[θ])^2;
```

In[ ]:=
```
D[ϕϕ[r], r] // FullSimplify
```

Out[ ]=

$$2.\, r + \frac{e^{-\frac{k}{r}}\, (-0.98\, r + 0.98\, k\, \text{Log}[e])}{r^3}$$

large output | show less | show more | show all | set size limit...

PLOTS:

Knowing the expression for ω, plot ω vs r with a=0.7, where:

In[ ]:=
```
Clear[k]
```

In[ ]:=
```
k = 0;
```

In[ ]:=

$$\omega[r\_] = \frac{-D[t\phi[r], r] + \sqrt{\left(D[t\phi[r], r]^2 - D[tt[r], r] \times D[\phi\phi[r], r]\right)}}{D[\phi\phi[r], r]}$$

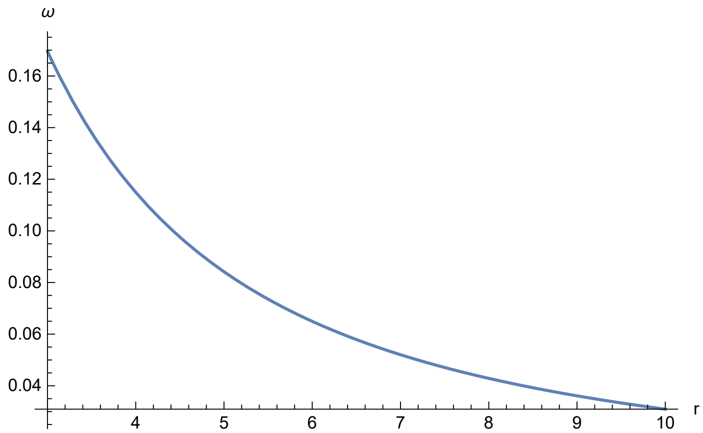
Out[ ]=

$$\left\{\frac{-\frac{2.8\, r^2}{(0.+r^2)^2} + \frac{1.4}{0.+r^2} + \sqrt{\left(\frac{2.8\, r^2}{(0.+r^2)^2} - \frac{1.4}{0.+r^2}\right)^2 - \left(2\, r - \frac{1.96\, r^2}{(0.+r^2)^2} + \frac{0.98}{0.+r^2}\right)\left(-\frac{4\, r^2}{(0.+r^2)^2} + \frac{2}{0.+r^2}\right)}}{2\, r - \frac{1.96\, r^2}{(0.+r^2)^2} + \frac{0.98}{0.+r^2}}\right\}$$

large output | show less | show more | show all | set size limit...

In[ ]:=
```
Plot[ω[r], {r, 3, 10}, AxesLabel → {"r", "ω"}]
```

Out[ ]=

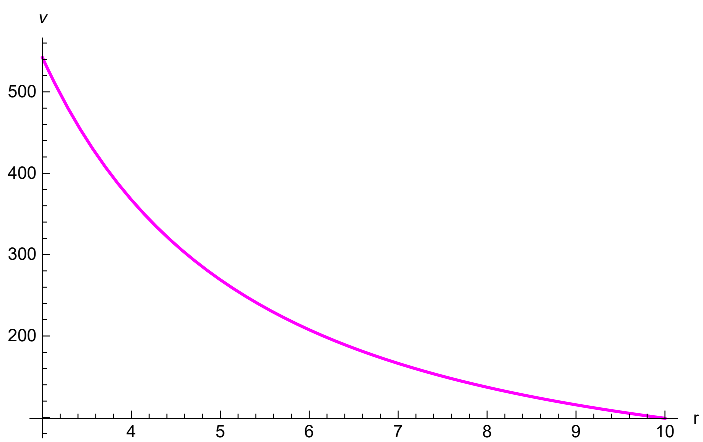


Knowing the expression for epicyclic frequencies, plot them , a=0.7, where:

1. For epicyclic frequency of Azimuthal component :

In[ ]:=
```
p1 = Plot[ω[r] * (0.2035 * 10^4) / 2 π, {r, 3, 10}, AxesLabel → {"r", "ν"}, PlotStyle → Magenta]
```

Out[ ]=



2. For epicyclic frequency of Radial component :

In[ ]:=
```
a = 0.7;
```

In[ ]:=
```
θ = π / 2;
```

In[ ]:=
```
ω1[r_] = ω[r] Sqrt[1 - 6 (r^-1) + 8 a (r^(-3/2)) - 3 a^2 (r^-2)]
```

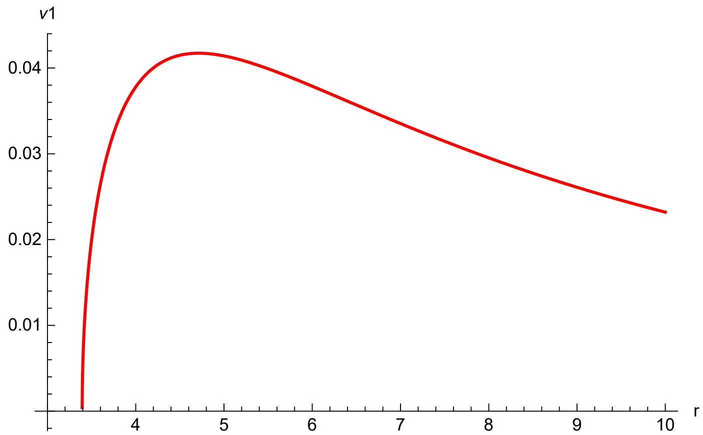
Out[ ]=

$$\left\{\frac{\sqrt{1-\frac{1.47}{r^2}+\frac{5.6}{r^{3/2}}-\frac{6}{r}}\left(-\frac{2.8\,r^2}{(0.+r^2)^2}+\frac{1.4}{0.+r^2}+\sqrt{\left(\frac{2.8\,r^2}{(0.+r^2)^2}-\frac{1.4}{0.+r^2}\right)^2-\left(2\,r-\frac{1.96\,r^2}{(0.+r^2)^2}+\frac{0.98}{0.+r^2}\right)\left(-\frac{4\,r^2}{(0.+r^2)^2}+\frac{2}{0.+r^2}\right)}\right)}{2\,r-\frac{1.96\,r^2}{(0.+r^2)^2}+\frac{0.98}{0.+r^2}}\right\}$$

large output | show less | show more | show all | set size limit...

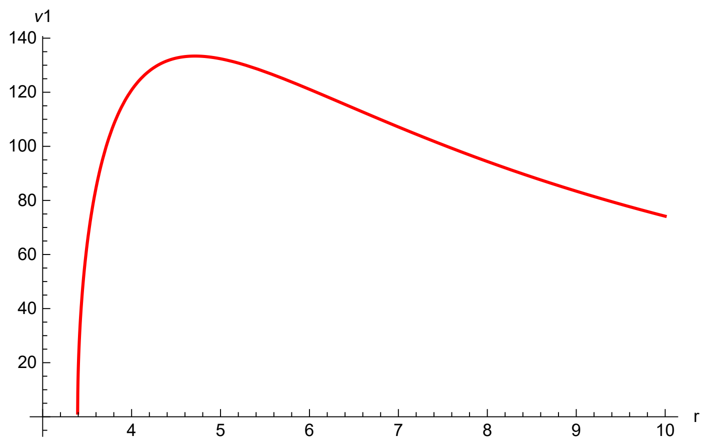
In[∘]:=
```
Plot[ω1[r], {r, 3, 10}, AxesLabel → {"r", "ν1"}, PlotStyle → Red]
```

Out[∘]=



In[∘]:=
```
p2 = Plot[ω1[r] * (0.2035 * 10^4) / 2 π, {r, 3, 10}, AxesLabel → {"r", "ν1"}, PlotStyle → Red]
```

Out[∘]=



3. For epicyclic frequency of vertical component :

In[∘]:=
```
a = 0.7;
```

In[∘]:=
```
θ = π / 2;
```

In[∘]:=
```
ω2[r_] = ω[r] Sqrt[1 - 4 a (r^(-3/2)) + 3 a^2 (r^-2)]
```

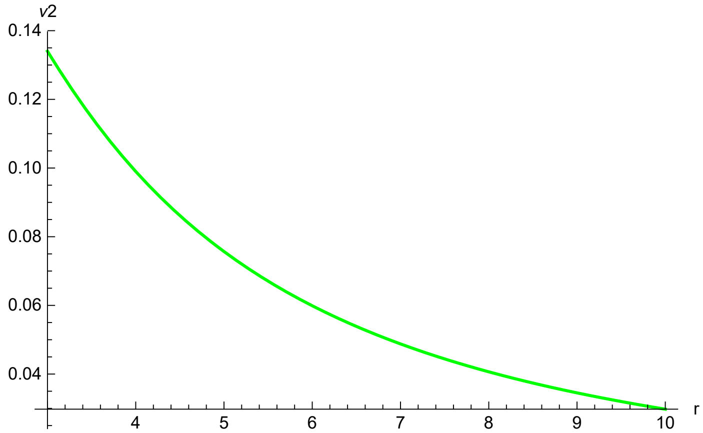
Out[∘]=

$$\left\{\frac{\sqrt{1+\frac{1.47}{r^2}-\frac{2.8}{r^{3/2}}}\left(-\frac{2.8\,r^2}{(0.+r^2)^2}+\frac{1.4}{0.+r^2}+\sqrt{\left(\frac{2.8\,r^2}{(0.+r^2)^2}-\frac{1.4}{0.+r^2}\right)^2-\left(2\,r-\frac{1.96\,r^2}{(0.+r^2)^2}+\frac{0.98}{0.+r^2}\right)\left(-\frac{4\,r^2}{(0.+r^2)^2}+\frac{2}{0.+r^2}\right)}\right)}{2\,r-\frac{1.96\,r^2}{(0.+r^2)^2}+\frac{0.98}{0.+r^2}}\right\}$$

large output | show less | show more | show all | set size limit...

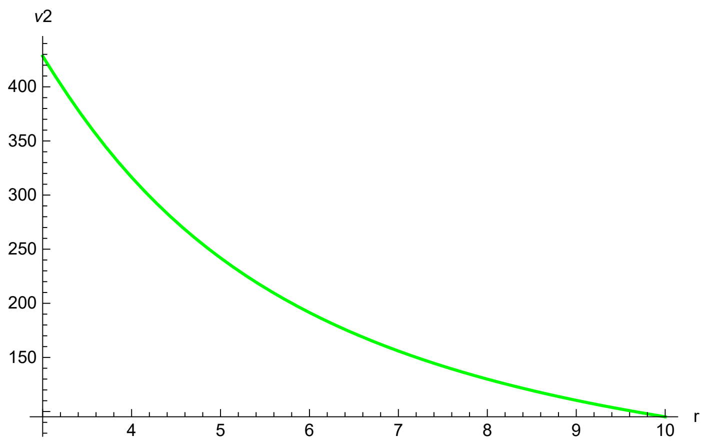
In[∘]:=
```
Plot[ω2[r], {r, 3, 10}, AxesLabel → {"r", "ν2"}, PlotStyle → Green]
```

Out[∘]=



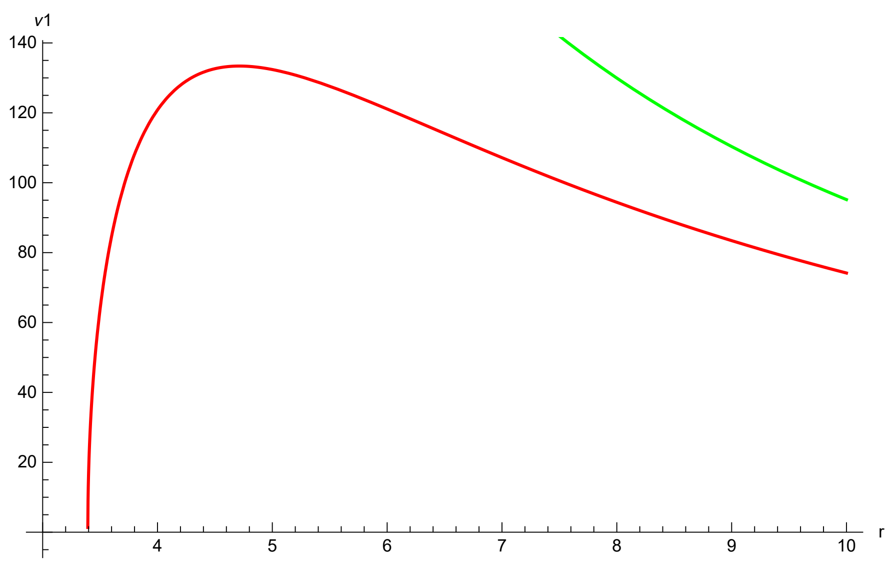
In[∘]:=
```
p3 = Plot[ω2[r] * (0.2035 * 10^4) / 2 π, {r, 3, 10}, AxesLabel → {"r", "ν2"}, PlotStyle → Green]
```

Out[∘]=



In[∘]:=
```
Show[p2, p3, AxesOrigin → Automatic]
```

Out[∘]=



For different values of “a”: Radial

For a=0:

Geodesic equations for the equatorial plane :

In[∘]:=
```
Clear[k]
```

1. dt/dτ →dtdτ[r, Em, L]

In[∘]:=
```
a = 0;
```

In[∘]:=
```
dtdτ [r_, Em_, L_] :=
  (1/Δ) [(r^2 + a^2 + (2 a^2 r e^(-k/r) / ρ[r])) Em - (-2 a r e^(-k/r) / ρ[r]) L]
```

In[∘]:=
```
dtdτ [r, Em, L]
```

Out[∘]=
$$\frac{1}{\Delta}\left[\mathrm{Em}\, r^2\right]$$

2. dϕ/dτ →dϕdτ [r, L, Em]

In[∘]:=
```
a = 0;
```

In[∘]:=
```
dϕdτ [r_, Em_, L_] := (1/Δ) [(1 - (2 r e^(-k/r) / ρ[r])) L + (-2 a r e^(-k/r) / ρ[r]) Em]
```

In[∘]:=
```
dϕdτ [r, Em, L]
```

Out[∘]=
$$\frac{1}{\Delta}\left[L\left(1-\frac{2\, e^{-\frac{k}{r}}}{r}\right)\right]$$

large output | show less | show more | show all | set size limit...

3. dr/dτ →drdτ [r, L, Em]

In[∘]:=
```
a = 0;
```

In[∘]:=
```
drdτ[r_, Em_, L_] :=
  √rr[r]^-1 [-1 - {Em^2 (tt[r] × (dtdτ [r, Em, L]) ^2)} + {2 L × Em × tϕ[r] × (dtdτ [r, Em, L]) ^2 ×
       (dϕdτ [r, Em, L]) ^2} - {L^2 × ϕϕ[r] × (dϕdτ [r, Em, L]) ^2}]
```

In[ ]:=
```
drdτ [r, Em, L]
```

Out[ ]=
$$\sqrt{\frac{-2\,e^{-\frac{k}{r}}\,r+r^2}{r^2}\left[\left\{\left\{-1-L^2\left(0.49+r^2+\frac{0.98\,e^{-\frac{k}{r}}\,r}{0.+r^2}\right)\frac{1}{\triangle}\left[L\left(1-\frac{2\,e^{-\frac{k}{r}}}{r}\right)\right]^2-\text{Em}^2\left(-1+\frac{2\,e^{-\frac{k}{r}}\,r}{0.+r^2}\right)\frac{1}{\triangle}\left[\text{Em}\,r^2\right]^2-\frac{2.8\,e^{-\frac{k}{r}}\,\text{Em}\,L\,r\,\frac{1}{\triangle}\left[L\left(1-\frac{2\,e^{-\frac{k}{r}}}{r}\right)\right]^2\frac{1}{\triangle}\left[\text{Em}\,r^2\right]^2}{0.+r^2}\right\}\right\}\right]}$$

large output | show less | show more | show all | set size limit...

In[ ]:=
```
drdτ [r, Em, L] ^2 // Expand
```

Out[ ]=
$$\frac{-2\,e^{-\frac{k}{r}}\,r+r^2}{r^2}\left[\left\{\left\{-1-L^2\left(0.49+r^2+\frac{0.98\,e^{-\frac{k}{r}}\,r}{0.+r^2}\right)\frac{1}{\triangle}\left[L\left(1-\frac{2\,e^{-\frac{k}{r}}}{r}\right)\right]^2-\text{Em}^2\left(-1+\frac{2\,e^{-\frac{k}{r}}\,r}{0.+r^2}\right)\frac{1}{\triangle}\left[\text{Em}\,r^2\right]^2-\frac{2.8\,e^{-\frac{k}{r}}\,\text{Em}\,L\,r\,\frac{1}{\triangle}\left[L\left(1-\frac{2\,e^{-\frac{k}{r}}}{r}\right)\right]^2\frac{1}{\triangle}\left[\text{Em}\,r^2\right]^2}{0.+r^2}\right\}\right\}\right]$$

large output | show less | show more | show all | set size limit...

For the epicyclic frequencies (*ω*):

In[ ]:=
```
a = 0;
```

In[ ]:=
```
θ = π / 2;
```

In[ ]:=
```
tt[r_] = -{1 - 2 r e^(-k / r) / ρ[r]};
```

In[ ]:=
```
D[tt[r], r] // FullSimplify
```

Out[ ]=
$$\left\{-\frac{2\,e^{-\frac{k}{r}}\,(r-k\,\text{Log}[e])}{r^3}\right\}$$

large output | show less | show more | show all | set size limit...

In[ ]:=
```
tϕ[r_] = -2 a r e^(-k / r) (Sin[θ]) ^2 / ρ[r];
```

In[ ]:=
```
D[tϕ[r], r] // FullSimplify
```

Out[ ]= 0

In[ ]:=
```
ϕϕ[r_] = (r^2 + a^2 + (2 a^2 r e^(-k / r) (Sin[θ]) ^2 / ρ[r])) (Sin[θ]) ^2;
```

In[∘]:=
```
D[ϕϕ[r], r] // FullSimplify
```
Out[∘]= $2\,r$

PLOTS:

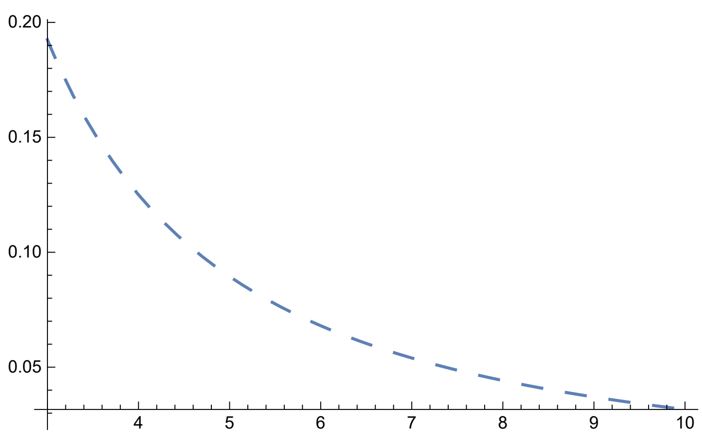
Knowing the expression for ω, plot “v vs r” with a=0, where:

In[∘]:=
```
Clear[k]
```
In[∘]:=
```
k = 0;
```
In[∘]:=
$$\omega[r\_] = \frac{-D[t\phi[r], r] + \sqrt{\left(D[t\phi[r], r]^2 - D[tt[r], r] \times D[\phi\phi[r], r]\right)}}{D[\phi\phi[r], r]}$$

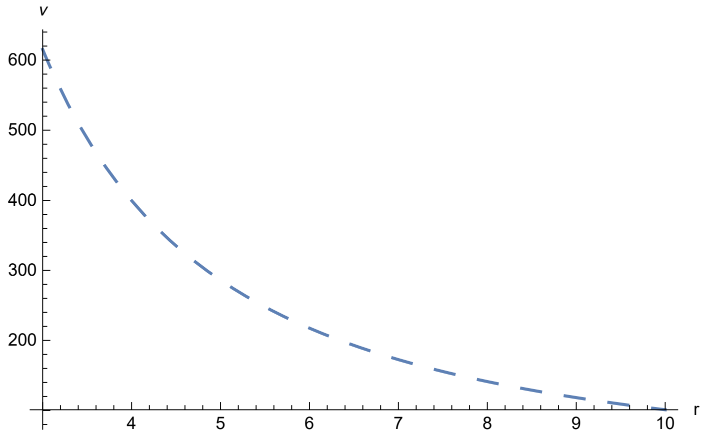
Out[∘]= $\left\{\left(\frac{1}{r}\right)^{3/2}\right\}$

For Figure 1 (Maselli 2017), Knowing the expression for epicyclic frequencies, plot them , a=0.7, where:

1. For epicyclic frequency of Azimuthal component :

In[∘]:=
```
Plot[ω[r], {r, 3, 10}, PlotStyle → {Dashing[Large]}]
```
Out[∘]=


In[ ]:=
```
p4 = Plot[ω[r] * (0.2035 * 10^4) / 2 π, {r, 3, 10},
  PlotStyle → {Dashing[Large]}, AxesLabel → {"r", "ν"}]
```

Out[ ]=


2. For epicyclic frequency of Radial component :

In[ ]:=
```
a = 0;
```

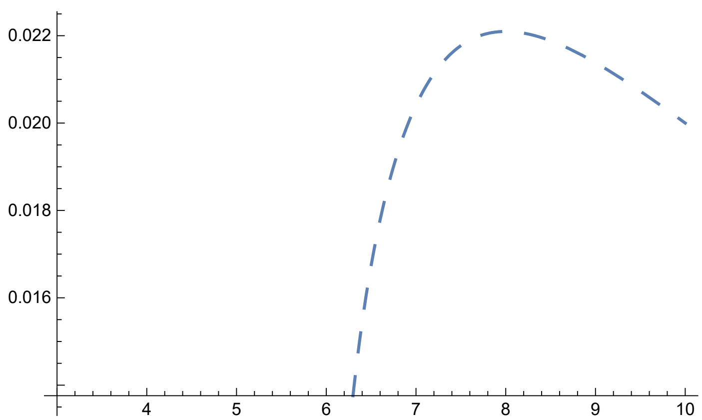
In[ ]:=
```
θ = π / 2;
```

In[ ]:=
```
ω1[r_] = ω[r] √(1 - 6 (r^-1) + 8 a (r^(-3/2)) - 3 a^2 (r^-2))
```

Out[ ]=
$$\left\{\sqrt{1-\frac{6}{r}}\left(\frac{1}{r}\right)^{3/2}\right\}$$

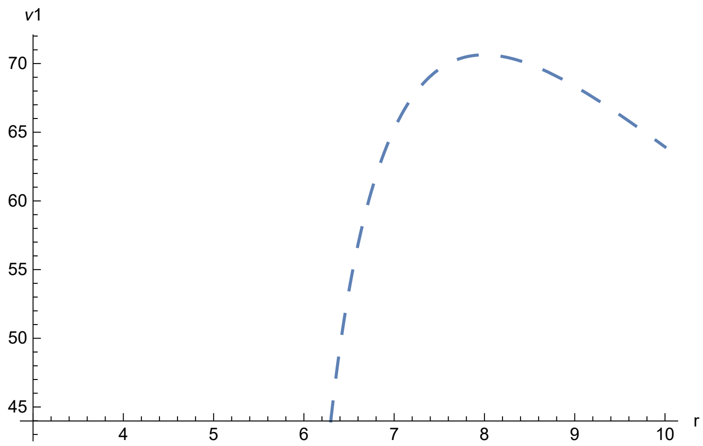
large output | show less | show more | show all | set size limit...

In[ ]:=
```
Plot[ω1[r], {r, 3, 10}, PlotStyle → {Dashing[Large]}]
```

Out[ ]=


In[ ]:=
```
p5 = Plot[ω1[r] * (0.2035 * 10^4) / 2 π, {r, 3, 10},
  PlotStyle → {Dashing[Large]}, AxesLabel → {"r", "ν1"}]
```

Out[ ]=



3. For epicyclic frequency of vertical component :

In[ ]:=
```
a = 0;
```

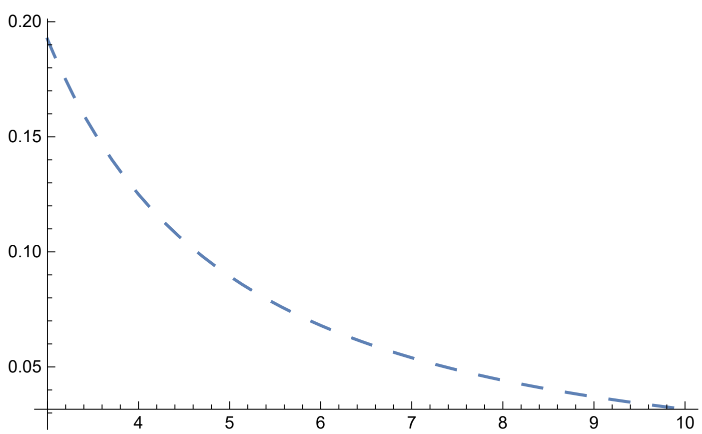
In[ ]:=
```
θ = π / 2;
```

In[ ]:=
```
ω2[r_] = ω[r] Sqrt[1 - 4 a (r^(-3/2)) + 3 a^2 (r^-2)]
```

Out[ ]= $\left\{\left(\frac{1}{r}\right)^{3/2}\right\}$

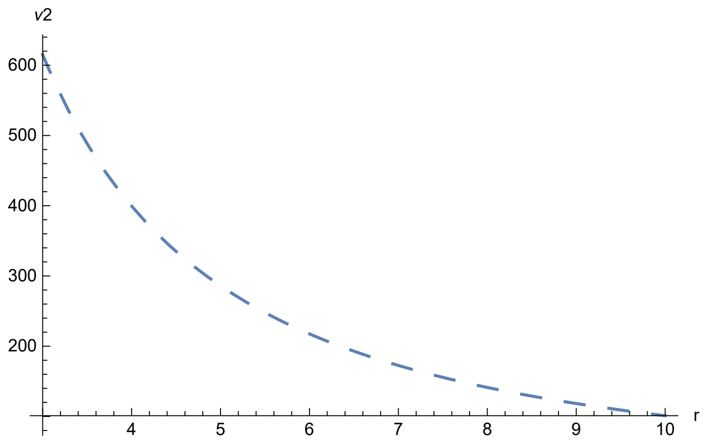
In[ ]:=
```
Plot[ω2[r], {r, 3, 10}, PlotStyle → {Dashing[Large]}]
```

Out[ ]=



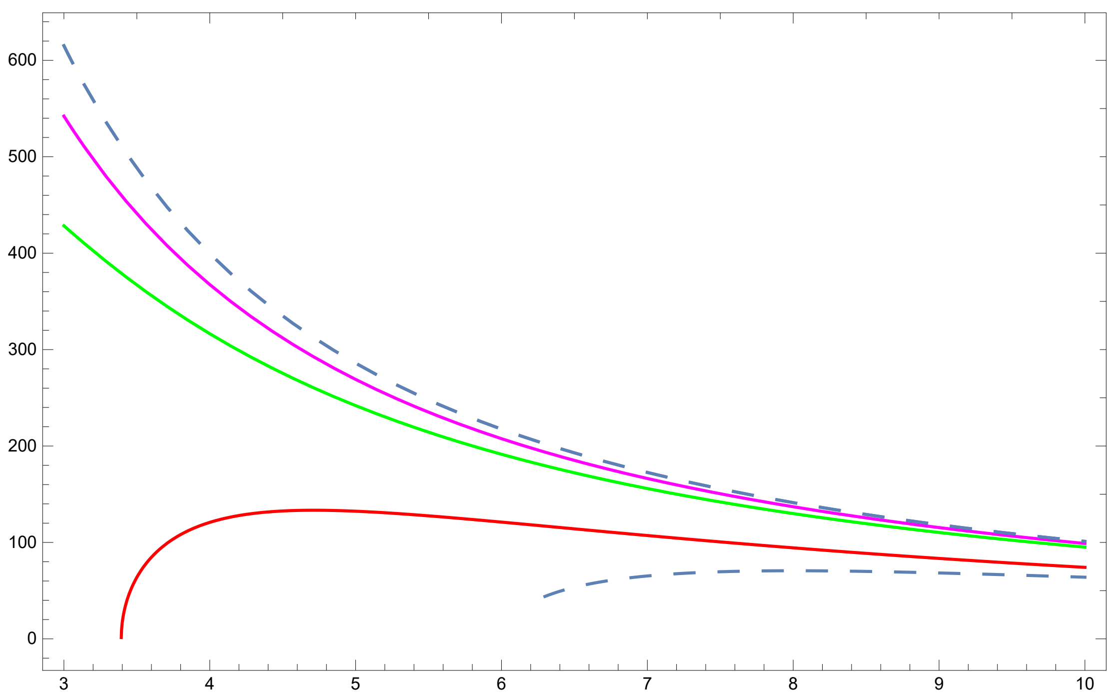
In[ ]:=
```
p6 = Plot[ω2[r] * (0.2035 * 10^4) / 2 π, {r, 3, 10},
  AxesLabel → {"r", "ν2"}, PlotStyle → {Dashing[Large]}]
```

Out[ ]=

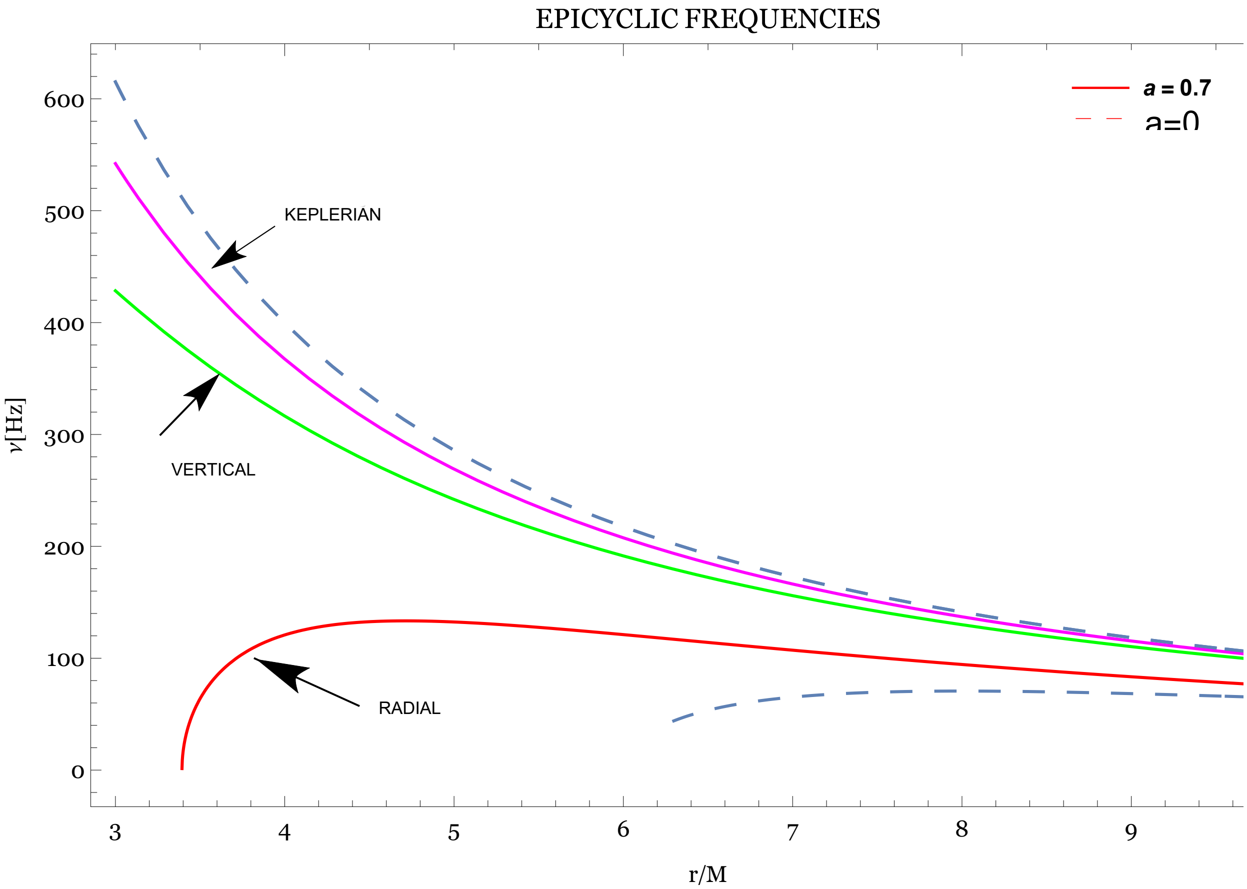


In[ ]:=
```
Show[p1, p2, p3, p4, p5, p6, PlotRange → All, Frame → True, AxesOrigin → Automatic]
```

Out[ ]=



In[ ]:=
```
Show[%88,
 FrameLabel → {{HoldForm[HoldForm["ν[Hz]"]], None}, {HoldForm[HoldForm["r/M"]], None}},
 PlotLabel → HoldForm[EPICYCLIC FREQUENCIES]]
```


EPICYCLIC FREQUENCIES
a = 0.7
a=0
KEPLERIAN
VERTICAL
RADIAL
ν[Hz]
r/M
600
500
400
300
200
100
0
3
4
5
6
7
8
9