

A connected graph G is Eulerian if and only if its edge set can be decomposed into Cycles.

Let graph G be decomposed into k cycles which are incident on vertex v. Then the degree of vertex will be $2k$. Similarly it can be for every vertex. Thus degree of each vertex in G will be even which satisfies the property of eulerian graph.

Also now let G graph be eulerian. Thus we know $\text{degree}(\text{vertex}) \geq 2$. Let us suppose we have cycles C_1, C_2, \dots, C_k whose in turn degree of each vertex will be 2 so as to satisfy the property of cycle. now $G - \text{edge}(c)$ will be a disconnected graph with each cycle C_1, C_2, \dots, C_k as shown above. By induction hypothesis we can say that each C_i is a disjoint union of cycles. Thus considering above two proves we can conclude that a connected graph G is eulerian iff its edge set can be decomposed into cycles.