Kindly try to solve all the problems your self with help of reading material on backpack and other places and following youtube links. Copying the solutions will not prepare you for quiz and end-semester exam.

Links for maximum likelihood estimate
https://www.youtube.com/watch?v=QBlERVSlFx4

cramer Rao lower bound
https://www.youtube.com/watch?v=i0JiSddCXMM

Bayesian inference
https://www.youtube.com/watch?v=5NMxiOGL39M

Maximum a posteriori
https://www.youtube.com/watch?v=EHqU9LE9tg8&spfreload=5

Links for Bayesian networks
https://www.youtube.com/watch?v=I3PHJZGot48

https://www.youtube.com/watch?v=zCWRTKnOYYg
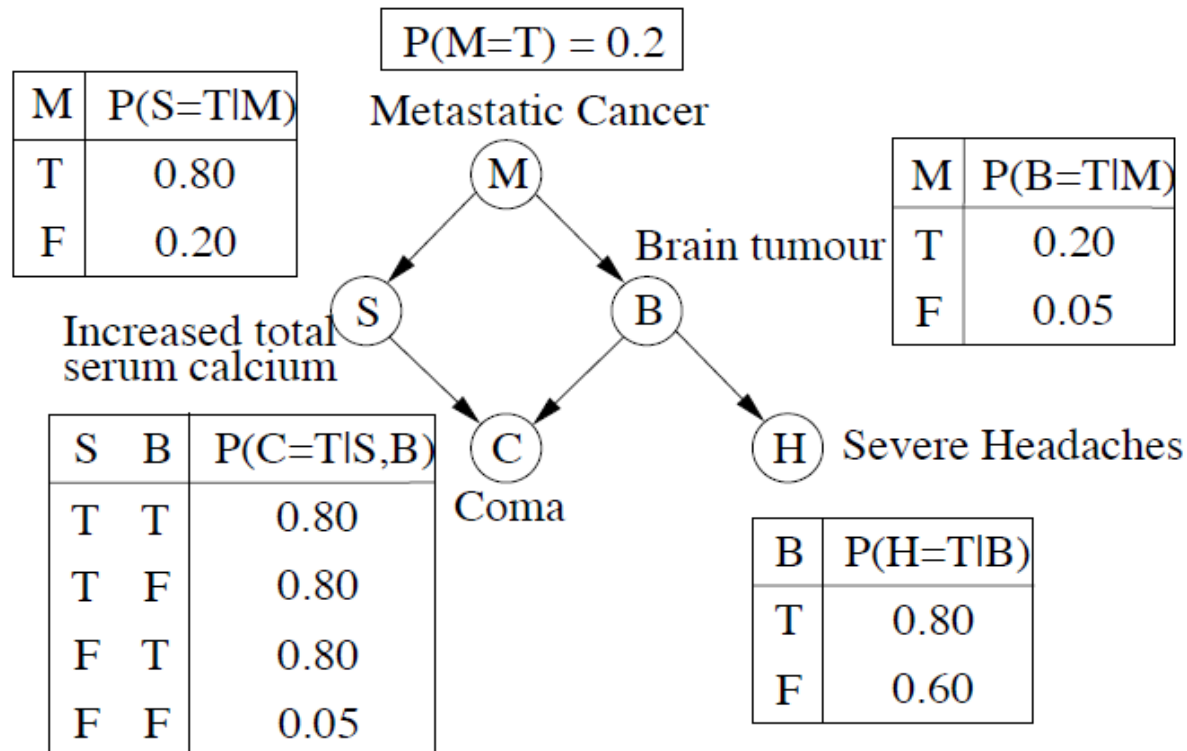https://www.youtube.com/watch?v=TuGDMj43ehw

1. Given below is the bayesian network for metastatic cancer and related effects. The conditional probability table is also given.
a) Find the probability that a person has metastatic cancer if he has severe Headache [i.e find P(M | H=T)].
b)What is probability for the person to have Brain tumour if he has severe Headache and Increased total serum calcium [i.e. find P(B | H=T, S=T)]

c) What is probability for the person to have Brain tumour if he has severe Headache and Increased total serum calcium and got coma [i.e. find P(B | H=T, S=T, C=T)]

P(M=T) = 0.2

Metastatic Cancer

M	P(S=T|M)
T	0.80
F	0.20

M	P(B=T|M)
T	0.20
F	0.05

Brain tumour

Increased total serum calcium

S	B	P(C=T|S,B)
T	T	0.80
T	F	0.80
F	T	0.80
F	F	0.05

Coma

Severe Headaches

B	P(H=T|B)
T	0.80
F	0.60

Question-2: Let $X1, X2, ..., Xn$ be an i.i.d. sample from an exponential distribution with the density function

$$f(x|\lambda) = \frac{1}{\lambda} e^{-x/\lambda}, \qquad , x > 0, \qquad \lambda > 0$$

a) Find the maximum likelihood estimate(MLE) of λ.
b) What is the sampling distribution of the MLE.
c) Show that the MLE is unbiased, and find its exact variance.
d) Is there any other unbiased estimate with smaller variance?

Question-3: Suppose that a random variable X has a binomial distribution for which the probability μ of success is unknown ($\mu > 0$). Find the Fisher information $I(\mu)$ in X.

Question-4: Suppose that a random variable X has a normal distribution for which the mean is 0 and the standard deviation σ is unknown ($\sigma > 0$). Find the Fisher information $I(\sigma)$ in X.

Question-5: Assume we have N samples, $x_1, \ldots, x_N$ independently drawn from a normal distribution with known variance σ^2 and unknown mean μ.

a) Now derive the MAP estimator for the mean μ. Assume that the prior distribution for the mean is itself a normal distribution with mean v and variance β.

b) Please comment on what happens to the MLE and MAP estimators as the number of samples N goes to infinity.

Question-6: Our data is n observations with one explanatory variable and one response variable. The model is that

$$y_i = \alpha + \beta x_i + \epsilon_i$$

(a) what is the MLE of β when ϵ_i are independent with x and has mean= 0 and normal distribution and (unknown) standard deviation is σ.

b) When ϵ_i follows beta distribution B(n, m) what is the MLE of β .