1. Philips designed new light bulbs. Philips company claimed that an average new light bulb lasts 300 days. Another company randomly selects 20 new bulbs from Philips for testing. The sampled bulbs last an average of 190 days, with a standard deviation of 50 days. If the Philip's claim were true,

a)what is the probability that 20 randomly selected bulbs would have an average life of no more than 190 days ?

b) what is the probability that 20 randomly selected bulbs have an average life of more than 400 days.

2. In the above problem if Philips company claimed that the average life of new bulbs is 200 days with standard deviation of 6 days.

A customer randomly select 20 bulbs. The standard deviation of the selected bulbs is 60 days. What would be the chi-square statistic represented by this test? What would be the P-value of claim made by Philip for standard deviation based on chi-square statistics.

3. For binomial distribution proove that mode is as such

$$\text{mode} = \begin{cases} \lfloor (n+1)\,p \rfloor & \text{if } (n+1)p \text{ is 0 or a noninteger}, \\ (n+1)\,p \;\text{ and }\; (n+1)\,p - 1 & \text{if } (n+1)p \in \{1, \ldots, n\}, \\ n & \text{if } (n+1)p = n+1. \end{cases}$$

where $\lfloor \cdot \rfloor$ is the floor function.

4. For beta distribution the probability distribution function is

$$f(x) = constant.x^{\alpha-1}(1-x)^{\beta-1}$$

Prove that the constant term comes out to be inverse of Beta function such that

$$constant = \frac{1}{B(\alpha, \beta)} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}$$

5. Prove that the mean for Beta distribution is

$$\mu = \frac{\alpha}{\alpha+\beta}$$

6. Estimate the integrals

$$\int_{0}^{\infty} e^{-t}\, dt \qquad\qquad \int_{-\infty}^{\infty} e^{-t^2}\, dt$$

7. Proove that gamma function is good approximation for factorial

$$n! = \Gamma(n+1) = \int_{0}^{\infty} t^{n} e^{-t}\, dt$$

8. From backpack Assignment-1 data choose any 5 out of 10 given data-set and predict their distribution function (Gaussian, Poisson, Binomial or Geometric) and the relevant parameters of the distribution function predicted.

9. For geometric distribution for x failures before 1 success

$Pr(x) = pq^x = p\ (1-p)^x$

Show that mean is q/p or (1-p)/p

And variance is $\dfrac{1-p}{p^2}$

10. proove stirling's approximation ($\log(n!) = n\log(n) - n + 1$)

Calculate log(200!) and log(20!) using program in R see what is the time difference when you do not assume stirling approximation and when you assume stirling's approximation.