

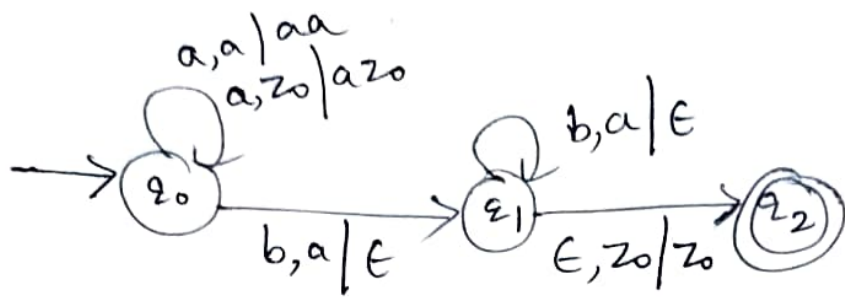
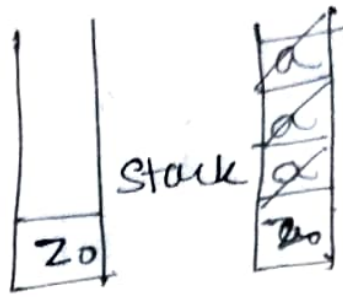
PDA examples

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1)

Give a PDA to accept the following language
 $L = \{a^n b^n \mid n \geq 1\}$

Ex: $n=3$,
 $\underline{a} \underline{a} \underline{a} \underline{b} \underline{b} \underline{b} \in$
 push pop



$$M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$$

$$Q = \{q_0, q_1, q_2\}$$

$$\Sigma = \{a, b\}$$

$$\Gamma = \{a, z_0\}$$

$$\delta: \begin{aligned} \delta(q_0, a, z_0) &= (q_0, a z_0) \\ \delta(q_0, a, a) &= (q_0, a a) \\ \delta(q_0, b, a) &= (q_1, \epsilon) \\ \delta(q_1, b, a) &= (q_1, \epsilon) \\ \delta(q_1, \epsilon, z_0) &= (q_2, z_0) \end{aligned}$$

$q_0 \in Q$ is the start state

$z_0 \in \Gamma$ is the initial symbol on the stack

$F = \{q_2\}$ — final state

Initial ID

15

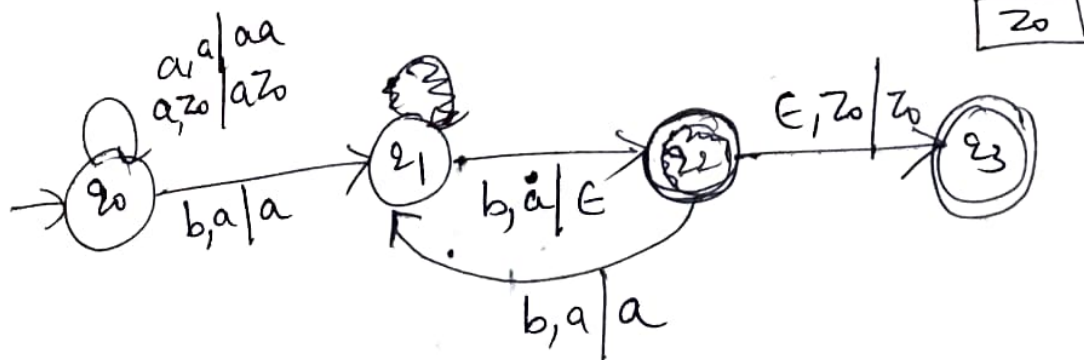
$(q_0, aabbbb, z_0) \vdash (q_0, aabbb, az_0)$
 $\vdash (q_0, abbb, aa z_0)$
 $\vdash (q_0, bbb, aaa z_0)$
 $\vdash (q_1, bb, aa z_0)$
 $\vdash (q_1, b, a z_0)$
 $\vdash (q_1, \epsilon, z_0)$
 $\vdash (q_2, \epsilon, z_0)$. accepted

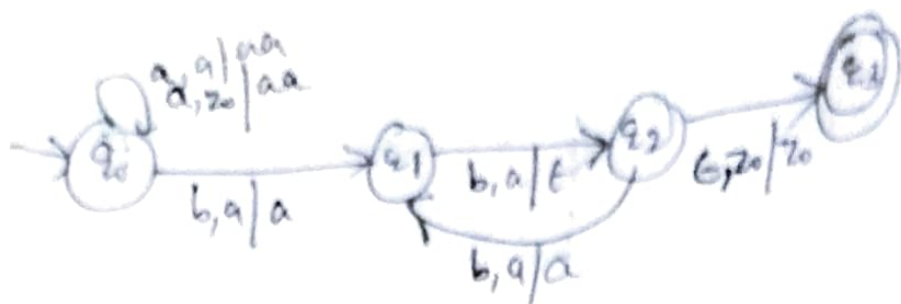
to reject the string

$(q_0, aabbbb, z_0) \vdash (q_0, abbbb, az_0)$
 $\vdash (q_0, bbb, aa z_0)$
 $\vdash (q_1, bb, a z_0)$
 $\vdash (q_1, b, z_0)$
 ~~\vdash~~ rejected

2] $L = \{a^n b^{2n} \mid n \geq 1\}$

Ex: aabbbb
 push skip pop skip pop





$$Q = \{q_0, q_1, q_2, q_3\}$$

$$\Sigma = \{a, b\}$$

$$\Gamma = \{z_0, a\}$$

$$\delta: \delta(q_0, a, z_0) = (q_0, az_0)$$

$$\delta(q_0, a, a) = (q_0, aa)$$

$$\delta(q_0, b, a) = (q_1, a)$$

$$\delta(q_1, b, a) = (q_2, \epsilon)$$

$$\delta(q_2, b, a) = (q_1, a)$$

$$\delta(q_1, b, a) = (q_2, \epsilon)$$

$$\delta(q_2, \epsilon, z_0) = (q_3, z_0)$$

q_0 : start state

z_0 : initial symbol on the stack

F : q_3 — final state

initial ID

$$(q_0, aabbbb, z_0) \vdash (q_0, abbbb, az_0)$$

$$\vdash (q_0, bbbb, aaz_0)$$

$$\vdash (q_1, bbb, aaz_0)$$

$$\vdash (q_2, bb, aaz_0)$$

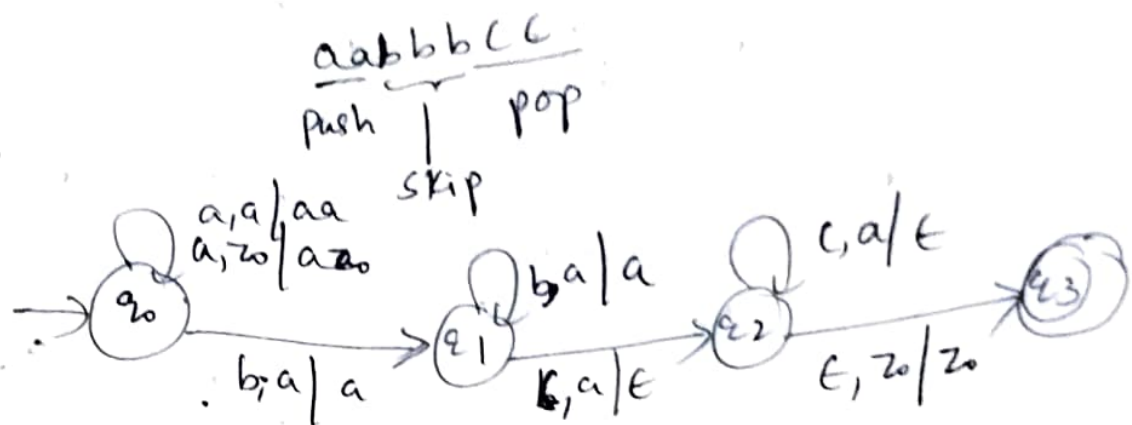
$$\vdash (q_1, b, aaz_0)$$

$$\vdash (q_2, \epsilon, z_0)$$

$$\vdash (q_3, \epsilon, z_0)$$

accepted.

$$3] L = \{a^n b^m c^n \mid n \geq 1, m \geq 1\}$$



$$Q = \{q_0, q_1, q_2, q_3\}$$

$$\Sigma = \{a, b\}$$

$$\Gamma = \{a, z_0\}$$

$$\delta: \delta(q_0, a, z_0) = (q_0, a z_0)$$

$$\delta(q_0, a, a) = (q_0, a a)$$

$$\delta(q_0, b, a) = (q_1, a)$$

$$\delta(q_1, b, a) = (q_1, a)$$

$$\delta(q_1, c, a) = (q_2, \epsilon)$$

$$\delta(q_2, c, a) = (q_2, \epsilon)$$

$$\delta(q_2, \epsilon, z_0) = (q_3, z_0)$$

$q_0 \rightarrow$ start state

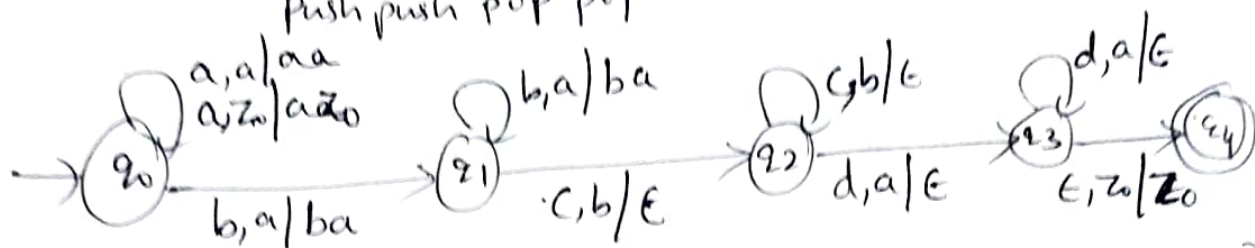
$z_0 \rightarrow$ initial symbol on the stack

$F \rightarrow q_3 \rightarrow$ final state.

$(q_0, aabbbcc, z_0) \vdash (q_0, abbbcc, az_0)$
 $\vdash (q_0, bbbcc, aa z_0)$
 $\vdash (q_1, bbcc, aa z_0)$
 $\vdash (q_1, bcc, aa z_0)$
 $\vdash (q_1, cc, aa z_0)$
 $\vdash (q_2, c, aa z_0)$
 $\vdash (q_2, \epsilon, z_0)$
 $\vdash (q_3, \epsilon, z_0)$
 accepted.

4) $L = \{a^n b^m c^m d^n \mid n \geq 1, m \geq 1\}$

$\underbrace{aa}_{\text{push push}} \underbrace{bbcc}_{\text{pop pop}} dd \quad n=2, m=3$



$Q = \{q_0, q_1, q_2, q_3, q_4\} \quad M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

$\Sigma = \{a, b, c, d\}$

$\Gamma = \{a, b, z_0\}$

$\delta:$
 $\delta(q_0, z_0) = (q_0, az_0)$
 $\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, a) = (q_1, ba)$

$\delta(q_1, b, a) = (q_1, ba)$

$\delta(q_1, c, b) = (q_2, \epsilon)$

$\delta(q_2, c, b) = (q_2, \epsilon)$

$\delta(q_2, d, a) = (q_3, \epsilon)$

$\delta(q_3, d, a) = (q_3, \epsilon)$

$\delta(q_3, \epsilon, z_0) = (q_4, z_0)$

$q_0 \rightarrow$ start state

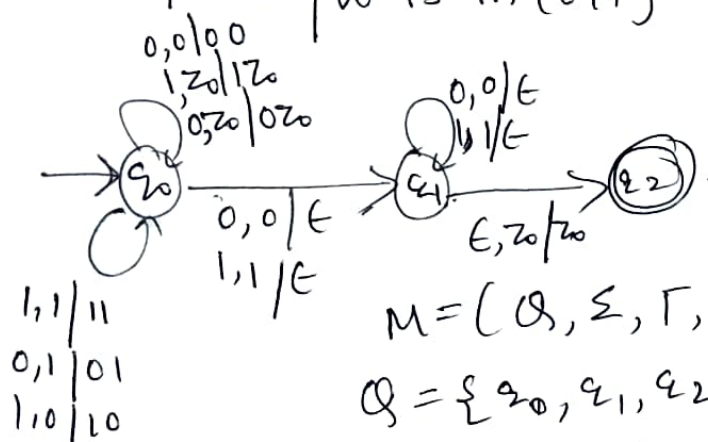
$z_0 \rightarrow$ initial symbol on the stack

$F \rightarrow q_4$ - final state

Initial z_0

$(q_0, aabbb(ccdd, z_0) \vdash (q_0, abbb(ccdd, az_0)$
 $\vdash (q_0, bbb(ccdd, aaz_0)$
 $\vdash (q_1, bbb(ccdd, baa z_0)$
 $\vdash (q_1, bccdd, bb aa z_0)$
 $\vdash (q_1, cccdd, bbb aa z_0)$
 $\vdash (q_2, cdd, bbb aa z_0)$
 $\vdash (q_2, dd, baa z_0)$
 $\vdash (q_3, d, aa z_0)$
 $\vdash (q_3, \epsilon, az_0)$
 $\vdash (q_4, \epsilon, z_0)$ accepted

5) $L = \{ w w^R \mid w \text{ is in } (0+1)^+ \}$



$$M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$$

$$Q = \{q_0, q_1, q_2\}$$

$$\Sigma = \{0, 1\}$$

$$\Gamma = \{0, 1, z_0\}$$

$$\delta: \delta(q_0, 0, z_0) = (q_0, 0z_0)$$

$$\delta(q_0, 1, z_0) = (q_0, 1z_0)$$

$$\delta(q_0, 0, 0) = (q_0, 00)$$

$$\delta(q_0, 1, 1) = (q_0, 11)$$

$$\delta(q_0, 0, 1) = (q_0, 01)$$

$$\delta(q_0, 1, 0) = (q_0, 10)$$

$$\delta(q_0, 1, 1) = (q_1, \epsilon)$$

$$\delta(q_0, 0, 0) = (q_1, \epsilon)$$

$$\delta(q_1, 1, 1) = (q_1, \epsilon)$$

$$\delta(q_1, 0, 0) = (q_1, \epsilon)$$

$$\delta(q_1, \epsilon, z_0) = (q_2, z_0) \quad q_0 - \text{start state}$$

$z_0 = \text{initial symbol on the stack}$

$$F = \{q_2\}$$

initial ID

$$\delta(q_0, abacaba, z_0) = (q_0, buabaa, az_0)$$

$$= (q_0, aabaa, baz_0)$$

$$= (q_0, aba, abaz_0)$$

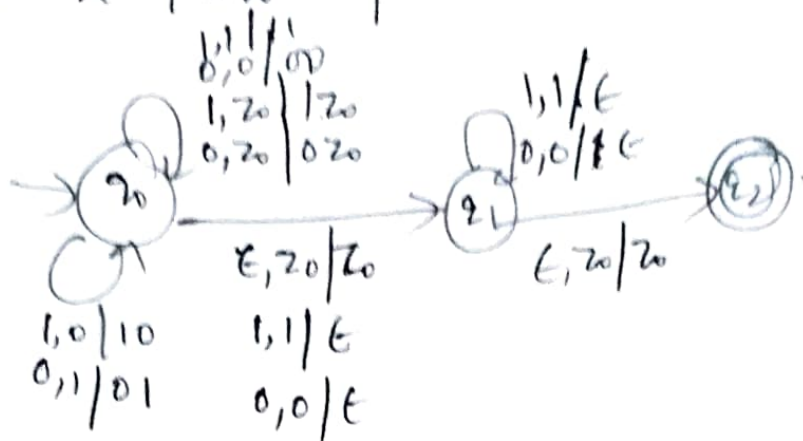
$$= (q_1, ba, baz_0)$$

$$= (q_1, a, az_0)$$

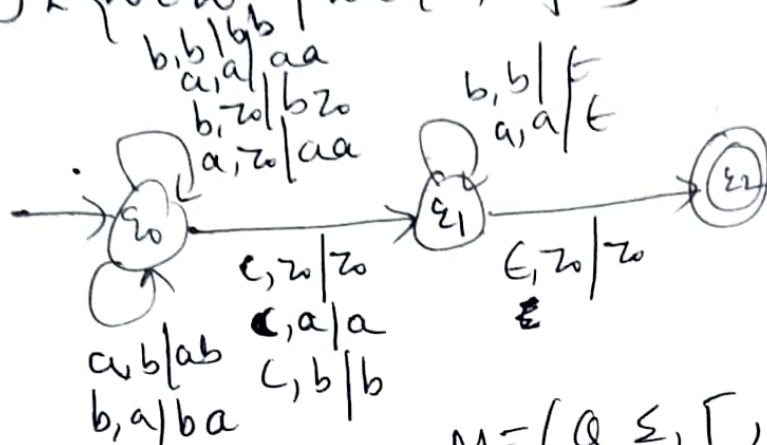
$$= (q_1, \epsilon, z_0)$$

$$= (q_2, z_0, z_0) \text{ accepted.}$$

$$6) L = \{ w w^R \mid w \text{ is in } (0+1)^* \}$$



$$7) L = \{ w c w^R \mid w \in \{a,b\}^* \}$$



$$M = (Q, \Sigma, \Gamma, \delta, q_0, 20, F)$$

$$Q = \{ q_0, q_1, q_2 \}$$

$$\Sigma = \{ a, b \}$$

$$\Gamma = \{ a, b, 20 \}$$

$$\delta : \begin{aligned} \delta(q_0, a, 20) &= (q_0, a20) \\ \delta(q_0, b, 20) &= (q_0, b20) \\ \delta(q_0, a, a) &= (q_0, aa) \\ \delta(q_0, b, b) &= (q_0, bb) \\ \delta(q_0, a, b) &= (q_0, ab) \\ \delta(q_0, b, a) &= (q_0, ba) \\ \delta(q_0, \epsilon, a) &= (q_1, a) \\ \delta(q_0, \epsilon, b) &= (q_1, b) \\ \delta(q_0, \epsilon, 20) &= (q_1, 20) \end{aligned}$$

$$\begin{aligned} \delta(q_1, a, a) &= (q_1, \epsilon) \\ \delta(q_1, b, b) &= (q_1, \epsilon) \\ \delta(q_1, \epsilon, 20) &= (q_2, 20) \end{aligned}$$

q_0 - start state
 20 - initial symbol on stack

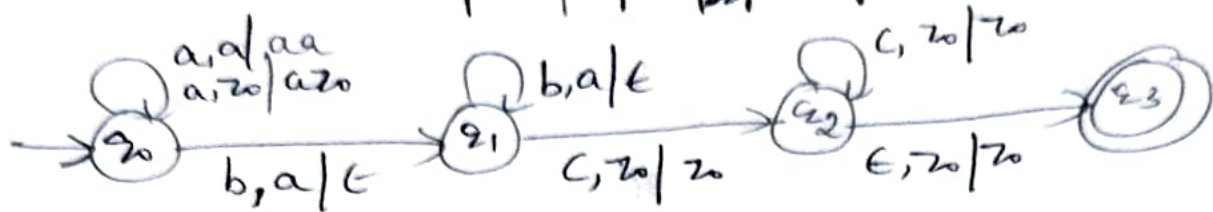
F - q_2 final state

9)

$$L = \{ a^n b^m c^m \mid n, m \geq 1 \}$$

aaabccc $n=2, m=3$

push pop ~~pop~~ skip

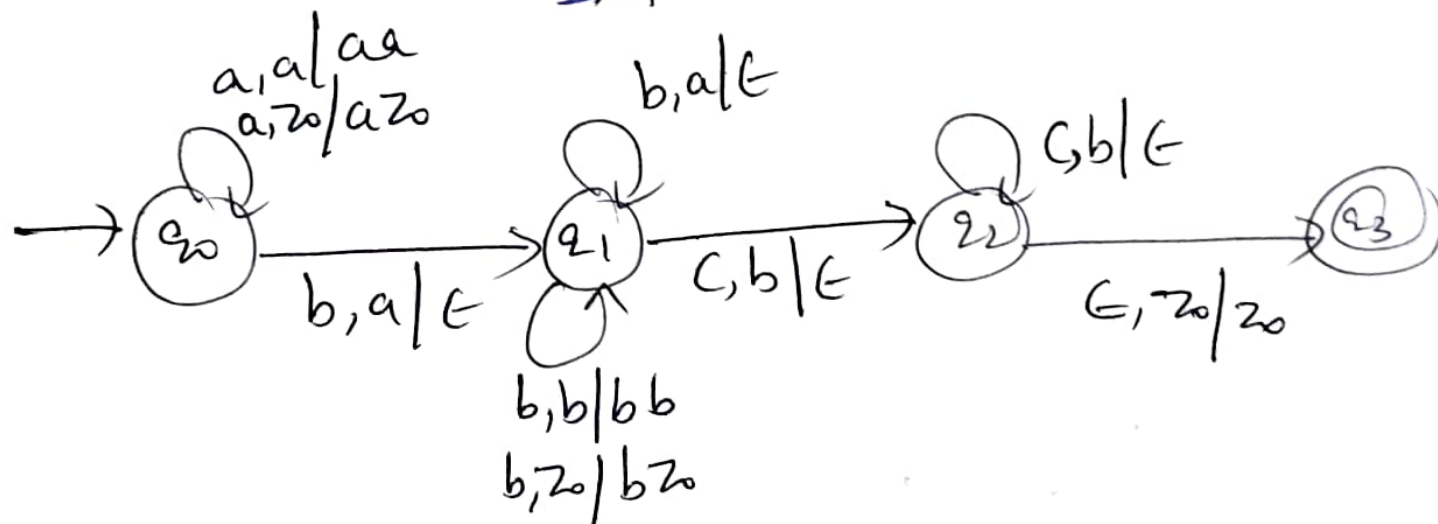
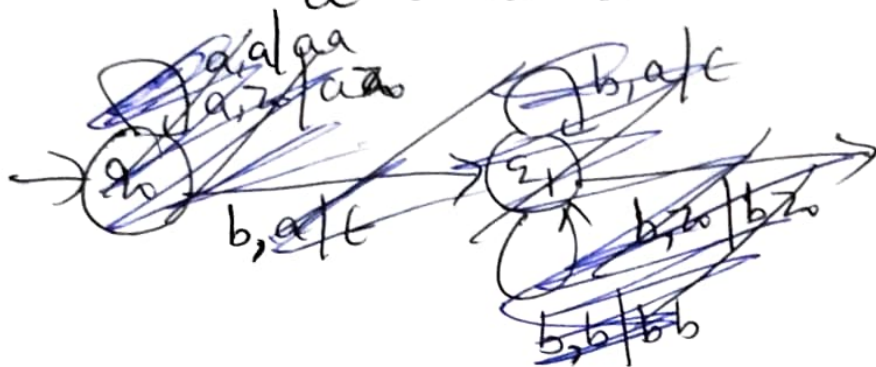


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$$L = \{a^n b^{m+n} c^m \mid n, m \geq 1\}$$

$$a^n b^m b^n c^m$$

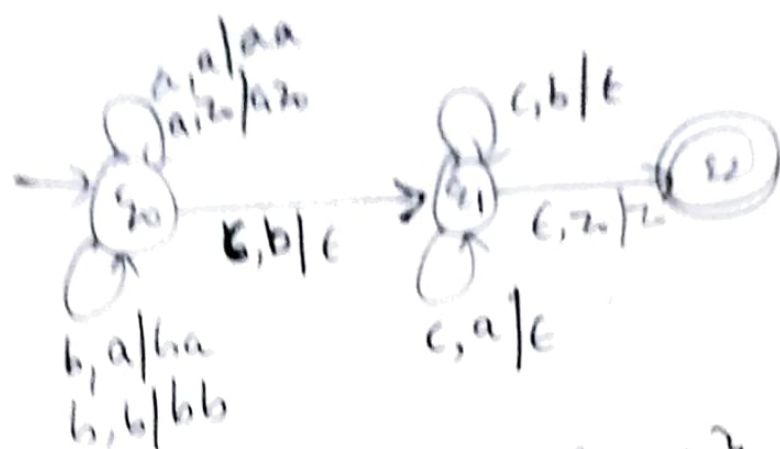
$$a^n b^n b^m c^m$$



$$10) L = \{a^n b^m c^{n+m} \mid n, m \geq 1\}$$

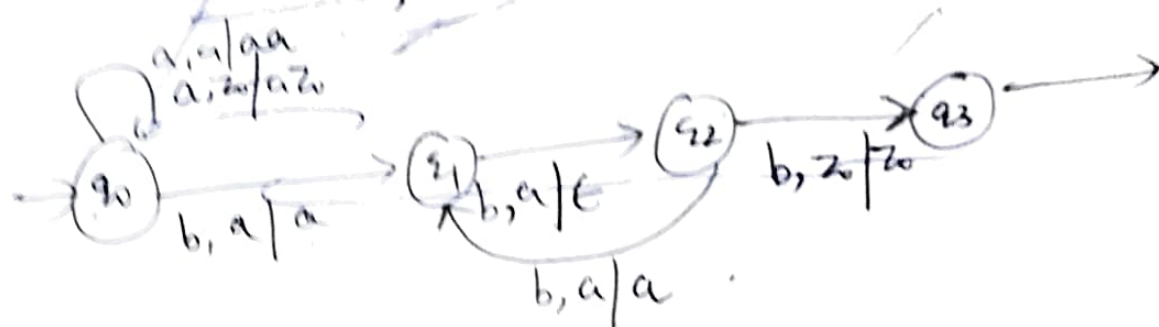
$$a^n b^m c^n c^m$$

$$a^n b^m c^m c^n$$



$$10) L = \{a^n b^{n+1} \mid \text{where } n \geq 1\}$$

$$n=2, aabbbbbb$$



$$11) L = \{a^i b^j c^k \mid i+j=k, i, j \geq 0\}$$

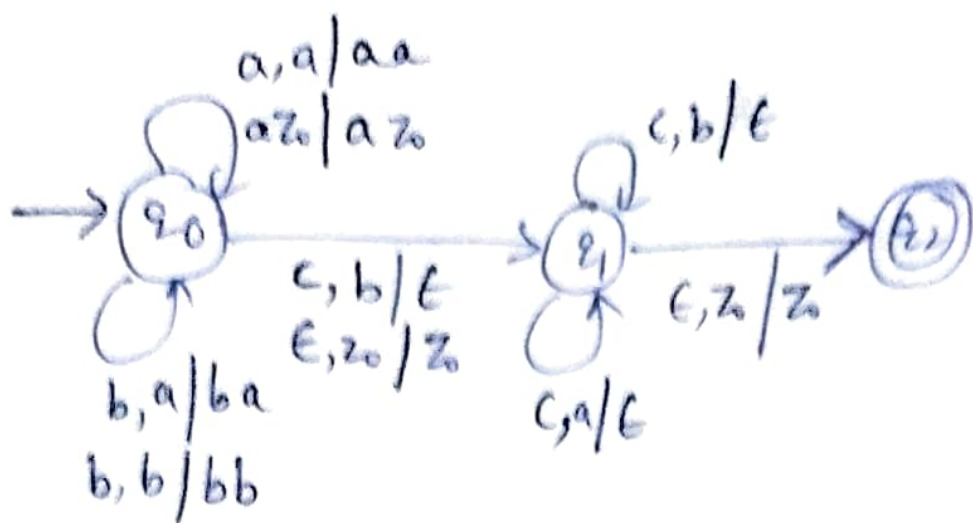
$$a^i b^j c^{i+j} \quad (\because k=i+j)$$

$$a^i b^j c^i c^j$$

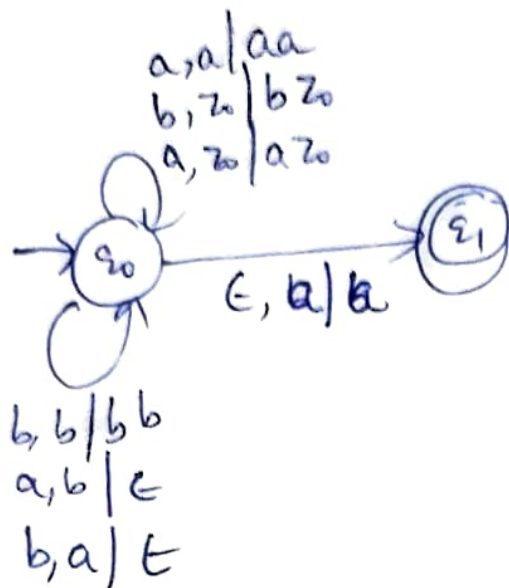
$$a^i b^j c^j c^i$$

$\uparrow \quad \uparrow \quad \nwarrow \quad \nwarrow$
 push push pop pop

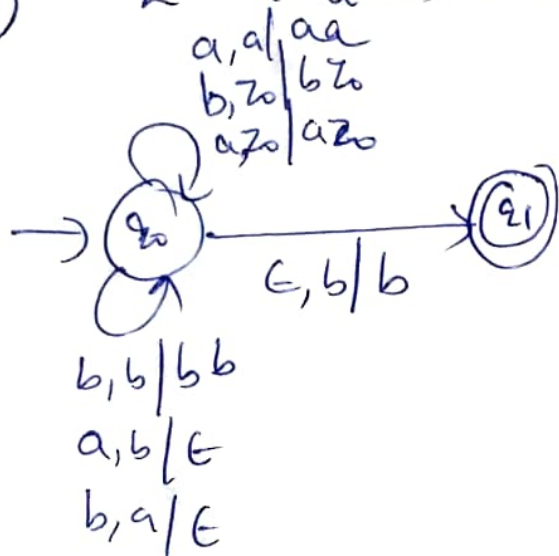
$$L = \{\epsilon, abcc, aabccc, \dots\}$$



12) $L = \{w \mid n_a(w) > n_b(w)\}$



13) $L = \{w \mid n_a(w) < n_b(w)\}$



$$13) L = \{ n_a(w) + n_b(w) \}$$

