ALGORITHM FOR SOLVING T-PERIOD EGSS. Start $T = 2$.

1. Guess $q(\theta_0)$ and Y
2. Solve for effort a_1 and a_2 using

$$a_t = \left[\frac{z_t a_t}{\psi_1 (Y - \frac{\kappa}{q(\theta)})} - \frac{\psi_t}{\epsilon_F} (h'(a_t)\sigma_\eta)^2 \right]^{\frac{\varepsilon}{1+\varepsilon}}$$

3. Check whether Y is consistent with the a_t's guessed, given a process for z's
4. Update Y to implied $\hat{Y}$

$$\hat{Y} = \mathbb{E}_0 \left[\sum_{t=0}^{T} (\beta(1-s))^t z_t a_t \right]$$

   (a) Discretize z on a number of values. Assign probability weights to them. Calculate $a_t(z)$. Take expectation numerically.

5. Repeat 2-4 until convergence of $Y(\theta_0)$
6. Solve for w_0 using (zero profit condition)

$$w_0 = \psi_1 \left(Y(\theta_0) - \frac{\kappa}{q(\theta_0)} \right)$$

7. Solve for path of $\mathbb{E}[\log w_t] : \ t = \{1, 2, \ldots, T\}$ using difference equation in EGSS_notes. (Can do for savings or no savings)

   (a) Again, do the discretizing of z trick from 4 to get the $h'(a_t)$ piece we need.

8. Then use this to calculate to value of participation constraint assuming log utility in consumption (so $\mathbb{E}[\log w_t]$ is exactly what we need).
9. If implied LHS of participation constraint (expected utility from contract) $> \omega$ then lower guess of θ_0 to $\hat{\theta}$ (check that this seems to be converging; make sure we don't need to raise guess of θ_0).

   (a) Dichotomy/bisection algorithm. E.g. start with range $\theta \in [\theta_L, \theta_H]$. For starters, let's take $\theta_L = 0$, $\theta_H = 1$. Start with guess $\hat{\theta}_0 = \frac{1}{2}(\theta_L + \theta_H) = 0.5$ (midpoint). If algo above reveals $\hat{\theta}_0 < \theta^*$, then update $\theta_L^1 \to 0.5$ and then $\hat{\theta}_1 = 0.5(\theta_L^1 + \theta_H)$

10. Iterate steps 2-??? till convergence.

UTILITY FUNCTIONS

$$u(c) - h(a) = \log c - \frac{a^{1+1/\epsilon}}{1 + 1/\epsilon}$$

setting $\epsilon = 0.5$ following Doligalski, Ndiaye and Werquin (2022)

0.0.1 With savings

$$\max_c \sum \beta^s(\log(c_s) - h(a_s))$$

BC and natural borrowing limit

I think in this income fluctuation problem agents consume a fixed fraction of NPV income

2 changes:

zero profit condition in step 7 and 4

- consumption (income fluctuation