Bargained vs. Incentive Wage Flexibility—Redux

February 16, 2023

- Define the expected present value of wages as

$$\mathcal{W}^* \equiv E_{0,a^*} \sum_{t=0}^{\infty} (\beta(1-s))^t w^* \left(\eta^t, z^t\right)$$

  and

$$\mathcal{E}(z_0) = \sum_{t=0}^{\infty} (\beta(1-s))^t \left[\log w^* \left(\eta^t, z^t\right) - h\left(a^* \left(\eta^t, z^t\right)\right) + \beta s \mathcal{E}(z_{t+1})\right]$$

- Then we can write

$$\mathcal{W}^* = \mathcal{W}(\mathcal{E}(z_0), z_0)$$

  where $\mathcal{W}$ is obtained substituting a^* into $\mathcal{E}(z_0)$ and inverting

  - **Note:** we don't need to know the analytical closed form, we are just explaining how one should think about this term

- Then by the chain rule

$$\begin{aligned} \frac{d\mathcal{W}(\mathcal{E}(z_0), z_0)}{dz_0} &= \frac{\partial \mathcal{W}(\mathcal{E}(z'), z_0)}{\partial z'}|_{z'=z_0} \quad \rightarrow \text{bargained wage flexibility} \\ &+ \frac{\partial \mathcal{W}(\mathcal{E}(z_0), z')}{\partial z'}|_{z'=z_0} \quad \rightarrow \text{incentive wage flexibility} \end{aligned}$$

- These terms are (rewritten) exactly the same bargained/incentive wage flexibility terms from our theorems.

  - **Probably we need to check this**

- We can compute each term numerically:

1. **Bargained wage flexibility** is

$$\frac{\partial \mathcal{W}(\mathcal{E}(z'), z_0)}{\partial z'}|_{z'=z_0} = \frac{\partial \mathcal{W}(\mathcal{E}, z_0)}{\partial \mathcal{E}}|_{\mathcal{E}=\mathcal{E}(z_0)} \frac{\partial \mathcal{E}(z_0)}{\partial z_0}.$$

   The first term on the RHS comes from taking the steady state of the model, and numerically perturbing by $\mathcal{E}$. That is: (i) take the steady state of the model, and calculate $\mathcal{W}^*$; (ii) change the value of $\mathcal{E}$ by a small amount (by varying γ) and recalculate $\mathcal{W}^*$; divide the change in $\mathcal{W}^*$ by the change in $\mathcal{E}$. The second term is obtained from $\mathcal{E}(z_0) = \gamma z^{\chi}$ by hand.

   Here we can directly see how bargained wage flexibility depends on χ.

2. **Incentive wage flexibility is**

$$\frac{\partial \mathcal{W}\left(\mathcal{E}\left(z_0\right), z'\right)}{\partial z'}|_{z'=z_0}.$$

To numerically calculate this object, we need to fix $\mathcal{E}$ at its steady state value and then perturb z by a small amount. In practice, this means setting $\chi = 0$, so $\mathcal{E}\left(z_0\right) = \gamma$ which is the correct steady state value. Then we can perturb by z_0.