CSCE 478/878 Recitation 7 Handout

Frequentist & Bayesian Learning: Binomial Distribution

February 26, 2019

Note: This is a zero-credit recitation. No submission is required. You may use the recitation time to work on these problems.

First, please carefully go through the jupyter notebook on "Frequentist and Bayesian Learning for Binomial Distribution":

https://github.com/rhasanbd/Frequentist-and-Bayesian-Learning-for-Binomial-Distribution/blob/master/Frequentist%20And%20Bayesian%20Learning-Binomial%20Distribution.ipynb

We will use a simple coin toss scenario to model binomial distribution. Imagine that we performed multiple trials of coin toss and recorded the observations (number of heads and tails). Assume that these observations are generated by some unknown binomial distribution that is governed by the parameter θ. Here θ represents the probability of observing heads. You will estimate the value of θ by using both the frequentist and Bayesian learning.

You will run 6 experiments and will compute the following parameters of the frequentist and Bayesian learning.

Frequentist Learning:

- θ_{MLE}

Bayesian Learning:
Use the Beta distribution as the conjugate prior for Bayesian learning.
Then, compute the mode (maximum a posteriori or MAP) and mean of the posterior distribution:

- θ_{MAP}
- $\bar{\theta}$

Also compute that standard deviation σ ("error bar") of the posterior distribution.

Report your results in the following table for all experiments.

	Number of trials: n Number of heads: α_H	θ_{MLE}	θ_{MAP}	$\bar{\theta}$	σ
Experiment 1	n = 10 α_H = 10				
Experiment 2	n = 10 α_H = 10				
Experiment 3	n = 10 α_H = 10				
Experiment 4	n = 10 α_H = 10				

Experiment 5	n = 10 α_H = 10				
Experiment 6	n = 10 α_H = 10				

Experiment 1:

Number of trials: 10
Number of heads: 10

Prior: uniform (see the lecture slide to find the shape and rate parameter values for a uniform distribution)

Experiment 2:

Number of trials: 10
Number of heads: 10

Prior: assume that the coin is biased towards tails. Choose the parameters a and b of the Beta distribution suitably so that you get a unimodal prior distribution that peaks towards the left corner.

Experiment 3:

Number of trials: 100
Number of heads: 85

Prior: assume that the coin is biased towards tails. Choose the parameters a and b of the Beta distribution suitably so that you get a unimodal prior distribution that peaks towards the left corner.

Experiment 4:

Number of trials: 100
Number of heads: 85

Prior: assume that the coin is **strongly** biased towards tails. Choose the parameters a and b of the Beta distribution suitably so that you get a unimodal prior distribution that peaks **sharply** towards the left corner. Hint: you may increase the rate parameters a and b by keeping a < b.

Experiment 5:

Number of trials: 1000

Number of heads: 850

Prior: assume that the coin is biased towards tails. Choose the parameters a and b of the Beta distribution suitably so that you get a unimodal prior distribution that peaks towards the left corner.

Experiment 6:

Number of trials: 1000000
Number of heads: 850000

Prior: assume that the coin is biased towards tails. Choose the parameters a and b of the Beta distribution suitably so that you get a unimodal prior distribution that peaks towards the left corner.

Answer the following questions.

1. What is the main limitation of the frequentist learning? When does this limitation arise? **Hint**: this limitation concerns the number of samples we have for learning.
2. How do we get an optimal estimate of θ by using the frequentist learning? **Hint**: see the two assumptions for the frequentist learning from the lecture slides.
3. How does the Bayesian learning overcome the limitation of the frequentist learning?
4. How do we reduce uncertainty of the posterior distribution? **Hint**: See the formula for the posterior standard deviation.