Worksheet 5

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1. We need to prove that

$$p(a, b, c|d, e) = p(a, b|c, d, e)p(c|d, e) \tag{1}$$

According to conditional probability, We know that

$$p(a|b) = \frac{p(a, b)}{p(b)} \tag{2}$$

Therefore,

$$p(a, b, c|d, e) = \frac{p(a, b, c, d, e)}{p(d, e)} \tag{3}$$

$$= \frac{p(a, b|c, d, e).p(c, d, e)}{p(d, e)} \tag{4}$$

From (2),

$$p(c, d, e) = p(c|d, e)p(d, e) \tag{5}$$

Substituting it (4), we get :

$$\frac{p(a, b|c, d, e).p(c|d, e).p(d, e)}{p(d, e)}$$

$$= p(a, b|c, d, e).p(c|d, e)$$

Hence, proved

2. We need to prove that

$$p(a|b, c) = \frac{p(b|a, c)p(a|c)}{p(b|c)}$$

According to conditional probability, We know that

$$p(a|b) = \frac{p(a, b)}{p(b)}$$

Therefore,

$$p(a|b, c) = \frac{p(a, b, c)}{p(b, c)}$$

$$= \frac{p(b|a,c).p(a,c)}{p(b,c)}$$

$$= \frac{p(b|a,c).p(a|c).p(c)}{p(b|c).p(c)}$$

$$= \frac{p(b|a,c).p(a|c)}{p(b|c)}$$

Hence, Proved

3.

$$p(cavity|\neg catch) = \frac{p(cavity \wedge \neg catch)}{p(\neg catch)}$$

$$= \frac{0.012}{(0.012 + 0.064)} = \frac{0.012}{0.076} = 0.1578$$

4. The given model is a head-to-tail connection. It means that Given X, Y and Z are independent of each other.i.e.,

$$p(Y, Z|X) = p(Y|X).p(Z|X)$$

We need to prove that

$$p(Y|X, Z) = p(Y|X)$$

$$p(Y, Z|X) = \frac{p(X, Y, Z)}{p(X)} = \frac{p(Y|Z, X).p(Z, X)}{p(X)}$$

$$=> \frac{p(Y|Z, X).p(Z, X)}{p(X)} = p(Y|X).p(Z|X)$$

$$=> \frac{p(Y|Z, X).p(Z|X).p(X)}{p(X)} = p(Y|X).p(Z|X)$$

$$=> p(Y|Z, X) = p(Y|X)$$

5.

$$p(W|R) = \sum_s p(W, S|R)$$

$$p(W, S|R) + P(W, \neg S|R) = \frac{p(W, S, R)}{p(R)} + \frac{p(w, \neg S, R)}{p(R)} = 0.19 + 0.72 = 0.91$$

6.

$$(a).p(W|\neg C, R) = \sum_s p(W, S|\neg C, R) = p(W, S|\neg C, R) + p(W, \neg S|\neg C, R)$$

$$= \frac{p(W, S, \neg C, R)}{p(\neg C, R)} + \frac{p(W, \neg S, \neg C, R)}{p(\neg C, R)}$$

$$p(W, S, \neg C, R) = p(\neg C).p(S|\neg C).p(R|\neg C).p(W|S, R)$$
$$= 0.5 * 0.5 * 0.1 * 0.95 = 0.02375$$
$$p(W, \neg S, \neg C, R) = p(\neg C).p(\neg S|\neg C).p(R|\neg C).p(W|\neg S, R)$$
$$= 0.5 * (1 - 0.5) * 0.1 * 0.9 = 0.0225$$
$$p(\neg C, R) = p(R|\neg C).p(\neg C) = 0.1 * 0.5 = 0.05$$
$$Result = \frac{0.04625}{0.05} = 0.925$$
$$(b).p(W|C, R) = \sum_s p(W, S|C, R) = p(W, S|C, R) + p(W, \neg S|C, R)$$
$$= \frac{p(W, S, C, R)}{p(C, R)} + \frac{p(W, \neg S, C, R)}{p(C, R)}$$
$$p(W, S, C, R) = p(C).p(S|C).p(R|C).p(W|S, R)$$
$$= 0.5 * 0.1 * 0.8 * 0.95 = 0.038$$
$$p(W, \neg S, C, R) = p(C).p(\neg S|C).p(R|C).p(W|\neg S, R)$$
$$= 0.5 * (1 - 0.1) * 0.8 * 0.9 = 0.324$$
$$p(C, R) = p(R|C).p(C) = 0.8 * 0.5 = 0.4$$
$$Result = \frac{0.38 + 0.324}{0.4} = 0.905$$

7. (a) P and Q have a head-to-head connection with R, and R is a descendant of S. hence, P and Q are not conditionally independent of S.

   (b) If we consider the path R→ S→ V, we can see that V is a descendant of R. P and Q have a head-to-head connection with R,and Since V is the descendant, P and Q are not conditionally independent, given V.

   (c) Yes, if we consider the path P→ R→ S→ V→ W, we can see that there is a head-to-tail connection at R. Hence, P and W are conditionally independent, given R.

   (d) If we consider P→R→S and P→R→T, there exists a head-to-tail connection from P to s and P to T.Hence, S and T are conditionally independent, given P.

   (e) U is dependant on V when S is not given, and W is dependant on V. Hence, U and W are not conditionally independent, given V.