CSCE 478/878 Recitation 6 Handout
Hidden Markov Model
February 19, 2019

Note: This is a zero-credit recitation. No submission is required. You may use the recitation time to work on these problems.

Rain Tracker Robot

Imagine a scenario that you are an unmindful scientist and work in a basement lab. You have two lab assistants: a human being and a robot.

To get to the lab you need to take long stairs. You use the stairs twice a day: first time when you come to the lab in the early morning, and the second (and last) time when you finish your work and go back to home.

Somedays it rains after you reach the lab. You have no way to know if it rains from your basement lab. You have an umbrella in the lab but you don't want to carry it everyday unless you are sure that it's raining outside.

Your human lab assistant comes to the lab around noon every day. She brings an umbrella with her if either it rains or if she anticipates that it might rain. You don't talk to her about anything other than your research. So, you cannot ask her whether it's raining outside so that you could carry umbrella on your way to home.

To make your life easier (i.e., not to get soaked) in case it rains you want your robot assistant to remind you to carry an umbrella. But the robot has to be sure about the rain, otherwise you will have to carry an umbrella unnecessarily, which you hate!

Since the robot cannot see directly whether it rains outside or not (because it never leaves the basement lab and are not in good terms with your human assistant), it has to assess/predict the probability of rain (or not rain) based on the actions of your human assistant (whether she brings an umbrella or not). In other words, the robot needs to evaluate/predict a hidden state (rain/not rain) based on some observations (umbrella or no umbrella).

You need to create a Hidden Markov Model (HMM) for your robot to aid its decision making. You have developed the following specification for the HMM.

A =

	Rainy	Dry
Rainy	0.7	0.3
Dry	0.3	0.7

B =

	Umbrella	No Umbrella
Rainy	0.9	0.1
Dry	0.2	0.8

PI =

Rainy	Dry
0.5	0.5

In particular, given the above HMM specification and an observation sequence, you need to compute the following. Show detail calculation.

1. Unnormalized forward variable matrix
2. Normalized forward variable matrix
3. Backward variable matrix
4. Evaluate of the observation sequence (use the normalized forward variable matrix and compute the log probability of the evaluation)
5. Compute the smoothed probability matrix by using the normalized forward variables and the backward variables
6. Compare the smoothed probability matrix (problem 5) with the normalized forward variable matrix (problem 2). What change do you notice about the smoothed estimate for rain on day 1 after observing umbrella on the second day? Based on this observation what can you conclude about smoothing?

The observation sequence is given as follows.

Observation = (Umbrella, Umbrella)